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WORKSHOP PRESENTATION Open Access

Efficient calculation of g-factors for CG-SENSE in high dimensions: noise amplification in random undersampling

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Background

SENSE [1,2] is one of the most used parallel imaging techniques. In [1], uniform undersampling was employed to efficiently reconstruct an unalised image, whereas in [2], a conjugate gradient-based method (CG-SENSE) was used for reconstruction with arbitrary trajectories. SENSE framework allows the calculation of g-factors, characterizing the noise amplification for a given k-space trajectory and coil configuration [1]. However, calculation of g-factors for arbitrary trajectories in high dimensions is time-consuming [3]. Furthermore, noise characteristics of random undersampling, used in compressed sensing, is not well-understood. In this work, we use a Monte-Carlo (MC) method for fast calculation of g-factors for CG-SENSE similar to [4,5] and apply it to random Cartesian undersampling trajectories. **Theory:** SENSE involves a pre-whitening step [1,2], thus without loss of generality, we assume white noise. SENSE reconstruction solves $\min_m ||\mathbf{y} - \mathbf{Em}||_2$, where $\mathbf{E}$ is the system matrix, and $\mathbf{y}$ are the undersampled measurements. The g-factor for the k^{th} voxel is given by $g_k = \sqrt{([\mathbf{E}^*\mathbf{E}]^{-1}_{k,k} [\mathbf{E}^*\mathbf{E}]_{k,k})}$. Inverting $\mathbf{E}^*\mathbf{E}$ is not feasible in high dimensions. Instead we note the g_k corresponds to the k^{th} diagonal of the reconstruction noise covariance matrix (for normalized coil sensitivities), where $\mathbf{n}_{recon} = (\mathbf{E}^*\mathbf{E})^{-1}\mathbf{E}^*\mathbf{n}_{meas}$, and $\mathbf{n}_{meas}$ is measurement noise with identity covariance matrix. We calculate the sample correlation matrix using a MC approach (since sample mean goes to 0), as $1/(p-1)\sum_p \mathbf{n}^p_{recon} (\mathbf{n}^p_{recon})^*$ for p instances of $\mathbf{n}_{recon}$. Note we only calculate and store the diagonal elements of this matrix, significantly increasing efficiency.

Methods

The MC method was first verified in a numerical simulation, where the g-factor was explicitly calculated for a 2D coil configuration, to determine how many MC simulations suffice. Whole-heart imaging was performed with an isotropic resolution of 1.3 mm using a 32-channel coil array. Two 4-fold accelerated acquisitions were performed, one with uniform undersampling (2 × 2 in the k_y-k_z plane) and one with random undersampling. Coil sensitivity maps were exported. Images were reconstructed using SENSE (for uniform) and CG-SENSE (for both). g-factors were also calculated with the proposed approach.

Results

Figure 1 shows the results of numerical simulations, indicating the method converges in ~50 MC simulations. Figure 2 shows the reconstructions associated with the two undersampling patterns and reconstructions, and the corresponding g-factors respectively. The results exhibit the semi-convergence property for random undersampling but not for uniform. Furthermore, the g-factor for random undersampling is smaller at its convergent point than for uniform.

Conclusions

g-factors for random undersampling is better than those for uniform at high k-space dimensions and high acceleration rates.

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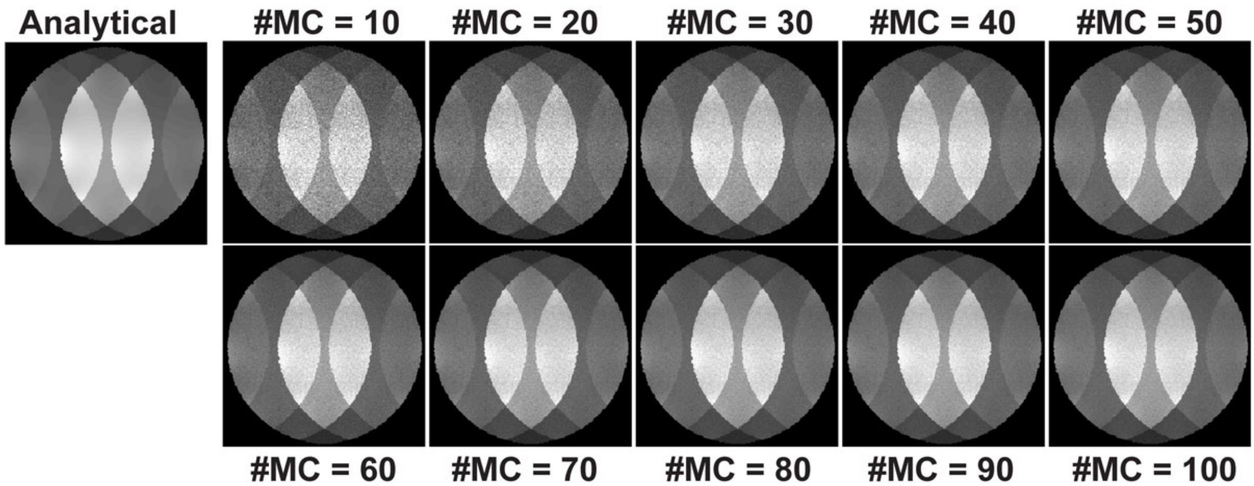

Figure 1 g-factor maps calculated from numerical simulations using point-by-point analytical evaluation, as well as the described Monte-Carlo method for various number of simulations (#MC). The Monte-Carlo based approach converges after ~50 simulations.

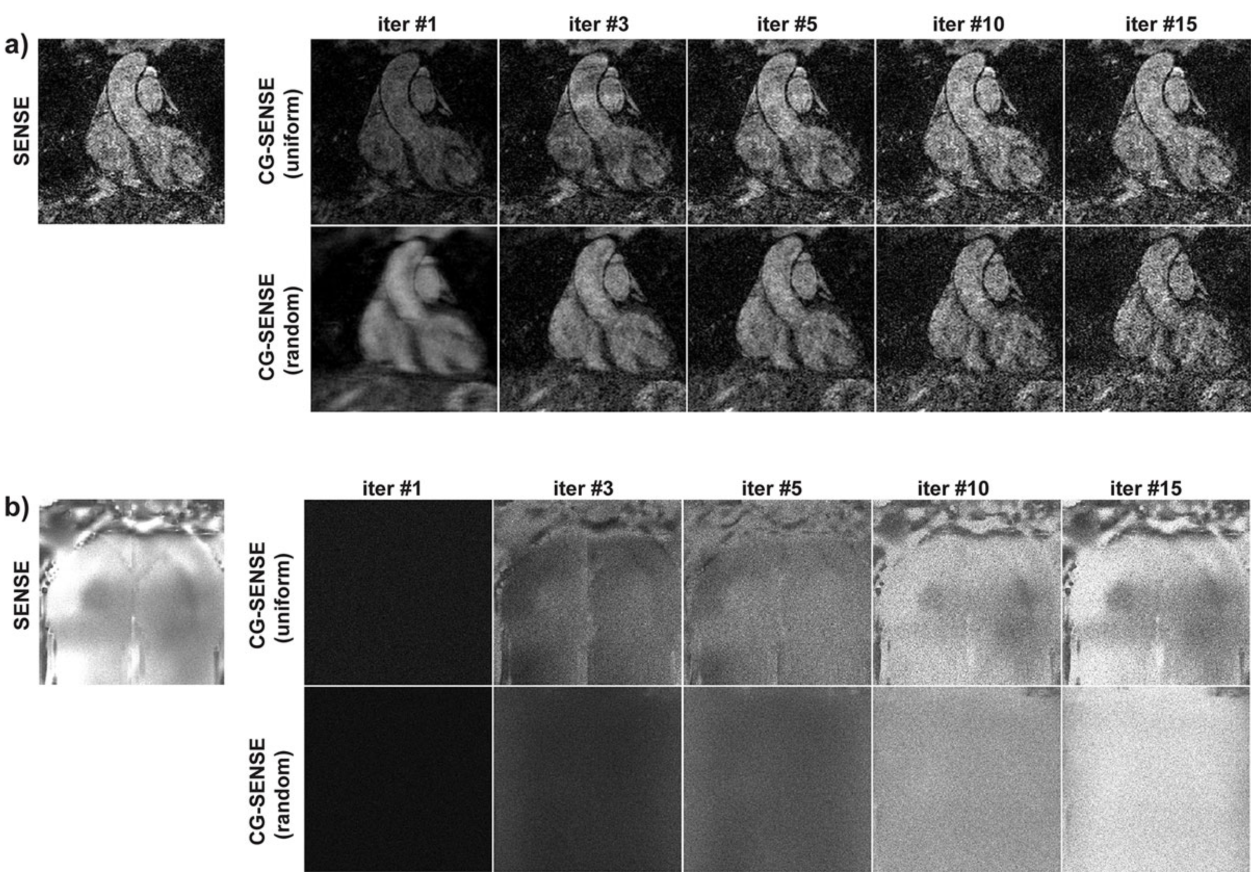

Figure 2 (a) Reconstructions from two 4-fold accelerated acquisitions with uniform and random sampling (zoomed into the heart). CG-SENSE with uniform undersampling converges in 10-15 iterations, whereas CG-SENSE with random undersampling converges in ~5 iterations (also exhibiting the semi-convergence property attributed to CG-SENSE). (b) The corresponding g-factor maps from 50 MC simulations (depicting the whole slice). g-factor maps for uniform undersampling with CG-SENSE converges to the SENSE maps in 10-15 iterations, exhibiting the folding patterns associated with SENSE reconstructions. g-factor maps for random undersampling are more homogenous, amenable to denoising with a fixed threshold (semi-convergence is also exhibited in these maps). g-factor values taken near the ascending aorta are 1.80 for SENSE; 0.55,1.26,1.60,1.80,1.79 for CG-SENSE with uniform undersampling (iterations 1,3,5,10,15 respectively); and 0.46,0.76,1.09,1.85,2.45 for CG-SENSE with random undersampling (iterations 1,3,5,10,15 respectively).

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