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Maximum entropy production and earthquake dynamics

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[1] We examine the consistency of natural and model seismicity with the maximum entropy production hypothesis for open, slowly-driven, steady-state, dissipative systems. Assuming the commonly-observed power-law feedback between remote boundary stress and strain rate at steady state, several natural observations are explained by the system organizing to maximize entropy production in a near but strictly *sub*-critical state. These include the low but finite seismic efficiency and stress drop, an upper magnitude cut-off that is large but finite, and the universally- observed Gutenberg-Richter *b*-value of 1 in frequency-magnitude data. In this state the model stress field organizes into coherent domains, providing a physical mechanism for retaining a finite memory of past events. This implies a finite degree of predictability, strongly limited theoretically by the proximity to criticality and practically by the difficulty of directly observing Earth's stress field at an equivalent resolution. **Citation:** Main, I. G., and M. Naylor (2008), Maximum entropy production and earthquake dynamics, *Geophys. Res. Lett.*, *35*, L19311, doi:10.1029/2008GL035590.

1. Introduction

[2] The idea that a complex system is thermodynamically driven to a state of maximum entropy production (MEP) has been suggested as a self-organizing mechanism for many physical, chemical, biological and geophysical phenomena [*Ozawa et al.*, 2003; *Whitfield*, 2005; *Martyushev and Seleznev*, 2006]. These include convection in the atmospheres of Earth, Mars and Titan [*Lorenz et al.*, 2001] and Earth's mantle [*Ozawa et al.*, 2003] and the phenomenon of self-organized criticality in sandpiles [*Dewar*, 2005], where slow steady loading of sand grains causes intermittent avalanches with a power-law size distribution near the critical angle of repose [*Bak et al.*, 1987]. In particular, *Dewar* [2005] showed that MEP occurred in sandpiles when avalanches of all sizes can occur, that is when the correlation length of avalanches diverges. Earthquakes are also thought to occur in a state of self-organized criticality [*Bak and Tang*, 1989], so here we examine whether the hypothesis can also be applied to natural and model seismicity. We use model seismicity since all aspects of the system are known and the relevant parameters can be calculated for a sufficiently long time-scale to observe steady state.

[3] Natural seismicity in the brittle part of Earth's lithosphere is produced by remarkably stationary loading from plate tectonics [*DeMets*, 1995], in turn driven by solid-state convection in the mantle as a consequence of heat generated by radioactive decay. This steady input of strain energy results ultimately in a rupture that releases energy, in the form of frictional heat on the sheared fault surfaces, that creates fresh fracture surfaces in the form of fault gouge and a fault damage zone, and that radiates elastic energy E_S, also ultimately converted into heat by anelastic attenuation [*Shipton et al.*, 2006]. In this paper we quantify analytically the entropy production from the energy budget and explore whether natural and model seismicity is consistent with a state of maximum entropy production.

2. Analytical Theory

[4] The radiated energy is a finite fraction of the total energy change ΔQ during an earthquake, $E_S = \eta\Delta Q$, where the seismic efficiency $0 < \eta < 1$, and is related to the ratio of stress drop to mean stress by $\eta = 0.5\Delta\sigma/\langle\sigma\rangle$, where angle brackets define an average [*Shearer*, 1999]. At steady state the energy lost to the system from a population of N earthquakes is

$$\Delta Q = N\langle E_S\rangle/\eta. \quad (1)$$

We use ΔQ to define the entropy production in seismogenic systems: $\Delta S = \Delta Q/T$, where T is a 'temperature term' [*Sornette*, 2006, chapter 7] defined not in terms of random thermal fluctuations, but from Boltzmann-like spatial and temporal fluctuations of strain energy within the seismogenic material, as observed in model systems [*Rundle et al.*, 1995; *Main et al.*, 2000].

[5] To investigate the temperature term and other aspects of entropy production we use the Olami-Feder-Christensen (OFC) multiple spring-block slider model [*Olami et al.*, 1992], a two-dimensional coupled-lattice model where each cell i represents a block of elementary area A_0 in contact with a stationary lower plate, connected to its nearest neighbors by connecting springs of stiffness K_C and length l_0. The blocks are driven through leaf springs of stiffness K_L and length l_0 by a rigid upper plate at a constant velocity V or strain rate $d\varepsilon/dt = V/l_0$ [*Main*, 1996, Figure 6]. When a single cell fails a proportion $\alpha = K_C/(K_L + 4K_C)$ of the stress drop is redistributed to each of its four nearest neighbors, with $\beta = 4\alpha$ defining a conservation factor. The connecting spring strain between the i'th and j'th neighboring blocks is given by $\delta\varepsilon_{i,j} = |\varepsilon_i - \varepsilon_j|$. For a large number of blocks q there are (to a good approximation) twice as many connecting springs as leaf springs on a two-dimensional lattice, so total strain energy in the system, expressed in the form of a temperature term with dimensions of energy, is

$$kT = \frac{1}{2}K_L\langle\varepsilon^2\rangle l_0^2 + K_C\langle\delta\varepsilon^2\rangle l_0^2. \quad (2)$$

This temperature term does not result from the random kinetic energy of molecules responsible for the dissipation

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of heat in thermal systems, but rather represents an additional potential source of heat production by dissipation in our problem, including mechanisms of friction, fracture and seismic radiation. Our results will confirm that a maximum entropy production rate (including the temperature term) is a consequence of a reduction in this random strain level, whose minimum coincides with the occurrence of correlated strain in the model system. If the maximum entropy production mechanism is a corollary of the overarching maximum entropy principle of statistical mechanics [*Dewar*, 2003] this provides a natural self-organizing mechanism for correlated domains in the model system [*Naylor and Main*, 2008], and perhaps in nature.

[6] The OFC model is used here because it is the simplest numerical model for earthquake dynamics that reproduces the observed Gutenberg-Richter (G-R) distribution of small and intermediate-magnitude earthquakes as an emergent property. The G-R relation is $\log F = a - bm$, where F here is an incremental frequency per unit time and m is magnitude, b is the Gutenberg-Richter 'b-value' and a is related to the total event rate $\Delta N/\Delta t$. This corresponds to a power-law probability density distribution of source rupture area $F \sim A^{-b-1}$ or energy $F \sim E^{-B-1}$, where $B = 2b/3$ [*Turcotte*, 1997]. The constraint of a finite flux of energy or seismic moment introduces an exponential truncation of the form $F \propto E^{-B-1} \exp(-E/\theta)$ where θ represents the finite upper cut-off [*Main and Burton*, 1984; *Kagan*, 1997; *Main et al.*, 2008]. The mean energy radiated per event is then

$$\langle E_S \rangle = \theta^{1-B} E_0^B B \Gamma(1-B), \tag{3}$$

where $\Gamma(x)$ is the gamma function and the absolute minimum for E_0 is the energy radiated by failure of an elementary block. In practice the lower cut-off $E_{\min} > E_0$ is determined by the level at which small events can be detected above the background seismic noise, but this is not usually a significant correction since the largest events dominate the energy budget.

[7] At steady state the rate of input of elastic energy must balance $\Delta Q/\Delta t$, so

$$q K_L \langle \varepsilon \rangle \frac{d\varepsilon}{dt} l_0^2 = (\Delta N/\Delta t)\langle E_S \rangle / \eta. \tag{4}$$

[8] Empirically, the driving stress is related to strain rate in Earth materials under semi-brittle conditions across a wide range of scales by a power law $d\varepsilon/dt = C\langle\sigma\rangle^n$ or $d\varepsilon/dt = D(K_L\langle\varepsilon\rangle)^n$, where C, D and the exponent n are constants for a given medium, with n in the range 2-6 [*Carter and Kirby*, 1978; *Newman and White*, 1997]. By combining the relations described above, the rate of entropy production at steady state is given by

$$\frac{1}{k}\frac{\Delta S}{\Delta t} = \frac{\frac{\Delta N}{\Delta t}\frac{\langle E_S \rangle}{\eta}}{\frac{1}{2}K_L\langle\varepsilon^2\rangle l_0^2 + K_C\langle\delta\varepsilon^2\rangle l_0^2} = \frac{qD(K_L\langle\varepsilon\rangle)^{n+1}}{\frac{1}{2}K_L\langle\varepsilon^2\rangle + K_C\langle\delta\varepsilon^2\rangle}. \tag{5}$$

3. MEP and Natural Seismicity

[9] From equation (5) entropy production is maximized by several competing processes. Some may even locally reduce entropy (produce order), as long as the net entropy production is positive, in accordance with the second law of thermodynamics. This is a fundamental mechanism for spontaneous self-organization in a wide variety of complex systems [*Nicolis and Prigogine*, 1989]. From the numerator terms entropy production is maximized when

[10] 1. The mean strain tends to its critical value $\langle\varepsilon\rangle \rightarrow \langle\varepsilon\rangle_C$.

[11] 2. The mean radiated energy or rupture area diverges, or the cut-off $\theta \rightarrow \infty$.

[12] 3. The seismic efficiency or stress drop is small: η, $\Delta\sigma/\langle\sigma\rangle \rightarrow 0$.

[13] The first three criteria (near critical strain, diverging correlation length and small fluctuations) are all observed in natural seismicity and are hallmarks of self-organized criticality [*Main*, 1996] - a marginally stable state maintained far from equilibrium. For a precise critical point system (second order phase transition) to be defined the difference in strain energy between 'broken' and 'intact' phases (before and rupture) would be zero (analogous to zero density difference between water droplets and vapour at the critical point [e.g., *Stanley*, 1971]), and hence $\Delta\sigma = 0$. In reality the competition between the criteria may ensure a small (relative to absolute stress) but finite stress drop [*Abercrombie and Leary*, 1993]. This is explored in more detail in the following section. In addition the numerator is maximized when $\Delta N/\Delta t$ is large, ensuring that the system remains complex, with a large population of events, also consistent with natural populations of earthquakes (high 10^a).

[14] From the denominator term in equation (5) entropy production is maximized when

[15] 4. $\langle\varepsilon^2\rangle \rightarrow 0$ or

[16] 5. $\langle\delta\varepsilon^2\rangle \rightarrow 0$. Criterion 4 is in competition with criterion 1, ensuring the system remains in a strictly *sub*-critical state, consistent with a finite magnitude cut-off and finite stress drop in natural seismicity [*Main*, 1996]. Criterion 5 provides a mechanism for spontaneous self-organization of the strain field into ordered 'domains' of correlated strain (where locally $\delta\varepsilon = 0$), separated by steep but localized boundaries. This is important since it provides a mechanism for memory of past events, a key spatial element of the 'characteristic' earthquake model [*Schwartz and Coppersmith*, 1984] that is explored in the next section.

4. Comparison With Model Seismicity

[17] We now consider to what extent these drivers exist in the OFC model (described in section 2), as a function of β, in order to examine the net effect of these competing processes. Variable β could occur in nature as K_C and K_L evolve due to mechanical softening or hardening processes as a result of damage accumulation and/or healing around and on the fault [*Kachanov*, 1986; *Shipton et al.*, 2006]. Computational time increases significantly as β decreases, restricting the numerical results here to the range $\beta \geq 0.6$. For $\beta = 0$ the nature of the model defines values of the parameters analytically (e.g. stress drop is total, mean rupture area is A_0).

[18] We initialize the OFC numerical model by assigning a random scalar strain ε_i to each cell at zero time. Strain is then accumulated by driving the upper plate at a constant strain rate until a single cell reaches its breaking strength

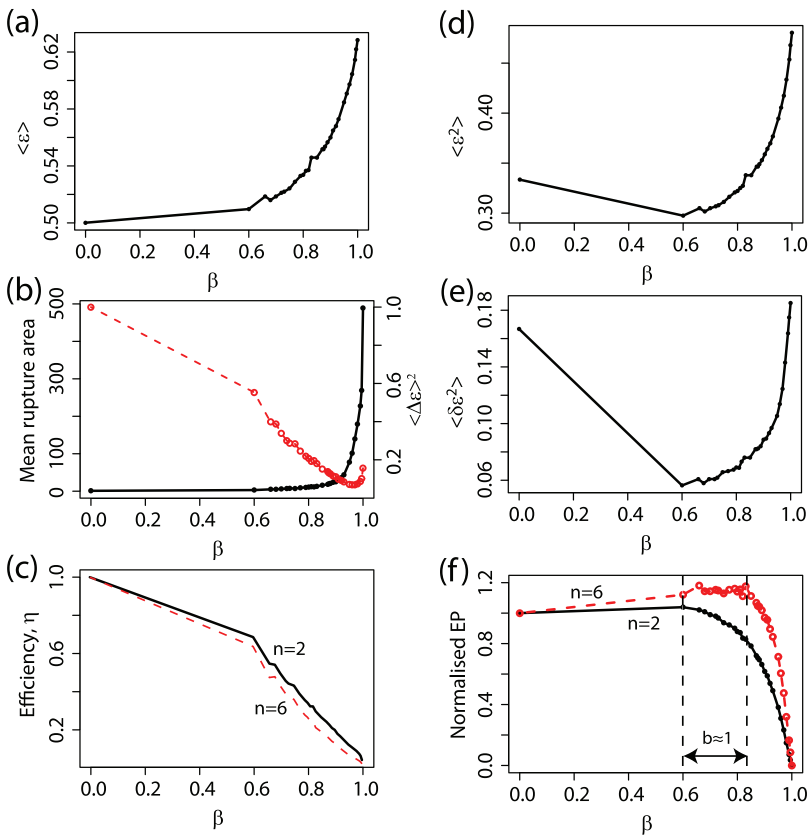

Figure 1. (a) Global strain, (b) radiated energy (red) and mean rupture area (black), (c) seismic efficiency, (d) leaf spring strain energy, (e) connecting spring strain energy and (f) net entropy production from equation (5), as a function of the conservation parameter β for the two values of the power-law rheology exponent n shown.

$\sigma_F = K_C\, \varepsilon_F\, l_0/A_0$ (normalized to 1 in this study) at which point this cell ruptures, resets its strain to zero, and redistributes a proportion of its stress as outlined in section 2. If any of these neighboring cells are now above threshold, they too rupture in the same manner until no cells are above threshold. The radiated energy is proportional to the area of rupture A_S multiplied by the average slip at a given site (number of failures in a single rupture). In the simulations here we use a square lattice of 200 × 200 elements, and run the model until steady-state is achieved, and then measure the relevant parameters only in the steady state.

[19] For the numerator terms, the OFC model output in Figure 1 shows increasing global strain (Figure 1a) diverging radiated energy (Figure 1b) and mean rupture area (Figure 1c), reducing seismic efficiency or stress drop as $\beta \rightarrow 1$. Apart from the effects of the finite system size (which maintain a finite stress drop and source correlation length) the system is precisely critical at $\beta \equiv 1$. For the denominator terms the strain energy in the leaf springs (Figure 1d) and the connecting springs (Figure 1e) both show minima at finite β, and increase at an accelerating rate to a finite value at $\beta = 1$. The entropy production at steady state (Figure 1f) shows a peak in the range $0.6 < \beta < 0.8$, for n in the observed range 2–6. The net effect of the competing processes is a maximum in entropy production at a near but strictly *sub*-critical state ($\beta < 1$). Within this range of β the Gutenberg-Richter b-value at steady state has been shown independently to be relatively constant at $b = 1$ (*Lise and Paczuski* [2001] with small deviations from a Universal value examined in more detail by *Boulter and Miller* [2003]). A universal b-value of near 1 is seen in natural seismicity at small and intermediate magnitudes [*Kagan*, 1997; *Main et al.*, 2008]. Entropy production is locally minimized in Figure 1f when $\beta = 0$, $K_C/K_L = 0$, and is an absolute minimum (zero) at the critical point $\beta = 1$, where $K_C/K_L = \infty$, and the temperature term $T \rightarrow \infty$ in equation (5) for finite K_L. The diverging spring stiffness ratio ensures that T is increasingly dominated by the energy in the connecting springs as $\beta \rightarrow 1$. The diverging temperature term (neglected by *Dewar* [2005]) ensures entropy production is maximized near but below the critical point.

5. Discussion: Memory and Predictability

[20] The minimum in connecting spring energy is associated with the occurrence of domains of correlated strain in the model at intermediate, associated with the memory of past events (Figure 2). For $\beta = 0$ the strain field retains the initial randomly-assigned structure (Figure 2a), with a correlation length of 1 block dimension. For $\beta = 1$, where the correlation length of the ruptures is a maximum (Figure 1b), the correlation length of the strain energy field is back to a few block sizes (Figure 2c), with large local fluctuations associated with high $\langle \delta\varepsilon^2 \rangle$, infinite K_C/K_L and consequently an effectively infinite temperature term and net resultant zero entropy production (Figure 1f). The state of maximum

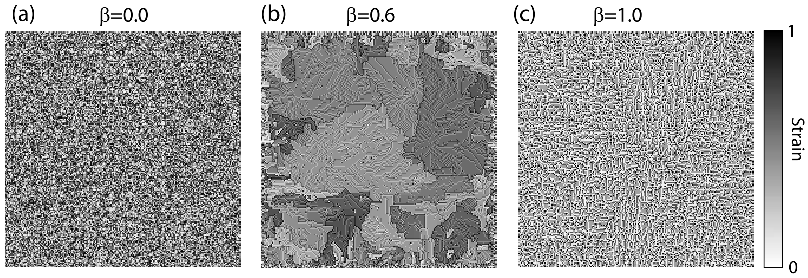

Figure 2. (a–c) Snap-shot images of the leaf spring strain field in the OFC model for various values of the conservation factor [*Naylor and Main*, 2008].

entropy production at intermediate β is associated with a finite spatial memory of past events in the form of strongly-correlated domains (Figure 2b) and a G-R *b*-value of 1. Previous attempts to explain the latter observation have largely been geometric, relying on analogies of source rupture area with a 'tiling' model that requires specific hierarchies of Euclidean tile boundaries [*Kanamori and Anderson*, 1975; *Main and Burton*, 1984; *Rundle*, 1989]. Here the observation emerges spontaneously in the dynamics with more generic tiles of complex and irregular (fractal) shape (Figure 2b).

[21] By comparing Figures 1 and 2, dissipative entropy production is maximized where self-organization in the internal strain energy of the system is also a maximum. This highlights the general point that internal self organization has to be 'paid for' by an increase in external entropy production, maintaining total entropy production positive. It is common to illustrate this principle of self-organization with the development of ordered Euclidean cells in Rayleigh-Bernard convection [*Nicolis*, 1989], although here the structure takes the form of fractal interlocking domains (Figure 2b).

[22] The finite spatial memory at a state of maximum entropy production provides a mechanism for forecasting the location and spatial extent of a large rupture, if not its exact timing, which would depend on the details of the rupture nucleation and dynamics, requiring high-resolution mapping of the stress field as in Figure 2b that is not foreseeable in real data in practice. Even in the purely deterministic OFC numerical model, accurate forecasting of event time and rupture process of extreme events becomes increasingly difficult for higher values of the conservation factor. Event time is difficult to forecast because energetic fluctuations have a Boltzmann character [*Rundle et al.*, 1995; *Main et al.*, 2000] rather than the more predictable 'saw-tooth' pattern [*Janosi and Keresz*, 1993] at lower β envisaged in the time-predictable version of the characteristic earthquake model [*Schwartz and Coppersmith*, 1984]. As a real example, in the recent hypothesis test of the characteristic earthquake model on the Parkfield segment of the San Andreas fault, the rupture location and size were as predicted, but the time and details of the rupture nucleation and subsequent dynamics were not [*Bakun et al.*, 2005].

[23] This paper has demonstrated that many natural observations validate the hypothesis of maximum entropy production. In the model system we have only examined one simple cellular automaton model that neglects long-range interactions and the tensor nature of stress interactions in nature. Further work on more realistic models is needed on a greater variety of models to establish the generality of this result, and in particular to examine whether or not MEP may drive fault localization processes at a more fundamental level.

6. Conclusion

[24] Many observations in natural and model seismicity are consistent with the hypothesis of maximum entropy production at steady state (equation (5)), including complexity (high event rate), broad-band scale invariance (high cut-off θ), the occurrence of spatially characteristic earthquakes (low $\langle \delta\varepsilon^2 \rangle$), and low but finite seismic efficiency and stress drop. When implemented in a numerical model entropy production overall is maximized in a state of self-organized *sub*-criticality, with $b \approx 1$, also consistent with observation. The results are consistent with entropy production as a thermodynamic driver for domain formation and self-organized (sub) criticality in natural and model seismicity.

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