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1 Deformation-induced melting in the margins of the
2 West-Antarctic ice streams

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3 **Abstract.** Flow of glacial ice in the West Antarctic Sheet localizes in nar-
4 row bands of fast flowing ice streams bordered by ridges of nearly stagnant
5 ice, but our understanding of the physical processes that generate this mor-
6 phology is incomplete. Here, we study the thermal and mechanical proper-
7 ties of ice-stream margins, where flow transitions from rapid to stagnant over
8 a few kilometers. Our goal is to explore under which conditions the intense
9 shear deformation in the margin may lead to deformation-induced melting.
10 We propose a 2D model that represents a cross-section through the ice-stream
11 margin perpendicular to the downstream flow direction. We limit temper-
12 ature to the melting point to estimate melt rates based on latent heat. Us-
13 ing rheology parameters as constrained by laboratory data and observations,
14 we conclude that a zone of temperate ice is likely to form in active shear mar-
15 gins.

1. Introduction

16 The West-Antarctic Ice Sheet is thought to lose over 80% of its mass [*Bamber et al.*,
17 2000] through outlet glaciers and arterial drainage routes called ice streams, which are
18 typically about a kilometer thick, tens of kilometers wide and hundreds of kilometers
19 long. Contrary to outlet glaciers, topography cannot fully explain the location of the fast-
20 flowing ice streams in the Ross Ice Shelf, Antarctica [*Shabtaie and Bentley*, 1987, 1988].
21 Further evidence that ice-stream width is not controlled by topography alone comes from
22 evidence that some margins have shifted in the past [*Jacobel et al.*, 1996; *Clarke et al.*,
23 2000; *Fahnestock et al.*, 2000; *Jacobel et al.*, 2000] or are migrating currently [*Bindschadler*
24 *and Vornberger*, 1998; *Harrison et al.*, 1998; *Echelmeyer and Harrison*, 1999]. These
25 observations suggest that a physical mechanism must exist that selects the location of the
26 margin and the flow speed of the stream self-consistently.

27 The Ross Ice Streams rest on weak and unconsolidated sediment, commonly referred to
28 as till, which overlays former seafloor [*Dreimanis*, 1988; *Tulaczyk et al.*, 1998]. Drilling
29 into several active ice streams has confirmed near-lithostatic fluid pressure in the till below
30 the ice streams [*Kamb*, 1991, 2001], and inverse methods have shown that the till layer
31 is nearly everywhere weak [*Joughin et al.*, 2004]. These findings imply that only a very
32 modest shear stress can be supported by the bed and highlight that till deformation is
33 probably the primary mechanism that allows ice streams to move rapidly despite relatively
34 small gravitational stresses [*Alley et al.*, 1986]. Measurements of shear stresses in the
35 margins of the Ross Ice Streams [*Joughin et al.*, 2002] and a laboratory study of ice cores
36 retrieved from depth [*Jackson and Kamb*, 1997] confirmed that a significant portion of

37 the driving stress is balanced by stresses on an approximately vertical interface parallel
38 to the edge of the ice stream. We refer to these lateral boundaries of ice streams, where
39 the surface velocity drops by two to three orders of magnitude over as little as a few
40 kilometers, as the shear margins.

41 While force-balance considerations clarify that shear margins play an important role
42 in ice-stream dynamics [*Whillans and Van Der Veen*, 1993; *Jackson and Kamb*, 1997;
43 *Harrison et al.*, 1998; *Whillans and Van der Veen*, 2001; *Joughin et al.*, 2004], if probably
44 to a different degree for different streams [*Raymond*, 2000], they do not offer any direct
45 insights into the mechanism through which the margin affects the ice-stream flow. One
46 possibility is that shear margins represent not only the transition from fast to slow flow,
47 but may also coincide with the boundary between temperate and frozen conditions at the
48 bed [*Jacobson and Raymond*, 1998; *Schoof*, 2012]. One problem with this idea is that
49 even a small perturbation in ice-stream width would lead to either run-away growth or to
50 stoppage of an ice stream [*Jacobson and Raymond*, 1998]. *Schoof* [2004] questioned the
51 assumption that the transition between a temperate and a frozen bed determines, or even
52 coincides with, the position of the shear margin. As previously suggested by *Raymond*
53 [1996], *Schoof* [2004] invokes a spatially variable yield stress in the till layer, analogous to
54 Barenblatt-Dugdale concepts in fracture mechanics (e.g., *Rice* [1968a, b]), but does not
55 offer an explanation for the assumed functional forms of the yield stress along the glacial
56 bed.

57 The goal of this study is to investigate the possibility of deformation-induced melting
58 in active shear margins. There are two reasons why melting and the associated presence
59 of meltwater in ice-stream margins might have important consequences for the dynamics

60 of ice streams. First, glacial till can be approximated as a Coulomb plastic material
61 with a yield strength that is strongly dependent on porosity, which is controlled by the
62 water content for full saturation [*Iverson et al.*, 1998; *Tulaczyk*, 1999; *Rathbun et al.*,
63 2008] and diminishes with effective pressure [*Kamb*, 1991; *Iverson et al.*, 1998; *Tulaczyk*
64 *et al.*, 2000] in agreement with critical-state soil mechanics (e.g., *Schofield and Wroth*
65 [1968]). The sensitive dependence of shear strength on water content suggests that the
66 spatial variability of basal stress and the positions of the shear margins could be intricately
67 linked to meltwater generation. Second, if significant quantities of meltwater are produced
68 in the margin, the water may accumulate in a channelized drainage system as pointed out
69 by *Perol and Rice* [2011]. The presence of a channel alters both the basal stresses outside
70 of it and the pore-pressure distribution in its vicinity, which could contribute to locking
71 of the bed of a widening stream.

72 The possibility that the shear margins of active ice streams may be temperate has been
73 pointed out before [*Jacobson and Raymond*, 1998; *Beem et al.*, 2010], but it remains un-
74 clear how pervasive melting is. *Perol and Rice* [2014] suggested that the shear-strain rates
75 measured by *Joughin et al.* [2002] for the five Ross Ice Streams are consistent with internal
76 melting for all stream margins except the currently inactive Kamb Ice Stream. However,
77 their model based on a 1D heat-transfer model was not versatile enough to include ice
78 advection perpendicular to the margin. While observational evidence constraining the
79 thermal properties of shear margins at depth is scarce, *Clarke et al.* [2000] identified a
80 prominent bottom diffractor extending to about 230 m above the bed in the ice sheet
81 close to Unicorn ridge (Figure 1), which they interpreted as a delineation of a zone of wet
82 and reflective ice. Further, drilling into the currently inactive Kamb Ice Stream margin

83 revealed flowing water in a 1.6 m cavity between the bottom of the ice sheet and its bed
84 [*Vogel et al.*, 2005].

85 Here, we devise a 2D thermomechanical model of an ice stream moving over a plastic bed
86 in steady state. We consider a cross-section through the ice-stream margin perpendicular
87 to the downstream flow direction and analyze the effect of the anti-plane shear stress
88 components on the mechanical equilibrium and energy dissipation. Our ice rheology
89 takes multiple creep mechanisms into account, which dominate at different stress levels.
90 In addition to diffusion and advection of heat, we include the temperature dependence
91 of material properties and a simplified representation of surface crevassing assuming a
92 temporally steady state of stress and flow velocity. To estimate melt rates based on latent
93 heat, we limit temperature to the melting point and estimate melt rates based on latent
94 heat. As detailed in Appendix A, we solve the governing equations numerically using finite
95 differences for a Cartesian grid with three refinement levels after carefully benchmarking
96 our computational technique against approximate analytical results.

97 Our model is intended as a representation of the Ross Ice Streams. Despite the general
98 scope of the model, we chose the southern margin of Whillans Ice Stream B2, commonly
99 referred to as Dragon margin (Figure 1), as a specific test case for our study. Dragon
100 margin is located near research camps and skiways and has been studied extensively.
101 The two most important data sets for our purposes are detailed observations of surface
102 velocities [*Echelmeyer et al.*, 1994; *Echelmeyer and Harrison*, 1999] and temperature
103 measurements from nine boreholes distributed across Dragon margin [*Harrison et al.*,
104 1998] (Figure. 2). In addition, *Jackson and Kamb* [1997] measured the enhancement factor
105 at Dragon margin to fit the observed rheology with the standard parametrization of Glen's

106 Law, and *Clarke et al.* [2000] provided valuable insights into the complex deformational
 107 history of the area over the last few hundred years.

2. Model setup

108 We consider a slab of ice flowing in the downstream direction x (Figure 3). The height
 109 coordinate, z , is taken positive upward from the base of the ice. In the transverse direction,
 110 the ice stream extends from the left boundary of the domain to its center ($-W/2 \leq y \leq 0$)
 111 and the ridge from the center to the right boundary of the domain ($0 \leq y \leq W/2$). The
 112 left boundary of our modeling domain thus coincides with the middle of the stream, and
 113 the origin of our coordinate system ($y = 0, z = 0$) represents the transition point from
 114 slipping to locking at the bed, indicated as a black dot in Figure 3. We do not explicitly
 115 model the process that would lead to such a transition. We distinguish the locked-to-
 116 sliding transition point from the shear margin itself, which constitutes the ice column at
 117 the lateral boundary of the stream where the surface velocity of ice increases rapidly over
 118 a few kilometers. We assume negligible variation of ice-sheet thickness in the transverse
 119 direction (along the y axis) and neglect topography at the bed such that the ice surface
 120 is parallel to the bed. We also neglect downstream variation of ice properties and flow
 121 speed u , which reduces our model to two dimensions, and assume a lithostatic pressure
 122 field. All notations used in this paper are summarized in Table 1 at the end of the paper.

2.1. Mechanical model

123 The only free variable of our mechanical model is the downstream velocity $u(y, z)$, which
 124 reduces the conservation of momentum (or static equilibrium in this case) to

$$125 \quad \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(\mu \frac{\partial u}{\partial z} \right) + \rho g \sin \alpha = 0 \quad (1)$$

126 where ρ is the ice density, g is the acceleration due to gravity, α is the inclination angle of
 127 the ice, and μ is the temperature- and strain-rate-dependent effective dynamic viscosity.
 128 Accordingly, we assume that the strain-rate tensor $\dot{\epsilon}$ and the deviatoric stress tensor $\boldsymbol{\tau}$
 129 have only two non-negligible components, the shear strain rates and shear stresses in
 130 horizontal ($\dot{\epsilon}_{xy}$ and τ_{xy}) and vertical ($\dot{\epsilon}_{xz}$ and τ_{xz}) directions on a face where x is constant.
 131 Note that by reducing our analysis to anti-plane deformation as in eq. 1, we inevitably
 132 neglect small components of in-plane deformation that must accompany the marginal
 133 melting and drainage that we later infer.

134 The stream-ridge system in Figure 3 is underlain everywhere by a thick layer of glacial
 135 till. Underneath the ice stream, the till is failing in shear, which justifies equating the
 136 basal shear stress, τ_{base} , underneath the ice stream with the yield stress. In the interest of
 137 simplicity, we assume that the basal stress is constant. Underneath the ridge, we assume
 138 that the shear stress at the ice-till interface does not exceed the yield stress, implying that
 139 failure and sliding do not occur. The full details that explain locking are yet to be fully
 140 understood. The appropriate boundary conditions at the bed are then

$$141 \quad \tau_{xz} = \tau_{base} \quad \text{at} \quad z = 0, y \leq 0 \quad (2)$$

$$142 \quad u = 0 \quad \text{at} \quad z = 0, y > 0. \quad (3)$$

144 The ice surface is assumed to be stress free. On the sides of the modeling domain, we
 145 use symmetric boundary conditions, implying an infinite juxtaposition of ice streams and
 146 ridges. Later, we estimate τ_{base} to best match measured surface flow rates and borehole
 147 temperature profiles.

148 The boundary conditions defined in eqs. 2 and 3 imply a stress singularity at the bed
 149 ($z = 0, y = 0$). This singularity is caused by the assumed till rheology, which posits
 150 that till transitions from locking to yielding almost discontinuously. The extreme stress
 151 concentration at the locked-to-sliding transition in the shear margin has important rami-
 152 fications for understanding ice-stream dynamics because it implies an inherent instability
 153 in which ice streams that experience slight perturbations might be prone to instability
 154 [*Jacobson and Raymond, 1998*]. That being said, it is possible that the singularity would
 155 be smoothed out by melting, as discussed later in detail, or a Dugdale-Barenblatt cohesive
 156 zone.

2.2. Ice rheology

157 There is no single mechanism that captures how ice deforms over a wide range of stresses
 158 [*Cuffey and Paterson, 2010*]. For our modeling domain, which encompasses both high and
 159 low stress conditions, we approximate the rheology of ice as a combination of diffusional
 160 creep $\dot{\epsilon}_D$ and Glen's Law $\dot{\epsilon}_G$

$$161 \quad \dot{\epsilon} = \dot{\epsilon}_D + \dot{\epsilon}_G. \quad (4)$$

162 Diffusional creep

$$163 \quad (\dot{\epsilon}_D)_{ij} = \frac{42\Omega}{k_B T d^2} B \exp\left(-\frac{Q}{RT}\right) \tau_{ij} \quad (5)$$

164 dominates the deformational behavior of ice at low stresses, and Glen's Law

$$165 \quad (\dot{\epsilon}_G)_{ij} = AE \exp\left[-\frac{Q}{R}\left(\frac{1}{T_h} - \frac{1}{T^*}\right)\right] \tau_E^2 \tau_{ij} \quad (6)$$

166 dominates at intermediate to high stresses, where τ_E is the effective shear stress defined
 167 by the second invariant of the deviatoric stress tensor, in our case $\tau_E^2 = \tau_{xy}^2 + \tau_{xz}^2$, and $\dot{\epsilon}_E$
 168 is the effective tensorial deviatoric strain rate, in our case $\dot{\epsilon}_E^2 = \dot{\epsilon}_{xy}^2 + \dot{\epsilon}_{xz}^2$.

169 The parameters specifying diffusional creep in eq. 5 are the molecular volume
 170 $\Omega = 3.27 \cdot 10^{-29} \text{ m}^3$, the Boltzmann constant $k_B = 1.38 \cdot 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$, the grain
 171 size $d = 1\text{-}10 \text{ mm}$, the exponential prefactor $B = 9.1 \cdot 10^{-4} \text{ m}^2 \text{ s}^{-1}$, the activation energy
 172 $Q = 59.4 \text{ kJ mol}^{-1}$ and the gas constant $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ [*Frost and Ashby, 1982*].
 173 For Glen's Law (eq. 6), we follow *Cuffey and Paterson [2010]* in using the pre-exponential
 174 constant $A = 3.5 \cdot 10^{-25} \text{ s}^{-1} \text{ Pa}^{-3}$, the temperature adjusted for the pressure effect on the
 175 melting point depression $T_h = T + p_0 P$ with $p_0 = 7 \times 10^{-8} \text{ K Pa}^{-1}$ and T in K and the
 176 activation energy $Q = 60 \text{ kJ mol}^{-1}$ for $T_h < T^*$ and $Q = 115 \text{ kJ mol}^{-1}$ for $T_h > T^*$ where
 177 $T^* = 263.15 \text{ K} = -10^\circ \text{C}$. The sum of the coefficients of τ_{ij} in eqs. 5 and 6 defines $1/(2\mu)$.

178 The rheological parameter that is most difficult to constrain is the enhancement factor.
 179 Enhancement is not a physical variable by itself but rather represents effects of grain
 180 size, impurities, fabrics and possibly other variables [*Cuffey and Paterson, 2010*]. As
 181 a consequence, estimates for enhancement vary widely. For the specific case of Dragon
 182 margin, *Jackson and Kamb [1997]* determined enhancement factors between $E \approx 1.12$
 183 and $E \approx 2.55$ for different ice specimens retrieved from Dragon margin. Because of the
 184 significant ambiguity introduced into our model results even by this moderate variation,
 185 we set $E = 1$ for most of our computations to allow for easier comparisons. We discuss
 186 the ramifications of varying enhancement in Section 4.

2.3. Thermal model

187 Our thermal model captures the effect of both diffusion and advection of heat. In
 188 addition, we cap temperature at the melting point to estimate melt production based on
 189 latent heat and include the effect of cold surface air pooling in crevasses [*Harrison et al.,*
 190 *1998*]. For a spatially-variable thermal conductivity k , the steady-state temperature field

191 is thus given by

$$192 \quad \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) - \rho c \left(v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) + 2\tau_E \dot{\epsilon}_E - L\dot{m} = 0, \quad (7)$$

193 where v and w are the lateral and vertical advection speeds, respectively, c is the specific
 194 heat of ice, τ_E is the effective shear stress, $\dot{\epsilon}_E$ is the effective tensorial shear strain rate,
 195 L is the latent heat of ice per unit mass and \dot{m} is the mass melting per unit time and
 196 unit volume. The thermal conductivity k and specific heat c vary with temperature as
 197 summarized in *Cuffey and Paterson* [2010]:

$$198 \quad k(T) = k_1 \exp(-k_2 \times 10^{-3} T) \quad (8)$$

$$199 \quad c(T) = c_1 + c_2 T, \quad (9)$$

201 where T is in Kelvin and the forefactors are $k_1 = 9.828 \text{ W m}^{-1} \text{ K}^{-1}$, $k_2 = 5.7 \text{ K}^{-1}$, $c_1 =$
 202 $152.5 \text{ J kg}^{-1} \text{ K}^{-1}$ and $c_2 = 7.122 \text{ J kg}^{-1} \text{ K}^{-2}$, respectively.

203 To compute the melt rate per unit volume \dot{m} in eq. 7, we assume that shear heating
 204 in the temperate zone is absorbed as latent heat. This assumption implies that the
 205 temperature in the temperate zone is capped at the melting point, which allows us to
 206 reduce eq. 7 to the following non-linear Poisson problem:

$$207 \quad \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) - \rho c \left(v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) + [1 - H(T - T_m)] 2\tau_E \dot{\epsilon}_E = 0, \quad (10)$$

208 where $H(T - T_m)$ denotes the Heaviside function. $H(T - T_m)$ is one in the temperate zone
 209 and zero outside such that $L\dot{m}$ is non-zero only in the temperate zone where it equals the
 210 shear heating $2\tau_E \dot{\epsilon}_E$.

211 The mechanical model set up in Section 2.1 solves only for the downstream velocity
 212 u . To include the horizontal and vertical velocities into our thermal model, we constrain

213 the functional forms for both v and w a priori, assuming that the associated strain rates,
 214 stresses and work rates are negligible. Considering horizontal and vertical velocities in the
 215 thermal but not the mechanical model is, of course, strictly inconsistent. This inaccuracy
 216 is warranted by the potentially important effect that the advection of cold ice into the
 217 shear margin is likely to have on deformation-induced melting [*Jacobson and Raymond,*
 218 1998]. In the mechanical model, however, the strain rates associated with the horizontal
 219 and vertical velocities are two orders of magnitude smaller than the anti-plane rates,
 220 implying that the strain rate components in these directions can be neglected in $\dot{\epsilon}_E$ of the
 221 creep law and the equilibrium equation.

222 For the vertical advection component we assume that w varies linearly with depth
 223 [*Zotikov, 1986*]

$$224 \quad w(y, z) = -a \frac{z}{H}, \quad (11)$$

225 where a is the surface accumulation rate of ice in m/yr, and a uniform contribution to
 226 w equal to the melt rate at the bed of the ice sheet is neglected. Eq. 11 implies that
 227 accumulation of ice at the surface is compensated by downslope stretching of ice and that
 228 basal melting or freeze-on are negligible [*Zotikov, 1986*], which is probably not strictly
 229 true in the shear margin.

230 Several studies have suggested that the position of Dragon margin has shifted in the
 231 past and may even be shifting currently with speeds on the order of 1 – 10 m/yr [*Harrison*
 232 *et al., 1998; Echelmeyer and Harrison, 1999*] to potentially ≈ 100 m/yr [*Bindschadler*
 233 *and Vornberger, 1998; Clarke et al., 2000*]. The simplest way to include ongoing margin
 234 migration at a constant rate into our ice-stream model is through the influx of cold ice
 235 from the ridge based on the rationale that in a coordinate system moving with the margin,

236 outward expansion of the stream is equivalent to influx of cold ice from the ridge (as also
 237 argued in *Schoof* [2012]). To be consistent with the zero-slip boundary condition at the
 238 bed underneath the ridge, we impose

$$239 \quad v(y, z) = v_0 \left[1 - \left(\frac{H - z}{H} \right)^4 \right] \quad (12)$$

240 instead of a depth-independent horizontal velocity as suggested by *MacAyeal* [1989]. We
 241 performed computations with both expressions for the horizontal velocity component and
 242 found only slight differences.

243 Similar to the mechanical model, we use a symmetric boundary condition on the sides
 244 of the domain. On the top, we specify the surface temperature of ice (see Section 2.5).
 245 Underneath the stream, we assume that the bed is at the melting point as supported by
 246 observations (e.g., *Engelhardt et al.* [1990]). Underneath the ridge, we do not specify a
 247 priori whether the bed is temperate or not. Instead, we adjust the geothermal heat flux G
 248 directly beneath the ice sheet such that the bed reaches a temperature of -5°C at large
 249 distances from the margin in agreement with observations (e.g., *Rose* [1979]). Depending
 250 on the computation, this condition typically requires geothermal heat fluxes on the order
 251 of $G = 48\text{-}85 \text{ mW/m}^2$. We assume the same geothermal heat flux underneath the stream.
 252 The assumption of a finite geothermal heat flux underneath the ice stream has no effect on
 253 the solution for the downstream velocity and temperature because the additional heating
 254 does not raise the base temperature above the melting point. It does, however, affect
 255 the total meltwater generation associated with both the geothermal heat flux and the
 256 frictional heating at the base of the ice stream (see Section 2.4).

2.4. Meltwater production

257 Despite the fact that our mechanical model (Section 2.1) entails a stress singularity, it is
 258 an integrable singularity from the standpoint of both force equilibrium and heat balance,
 259 which allows us to use the dissipation in the temperate zone to obtain an estimate for the
 260 melt rate per unit volume (see Section 2.3). In 2D, the mass balance for the meltwater
 261 produced in the temperate zone is

$$262 \quad \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = -\frac{2\tau_E \dot{\epsilon}_E}{L\rho_w}, \quad (13)$$

263 where q_y and q_z represent the meltwater flux in horizontal and vertical directions, respec-
 264 tively, and ρ_w is the density of water.

265 For simplicity, we neglect meltwater flux in the horizontal direction, $q_y = 0$, noting that
 266 we have previously assumed a hydrostatic pressure in our mechanical model (Section 2.1).
 267 We then integrate eq. 13 numerically to obtain the basal meltwater flux due to shear
 268 heating in the temperate zone very simply as

$$269 \quad q_{temp} = -\int_0^{H_m} \frac{\dot{m}}{\rho_w} dz = -\int_0^{H_m} \frac{2\tau_E \dot{\epsilon}_E}{L\rho_w} dz, \quad (14)$$

270 where H_m indicates the height of the temperate zone measured from the bed upwards. The
 271 possibility of a finite meltwater flux in vertical direction raises the question whether the
 272 permeability entailed by the flux we compute is consistent with measurements. Assuming
 273 that meltwater percolation occurs through Darcy's law [*Lliboutry, 1996*] and that the
 274 water pressure in veins equals the ice overburden pressure, $dp/dz = -\rho g$, we estimate the
 275 permeability of the temperate ice as [*Perol and Rice, 2011*]

$$276 \quad \kappa = \frac{q_{temp} \mu_w}{(\rho - \rho_w)g}. \quad (15)$$

277 The meltwater fluxes we later infer imply temperate-ice permeabilities on the order of
 278 10^{-16} m^2 , which agrees with experimental results (e.g., *Jordan and Stark* [2001]).

279 Permeabilities in the estimated range are sufficiently high to allow for meltwater per-
 280 colation to the base over the time scales considered here, and we will thus assume for the
 281 remainder of the paper that all of the water created in the temperate zone rains down to
 282 the bed. In addition to melt production from shear heating, frictional heat dissipation at
 283 the bed and the difference between heat in- and out-flux may contribute to the meltwater
 284 flux, yielding

$$285 \quad q_{base} = q_{temp} + \frac{1}{L\rho_w} \left[u(y, 0)\tau_{base} + G - k \frac{\partial T}{\partial z} \right]. \quad (16)$$

2.5. Surface crevassing

286 The crevassed zone at Dragon margin consists of an approximately 2 km-wide zone
 287 of intense, chaotic crevassing [*Vornberger and Whillans*, 1990; *Harrison et al.*, 1998;
 288 *Echelmeyer and Harrison*, 1999]. On the ridge side of the margin, the chaotic crevasses
 289 transition to large arcuate crevasses and on the stream-side to somewhat organized and
 290 widely spaced crevasses that tend upstream [*Harrison et al.*, 1998; *Echelmeyer and Har-*
 291 *rison*, 1999]. The crevasses are thought to extend about 30 m into the ice [*Harrison et al.*,
 292 1998]. The location of the crevassed zone and the positions of the nine boreholes for which
 293 *Harrison et al.* [1998] reported temperature measurements are reproduced in Figures 1A
 294 and B. *Harrison et al.* [1998] also indicate the transverse derivatives of the surface veloc-
 295 ities at the three boreholes, “Dragon Pad”, “Lost Love” and “Chaos” (Figure 2), which
 296 indicates that borehole “Dragon Pad” is located closest to the locked-to-sliding transition
 297 but on the ridge side at approximately $y = 200$ in our modeling domain (Figure 3).

298 We represent the zone of chaotic crevassing as a 2 km-wide rectangular area in our y, z -
299 plane that extends from the surface 30 m into the ice. We assume that the dense spacing of
300 open crevasses lowers the effective viscosity in this area by an order of magnitude as com-
301 pared to uncrevassed ice (i.e., from μ to $\mu/10$). We refer to the drop in effective viscosity
302 as the mechanical-weakening factor. While this choice is somewhat arbitrary, we verified
303 that the results are not sensitive to the assumed value, mostly because the crevassed zone
304 is relatively shallow. To capture the zones of arcuate and upstream crevassing on the ridge
305 and stream sides of the chaotic zone, we gradually lower the effective viscosity unaffected
306 by crevassing to the mechanically weakened effective viscosity representative of chaotic
307 crevassing over 400 m on both the stream and the ridge sides. We include the effect of
308 cool winter air pooling in the crevasses by enforcing a surface temperature of -34°C in
309 the crevassed zone, which gradually increases to -26°C in the uncrevassed ice.

310 There is no doubt that the representation of surface crevassing considered here is ex-
311 tremely simplified. A more complete model would allow the crevassed zone to evolve
312 self-consistently instead of specifying its extent a priori. Nonetheless, we argue that the
313 approach we have chosen here provides a reasonable first step for considering crevassing in
314 the context of a steady-state model and is preferable to not including crevassing at all be-
315 cause it allows us to compare computed temperature fields to the borehole measurements
316 from *Harrison et al.* [1998].

3. Results

317 To gain a better understanding of the different effects that contribute to the thermome-
318 chanics of ice streams, we start by reducing our model to its bare minimum. In its most
319 simplistic form, the model is reminiscent of models that are amenable to analytical solution

320 like *Schoof* [2004]. Then, we add in one term after another, gradually obtaining a more
321 realistic representation of the behavior of a stream-ridge system. The sequence of effects
322 that we consider is as follows: First, we investigate the ramifications of a temperature-
323 dependent creep rheology. Second, we consider the ramifications of vertical and horizontal
324 advection of cold ice into the margin. Third, we take into account crevasses along the
325 surface expression of the margin. To quantify the explanatory potential of this sequence
326 of approximations, we attempt to reproduce the surface velocities measured for Dragon
327 margin [*Echelmeyer and Harrison, 1999*] at each step and compare the respective fits.
328 We limit the ambiguity introduced into our model by a large number of parameters by
329 varying only the basal stress to fit observed surface velocities in Sections 3.1 and 3.2 while
330 all other model parameters are fixed. We find that the best-fitting basal stresses depend
331 sensitively on the model assumptions, which highlights the strongly nonlinear nature of
332 the thermomechanical behavior of ice streams. When comparing our computations to
333 the temperature data by *Harrison et al.* [1998] in Section 3.3, we additionally adjust the
334 accumulation rate and the speed of margin migration because we are unable to obtain a
335 satisfactory fit by focusing exclusively on basal stress.

3.1. The importance of a temperature-dependent rheology

336 The most significant assumption in analytical models of ice-stream dynamics is prob-
337 ably the usage of a simplified rheology, such as a constant Newtonian viscosity [*Schoof,*
338 2004, 2012] or a temperature-independent power-law rheology. In this section, we quantify
339 the ramifications of using a simplified rheology by comparing our model predictions to
340 the surface velocities at Dragon margin observed by *Echelmeyer and Harrison* [1999]. All

341 computations in this section are based on the reduced thermal model

$$342 \quad \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + [1 - H(T - T_m)] 2\tau_E \dot{\epsilon}_E = 0, \quad (17)$$

343 which neglects the advective terms and the effect of surface crevassing as compared to the
 344 full thermal model defined in Section 2.3. The latter two effects are discussed in detail in
 345 the subsequent sections.

346 In Figure 4, we compare the temperate zones and predicted surface velocities for three
 347 different rheologies. We adjust the basal stress to reproduce the surface velocity in the
 348 center of the stream exactly, and then we align the computed velocity profile to minimize
 349 the mean square error in the shear margin. The computation on top (Figures 4A1 and
 350 B1) is based on a constant Newtonian rheology of $\mu = 10^{14}$ Pa s, which clearly gives the
 351 worst fit to observational data. The computation in the middle (Figures 4A2 and B2)
 352 takes the strain-rate dependence of the rheology into account but neglects the temperature
 353 dependence. While this scenario is clearly more realistic than the Newtonian case, the
 354 width of the shear margin is overestimated by approximately a factor of two. Considering
 355 a realistic rheology that takes both the strain-rate and the temperature dependence into
 356 account improves the fit to observations dramatically (Figures 4A3 and B3). The extent of
 357 the temperate zone in the last computation (Figures 4A3 and B3) is roughly comparable
 358 to the observations by *Clarke et al.* [2000].

359 Figure 5 demonstrates why the temperature dependence of the ice rheology is such
 360 an important effect. It shows the spatial variation in shear heating in the vicinity of
 361 the singularity for the strain-rate dependent rheologies (Figures 4A2, B2 and A3, B3).
 362 Both cases show a pronounced peak in shear heating at the slip singularity. When taking

363 temperature into account, we find a second local maximum in shear heating at the ice
 364 surface above the singularity. The reason is that the ice is coldest on the surface, which
 365 translates into higher effective viscosity and higher shear heating than in the ice below.
 366 We conclude that the width of the high-strain region at the margin is controlled primarily
 367 by differences in the temperature of ice and thereby in its effective viscosity. To reproduce
 368 the rapid increase in surface speeds observed at Dragon margin [*Echelmeyer and Harrison,*
 369 1999], the ice in the margin has to be significantly warmer and thus weaker than the ice
 370 outside.

3.2. The effect of advection

371 The computations in Figure 4 indicate that a significant portion of the ice in Dragon
 372 margin is temperate. However, both models may overestimate the volume of temperate
 373 ice because they do not consider the effect of cold ice being advected into the margin both
 374 from the surface and from the ridge. To better isolate the effect of horizontal as opposed
 375 to vertical advection, we study them through separate simulations. The thermal model
 376 we consider is thus eq. 10 with $v = 0$ and $w = 0$, respectively.

377 To test the effect of vertical advection only, we first assume the same basal stress ($\tau_{base} =$
 378 4.07 kPa) as in Figures 4A3 and B3. We add vertical advection assuming an accumulation
 379 of $a = 0.1$ m/yr, which is on the low end of the estimated accumulation of 0.1 – 0.2 m/yr
 380 for Antarctica [*Giovinetto and Zwally, 2000; Joughin et al., 2002; Spikes et al., 2004*]. We
 381 find that the temperate zone vanishes almost entirely. The associated surface velocities
 382 in the ice stream, however, are now much lower than observations suggest [*Echelmeyer*
 383 *and Harrison, 1999*]. The reason is that by cooling the margin, vertical advection makes
 384 the margin stronger, and a strong margin provides more resistance against gravity than

385 a weak one. To reproduce the observed surface velocities, we have to lower the assumed
386 basal stress underneath the ice stream to $\tau_{base} \approx 3.17$ kPa. Figures 6A1 and B1 show
387 the resulting temperature field and surface velocities. Interestingly, the extent of the
388 temperate zone is now comparable to that in Figure 4A3. We thus conclude that the main
389 effect of vertical advection, if we insist that the surface deformation data be matched, is
390 to slightly shift the force balance between the resistance to flow provided by the shear
391 margin as compared to basal friction. The extent of the temperate zone in the margin
392 changes only slightly after accounting for the different basal stress required to balance
393 gravity. We have verified that this conclusion remains valid for accumulation rates of
394 $a = 0.2$ m/yr.

395 Observations suggest that Dragon margin is currently moving outwards at a constant
396 speed of 7.3 m/yr [*Harrison et al.*, 1998], which is equivalent to including horizontal ad-
397 vection at -7.3 m/yr, and possibly more rapidly [*Echelmeyer and Harrison*, 1999]. The
398 effect of including horizontal advection at -7.3 m/yr is shown in Figures 6A2 and B2. As
399 a consequence of the lateral influx of cold ice from the ridge, the temperate zone vanishes
400 entirely, and it becomes impossible to reproduce the rapid increase in the observed surface
401 speed of ice in our computations. The finding that horizontal advection on the order of
402 m/yr precludes the formation of a temperate zone is consistent with the previous study by
403 *Jacobson and Raymond* [1998]. Our simulations indicate that horizontal advection with
404 speeds on the order of 0.1 m/yr are associated with finite temperate zones, albeit smaller
405 ones than in the absence of a horizontal influx of cold ice.

406 Our inability to reproduce observed surface velocities for the estimated migration speeds
407 is clearly an important caveat. We argue that the main problem is that our simplified

408 representation of horizontal advection does not adequately represent the physics of mar-
409 gin migration. The two most important concerns are: First, it is highly questionable
410 that margin migration as rapid as several m/yr can be treated through a steady-state
411 model. Indeed, the evidence in favor of migration of Dragon margin comes primarily
412 from surface lineations and subsurface diffractors that indicate a complex deformational
413 history [*Harrison et al.*, 1998; *Echelmeyer and Harrison*, 1999; *Clarke et al.*, 2000] and
414 both inward and outward migration of the margin [*Clarke et al.*, 2000]. The observational
415 evidence thus suggest that the system has not reached a steady state. Second, Dragon
416 margin is located close to the confluence between Ice Streams B1 and B2. The interaction
417 between the two streams and the interjacent Unicorn ridge may be an important factor to
418 consider in modeling the evolution of the system. An example of observational evidence
419 that supports a non-trivial geometric connection between the two branches of Whillans
420 Ice Stream is a hook-shaped surface lineation called “Fishhook”, which connects the two
421 streams and correlates with several near-surface strain features [*Clarke et al.*, 2000].

422 Nonetheless, the simulations including horizontal advection lend additional support to
423 our previous conclusion that Dragon margin is at least partially temperate. Our compu-
424 tations show that only temperate ice is weak enough to concentrate strain to the degree
425 necessary for the surface speed to increase from approximately zero to its maximum value
426 over as little as 8 km. Cold ice as predicted in Figures 6A2 and B2 is more rigid than
427 temperate ice and is consequently associated with a much wider margin than observed.
428 The finding that Dragon margin is likely temperate over a significant portion of its thick-
429 ness is consistent with *Perol and Rice* [2011]. Albeit not including horizontal advection,
430 they predicted temperate zones in the shear margins of the active Siple-Coast Ice Streams

431 using a 1D heat-transfer model in conjunction with the shear-strain rates measured by
432 *Joughin et al.* [2002] to constrain shear heating. In fact, the size of the temperate zone
433 estimate here for Dragon margin and the results by Perol and Rice match moderately well
434 (Figure 7).

3.3. The ramifications of surface crevassing

435 The ice streams of West Antarctica were first identified by radar detection of their
436 crevassed margins [*Rose, 1979*], which highlights that intense surface crevassing is a char-
437 acteristic feature of the Ross Ice Streams. This observation raises the question of how
438 surface crevasses affect the thermomechanics of ice-stream margins. Within the framework
439 of our model, surface crevassing has two competing effects: On the one hand, crevasses
440 lower the creep resistance of the ice because of the void space they introduce into the ice.
441 We refer to this effect as mechanical weakening. On the other hand, crevasses lower the
442 temperature in the ice through cool winter air pooling in the crevasses [*Harrison et al.,*
443 1998] thus increasing the creep resistance. We call this effect thermal strengthening.

444 While we are able to reproduce the velocity data for Dragon margin [*Echelmeyer and*
445 *Harrison, 1999*] with a wide range of modeling parameters, compatibility with temperature
446 measurements [*Harrison et al., 1998*] poses more constraints. Figures 8 and 9 illustrate
447 a computation that attempts to match both data sets simultaneously. To facilitate the
448 comparison with our simulations, we briefly summarize four key observations by *Harrison*
449 *et al.* [1998]. In the interest of consistency with the rest of this manuscript, we continue to
450 use “depth” to describe distance from the bed despite the fact that “depth” in boreholes
451 is typically measured from the surface. First, the temperature for the two boreholes in
452 the ridge, named “OutB” and “Stage” (see Figure 1 for approximate borehole locations),

453 is approximately constant at -26°C over the depth range measured (Figure 8). Second,
454 out of the nine boreholes, the boreholes “Remote”, “Intermediate” and “Pad” exhibit
455 the warmest temperature at depth (-22°C at a depth of approximately 700 m) with
456 ”Intermediate” being slightly cooler than the other two. Third, borehole “UpB,” located
457 well into the stream, is slightly cooler yet (-25°C at a depth of approximately 700 m, see
458 Figure 8). Fourth, the -26°C contour extends from borehole “Dragon Pad” to borehole
459 “Intermediate” and attains the largest depths (≈ 830 m) at boreholes “Lost Love” and
460 “Chaos” (Figure 9).

461 Supposing that the temperatures measured by *Harrison et al.* [1998] resemble a steady
462 state, these four observations translate into the following constraints for our modeling.
463 First, the finding that the boreholes in the ridge maintain a typical surface temperature of
464 -26°C over a 300 m depth interval suggests high rates of vertical advection. High vertical
465 advection in turn implies high accumulations on the order of $a = 0.20 - 0.24$ m/yr, which
466 is reasonable for Antarctica [*Giovinetto and Zwally, 2000; Spikes et al., 2004*]. Second, the
467 relatively warm temperatures measured for “Remote”, “Intermediate” and “Pad” points
468 to internal heating at these locations. The offset between the locations of highest strain
469 rate and highest internal heating may indicate horizontal advection probably relating to
470 margin migration [*Harrison et al., 1998; Echelmeyer and Harrison, 1999*]. Third, the
471 comparatively cooler temperatures at “UpB” are consistent with a localized heat source
472 close to “Remote”, “Intermediate” and “Pad” that does not extend far into the stream.
473 Fourth, we take the -26°C contour as a proxy for the extent and location of the crevassed
474 zone at Dragon margin, which stretches from “Dragon Pad” to “Intermediate”.

475 Figure 8A shows the computed temperature in our modeling domain. The cooling effect
476 of crevassing depresses the temperature notably in the vicinity of the shear margin. We
477 also compare computed and measured temperatures at a depth of 700 m for the three
478 boreholes that are located far from the margin. The temperatures we estimate are $1 - 2^\circ$
479 higher than the measured values. The agreement with surface velocities (Figure 8B) is
480 slightly less satisfactory than in previous simulations (e.g., Figures 4B3 and 6B1). While
481 it is certainly possible to improve the agreement with surface velocities, doing so comes
482 at the cost of deteriorating the consistency with available temperature data. Figure 9
483 compares the computed and measured temperatures for the boreholes in the vicinity
484 of the shear margin at a depth of 700 m for the computation shown in Figure 8. We
485 successfully reproduce the extent and approximate depths of the -26°C contour from
486 *Harrison et al.* [1998], highlighted on the boreholes “Dragon Pad” to “Intermediate” as
487 short horizontal dashes. Our computations are also consistent with the observation that
488 the three leftmost boreholes are warmest and that the borehole “Intermediate” is colder
489 than its two neighboring boreholes. We suggest that the relatively warmer temperatures
490 at these three boreholes could result from their proximity to a temperate zone instead
491 of non-steady state effects as argued in *Harrison et al.* [1998]. Despite being able to
492 reproduce these key features, the temperatures we compute are systematically too high
493 by $1 - 6^\circ\text{C}$ with the highest errors occurring at the boreholes closest to the margin (i.e.,
494 “Dragon Pad”, “Lost Love” and “Chaos”).

495 Figure 10 summarizes the relationship between the average absolute errors in reproduc-
496 ing the observational data and the properties of the temperate zones for various speeds
497 of horizontal advection v_0 . The speed of horizontal advection is the most consequential

498 parameter when reproducing both data sets simultaneously because it has a strong effect
499 on the position and extent of the temperate zone and thereby on the surface velocities.
500 The speed of vertical advection, in comparison, is reasonably well constrained by fitting
501 the temperature measurements in boreholes “OutB” and “Stage” (Figure 8A) and does
502 not affect the shear margin as sensitively (see also the discussion in Section 3.2). Figure 10
503 highlights that the maximum height and the shape of the temperate zones are highly de-
504 pendent on the assumed speed of horizontal advection. It is also evident from Figure 10
505 that large temperate zones entail more realistic surface velocities, while small temperate
506 zones produce more realistic temperatures. We argue that horizontal advection on the
507 order of $v_0 = -0.3$ to -0.35 m/yr probably provide a reasonable compromise (Figures 8
508 and 9 show the case where $v_0 = -0.35$ m/yr). Increasing the speed of horizontal advection
509 beyond $v_0 \approx -0.35$ m/yr deteriorates the fit to observed surface velocities notably (similar
510 to Figure 6B2) while improving the match to measured temperatures only minimally.

511 The temperate zones for horizontal-advection speeds between $v_0 = -0.3$ and -0.35 m/yr
512 reach a maximum height of 120 – 160 m. In the vicinity of the locked-to-sliding transition
513 point, the height of the temperate zone has a second maximum of approximately 80 m.
514 The widths of the temperate zones, which we define as the maximum width range over
515 which ice is temperate not only at the bed but at a finite depth, are approximately 2.2
516 and 2.3 km. We obtain the total meltwater production by integrating the basal meltwater
517 flux over the temperate zone, which extends from about $y = -16.94$ km to $y = 512$ m for
518 $v_0 = -0.30$ m/yr and from $y = -19.30$ km to $y = 413$ m for $v_0 = -0.35$ m/yr, is 26.5 and
519 25 m^3 per year and per meter in downstream direction, respectively. Increasing the rate
520 of outward margin migration shifts the temperate zone further into the stream, depresses

521 the maximum depth to which temperate ice extends and decreases the total meltwater
522 production (Figure 10).

523 When weighing how to prioritize the two data sets, it is important to keep in mind
524 that a steady-state approach to modeling the temperatures throughout Dragon margin
525 is in itself highly questionable (see also Section 3.2). A simple scaling analysis shows
526 that the time it takes for the full ice thickness to reach steady state, $t = H^2/\alpha_{th}$, is
527 approximately 10^4 years. In contrast, *Harrison et al.* [1998] estimated that the residence
528 time of ice in Dragon margin is approximately a half century. It is thus clear that the
529 stream-ridge system can not possibly be in steady state. This insight is supported by
530 field observations that indicate a complex deformational history dating back over the last
531 few hundred years [*Harrison et al.*, 1998; *Echelmeyer and Harrison*, 1999; *Clarke et al.*,
532 2000]. Non-steady state effects should be most significant for the boreholes closest to
533 the margin because of the rapid margin migration in recent years [*Harrison et al.*, 1998;
534 *Echelmeyer and Harrison*, 1999]. The observation that the deviations between computed
535 and observed temperatures are indeed highest in the shear margin lends support to the
536 interpretation that non-steady effects are probably the main reason for the temperature
537 mismatch between model and data.

4. Discussion

538 While several previous models of ice streams [*Jacobson and Raymond*, 1998; *Schoof*,
539 2004; *Beem et al.*, 2010; *Perol and Rice*, 2011] have raised the possibility of melting in
540 active ice-stream margins, only *Perol and Rice* [2011] discuss the extent of a potential
541 temperate zone in detail (see Figure 7 for a comparison with their results). In this study,
542 we provide additional constraints on how different factors such as the assumed ice rheol-

ogy (Section 3.1), horizontal and vertical advection (Section 3.2) and surface crevassing
(Section 3.3) affect deformation-induced melting and the size of a potential temperate
zone.

While the simulations presented in Sections 3.1 and 3.2 raise the possibility that a
temperate zone may extend several hundred meters above the bed at Dragon margin, the
borehole measurements by *Harrison et al.* [1998] indicate variable but cold temperatures in
the upper 300–400 m of the ice. In Section 3.3 we thus attempt to fit velocity [*Echelmeyer
and Harrison, 1999*] and temperature data [*Harrison et al., 1998*] simultaneously. We
find that the two data sets provide somewhat conflicting constraints on the size of a
potential temperate zone: While the agreement with surface velocities improves for a
large temperate zone, the agreement with temperature measurements improves for a small
temperate zone. We suggest that the computation shown in Figures 8 and 9 represents a
reasonable compromise between the two constraints. This particular computation entails
a temperate zone with a maximum height of ≈ 120 m and entails a meltwater production
of $25 \text{ m}^3/\text{yr}$ per meter in the downstream direction (Figure 9).

As discussed in more detail in Section 3.3, it is unreasonable to expect that temper-
ature data can be reproduced exactly within the confines of a steady-state model (see
Section 3.3), particularly for a location with a complex deformational history like Dragon
margin [*Clarke et al., 2000*]. To remedy the difficulties associated with replicating mea-
sured surface velocities without allowing for melting, *Echelmeyer et al.* [1994] and *Scambos
et al.* [1994] have chosen to adjust the enhancement factor and its spatial variability in
the model. It is certainly possible to match the surface velocity *Echelmeyer and Harrison*
[1999] and temperature [*Harrison et al., 1998*] data for Dragon margin only by varying

566 enhancement in different parts of the stream-ridge system. However, we require very high
567 enhancement values on the order of 15 – 20 in the shear margin to obtain a satisfactory
568 fit to observational data, which conflicts with the results of *Jackson and Kamb* [1997],
569 who determined enhancement factors between $E \approx 1.12$ and $E \approx 2.55$ for different ice
570 specimens retrieved from Dragon margin.

571 Our simulations show that it is not necessary to resort to very high enhancement factors
572 to reproduce rapid velocity increases in shear margins. We obtain excellent agreement
573 with surface velocities even when neglecting enhancement entirely. This insight is not
574 meant to imply that varied effects such as fabric, impurities or grain-size variations, which
575 are usually integrated into a single enhancement factor, are not important. Instead, we
576 argue that the relatively small enhancement factors measured at Dragon margin [*Jackson*
577 *and Kamb*, 1997], in combination with our simulations, suggest that enhancement effects
578 are probably not the whole story and that deformation-induced melting may play an
579 important role in the thermomechanics of ice-stream margins, as is also supported by
580 some observations [*Clarke et al.*, 2000; *Vogel et al.*, 2005].

5. Conclusion

581 In this study, we investigate the possibility of deformation-induced melting in active
582 shear margins. Melting and the associated presence of meltwater in ice-stream margins
583 might have important consequences for the dynamics of ice streams, primarily because the
584 yield strength of glacial till depends sensitively on porosity, which is controlled by the wa-
585 ter content if saturated. The position of the shear margins could thus be intricately linked
586 to meltwater production [*Raymond*, 1996, 2000], which remains poorly constrained. We
587 devise a 2D thermomechanical model of an ice stream moving over a plastic bed in steady

588 state. We solve our model numerically after carefully benchmarking our computational
589 approach against an asymptotic analytic solution. In combination with previous studies
590 [*Jacobson and Raymond, 1998; Schoof, 2004; Perol and Rice, 2014*], our simulations lend
591 theoretical support to the hypothesis that active shear margins are partially temperate.
592 For Dragon margin, we estimate a temperate zone with a maximum height of ~ 120 to
593 150 m that produces approximately 25 to 26.5 m³ meltwater per year and per meter in
594 the downstream direction. This estimate for the extent of a temperate zone is roughly
595 comparable to the height of the bottom diffractor identified by *Clarke et al. [2000]* under
596 Unicorn ridge. Despite focusing primarily on Dragon margin, we argue that our insights
597 may generalize to the other active Ross Ice Streams.

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Appendix A: Numerical methodology

740 Our numerical implementation uses centered finite-differences on a Cartesian grid, yield-
741 ing a second-order accurate approximation to the governing equations. Our grid setup
742 allows for three refinement levels in the vicinity of the singularity at the transition point
743 between stream and ridge in the mechanical model (see Section 2.1). We do not use grid
744 refinement in our solution to the thermal model (see Section 2.3) because our temperature
745 field is capped at the melting point, which implies constant temperature in the vicinity
746 of the stress singularity.

747 We solve the mechanical and thermal models iteratively by repeating the following
748 computational steps: First, we seek an approximate solution to our mechanical model
749 (eq. 1). Second, we compute the dissipative heating associated with the estimated velocity
750 field and interpolate it to coincide with the temperature grid. Third, we compute the
751 temperature field resulting from this dissipative heating term by solving eq. 10. Fourth,
752 we update our estimates for the effective viscosity, thermal conductivity and specific heat
753 based on the new temperature field. Once the coupled mechanical and thermal models
754 have converged to a stable solution for the temperature and velocity fields of the stream-
755 ridge system, we estimate melt production (eq. 16).

756 Solving our thermal model (eq. 10) requires an iterative procedure in itself because the
757 extent of the temperate zone depends on temperature. At each iteration, we update the
758 source term based on the revised estimate for the extent of the temperate zone. We also
759 shift the transition between the boundary condition at the base of the domain such that
760 geothermal heat flux is imposed only underneath the portion of the ridge that is frozen
761 (although we continue to maintain a no-slip condition underneath the entire ridge). We

762 construct two Heaviside distributions, the first representative of the temperate zone and
 763 the second representative of the crevassed zone as described in Section 2.5. In both cases,
 764 we use a standard trigonometric taper to regularize the Heaviside distribution. We solve
 765 eq. 10 through a multigrid solver [*Briggs et al.*, 2000] in which we cycle through a hierarchy
 766 of four grids. We have also found satisfactory results with classical iterative techniques
 767 such as successive overrelaxation, but the multigrid is computationally more efficient and
 768 less prone to error oscillations, particularly along the bed.

769 An important concern when solving our mechanical model numerically for a creep-type
 770 rheology is to sufficiently resolve the singularity at the base between stream and ridge. To
 771 validate our computational approach, we benchmark it against an approximate analytic
 772 result valid for very wide ice streams with a temperature-independent viscosity sliding
 773 over a stress-free bed. Our analytic derivation is based on the insight that the transition
 774 from a slipping ice stream to a locked ridge is analogous to a crack problem. In this
 775 analogy, the singularity at the bed between stream and ridge represents the crack tip and
 776 the base of the ice stream can be thought of as a shear crack. Using this parallel, we can
 777 study the dissipation in the near-tip field with the analytical techniques developed for
 778 crack mechanics.

779 Our strategy for solving for the stress, strain-rate and dissipation in the near-tip field
 780 consists of two steps: First, we can deduce from *Rice* [1967], reinterpreted for nonlinear
 781 viscous flow, and *Rice* [1968b] that the shear heating in the near-tip region is given by

$$782 \quad 2\tau_E \dot{\epsilon}_E = \frac{3J_{tip}}{2\pi r} \left(\sqrt{4 - \sin^2 \theta} + \cos \theta \right), \quad (A1)$$

783 where r and θ are the polar coordinates centered on the slip singularity such that r is
 784 the radial distance from the slip singularity and θ the angle taken to be zero at the bed
 785 underneath the ridge (see Appendix B for details). Second, we derive an approximate
 786 estimate of the unspecified constant, J_{tip} in eq. A1, which captures the intensity of
 787 straining at the crack tip due to far-field loading, using the path-independence of J-type
 788 integrals as detailed in Appendix C. We estimate J_{tip} as

$$789 \quad J_{tip} = H\tau_{lat}\dot{\epsilon}_{lat}, \quad (A2)$$

790 where τ_{lat} is the average shear stress, τ_{xy} , at the margin that would balance the gravita-
 791 tional load of the ice stream and $\dot{\epsilon}_{lat}$ is the strain rate associated with that average lateral
 792 shear stress. Note that contrary to the exact result for the near-tip field (see eq. A1),
 793 the evaluation of the J-integral in eq. A2 is approximate and based on the simplifying
 794 assumptions that (1) the rheological behavior of ice is independent of position, hence
 795 neglecting the effect of temperature variations and (2) ice streams are much wider than
 796 thick ($H \ll W$).

797 It is unreasonable to expect the numerical and analytical solutions to match exactly for
 798 two reasons: First, the analytical solution is an approximate result that applies strictly
 799 only to very wide ice streams. We can still use it to benchmark our computational
 800 technique, however, by verifying that the error between analytical and numerical results
 801 decreases with increasing ice-stream width. Second, the analytical treatment only captures
 802 the contribution to shear heating that results from the stress singularity at the bed. The
 803 effect of the nonsingular, but still intense, shear heating throughout the entire depth extent
 804 of the ice is not accounted for by the singular term only. The numerically estimated

805 shear heating will thus never agree exactly with the analytical estimate, but the error
806 should be smallest in the immediate vicinity of the singularity ($r \rightarrow 0$) and increase
807 with increasing distance. To ensure comparability between our numerical model and the
808 analytical benchmark results, we neglect crevassing, the temperature-dependence of the
809 ice rheology and thermal conductivity and the advection terms for all computations in
810 this section. We also assume zero basal stress underneath the ice stream.

811 Figure 11A confirms that the percentage error between the numerical and the analytical
812 estimates for the non-singular dissipation, $2\tau_E \dot{\epsilon}_E \times r$ at $\theta = 0^\circ$, decreases with increasing
813 ice-stream width. To isolate the asymptotic effect of ice-stream width, all computations
814 were performed at the same grid resolution of 0.1 m in the vicinity of the singularity.
815 The flattening of the error curve in Figure 11A indicates that ice-streams with widths of
816 about $100 H$ are well approximated by the assumption of a very wide stream and suggests
817 that the remaining error is dominated by the finite resolution of our computations. In
818 Figure 11B, we demonstrate that the percentage error between numerical and analytical
819 results for an ice stream of $W = 80 H$ at $\theta = 0^\circ$ drops well below 1% for grid resolutions
820 of < 0.1 m, highlighting that we can accurately reproduce the analytical behavior with a
821 sufficiently refined grid.

822 Beyond serving as a benchmark, the comparison between analytical and numerical re-
823 sults also allows us to verify if and when the far-field contribution to shear heating starts
824 dominating the total dissipation in the shear margin. Figure 12 shows that for a wide ice
825 stream with $W = 80 H$ and a fine-grid resolution of 0.1 m, the far-field contribution to
826 shear heating already becomes important at radial distances of just a few meters, render-
827 ing the analytical solution that only captures heating in the near-field of the singularity

828 less adequate. As the radius approaches zero, however, the numerical solutions become
 829 increasingly dominated by numerical errors associated with the quickly diverging stresses
 830 and are less reliable. As demonstrated in Figure 11B, the singularity can be captured
 831 more accurately with sufficient computational expense, but ultimately there will always
 832 be a finite, if vanishingly small, radius below which the analytical result is more accurate.

Appendix B: The near-tip field parametrized by J_{tip}

833 By reinterpreting the material rheology, a parallel can be drawn between the down-
 834 stream velocity profile in a two-dimensional margin and an anti-plane crack problem. In
 835 an elastic (or “deformation theory” elastic-plastic) body, the stress depends on the strain,
 836 while in the viscous material considered here the stress depends on the strain rate. The
 837 stress fields in the elastic and viscous problems are identical. Strain rate in the viscous
 838 problem is analogous to strain in the elastic problem and downstream velocity is analogous
 839 to displacement.

840 In this section, we will solve analytically for the shear heating profile near the point
 841 where the bed transitions from slipping to locking. To develop the analytic solution, we
 842 must neglect the temperature dependence of the rheology and assume a relationship of
 843 the form

$$844 \quad \dot{\epsilon}_E = A\tau_E^n, \quad (\text{B1})$$

845 where A and n are constant. Our problem is now mathematically equivalent to the anti-
 846 plane crack problem solved in *Rice* [1967, 1968b], which solved for the stress field in a
 847 material with a linear stress-strain relationship up to a given yield stress and an arbitrary
 848 nonlinear stress-strain relationship (including a power-law relationship as a special case)

849 above the yield stress. The solution in *Rice* [1967] relies on a transformation to the hodo-
 850 graph plane. This transformation interchanges the dependent and independent variables,
 851 allowing us to solve for y and z as a function of the strain rate components $\dot{\epsilon}_{xz}$ and $\dot{\epsilon}_{yz}$.
 852 As shown in *Rice* [1967], the solution for the field near the crack tip takes the form

$$853 \quad y = X(\dot{\epsilon}_E) + F(\dot{\epsilon}_E) \cos 2\phi \quad \text{and} \quad z = F(\dot{\epsilon}_E) \sin 2\phi, \quad (\text{B2})$$

854 where we have used the polar coordinates

$$855 \quad \dot{\epsilon}_{xy} = -\dot{\epsilon}_E \sin \phi \quad \text{and} \quad \dot{\epsilon}_{xz} = \dot{\epsilon}_E \cos \phi. \quad (\text{B3})$$

856 For the power-law rheology given in equation (B1) the functions $X(\dot{\epsilon}_E)$ and $F(\dot{\epsilon}_E)$ are

$$857 \quad F(\dot{\epsilon}_E) = \frac{J_{tip}}{2\pi\dot{\epsilon}_E\tau_E(\dot{\epsilon}_E)} \quad \text{and} \quad X(\dot{\epsilon}_E) = \frac{n-1}{n+1}F(\dot{\epsilon}_E), \quad (\text{B4})$$

858 where the constant J_{tip} is determined by the far-field loading on the margin, and the
 859 evaluation of J_{tip} will be discussed in detail in Appendix C. The function $F(\dot{\epsilon}_E)$ can then
 860 be related to the shear-heating rate through

$$861 \quad 2\tau_E\dot{\epsilon}_E = \frac{J_{tip}}{\pi F(\dot{\epsilon}_E)}. \quad (\text{B5})$$

862 To solve for $F(\dot{\epsilon}_E)$, we first eliminate ϕ from eq. B2, which uncovers the equation for a
 863 circle

$$864 \quad (y - X(\dot{\epsilon}_E))^2 + z^2 = F(\dot{\epsilon}_E)^2. \quad (\text{B6})$$

865 Thus, lines of constant shear heating form circles in the (y, z) -plane with a radius of $F(\dot{\epsilon}_E)$
 866 and a center at $y = X(\dot{\epsilon}_E), z = 0$.

867 For a Newtonian rheology ($n = 1$), $X(\dot{\epsilon}_E) = 0$ and thus eq. B6 simplifies to

$$868 \quad 2\tau_E\dot{\epsilon}_E = \frac{J_{tip}}{\pi r}, \quad (\text{B7})$$

869 where $r = \sqrt{y^2 + z^2}$. This means that lines of constant frictional heating form circles
 870 about the point where the bed transitions from slipping to locking. Consequently, the
 871 shear heating within the ridge and the ice stream is the same. For the more realistic
 872 Glen's Law ($n = 3$), the solution is more complicated. In this case, $X(\dot{\epsilon}_E) = F(\dot{\epsilon}_E)/2$ and
 873 therefore eq. B6 is a quadratic equation in $F(\dot{\epsilon}_E)$. Noting that $F > 0$ is required for a
 874 physically relevant frictional heating profile, the only solution is

$$875 \quad F(\dot{\epsilon}_E) = \frac{2}{3} \left(\sqrt{4y^2 + 3z^2} - y \right). \quad (\text{B8})$$

876 This can be simplified by using polar coordinates centered on the transition point,

$$877 \quad y = r \cos \theta \quad \text{and} \quad z = r \sin \theta \quad (\text{B9})$$

878 leading to the final form of the frictional heating profile

$$879 \quad 2\tau_E \dot{\epsilon}_E = \frac{3J_{tip}}{2\pi r} \left(\sqrt{4 - \sin^2 \theta} - \cos \theta \right). \quad (\text{B10})$$

880 For Glen's law, the circles that show the lines of constant shear heating are no longer
 881 centered on the transition point, but are shifted towards the ridge by an amount $X(\dot{\epsilon}_E) =$
 882 $J_{tip}/(4\pi\tau_E\dot{\epsilon}_E)$ that varies with the magnitude of shear heating. This means that the shear
 883 heating is skewed, with more intense heating in the ridge than in the ice stream.

Appendix C: J-integral evaluation for an anti-plane flow

884 In this section, we use a path independent integral to evaluate the constant J_{tip} from
 885 Appendix B, linking the far-field loading on the ice stream to the shear heating singularity
 886 near the transition from a slipping to a locking bed. This is an extension to J-type
 887 integrals, which were pioneered by *Rice* [1968a], *Rice* [1968b], *Cherepanov* [1968] and *Bilby*

888 *and Eshelby* [1968] in the context of cracks in elastic solids, but have been generalized to
 889 more complex, nonlinear creep rheologies (e.g. *Goldman and Hutchinson* [1975]; *Landes*
 890 *and Begley* [1976]; *Kubo et al.* [1979]; *Ben Amar and Rice* [2002]) relevant for our case
 891 and applied to glaciers by *McMeeking and Johnson* [1986].

892 Our domain of ice is made to coincide with that of a classical anti-plane crack problem,
 893 for an ice slab of thickness $2H$, when we add to our domain its mirror image about the
 894 base. We thus have a classical crack problem, with $u = 0$ along the prolongation of the
 895 slipping zone into the locked zone $z = 0$ and $y > 0$, gravity loadings in the respective
 896 domains $z > 0$ and $z < 0$, traction-free surfaces, $\tau_{zx} = 0$, at $z = \pm H$ and with $\tau_{zx} = \tau_{base}$
 897 on both sides of $z = 0$ where $y < 0$.

898 Assuming that the properties of ice do not change with temperature (or that tem-
 899 perature varies only with z), the appropriate path-independent integral for our problem
 900 is

$$901 \quad J = \int_{\Gamma} (\Phi(\dot{\epsilon}_E, z) - \rho g \sin \alpha u) dz - \boldsymbol{\tau} \cdot \mathbf{n} \frac{\partial u}{\partial y} ds, \quad (C1)$$

902 where $\boldsymbol{\tau} = (\tau_{xy}, \tau_{xz})$, Φ is analogous to the strain energy density function from elasticity
 903 and is defined for a creeping solid as

$$904 \quad \Phi(\dot{\epsilon}_E) = 2 \int_0^{\dot{\epsilon}_E} \tau_E(\xi) d\xi \quad (C2)$$

905 \mathbf{n} is the outward unit normal to the curve Γ , and ds is evaluated in a counter-clockwise
 906 fashion. For any closed curve Γ that does not enclose the transition point (i.e., the crack
 907 tip), $J = 0$ so that the integral is path-independent.

908 For classical crack problems, Γ is taken to start on the lower crack surface $z = 0^-$,
 909 $y < 0$, encircle the crack tip and end on the upper crack surface $z = 0^+$, $y < 0$. J is

910 independent of path Γ for all paths with the same starting and ending points. If there
 911 is no traction on the crack faces ($\tau_{base} = 0$), J is independent of where we start and end
 912 along the faces. When $\tau_{base} \neq 0$, we start and end at a point close to the tip on the crack
 913 face where $y < 0$. Then we take the limit $y \rightarrow 0$ on both faces and define J_{tip} as the value
 914 of J . Subsequently, we focus on the part of any path Γ in the domain $z \geq 0$ so that the
 915 result of eq. C1, taken along that part of the path, is $J_{tip}/2$.

916 We now evaluate J_{tip} along the two curves Γ_{tip} and Γ_{far} shown in Figure 13, which meet
 917 these specifications, with Γ_{tip} sufficiently close to the transition point that the deformation
 918 can be described by the solution in Appendix B. Evaluation along Γ_{tip} just confirms
 919 the relations involving J_{tip} in that appendix. For a typical ice-stream geometry, the
 920 contribution to J from the portion of Γ_{far} in the center of the ridge is negligible, so for
 921 the boundary conditions highlighted in Figure 13

$$922 \quad \frac{J_{tip}}{2} = \int_H^0 [\Phi(\dot{\epsilon}_{xz}) - \rho g \sin \alpha u]_{y=-W/2} dz + \int_{-W/2}^0 \tau_{base} \frac{\partial u(y, 0)}{\partial y} dy. \quad (C3)$$

923 To calculate the two integrals in eq. C3, we need to know u within the ice stream. An
 924 approximate evaluation can be produced using a simple one-dimensional model with a
 925 constant basal stress τ_{base} beneath the ice stream. Assuming that τ_{xy} and u are functions
 926 of y alone, we integrate the equation for mechanical equilibrium from $z = 0$ to $z = H$,
 927 arriving at

$$928 \quad \frac{d\tau_{xy}}{dy} = - \left(\rho g \sin \alpha - \frac{\tau_{base}}{H} \right) \quad \text{and} \quad \dot{\epsilon}_{xy} = A\tau_{xy}^n, \quad (C4)$$

929 where it may be noted that the first of these is exact if we reinterpret τ_{xy} as its average
 930 over the thickness H in the z direction. This average becomes arbitrarily larger than the
 931 average of τ_{xz} as W/H becomes increasingly large. In that same limit, $\dot{\epsilon}_{xy}$ becomes much

larger than $\dot{\epsilon}_{xz}$, and the flow law reduces to the second equation. Treating the problem in that large W/H limit, we integrate outwards from the stress free boundary at $y = -W/2$ to calculate $\tau_{xy}(y)$ and hence the strain-rate profile

$$\frac{du}{dy} = -2A \left(\rho g \sin \alpha - \frac{\tau_{base}}{H} \right)^n \left(y + \frac{W}{2} \right)^n. \quad (C5)$$

To create the single boundary condition needed to integrate this equation, we assume that the downstream velocity vanishes at the margin, as is appropriate to the large W/H limit, and find

$$u = \frac{2A}{n+1} \left(\rho g \sin \alpha - \frac{\tau_{base}}{H} \right)^n \left[\left(\frac{W}{2} \right)^{n+1} - \left(y + \frac{W}{2} \right)^{n+1} \right]. \quad (C6)$$

The boundary condition used here is an approximation since the velocity field in the vicinity of the margin will be a function of both y and z and will not completely vanish. However, comparing the predictions for velocity in the center of the ice stream from eq. C6 and two-dimensional computational models, we find that in the limit $H \ll W/2$, eq. C6 is the asymptotic limit of such models.

Using the one-dimensional model to evaluate J_{tip} , which relies only on the value of u at the center of the ice stream, eq. C3 gives:

$$\frac{J_{tip}}{2} = \frac{2AH}{n+1} \left(\rho g \sin \alpha - \frac{\tau_{base}}{H} \right)^{n+1} \left(\frac{W}{2} \right)^{n+1} \quad (C7)$$

Defining the average lateral shear stress at the margins and the corresponding equivalent strain rate

$$\tau_{lat} = \left(\rho g \sin \alpha - \frac{\tau_{base}}{H} \right) \frac{W}{2} \quad \text{and} \quad \dot{\epsilon}_{lat} = A \tau_{lat}^n, \quad (C8)$$

we can simplify the equation for J_{tip} to

$$J_{tip} = \frac{4H}{n+1} \tau_{lat} \dot{\epsilon}_{lat} \quad (C9)$$

⁹⁵³ when $W \gg H$.

Tables

Table 1. Overview of the notation used in this study.

Symbol	Variable	Unit
a	surface accumulation rate of ice	m yr^{-1}
A	temperature-dependent creep parameter	$\text{s}^{-1} \text{Pa}^{-3}$
α	inclination angle	rad
c	specific heat of ice	$\text{J kg}^{-1} \text{K}^{-1}$
c_1	empirical forefactor	$\text{J kg}^{-1} \text{K}^{-1}$
c_2	empirical forefactor	$\text{J kg}^{-1} \text{K}^{-2}$
d	grain size	mm
E	enhancement factor	
$\dot{\epsilon}$	strain rate tensor	yr^{-1}
$\dot{\epsilon}_D$	strain resulting from diffusional creep	yr^{-1}
$\dot{\epsilon}_E$	effective strain rate	yr^{-1}
$\dot{\epsilon}_G$	strain resulting from Glen's Law	yr^{-1}
$\dot{\epsilon}_{lat}$	average shear strain at the margin	yr^{-1}
F	variable used to calculate transition point stress field	m
g	gravitational acceleration	m s^{-2}
G	Geothermal heat flux at the ice sheet base	m W m^{-2}
Γ	curve along which J-integral is evaluated	
Γ_{tip}	curve through transition point stress field	
Γ_{far}	curve around border of domain	
H	ice sheet thickness	m
H_m	height of the temperate zone	m
J_{tip}	J-integral evaluated along Γ_{tip}	$\text{J m}^{-2} \text{yr}^{-1}$
k	thermal conductivity of ice	$\text{W m}^{-1} \text{K}^{-1}$
k_1	empirical factor	$\text{W m}^{-1} \text{K}^{-1}$
k_2	empirical factor	K^{-1}
k_B	Boltzmann constant	$\text{m}^2 \text{kg s}^{-2} \text{K}^{-1}$
L	latent heat per unit mass	J kg^{-1}
\dot{m}	melt rate per unit volume	$\text{kg s}^{-1} \text{m}^{-3}$
n	exponent in ice rheology	
\mathbf{n}	unit normal vector to Γ	
μ	effective viscosity of ice	Pa s
μ_w	viscosity of water	Pa s
Ω	molecular volume	m^3
ϕ	angle between principal strain rate direction and z	rad
Φ	strain energy density analog	Pa yr^{-1}
q	meltwater flux	m s^{-1}
q_{base}	meltwater flux at the bed	m s^{-1}
q_{temp}	meltwater flux in the temperate zone	m s^{-1}
Q	activation energy	J mol^{-1}

Symbol	Variable	Unit
r	radial distance from the singularity	m
R	gas constant	$\text{J K}^{-1} \text{mol}^{-1}$
ρ	density of ice	kg m^{-3}
ρ_w	density of water	kg m^{-3}
s	arc length of curve Γ	m
T	temperature	C
T_{melt}	melting temperature of ice	C
T_h	temperature adjusted for melting point depression	K
T^*	cutoff temperature in Glen's Law	K
$\boldsymbol{\tau}$	stress tensor	Pa
τ_E	effective stress	Pa
τ_{lat}	average shear stress at the margin	Pa
τ_{base}	basal shear stress	Pa
θ	angle from the base of the ridge	rad
u	downstream velocity component	m s^{-1}
v	transverse velocity component	m s^{-1}
v_0	speed of lateral margin migration	m s^{-1}
v_{out}	velocity boundary condition in local model	m s^{-1}
w	vertical velocity component	m s^{-1}
W	ice stream width	m
x	downstream cartesian coordinate	m
X	variable used to calculate transition point stress field	m
ξ	dummy integration variable used to define Φ	
y	transverse cartesian coordinate	m
z	vertical cartesian coordinate	m

Figures

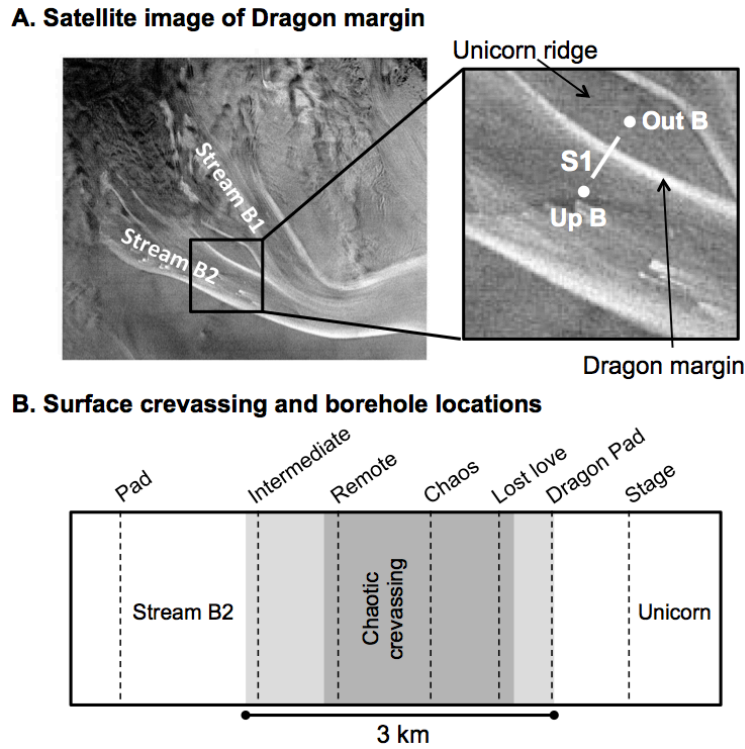


Figure 1. A: The left panel shows a satellite image of the confluence between ice streams B1 and B2 in the upstream portion of Whillans Ice Stream, taken by the Radarsat Antarctic Mapping Project. The right panel is a detailed view of Dragon margin, Unicorn ridge and the profile S1 along which *Echelmeyer and Harrison* [1999] measured surface velocities and *Harrison et al.* [1998] reported temperature for the upper few hundred meters. We have highlighted the positions of the two outermost boreholes, “Out B” and “Up B”, used in *Harrison et al.* [1998]. B: Approximate locations of the seven boreholes in the vicinity of the shear margin with respect to surface crevassing (after *Harrison et al.* [1998]). The dark grey area represents the roughly 2 km-wide zone of chaotic crevassing. The light grey zones exhibit less intense crevassing.

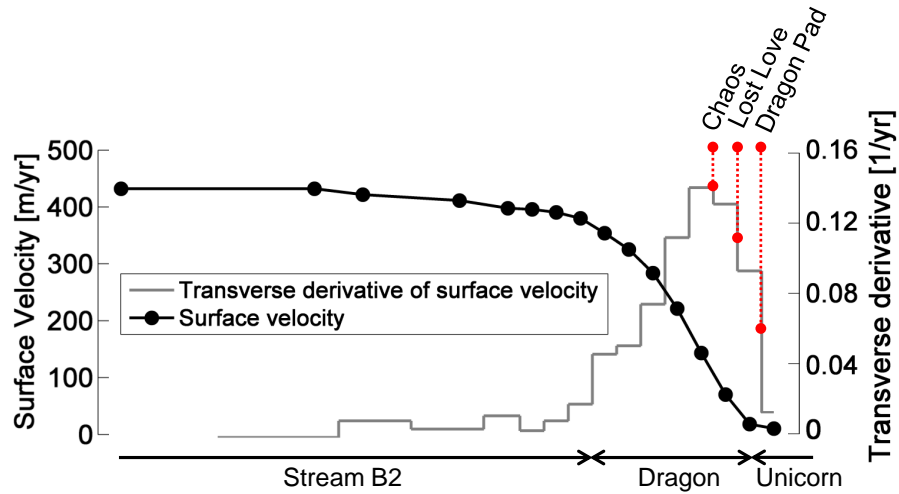


Figure 2. Surface velocities across Dragon margin as measured by *Echelmeyer and Harrison* [1999] and the transverse derivative of surface velocities, du/dy , computed from the measured surface velocities. Highlighted in red are the approximate transverse velocity derivatives for boreholes “Dragon Pad”, “Love Love”, and “Chaos” quoted from *Harrison et al.* [1998].

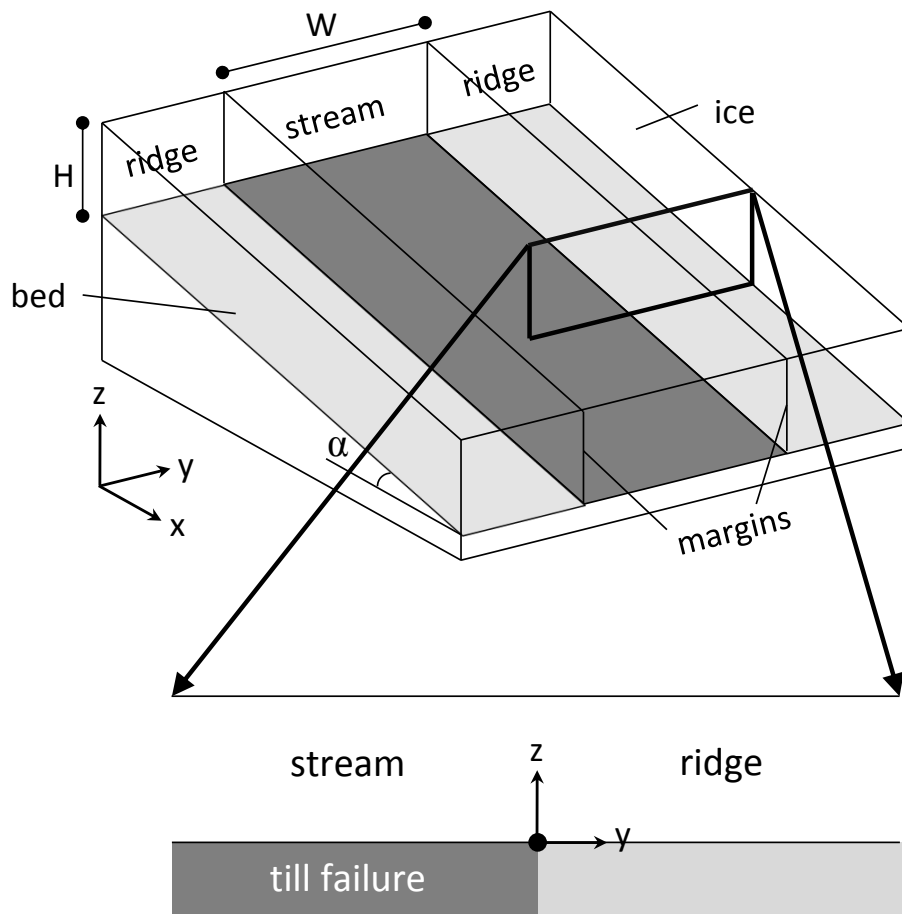


Figure 3. Simplified geometry of our model setup. The ice thickness is H and the total width of the stream is W . The 2D setup (bottom) is equivalent to assuming a 3D stream-ridge geometry with no downstream variation (top).

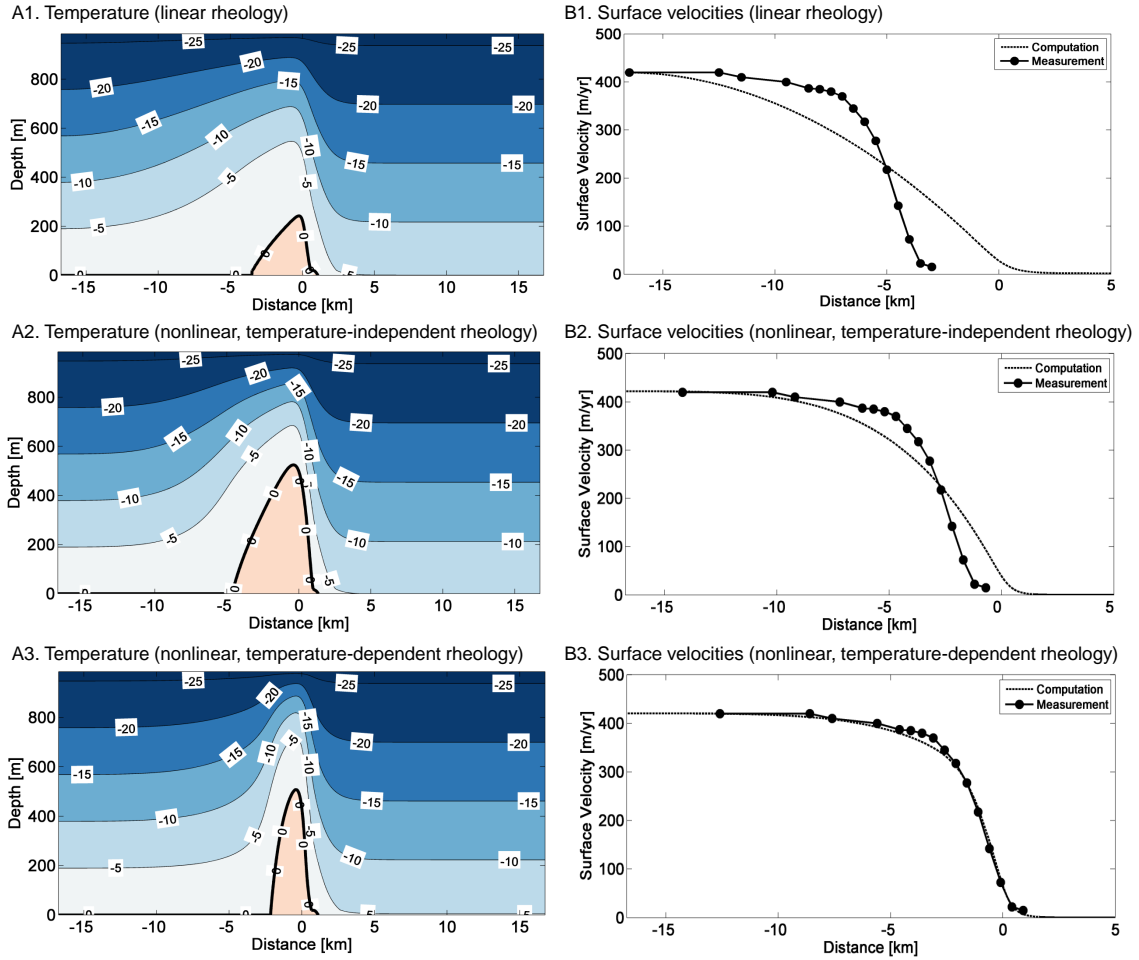


Figure 4. Top: Temperature field (A1) and surface velocities (B1) assuming a Newtonian rheology of $\mu = 10^{14}$ Pa s. Middle: Temperature field (A2) and surface velocities (B2) for a power-law rheology that accounts for the strain-rate dependence of the effective viscosity, but neglects the temperature dependence. Bottom: Temperature field (A3) and surface velocities (B3) for a realistic rheology that captures both strain-rate and temperature dependence. The best fitting basal stresses are $\tau_{base} = 2.56$ kPa (A1, B1), $\tau_{base} = 1.12$ kPa (A2, B2) and $\tau_{base} = 4.07$ kPa (A3, B3), respectively. All computations neglect advection and surface crevassing. Measured surface velocities are from *Echelmeyer and Harrison [1999]*.

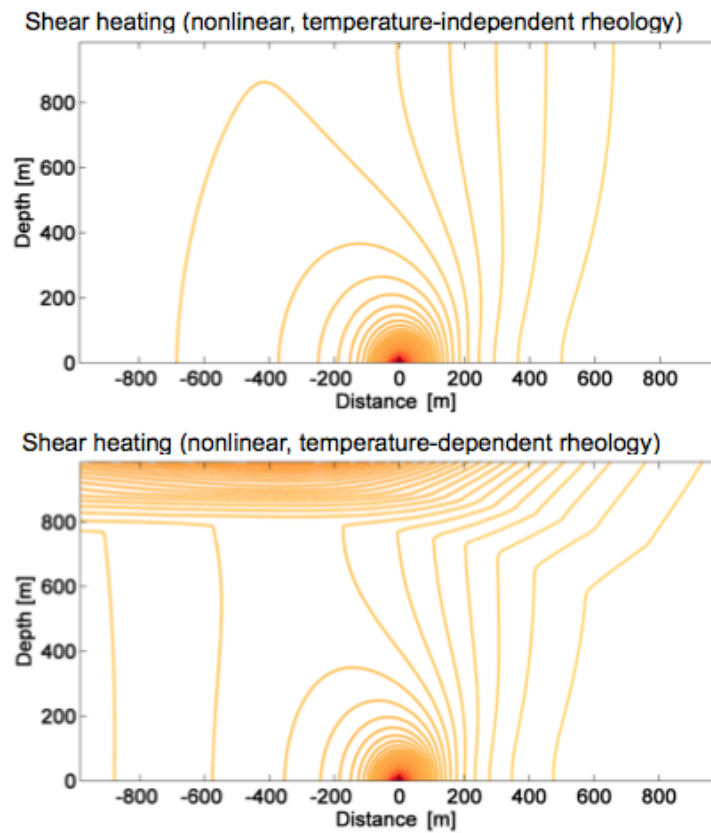


Figure 5. Eighty contours of constant shear heating in units of [Pa/yr] in the vicinity of the singularity when neglecting (top) and including (bottom) the temperature-dependence of the rheology.

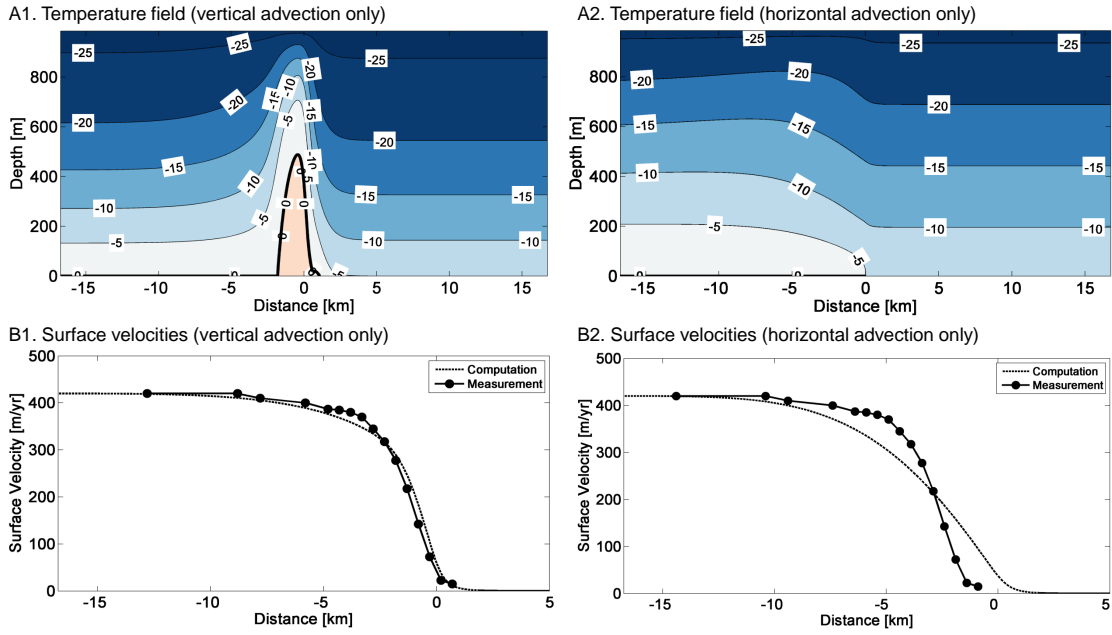


Figure 6. Temperature fields and surface velocities for Dragon margin when including only vertical advection (A1 and B1) with $a = 0.1$ m/yr and only horizontal advection (A2 and B2) with $v = -7.3$ km/yr, respectively. The best fitting basal stresses are $\tau_{base} = 5.31$ kPa (A1 and B1) and $\tau_{base} = 0.94$ kPa (A2 and B2), respectively. Both computations neglect surface crevassing.

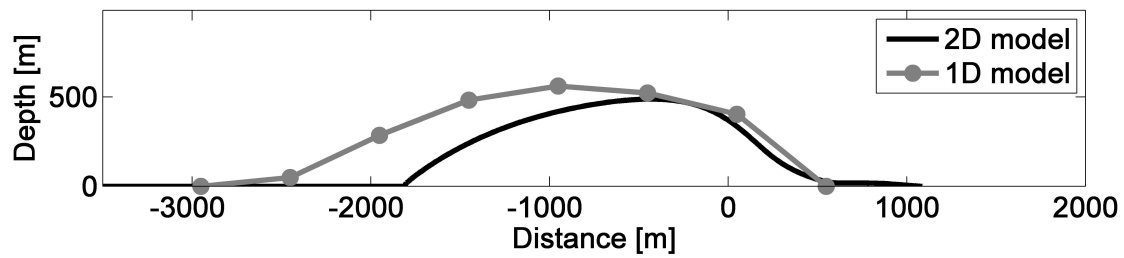


Figure 7. Comparison of the temperate zone from Figures 6A1 and B1, replotted on a 1:1 scale, with the simplified 1D model by *Perol and Rice* [2011] using measured surface velocities *Echelmeyer and Harrison* [1999] with a surface accumulation of $a = 0.1$ m/yr in both cases.

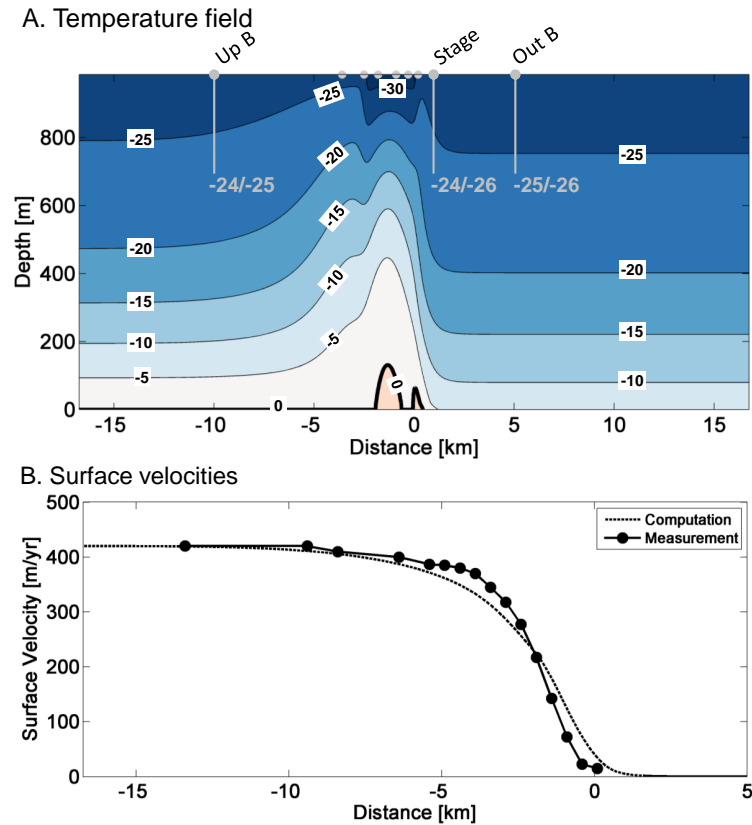


Figure 8. Temperature field (A) and surface velocities (B) for Dragon margin when attempting to match the observed borehole temperatures [Harrison *et al.*, 1998], see Figures 1 and 2 for borehole locations, and surface velocities [Echelmeyer and Harrison, 1999] simultaneously. The computation is based on the model parameters $\tau_{base} = 1.22$ kPa, accumulation $a = 0.23$ m/yr, geothermal heat flux $G = 85$ mW/m² and horizontal advection at $v = -0.35$ m/yr. The approximate locations of the nine boreholes considered in Harrison *et al.* [1998] are indicated as grey dots. We highlight the boreholes located far from margin as grey lines in accordance with their depth. The left numbers represent the computed value and the right number the measured value at maximum depth. The corresponding temperature estimates for the boreholes in the vicinity of the margin are shown in the next figure.

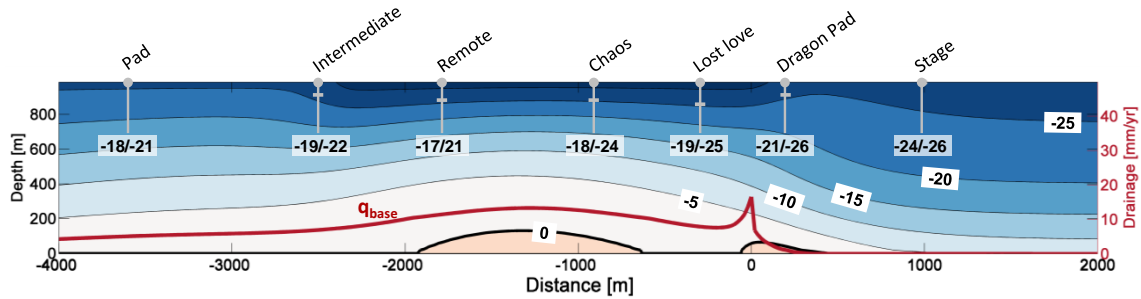


Figure 9. Extent of a potential temperate zone at Dragon margin plotted on a 1:1 scale and meltwater flux at the base of the ice, q_{base} , in mm/yr (grey line) for the computation also shown in Figure 8. The total meltwater produced in the temperate zone is $25 \text{ m}^2/\text{yr}$. The approximate locations of the boreholes from *Harrison et al.* [1998] are highlighted in grey with the left number representing the computed temperature and the right value the measured temperature at a depth of approximately 700 m. Small horizontal dashes along the boreholes in the vicinity of the shear margin indicate the approximate position of the -26°C contour.

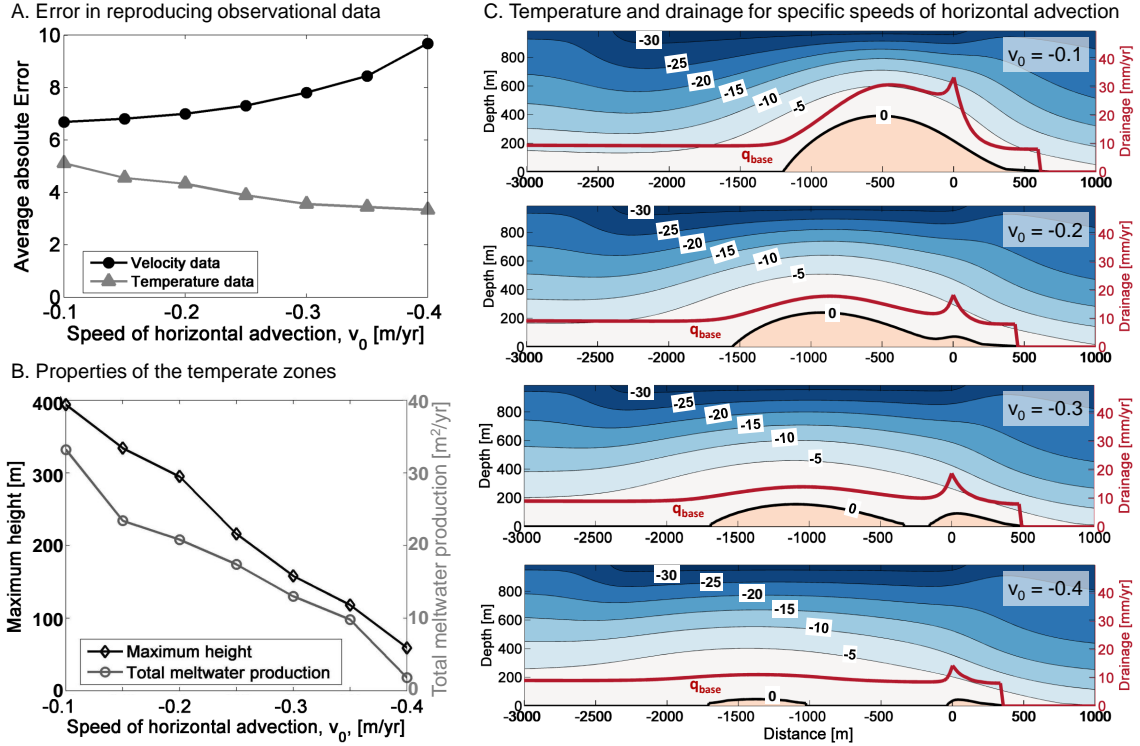


Figure 10. A. Average absolute error in reproducing observed temperatures and velocities for horizontal advection speeds between $v_0 = -0.1$ and -0.4 m/yr. B. Maximum height of the temperate zones and total meltwater production for horizontal advection speeds between -0.1 and -0.4 m/yr. The total meltwater production is computed by integrating the basal meltwater flux, q_{base} , over the width of the zone where ice is temperate not only at the bed but at finite depth. C. Temperature fields and drainage curves for the four horizontal advection speeds $v_0 = -0.1, -0.2, -0.3$ and -0.4 m/yr, respectively. The best-fitting basal stresses for the four computations are $\tau_{base} = 1.57, 1.44, 1.31$ and 1.13 kPa from the top to the bottom. Apart from the horizontal-advection speed and basal stress, all computations are based on the same model parameters, most importantly $a = 0.23$ m/yr and $G = 85$ mW/m².

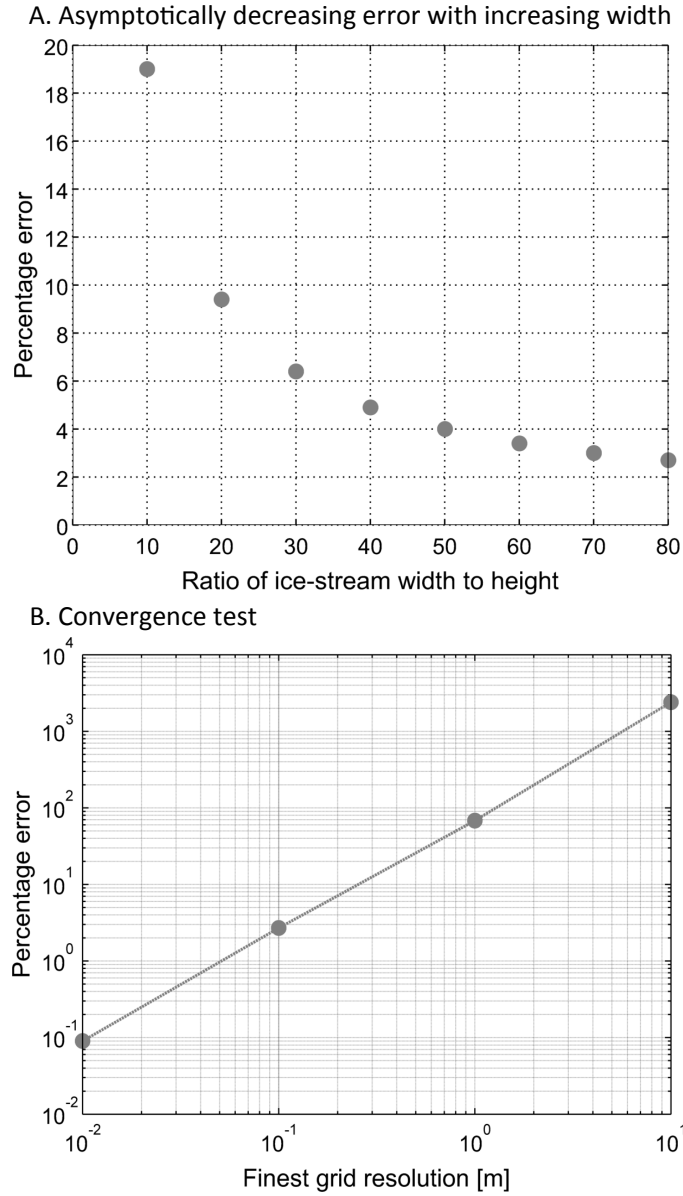


Figure 11. Validation of our computational approach. Panel A shows that the percentage error between the numerical and analytical estimates for the nonsingular shear heating, $2\tau_E \dot{\epsilon}_E \times r$, at $\theta = 0^\circ$, decreases as the ice-stream width increases. The grid resolution in the vicinity of the singularity is 0.1 m for all computations. Panel B summarizes the results of a convergence test performed for a wide ice stream with $W/H \approx 80$ at $\theta = 0^\circ$.

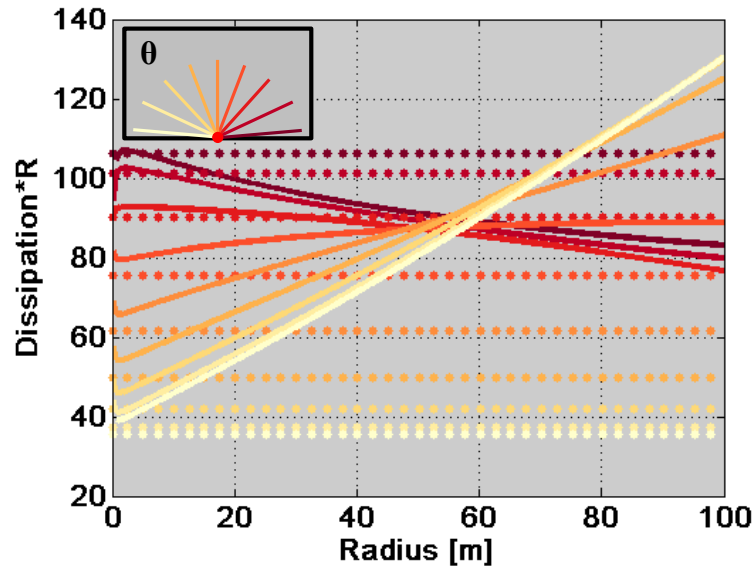


Figure 12. Plot of the angular dependence of dissipation for nine different angles from $5^\circ - 175^\circ$ represented by a specific color as detailed in the inset on the upper left. For each angle, the dotted lines represent the analytic and the full lines the numeric result. Evidently, the importance of the far-field contribution to shear heating depends primarily on the distance from the singularity, but also on the angle.

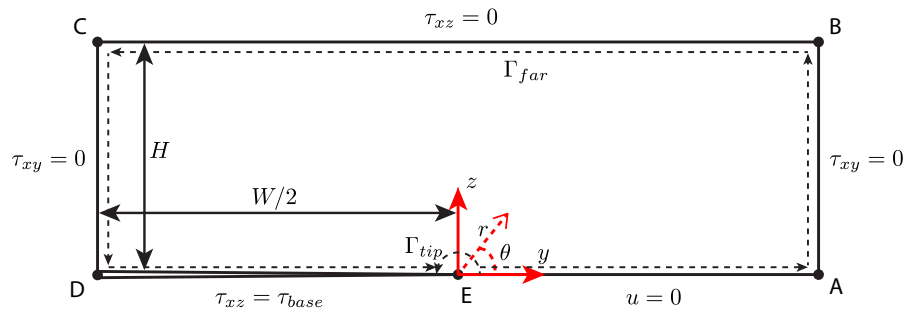


Figure 13. Sketch showing the two paths Γ_{tip} and Γ_{far} used to evaluate the path independent integral defined in eq. C1. Γ_{tip} is taken sufficiently close to the transition point so that the stress field is described by the solution in Appendix B, and Γ_{far} is evaluated along the border of the domain.