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## An infrared approach to Reggeization

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We present a new approach to Reggeization of gauge amplitudes based on the universal properties of their infrared singularities. Using the “dipole formula”, a compact ansatz for all infrared singularities of massless amplitudes, we study Reggeization of singular contributions to high-energy amplitudes for arbitrary color representations, and any logarithmic accuracy. We derive leading-logarithmic Reggeization for general cross-channel color exchanges, and we show that Reggeization breaks down for the imaginary part of the amplitude at next-to-leading logarithms and for the real part at next-to-next-to-leading logarithms. Our formalism applies to multiparticle amplitudes in multi-Regge kinematics, and constrains possible corrections to the dipole formula starting at three loops.

**Introduction.** The high-energy limit of gauge theory scattering amplitudes is of great interest both from a theoretical standpoint and in view of phenomenological applications. It has, therefore, been studied in depth for many years, starting with work done before the development of the Standard Model of particle physics [1]. The key feature of the high-energy limit is the phenomenon of Reggeization, which can be economically described in the simple case of four-point massless gauge theory amplitudes, characterized by the Mandelstam invariants  $s$  (the center-of-mass energy),  $t$  and  $u$ , satisfying  $s + t + u = 0$ . In the high-energy limit,  $|s/t| \rightarrow \infty$ , amplitudes which are dominated at the lowest perturbative order by the exchange of a given state in the  $t$  channel, are found to receive logarithmic corrections in powers of  $\ln(s/(-t))$ . These corrections can be resummed to all orders in perturbation theory by the simple prescription of replacing the tree-level  $t$ -channel propagator according to

$$\frac{1}{t} \longrightarrow \frac{1}{t} \left( \frac{s}{-t} \right)^{\alpha(t)}, \quad (1)$$

where  $\alpha(t)$  is the Regge trajectory, which can be expanded in powers of the gauge coupling  $\alpha_s$ . The Regge trajectory is infrared divergent, since its formal definition involves integrations over the transverse momentum of virtual gauge bosons. It is therefore appropriate to resort to dimensional regularization, and express the trajectory in terms of the  $d$ -dimensional coupling (with  $d = 4 - 2\epsilon$ ,  $\epsilon < 0$ ), evaluated at the scale  $\mu^2 = -t$ . One then writes

$$\alpha(t) = \frac{\alpha_s(-t, \epsilon)}{4\pi} \alpha^{(1)} + \left( \frac{\alpha_s(-t, \epsilon)}{4\pi} \right)^2 \alpha^{(2)} + \mathcal{O}(\alpha_s^3), \quad (2)$$

where  $\alpha_s(-t, \epsilon) = (\mu^2/(-t))^{-\epsilon} \alpha_s(\mu^2) + \mathcal{O}(\alpha_s^2)$ . The replacement rule in eq. (1) can be generalized to the case of  $2 \rightarrow n$  amplitudes in the so-called ‘multi-Regge’ kinematic (MRK) regime [2, 3]. When the  $n$  emitted partons are strongly ordered in rapidity, but have comparable

transverse momenta, if the tree-level amplitude is dominated by the  $t$ -channel exchange of a given color state, then leading logarithmic (LL) virtual corrections to the amplitude are resummed to all orders by replacing the  $t$ -channel propagator between the emission of particle  $k$  and particle  $k + 1$  according to

$$\frac{1}{t_k} \longrightarrow \frac{1}{t_k} e^{\alpha(t)(y_k - y_{k+1})} \equiv \frac{1}{t_k} \left( -\frac{s_{k,k+1}}{t_k} \right)^{\alpha(t)}, \quad (3)$$

where  $y_k$  are the rapidities of the emitted particles, and  $s_{k,k+1} \equiv (p_k + p_{k+1})^2 = 2p_k \cdot p_{k+1}$ . Reggeization has been proved for  $t$ -channel gluon exchange at LL [4] and NLL accuracy [5], and for quark exchange at LL accuracy [6]. Furthermore, the quark [7] and gluon [8–10] Regge trajectories have been determined up to two loops.

Reggeization proofs typically rely upon intricate recursive arguments, and depend upon the specific state that dominates  $t$ -channel exchanges for the process at hand. We propose a different viewpoint, based upon recent advances in our understanding of the all-order structure of infrared divergences in massless gauge theories. This approach is made possible [11, 12] by the fact that the Regge trajectory  $\alpha(t)$  is infrared divergent, and indeed is largely determined by infrared singularities (non-trivial finite contributions start arising only at two loops). Furthermore, singular terms have been empirically shown to be universal: for quarks and gluons at one loop one finds that  $\alpha^{(1)} = 2C_R/\epsilon$ , where  $C_R$  is the Casimir of the appropriate color representation,  $C_A$  for gluons and  $C_F$  for quarks. This suggests that Reggeization can be efficiently and generally studied from an infrared point of view.

**The infrared approach.** Our main tool is the ‘dipole formula’ [13–15], an all-order ansatz for infrared divergences of fixed-angle massless gauge theory amplitudes. Such amplitudes can be expressed in the factorized form

$$\mathcal{M}(\{p_i\}, \alpha_s, \epsilon) = Z(\{p_i\}, \alpha_s, \epsilon) \mathcal{H}(\{p_i\}, \alpha_s, \epsilon), \quad (4)$$

where  $p_i$ ,  $i = 1, \dots, L$ , are the hard parton momenta,  $\mathcal{H}$  is the hard part of the amplitude, finite as  $\epsilon \rightarrow 0$ , and  $Z$  is the operator responsible for all infrared and collinear divergences. The dipole formula is a compact expression for the  $Z$  operator, stating that only two-parton correlations appear at the level of the exponent. In the color generator notation [17], one finds

$$Z(\{p_i\}, \alpha_s, \epsilon) = \exp \left\{ \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \left[ \frac{1}{4} \hat{\gamma}_K(\alpha_s(\lambda^2, \epsilon)) \times \sum_{i < j} \ln \left( \frac{-s_{ij}}{\lambda^2} \right) \mathbf{T}_i \cdot \mathbf{T}_j - \frac{1}{2} \sum_{i=1}^L \gamma_i(\alpha_s(\lambda^2, \epsilon)) \right] \right\}. \quad (5)$$

Here  $-s_{ij} = 2|p_i \cdot p_j| e^{-i\pi\lambda_{ij}}$ , with  $\lambda_{ij} = 1$  if partons  $i$  and  $j$  both belong to either the initial or the final state,  $\lambda_{ij} = 0$  otherwise;  $\mathbf{T}_i$  are color generators in the representation of parton  $i$ , acting on the color indices of the amplitude as described in [17];  $\hat{\gamma}_K(\alpha_s)$  is the cusp anomalous dimension [18], with the Casimir of the appropriate representation scaled out;  $\gamma_i$  are the anomalous dimensions of the fields associated with external particles.

The dipole formula [13, 15] is the simplest solution to a set of exact equations governing infrared singularities for massless particles, which in turn follow from factorization properties of fixed-angle matrix elements in soft and collinear limits, and from rescaling invariance of light-like Wilson lines. The dipole formula is known to be exact up to two loops in the exponent [16], and corrections can only arise, starting at three loops, in the form of highly constrained functions of conformal invariant cross-ratios of external momenta [13], studied in [15, 19], or, at even higher orders, if the cusp anomalous dimension receives contributions from higher-order Casimir operators [13].

In the present context, it is important to note that the fixed-angle assumption, which underlies eq. (4), and which amounts to the requirement that all kinematic invariants  $s_{ij}$  be of comparable size, breaks down in the Regge limit  $s \gg |t|$ . This does not affect our reasoning concerning infrared singularities: indeed, all logarithms of  $s/t$  which are accompanied by infrared poles are correctly captured by eq. (4). We will, however, not be able to control logarithms with coefficients that remain finite as  $\epsilon \rightarrow 0$ , and thus our approach will only yield the divergent contributions to the Regge trajectory  $\alpha(t)$ .

With this important proviso, we may study the high-energy limit of eq. (5), starting with the simple but crucial case of the four-point amplitude. Expanding the exponent in powers of  $t/s$ , and enforcing color conservation by means of the constraint  $\sum_i \mathbf{T}_i = 0$ , we find that to leading power in  $t/s$ , and thus to *any* logarithmic accuracy, the infrared operator  $Z$  can be factorized as

$$Z(\{p_i\}, \alpha_s, \epsilon) = \tilde{Z}\left(\frac{s}{t}, \alpha_s, \epsilon\right) Z_1(t, \alpha_s, \epsilon), \quad (6)$$

where the factor  $Z_1$  is proportional to the identity matrix in color space, and is independent of  $s$ , while the non-trivial color structure and  $s$  dependence are contained

in the Reggeization operator  $\tilde{Z}$ . Introducing the color operators [12]  $\mathbf{T}_s = \mathbf{T}_1 + \mathbf{T}_2$  and  $\mathbf{T}_t = \mathbf{T}_1 + \mathbf{T}_3$  the Reggeization operator can be compactly written as,

$$\tilde{Z}\left(\frac{s}{t}, \alpha_s, \epsilon\right) = \exp \left\{ K(\alpha_s, \epsilon) \left[ \ln \left( \frac{s}{-t} \right) \mathbf{T}_t^2 + i\pi \mathbf{T}_s^2 \right] \right\}, \quad (7)$$

while the color-trivial factor  $Z_1$  can be written as

$$Z_1(t, \alpha_s, \epsilon) = \exp \left\{ \sum_{i=1}^4 B_i(\alpha_s, \epsilon) + \frac{1}{2} \left[ K(\alpha_s, \epsilon) \left( \ln \left( \frac{-t}{\mu^2} \right) - i\pi \right) + D(\alpha_s, \epsilon) \right] \sum_{i=1}^4 C_i \right\}, \quad (8)$$

where  $C_i$  are the Casimir invariants of the appropriate representations, which in the present notation are given by  $C_i = \mathbf{T}_i \cdot \mathbf{T}_i$ . In eqs. (7) and (8) we have introduced a set of functions encoding the dependence on the coupling and on  $\epsilon$ , through integrals over the scale of the  $d$ -dimensional running coupling. They are defined by

$$K(\alpha_s, \epsilon) \equiv -\frac{1}{4} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \hat{\gamma}_K(\alpha_s(\lambda^2, \epsilon)), \quad (9)$$

$$D(\alpha_s, \epsilon) \equiv -\frac{1}{4} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \hat{\gamma}_K(\alpha_s(\lambda^2, \epsilon)) \ln \left( \frac{\mu^2}{\lambda^2} \right), \quad (10)$$

$$B_i(\alpha_s, \epsilon) \equiv -\frac{1}{2} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \gamma_{J_i}(\alpha_s(\lambda^2, \epsilon)); \quad (11)$$

and they can be evaluated order by order in the coupling, yielding for example

$$K(\alpha_s, \epsilon) = \frac{\alpha_s}{\pi} \frac{1}{2\epsilon} + \left( \frac{\alpha_s}{\pi} \right)^2 \left( \frac{\hat{\gamma}_K^{(2)}}{8\epsilon} - \frac{b_0}{16\epsilon^2} \right) + \mathcal{O}(\alpha_s^3), \quad (12)$$

where

$$b_0 = \frac{11C_A - 2n_f}{3}, \quad \hat{\gamma}_K^{(2)} = \left( \frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{5}{9} n_f, \quad (13)$$

and we used the fact that  $\hat{\gamma}_K^{(1)} = 2$  in our normalization.

Equation (7) is one of our central results: it allows us to study Reggeization, for singular contributions to the amplitude, on completely general grounds. In particular, at LL accuracy we observe that Reggeization is a general property of any amplitude which at leading order and high energy is dominated by the  $t$ -channel exchange of a state belonging to a given color representation. Indeed, at LL level we can neglect the imaginary part of the exponent of eq. (7). Equation (4) can then be written as

$$\mathcal{M}|_{\text{LL}} = \left( \frac{s}{-t} \right)^{K \mathbf{T}_t^2} Z_1 \mathcal{H}. \quad (14)$$

If the hard function  $\mathcal{H}$ , at tree level and at leading power in  $t/s$ , is given by the exchange of a specific color representation  $R$  in the  $t$  channel, then it is an eigenstate of

the  $t$ -channel operator  $\mathbf{T}_t^2$ , and LL Reggeization follows, since one can simply replace the operator  $\mathbf{T}_t^2$  in (14) by its eigenvalue  $C_R$ , the Casimir operator of the appropriate representation. This implies the universality of the LL Regge trajectory, which for any process of the kind just described can be read off using eq. (14), together with eqs. (2) and (12), and is given by  $\alpha^{(1)} = 2C_R/\epsilon$ . We note in passing that an identical reasoning can be repeated for  $u$ -channel exchanges, in all cases in which they are responsible for logarithmic enhancements: Reggeization follows, with the same universal form of the Regge trajectory, given the symmetry of the dipole formula under  $t \leftrightarrow u$ . By the same token, it is possible to write the Reggeized amplitude so that it displays the correct ‘signature’, namely the symmetry under  $s \leftrightarrow u$  exchange.

**Reggeization breaking.** We see that LL Reggeization is a general feature of massless gauge amplitudes, dictated by the universal structure of infrared divergences. The dipole ansatz (5), however, is an all-order statement on the perturbative exponent, allowing for a much deeper analysis. In particular, eq. (7) is valid to any logarithmic accuracy, and it can be used to study Reggeization beyond LL. It is immediately evident from eq. (7) that already at NLL level the simple picture of Reggeization given by eq. (14) will break down for the imaginary part of the amplitude: indeed, if the hard part  $\mathcal{H}$  is an eigenstate of the operator  $\mathbf{T}_t^2$ , it will generically not be an eigenstate of  $\mathbf{T}_s^2$ , which does not commute with  $\mathbf{T}_t^2$ . Next-to-leading logarithms will then mix different color structures, and a simple resummation of the form of eq. (1) will fail. We verified that at NLL level only the imaginary part of the amplitude is affected, confirming and extending to general color exchanges the property of NLL Reggeization of the real part of the amplitude, with a universal NLO Regge trajectory given by eq. (12), multiplied by the appropriate Casimir eigenvalue. Proceeding to NNLL [10], Reggeization generically breaks down also for the real part of the amplitude. The leading color operator responsible for this breakdown is

$$\mathcal{E}\left(\frac{s}{t}, \alpha_s, \epsilon\right) \equiv -\frac{\pi^2}{3} K^3(\alpha_s, \epsilon) \ln\left(\frac{s}{-t}\right) \left[\mathbf{T}_s^2, [\mathbf{T}_t^2, \mathbf{T}_s^2]\right], \quad (15)$$

and it will generate a tower of non-Reggeizing logarithms, starting at three loops with terms of the form  $-(\pi^2/3)(\alpha_s/(2\pi\epsilon))^3 \ln(s/(-t))$ . The specific nature of the color mixing responsible for the breakdown of Reggeization can be studied case by case by applying the color operator  $[\mathbf{T}_s^2, [\mathbf{T}_t^2, \mathbf{T}_s^2]]$  to the hard amplitude  $\mathcal{H}$ : while it is conceivable that in some specific cases the operator will have a vanishing eigenvalue, this will not happen for generic representations, and the resulting effect can be quantified using eq. (15). Note that Reggeization breaking is a subleading effect in the large- $N_c$  limit. Finally, we emphasize that although  $\mathcal{O}(1/\epsilon)$  corrections going beyond the dipole formula (5) may arise at  $\mathcal{O}(\alpha_s^3)$  [13, 15, 19], these cannot influence Reggeization breaking (15), which is  $\mathcal{O}(1/\epsilon^3)$ .

**Constraining soft singularities.** One may also re-

verse the logic, and use what is known about the Regge limit to constrain possible corrections to the dipole formula. Such corrections may first arise at three loops. These are highly constrained even before considering the Regge limit: they may only depend on kinematics via conformally-invariant cross ratios [13] and they must vanish in all collinear limits [15, 19]. Nevertheless, a few explicit examples, consistent with all constraints were constructed [19]. Specialising these examples to the high-energy limit, one finds that they are all characterized by super-leading logarithms  $\mathcal{O}(\alpha_s^3 \log^4(s/-t))$ , which are inconsistent with known results on LL Reggeization. Thus, consistency with the Regge limit is a powerful constraint: it rules out all existing examples for potential three-loop corrections which go beyond the dipole formula. Note, however, that such corrections may still be present: the unphysical logs could cancel in linear combinations, or there might be other functions satisfying all constraints.

**Extension to  $2 \rightarrow n$  scattering.** So far we have concentrated on virtual corrections to four-point amplitudes in the high-energy limit, to illustrate the basic features of our approach. We note however that the dipole formula applies to amplitudes with any number of partons, and can therefore be used also to study Reggeization in the general case of multiparticle amplitudes. Consider specifically a scattering process  $p_1 + p_2 \rightarrow p_3 + \dots + p_L$ , in multi-Regge kinematics, where the parton rapidities are strongly ordered,  $y_3 \gg y_4 \gg \dots \gg y_L$ , while their transverse momenta  $k_i^\perp \simeq k_j^\perp, \forall i, j$  are comparable. In this situation the Mandelstam invariants are also hierarchically ordered, and may be approximated by  $-s_{ij} \simeq e^{-i\pi\lambda_{ij}} k_i^\perp k_j^\perp e^{y_i - y_j}$ . It is then possible to show that in multi-Regge kinematics the infrared operator  $Z$  expressed by the dipole formula in eq. (5) also factorizes, generalizing eq. (6). One finds

$$Z(\{p_i\}) = \tilde{Z}^{\text{MR}}(\{\Delta y_i\}) Z_1^{\text{MR}}(\{k_i^\perp\}), \quad (16)$$

where  $\Delta y_i \equiv y_i - y_{i+1}$ , and where we omitted, for simplicity, the arguments  $\alpha_s$  and  $\epsilon$  in the three functions. The multi-Regge operator  $\tilde{Z}^{\text{MR}}$  and the singlet factor  $Z_1^{\text{MR}}$  generalize eqs. (7) and (8), respectively. To write them down, it is useful to define the color operators

$$\mathbf{T}_{t_i} \equiv \mathbf{T}_1 + \sum_{p=1}^i \mathbf{T}_{p+2}, \quad (17)$$

whose eigenstates are definite  $t$ -channel exchanges between partons  $i+2$  and  $i+3$ . Using color conservation, one can then derive an explicit expression for  $\tilde{Z}^{\text{MR}}$ , valid for any number of emitted partons. It is given by

$$\tilde{Z}^{\text{MR}}(\{\Delta y_i\}) = \exp\left\{K \left[ \sum_{i=3}^{L-1} \mathbf{T}_{t_{i-2}}^2 \Delta y_i + i\pi \mathbf{T}_s^2 \right]\right\}. \quad (18)$$

The form of  $Z_1^{\text{MR}}$  will be given elsewhere. In eq. (16) the rapidity (and non-trivial colour) dependence is concentrated in the multi-Regge operator  $\tilde{Z}^{\text{MR}}$ , whereas the

factor  $Z_1^{\text{MR}}$ , which is proportional to the unit matrix in color space, depends only on the transverse momenta of the emitted partons. The requirement for Reggeization of the amplitude is then that the hard interaction be dominated by a series of  $t$ -channel exchanges in the Regge limit. If this is the case, then each  $t$ -channel exchange between, say, the emissions of partons  $i+2$  and  $i+3$ , will be an eigenstate of the color operator  $\mathbf{T}_{t_i}^2$ , and the corresponding rapidity dependence will enter the exponent of the amplitude with the relevant eigenvalue. The simplest case is when a single particle species is exchanged in the  $t$ -channel, which effectively radiates all other final state particles. One then recovers the well-known Reggeization of leading logarithms in the form of eq. (3). Reggeization, however, is more general than this, as is clear from eq. (18): in principle, different  $t$ -channel exchanges may occur, so that different rapidity intervals will exponentiate with distinct eigenvalues. Note that, as in the four-point case, the  $s$ -channel color operator  $\mathbf{T}_s^2$  in eq. (18) will generically lead to a breakdown of Reggeization for the imaginary part of the amplitude at NLL accuracy, and for the real part of the amplitude at NNLL level.

**Conclusions.** We have proposed a general approach to Reggeization of massless gauge-theory amplitudes, which uses the universal structure of infrared divergences embodied in the dipole formula, eq. (5). The drawback of our approach is that we have no direct control of finite contributions to the Regge trajectory. The dipole formula may also receive corrections, starting at three loops, but we have been able to show that such corrections are irrelevant both for deriving Reggeization, and for understanding its breaking. Conversely, the Regge limit provides a useful constraint on potential correc-

tions going beyond the dipole formula, which are already highly constrained by other means. This strengthens the evidence for the validity of the dipole formula beyond two loops. Our approach shows the complete generality of the Reggeization phenomenon at LL level, explaining the universality of divergent contributions to the Regge trajectory at LL and NLL accuracy. Furthermore, we give compact expressions for the operators which generate all high-energy logarithms associated with infrared divergences, both for the four-point amplitude and for the multiparticle case in multi-Regge kinematics. This also allows us to identify the color operator responsible for the breakdown of Reggeization at NLL for the imaginary part of the amplitude, and to determine the operator (15) that breaks Reggeization for the real part at NNLL. We believe that our results, to be discussed in detail elsewhere, pave the way for further progress: corrections to the dipole formula may be further constrained; Reggeization of finite contributions to the amplitude may be studied from an infrared viewpoint; our results could be used to test the breakdown of Reggeization at NNLL and gauge its impact on phenomenology; finally, the infrared singularity structure of amplitudes may be used to study the high-energy limit beyond the realm of Reggeization, as well as other kinematic limits.

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- [1] R. J. Eden, P. V. Landshoff, D. I. Olive and J. C. Polkinghorne, ‘‘*The Analytic S-Matrix*’’, Cambridge (1966).
- [2] E. A. Kuraev, L. N. Lipatov, V. S. Fadin, Sov. Phys. JETP **44** (1976) 443-450.
- [3] V. Del Duca, [hep-ph/9503226](#).
- [4] I. I. Balitsky, L. N. Lipatov, V. S. Fadin, In Leningrad 1979, Proceedings, Physics Of Elementary Particles, in Russian, Leningrad 1979, 109-149.
- [5] V. S. Fadin, R. Fiore, M. G. Kozlov, A. V. Reznichenko, Phys. Lett. **B639** (2006) 74-81. [hep-ph/0602006](#).
- [6] A. V. Bogdan, V. S. Fadin, Nucl. Phys. **B740** (2006) 36-57. [hep-ph/0601117](#).
- [7] A. V. Bogdan, V. Del Duca, V. S. Fadin, E. W. N. Glover, JHEP **0203** (2002) 032. [hep-ph/0201240](#).
- [8] V. S. Fadin, M. I. Kotsky, R. Fiore, Phys. Lett. **B359** (1995) 181-188; V. S. Fadin, R. Fiore, M. I. Kotsky, Phys. Lett. **B387** (1996) 593-602. [hep-ph/9605357](#); V. S. Fadin, R. Fiore, A. Quartarolo, Phys. Rev. **D53**, 2729-2741 (1996). [hep-ph/9506432](#).
- [9] J. Blumlein, V. Ravindran, W. L. van Neerven, Phys. Rev. **D58** (1998) 091502. [hep-ph/9806357](#).
- [10] V. Del Duca, E. W. N. Glover, JHEP **0110** (2001) 035. [\[hep-ph/0109028\]](#).
- [11] G. P. Korchemsky, Phys. Lett. **B325** (1994) 459-466. [\[hep-ph/9311294\]](#); I. A. Korchemskaya, G. P. Korchemsky, Phys. Lett. **B387** (1996) 346-354. [\[hep-ph/9607229\]](#); Nucl. Phys. **B437** (1995) 127-162. [\[hep-ph/9409446\]](#).
- [12] Y. L. Dokshitzer, G. Marchesini, JHEP **0601**, 007 (2006). [hep-ph/0509078](#).
- [13] E. Gardi, L. Magnea, JHEP **0903** (2009) 079. [arXiv:0901.1091 \[hep-ph\]](#).
- [14] T. Becher, M. Neubert, Phys. Rev. Lett. **102** (2009) 162001. [arXiv:0901.0722 \[hep-ph\]](#).
- [15] T. Becher, M. Neubert, JHEP **0906** (2009) 081. [arXiv:0903.1126 \[hep-ph\]](#).
- [16] S. M. Aybat, L. J. Dixon, G. F. Sterman, Phys. Rev. **D74** (2006) 074004. [hep-ph/0607309](#).
- [17] S. Catani and M. H. Seymour, Nucl. Phys. B **485**, 291 (1997) [Erratum-ibid. B **510**, 503 (1998)], [hep-ph/9605323](#).
- [18] G. P. Korchemsky and A. V. Radyushkin, Phys. Lett. B **171**, 459 (1986); G. P. Korchemsky and A. V. Radyushkin, Nucl. Phys. B **283**, 342 (1987);
- [19] L. J. Dixon, E. Gardi, L. Magnea, JHEP **1002** (2010) 081. [arXiv:0910.3653 \[hep-ph\]](#).