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FAST TRACK PAPER

A statistical evaluation of a ‘stress-forecast’ earthquake

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SUMMARY

The goodness of fit for competing statistical models with different numbers of degrees of freedom cannot be assessed solely by the residual sum of squares, because more complex models will naturally have lower residuals. A standard approach to hypothesis testing for large data sets is to use the objective Bayesian information criterion (BIC), which penalizes models with larger numbers of free parameters appropriately. We apply this method to the analysis of time delays from data on seismic shear-wave splitting in SW Iceland. The same data set has previously been used to estimate the time at which stress-modified micro-cracking reaches an inferred state of fracture criticality. The method does not forecast the location of the event, however, the time and magnitude were consistent with the actual occurrence of an $M = 5$ earthquake in the region. The forecast was based on a multi-line model with 17 degrees of freedom (five straight lines, each with a start and end point, plus the variance, with four endpoints being fixed by the occurrence of a main shock), and a signal-to-noise ratio near unity. The BIC is used to assess this forecast in comparison with competing curve fits for Poisson, multiline, sinusoidal, or polynomial (truncated Taylor expansion) hypotheses. The null hypothesis of random occurrence can only be rejected formally for the sinusoidal model, implying cyclical recurrence with a period of 134.6 days. All other models we consider have a lower BIC. We also analyse the selected portion of the data set used to make the forecast of fracture criticality using Gaussian statistics. The 95 per cent confidence intervals on the predicted main shock magnitude range between magnitudes of 3.9 and 6.7. The time range indicated by the same confidence limits starts 42 days before the actual event; a clear end cannot be located. The relation between predicted magnitude and waiting time is not significantly different from that inferred from the background Gutenberg–Richter frequency–magnitude relation within the model uncertainties. Thus, it is not possible, based on the data, to formally reject the hypothesis that the magnitude 5 event was part of the normal background seismicity.

Key words: earthquake prediction, fractures, shear-wave splitting, statistical methods.

1 INTRODUCTION

It has been suggested that analysis of time delays resulting from seismic shear wave splitting can be used to establish an estimate of the time and magnitude at which stress-modified microcracking reaches fracture criticality (Crampin 1994). This technique has been applied to ‘stress-forecast’ an $M = 5$ earthquake in Iceland (Crampin *et al.* 1999). However, statistical models like that used to make the forecast have many degrees of freedom and the data is relatively noisy, both in terms of errors of measurement and statistical fluctuation between individual data points. In fact, the effective signal-to-noise ratio is near unity, even for the single station (out of a

total of four) chosen to make the quantitative forecast. The statistical significance of such forecasts needs to be analysed and, in particular, there must be an evaluation of the model or prediction error compared to the null hypothesis of random occurrence. Obviously, adding extra parameters to a model will improve the curve fit to the data, as it reduces the sum of squares of the residuals between the best-fitting curve and the data-points. It is, therefore, necessary to introduce a penalty parameter for the extra degrees of freedom in a more complex model. This approach is commonly used in a variety of applications in science, engineering, demography etc. For large data sets, the appropriate criterion to use is the Bayesian information criterion (BIC) of Leonard & Hsu (1999). This approach has

recently been applied to detecting statistically significant breaks of slope in geophysical data by Main *et al.* (1999). Here we apply this technique to assess whether there is a statistically significant signal with predictive power in the time-delay data used by Crampin *et al.* (1999). To complement this exercise in inference, we also apply standard Gaussian curve fitting techniques to calculate the errors of extrapolation associated with the time-magnitude window for the earthquake.

2 STRESS FORECASTING

According to Geller (1997) earthquake prediction is a term that should be reserved for warnings on a timescale of, at most, a few days. The prediction should be based on a scientific hypothesis and should specify time, magnitude and a spatial window. Furthermore, the author's level of confidence in the prediction should be clearly stated and the chances of the earthquake to occur as a random event should be calculated.

In this sense, the notion of stress-forecasting is not the same as earthquake prediction, but is more like weather forecasting, where one uses observable parameters to forecast the probabilistic behavior of a non-linear system. One reason why individual earthquakes may be so hard to predict is the fact that the Earth's brittle crust seems to be maintained close to a critical state by constant tectonic forcing. This notion is referred to as self-organized criticality (Bak *et al.* 1988) and is consistent with the first-order scaling features of faulting and seismicity (Main 1996). One aspect of this criticality may be the narrow range of crack densities inferred from analysis of shear wave splitting and interpretation of the results in terms of a distribution of stress-aligned microcracks just below the percolation threshold (Crampin 1994). The ongoing debate on the degree of earthquake predictability then rests on the magnitude of stress fluctuations between events relative to the critical state and the ability to detect precursors with statistical significance (Scholz 2002).

In appropriate seismic data shear wave splitting (seismic birefringence) is observed for almost all kinds of rocks in the Earth. Crampin & Lovell (1991) suggest anisotropy induced by cracks, which have been aligned by external stress, as the most likely cause in the brittle part of the crust. Laboratory studies under true triaxial stress conditions ($\sigma_1 > \sigma_2 > \sigma_3$) (Crawford *et al.* 1995) or under uniaxial loading with anisotropic material (Gao 2001) have confirmed that an increase in time delay between the fast and slow split shear waves is systematically correlated with an increase in stress.

In order to examine the possibility of such observations scaling to conditions in the Earth, Crampin *et al.* (1999) monitored the shear wave splitting recorded by the South Iceland Lowland (SIL) seismic network (maintained by the Iceland Meteorological Office) over a period of 23 months. Of the four stations used to calculate time delays in this network, one, the BJA station, appeared to show systematic fluctuations that were most closely correlated with the occurrence times of the four largest main shocks that occurred in this period. The selected data set was interpreted in terms of the approach to and retreat from fracture criticality and used in prospective mode to stress-forecast a future earthquake of magnitude between $M = 5$ and $M = 6$ within an 108 day time range. The method does not predict location and no attempt was made formally to demonstrate that the forecast was beyond chance. In fact an $M = 5$ earthquake did occur within the specified time and magnitude window. A later earthquake (2000 June 17, $M = 5.6$, in southwest Iceland) was not forecast following a microseismicity gap of 2–3 months where no data were available to test the criticality hypothesis. The details of the published forecast, especially the calculation of the time-magnitude window, are described by Crampin *et al.* (1999). There, no attempt

was made to calculate the statistical significance of the empirical line fit to the data or any formal errors of extrapolation associated with the forecast. The aim of this paper is to assess these questions using the same data set, except for 14 additional points provided by T. Volti. The exercise is purely statistical: all models are treated with equal potential merit and, for clarity, the limitations of such analyses are also noted.

3 METHOD

To model the time-delay data from the BJA seismic station we generated models using a least-squares approach. We fitted Poisson, multi-line, sinusoidal and polynomial (truncated Taylor expansion) models with varying dimensions to the data. A linear model for example has three parameters: slope, intercept and variance. We reproduced the model of the increase in normalized time delays suggested by Crampin *et al.* (1999) and calculated the residual sum of squares for all models. The confidence limits were derived from the uncertainty as outlined by Draper & Smith (1998).

A complex model with many degrees of freedom, applied to noisy data, may result in an over-interpretation of the data. For example, any estimate based on a method that minimizes the residual sum of squares will select the model with the highest permissible number of parameters, even when not justified by the data. A critical problem is the primary determination of the appropriate dimensionality of the model. Here we used a selection criterion to balance model fit and complexity, choosing a model that maximizes the BIC proposed by Leonard & Hsu (1999):

$$\text{BIC} = L(\hat{\theta} | \mathbf{y}) - \frac{p}{2} \ln \frac{N}{2\pi}, \quad (1)$$

where \mathbf{y} stands for the vector of measurements $\mathbf{y} = (\mathbf{y}_1, \dots, \mathbf{y}_N)^T$, $\hat{\theta}$ denotes the maximum likelihood estimate of the vector of unknown parameters $\theta = (\theta_1, \dots, \theta_p)^T$, N is the number of measurements and p is the number of parameters. $\hat{\theta}$ denotes the vector θ that, out of all possible choices for the parameters θ_i , best fits the data. The number of unknown parameters is the number of degrees of freedom for the model plus one for the variance. For the likelihood function $l(\theta | \mathbf{y})$ the following applies:

$$l(\theta | \mathbf{y}) \propto (\sigma^2)^{-N/2} \exp \left\{ - \sum_{i=1}^N \frac{[\mathbf{y}_i - f(\theta | x_i)]^2}{2\sigma^2} \right\}, \quad (2)$$

where σ stands for the standard deviation. Main *et al.* (1999) take the natural logarithm and maximize with respect to the unknown parameters, which gives the maximized logarithmic likelihood $L(\hat{\theta} | \mathbf{y})$:

$$L(\hat{\theta} | \mathbf{y}) = -(N/2) \ln \left\{ \sum_{i=1}^N [\mathbf{y}_i - f(\hat{\theta} | x_i)]^2 \right\}, \quad (3)$$

where the term in the curly brackets is the residual sum of squares that we calculated for each of our models. The likelihood function should include terms that are independent of p and the fit to the data, but in practice this simplified form is used.

The BIC is preferable to the more commonly used Akaike information criterion (AIC) when the number of data points is greater than 46 (Leonard & Hsu 1999; Main *et al.* 1999). This is because although AIC finds the best statistical model in synthetic tests for small data sets, it does not for larger ones. When $n = 46$, AIC = BIC, therefore, this number represents the point of transition. The number of data points used here was 145. From eq. (1) a model with a high BIC is preferable to one with a lower value. We regard a change in BIC of a value equivalent to increasing p by one unit to be significant.

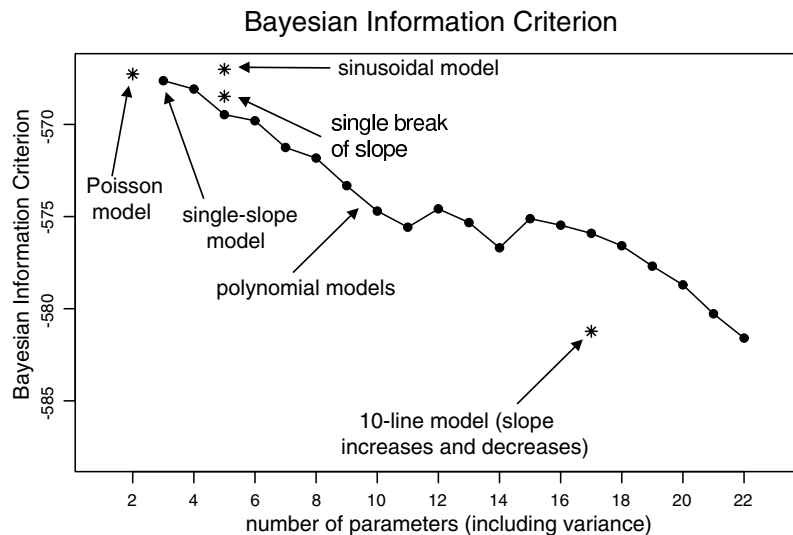


Figure 1. Comparison of the BIC for various models as a function of the number of degrees of freedom. The variance is included as one parameter. The 10-line model has 17 parameters, including the start and endpoint of each line minus the four previous main shocks whose occurrence time was known precisely. This is the model of Crampin *et al.* (1999) except for the connecting lines between the end point of one and the start point of another increase.

4 RESULTS

A comparison of BIC as a function of the number of model parameters for the different statistical models is shown in Fig. 1. The simplest model was the constant (time-independent or Poisson) model, which had one of the highest BIC values. This has two parameters: mean and variance. We consider that the five-line model using the additional earthquake information suggested by Crampin *et al.* (1999) has 17 degrees of freedom (five lines with start and endpoint plus one for the variance, minus the four endpoints where the time of occurrence of the main shocks could be considered known). We do not consider the start point fixed because this will have a finite degree of error in its estimate as a local minimum in a nine-point moving average. The BIC for this model was calculated first assuming a linear interpolation between the start and endpoints to generate 10 lines. This assumption uses all of the data points, not just those in the parts of the model with positive slope, and hence minimizes the residual sum of squares without introducing any additional free parameters. This model had a slightly lower BIC than other models having similar numbers of parameters. Furthermore, its BIC was significantly lower than the BIC for the constant (Poisson) model implying that the Poisson model is a better statistical model in this direct comparison. We fitted a sinusoidal model (five degrees of freedom: amplitude, intercept, frequency, phase and variance) to the data. The BIC was -567.3 for the constant model and -567.0 for the best-fitting sinusoidal model. This difference was small and corresponds to less than the equivalent for one degree of freedom ($\Delta\text{BIC} = (1/2) \ln(145/2\pi) \approx 1.57$), therefore, it is not significant by our criterion above. The various polynomial models showed a systematic decrease of the BIC, when we increased the number of parameters.

Crampin *et al.* (1999) used only the increasing parts of the apparent slope to the data curves, whereas in Figs 1 and 2 we have examined all of the data assuming the simplest linear connection between end points. If the segments of the data with linear decreasing trend are removed from the analysis, the BIC from the Poisson model ($p = 2$ on the reduced data set of $N = 82$ data points) is -298.1 . This compares with -304.9 for $p = 17$ parameters for the five-line model of Crampin *et al.* (1999). Once again, an appropriate

penalty for the large number of parameters overwhelms any small statistical gain in reducing the residual sum of squares achieved by segmenting the data.

The fit of the Poisson model to the time-delay data set is shown in the upper graph of Fig. 2. The model has two degrees of freedom: intercept with the vertical axis and variance. This was our simplest model and assumed that the data points are scattered randomly around the mean value. The middle graph of Fig. 2 shows the best fitting sinusoidal model. In the bottom graph (Fig. 2) we examine the model for the increase in normalized time delays prior to an earthquake by Crampin *et al.* (1999). Every increase started at the minimum of the nine-point moving-average filtered data and ended with the occurrence of a major earthquake. The endpoint and the start point of two line segments were joined to calculate the BIC in the simplest statistical model, i.e. without adding extra degrees of freedom associated with a more physically-based stress relaxation model. Given the variance in the data, it is unlikely that such a model could be validated in this data set. The confidence limits for the true mean value of every line segment and for values at a certain point in time as given by the model are shown.

The result of fitting a straight line to the specific part of the data set that was used to make the stress forecast by Crampin *et al.* (1999) is shown with the appropriate confidence limits in Fig. 3. It is important to clarify that this part of the analysis was only a standard least-squares analysis of errors consistent with the method of line fitting used by Crampin *et al.* (1999) and does not rely on the concept of an information criterion. The minimum in the moving-average filtered data was used as a starting point for the line. We then extrapolated this line assuming that the linear increase in normalized time delays will continue until the limit of fracture criticality will be reached and an earthquake will occur, exactly as Crampin *et al.* (1999) have done. We used the linear relationship between waiting time and magnitude as Crampin *et al.* (1999) to generate the equivalent magnitude scale shown on the right side of the plot. This magnitude and waiting time correlation is based only on four points, and hence it is not possible to validate the relationship, due to the small number of data points. Ignoring any, possibly gross, uncertainty associated with such a poor calibration, we then calculated the time-magnitude window from the slope of the increase in

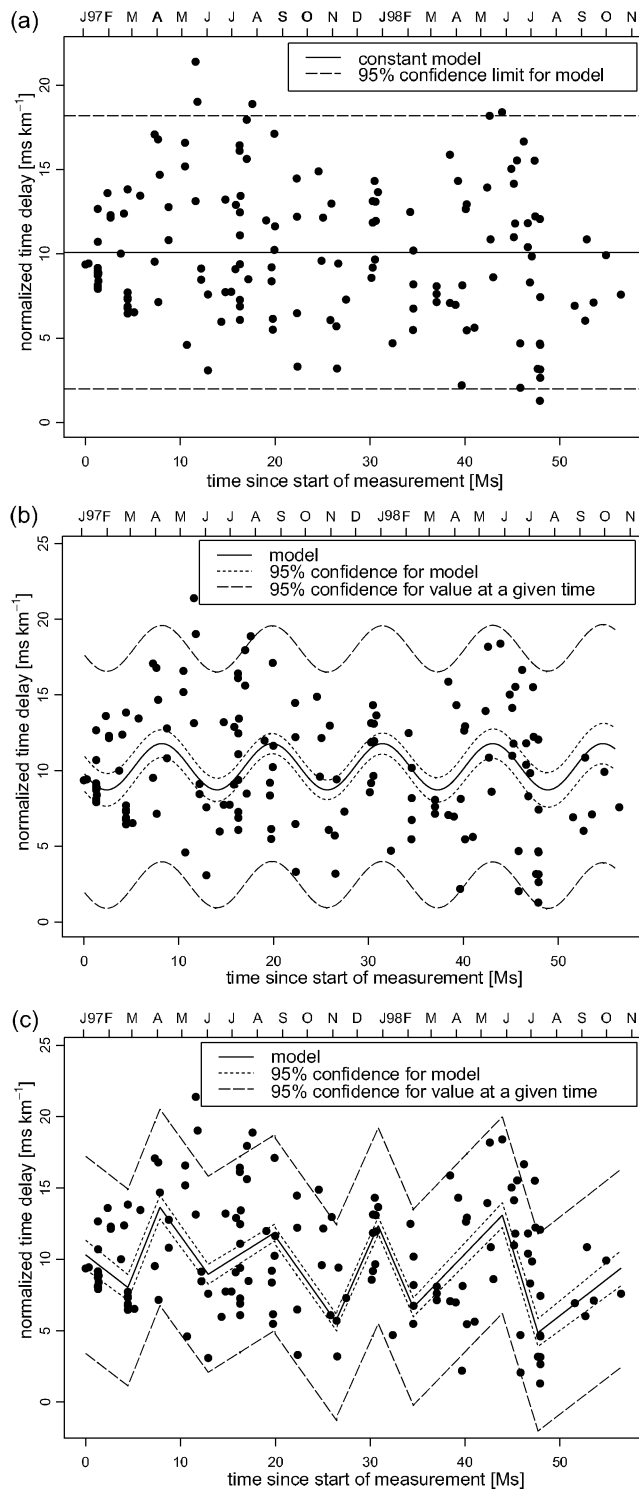


Figure 2. Normalized time delays between split shear waves versus time. The lower time axis is in units of 10^6 s from the start of the data set, the upper axis shows the actual time in months/years. Graph (a) shows the constant model (the Poisson model) with its 95 per cent confidence limits. Graph (b) shows the sinusoidal model and (c) shows the model of Crampin *et al.* (1999) plus the connecting lines. In the text a similar comparison is made for the case when the data for the connecting lines are ignored.

normalized time delays and the inferred level of fracture criticality at which previous earthquakes occurred. Using the equivalent magnitude scale the $M = 5$ earthquake of 1998 November 13 lies inside the suggested time-magnitude window (Fig. 3). When the 95 per cent confidence limits are included, the uncertainty in magnitude and particularly waiting time is much larger than the range specified solely by the least squares line used by Crampin *et al.* (1999). The forecast by Crampin *et al.* (1999) gives an early and a late time estimate for time and magnitude but no estimate of the uncertainty. According to their analysis the earliest the earthquake could occur was on 1998 November 13 with a magnitude of $M = 5$. The latest the earthquake could occur was on 1999 February 28 with a magnitude of $M = 6$. Explicitly ignoring any variance in the data, their time magnitude window spans 108 days and one magnitude unit. When the confidence limits are calculated formally in Fig. 3, at the time of the earthquake the errorbar starts at magnitude $M = 3.9$ and ends at magnitude $M = 6.7$. The errorbar for the time starts 42 days before the event; a clear end cannot be located. This gives a range of 2.8 magnitude units and a time range, that lasts significantly longer than 108 days. We conclude that the large error of line fitting and extrapolation precludes a formal validation of the forecast made. In summary, the close correspondence of the predicted magnitude and time of the 13 November 1998 event with the centre of this large cross, although appealing, is simply not statistically significant when the large formal error bars in magnitude and time are taken into account.

Finally we compared the linear waiting time-magnitude relation with the background seismicity and its 95 per cent confidence limits in Fig. 4. The former plots as a curve on this graph. On this log-linear graph the recurrence data plot as a straight line following the Gutenberg–Richter law. The graph also shows the estimated waiting time that might be predicted from the background seismicity (inverse frequency) so that a direct comparison can be made. The two are statistically indistinguishable from each other within the magnitude range used, implying that the waiting time–magnitude relation used to make the forecast in Crampin *et al.* (1999) is not statistically distinguishable from a Poisson process over the magnitude range of interest.

5 DISCUSSION

In general, simple models with fewer parameters are better statistical models for the whole data set analysed here whether or not the data are split into sections. The time delays of shear-wave splitting do show some correlation with the four largest mainshocks. The marginally optimal model is a simple sinusoid rather than the 17-parameter model of Crampin *et al.* (1999). However, it is important to note that many filters have been applied to the data prior to our analysis. First, the data have been filtered with respect to earthquake location to be within the shear-wave window. This is an objective filter based on independent knowledge of the velocity structure in the region. Second, only data from a part of the shear-wave window have been used, so there is a potential directional bias to the data. However, we found no systematic correlation between event location and time delay, so this filter is unlikely to propagate into spurious time-delay anomalies. Third, only reliable time delays (small errors) have been used. Again this seems appropriate, although the choice of cut-off introduces an additional free parameter. Fourth, only data from one station were used to make the quantitative forecast. Overall these spatial and temporal filtering exercises all potentially introduce additional degrees of freedom, that have not been statistically accounted for in our analysis. As a consequence our BIC values for the non-Poissonian models, if anything, are probably overestimates.

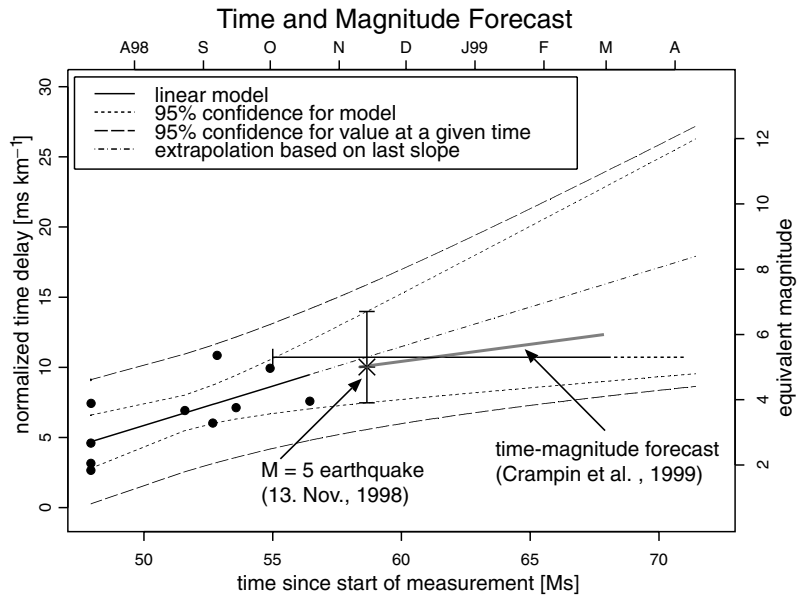


Figure 3. Extrapolation of the increase in normalized time delays for the section of data prior to the $M = 5$ earthquake of 1998 November 13. The lower time axis is in units of 10^6 s from the start of the data set, the upper axis shows the actual time in months/years. The graph shows the error bars for the time of the earthquake.

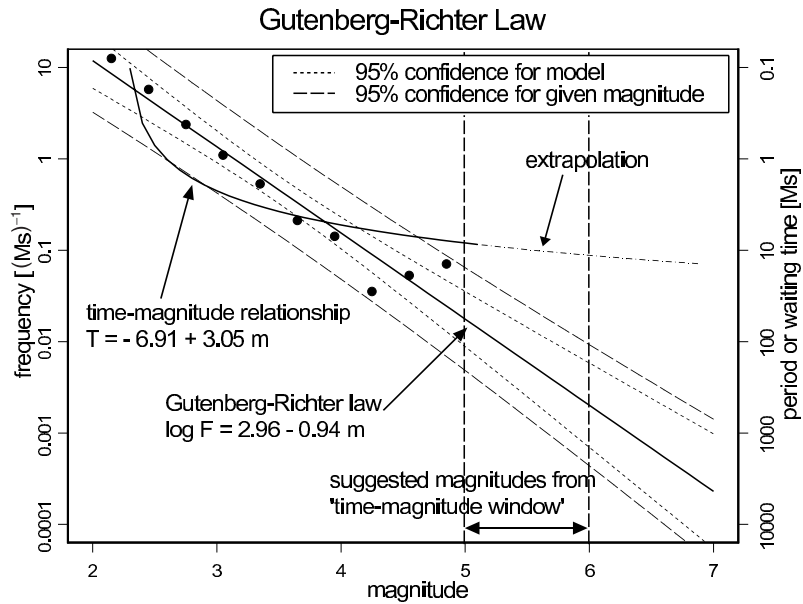


Figure 4. Incremental frequency magnitude relationship, compared to the time-magnitude relationship of Crampin *et al.* (1999). The bins were of 0.3 magnitudes each.

When re-examining the data we were able to reproduce some of the results published by Crampin *et al.* (1999). Prior to the four major mainshocks the moving-average filtered data showed an apparent increase in normalized time delays. All the higher order models (in retrospective mode) showed maxima at the same time as one or more of these earthquakes. The earthquake of 1998 November 13 lies in the (rather large) time-magnitude window predicted by extrapolation. This window was calculated from data for only four previous earthquakes and neglected the effect of the uncertainties involved in the curve fit and its extrapolation. With only four events, the purely empirical linear relation between waiting time and magnitude cannot be established with any confidence. Our extrapolations of the model for the selected last part of the data set are consistent with a crust approaching the limit of fracture criticality, however, the uncertainties

involved are too great to allow the null hypothesis of background random occurrence to be clearly rejected. Perhaps this is not surprising given the data scatter. More accurate measurements of time-dependent anisotropy would have to be made to reduce this scatter, most obviously by using controlled sources in quiet borehole sites. Given the uncertainty in all aspects listed above, it is not possible at this stage to estimate the degree of accuracy required, except to say that an order of magnitude increase in signal-to-noise ratio would be desirable.

Our main conclusion is that the data are not yet adequate to make forecasts of individual events. However, the laboratory data indicate that higher stresses are associated with higher time delays (Crawford *et al.* 1995; Gao 2001). We plotted time delay against magnitude on Fig. 5 to see if there are systematic effects

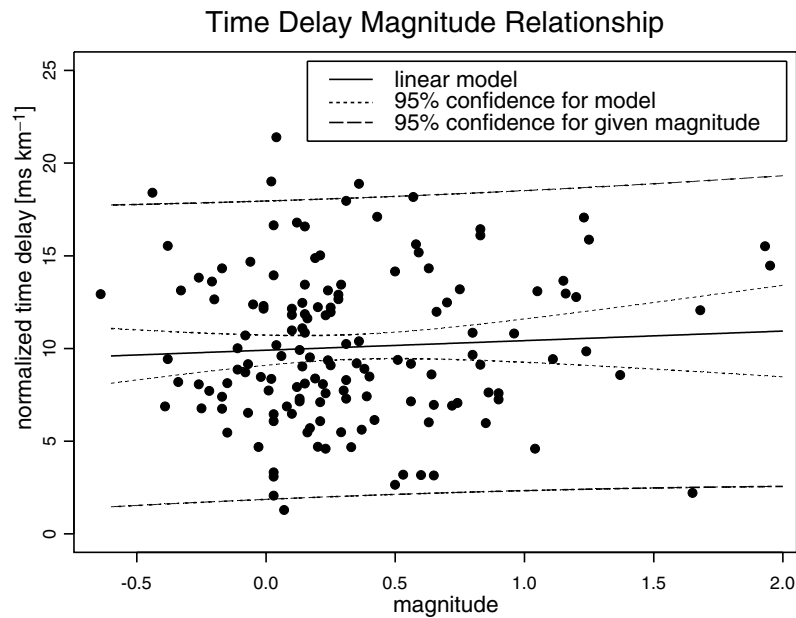


Figure 5. Relationship between normalized time delay and magnitude for the data set used to generate the five-line model.

in the population. In fact, the results show essentially no correlation of the time delay between the fast and slow shear wave and the magnitude of the event ($r^2 = 0.004$).

6 CONCLUSIONS

From the whole data set used by Crampin *et al.* (1999), it is not possible formally to reject the null hypothesis of a Poisson process in favour of any multi-line model. Excluding the effect of hidden degrees of freedom from data filtering, the only model comparable with the Poisson hypothesis was a periodic signal with a period of 134.6 days. For the selected portion of the data used to make the forecast of fracture criticality, the statistical errors of extrapolation were significantly bigger than the suggested deterministic time-magnitude window. The forecast individual event could not be distinguished from the background frequency of occurrence within the uncertainties of the two curve fits. We conclude that at least an order of magnitude increase in signal-to-noise ratio would be required to validate forecasts for individual events. The population shows essentially no correlation between time delay and magnitude. A positive correlation would be consistent with laboratory results if large-magnitude events are more likely to occur when the stress is higher. Again, the basic scatter in the data is the main reason for the lack of definitive results. The main source of scatter is probably the use of ray paths with different azimuths, thereby mixing transverse and azimuthal anisotropy. This problem could, in principle, be surmounted by using a narrow range of azimuths. For natural seismicity this is not possible as a result of the large reduction in the number of data points. A more definitive test of the fracture criticality hypothesis awaits the application of controlled source, constant azimuths, borehole-derived data. Fundamentally, the signal-to-noise ratio needs to be improved before firmer conclusions can be drawn, most likely by using borehole data and a controlled source.

NOTE ADDED IN PROOF

We note that Crampin *et al.* (2004) have published a comment on our paper. We respectfully disagree with their conclusions, but feel we

have nothing to add to the clear case made above. We simply invite the reader to examine the two publications carefully and decide which is the more appropriate analysis.

ACKNOWLEDGMENTS

We thank Stuart Crampin and Theodora Volti for permission to analyse the data and responding openly to our questions on the data analysis carried out in their previous work. Our conclusions differ from theirs and we genuinely appreciate their unselfish cooperation in research. We are also grateful to Sebastien Chastin and to reviewers Stuart Crampin and John Haines for their constructive comments on an earlier draft.

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