Essays in Monetary Policy

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presented by **Gaoyan Tang**

candidate for the degree of Doctor of Philosophy and hereby certify that it is worthy of acceptance.

Signature ____________________

Typed name: Prof. Emmanuel Farhi

Signature ____________________

Typed name: Prof. Gita Gopinath

Signature ____________________

Typed name: Prof. N. Gregory Mankiw

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Essays in Monetary Policy

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Gaoyan Tang

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Abstract

This dissertation presents three chapters addressing issues pertaining to monetary policy, information, and central bank communication. The first chapter studies optimal monetary policy in an environment where policy actions provide a signal of economic fundamentals to imperfectly informed agents. I derive the optimal discretionary policy in closed form and show that, in contrast to the perfect information case, the signaling channel leads the policymaker to be tougher on inflation. The strength of the signaling effect of policy depends on relative uncertainty levels. As the signaling effect strengthens, the optimal policy under discretion approaches that under commitment to a forward-looking linear rule, thereby decreasing the stabilization bias. This contributes to the central bank finding it optimal to withhold its additional information from private agents. Under a general linear policy rule, inflation and output forecasts can respond positively to a positive interest rate surprise when the signaling channel is strong. This positive response is the opposite of what standard perfect information New Keynesian models predict and it matches empirical patterns found by previous studies. Chapter 2 provides new empirical evidence supporting the predictions of the model presented in Chapter 1. More specifically, I find that the responses of inflation forecasts to interest rate surprises is especially positive when there is greater uncertainty regarding the previous forecast. Finally, Chapter 3 examines whether communications by the Federal Open Market Committee might have the ability to influence financial market responses to macroeconomic news. In particular, I am able to relate labor-related word use in FOMC statements and meeting minutes to the amount by which interest rates' response to labor-related news exceeds their response to other news.

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Chapter 1

Uncertainty and the Signaling Channel of Monetary Policy: Theoretical Analysis

1.1 Introduction

It has become widely accepted that expectations play a key role in the decisions that drive economic fluctuations. How these expectations are formed has been a subject of much debate. With a few exceptions, the majority of macroeconomic models feature private agent expectations of economic fundamentals that are formed independently of policy actions. However, there is a growing body of both anecdotal and empirical evidence supporting the view that monetary policy actions, in fact, communicate information about the economy to the public, and thereby affect agents' expectations. Thus, it follows that optimal policy may be altered when policy actions also influence the economy through this channel.

In this paper, I study a setting where asymmetric information exists between the policymaker and private agents. I assume that the policymaker has more information about the state of the economy than private agents. This assumption captures the central bank's private information about policy targets and its access to some confidential data. A central bank can also be better informed due to devoting more resources to processing data available to all agents[1]. In this environment, rational private agents gain information from observations of monetary policy actions that respond to these fundamentals. This process through which policy affects private agents' beliefs about the state of the economy is what I refer to as the signaling channel.

My first key result is that, for a given monetary policy, the model can produce positive responses of inflation and output forecasts to positive interest rate surprises. Second, I provide a closed-form solution for optimal policy under no commitment. The main conclusion is that the signaling channel alters the policymaker's tradeoff in a way that allows him to credibly implement an equilibrium closer to the one possible under commitment. This is one of the reasons behind my third key result showing that it can be beneficial for the policymaker to withhold his extra information from the public.

The analysis is conducted using a standard New Keynesian model with consumers and firms who have homogeneous, but imperfect information about exogenous shocks. Firms are monopolistically competitive and face a nominal price-setting friction. These two elements lead to a standard monopoly distortion and relative price distortions when there are fluctuations in nominal marginal costs. The allocative distortion resulting from inflation means that zero inflation is efficient. In this setting, welfare is maximized when inflation and output are stabilized around their efficient levels[2]. The central bank can always achieve zero inflation and bring output to the level that would prevail under flexible prices by stabilizing nominal marginal costs so that firms never want to change prices. Therefore, the policymaker is able to achieve the first-best outcome when flexible-price output coincides with the efficient level. However, shocks that drive a wedge between the flexible-price and efficient output levels create an inflation-output tradeoff for the policymaker.

[1] With costly information processing, a central bank that devotes more resources to information processing, relative to private agents, is ultimately better informed about relevant economic fundamentals.

[2] I assume a constant wage bill subsidy financed by lump sum taxes that offsets the average monopoly distortion and that there is no inherited price dispersion in the economy.

In the baseline model, there are two exogenous shocks: government demand and a time-varying target for the gap between actual output and the flexible-price level[3]. The output gap target summarizes exogenous variation in the wedge between the efficient and flexible-price levels of output coming from real imperfections not otherwise captured by the model. It can also represent exogenous variation in a politically-motivated output target that differs from the socially efficient level. The government demand shock does not create an inflation-output tradeoff for the policymaker while variations in the output gap target do. The policymaker is better informed than private agents about both shocks and sets the nominal interest rate conditional on this extra knowledge, thus making it a signal to private agents about these fundamentals. This setup reflects a narrative often seen in the popular press: upon seeing a negative interest rate surprise, private agents can interpret this as a countercyclical response to weakness in the economy (lower demand) or a desire to further boost activity (a higher output gap target).

Private agents form beliefs through a signal extraction problem, thus making the signaling effect of policy actions dependent on the relative uncertainty over the two shocks. When uncertainty about demand is high relative to uncertainty about the policy target, interest rate surprises lead to larger belief revisions about demand and smaller revisions to beliefs about the output gap target. The recent crisis provides a good example of a time when uncertainty about economic strength was particularly high and indeed, the press has interpreted many recent policy actions as indicators of economic strength. Following the release of the December 2007 FOMC meeting minutes, a New York Times story entitled "Discussion of a Fed Cut Only Stirs Up Concerns About a Weak Economy"**?** stated that "while investors usually cheer an impending rate cut, the minutes only fueled anxiety that the economy would fall into a recession". Later on, after the February 2010 decision to raise the discount rate, the Financial Times released an article entitled "Fed Discount Rate Rise Sends Recovery Signal"**?**. Interestingly, this was despite the Federal Reserve's press release

[3] Similar policy target shocks have been used by Faust and Svensson (2001) and Mertens (2011).

explicitly stating that "the modifications [...] do not signal any change in the outlook for the economy or for monetary policy".

My first key result is that when the policy response to demand shocks is inadequate and positive interest rate surprises are a strong enough signal of higher demand, the model produces a positive response of inflation and output gap forecasts to these surprises. For this result, the output gap target shock merely acts as a source of noise preventing agents from perfectly inferring the demand shock from the interest rate. This mechanism can explain the empirical patterns documented by Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2013) which show small increases in forecasts of inflation and real economic activity following positive federal funds rate surprises. The model further predicts that the responses of inflation and output gap forecasts to interest rate surprises will vary with uncertainty levels in the economy and Tang (2014a) provides empirical evidence of this fact.

Turning to the question of optimal discretionary interest rate policy, I show in closed form that the interest rate's signaling effect on private agents' beliefs about the output gap target makes accommodation of these target shocks more costly. That is, bringing output closer to its target now leads to larger inflation fluctuations compared to the perfect information case. This change in the inflation-output tradeoff reduces the stabilization bias that typically exists when the policymaker cannot commit and private agents are forward-looking. This stabilization bias generally results in excessively large inflation fluctuations. To better understand the source of this bias and the intuition behind the result, note that raising the output gap in response to a positive target shock incurs short-run inflation determined by the price-setting behavior of firms. This inflation-output tradeoff is summarized by a New Keynesian Phillips curve linking inflation to the output gap and expected future inflation. A discretionary policy typically accommodates output gap target changes too much relative to the optimal response under commitment due to contrasting effects of policy on this expected future inflation[4].

Inflation expectations can be split into agents' expectations of two components: (i) future fundamentals and (ii) future policy responses to those fundamentals. In a perfect information setting, a policymaker who cannot commit to future policy has no effect on either part. Therefore, he does not account for the effect of his current actions on previous periods' inflation expectations. On the other hand, a central banker who commits to a policy rule internalizes this intertemporal effect. He recognizes that committing to maintain smaller responses of inflation to shocks will reduce inflation expectations in prior periods and allow for greater stabilization.

When the policymaker has an information advantage, a discretionary policymaker's choice of the interest rate level now affects inflation expectations through a signaling effect on expectations of future fundamentals. I show that greater accommodation of output gap target shocks gives rise to greater belief revisions, thus leading to larger changes in inflation expectations. This tilts the discretionary policymaker's short-run inflation-output tradeoff in favor of accommodating these shocks less and maintaining relatively smaller inflation fluctuations. Hence, the signaling channel allows a policymaker to be credibly tougher on inflation without making explicit policy commitments. The policy's departure from the optimal discretionary policy under perfect information depends on the size of policy's signaling effect on private agents' beliefs about the output gap target shock. As this effect approaches its largest possible value, I show that the optimal discretionary policy becomes equivalent to the policy under commitment to a forward-looking interest rate rule. Therefore, maintaining an information advantage imposes welfare-improving discipline on discretionary interest rate policy[5] which contributes to my next result on communication policy.

[4] This is in contrast to the positive average inflation bias that occurs when the policymaker targets a level of output that is above the flexible-price level on average. Clarida, Galí, and Gertler (1999) and Woodford (2003) provide explanations of both the stabilization and average inflation biases in similar New Keynesian models.

[5] The signaling channel will generally not allow optimal policy under discretion to achieve the same welfare possible under an unrestricted commitment. In particular, optimal discretionary policy continues to be forward-looking with the interest rate responding to past shocks only through their effect on current beliefs. This contrasts with an explicit commitment of responding to lagged shocks for the purpose of

Using this model, I address communication policy by examining whether direct communication of the policymaker's additional information to the public improves welfare. In addition to the baseline no direct communication case, I consider noiseless communication of both or either one of the exogenous states to private agents. I assume that the interest rate follows the optimal discretionary policy corresponding to each case. Here, I find that the welfare is lowest under full communication of both states so that there is a benefit of maintaining some information advantage. The gains from intransparency come from two sources: (i) a reduction in the stabilization bias as discussed above, and (ii) smaller overall fluctuations under imperfect information even absent a reduction in the stabilization bias. Keeping information away from firms reduces the effects of shocks on firms' expectations of future marginal cost changes which reduces inefficient fluctuations on average. Thus, some form of intransparency is always beneficial in this setting[6]. I also show that the current welfare effect of choosing partial versus no communication will always depend on the current realizations of shocks. Therefore, the communication policy problem in this environment will generally exhibit time-inconsistency.

Lastly, I explore a few extensions of the model which illuminate some general properties of optimal policy when the interest rate has a signaling role. One property is that, if the policymaker only has superior information about shocks that do not generate an inflation-output tradeoff, the optimality condition characterizing interest rate policy is invariant to the presence of a signaling channel. I also show using a different set of shocks that when the interest rate has a signaling role, optimal policy responses to shocks can change even for shocks that are common knowledge to all agents. This occurs because the signaling channel incentivizes the policymaker to maintain smaller inflation deviations *conditional on any shock* to the economy. With an added shock to the firms' price-setting condition, the signaling

improving the set of achievable outcomes intertemporally which has been shown to lead to higher welfare in the perfect information setting (Woodford (2003)).

[6] Note that I show this under the assumption that direct communication by the central bank is noiseless and costless. Gains from intransparency would only increase if this communication were obscured by signal noise or a friction such as sticky information a la Mankiw and Reis (2002) or rational inattention as in Sims (2003).

channel can lead the policymaker to be too tough on inflation relative to a policymaker who commits to a forward-looking interest rate rule.

In another extension, I explore the case where the central bank's information advantage lies in a time-varying inflation target rather than an output gap target. I again show that it's possible under certain conditions to observe increases in inflation and output following interest rate surprises. Optimal discretionary interest rate policy continues to be characterized by smaller inflation fluctuations and a reduction in the stabilization bias arising from a lack of commitment. However, the implications differ for communication policy since, in this case, the central bank is better able to achieve its stabilization goals when private agents know the true inflation target in equilibrium. Here, I show that the expected future welfare loss is lowest when the central bank communicates only the level of demand to private agents while allowing them to perfectly infer the inflation target from the realization of the interest rate.

The next subsection reviews the related literature. Section 1.2 sets up the model. I discuss equilibrium dynamics under a general linear interest rate rule in Section 1.3 to build intuition about the interest rate's signaling effect. I turn to the main question of optimal discretionary interest rate policy in Section 1.4 with a discussion on the value of information in Section 1.5. Section 1.6 outlines extensions of the model and Section 1.7 concludes.

1.1.1 Related literature

This paper is related to several literatures. My theoretical results complement previous work on the signaling effect of monetary policy actions by Cukierman and Meltzer (1986), Faust and Svensson (2001), Geraats (2007), Walsh (2010), Berkelmans (2011), and Mertens (2011). Cukierman and Meltzer (1986), Faust and Svensson (2001), and Geraats (2007) focus on how the signaling channel ameliorates the average inflation bias present when the central bank has a positive average output target. In this paper, I show that the signaling channel can also lessen the stabilization bias present when there is no average inflation bias.

Another difference lies in the fact that Cukierman and Meltzer (1986) and Faust and Svensson (2001) both use models where agents' behavior depends on lagged expectations which are a function only of past policy actions. Thus, the presence of a signaling channel does not affect the policymaker's short-run incentives in their models as it does here. Walsh (2010) and Berkelmans (2011) focus on using numerical methods to study the signaling channel in models where agents have heterogeneous information.

The paper closest to mine is Mertens (2011). However, he focuses on a case where the central bank is more informed only about their policy objective and not other economic fundamentals. By allowing the central bank to also have an information advantage regarding a demand shock, I show that this framework is able to produce the empirical results found in Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2013). Furthermore, this paper sharpens the intuitions given for the numerical simulations in Mertens (2011) by providing closed-form expressions and illustrating links between discretionary and commitment policies.

My result on the benefits of central bank intransparency are consistent with the numerical analyses in Faust and Svensson (2001), Walsh (2010), and Mertens (2011). In contrast to these papers, I precisely characterize the sources of gains from intransparency. This finding differs from the conclusions reached in models where private agents' lack of perfect information is the only friction such as those in the spirit of Lucas Jr. (1972) and Barro (1976)[7]. In a more stylized setting, Angeletos and Pavan (2007) shows that less information can be beneficial in an economy that is inefficient under perfect information.

[7] Even when information frictions are the only frictions, full communication may be suboptimal if the central bank cannot give perfect, homogeneous information to all agents (Adam (2007), Baeriswyl and Cornand (2010)).

1.2 Model

1.2.1 Setup

I study the signaling channel of monetary policy in a standard New Keynesian economy with monopolistically competitive firms and sticky prices in the style of Calvo (1983). Fluctuations are driven by two shocks: an exogenous government spending shock and a shock to the policy target for the output gap. I assume that the monetary authority has perfect information while consumers and firms have homogeneous but imperfect information regarding these shocks. Private agents observe shocks perfectly with a one-period lag and get information about current values from observations of a nominal interest rate that responds linearly to current state variables. I first describe the model structure and then provide details on the information structure and belief formation.

Consumers

There is a representative household who maximizes utility that is additively separable in time, labor, and consumption of a composite good made up of a continuum of varieties

$$\max E \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t\right) - V\left(L_t\right) \right], \text{ where } C_t \equiv \left[\int_0^1 C_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1$$

The economy is cashless. Each consumer gets profits from all firms, pays a lump sum tax, and can trade in a riskless nominal one-period bond so that the budget constraint is

$$\int_0^1 P_{jt} C_{jt} dj + B_t \leq R_{t-1} B_{t-1} + W_t L_t - T_t + \int_0^1 \Pi_{jt} dj$$

Consumer optimization results in a standard intertemporal Euler equation and an intratemporal labor supply relation involving the price of the composite good

$$U_{C,t} = \beta R_t E\left[U_{C,t+1}\frac{P_t}{P_{t+1}}\middle|\mathcal{I}_t\right]$$
$$\frac{V_{L,t}}{U_{C,t}} = \frac{W_t}{P_t}$$

where $\mathcal{I}_t$ is a time-t information set to be defined below.

The resulting consumer demand for each variety j is

$$C_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} C_t$$

and the price of the composite good becomes

$$P_t = \left[\int_0^1 P_{jt}^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}$$

Firms

There is a continuum of firms producing differentiated goods that each maximize profits subject to demand from consumers and the government. I assume that the government consumes the same composite good as consumers and allocates their demand across varieties in the same way. Then, firm j faces total demand of

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} Y_t$$

where Y_t is aggregate real output defined as

$$Y_t \equiv \frac{1}{P_t}\int P_{jt}\left(C_{jt} + G_{jt}\right) dj = C_t + G_t$$

Production technologies are identical across firms and linear in each firms' labor

$$Y_{jt} = AL_{jt}$$

The labor market is perfectly competitive while firms also receive a constant proportional subsidy τ on their wage bills so that each firm's total cost of production is

$$\psi\left(Y_{jt}\right)=(1-\tau)\frac{W_t}{A}Y_{jt}$$

Each firm faces a $1-\theta$ probability of being able to reset their prices in each period. Firms who cannot reset prices charge their previous price. Each resetter maximizes the net present value of profits discounted according to the consumer-owners' stochastic discount factor $\beta^k\frac{\lambda_{t+k}}{\lambda_t}$ where λ_{t+k} is the Lagrange multiplier on the consumers' budget constraint which reflects the shadow value of wealth in period $t+k$.

$$P_{jt}^{*}=\arg\max_{P}\sum_{k=0}^{\infty}(\theta\beta)^k E\left[\frac{\lambda_{t+k}}{\lambda_t}\left[PY_{j,t+k}-\psi\left(Y_{j,t+k}\right)\right]\middle|\mathcal{I}_t\right]$$

Since firms employ identical technologies and hire workers from a centralized labor market, all resetters choose the same optimal price in a given period (i.e., $P_{jt}^{*}=P_t^{*}\ \forall j$). Then, the aggregate price level evolves as

$$P_t=\left[(1-\theta)\left(P_t^{*}\right)^{1-\varepsilon}+\theta P_{t-1}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

1.2.2 Equilibrium conditions

Unless otherwise noted, let lower-case letters represent log deviations from steady-state values (i.e., $x_t\equiv\ln\left(X_t/X\right)$) and let private agents' expectations be denoted by $x_{t'|t}\equiv E\left[x_{t'}|\mathcal{I}_t\right]$. Then, log-linearizing the above optimality conditions around the deterministic steady state leads to two equations characterizing aggregate output and inflation dynamics.

$$\tilde{y}_t=\tilde{y}_{t+1|t}-\frac{1}{\sigma}\left(i_t-\pi_{t+1|t}\right)+d_t-d_{t+1|t}\tag{1.1}$$

$$\pi_t=\beta\pi_{t+1|t}+\kappa\tilde{y}_t\tag{1.2}$$

where d_t is an aggregate demand shock that originates from government spending

$$d_t \equiv \frac{\varphi}{\sigma + \varphi} \left(1 - \frac{C}{Y}\right) g_t$$

$\tilde{y}_t$ represents the gap between output and its natural (i.e., flexible-price) level

$$\tilde{y}_t \equiv y_t - y_t^n \quad \text{where } y_t^n = \frac{\sigma}{\varphi} d_t$$

The coefficients can be expressed in terms of steady-state values and structural parameters.

$$\sigma \equiv -\frac{U_{cc} Y}{U_c}, \quad \varphi \equiv \frac{V_{ll} L}{V_l}, \quad \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\sigma + \varphi) \tag{1.3}$$

The first equilibrium condition in equation (1.1) stems from the resource constraint and the consumers' Euler equation

$$c_t = c_{t+1|t} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right)$$

The real interest rate is the price of consumption today relative to tomorrow and so its level determines the difference between period t and expected $t+1$ consumption. When it is kept at zero, consumption stays at its steady-state level. To determine the relationship between the output gap in period t and the expected $t+1$ output gap, the expected growth rate of government spending net of variations in the natural level of output also has to be accounted for. This is captured by the natural real rate of interest

$$r_t^n \equiv \sigma\left(d_t - d_{t+1|t}\right)$$

The New Keynesian Phillips curve in equation (1.2) is derived from firms' pricing behavior, consumers' labor supply, the resource constraint and the evolution of aggregate prices.

I now define a real interest rate gap between the actual real rate and the natural rate

$$\tilde{r}_t \equiv i_t - \pi_{t+1|t} - r_t^n$$

This real interest rate gap affects the output gap in the same way that the real interest rate affects consumption. When it is kept at zero, output stays at its natural level. In this model, this also gives zero inflation. When I examine output gap and inflation responses to interest rate surprises in the next section, it will be convenient to do so through the lens of $\tilde{r}_t$. Equations (1.1) and (1.2) can be rearranged to show that shocks affect current outcomes through expectations of next period's outcomes and the real interest rate gap

$$\tilde{y}_t = \tilde{y}_{t+1|t} - \frac{1}{\sigma}\tilde{r}_t$$

$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_{t+1|t} - \frac{\kappa}{\sigma}\tilde{r}_t$$

The model is closed with specifications for the nominal interest rate $i_t \equiv \ln(R_t/R)$ and the shocks. For now, I assume that the interest rate responds linearly to the demand shock, an output gap target shock $\bar{y}_t$ and private agents' beliefs.

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{y}}\bar{y}_t + f_{\bar{y},b}\bar{y}_{t|t} \tag{1.4}$$

$\bar{y}_t$ is the policymaker's time-varying target for the output gap. The role of this target will be clarified when I present the optimal policy problem. For now, it should be apparent that this shock affects equilibrium output and inflation in a way similar to an exogenous interest rate shock since it only enters the model's equilibrium conditions through the interest rate. I will first characterize the equilibrium under general policy coefficients to illustrate the effect of the interest rate signaling mechanism in this model. I later show a case where optimal discretionary monetary policy results in interest rate setting behavior that matches the form in (1.4).

I assume that both shocks follow AR(1) processes

$$d_t = \rho_d d_{t-1} + \epsilon_{d,t} \tag{1.5}$$

$$\bar{y}_t = \rho_{\bar{y}}\bar{y}_{t-1} + \epsilon_{\bar{y},t} \tag{1.6}$$

where $\epsilon_{d,t}$ is serially uncorrelated and normally distributed with mean zero and variance $\sigma^2_{d,t-1}$. Similarly, $\epsilon_{\bar{y},t}$ is serially uncorrelated and normally distributed with mean zero and variance $\sigma^2_{\bar{y},t-1}$. The two shocks are uncorrelated with each other and I do not restrict the stochastic properties of $\sigma^2_{d,t-1}$ and $\sigma^2_{\bar{y},t-1}$ for now. This timing of the variances is chosen so that the one-period-ahead conditional distributions of the levels remain normal with known variances. This timing is also used in the uncertainty shock literature by Bloom (2009).

1.2.3 Information structure and belief formation

I assume that agents know the structure of the model and the true values of all parameters, including those in the interest rate rule. However, they do not see the true current values of shocks. This implies that private agents cannot see the true current values of $\tilde{y}_t$ and π_t (otherwise, they can infer d_t). My preferred explanation of this setup is that it describes a situation where individuals face idiosyncratic shocks and are not aware of current aggregate conditions. They also do not see current aggregate outcomes as these are based on decisions made simultaneously by other individuals. The Appendix provides a derivation of the equilibrium conditions for aggregate variables in this type of environment and shows that the only differences are extra terms in the aggregate inflation equation which depend on the exogenous shocks $\epsilon_{d,t}$ and $\epsilon_{\bar{y},t}$. I choose not to proceed with a setup using idiosyncratic shocks in order to abstract from the issues involved with an interest rate providing public information when private agents have heterogenous information.[8]

I assume that they observe lagged state variables perfectly (perhaps through observations of lagged aggregate outcomes) which mimics the information setup used in Lucas (1973) and many subsequent papers. They also observe i_t which gives an additional piece of information about the current shocks. Formally, the information set of private agents in period t is

$$\mathcal{I}_t = \left\{ i^t, d^{t-1}, \bar{y}^{t-1}, \left(\sigma^2_d\right)^t, \left(\sigma^2_{\bar{y}}\right)^t \right\}$$

[8] Morris and Shin (2002), Angeletos and Pavan (2007), and Lorenzoni (2010) examine these issues in other settings.

Meanwhile, I assume that the central bank has perfect information about the entire history of exogenous variables up to time t. Thus, the central bank's information advantage is captured by knowledge of the current shocks $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$. A benefit of assuming that agents can see lagged true values is that it limits the signaling effect of the interest rate to current beliefs and allows me to focus on changes to the short-run incentives that are central to the optimal discretionary policy problem. I discuss the case where lagged true values cannot be seen as an extension in Section 1.6.3.

Since the shocks are AR(1) and past shocks are perfectly observed, previous observations of the interest rate do not give additional information. Beliefs are optimally formed through a static Gaussian signal extraction problem. There is a slight departure due to the dependence of the interest rate on current private agent beliefs. This introduces circularity into the belief formation problem which I resolve using the method outlined in Svensson and Woodford (2003). The basic approach is to posit a form of beliefs and then to re-express the belief formation problem in terms of errors from expectations made absent the interest rate signal. In this form, there is no circularity issue and beliefs can be found using standard signal extraction results. Here, I posit that beliefs take the form

$$
\begin{aligned}
d_{t|t} &= \rho_d d_{t-1} + K_{d,t}\left(i_t - f_d \rho_d d_{t-1} - f_{d,b} d_{t|t} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1} - f_{\bar{y},b} \bar{y}_{t|t}\right) \\
\bar{y}_{t|t} &= \rho_{\bar{y}} \bar{y}_{t-1} + K_{\bar{y},t}\left(i_t - f_d \rho_d d_{t-1} - f_{d,b} d_{t|t} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1} - f_{\bar{y},b} \bar{y}_{t|t}\right)
\end{aligned}
$$

for some $K_{d,t}, K_{\bar{y},t}$ that I will later solve for. Then, I can write the evolution of the shocks and the interest rate in terms of expectational errors defined as $x_t^{err} \equiv x_t - E\left[x_t|\mathcal{I}_t \setminus i_t\right]$. Note that this error for i_t corresponds to an interest rate surprise defined as the difference between the observed interest rate and the one expected based on all period t information *except* for the interest rate itself. Thus, I use the notation i_t^{surp} to denote this expectational error.

$$
\begin{aligned}
d_t^{err} &= \epsilon_{d,t} \\
\bar{y}_t^{err} &= \epsilon_{\bar{y},t} \\
i_t^{surp} &= \left(1 + f_{d,b}K_{d,t} + f_{\bar{y},b}K_{\bar{y},t}\right)\left(f_d\epsilon_{d,t} + f_{\bar{y}}\epsilon_{\bar{y},t}\right) \qquad (1.7)
\end{aligned}
$$

This is now a standard signal extraction problem which gives

$$
\begin{aligned}
d_{t|t}^{err} &= \frac{f_d\sigma_{d,t-1}^2}{f_d^2\sigma_{d,t-1}^2 + f_{\bar{y}}^2\sigma_{\bar{y},t-1}^2}\frac{1}{1 + f_{d,b}K_{d,t} + f_{\bar{y},b}K_{\bar{y},t}}i_t^{surp} \\
\bar{y}_{t|t}^{err} &= \frac{f_{\bar{y}}\sigma_{\bar{y},t-1}^2}{f_d^2\sigma_{d,t-1}^2 + f_{\bar{y}}^2\sigma_{\bar{y},t-1}^2}\frac{1}{1 + f_{d,b}K_{d,t} + f_{\bar{y},b}K_{\bar{y},t}}i_t^{surp}
\end{aligned}
$$

Since $x_{t|t} = x_{t|t}^{err} + E\left[x_t|\mathcal{I}_t \setminus i_t\right]$, beliefs will fit the form assumed above so that, in equilibrium, they depend on lagged true states and current shocks

$$
d_{t|t} = \rho_d d_{t-1} + \underbrace{\frac{f_d\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}}{f_d^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2} + f_{\bar{y}}^2}}_{K_{d,t}}\left(f_d\epsilon_{d,t} + f_{\bar{y}}\epsilon_{\bar{y},t}\right) \qquad (1.8)
$$

$$
\bar{y}_{t|t} = \rho_{\bar{y}}\bar{y}_{t-1} + \underbrace{\frac{f_{\bar{y}}}{f_d^2\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2} + f_{\bar{y}}^2}}_{K_{\bar{y},t}}\left(f_d\epsilon_{d,t} + f_{\bar{y}}\epsilon_{\bar{y},t}\right) \qquad (1.9)
$$

The AR(1) form of d_t and $\bar{y}_t$ then implies that $d_{t+h|t} = \rho_d^h d_{t|t}$ and $\bar{y}_{t+h|t} = \rho_{\bar{y}}^h\bar{y}_{t|t}$.

Note the following properties of $K_{d,t}$ and $K_{\bar{y},t}$:

1. $f_dK_{d,t} + f_{\bar{y}}K_{\bar{y},t} = 1$
2. $\frac{K_{d,t}}{K_{\bar{y},t}} = \frac{f_d}{f_{\bar{y}}}\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}$

The first property is equivalent to the expression

$$
f_d d_{t|t} + f_{\bar{y}}\bar{y}_{t|t} = f_d d_t + f_{\bar{y}}\bar{y}_t
$$

The linear combination on the right can be perfectly inferred through i_t so the same linear combination of their beliefs has to match the observed sum on the right. Then the belief formation process can be understood as agents observing a sum of two unknown shocks and assigning a portion of this value to each shock. The relative fraction assigned to each underlying shock depends on the relative importance of that shock in the sum. The second property shows that more of this observed sum is attributed to a demand shock when the interest rate rule responds relatively more to demand shocks ($\frac{f_d}{f_{\bar{y}}}$ is high) or when the demand shock is more variable ($\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ is high). When agents are relatively more unsure about the current demand level versus the central bank's output gap target, then they find it likely that the policy surprise is due mostly to a change in demand conditions.

1.3 Equilibrium dynamics

The model is described by a system of equations which summarize private agent optimization ((1.1) and (1.2)), policy (equation (1.4)), shock evolution ((1.5) and (1.6)), and beliefs ((1.8) and (1.9)). This system of linear stochastic difference equations can be solved by conjecturing that $\tilde{y}_t$ and π_t are linear in the true states and current private agent beliefs $\{d_t, \bar{y}_t, d_{t|t}, \bar{y}_{t|t}\}$ with unknown coefficients[9]. This allows $\tilde{y}_{t+1|t}$ and $\pi_{t+1|t}$ to be expressed in terms of current beliefs. Then, substituting (1.4) into (1.1) and (1.2) gives two equations in terms of $\{d_t, \bar{y}_t, d_{t|t}, \bar{y}_{t|t}\}$ which are used to solve for the unknown coefficients.

With this linear solution, the response of a given outcome x_t to the two structural shocks can each be broken down into three parts

$$\frac{dx_t}{d\epsilon_{\bar{y},t}} = \frac{\partial x_t}{\partial \bar{y}_t} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial x_t}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}}$$
$$\frac{dx_t}{d\epsilon_{d,t}} = \frac{\partial x_t}{\partial d_t} + \frac{\partial x_t}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{d,t}} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}}$$

[9] An interest rate rule of the form given in (1.4) will not guarantee that this equilibrium is unique. See the latter part of Corollary 3 for an illustration of how the interest rate rule can be rewritten to guarantee uniqueness while maintaining the same equilibrium behavior vis-à-vis the state variables.

The first term captures the direct effects of shocks on equilibrium conditions or the interest rate. The last two terms capture an indirect expectational effect which works through forward-looking terms in the equilibrium conditions as well as the interest rate's response to private agents' beliefs. In this model, the serially correlated nature of the state variables cause agents to form expectations of future outcomes based on today's beliefs of demand and output gap target levels. These revised expectations affect current outcomes through the standard consumption smoothing and Calvo pricing mechanisms. It is predominantly this expectational effect that is altered when information becomes imperfect. In the perfect information case, beliefs are correct so that $\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} = \frac{d\bar{y}_t}{d\epsilon_{\bar{y},t}} = 1$ and $\frac{dd_{t|t}}{d\epsilon_{d,t}} = \frac{dd_t}{d\epsilon_{d,t}} = 1$ while $\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} = \frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} = 0$. Here, these effects become

$$
\begin{aligned}
\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} &= f_{\bar{y}} K_{\bar{y},t} \in (0,1)\,, \quad \frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} = f_{\bar{y}} K_{d,t} \in \left(\frac{f_{\bar{y}}}{f_d}, 0\right) \\
\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} &= f_d K_{\bar{y},t} \in \left(\frac{f_d}{f_{\bar{y}}}, 0\right), \quad \frac{dd_{t|t}}{d\epsilon_{d,t}} = f_d K_{d,t} \in (0,1)
\end{aligned}
$$

Thus, the expectational effects of the two shocks now "spill over" into each other. When a shock hits the economy, agents observe this through an unexpected change in the interest rate. This observation does not allow them infer the source of the shock and so they update their beliefs of both the current demand level and the output gap target by a fraction of the interest rate surprise.

The marginal responses of forecasts behave similarly

$$
\begin{aligned}
\frac{dx_{t+1|t}}{d\epsilon_{\bar{y},t}} &= \rho_{\bar{y}} \left(\frac{\partial x_t}{\partial \bar{y}_t} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\right) \frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \rho_d \left(\frac{\partial x_t}{\partial d_t} + \frac{\partial x_t}{\partial d_{t|t}}\right) \frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} \\
\frac{dx_{t+1|t}}{d\epsilon_{d,t}} &= \rho_d \left(\frac{\partial x_t}{\partial d_t} + \frac{\partial x_t}{\partial d_{t|t}}\right) \frac{dd_{t|t}}{d\epsilon_{d,t}} + \rho_{\bar{y}} \left(\frac{\partial x_t}{\partial \bar{y}_t} + \frac{\partial x_t}{\partial \bar{y}_{t|t}}\right) \frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}}
\end{aligned}
$$

In the remainder of this section, I examine the comovement between current outcomes, forecasts, and interest rate surprises. The interest rate surprise defined in (1.7) is linear in $\{\epsilon_{\bar{y},t}, \epsilon_{d,t}\}$ so I can characterize the comovements using the responses to these shocks.

I build intuition for the general case by first examining two benchmark cases.

1.3.1 Benchmark 1: Perfect information with an exogenous interest rate shock

The model above can be made isomorphic to a perfect information model with an exogenous interest rate shock by allowing agents to see the current value of d_t. That is, I suppose for this subsection that the agents' information set is $\mathcal{I}_t = \left\{i^t, d^t, \bar{y}^{t-1}, \left(\sigma_d^2\right)^t, \left(\sigma_{\bar{y}}^2\right)^t\right\}$. Then, with $f_{\bar{y}} \neq 0$, the interest rate perfectly reveals $\bar{y}_t$ so that beliefs are correct in equilibrium

$$d_{t|t} = d_t \quad \text{and} \quad \bar{y}_{t|t} = \bar{y}_t$$

Interest rate behavior simplifies to

$$i_t = \left(f_d + f_{d,b}\right) d_t + \left(f_{\bar{y}} + f_{\bar{y},b}\right) \bar{y}_t$$

and the interest rate surprise is a scaled output gap target shock

$$i_t^{surp} = \left(f_{\bar{y}} + f_{\bar{y},b}\right) \epsilon_{\bar{y},t}$$

Since agents are perfectly informed after observing i_t, the resulting responses of outcomes to the interest rate surprise are the same familiar results obtained under perfect information. In other words, this case gives a model that's isomorphic to a perfect information model in which $\left(f_{\bar{y}} + f_{\bar{y},b}\right) \bar{y}_t$ is an autocorrelated exogenous component of the nominal interest rate. To get impulse responses that have the usual signs, I make the following assumption that the shocks are not too persistent

Assumption 1 $\rho_d, \rho_{\bar{y}} \in [0, \bar{\rho})$ *where* $\bar{\rho} \leq \theta$*. (See Appendix for the exact expression for* $\bar{\rho}$*.)*

Under Assumption 1, the familiar perfect information channels of a positive interest rate surprise are at work. First, it raises the current real interest rate gap which lowers the current output gap and inflation holding expectations fixed.

$$\frac{d\tilde{r}_t}{di_t^{surp}} = \frac{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)}{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)-\frac{\kappa}{\sigma}\rho_{\bar{y}}} > 0$$

Secondly, the persistent nature of the output gap target shock means that future real interest rate gaps also increase following a positive interest rate surprise.

$$\frac{d\tilde{r}_{t+h}}{di_t^{surp}} = \rho_{\bar{y}}^h \frac{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)}{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)-\frac{\kappa}{\sigma}\rho_{\bar{y}}} \geq 0$$

This contributes to lower expectations of future output gaps and inflation

$$\frac{d\tilde{y}_{t+1|t}}{di_t^{surp}} = -\rho_{\bar{y}} \frac{\frac{1}{\sigma}\left(1-\beta\rho_{\bar{y}}\right)}{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)-\frac{\kappa}{\sigma}\rho_{\bar{y}}} \leq 0$$
$$\frac{d\pi_{t+1|t}}{di_t^{surp}} = -\rho_{\bar{y}} \frac{\frac{\kappa}{\sigma}}{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)-\frac{\kappa}{\sigma}\rho_{\bar{y}}} \leq 0$$

which pushes current values down further. In sum, both the current real interest rate gap and future expectations channels push the current output gap and inflation down following a positive interest rate surprise

$$\frac{d\tilde{y}_t}{di_t^{surp}} = -\frac{\frac{1}{\sigma}\left(1-\beta\rho_{\bar{y}}\right)}{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)-\frac{\kappa}{\sigma}\rho_{\bar{y}}} < 0$$
$$\frac{d\pi_t}{di_t^{surp}} = -\frac{\frac{\kappa}{\sigma}}{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)-\frac{\kappa}{\sigma}\rho_{\bar{y}}} < 0$$

The important properties of this benchmark case which contrast with the cases below are that: (1) both the current output gap and inflation as well as agents' forecasts of future outcomes respond negatively to an interest rate surprise and (2) the responses do not vary with the relative variance $\frac{\sigma_{d,t-1}^2}{\sigma_{\bar{y},t-1}^2}$. Moreover, these responses do not depend on the values of policy response coefficients.

1.3.2 Benchmark 2: The policymaker perfectly offsets d_t

For this case, recall that fluctuations in the natural real rate only affect the equilibrium output gap and inflation if they are passed through to fluctuations in the real rate gap. The policymaker can prevent this by setting $f_d = \sigma$ and $f_{d,b} = -\sigma\rho_d$ which results in a nominal interest rate that moves one-for-one with changes in the natural real rate of interest while also responding to fluctuations in the output gap target and agents' belief about it

$$i_t = r_t^n + f_{\bar{y}}\bar{y}_t + f_{\bar{y},b}\bar{y}_{t|t}$$

This creates an equilibrium where there are no fluctuations associated with changes in the natural real rate (coming from d_t or $d_{t|t}$) and all movements are due to changes in the output gap target and agents' belief about its current level. That is,

$$\frac{\partial\tilde{y}_t}{\partial d_t} = \frac{\partial\tilde{y}_t}{\partial d_{t|t}} = \frac{\partial\pi_t}{\partial d_t} = \frac{\partial\pi_t}{\partial d_{t|t}} = 0$$

Demand shocks only affect outcomes through agents' belief about the output gap target.

Here, the responses of a given outcome x_t to the shocks become

$$\begin{aligned}\frac{dx_t}{d\epsilon_{\bar{y},t}} &= \frac{\partial x_t}{\partial\bar{y}_t} + \frac{\partial x_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} \\ \frac{dx_t}{d\epsilon_{d,t}} &= \frac{\partial x_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}}\end{aligned}$$

while the interest rate surprise is linear in the two shocks

$$i_t^{surp} = \iota_d\epsilon_{d,t} + \iota_{\bar{y}}\epsilon_{\bar{y},t}$$

Since the interest rate surprise is now made up of two independent shocks, there are two ways that I can analyze how outcomes move with interest rate surprises. I can look at the "response" of some outcome x_t to an interest rate surprise conditional on a shock to $s \in \{d, y\}$ using the ratio $\frac{dx_t/d\epsilon_{s,t}}{di_t^{surp}/d\epsilon_{s,t}}$. Alternatively, I can also look at the statistic $\frac{Cov_{t-1}\left(x_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}$ for a given outcome variable x_t. This scaled covariance is analogous to the statistic that is estimated by OLS regressions of the outcome variable on interest rate surprises with the exception that I evaluate the moments using one-period-ahead conditional distributions due to the presence of time-varying uncertainty.

I now state three additional coefficient restrictions which help me to sign responses.

Assumption 2 $f_{\bar{y}} \leq 0,\ f_{\bar{y},b} + f_{\bar{y}} \leq 0$

Assumption 3 $f_{\bar{y}} \leq 0,\ f_{\bar{y},b} \leq -\rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right) f_{\bar{y}},\ \rho_d \in \left(0, \rho_{\bar{y}}\left(1 - \beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\right)$

Assumption 4 $f_{\bar{y}} \leq 0,\ f_{\bar{y},b} \leq -\rho_{\bar{y}}\left(1 + \frac{\frac{\kappa}{\sigma}}{1-\beta\rho_{\bar{y}}}\right) f_{\bar{y}},\ \rho_d \in \left(0, \rho_{\bar{y}}\left(1 + \frac{\frac{\kappa}{\sigma}}{1-\beta\rho_{\bar{y}}}\right)\right)$

The first assumption can be understood as policy responding the "right way" to output gap target shocks. Holding constant agents beliefs, $f_{\bar{y}} < 0$ means that the nominal interest rate is reduced when the output gap target is high. Additionally, $f_{\bar{y},b} < -f_{\bar{y}}$ ensures that inflation and the output gap are increasing in the output gap target shock in the perfect information version of this model presented above. The second and third assumptions place successively tighter bounds on the nominal rate's response to private beliefs about the output gap target and analogous bounds on ρ_d which are needed to sign some of the responses below.

Turning first to the responses under each individual shock, I can show the following:

1. Under Assumption 2, $\frac{di_t^{surp}}{d\epsilon_{\bar{y},t}} = \iota_{\bar{y}} < 0 < \iota_d = \frac{di_t^{surp}}{d\epsilon_{d,t}}$
2. Under Assumptions 1 and 4, $\frac{d\tilde{y}_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} < 0$ and $\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} < 0$; both increase with $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.
3. Under Assumptions 1 and 3, $\frac{d\pi_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} < 0$ and $\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} < 0$; both increase with $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

To explain each of these, I again turn to the corresponding responses of the expected future variables and the real interest rate gap. First, under Assumptions 1 and 2:

- $\frac{d\tilde{y}_{t+1|t}/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} = \frac{d\tilde{y}_{t+1|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} \leq 0$ and approach zero as $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ increases.

- $\frac{d\pi_{t+1|t}/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} = \frac{d\pi_{t+1|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} \leq 0$ and approach zero as $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ increases.

Since the demand shock is perfectly offset, expectations of future outcomes depend only on expectations of the future output gap target level. A positive interest rate surprise originating from either underlying shock results in a weakly negative revision to this expectation which results in negative responses of output gap and inflation expectations. Forward-looking behavior in this economy means that this negatively affects current outcomes. As $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ increases, interest rate surprises result in smaller revisions to the believed value of the output gap target and so this negative effect moves towards zero.

In terms of the real interest rate gap, I can show that

- $\frac{d\tilde{r}_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}$ approaches zero as $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ increases.

- $\frac{d\tilde{r}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} \geq 0$ and remains positive as $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} \to \infty$.

Signing the effect of a higher $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ on $\frac{d\tilde{r}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ requires further restrictions on $f_{\bar{y},b}$, but on net, these two channels produce the effects on current outcomes presented above.

Turning to the scaled conditional covariance between outcomes and interest rate surprises, I obtain the following under Assumptions 1 and 2:

1. $\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} < 0$ and is increasing in $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. $\frac{Cov\left(\pi_t, i_t^{surp}\right)}{Var\left(i_t^{surp}\right)} \to 0$ as $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} \to \infty$. The same is true for the output gap.

2. $\frac{Cov\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var\left(i_t^{surp}\right)} < 0$ and is increasing in $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. $\frac{Cov\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var\left(i_t^{surp}\right)} \to 0$ as $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} \to \infty$. The same is true for output gap forecasts.

This statistic is a weighted average of the responses to individual underlying shocks so the intuition behind the signs of the individual shocks' effects underlie the sign of this statistic. An increase in $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ affects the responses to individual shocks as outlined above, but also results in greater weights on the responses to $\epsilon_{d,t}$ in this statistic. As $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} \to \infty$, $\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}$ approaches the response measured by $\frac{d\pi_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}$ which itself is zero in this limit. The same logic applies to the output gap.

The main departure from the first benchmark case above is the responses' dependence on the relative uncertainty $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. In this case, the interest rate policy is such that the true level and agents' belief about demand have no impact on current or future outcomes in equilibrium. Thus, upon observing a positive interest rate surprise, private agents attribute this partly to an increase in demand which has no effect in equilibrium, and partly to a decrease in the output gap target, which has a negative effect on current outcomes and forecasts (under appropriate coefficient restrictions). Then, the net effect is always negative but it is weaker when more of the interest rate surprise is attributed to a change in demand. With the information structure in this model, this happens when uncertainty about demand is high relative to uncertainty about the output gap target.

1.3.3 The general case

For the general case, I use the following restrictions on the interest rate's response to demand and agents' belief about the current demand level.

Assumption 5 $f_d \in (0, \infty),\ f_d + f_{d,b} \in (0, \sigma(1-\rho_d))$

Assumption 6 $f_d \in (0, \infty),\ f_d + f_{d,b} \in \left(0, \sigma\left(\frac{\frac{\kappa}{\sigma}\rho_d}{(1-\rho_d)(1-\beta\rho_d)} - \rho_d\right)\right)$

The additional feature present under Assumption 5 is that the policy response to demand shocks is not strong enough. Then, positive changes in true demand and agents' belief about it retain expansionary effects in equilibrium. This allows the model to produce positive responses of current and expected outcomes to positive interest rate surprises.

Proposition 1 *Given Assumptions 1, 2, and 5*

1. $\frac{di_t^{surp}}{d\epsilon_{\bar{y},t}} = \iota_{\bar{y}} < 0 < \iota_d = \frac{di_t^{surp}}{d\epsilon_{d,t}}$

2. $\frac{d\tilde{y}_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}$ *and* $\frac{d\pi_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}$ *can both be positive for large* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

3. $\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ *and* $\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ *can both be positive for large* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ *under Assumption 6.*

4. $\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}$ *is increasing in* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ *and can be positive for a large enough* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. *The same is true for the output gap.*

5. $\frac{d\tilde{y}_{t+h|t}/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} = \frac{d\tilde{y}_{t+h|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ *can be positive and are increasing in* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

6. $\frac{d\pi_{t+h|t}/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} = \frac{d\pi_{t+h|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ *can be positive and are increasing in* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

7. $\frac{Cov\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var\left(i_t^{surp}\right)}$ *is increasing in* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ *and can be positive for a large enough* $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$. *The same is true for output gap forecasts.*

Proof. See Appendix. ■

Again, the signs of effects on current outcomes can be understood by looking at the effects on one-period-ahead expectations and the real interest rate gap:

- $\frac{d\tilde{r}_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}}$ can be negative for large enough $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.
- $\frac{d\tilde{r}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ can be negative for large enough $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ under Assumption 6.

The part of the interest rate surprise that agents interpret as a demand increase now has a positive effect on current outcomes and forecasts. When uncertainty about demand is relatively high, this positive part of the interest rate surprises' signaling effect on current outcomes and forecasts is large so the total response can become positive.

This mechanism has been discussed as one reason behind the expansionary responses of inflation and unemployment forecasts to positive interest rate surprises found in Romer and Romer (2000) and Campbell, Evans, Fisher, and Justiniano (2012). The theory presented here also implies that this is particularly likely to be the case when (i) the policy response to fundamental shocks is inadequate and (ii) private agents are relatively more uncertain about the strength of the economy than they are about policy objectives. The recent recession was a period of time where these conditions were plausibly present since the federal funds target effectively reached zero at the end of 2008 and there is also evidence of high economic uncertainty prior to and during the recession, such as the influential work by Bloom (2009). Tang (2014a) also presents new empirical evidence that the response of inflation forecasts to interest rate surprises does indeed have a significant interaction with forecasters' subjective uncertainty.

1.4 Optimal discretionary interest rate policy

In this section, I turn to the question of optimal discretionary interest rate policy. For now, I do not allow the central bank to directly communicate their additional information to the public aside from the information embodied in the interest rate. To retain tractability, I limit attention to the case where variances are constant parameters and consider comparative statics with respect to the relative variance $\frac{\sigma_d^2}{\sigma_{\tilde{y}}^2}$. I discuss the implications of time-varying uncertainty for the optimal policy problem in Section 1.6.4. I also assume that the constant wage bill subsidy τ offsets the average monopolist pricing inefficiency so that the steady state is undistorted. Then, a second-order log approximation around the deterministic steady state gives that the consumers' lifetime utility from date t_0 onwards is proportional to

$$\mathbb{U}_{t_0,\infty} = -\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\tilde{y}_t^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right) + h.o.t.$$

where I've omitted constants and terms independent of policy.

I then consider a monetary authority that maximizes welfare derived from consumer utility but with an exogenous time-varying target for the output gap. A similar time-varying target has been used in other papers studying optimal policy in an imperfect information context such as Mertens (2011) and Faust and Svensson (2001). My preferred interpretation of this shock is that it summarizes short-run deviations of the efficient level of output from the natural flexible-price level of output which are not captured by the above microfoundations. Then, $\tilde{y}_t - \bar{y}_t$ represents the deviation of actual output from the efficient level. The policymaker's objective is to minimize the following loss

$$\mathcal{L}_{t_0} = E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right) \tag{1.10}$$

where the expectation is evaluated according to his own information set[10].

In the imperfect information case, a policymaker who cannot commit chooses the interest rate level in each period to minimize this loss subject to equilibrium conditions (1.1) and (1.2) and taking private agents' beliefs regarding future policy and the form of current policy as given.

Beliefs regarding future policy affect the expectations $\left\{ \tilde{y}_{t+1|t}, \pi_{t+1|t} \right\}$. Since the equilibrium of this model is linear in $\left\{ d_t, d_{t|t}, y_t, y_{t|t} \right\}$ while beliefs satisfy $d_{t+1|t} = \rho_d d_{t|t}$ and $\bar{y}_{t+1|t} = \rho_{\bar{y}} \bar{y}_{t|t}$, these expectations can be written in matrix form as

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} d_{t+1|t} \\ \bar{y}_{t+1|t} \end{bmatrix} \tag{1.11}$$

In equilibrium, the coefficients in the matrix $\mathbf{M}$ are determined by the behavior of future nominal interest rates. Then, taking private agents' beliefs about future policy as given amounts to the policymaker recognizing that his current choice does not have an effect on this $\mathbf{M}$ matrix. However, the policymaker does recognize that his choice impacts $\left\{ d_{t+1|t}, \bar{y}_{t+1|t} \right\}$

[10] The model equations can be rearranged into the canonical form studied in Clarida, Galí, and Gertler (1999) where the output gap target shock shows up as both a positive cost-push shock and a negative component of the demand shock.

$$L_{t_0} = E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\left(\tilde{y}_t^{CB} \right)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right)$$
$$\tilde{y}_t^{CB} = \tilde{y}_{t+1|t}^{CB} - \frac{1}{\sigma} \left[i_t - \pi_{t+1|t} - r_t^{CB} \right]$$
$$\pi_t = \beta \pi_{t+1|t} + \kappa \tilde{y}_t^{CB} + v_t$$
$$\text{where } \tilde{y}_t^{CB} \equiv \tilde{y}_t - \bar{y}_t, \; r_t^{CB} = \sigma \left[(d_t - \bar{y}_t) - \left(d_{t+1|t} - \bar{y}_{t+1|t} \right) \right] \quad \text{and} \quad v_t = \kappa \bar{y}_t$$

and therefore has a marginal effect on current outcomes through $\left\{\tilde{y}_{t+1|t}, \pi_{t+1|t}\right\}$. This is in contrast to the discretionary policy problem under perfect information where the interest rate level chosen today has zero impact on these expectations.

Unlike the perfect information case, private agents' beliefs about the form of current policy is now relevant since it determines private agents' belief formation process. When private agents suppose that the behavior of the current interest rate can be described by

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{y}} \bar{y}_t + f_{\bar{y},b} \bar{y}_{t|t} \tag{1.12}$$

then beliefs follow

$$d_{t|t} = \rho_d d_{t-1} + K_d i_t^{surp} \tag{1.13}$$

$$\bar{y}_{t|t} = \rho_{\bar{y}} \bar{y}_{t-1} + K_{\bar{y}} i_t^{surp} \tag{1.14}$$

$$\text{where } i_t^{surp} = i_t - f_d \rho_d d_{t-1} - f_{d,b} d_{t|t} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1} - f_{\bar{y},b} \bar{y}_{t|t}$$

as shown above, where K_d and $K_{\bar{y}}$ take the forms given in (1.8) and (1.9) with $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$ now being constant. To get around the circularity issue introduced by the interest rate surprise i_t^{surp} itself being a function of beliefs, I redefine the policy problem as a choice of a component of the interest rate i_t^{dis} where the realized nominal rate is

$$i_t = i_t^{dis} + f_{d,b} d_{t|t} + f_{\bar{y},b} \bar{y}_{t|t}$$

Since the policymaker is free to choose any value of i_t^{dis}, this still gives him full control over the resulting behavior of i_t and so it does not impose any additional constraint on the policy problem. The benefit of this relabeling is that beliefs can now be written neatly as a function of i_t^{dis} and lagged exogenous state variables.

$$d_{t|t} = \rho_d d_{t-1} + K_d \left(i_t^{dis} - f_d \rho_d d_{t-1} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1}\right)$$

$$\bar{y}_{t|t} = \rho_{\bar{y}} \bar{y}_{t-1} + K_{\bar{y}} \left(i_t^{dis} - f_d \rho_d d_{t-1} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1}\right)$$

Then, a policymaker who takes private agents' beliefs about current policy as given considers a change in i_t^{dis} to have marginal effects of K_d and $K_{\bar{y}}$ on beliefs $d_{t|t}$ and $\bar{y}_{t|t}$, respectively.

To summarize, a policymaker who can only choose the interest rate level today and cannot make credible commitments about future policy does not internalize the effect of equilibrium interest rate behavior on the following objects: (i) the $\mathbf{M}$ matrix which captures the relationship between beliefs about state variables and expectations $\left\{\tilde{y}_{t+1|t}, \pi_{t+1|t}\right\}$ as well as (ii) the belief coefficients K_d and $K_{\bar{y}}$ which capture the marginal effects of the interest rate on beliefs. This is consistent with the notion that the policymaker chooses the current *level* of the nominal interest rate but cannot commit to implementing a particular interest rate *rule.* The main difference from the perfect information discretionary policy problem is that the policymaker recognizes that he can influence expectations of future outcomes through the beliefs in the vector $\left[d_{t|t} \quad \bar{y}_{t|t}\right]'$ in equation (1.11).

Because the policymaker minimizes a quadratic loss function subject to linear constraints of the same form in each period, the optimal interest rate ends up having the same form as (1.12). Solving for an equilibrium under optimal policy then consists of finding a solution to the set of linear stochastic difference equations given by (1.1), (1.2), (1.5), (1.6), (1.13), (1.14), and the policymaker's optimality condition.

Proposition 2 *The policymaker's optimality condition is*

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}\frac{\varepsilon}{\kappa}\pi_t \tag{1.15}$$

$$\text{where } \mathcal{R} \equiv \frac{\frac{d\pi_t}{di_t^{dis}}}{\frac{d\tilde{y}_t}{di_t^{dis}}} = \frac{\frac{\partial \pi_t}{\partial i_t^{dis}} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}}K_{\bar{y}}}{\frac{\partial \tilde{y}_t}{\partial i_t^{dis}} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}}K_{\bar{y}}} \text{ in equilibrium}$$

$\mathcal{R}$ is itself a function of interest rate response coefficients and is therefore determined in equilibrium. There may be multiple equilibrium values for $\mathcal{R}$ but those that satisfy $\mathcal{R} \geq 0$ exhibit the following properties when $\beta\rho_{\bar{y}} > 0$:

1. $\mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$

2. $\mathcal{R}$ is decreasing in $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$

- *As $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to \infty$, $K_{\bar{y}} \to 0$ and $\mathcal{R} \to \kappa$. In this limit, the interest rate has no effect on $\bar{y}_{t|t}$ and the optimality condition for policy becomes equivalent to that in the case of optimal discretionary policy when agents have perfect information.*
- *As $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$, $K_{\bar{y}} \to \frac{1}{f_{\bar{y}}}$ and $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$. In this limit, the interest rate has its largest possible effect on $\bar{y}_{t|t}$ and the optimality condition for policy becomes equivalent to that in the case of commitment to a rule of the form*

$$i_t = r_t^n + f_{\bar{y}}^c \bar{y}_t + f_{\bar{y},b}^c \bar{y}_{t|t}$$

3. When $\beta = 0$ or $\rho_{\bar{y}} = 0$, $\mathcal{R} = \kappa$ in equilibrium for any value of $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$.

This optimal policy solution is unique under any initial supposed private sector belief about current policy that results in beliefs $d_{t|t}$ and $\bar{y}_{t|t}$ that are linear in i_t^{dis}. More specifically, the same solution is obtained if (1.12) is replaced with a belief that the current interest rate may also respond linearly to the entire history of past fundamentals.

Proof. See Appendix. ■

The optimal policy results in this environment can be understood by noting that the signaling channel tilts the policymaker's short-run tradeoff between inflation and deviations of the output gap from its target. To better understand this, note that the policymaker's problem can be recast as one in which he chooses $\tilde{y}_t$ since there is a one-to-one mapping between the nominal interest rate and $\tilde{y}_t$ through equation (1.1). Then, the only remaining constraint imposed on the policymaker is the second equilibrium condition, equation (1.2), which I rewrite here in terms of the output gap deviation from its target.

$$\pi_t = \beta\pi_{t+1|t} + \kappa\left(\tilde{y}_t - \bar{y}_t\right) + \kappa\bar{y}_t$$

This New Keynesian Phillips curve then summarizes the policymaker's tradeoff between π_t

and $\tilde{y}_t - \bar{y}_t$. In the perfect information setting, the discretionary policymaker has no impact on $\pi_{t+1|t}$. Therefore, the slope of this constraint is

$$\mathcal{R}^{PI} = \frac{\partial \pi_t / \partial i_t^{dis}}{\partial \tilde{y}_t / \partial i_t^{dis}} = \kappa$$

When the policymaker has an information advantage, the nominal interest rate now impacts the expectation $\pi_{t+1|t}$ through the belief $y_{t|t}$ since the policymaker recognizes that

$$\pi_{t+1|t} = M_{22} \rho_{\bar{y}} \bar{y}_{t|t} \text{ and } \frac{d\bar{y}_{t|t}}{di_t^{dis}} = K_{\bar{y}}$$

where M_{22} is the lower right element of the matrix $\mathbf{M}$ that appears in (1.11). This changes the slope of the policymaker's constraint to

$$\mathcal{R} = \frac{\frac{\partial \pi_t}{\partial i_t^{dis}} + \frac{\partial \pi_t}{\partial d_{t|t}} \frac{dd_{t|t}}{di_t^{dis}} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}} \frac{d\bar{y}_{t|t}}{di_t^{dis}}}{\frac{\partial \tilde{y}_t}{\partial i_t^{dis}} + \frac{\partial \tilde{y}_t}{\partial d_{t|t}} \frac{dd_{t|t}}{di_t^{dis}} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}} \frac{d\bar{y}_{t|t}}{di_t^{dis}}}$$

Thus, the policymaker's optimality condition retains the same form as the perfect information setting where the goal is to maintain an optimal ratio between output and inflation deviations. The key difference is that the slope $\mathcal{R}$ governing this ratio now depends crucially on the size of the effects that the interest rate has on beliefs.

In equilibrium, $\mathcal{R}$ depends only on the effect that interest rates have agents' belief about the output gap target and not their belief about demand. This is because the policymaker perfectly offsets the effects of changes in the belief about demand on outcomes so that $\frac{\partial \pi_t}{\partial d_{t|t}} = \frac{\partial \tilde{y}_t}{\partial d_{t|t}} = 0$ in equilibrium. Then, the interest rate still affects $d_{t|t}$, but inflation expectations are not ultimately affected through this channel. On the other hand, changes in the true level and belief about the output gap target will affect inflation expectations under the optimal policy. Thus, what ultimately matters for optimal policy is how much influence the policymaker has on this belief.

Solving for the equilibrium value of $\mathcal{R}$ reveals that $\mathcal{R} \geq \kappa$, meaning that it's optimal to maintain smaller inflation deviations relative to output deviations when policy has a larger signaling effect on $\bar{y}_{t|t}$. This reduces the usual stabilization bias that occurs in perfect information New Keynesian models where short-run inflation fluctuations are inefficiently large when a policymaker is not able to commit. As uncertainty about the output gap target grows relative to uncertainty about demand shocks, policy's signaling effect on $\bar{y}_{t|t}$ becomes larger and this stabilization bias is further reduced. In a more general setting where there may be additional shocks to the rate-setting process, the key measures are uncertainty about the output gap target relative to uncertainty about all other unobserved components of i_t.

At the limits of the interest rate's influence on beliefs, the optimal discretionary policy in this imperfect information model corresponds with some familiar benchmarks. When $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to \infty$, the interest rate has no effect on beliefs about the output gap target shock and the optimal discretionary policy under imperfect information coincides with that under perfect information. When $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$, the interest rate has its largest possible effect on beliefs about the output gap target shock and the optimal discretionary policy coincides with the optimal policy when the policymaker can commit to an interest rate rule of the form given above. In other words, there is no benefit to this type of commitment at this limit.

In this particular example, the optimal discretionary policy at this limit also coincides with the optimal policy under perfect information when the policymaker can commit to a rule of the form considered in section 4.2.1 of Clarida, Galí, and Gertler (1999) which is

$$i_t = r_t^n + f_{\bar{y}}^c \bar{y}_t$$

Lastly, there are two special cases where the equilibrium ratio $\mathcal{R}$ does not depend on relative variances levels. This happens when $\rho_{\bar{y}} = 0$ or $\beta = 0$.

1. In the $\rho_{\bar{y}} = 0$ case, the output gap target becomes white noise so expectations about future levels are always zero. The policymaker only affects agents' belief about the current output gap target which has no direct impact on current outcomes.

2. In the case of $\beta = 0$, inflation expectations no longer affect the current policy tradeoff

since prices are set by firms who no longer take the future into account. Note that the key discount factor that β is capturing in this special case is the one used by firms in their price-setting decision. This result still holds if I assume that consumers, and hence the central bank, maintain a positive discount factor different from the firms'.

The stationary equilibrium under this optimality condition features an output gap and inflation which are linear in $\bar{y}_t$ and $\bar{y}_{t|t}$

$$\tilde{y}_t - \bar{y}_t = -\frac{\mathcal{R}\varepsilon}{1+\mathcal{R}\varepsilon}\bar{y}_t - \frac{\mathcal{R}\varepsilon\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}\bar{y}_{t|t} \tag{1.16}$$

$$\pi_t = \frac{\kappa}{1+\mathcal{R}\varepsilon}\bar{y}_t + \frac{\kappa\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}\bar{y}_{t|t} \tag{1.17}$$

The next result characterizes the interest rate which implements this equilibrium.

Corollary 3 *A nominal interest rate which can implement this policy is given by*

$$i_t^* = r_t^n + f_{\bar{y}}^*\left(\mathcal{R}\right)\bar{y}_t + f_{\bar{y},b}^*\left(\mathcal{R}\right)\bar{y}_{t|t}$$

The interest rate moves one-for-one with the natural rate of interest while $f_{\bar{y}}^*$ *and* $f_{\bar{y},b}^*$ *are functions of* $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$ *through* $\mathcal{R}$*. This interest rate behavior matches that assumed in the second benchmark case above with coefficients on* $\bar{y}_t$ *and* $\bar{y}_{t|t}$ *that satisfy Assumption 3. The exact expressions for the functions* $f_{\bar{y}}^*(\cdot)$ *and* $f_{\bar{y},b}^*(\cdot)$ *are given in the Appendix.*

This can be compared to the nominal interest rate under optimal discretionary policy in the perfect information case which can be written as

$$i_t^{*,PI} = r_t^n + \left(f_{\bar{y}}^*\left(\kappa\right) + f_{\bar{y},b}^*\left(\kappa\right)\right)\bar{y}_t$$

To ensure unique implementation, the interest rate specification can be augmented by a term that reacts more than one-for-one to deviations of inflation from its intended path

$$i_t^* = r_t^n + \left(f_{\bar{y}}^*\left(\mathcal{R}\right) - \phi_\pi\Gamma_{\bar{y}}\right)\bar{y}_t + \left(f_{\bar{y},b}^*\left(\mathcal{R}\right) - \phi_\pi\Gamma_{\bar{y},b}\right)\bar{y}_{t|t} + \phi_\pi\pi_t$$

where $\Gamma_{\bar{y}}, \Gamma_{\bar{y},b}$ are the coefficients on $\bar{y}_t$ and $\bar{y}_{t|t}$ in the equilibrium solution for π_t. Choosing $\phi_\pi > 1$ ensures that the intended equilibrium is the unique solution in the system of equations defined by (1.1), (1.2), (1.5), (1.6), (1.13), (1.14), and this interest rate rule.

Proof. See Appendix. ■

A necessary element in these results is that the policymaker has an information advantage regarding an outcome-relevant state variable that has some persistence. I use the term "outcome-relevant" to mean that it creates an inflation-output tradeoff and therefore affects equilibrium outcomes under the optimal policy. This provides the channel through which the current interest rate level can affect expectations $\{\tilde{y}_{t+1|t}, \pi_{t+1|t}\}$. Without a state variable that has these features, optimal policy becomes invariant to the signaling channel.

To be precise, consider a model analogous to the one proposed above but with a more general set of shocks. I denote the set of exogenous state variables with a vector $\mathbf{z}_t$ that evolves as a VAR(1) process with independent shocks

$$\mathbf{z}_t = \boldsymbol{\Upsilon}\mathbf{z}_{t-1} + \mathbf{e}_t,\ \mathbf{e}_t \sim \text{iid } N(0, \boldsymbol{\Sigma}) \text{ where } \boldsymbol{\Sigma} \text{ is diagonal}$$

I partition this vector into two subvectors $\mathbf{z}_{1,t}, \mathbf{z}_{2,t}$ where $\mathbf{z}_{1,t}$ is perfectly observed by private agents while they can only see the true value of $\mathbf{z}_{2,t}$ with a lag. I also restrict $\boldsymbol{\Upsilon}$ so that $\mathbf{z}_{1,t}$ does not depend on lags of $\mathbf{z}_{2,t-1}$ (i.e., $\Upsilon_{12} = 0$) and assume that the eigenvalues of $\boldsymbol{\Upsilon}$ are less than one in absolute value.

Again, the central bank's information advantage is that they can observe the current $\mathbf{z}_{2,t}$ while private agents cannot. I then let private agents suppose that the interest rate i_t is linear in $\{\mathbf{z}_{1,t}, \mathbf{z}_{2,t}, \mathbf{z}_{2,t|t}\}$ which is the case under the optimal discretionary policy. Let the equilibrium conditions in this model be

$$\tilde{y}_t^{CB} = \tilde{y}_{t+1|t}^{CB} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right) + \boldsymbol{\Xi}_{\tilde{y}}\mathbf{z}_t$$

$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t^{CB} + \boldsymbol{\Xi}_\pi\mathbf{z}_t$$

where I now use $\tilde{y}_t^{CB}$ to denote the welfare-relevant output gap under this alternate configuration of shocks. Then, I obtain the following

Proposition 4 *Suppose that the shocks in $\mathbf{z}_{2,t}$ do not impose an output-inflation tradeoff. That is, suppose that $\Xi_\pi \mathbf{z}_t = \Xi_{\pi,1}\mathbf{z}_{1,t}$ so that only shocks in $\mathbf{z}_{1,t}$ enter into the inflation equilibrium condition. Then the equilibrium under the discretionary optimal policy features $\frac{d\tilde{y}_t^{CB}}{d\mathbf{z}_{2,t}} = \frac{d\pi_t}{d\mathbf{z}_{2,t}} = \frac{d\tilde{y}_t^{CB}}{d\mathbf{z}_{2,t|t}} = \frac{d\pi_t}{d\mathbf{z}_{2,t|t}} = 0$ while the policymaker's optimality condition becomes the same as the perfect information case*

$$\tilde{y}_t^{CB} = -\varepsilon \pi_t$$

Proof. See Appendix. ■

In the language of New Keynesian models, this result show that if the policymaker only has an information advantage regarding demand or natural real interest rate shocks while not having superior knowledge regarding cost-push-type shocks, then the policymaker optimally maintains the same ratio between output gap and inflation deviations as in the perfect information case. While changes in the interest rate still have an effect on private agents' beliefs $\mathbf{z}_{2,t|t}$, the presence of this signaling channel does not impact optimal discretionary policy when the information advantage is limited to this class of shocks.

1.5 The value of information

In this section, I consider whether it would be beneficial for the policymaker to directly communicate information to private agents. I will first compare the welfare losses under the two extremes of no communication and full communication. Later on in this section, I examine the case of partial communication.

The no communication case is the one analyzed above where the policymaker can only choose the interest rate under the given asymmetric information structure. Under full communication, the central bank costlessly and noiselessly discloses the true values of both current exogenous states $\{d_t, \bar{y}_t\}$ to all private agents so that the setting is equivalent to the standard perfect information case. In each of these cases, I presume that the central bank is implementing the optimal discretionary interest rate policy.

The loss under no communication can be evaluated using the equilibrium shown in the previous section. Meanwhile, optimal discretionary policy under full communication is

$$\tilde{y}_t^{PI} - \bar{y}_t = -\varepsilon \pi_t^{PI}$$

Substituting this into (1.2) and solving forward gives the equilibrium solutions

$$\tilde{y}_t^{PI} - \bar{y}_t = \frac{-\varepsilon\kappa}{1 - \beta\rho_{\bar{y}} + \varepsilon\kappa}\bar{y}_t \quad \text{and} \quad \pi_t^{PI} = \frac{\kappa}{1 - \beta\rho_{\bar{y}} + \varepsilon\kappa}\bar{y}_t$$

The period t welfare loss consists of a current period loss and an expected future loss

$$\mathcal{L}_t = \underbrace{\frac{1}{2}\left[(\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right]}_{l_t} + \underbrace{E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-t}\frac{1}{2}\left((\tilde{y}_s - \bar{y}_s)^2 + \frac{\varepsilon}{\kappa}\pi_s^2\right)}_{\beta E_t^{CB}\mathcal{L}_{t+1}}$$

Proposition 5 *Under an equilibrium where $\mathcal{R} \geq 0$,*

1. *The expected future loss is always higher under full communication*

$$E_t^{CB}\mathcal{L}_{t+1} \leq E_t^{CB}\mathcal{L}_{t+1}^{PI}$$

2. *The current period loss under no communication may be higher or lower than the full communication case. The difference depends on the current realizations of shocks $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$.*

Proof. See Appendix. ■

The gains from no communication relative to full communication comes from two sources. The first is the reduction in the stabilization bias when the interest rate's signaling effect on inflation expectations leads a discretionary policymaker to be tougher on inflation. The second benefit comes from imperfect information resulting in smaller inflation and output fluctuations even absent a reduction in the stabilization bias. To understand this better, first note that the policymaker is always able to fully offset the effects of changes in demand. Now, consider a positive shock to the output gap target which leads the policymaker to boost output by lowering the interest rate. The inflation fluctuations created by this action depend on both its impact on firms' current marginal costs as well as their forecasts of future marginal costs where the latter depends on firms' beliefs. In the perfect information setting, these components move in tandem since they both depend only on the true output gap target. When firms are imperfectly informed, their forecasts of future marginal costs depend on their beliefs about the output gap target which now moves less than one-for-one with true output gap target shocks while now also moving with demand shocks. Thus, for a given deviation of output away from its efficient level, the resulting inflation fluctuation is now spread across both shocks and ends up being smaller on average. As an extreme example, suppose that after setting the interest rate, the central bank can independently manipulate beliefs by choosing any value of $\bar{y}_{t|t}$. Then, it's clear from the equilibrium in (1.16) and (1.17) that it's always optimal to choose $\bar{y}_{t|t}$ in a way that offsets $\bar{y}_t$. Maintaining imperfect information helps the policymaker to get closer to this ideal.

I can also show that these two benefits of no communication operate independently.

Corollary 6 *To isolate the benefit from an interest rate policy that now exhibits a smaller stabilization bias, I exogenously impose that $\bar{y}_{s|s} = \bar{y}_s$ for $s > t$ in evaluating the welfare losses. In this case,*

$$E_t^{CB}\mathcal{L}_{t+1} \leq E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \textit{for } \mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$$

To isolate the benefit of beliefs that do not correlate perfectly with true states, I exogenously *impose $\mathcal{R} = \kappa$. In this case,*

$$E_t^{CB} \mathcal{L}_{t+1} \leq E_t^{CB} \mathcal{L}_{t+1}^{PI}$$
$$\text{when } Var_t^{CB}\left(\bar{y}_{s|s}\right) \leq Var_t^{CB}\left(\bar{y}_s\right) \text{ and } Corr_t^{CB}\left(\bar{y}_{s|s}, \bar{y}_s\right) \leq 1 \text{ for } s > t$$

which is satisfied in this model.

Proof. See Appendix. ■

As a second exercise, I now consider partial communication where the central bank perfectly communicates the true value of one of the current exogenous states to private agents. The true value of the remaining exogenous state is then perfectly inferred from the interest rate so that all agents are perfectly informed in equilibrium as in the full communication case. The key difference from the full communication case is that the interest rate retains a signaling effect on private agents' beliefs since it is used to infer the remaining exogenous state which was not directly communicated.

I will first consider the case of the central bank communicating the true current state of demand to agents. Then their belief about the current level of the output gap target is inferred from the interest rate as

$$\bar{y}_{t|t} = \frac{1}{f_{\bar{y}}}\left(i_t^{dis} - f_d d_t\right)$$

Thus, a discretionary policymaker still faces a signaling effect of $K_{\bar{y}} \equiv \frac{d\bar{y}_{t|t}}{di_t^{dis}} = \frac{1}{f_{\bar{y}}}$ when choosing the interest rate though private agents' beliefs will be correct in equilibrium. This maximizes the marginal effect of the discretionary policymaker's interest rate choice on inflation expectations and results in an inflation-output tradeoff characterized by $\mathcal{R} = \frac{\kappa}{1-\beta\rho_{\bar{y}}}$. This achieves the largest possible reduction in the stabilization bias through the signaling channel and raises welfare compared to both the no communication and full communication cases. However, because agents are perfectly informed in equilibrium, beliefs about the output gap target will now move in sync with true shocks which lowers welfare compared to the no communication case. On net, partial communication of only the demand shock is always preferable to full communication but is not unambiguously preferable to no communication.

Proposition 7 *Under an equilibrium where $\mathcal{R} \geq 0$ and with partial communication of only the demand shock denoted by a ^{d} superscript,*

1. *Both the current and expected future welfare losses are higher under full communication than under partial communication of only the demand shock*

$$E_t^{CB}\mathcal{L}_{t+1}^d \leq E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \textit{and} \quad l_t^d \leq l_t^{PI} \quad \textit{for any realization of shocks } \{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$$

2. *The expected future welfare loss under no communication may be higher or lower than under partial communication of only the demand shock. The difference cannot be unambiguously signed and depends on parameter values.*

3. *The current period loss under no communication may be higher or lower than under partial communication of only the demand shock. The difference depends on the current realizations of shocks $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$ even for a fixed set of parameter values.*

Proof. See Appendix. ■

Partial communication of only the true current output gap target results in the same optimal discretionary interest rate policy and welfare loss as full communication. In this case, the interest rate's signaling effect is only on agents beliefs about demand. As discussed in Section 1.4, demand shocks are perfectly offset by the policymaker and do not affect inflation in equilibrium. Therefore, the interest rate does not have a signaling effect on inflation expectations through beliefs about demand which results in no reduction of the stabilization bias.

The fact that the current period loss is not unambiguously lower under either no communication or partial communication of only the demand shock implies that this choice features time inconsistency. For a fixed set of parameter values, the central bank always wants to commit to one of these communication policies for future periods. However, there may be realizations of shocks that make the alternate communication policy preferable after taking into account current welfare, which would go against the policymakers' commitment. This property also suggests that a full analysis of optimal discretionary communication policy in this setting would involve private agents' beliefs that are formed by a non-Gaussian signal extraction problem. When it's optimal for the policymaker to communicate only in certain states, then a decision to withhold information is itself informative.

1.6 Extensions

1.6.1 Adding more structural shocks

In this section, I explore how the above results may change in environments with a richer set of structural shocks. The optimal discretionary policy is affected by the existence of a signaling channel only through a change in the slope of the short-run inflation-output tradeoff which, in turn, determines the optimal ratio maintained between output gap and inflation deviations. An immediate consequence of this property is that the interest rate should still perfectly offset shocks that affect only the natural real rate of interest regardless of whether the policymaker possesses an information advantage on these shocks.

On the other hand, the presence of additional cost-push-type shocks, which the policymaker cannot perfectly offset, produces more interesting results. First, consider the case of adding a shock v_t to the firms' price-setting equation so that it becomes

$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t + v_t$$

$$\text{where } v_t = \rho_v v_{t-1} + \epsilon_{v,t} \text{ with } \epsilon_{v,t} \sim \text{iid } N\left(0, \sigma_v^2\right) \text{ and } \rho_v \in [0, \bar{\rho})$$

I first assume that both private agents and the policymaker can see the entire history v^t at time t so that the policymaker has no information advantage regarding this shock. Then, I obtain the following

Proposition 8 *The optimal interest rate under discretionary policy with an additional cost-push shock which the policymaker does not have an information advantage for is*

$$i_t^* = r_t^n + f_{\bar{y}}^* (\mathcal{R}) \, \bar{y}_t + f_{\bar{y},b}^* (\mathcal{R}) \, \bar{y}_{t|t} + f_v^* (\mathcal{R}) \, v_t$$

where $\mathcal{R}$ depends on underlying parameters in the same way as in the baseline model.

This can be compared to the optimal interest rate under perfect information

$$i_t^{*,PI} = r_t^n + \left(f_{\bar{y}}^* (\kappa) + f_{\bar{y},b}^* (\kappa) \right) \bar{y}_t + f_v^* (\kappa) \, v_t$$

The expression for the function $f_v^* (\cdot)$ is given in the Appendix.

Proof. See Appendix. ■

Despite the policymaker not having an information advantage about the cost-push shock v_t, the optimal response to this shock is still influenced by the signaling effect that the interest rate has on private agents' belief about the output gap target. The presence of that signaling effect tilts the short-run inflation-output tradeoff in a way that leads the policymaker to enforce smaller inflation deviations *conditional on any shock* to the economy.

Another result of adding a cost-push shock is that the optimal discretionary policy in the limit when the interest rate has its largest effect on expectations no longer corresponds to the optimal commitment to a rule of the form

$$i_t = r_t^n + f_{\bar{y}}^c \bar{y}_t + f_{\bar{y},b}^c \bar{y}_{t|t} + f_v^c v_t$$

in this limit. The Appendix shows that an optimal commitment to this type of rule implies the same response coefficients for $\bar{y}_t$ and $\bar{y}_{t|t}$ but a different response to v_t given by

$$f_v^{*,c} = f_v^* \left(\frac{\kappa}{1 - \beta \rho_v} \right) \neq f_v^* \left(\frac{\kappa}{1 - \beta \rho_{\bar{y}}} \right)$$

where the last term is the optimal discretionary response to v_t in this limit as $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$. Since $f_v^*(\cdot)$ is increasing in its argument, then if $\rho_v < \rho_{\bar{y}}$, the policymaker operating without commitment actually chooses an interest rate that *overreacts* to the cost-push shock v_t relative to the policymaker who can commit to a rule of the form given above. Due to this overreaction, it's possible for full communication to be welfare-improving in this case depending on the relative importance of the different shocks.

I can also consider the case where the policymaker has an information advantage about v_t in addition to $\{d_t, \bar{y}_t\}$. Moreover, beliefs are formed under the following supposed current interest rate behavior which replaces equation (1.12)

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{y}} \bar{y}_t + f_{\bar{y},b} \bar{y}_{t|t} + f_v v_t + f_{v,b} v_{t|t}$$

Now there are three private agent beliefs $\left\{d_{t|t}, \bar{y}_{t|t}, v_{t|t}\right\}$ all of which are linear in i_t^{dis}. If I define $K_v \equiv \frac{dv_{t|t}}{di_t^{dis}}$, then the optimal discretionary policy can be shown to be equivalent to the one derived above in the baseline model with the exception that now, the equilibrium $\mathcal{R}$ depends on K_v as follows:

$$\mathcal{R} \equiv \frac{\frac{d\pi_t}{di_t^{dis}}}{\frac{d\tilde{y}_t}{di_t^{dis}}} = \frac{\frac{\partial \pi_t}{\partial i_t^{dis}} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}} K_{\bar{y}} + \frac{\partial \pi_t}{\partial v_{t|t}} K_v}{\frac{\partial \tilde{y}_t}{\partial i_t^{dis}} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}} K_{\bar{y}} + \frac{\partial \tilde{y}_t}{\partial v_{t|t}} K_v}$$

where $K_{\bar{y}}$ and K_v will now depend on $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$, $\frac{\sigma_v^2}{\sigma_{\bar{y}}^2}$, and the policy coefficients.

1.6.2 Time-varying inflation target

Here, I will show the case of an inflation target $\bar{\pi}_t$ rather than the time-varying output gap target. That is, suppose that the policy objective in (1.10) is replaced with

$$\mathcal{L}_{t_0} = E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\tilde{y}_t^2 + \frac{\varepsilon}{\kappa} \left(\pi_t - \bar{\pi}_t\right)^2 \right)$$

where

$$\bar{\pi}_t = \rho_{\bar{\pi}} \bar{\pi}_{t-1} + \epsilon_{\bar{\pi},t} \tag{1.18}$$

with $\epsilon_{\bar{\pi},t}$ being serially uncorrelated and normally distributed with mean zero and variance $\sigma^2_{\bar{\pi},t-1}$. All other aspects of the setup remain parallel with the baseline case of an output gap target. In particular, the central bank continues to have perfect information while the information set of private agents is given by

$$\mathcal{I}_t = \left\{ i^t, d^{t-1}, \bar{\pi}^{t-1}, \left(\sigma^2_d\right)^t, \left(\sigma^2_{\bar{\pi}}\right)^t \right\}$$

For equilibrium dynamics under a general linear interest rate rule, suppose that the interest rate in (1.4) is replaced with the following expression which is now linear in the inflation target along with beliefs about the inflation target

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{\pi}} \bar{\pi}_t + f_{\bar{\pi},b} \bar{\pi}_{t|t}$$

Then, belief formation will mirror the baseline case so that they are given by

$$d_{t|t} = \rho_d d_{t-1} + \underbrace{\frac{f_d \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{\pi},t-1}}}{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{\pi},t-1}} + f^2_{\bar{\pi}}}}_{K_{d,t}} \left(f_d \epsilon_{d,t} + f_{\bar{\pi}} \epsilon_{\bar{\pi},t} \right)$$

$$\bar{\pi}_{t|t} = \rho_{\bar{\pi}} \bar{\pi}_{t-1} + \underbrace{\frac{f_{\bar{\pi}}}{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{\pi},t-1}} + f^2_{\bar{\pi}}}}_{K_{\bar{\pi},t}} \left(f_d \epsilon_{d,t} + f_{\bar{\pi}} \epsilon_{\bar{\pi},t} \right)$$

The equilibrium is now characterized by the system of equations given by (1.1), (1.2), (1.5) and (1.18) along with the above policy rule and belief formation equations. Since $\bar{\pi}_t$ and $\bar{\pi}_{t|t}$ enter into this system of equations in the exact same way as $\bar{y}_t$ and $\bar{y}_{t|t}$ in the baseline model, the results related to the output gap target in Section 1.3 continue to hold here with the inflation target.

In terms of the optimal discretionary policy problem, assuming now that the variances of shocks are constant and following the same steps as in Section 1.4 yields the following optimality condition

$$\tilde{y}_t = -\mathcal{R}\frac{\varepsilon}{\kappa}\left(\pi_t - \bar{\pi}_t\right)$$
$$\text{where } \mathcal{R} \equiv \frac{\frac{d\pi_t}{di_t^{dis}}}{\frac{d\tilde{y}_t}{di_t^{dis}}} = \frac{\frac{\partial \pi_t}{\partial i_t^{dis}} + \frac{\partial \pi_t}{\partial \bar{\pi}_{t|t}} K_{\bar{\pi}}}{\frac{\partial \tilde{y}_t}{\partial i_t^{dis}} + \frac{\partial \tilde{y}_t}{\partial \bar{\pi}_{t|t}} K_{\bar{\pi}}} \text{ in equilibrium}$$

It can again be shown that $\mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right]$ where $\mathcal{R}$ approaches its lower bound as $\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} \to \infty$ so that private agents attribute any change in the interest rate to a demand shock. When $\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} \to 0$, interest rate changes have their largest possible effect on inflation target beliefs and $\mathcal{R}$ approaches its largest possible equilibrium value. In fact, since this optimality condition is identical to (1.15) with $\mathcal{R}\frac{\varepsilon}{\kappa}\bar{\pi}_t$ in place of $\bar{y}_t$, the implied equilibrium interest rate behavior will also mirror the case of an output target shock with this change of variables.

The stationary equilibrium under this optimality condition is given by

$$\tilde{y}_t = \frac{\mathcal{R}\frac{\varepsilon}{\kappa}}{1+\mathcal{R}\varepsilon}\bar{\pi}_t - \frac{\frac{1}{\kappa}\left(\mathcal{R}\varepsilon\right)^2\beta\rho_{\bar{\pi}}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}\bar{\pi}_{t|t}$$
$$\pi_t - \bar{\pi}_t = -\frac{1}{1+\mathcal{R}\varepsilon}\bar{\pi}_t + \frac{\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}\bar{\pi}_{t|t}$$

The results so far have coincided with the output gap target case. The main differences in these two cases are in the implications for communication policy. In the case of an inflation target, partial communication of only demand now becomes unambiguously optimal for the expected future loss. The best communication strategy for the current period loss will still depend on the realizations of shocks. The following proposition states these results where I again denote the case of partial communication of only the demand shock by a superscript ^{d}.

Proposition 9 *Under an equilibrium where $\mathcal{R} \geq 0$,*

1. *The expected future loss is always lowest under communication of only d_t, that is*

$$E_t^{CB}\mathcal{L}_{t+1}^{d} \leq E_t^{CB}\mathcal{L}_{t+1}^{PI} \;\; \text{and} \;\; E_t^{CB}\mathcal{L}_{t+1}^{d} \leq E_t^{CB}\mathcal{L}_{t+1}$$

2. *For the current period loss, communication of only d_t is always preferable to full communication.*

$$l_t^{d} \leq l_t^{PI} \;\; \text{for any realization of shocks } \{\epsilon_{d,t}, \epsilon_{\bar{\pi},t}\}$$

However, whether it is preferable to no communication depends on the current realizations of shocks $\{\epsilon_{d,t}, \epsilon_{\bar{\pi},t}\}$.

Proof. See Appendix. ■

The reason for this difference is that, in contrast to the output gap target case, it's less costly for the central bank to bring inflation closer to the inflation target when this target is known by private agents. To better understand the intuition, consider the case of a positive shock to the output gap target. If firms are aware of this higher target, they will raise prices more today in anticipation of equilibrium output being higher for some time. This increased inflation will have a negative effect on consumer demand, thus undermining the central bank's efforts to boost output towards the higher target. In the case of a positive shock to the inflation target, making firms aware of this elevated target will also lead them to raise prices more today for a given level of current output. However, this is now beneficial to the central bank's efforts to achieve a higher inflation target.

In summary, when interest rate changes have an effect on private agents' beliefs about either an output gap target and or inflation target, it's possible to observe increases in inflation and output following interest rate surprises. In addition, signaling effects about either type of shock will lead a discretionary policymaker to choose to maintain smaller inflation deviations from target than he would under perfect information, thus resulting in a reduction in the stabilization bias arising from a lack of commitment. However, the implications differ for communication policy in that the central bank is better able to achieve its stabilization goals when private agents' beliefs about the inflation target move with the true inflation target.

1.6.3 Lagged states not observed

When agents cannot see the true lagged states, then beliefs are formed through a Kalman filter rather than a static signal extraction problem. This is the information structure which is more commonly found in the recent literature studying imperfect information in New Keynesian models such as Lorenzoni (2009), Mertens (2011), Berkelmans (2011). The same technique from Svensson and Woodford (2003) used above to deal with the circularity issue present in the belief formation problem can also be applied here. With $\rho_d, \rho_{\bar{y}} < 1$ and constant variances, this Kalman filter converges to a steady state where beliefs are given by

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} \left(i_t^{dis} - f_d d_{t|t-1} - f_{\bar{y}} \bar{y}_{t|t-1} \right)$$

where $d_{t+1|t} = \rho_d d_{t|t}$ and $\bar{y}_{t+1|t} = \rho_{\bar{y}} \bar{y}_{t|t}$. In this steady state, the $\hat{K}_d, \hat{K}_{\bar{y}}$ coefficients are functions of the parameters $\left\{ \rho_d, \rho_{\bar{y}}, f_d, f_{\bar{y}}, \sigma_d^2, \sigma_{\bar{y}}^2 \right\}$. The main difference now is that agents' prior beliefs are no longer reset based on observations of the true lagged values in each period. Rather, beliefs from period t form the prior belief for period $t+1$. In essence, this change in the information structure turns private agents' beliefs into an additional set of endogenous state variables which policy influences.

This adds another dimension to the interest rate's signaling effect. When agents can see lagged true fundamentals, the interest rate's signaling effect is limited to private agents' current expectations. When agents cannot see lagged fundamentals, the policymaker's choice of the current interest rate now also affects future beliefs and thereby, future outcomes. This additional effect adds a set of new terms to the policymaker's optimality condition

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}\frac{\varepsilon}{\kappa}\pi_t - \beta\left(E_t^{CB}\left[\tilde{y}_{t+1} - \bar{y}_{t+1}\right]\frac{d\tilde{y}_{t+1}/di_t^{dis}}{d\tilde{y}_t/di_t^{dis}} + \frac{\varepsilon}{\kappa}E_t^{CB}\left[\pi_{t+1}\right]\frac{d\pi_{t+1}/di_t^{dis}}{d\tilde{y}_t/di_t^{dis}}\right)$$

In equilibrium, this optimality condition still implies a forward-looking optimal interest rate level which is linear in $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$. When expressed in this form, the optimal interest rate no longer moves one-for-one with the natural real rate and a part that's linear in $\left\{\bar{y}_t, \bar{y}_{t|t}\right\}$. To be precise, I denote the optimal interest rate and policy coefficients under this altered information structure by a superscript ** and show that

Proposition 10 *In general, when agents cannot see lagged true states*

$$i_t^{**} \neq r_t^n + f_{\bar{y}}^{**}\bar{y}_t + f_{\bar{y},b}^{**}\bar{y}_{t|t} \quad \textit{for any } f_{\bar{y}}^{**}, f_{\bar{y},b}^{**}$$

Proof. See Appendix. ■

To understand the intuition behind this property, suppose instead that the interest rate continues to respond one-for-one to $r_t^n = \sigma\left(d_t - \rho_d d_{t|t}\right)$. This offsets the contemporaneous effects of the natural real rate on outcomes so that ultimately, $\tilde{y}_t$ and π_t move only with variations in the true level and belief about the output gap target. However, now that agents cannot see lagged true states, the current forecast error made about demand carries through to the next period and affects future outcomes through $\bar{y}_{t+1|t+1}$. Thus, d_t and $d_{t|t}$ have a new intertemporal effect on future outcomes through the forecast error $d_t - d_{t|t}$. A policymaker with an information advantage can detect this forecast error and foresee this effect. This introduces a new element to the tradeoff he faces when deciding how to respond to d_t and $d_{t|t}$, which alters the resulting optimal response. The following corollary gives special cases where this new consideration does not apply and the policymaker again finds it optimal to set a nominal interest rate that moves one-for-one with the natural real rate.

Corollary 11 *(i) Under $\hat{K}_d = 0$, $\hat{K}_{\bar{y}} = 0$, or $\rho_{\bar{y}} = \rho_d$, the interest rate does not affect future* *beliefs and optimal policy is the same as the case where agents could see lagged true states.*

$$i_t^{**} = r_t^n + f_{\bar{y}}^* \bar{y}_t + f_{\bar{y},b}^* \bar{y}_{t|t}$$

(ii) When $\rho_d = 0$, the optimal interest rate responds one-for-one to the natural real rate, but the responses to the output gap target and private agents' belief about it differ.

$$i_t^{**} = r_t^n + f_{\bar{y}}^{**} \bar{y}_t + f_{\bar{y},b}^{**} \bar{y}_{t|t}, \text{ where } f_{\bar{y}}^{**} \neq f_{\bar{y}}^* \text{ and } f_{\bar{y},b}^{**} \neq f_{\bar{y},b}^*$$

Proof. See Appendix. ■

In the first set of special cases, beliefs become a function only of the current interest rate in equilibrium so there is no effect of a marginal change in the interest rate on future outcomes. In the second special case with $\rho_d = 0$, though the current interest rate still affects future outcomes through prior beliefs that agents carry into the next period, the current forecast error for the demand shock has no intertemporal effect on future beliefs. Then, the tradeoff with respect to d_t and $d_{t|t}$ becomes equivalent to the case above where they only have contemporaneous effects.

1.6.4 Optimal policy under dynamic time-varying uncertainty

Here, I consider optimal policy under dynamically varying demand and output gap target uncertainty of the kind assumed in Section 1.2. To review, in this specification, the shocks $\epsilon_{d,t}$ and $\epsilon_{\bar{y},t}$ are serially uncorrelated, uncorrelated with each other, and normally distributed with means zero and variances $\sigma_{d,t-1}^2$ and $\sigma_{\bar{y},t-1}^2$, respectively. In the case of static variances, I showed that the optimal policy features policy coefficients $f_{\bar{y}}^*$ and $f_{\bar{y},b}^*$ that depend on the relative variance $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$. Because of this, I conjecture an equilibrium where policy coefficients are now time-varying through a dependence on the time-varying relative variances. I assume that private agents know the entire history of variances so that they still know the true current value of the policy coefficients. Then, their beliefs take the same form as above with the only difference being time subscripts on the policy coefficients. Due to this time dependence, I conjecture that in equilibrium, $\tilde{y}_t$ and π_t are linear in $\left\{d_t, \bar{y}_t, d_{t|t}, \bar{y}_{t|t}\right\}$ with time-varying coefficients. This means that the policymaker now takes as given that agents' expectations of future outcomes are linear in beliefs with time-varying coefficients that he takes as given.

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M}_t \begin{bmatrix} d_{t+1|t} \\ \\ \bar{y}_{t+1|t} \end{bmatrix}$$

Beliefs are formed as follows

$$d_{t|t} = \rho_d d_{t-1} + K_{d,t}\left(i_t^{dis} - f_d \rho_d d_{t-1} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1}\right)$$
$$\bar{y}_{t|t} = \rho_{\bar{y}} \bar{y}_{t-1} + K_{\bar{y},t}\left(i_t^{dis} - f_d \rho_d d_{t-1} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t-1}\right)$$
$$\text{where } K_{d,t} = \frac{f_{d,t} \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f^2_{d,t} \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f^2_{\bar{y},t}} \quad \text{and} \quad K_{\bar{y},t} = \frac{f_{\bar{y},t}}{f^2_{d,t} \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f^2_{\bar{y},t}}$$

and the policymaker also takes $K_{d,t}$ and $K_{\bar{y},t}$ as given.

In this setting, the policymaker's optimality condition has the same form as before

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}_t \frac{\varepsilon}{\kappa} \pi_t$$

where $\mathcal{R}_t$ is now characterized by a nonlinear stochastic difference equation whose forcing variable is $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ (see Appendix). Furthermore, the optimal interest rate is

$$i_t^* = r_t^n + f^*_{\bar{y},t} \bar{y}_t + f^*_{\bar{y},b,t} \bar{y}_{t|t}$$

where $f^*_{\bar{y},t}$ is a function of $\mathcal{R}_t$ alone and $f^*_{\bar{y},b,t}$ can be written as

$$f^*_{\bar{y},b,t} = E\left[\mathcal{F}\left(\mathcal{R}_t, \mathcal{R}_{t+1}, \ldots\right) | \mathcal{I}_t\right]$$

1.7 Conclusion

In this paper, I explored the impact of a signaling channel on the conduct of optimal interest rate policy as well as equilibrium responses to policy surprises. I found that a discretionary policymaker who is better informed about an output gap target can influence inflation expectations in a way that tilts the short-run inflation-output tradeoff toward a policy that maintains smaller inflation fluctuations. This effect is stronger when the policymaker has a larger impact on inflation expectations. As this influence grows, the optimal discretionary policy approaches the optimal policy under commitment to a forward-looking interest rate rule. Compared to the perfect information case, the signaling effect reduces the stabilization bias which typically exists when the policymaker is unable to commit. This contributes to the finding that it is optimal for the policymaker to maintain some information advantage, which helps to rationalize the Federal Reserve's policy of publishing staff economic projections with a five-year lag.

For a general interest rate rule, I showed that when the policymaker is better informed about demand shocks (or shocks to the natural real rate of interest) and the policy response to these shocks is inadequate, the it is possible to see positive responses of current economic activity and forecasts to interest rate tightening. This matches the empirical patterns found in the present paper as well as previous work in Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2013). Tang (2014a) provides new empirical evidence showing that the responses of inflation forecasts to positive interest rate surprises are strongly positive when prior uncertainty about inflation is high, as predicted under this information setup.

Though this paper examined a model of monetary policy, the logic behind the optimal policy results is generalizable to other settings where a policymaker possesses superior information and has the potential to influence outcomes through expectations. The positive results showing that the economy can sometimes grow in response to a supposedly contractionary policy action can also manifest in other scenarios where policy is intended to be countercyclical, such as fiscal policy.

A natural extension of this paper which I reserve for future work is a study of the impact of incorporating a zero lower bound. In this environment, even optimal policy will not be able to adequately respond to fluctuations in the natural real rate, thus making it more likely that supposedly expansionary policy actions, taken when the economy is close hitting to the ZLB, can lead to further declines in economic activity.

Lastly, though the linearized form of the model used in this paper was crucial for obtaining closed-form results on optimal policy, I plan to revisit the optimal communication policy question in a more realistic framework that includes higher-order welfare effects of uncertainty as well as a channel for communication to impact the transmission of interest rate policy through its effects on risk premia.

Chapter 2

Uncertainty and the Signaling Channel of Monetary Policy: Empirical Evidence

2.1 Introduction

The information advantage of the Federal Reserve has received an increasing amount of empirical support over the past few decades. Media accounts of Federal Reserve policy actions often interpret them as being indicative of the strength of the economy. The Federal Reserve may have an information advantage for a few reasons. A direct source of information advantage could come through access to confidential data. For example, the Federal Reserve produces data on industrial production as well as many series related to the banking sector so it's very likely that, at the very least, the central bank has more detailed or more timely information than the public regarding these variables. Furthermore, the Federal Reserve Board employs nearly three hundred Ph.D. economists, not to mention those employed at the various regional Reserve Banks. The vast resources dedicated to processing data which is available to all agents could afford the Federal Reserve an advantage in forecasting relevant economic fundamentals. In this setting, monetary policy actions that respond to these fundamentals will convey information to financial market participants in a process which I will refer to as the signaling channel.

My previous work in Tang (2014b) explored the theoretical implications of the signaling effect of monetary policy in a New Keynesian model where the central bank has an information advantage over the demand level and an output gap target. That model is able to produce positive responses of forecasts of economic activity to interest rate surprises when the policy response to demand shocks is inadequate and positive interest rate surprises are a strong enough signal of higher demand. In particular, the model produces implications regarding the interaction between uncertainty and the effect that interest rate surprises have on inflation and output forecasts.

In this paper, I present new empirical evidence in support of this type of interaction effect. More specifically, I present empirical evidence of a positive effect of interest rate surprises on inflation forecasts which is concentrated in periods when forecasters reported high uncertainty over the previously made forecast. This result adds to the existing empirical evidence of a monetary policy signaling effect found in Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2013).

The main focus of my analysis is inflation forecasts since they play a key role in many macroeconomic models. Consequently, there is also a large body of empirical work on inflation forecasts serving as precedent for the following analysis. I use forecasts from the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia. I measure federal funds rate surprises using futures prices following Kuttner (2001) and estimate a slightly positive effect of these surprises on inflation forecasts over the 1989Q1-2011Q1 period. This echoes a result from an earlier sample in Romer and Romer (2000). I then decompose this overall effect by showing that the effect is especially strong in periods when forecasters had high uncertainty regarding their previous forecast. This further substantiates an explanation based on a signaling effect of these policy actions. Competing explanations for the positive overall effect, such as a cost channel where higher interest rates raise firms' financing costs, do not naturally generate this type of interaction[1]. I repeat the analysis for real output growth forecasts, also from the SPF, and find similar qualitative results though these estimates are less precise.

In another set of empirical results, I estimate time-varying gain coefficients measuring the response of inflation forecasts to general news about inflation. I estimate the coefficients at an annual frequency for the 1971-2012 period and show that there is substantial variation in this coefficient over time. Furthermore, I show that these estimates are negatively correlated with forecast dispersion and positively correlated with subjective uncertainty in a way that is consistent with the predictions of the noisy information framework. This adds to the evidence found in Coibion and Gorodnichenko (2012a) and Coibion and Gorodnichenko (2012b) in support of the noisy information framework.

The next subsection reviews the related literature. Section 2.2 sets up the model while 2.3 describes the data used. I present general time-varying estimates of the response of inflation forecasts to news about inflation in Section 2.4. Section 2.5 estimates the effect of interest rate surprises on inflation forecast revisions and shows that the effects are indeed more positive when prior uncertainty about inflation is high. Section 2.6 repeats this analysis using forecasts of real output growth and Section 2.7 concludes.

2.1.1 Related literature

This paper refines the work in Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2013) which suggest that interest rate surprises convey information regarding the state of the economy. I show that the slightly positive responses of inflation forecasts to policy actions found in Romer and Romer (2000) are also present in a later sample and are robust to using interest rate surprises derived from federal funds futures prices. I relate this result more directly to a signaling effect of

[1] I show below that forecasters' subjective uncertainty is not highly correlated with other measures of economic activity or uncertainty.

monetary policy by showing a positive interaction between these responses and subjective uncertainty over previous inflation forecasts. I also obtain similar results for real output forecasts although the estimates are less precise. My results also relate to the work of Coibion and Gorodnichenko (2012a) and Coibion and Gorodnichenko (2012b) on the estimation of noisy information models. I estimate higher frequency time-variation in the responses of inflation forecasts to news and show that these responses correlate with forecast dispersion and prior uncertainty in the directions suggested by noisy information models.

Ellingsen and Söderström (2001), Erceg and Levin (2003), and Gürkaynak, Sack, and Swanson (2005b) use an interest rate signaling effect to explain various features of macroeconomic data including inflation persistence and the response of the yield curve to monetary policy actions. Melosi (2013) structurally estimates a dispersed information DSGE model where monetary policy is assumed to follow a Taylor rule that responds to aggregate variables which individual firms cannot observe. He shows that allowing for a monetary policy signaling effect enables the model to fit inflation forecast data from the SPF better than the corresponding perfect information model. However, he does not allow for time-varying uncertainty in his estimation.

2.2 Empirical model

The regressions below are motivated using a model that assumes an AR(1) reduced form for inflation along with a Taylor-style interest rate rule that responds directly to inflation[2]. Coibion and Gorodnichenko (2012a) and Coibion and Gorodnichenko (2012b) show that this type of reduced-form framework characterizes inflation forecast data well.

Suppose that inflation follows an AR(1) process

$$\pi_t = \rho_\pi \pi_{t-1} + \varepsilon_t$$

[2] I show in the Appendix that the New Keynesian structural model in Tang (2014b) can be modified slightly to give similar empirical relationships as the ones tested below.

where $\varepsilon_t \sim N\left(0, \sigma^2_{\varepsilon,t-1}\right)$ is serially uncorrelated and normally distributed with time-varying variances. Agents cannot observe π_t directly but instead receive two signals: one from the observed interest rate which responds to true inflation and another composite signal which contains idiosyncratic noise.

$$i_t = \phi\pi_t + u_t$$

$$s_{jt} = \pi_t + e_{jt}$$

I assume $\phi > 0$ and that the two signal noise terms $\{u_t, e_{jt}\}$ are also serially uncorrelated and normally distributed with variances that are identical across agents and possibly time-varying. Agents additionally observe lagged inflation without noise. This is a departure from the empirical models used in previous studies which generally assume that agents cannot see true inflation at any lag. Another difference is the explicit inclusion of an interest rate signal containing additional information about inflation. A main element of this formulation is the interest rate's response to true inflation. If, for example, the interest rate was a function only of private beliefs about π_t, then it would not convey any additional information to private agents and i_t would not enter independently into forecasts.

Each agent j has the information set $\mathcal{I}_{jt} = \left\{\pi^{t-1}, i^t, s^t_j, \left(\sigma^2_\varepsilon\right)^t\right\}$ and forms his conditional expectation of current inflation via a static Gaussian signal extraction problem which yields

$$\pi_{t|jt} = \rho_\pi \pi_{t-1} + K^i_t \left(i_t - E\left[i_t|\pi_{t-1}\right]\right) + K^s_t \left(s_{jt} - E\left[s_{jt}|\pi_{t-1}\right]\right)$$

where $K^i_t \in \left(0, \phi^{-1}\right)$ and $K^s_t \in (0,1)$ are increasing in $\sigma^2_{\varepsilon,t-1}$, which captures prior uncertainty. This expression can be transformed into two different testable relationships.

First, the news from both the interest rate and composite signal s_{jt} can be combined into a current nowcast error term which reflects all current period news. This gives the following equation for forecast revisions for different horizons $h \geq 0$

$$\pi_{t+h|jt} - \pi_{t+h|j,t-1} = K_t \rho^h_\pi \left(\pi_t - \pi_{t|j,t-1}\right) + \left(1 - K_t\right) \rho^{h+1}_\pi \left(\pi_{t-1} - \pi_{t-1|j,t-1}\right) + error_{jht} \quad (2.1)$$

where $K_t \equiv \phi K_t^i + K_t^s \in (0,1)$ and is decreasing in signal noise and increasing in prior uncertainty $\sigma^2_{\varepsilon,t-1}$. $error_{jht}$ may be correlated across individuals and horizons but are uncorrelated across time and with the other RHS variables. This expression states that the effect of general inflation news on forecasts will be time-varying. In particular, the effect of current nowcast errors on inflation forecast revisions is increasing in prior uncertainty and the effect of lagged forecast errors will be decreasing in prior uncertainty.

Secondly, the model also makes predictions about the effect of interest rate surprises on inflation forecast revisions. With additional data on aggregate interest rate surprises, one can test the following relationship for aggregate forecast revisions

$$\begin{aligned} \overline{\pi_{t+h|t}} - \overline{\pi_{t+h|t-1}} &= \rho_\pi^h K_t^i \left(i_t - \overline{E\left[i_t|\pi_{t-1}\right]} \right) + \rho_\pi^h K_t^s \left(\pi_t - \overline{\pi_{t|t-1}} \right) \\ &+ \rho_\pi^{h+1} \left(1 - K_t^s\right) \left(\pi_{t-1} - \overline{\pi_{t-1|t-1}} \right) + error_{ht} \end{aligned} \tag{2.2}$$

The last error term in this equation is a function of the average noise in s_t and is not correlated with the other RHS terms. This gives a regression equation that is nearly identical to equation (5) in Romer and Romer (2000). The main difference is that while they use the Federal Reserve's forecasts to control for other inflation-related news, all relevant news in this model is captured by the lagged forecast and nowcast errors.

Extensions of the empirical model

I can allow for a standard direct negative effect of i_t on π_t of the following form

$$\pi_t = \rho_\pi \pi_{t-1} - \delta i_t + \varepsilon_t$$

where $\delta > 0$ and the expressions for i_t and s_{jt} continue to be those given above. This yields a solution for π_t that is similar to the above model

$$\pi_t = \check{\rho}_\pi \pi_{t-1} + \frac{1}{1+\delta\phi}\varepsilon_t - \frac{\delta}{1+\delta\phi}u_t \quad \text{where } \check{\rho}_\pi \equiv \frac{\rho_\pi}{1+\delta\phi}$$

The main difference here is the covariance between true inflation and the interest rate. In this case, forecast revisions evolve as

$$\overline{\pi_{t+h|t}} - \overline{\pi_{t+h|t-1}} = \check{\rho}_\pi^h \check{K}_t^i \left(i_t - \overline{E\left[i_t|\pi_{t-1}\right]} \right) + \check{\rho}_\pi^h \check{K}_t^s \left(\pi_t - \overline{\pi_{t|t-1}} \right)$$
$$+ \check{\rho}_\pi^{h+1} \left(1 - \check{K}_t^s \right) \left(\pi_{t-1} - \overline{\pi_{t-1|t-1}} \right) + error_{ht}$$

where $\check{K}_t^i$ may now take on negative values but both $\check{K}_t^i$ and $\check{K}_t^s$ are still increasing in $\sigma^2_{\varepsilon,t-1}$.

If I do not allow agents to observe lagged inflation, then agents' forecasts are described by a Kalman filter[3]. In this case, aggregate forecast revisions evolve as

$$\overline{\pi_{t+h|t}} - \overline{\pi_{t+h|t-1}} = \rho_\pi^h \hat{K}_t^i \left(i_t - \overline{i_{t|t-1}} \right) + \rho_\pi^h \hat{K}_t^s \left(\pi_t - \overline{\pi_{t|t-1}} \right) + error_{ht}$$

where $\hat{K}_t^i \in \left(0, \phi^{-1}\right)$ and $\hat{K}_t^s \in (0,1)$ are now increasing in prior uncertainty, $Var_{t-1}(\pi_t)$, which itself is increasing in $\sigma^2_{\varepsilon,t-1}$. The lagged nowcast term drops out of the regression equation. However, this term enters significantly in the regressions below, suggesting that the assumption that agents can see lagged inflation is valid.

2.3 Data

For aggregate inflation forecasts, I use median forecasts from the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia. The survey starts in 1968Q4 and is quarterly with about 40 respondents in each quarter. I look specifically at quarterly forecasts of the GNP/GDP deflator (GDP starting in 1992). Real GNP/GDP growth and unemployment forecasts are used for some robustness checks. One unique feature of the SPF is that, in addition to point forecasts, it also asks respondents to report forecasted probability distributions for annual inflation. This allows me to impute a measure of subjective uncertainty over inflation.

[3]This is the linear least-squares forecast which is also optimal if we additionally assume that agents' prior beliefs about the initial state π_0 are normally distributed.

For some specifications, I also use the Federal Reserve's Greenbook forecasts of the GNP/GDP deflator[4] which are published with a five year lag starting in December 1965.

For actual data, I use real-time data from the Federal Reserve Bank of Philadelphia taking values from a two-quarters ahead vintage (e.g., the 2001Q1 observation for inflation is taken from the 2001Q3 vintage). This timing is chosen to correspond to the final published NIPA estimates prior to annual or benchmark revisions.

To measure policy surprises, I use prices for 30-day federal funds futures obtained from Bloomberg which start in December 1988. I use the method described in Kuttner (2001) to construct surprises on policy news days. I define these as days when the target rate changed or scheduled Federal Open Market Committee meeting days starting in 1994 (some dating adjustments were made following Kuttner (2003)). As described in Swanson (2006), the FOMC only began issuing post-meeting press releases in 1994. Additionally, rate changes were not strongly associated with meeting days prior to 1994. For instance, only 31% of actual target changes from the start of 1989 to the end of 1993 were associated with scheduled meetings compared to 86% starting in 1994 until the target effectively hit zero in late 2008. Thus, pre-1994 meeting days when no change was made are not categorized as news days, but the results are not sensitive to this choice. To get a measure of policy surprises that corresponds to the quarterly SPF timing, I sum one-day policy surprises between SPF deadlines[5].

Finally, in the regressions estimating the effect of news from interest rate surprises, I exclude dates after 2011Q1 due the Fed's decision to begin regularly releasing economic projections of Federal Reserve Board members and Bank presidents in conjunction with post-meeting press releases. The results are not sensitive to this choice.

[4] The Greenbook switches to forecasting the GDP deflator measure five months after the SPF switched so these observations are excluded.

[5] Deadline dates are available starting in 1990Q2. Prior to that, I use the 15th of the middle month of each quarter.

2.3.1 Imputing subjective uncertainty

I proxy subjective uncertainty using the SPF's probability forecasts for the GNP/GDP deflator where agents report probabilities of inflation being in pre-defined ranges. Starting in 1981Q3, the survey consistently contains these reports for both the current and following years' inflation as measured by the percentage change in the annual averages of the price index. To impute the variance associated with these forecasts, I fit a normal distribution to the data by minimizing the sum of squared differences between the reports and the probabilities for the same ranges implied by a normal distribution following Giordani and Söderlind (2003) and Lahiri and Liu (2006). More formally, for a given set of reported probabilities $\{q_n\}_{n=1}^{N}$ corresponding to ranges $\{[a_n, b_n)\}_{n=1}^{N}$, I solve the problem

$$\min_{\mu,\sigma} \sum_{n=1}^{N} \left\{ q_i - \left[\Phi\left(\frac{b_n - \mu}{\sigma}\right) - \Phi\left(\frac{a_n - \mu}{\sigma}\right) \right] \right\}^2$$

I remove individual-level post-1991 means from these variances to account for a switch from GNP to GDP measures and a change in the number of ranges provided in the survey from 6 to 10. In the analysis below, I use the median of the adjusted variances of forecasts for the next year's inflation as a proxy of subjective forecast uncertainty, denoted as $\overline{Std_t^{\pi}}$. The following table shows that this measure is not highly correlated with macroeconomic variables or other measures of uncertainty commonly used in the literature on uncertainty shocks[6]. This low correlation with other uncertainty measures is not surprising since they capture many aspects of economic uncertainty and not just those related to inflation. The low correlation with macroeconomic variables indicates that regressions containing interactions with this measure of subjective uncertainty are unlikely to be picking up nonlinearities or state-dependence related to the business cycle.

[6] Uncertainty measures are from the dataset accompanying Bachmann, Elstner, and Sims (2013) as well as the policy-related economic uncertainty described in Baker, Bloom, and Davis (2013) and available at www.policyuncertainty.com.

Table 2.1: Correlations between $\overline{Std_t^\pi}$ and macro variables

x	x_{t-1}	x_t	x_{t+1}
Macro Variables			
Inflation	-0.02	0.12	-0.09
Real GNP/GDP growth	-0.08	0.02	0.10
Uncertainty Measures			
Google econ uncertainty index	0.24**	0.13	0.12
Stock volatility	0.02	-0.11	-0.10
Policy uncertainty index	0.07	-0.05	-0.05

Notes: These correlations are computed with the longest samples available in the data. The sample sizes vary between 110 and 124 quarters. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

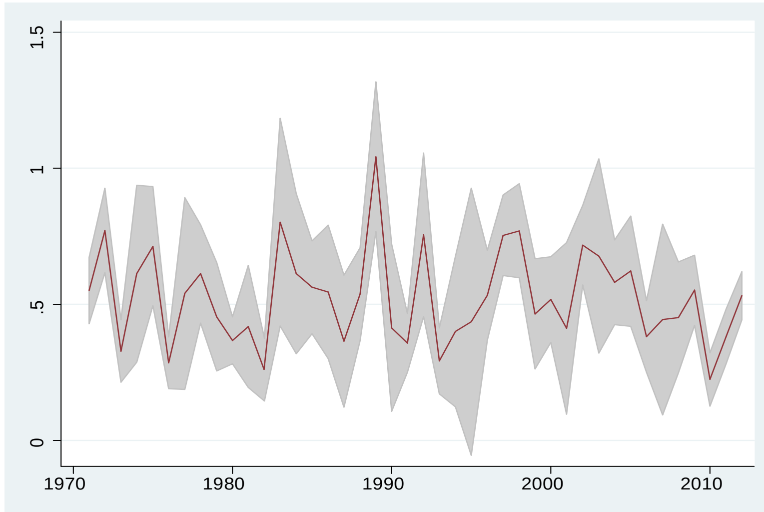
2.4 Time-variation in sensitivity of inflation forecasts to news

I first examine the overall effect of all inflation news on forecasts given in (2.1). Using 17,716 observations of individual level quarterly data over the period 1971-2012, I obtain annual estimates using a nonlinear least squares estimation of the following equation with standard errors clustered within quarters[7].

$$\pi_{t+h|jt} - \pi_{t+h|j,t-1} = \alpha_{ht} + K^{FE}_{year_t}\rho^h_\pi\left(\pi_t - \pi_{t|j,t-1}\right) + K^{NE}_{year_t}\rho^{h+1}_\pi\left(\pi_{t-1} - \pi_{t-1|j,t-1}\right) + error_{jht}$$

Figure 2.1 shows estimates of my main coefficients of interest which are the time-varying responses of inflation forecasts to current news.

[7] Coibion and Gorodnichenko (2012a) also estimates time-varying sensivity of forecasts to news using a different empirical approach. They discuss low frequency changes in this parameter associated with the Great Moderation.



Notes: Gray area represents 90% confidence intervals

Figure 2.1: Annual estimates of $K^{FE}_{year_t}$

There is substantial time-variation in this coefficient. Table 2.2 shows that the estimates correlate negatively with forecast dispersion (an imperfect proxy for idiosyncratic signal noise[8]) and positively with my measure of prior uncertainty as predicted by the model.

Table 2.2: Correlations between $\hat{K}^{FE}_{year_t}$ and signal noise or prior uncertainty

Variable	Correlation
Dispersion: $h = 0$	-0.39**
Dispersion: $h = 1$	-0.30*
Dispersion: $h = 2$	-0.36**
Dispersion: $h = 3$	-0.15
Dispersion: $h = 4$	-0.13
Lagged current year uncertainty	0.40**
Lagged next year uncertainty	0.38**

Notes: Correlations are calculated between annual coefficient estimates and annual means of the variables. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

[8] The proxy is imperfect due to a nonmonotonic relationship between idiosyncratic signal noise and forecast dispersion. Forecast dispersion becomes decreasing in idiosyncratic signal noise when it is high relative to the variability of inflation innovations and the exogenous component of the interest rate. As s_{jt} becomes dominated by noise, agents optimally ignore these signals and forecast dispersion approaches zero.

Meanwhile, time-variation in these estimates does not seem to be associated with macroeconomic variables or other common measures of uncertainty as shown in Table 2.3. The fact that these correlations are lower than the ones in Table 2.2 suggests that the variation in inflation forecast sensitivity to news is more related to an information story than other explanations.

Table 2.3: Correlations between $\hat{K}^{FE}_{year_t}$ and macro variables

x	x_{year_t-1}	x_{year_t}	x_{year_t+1}
Macro Variables			
Inflation	−0.03	−0.07	−0.09
Real GNP/GDP growth	−0.05	0.28*	0.21
Uncertainty Measures			
Google econ uncertainty index	−0.18	−0.07	−0.14
Stock volatility	0.20	0.00	−0.05
Policy uncertainty index	−0.02	−0.22	−0.18

Notes: Correlations are calculated between annual coefficient estimates and annual means of the variables. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

2.5 Effect of interest rate surprises on inflation forecasts

In this section, I separately estimate the impact of interest rate news on inflation forecasts and present the main empirical result in support of the interest rate's signaling effect. My estimates echo the findings in Table 8 of Romer and Romer (2000) which shows that monetary policy tightening seems to have a mildly positive (though not statistically significant) effect on inflation forecasts. This can be seen as estimating a version of (2.2) with constant coefficients.

My analysis differs from theirs in several ways. First, my sample period is 1989:Q1 to

2011:Q1 which has little overlap with their sample of 1974:Q3 to 1991:Q4 with the Volcker years removed. Secondly, I use lagged forecast and nowcast errors as my summary measures of "other news" as implied by the above empirical model while they used changes in the Federal Reserve's Greenbook forecast. Lastly, they used federal funds rate changes or a dummy variable based on articles in the *Wall Street Journal* following Cook and Hahn (1989a) and Cook and Hahn (1989b) to measure monetary policy actions. For my regressions, I instead use interest rate surprises measured using daily federal funds futures prices which arguably has less of an endogeneity problem.

Despite these differences, I am able to qualitatively replicate their result as shown in Table 2.4. In fact, the regressions show that this positive effect of surprise interest rate tightening on inflation forecast revisions is actually significant at a 10% or better level for all four forecast horizons. The coefficients are larger than those estimated by Romer and Romer (2000) owing to the fact that the average magnitude of interest rate surprises is only about one-third the average size of target changes.

Table 2.4: Baseline effect of federal funds rate surprises on inflation forecasts

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	0.304*	0.267**	0.332***	0.181*
	[1.81]	[2.14]	[2.76]	[1.79]
$\pi_t - \overline{\pi_{t\|t-1}}$	0.101***	0.020	0.028	0.030
	[2.69]	[0.89]	[1.27]	[1.32]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}}$	0.191***	0.143***	0.067***	0.095***
	[3.79]	[4.30]	[2.94]	[3.55]
Adjusted R^2	0.325	0.278	0.204	0.216
N	88	88	88	88

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 and 1996:Q1 dropped due to switches in the SPF from the GNP to GDP deflator and then subsequently to the GDP price index making the lagged forecast unavailable in those periods. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

To further build upon this and test the main prediction that K_t^i is higher when agents have more uncertainty over the last forecast they made, I interact the news variables in this regression with the measure of subjective prior uncertainty described above. Table 2.5 shows the results of interacting each news variable with a dummy indicating whether $\overline{Std^{\pi}_{t-1}}$ is below or above its median.

Table 2.5: Effect of federal funds rate surprises on inflation forecasts with a high vs low prior uncertainty interaction

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.081	0.110	0.114	0.144
	[0.45]	[0.85]	[1.20]	[1.49]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.666**	0.428**	0.756***	0.212
	[2.37]	[2.05]	[4.52]	[0.84]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.064	-0.023	-0.007	0.026
	[1.01]	[-0.61]	[-0.21]	[0.73]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.116**	0.043	0.039	0.029
	[2.35]	[1.52]	[1.54]	[1.11]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.0.230***	0.199***	0.097***	0.112***
	[3.13]	[4.45]	[3.21]	[3.11]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.141**	0.071*	0.042	0.066
	[2.60]	[1.93]	[1.49]	[1.65]
$\overline{Std^{\pi}_{t-1}}$ high	0.113*	0.068	0.082**	0.022
	[1.82]	[1.64]	[2.26]	[0.57]
Adjusted R^2	0.335	0.313	0.276	0.189
N	88	88	88	88
P-value of F-test of difference in $i_t - \overline{i_{t\|t-1}}$ coef	0.083	0.199	0.001	0.801

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 and 1996:Q1 dropped due to switches in the SPF from the GNP to GDP deflator and then subsequently to the GDP price index making the lagged forecast unavailable in those periods. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

Compared to the baseline results, the coefficient on interest rates surprises in periods of low prior uncertainty are smaller and not statistically significant while the coefficients in periods of high uncertainty are higher and statistically significant (save for the farthest horizon). F-tests show that the differences in these coefficients are statistically significant in a few of the horizons as well. In addition, the interactions on the news captured by the lagged forecast and nowcast errors also go in the predicted directions.

Table 2.6 shows that estimating a continuous interaction with prior uncertainty produces the same qualitative results. Here, the prior uncertainty measure is standardized to have zero mean and standard deviation of one. Thus, the coefficients for the main effects of interest rate surprises, lagged forecast errors, and nowcast errors may be interpreted as the average effect when prior uncertainty is at its mean value. In this set of results, it's evident that the interaction effect is stronger at shorter horizons. One candidate explanation of this is that the Federal Reserve's information advantage in forecasting inflation is stronger at lower horizons. Some evidence supporting this possibility is presented in Table 4 of Sims (2003) which shows results of a test of whether the Federal Reserve's inflation forecast has a lower RMSE than the SPF's average forecast. The evidence presented there is stronger for one-quarter-ahead forecasts than for four-quarter-ahead forecasts.

Lastly, comparing the adjusted R^2 values to the baseline case indicates that allowing for this interaction improves the model's ability to explain forecast revisions.

Table 2.6: Effect of federal funds rate surprises on inflation forecasts with a continuous prior uncertainty interaction

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	0.452***	0.254	0.352**	0.147
	[2.92]	[1.63]	[2.19]	[1.07]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$	0.422**	0.235*	0.187	−0.098
	[2.07]	[1.70]	[1.64]	[−0.77]
$\pi_t - \overline{\pi_{t\|t-1}}$	0.091**	0.022	0.028	0.034
	[2.60]	[0.99]	[1.31]	[1.48]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$	0.070*	0.062**	0.038**	0.005
	[1.73]	[2.38]	[2.15]	[0.20]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}}$	0.215***	0.144***	0.065***	0.090***
	[3.99]	[4.30]	[2.84]	[3.07]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$	−0.048	−0.071*	−0.027	0.023
	[−0.79]	[−1.73]	[−0.93]	[0.63]
$\overline{Std^{\pi}_{t-1}}$	0.015	0.019	0.046***	0.004
	[0.41]	[0.88]	[2.69]	[0.22]
Adjusted R^2	0.347	0.296	0.239	0.193
N	88	88	88	88

Notes: $\overline{Std^{\pi}_{t-1}}$ is standardized to have zero mean and standard deviation of one. The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 and 1996:Q1 dropped due to switches in the SPF from the GNP to GDP deflator and then subsequently to the GDP price index making the lagged forecast unavailable in those periods. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

2.5.1 Robustness checks

One might be concerned that forecasters take into account other variables when making inflation forecasts. To address this issue, I also run specifications with added measures of news about either real output growth or unemployment. These news terms are proxied analogously with lagged forecast and nowcast errors. The tables given in the Appendix show that the results remain unchanged. In fact, with these additional controls, the interaction effect of prior uncertainty on the response to interest rate surprises becomes stronger.

I get similar results using the Federal Reserve's Greenbook forecast revisions as the proxy for other news (following Romer and Romer (2000)) though I lose some observations due to the Greenbook's five year publication lag. The estimates are also almost identical with the lagged SPF forecast on the right hand side with a coefficient that is not constrained to one.

2.6 Effect of interest rate surprises on output forecasts

In this section, I repeat the exercises in Section 2.5 for real output forecasts. Romer and Romer (2000) finds that the Federal Reserve also possesses an information advantage in forecasting real output relative to the SPF though the evidence seems to be weaker than that for inflation forecasts (Sims (2003) confirms this difference as well). Thus, it may be possible that a signaling effect of interest rate surprises also exists for real output.

All the variables used in these exercises are constructed in the same way as those corresponding to the above inflation measures. Table 2.7 shows that the prior uncertainty measure for output exhibits slightly stronger, but still small, correlations with macroeconomic variables and other measures of uncertainty than the prior uncertainty measure for inflation. The contemporaneous correlation between $\overline{Std_t^\pi}$ and $\overline{Std_t^y}$ is .55.

Table 2.7: Correlations between $\overline{Std_t^y}$ and macro variables

x	x_{t-1}	x_t	x_{t+1}
Macro Variables			
Inflation	−0.12	−0.05	−0.19**
Real GNP/GDP growth	−0.22**	−0.05	−0.01
Uncertainty Measures			
Google econ uncertainty index	0.28**	0.22**	0.12
Stock volatility	0.12	0.02	−0.09
Policy uncertainty index	0.17*	0.13	−0.04

Notes: These correlations are computed with the longest samples available in the data. The sample sizes vary between 110 and 124 quarters. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table 2.8 shows the baseline effect of surprise interest rate tightening on real output forecast revisions. The coefficients are large and positive for shorter forecast horizons but turn negative at the farthest forecast horizon. Nakamura and Steinsson (2013) also find a positive overall effect of interest rate surprises on real output forecasts from the *Blue Chip Economic Indicators* survey that is generally larger and more statistically significant for shorter horizons.

Table 2.8: Baseline effect of federal funds rate surprises on output forecasts

Dependent variable: $\overline{y_{t+h\|t}} - \overline{y_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	1.245*	0.763	0.014	−0.314**
	[1.94]	[1.40]	[0.07]	[−2.11]
$y_t - \overline{y_{t\|t-1}}$	0.205***	0.115***	0.060**	0.027
	[4.21]	[2.92]	[2.07]	[1.25]
$y_{t-1} - \overline{y_{t-1\|t-1}}$	0.204***	0.096**	0.030	0.002
	[3.73]	[2.40]	[1.47]	[0.14]
Adjusted R^2	0.468	0.315	0.097	0.027
N	89	89	89	89

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from real GNP to real GDP making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

Table 2.9 estimates the same equation with the addition of interactions with a variable indicating whether $\overline{Std^y_{t-1}}$ is below or above its median. Compared to the baseline results, the coefficients on interest rates surprises in periods of high uncertainty are much larger except for the farthest horizon. However, unlike the estimates for inflation, the difference in the effect is not statistically significant. Moreover, the interactions on the news captured by the lagged forecast goes in the direction predicted by the model while the interactions for nowcast errors do not.

Table 2.9: Effect of federal funds rate surprises on output forecasts with a high vs low prior uncertainty interaction

Dependent variable: $\overline{y_{t+h\|t}} - \overline{y_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^y_{t-1}}$ low	1.022*	0.252	−0.140	−0.321**
	[1.98]	[0.54]	[−0.63]	[−2.25]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^y_{t-1}}$ high	2.058	1.921*	0.309	−0.338
	[1.21]	[1.69]	[0.70]	[−0.86]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^y_{t-1}}$ low	0.249***	0.129**	0.068	0.041
	[3.81]	[2.22]	[1.63]	[1.30]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^y_{t-1}}$ high	0.123**	0.059	0.039	0.009
	[2.04]	[1.54]	[1.01]	[0.38]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^y_{t-1}}$ low	0.220***	0.150***	0.043	−0.005
	[3.36]	[3.02]	[1.48]	[−0.23]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^y_{t-1}}$ high	0.174**	0.044	0.016	0.003
	[2.24]	[0.87]	[0.55]	[0.13]
$\overline{Std^y_{t-1}}$ high	−0.078	0.109	0.077	0.056
	[−0.46]	[0.90]	[0.77]	[0.90]
Adjusted R^2	0.468	0.337	0.067	0.005
N	89	89	89	89
P-value of F-test of difference in $i_t - \overline{i_{t\|t-1}}$ coef	0.562	0.178	0.368	0.967

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from real GNP to real GDP making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

Table 2.10 shows that similar results can be obtained from an estimation with a continuous interaction with prior uncertainty. Again, I standardize the prior uncertainty measure to have zero mean and standard deviation of one. The point estimates on the interaction between interest rate surprises and prior uncertainty are all positive as predicted by the model, but none are statistically significant at standard levels. One possible explanation for the evidence being weaker here is the above-mentioned fact that the Federal Reserve's information advantage is less strong for output than it is for inflation. Another explanation is that real output growth is not characterized as well by an AR(1) process as inflation is. This could imply that there are omitted variables in the above regressions. This issue will be addressed in future work.

Table 2.10: Effect of federal funds rate surprises on output forecasts with a continuous prior uncertainty interaction

Dependent variable: $\overline{y_{t+h\|t}} - \overline{y_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	1.266*	0.864*	0.026	−0.297*
	[1.77]	[1.68]	[0.12]	[−1.79]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^y_{t-1}}$	0.166	0.809	0.325	0.201
	[0.21]	[1.17]	[1.64]	[1.27]
$y_t - \overline{y_{t\|t-1}}$	0.199***	0.104**	0.054*	0.025
	[3.94]	[2.60]	[1.77]	[1.12]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^y_{t-1}}$	−0.033	−0.019	−0.012	−0.016
	[−0.58]	[−0.48]	[−0.36]	[−0.72]
$y_{t-1} - \overline{y_{t-1\|t-1}}$	0.197***	0.091**	0.025	−0.002
	[3.39]	[2.36]	[1.38]	[−0.10]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^y_{t-1}}$	−0.022	−0.077***	−0.044**	−0.010
	[−0.51]	[−2.70]	[−2.16]	[−0.65]
$\overline{Std^y_{t-1}}$	0.033	0.146**	0.108***	0.060*
	[0.39]	[2.63]	[2.80]	[1.87]
Adjusted R^2	0.446	0.340	0.126	0.023
N	89	89	89	89

Notes: $\overline{Std^y_{t-1}}$ is standardized to have zero mean and standard deviation of one. The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from real GNP to real GDP making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

2.7 Conclusion

In this paper, I presented a reduced-form model of inflation where the nominal interest rate responds directly to the true level of inflation which is itself only seen by agents with a one-period lag. Using this model, I derived testable empirical implications for inflation forecast revisions. First, the model predicts that inflation forecast revisions will respond more to general inflation news when prior uncertainty is high and news is less noisy. Since the interest rate conveys information about the true level of inflation in this model, it also predicts that interest rate surprises can have a positive impact on inflation forecast revisions and that this effect will be increasing in forecasters' prior uncertainty.

To test these implications, I constructed measures of subjective forecast uncertainty using the responses to a question in the SPF asking forecasters to report probabilities that future inflation and output growth would fall within given ranges. First, I estimated general time-variation in the response of inflation forecasts to overall news and found that this does indeed correlate positively with the imputed prior uncertainty measure. The estimates also correlate negatively with forecast dispersion which acts as a proxy for noisiness of news.

Second, I estimated the effect of interest rate surprises on inflation forecasts. Without any interactions with prior uncertainty, the baseline effects match the small positive effects found in Romer and Romer (2000) and Campbell, Evans, Fisher, and Justiniano (2012). Adding interactions with prior uncertainty confirm the model's prediction that the effect is larger when prior uncertainty is high. This decomposition of the effect of interest rates on inflation forecasts further substantiates the existence of a signaling effect. While other theories, such as a cost channel, can explain the small positive baseline effect found in previous studies, they do not naturally explain this interaction with prior uncertainty.

Lastly, I repeat the exercises using real output growth forecasts and find similar conclusions though these estimates are not as precise. In the future, it would be interesting to see whether this empirical relationship also exists for expectations derived from asset prices.

Chapter 3

FOMC Communication and Interest Rate Sensitivity to News

3.1 Introduction

Over the past few decades, it has become widely accepted that central bank communication can be a valuable monetary policy tool. The aspect of central bank communications that has received the most attention is the use of forward guidance since many studies have shown that it may be used to stimulate demand when nominal interest rates are close to zero which is a situation that many advanced economies currently find themselves in[1]. Much of the empirical work on central bank communication has, likewise, also focused on the effect of communications on interest rate expectations.

There has, however, been less attention paid to other dimensions of central bank communication such as its ability to convey information regarding the policy reaction function. In this paper, I take a step in this direction by exploring the relationship between the language used in FOMC texts and financial market responses to different types of macroeconomic news. In particular, this study looks for the existence of an interaction effect where empha-

[1] Some notable examples are Krugman (1998) and Eggertsson and Woodford (2003) while Woodford (2012) summarizes many of the key issues covered in this literature.

sis of certain economic topics within FOMC communications might lead to stronger responses of interest rates to news related to those topics. As central banks are increasingly taking steps to improve the public's understanding of their objectives and operations, it is important to gain a better understanding of the potential for financial markets to glean information about policy reaction functions from central banks' communications.

In the current analysis, I focus on the topic of labor due to some relevant recent developments in Fed communication. As the analysis below shows, the extent of discussion regarding labor market conditions in FOMC minutes and statements has grown rapidly during the recent recession. One especially salient event occurred in December 2012, when the FOMC decided to start including an explicit unemployment threshold in their statements.

However, rather than taking an event study approach, I instead make use of the large amount of information available in FOMC texts by constructing a continuous measure capturing the extent of labor-related discussion within these texts. I then relate financial market responses to different types of macroeconomic news to this measure using two different approaches (both are inspired by the analysis in Swanson and Williams (2014)). The first approach involves an initial step where I allow for unrestricted time-variation in the sensitivity of interest rates to labor news as well as all other news. I then relate my measures of labor word use in FOMC texts to the differential sensitivity to labor news versus other news. The second approach is a more parametric procedure where I estimate an equation expressing changes in interest rates as a function of news where the differential response to labor-related news is restricted to be a function of labor word use in FOMC texts. Both methods show a positive relationship, and furthermore, this relationship is stronger for interest rates of longer maturities.

The next subsection reviews the related literature. Section 3.2 gives background on FOMC texts and describes the word use measure. Section 3.3 describes the estimation of time-varying sensitivity to news while the relationship between these estimates and the word use measures are explored in Section 3.4. Section 3.5 presents the more parametric approach and 3.6 discusses robustness checks. In Section 3.7, I outline some issues that are left for future work and Section 3.8 concludes.

3.1.1 Related literature

There are several existing papers measuring the sensitivity of interest rates to macroeconomic news. Two recent examples are Gürkaynak, Sack, and Swanson (2005b) and Faust, Rogers, Wang, and Wright (2007). Swanson and Williams (2014) estimate time-varying sensitivity to general macroeconomic news with an emphasis on showing its decline during the current zero lower bound episode. In this paper, I largely follow their estimation procedures with the key difference being a division of news into two categories: labor-related and other.

Thus far, there have been few attempts to relate changes in interest rate sensitivity to news to central bank communications. One exception is an event study by Bernanke, Reinhart, and Sack (2004) of the August 2003 introduction of the phrase "considerable period" into the FOMC statement which is interpreted as indicating concern for the "jobless" nature of the recovery. They find that sensitivity of 10-year Treasury yields to news regarding nonfarm payroll employment is higher after this change.

The existing empirical work examining FOMC communications more generally has followed a natural progression[2]. Some of the first papers in this area focused on assessing whether central bank communications move markets per se by examining their effect on financial market volatility. Two notable papers in this category are Kohn and Sack (2004) and Gürkaynak, Sack, and Swanson (2005a) which both provide evidence that the statements that accompany FOMC meetings have an effect on financial market variables beyond the target change itself.

Once it was established that financial markets do indeed respond to communications, attention was turned towards the question of whether these responses are in the expected directions. The earlier studies categorized communications as "hawkish" versus "dovish"

[2] Much of this literature is reviewed in Blinder, Ehrmann, Fratzscher, Haan, and Jansen (2008).

through authors' readings of FOMC communications (Bernanke, Reinhart, and Sack (2004)). Recent papers have turned to more objective methods used in computer science to perform this quantification (Lucca and Trebbi (2011)).

More recently, some authors have made efforts to quantify FOMC communications along more dimensions than just monetary policy stance. Boukus and Rosenberg (2006) uses latent semantic analysis to extract themes from FOMC meeting minutes and show that the prevalence of these themes have an effect on Treasury yields beyond just the release of minutes. One drawback of this method is that the extracted themes are not readily interpretable since they are linear combinations of underlying topics that explain the most variation in the prevalence of words across documents. The true underlying topics are not separately identified. In this paper, I measure the extent of discussion about the topic of labor within FOMC communications by enumerating usage of labor-related words which I later define[3]. This improves interpretability of the measure at the expense of objectivity.

Text analysis methods from computer science have been used more extensively in other contexts in economics. Gentzkow and Shapiro (2010) construct an index of media slant of newspapers by comparing the language to that in the 2005 *Congressional Record*. Antweiler and Frank (2004) and Tetlock (2007) construct measures of investor sentiment form stock market message boards and the *Wall Street Journal's* daily "Abreast of the Market" column.

3.2 FOMC text data

3.2.1 Background on FOMC communications

The primary texts to be analyzed in this paper are the FOMC meeting minutes and policy statements. The most timely communication issued by the FOMC regarding monetary policy is the post-meeting policy statement. This document first appeared following the February

[3]Gorodnichenko and Shapiro (2007) use a similar method to measure the Fed's commitment to price-level targeting rather than inflation targeting during the tenures of recent FOMC chairmans.

1994 meeting. In mid-1999, the Committee began issuing statements following meetings in which there had not been a policy change and it was announced in January 2000 that statements would be issued following all regularly scheduled meetings.

Meeting minutes give a more extensive summary of the issues discussed at each FOMC meeting. The publication of minutes in their present form began with those of the February 1993 meeting. The current minutes combine material previously covered by two separate documents: the *Record of Policy Actions* and the *Minutes of Actions*. Prior to December 2004, minutes were published approximately three days following the next meeting. Since then, publication has been accelerated to three weeks following the meeting.

In addition to these documents, the FOMC also releases lightly edited meeting transcripts which are the most detailed record of meeting proceedings available. Due to their five-year publication lag, transcripts are not being examined in the current paper as a form of FOMC communication.

Lastly, some papers have also looked at central bankers' speeches, interviews, congressional testimonies, papers, and books as forms of central bank communication. Kohn and Sack (2004) show that congressional testimony by Chairman Greenspan has a significant effect on the unexplained variance of changes in various Treasury yields and interest rate futures while his speeches do not. Ehrmann and Fratzscher (2005) find that asset markets reacted more strongly to speeches, interviews, and testimony by Chairman Greenspan than those by other FOMC members.

3.2.2 Processing text

For the analysis, I will be using statements and minutes from meetings occurring between January 1996 and January 2014[4]. For statements, I remove the title of the press release and procedural statements from the text. The procedural statements that are removed

[4]Some of the original texts were obtained from the data accompanying Zadeh and Zollmann (2009) which is available from the authors' websites. The remaining texts were downloaded from the Federal Reserve's website.

include the sentences indicating members who voted in favor of the policy action as well as member absences. Sentences describing dissenting votes are kept because these sometimes contain information regarding the reason for dissent which may have economic content. Sentences stating the discount rate action and the associated requests made by various Reserve Banks are also removed as these do not contain economic content. For minutes, I follow Boukus and Rosenberg (2006) in removing administrative items and only keeping sections of text containing economic content. This text mainly consists of the part of the minutes starting with a phrase similar to “The Committee then turned to a discussion of the economic outlook," or, “The information reviewed at this meeting...”, but also includes discussion regarding special studies conducted by the Federal Reserve staff or statements regarding unconventional policy during the recent period.

After this pre-processing, I transform the remaining text into numeric data using techniques common to many natural language processing procedures. First, I remove formatting, punctuation, capitalization, and numbers. I then remove *stop words* which are commonly used words such as "the", "and", "a", "that", etc.[5]. Next, the remaining words are stemmed using the Porter Stemmer[6] to reduce them to their roots. Finally, for each word within a document, I calculate its proportion of use within the document so that each document is ultimately represented by a vector of word use proportions which sums to 1.

Figure 3.1 shows an original FOMC statement and the list of words that remains after this procedure.

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[5] I use the list of stop words provided by Jason Chen and Siamak Faridani as part of their Natural Language Processing toolbox for Matlab. The full list is available at https://github.com/faridani/MatlabNLP/blob/master/nlp%20lib/corpora/English%20Stop%20Words/english.stop.

[6] Implementations of this algorithm in various programming languages are available at http://tartarus.org/martin/PorterStemmer/index.html.

FRB: Press Release–FOMC statement–January 22, 2008

The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent.

The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth. While strains in short-term funding markets have eased somewhat, broader financial market conditions have continued to deteriorate and credit has tightened further for some businesses and households. Moreover, incoming information indicates a deepening of the housing contraction as well as some softening in labor markets.

The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully.

Appreciable downside risks to growth remain. The Committee will continue to assess the effects of financial and other developments on economic prospects and will act in a timely manner as needed to address those risks.

Voting for the FOMC monetary policy action were: Ben S. Bernanke, Chairman; Timothy F. Geithner, Vice Chairman; Charles L. Evans; Thomas M. Hoenig; Donald L. Kohn; Randall S. Kroszner; Eric S. Rosengren; and Kevin M. Warsh. Voting against was William Poole, who did not believe that current conditions justified policy action before the regularly scheduled meeting next week. Absent and not voting was Frederic S. Mishkin.

In a related action, the Board of Governors approved a 75-basis-point decrease in the discount rate to 4 percent. In taking this action, the Board approved the requests submitted by the Boards of Directors of the Federal Reserve Banks of Chicago and Minneapolis.

↓

feder open market committe decid lower target feder fund rate basi point percent committe action view weaken econom outlook increas downsid risk growth strain short term fund market eas broader financi market condit continu deterior credit tighten busi household incom inform deepen hous contract soften labor market committe expect inflat moder quarter continu monitor inflat develop carefulli appreci downsid risk growth remain committe continu assess effect financi develop econom prospect act time manner address risk vote william pool current condit justifi polici action regularli schedul meet week

Figure 3.1: Example of text processing

Table 3.1 shows some properties of the processed texts. As can be seen from these word counts, FOMC statements are much more succinct than meeting minutes and also use less variety of language.

Table 3.1: Properties of FOMC minutes and statements

	Meeting minutes	Statements
Total # of documents	145	134
Words per document		
Mean	2571	144
Median	2196	95
Min	1521	42
Max	4444	454
Number of unique words across all documents	3500	882

Furthermore, statements have evolved more over time than minutes. Since the start of the recent recession, the FOMC has been trying to convey more information in the post-meeting statements. This change in the nature of FOMC statements is apparent in Figure 3.2. This graph plots the word counts of both texts as ratios of their pre-2008 averages. The figure shows that the word counts of statements were relatively stable until the recent period, but are now more than 5 times as high as the pre-2008 average. Word counts of meeting minutes have also grown during the recent recession, but not nearly as dramatically.

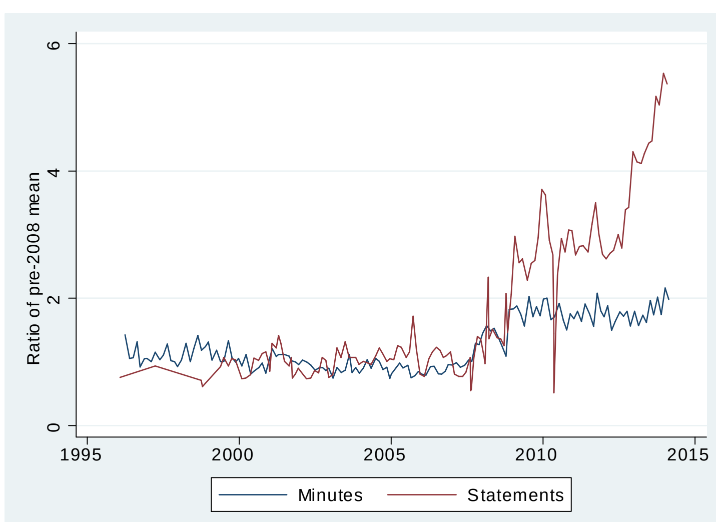

Figure 3.2: Word counts as a ratio of pre-2008 averages

Due to the greater detail included in FOMC meeting minutes, it may be reasonable to believe that financial market participants extract more information regarding the Committee's decision-making process from these documents. For both this reason and the apparent evolution of the nature of FOMC statements, I will place greater emphasis on the results below which involve minutes though results involving statements will also be presented.

3.2.3 Prevalence of labor-related words

To obtain a measure of the extent to which FOMC texts emphasized the topic of labor, I define a set of labor-related words from the set of unique (stemmed) words across both minutes and statements which are displayed in the following table[7].

Table 3.2: Labor-related words

emploi	job	nonemploye	unemploi	work
employ	jobless	nonlabor	unemploy	worker
employe	jobseek	payrol	vacanc	workforc
hire	labor	underemploi	vacant	workweek

I then calculate the proportions of words in each document which appear within this set to arrive at an index of labor word use for FOMC minutes and statements. The index is dated according to the release date of each document[8]. Figure 3.3 shows the evolution of this index for both FOMC minutes and statements.

[7] This method is admittedly highly subjective and alternative methods are discussed in Section 3.7.1.

[8] The release dates for documents were scraped from the historical calendar of monetary policy press releases available at http://www.federalreserve.gov/newsevents/press/monetary/2014monetary.htm. One correction was made for the publication of the December 19, 2000 meeting minutes which took place on February 1, 2001 rather than January 4, 2001.

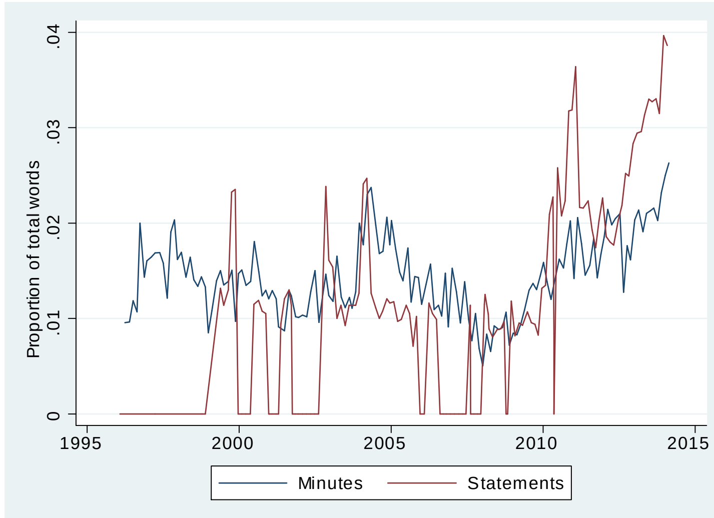

Figure 3.3: Labor word use in FOMC texts

As can be seen, the index for FOMC statements displays higher variance than that for minutes. 40 statements out of a total of 134 contain none of the words in the above set of labor-related words and the prevalence of labor-related language in statements has grown dramatically during the most recent recession. Labor word use in FOMC minutes evolves more smoothly while reflecting the same broad patterns of a decline in labor-related discussion over the 2004-2008 period with a steady increase over the recent recession.

3.3 Estimating interest rate sensitivity to news

I follow the methods of Swanson and Williams (2014) to estimate interest rate sensitivity to news. The main departure is that I categorize news as being related to labor or not and separately estimate sensitivity to labor-specific news versus other news. For the first exercise, I estimate these time-varying sensitivities and then relate the FOMC's use of labor-related language to the estimated differential sensitivity to labor-specific news relative to other news. In Section 3.5, I take a more parametric approach and estimate a single equation that models sensitivity to labor-specific news as linear in the FOMC's labor-related word use while controlling for more general time-variation in sensitivity to all news.

3.3.1 Interest Rates and News Data

The interest rates I will examine include the secondary market rate on the 6-month Treasury bill (obtained from the St. Louis Fed's FRED database), a 1-year forward rate 4 years ahead, as well as 1, 2, 5, and 10 year Treasury yields from Gürkaynak, Sack, and Wright (2007)[9]. I consider the same twelve macroeconomic data releases as Swanson and Williams (2014) and these are: initial jobless claims, nonfarm payroll employment, the unemployment rate, core CPI inflation, core PPI inflation, consumer confidence, capacity utilization, new home sales, leading indicators index, ISM manufacturing index, real GDP growth (advance), and retail sales. The first three items in this list are categorized as being labor-specific. To measure the news content within these data releases, I take the difference between the actual release and the median forecasts reported by Money Market Services[10]. Each individual news series is divided by its standard deviation to facilitate comparison of coefficients across different news series. As a robustness check, I sometimes include federal funds target surprises (computed following the method in Kuttner (2001)) in the other news category. Each news series is set to zero on days when there is no data release. The signs of unemployment rate and initial jobless claims surprises are flipped so that all positive surprises represent favorable news.

3.3.2 Time-varying sensitivity to labor news

In this section, I follow the two-step estimation process used in Swanson and Williams (2014) to arrive at daily estimates of sensitivity to labor and all other news. The first step involves estimating the following equation using nonlinear least squares.

$$\Delta i_t = \alpha_s + \delta_s \beta News_t^{labor} + \zeta_s \gamma News_t^{other} + \varepsilon_t \tag{3.1}$$

[9] Daily yields data are updated regularly and available from http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html

[10] I'd like to thank Ali Ozdagli and Michelle Barnes for their help in obtaining this data.

where Δi_t is the one-day change in the relevant interest rate, $News_t^{labor}$ is a vector of the three labor-related news realizations on date t, and $News_t^{other}$ is a vector of the nine other news realizations on date t. α, δ, and ζ are allowed to vary over calendar years s with δ and ζ normalized to average to 1 over the 1990-2000 period. Then, β and γ can be interpreted as coefficients representing the average contribution of individual news releases to interest rate changes during the baseline 1990-2000 period.

In the second stage of this estimation, the estimated $\hat{\beta}$ and $\hat{\gamma}$ vectors from the first step are used to construct one-dimensional series of labor and other news

$$\widehat{News_t^{labor}} \equiv \hat{\beta} News_t^{labor} \quad \text{and} \quad \widehat{News_t^{other}} \equiv \hat{\gamma} News_t^{other}$$

Then, rolling regressions with one year windows[11] whose midpoint is date τ are estimated at a daily frequency to obtain daily estimates of sensitivity to labor and other news.

$$\Delta i_t = \gamma_\tau + \delta_\tau \widehat{News_t^{labor}} + \zeta_\tau \widehat{News_t^{other}} + \varepsilon_t \tag{3.2}$$

As shown in Swanson and Williams (2014), there is time-variation in sensitivity of interest rates to macroeconomic news in general, even prior to the recent zero lower bound episode. This method allows me to focus on the differential sensitivity to labor news in particular while controlling for time-variation in interest rate sensitivity to all news in general.

All the regressions in this section are run over the January 1, 1990 to September 30, 2012 sample (excluding the week following September 11, 2001) where only days containing at least one news item are included.

Table 3.3 shows the nonlinear least squares estimates for β and γ in (3.1). These estimates are broadly in line with those presented in Swanson and Williams (2014) which come from estimating (3.1) with the restrictions $\delta_s = \zeta_s \ \forall s$. There are some patterns evident from these estimates such as the relatively greater sensitivity of higher maturity yields to core

[11] Strictly speaking, the windows include the most recent 252 trading days which only approximately corresponds to a calendar year.

PPI inflation and the relatively greater sensitivity of lower maturity yields to the leading indicators index and real GDP growth.

Table 3.3: Interest rate sensitivity to individual news items

	6-Month	1-Year	2-Year	5-Year	10-Year	1-Year Fwd 4 Yrs Ahead
Initial Claims	0.57***	0.91***	1.02***	0.96***	0.77***	0.70***
	[3.74]	[4.91]	[4.97]	[4.55]	[3.86]	[3.14]
Nonfarm Payrolls	3.30***	3.63***	4.12***	3.49***	2.46***	2.19***
	[7.64]	[7.98]	[7.82]	[6.51]	[5.07]	[3.96]
Unemployment Rate	0.94***	1.05***	1.08**	0.86**	0.52*	0.45
	[2.88]	[2.83]	[2.57]	[2.28]	[1.79]	[1.50]
Core CPI Inflation	0.99***	2.13***	2.54***	2.63***	2.28***	2.56***
	[3.04]	[4.98]	[5.05]	[5.03]	[4.52]	[4.37]
Core PPI Inflation	−0.02	0.41	0.57	0.65	0.96**	0.33
	[−0.09]	[0.88]	[1.19]	[1.44]	[2.27]	[0.77]
Consumer Confidence	0.83***	1.79***	1.90***	1.79***	1.64***	1.23**
	[2.98]	[4.07]	[3.90]	[3.28]	[3.22]	[2.09]
Capacity Utilization	−0.23	1.73**	2.35***	2.46***	1.64***	1.94***
	[−0.29]	[1.98]	[3.05]	[3.76]	[3.05]	[3.34]
New Home Sales	0.75**	1.26***	1.48***	1.63***	1.57***	1.58***
	[2.11]	[2.95]	[3.15]	[3.56]	[3.80]	[3.32]
Leading Indicators	2.08***	0.76	0.37	0.07	0.15	−0.53
	[2.66]	[1.25]	[0.66]	[0.12]	[0.29]	[−0.83]
ISM Manufacturing	1.07***	2.91***	3.77***	3.82***	3.27***	3.30***
	[2.60]	[6.49]	[7.34]	[7.37]	[6.79]	[5.77]
Real GDP (adv)	0.72**	1.03*	1.05	0.71	0.41	0.72
	[2.02]	[1.80]	[1.36]	[0.78]	[0.52]	[0.78]
Retail Sales	1.07**	2.12***	2.45***	2.45***	2.49***	1.77***
	[2.55]	[3.65]	[3.99]	[3.91]	[3.90]	[2.64]
Adjusted R^2	0.16	0.16	0.16	0.12	0.1	0.07
N	2801	2801	2801	2801	2801	2801

Notes: ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are given in brackets.

Figure 3.4 plots daily estimates of sensitivity to labor news and other news obtained from rolling regressions of (3.2). The solid portions of each line indicate that the estimates are positive and significant at the 10% level while dotted portions indicate estimates that are not significantly different from zero. None of the estimates are significantly negative[12]. Due to the normalization of δ_s and ζ_s in the estimation of (3.1), the magnitudes of these estimates can be interpreted as sensitivity relative to the "normal" 1990-2000 period. These estimates show appreciable differences in interest rates' sensitivity to labor news apart from other news. This is especially apparent for longer maturity yields where the sensitivity to labor news rose to more than four times the baseline level in the post-2004 period while sensitivity to other news was rarely more than double its baseline level.

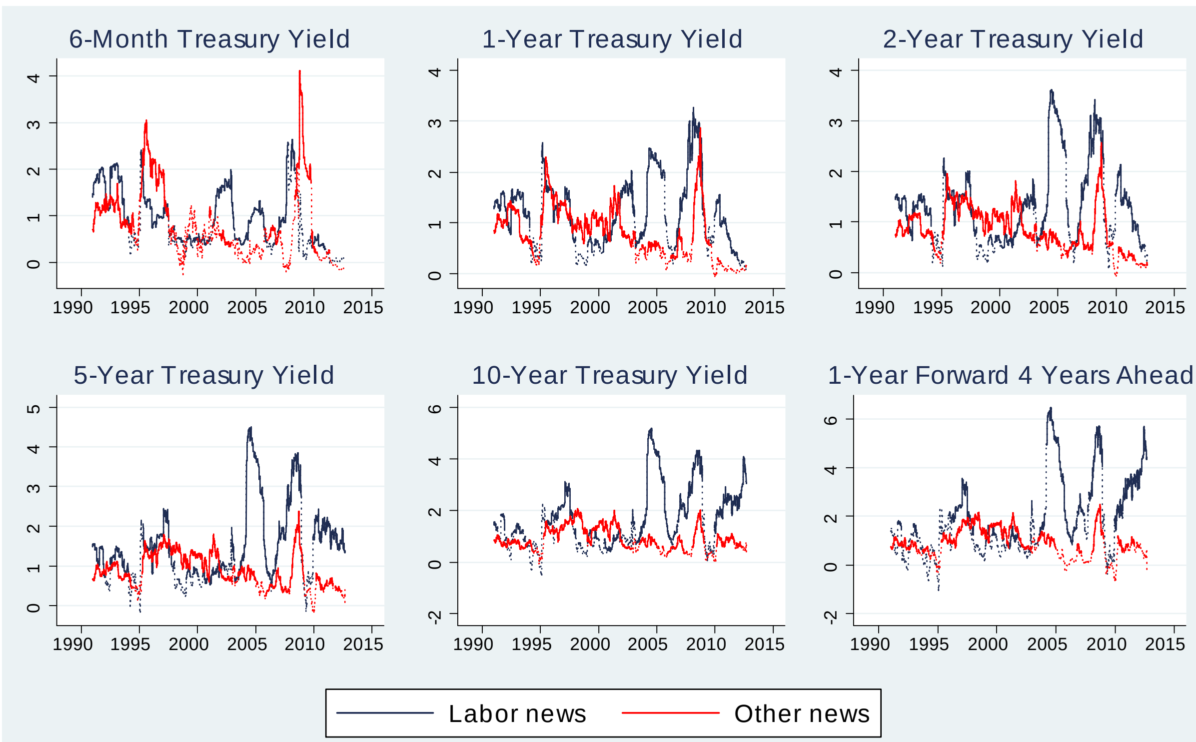

Figure 3.4: Time-varying sensitivity of interest rates to labor news and other news

[12] I follow Swanson and Williams (2014) in adjusting the standard errors for the use of generated regressors by using the estimated standard errors of the $\{\delta_s\}$ and $\{\zeta_s\}$ from the estimation of (3.1) as benchmarks. Since these annual estimates will correspond with those of (3.2) when the rolling window covers the calendar year, the difference in standard errors of these estimates is attributable to the use of generated regressors in (3.2). This gives a scaling factor for those dates where the window corresponds to calendar years. Linear interpolation is then used to obtain scaling factors for the intervening dates.

3.4 Relating sensitivity to FOMC labor word use

In this section, I explore the relationship between the FOMC's labor word use and the estimated differential sensitivity of interest rates to labor news over other news. Figure 3.5 plots these series with labor word use from FOMC minutes. In the plots, the relationships appear tighter for interest rates of longer maturities and this feature is confirmed below.

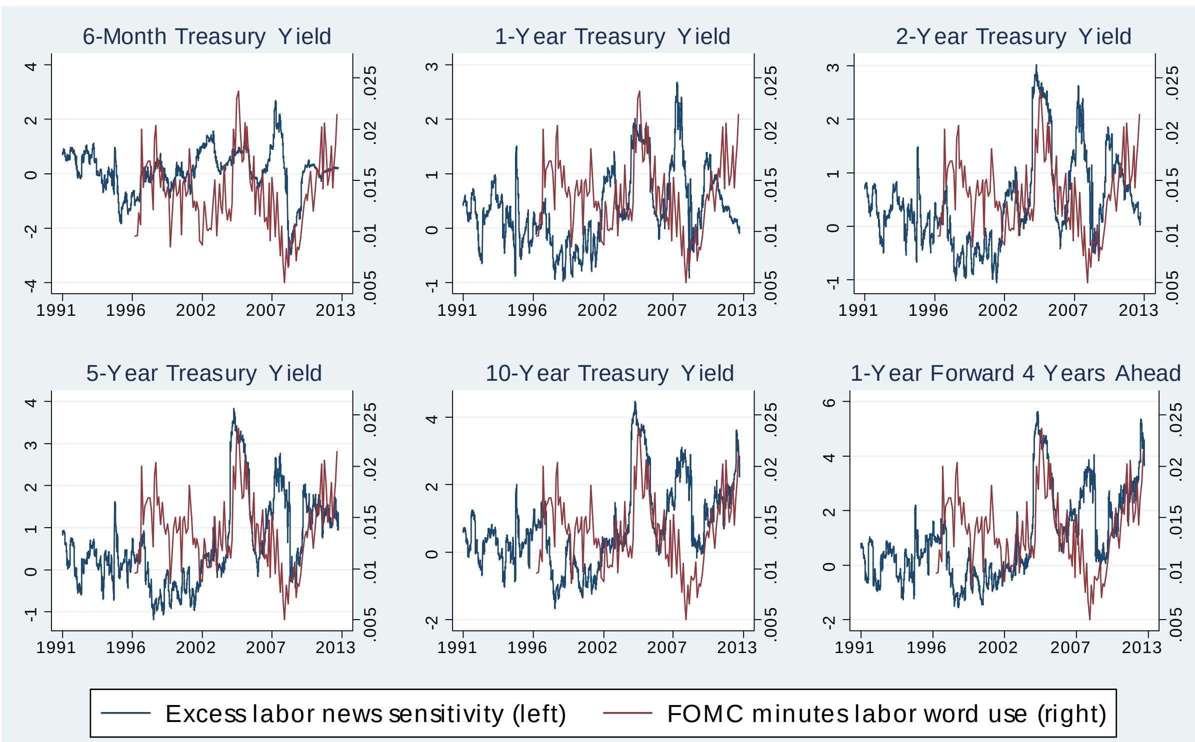

Figure 3.5: FOMC minutes labor word use and differential sensitivity to labor news

For the remainder of this section, the units of observation are periods of time between either FOMC minutes or statements. The sensitivity measures are averages of the daily estimates between the current and next publication date of the relevant text. Unless otherwise indicated, the analyses are run over the January 1996 to September 2012 period[13].

The next two tables give correlations which show more clearly that the relationship between FOMC word use and differential sensitivity to labor news is closer for interest rates

[13] The results are not sensitive to starting the analysis for FOMC statements in January 2000, when the committee began regularly issuing statements following every meeting.

of longer maturities. In each table, the first three lines show correlations with labor word use in the current release using samples starting in different years. The last three lines do the same with a moving average of word use from the four most recent releases. Correlations for FOMC minutes grow stronger towards the latter part of the sample which may be due to increased financial market attention to the meeting minutes.

Table 3.4: Correlations of differential sensitivity to labor news and labor word use in FOMC minutes

	6-Month	1-Year	2-Year	5-Year	10-Year	1-Year Fwd 4 Yrs Ahead	N
Current release							
Starting in 1996	0.23***	0.04	0.14	0.19**	0.18**	0.24***	129
Starting in 2000	0.22**	0.12	0.25**	0.32***	0.32***	0.39***	103
Starting in 2004	0.37***	0.18	0.30**	0.38***	0.39***	0.46***	70
4-release MA							
Starting in 1996	0.28***	−0.01	0.09	0.15*	0.15*	0.21**	126
Starting in 2000	0.31***	0.12	0.23**	0.32***	0.32***	0.39***	103
Starting in 2004	0.45***	0.19	0.29**	0.38***	0.39***	0.47***	70

Notes: ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table 3.5: Correlations of differential sensitivity to labor news and labor word use in FOMC statements

	6-Month	1-Year	2-Year	5-Year	10-Year	1-Year Fwd 4 Yrs Ahead	N
Current release							
Starting in 1996	−0.13	−0.08	0.09	0.20**	0.23**	0.33***	118
Starting in 2000	−0.17*	−0.14	0.04	0.17*	0.20**	0.31***	112
Starting in 2004	−0.09	−0.25**	−0.07	0.07	0.12	0.31***	76
4-release MA							
Starting in 1996	−0.17*	−0.12	0.07	0.21**	0.26***	0.38***	115
Starting in 2000	−0.19**	−0.17*	0.03	0.18*	0.23**	0.36***	112
Starting in 2004	−0.11	−0.32***	−0.12	0.04	0.13	0.33***	76

Notes: ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Next, I present results of regressing differential labor news sensitivity on the word use measures. In Figure 3.5, it appears that there may be some growth in the differential sensitivity to labor news over time (particularly in the longer maturity interest rates) so I also present results of regressing differential sensitivity to labor news on the word use measures while additionally controlling for a time trend. The coefficients presented are standardized so that the magnitudes of the effect are in standard deviation units for both the left- and right-hand side variables.

Table 3.6 presents the results for labor word use in FOMC minutes with interest rates of different maturities in columns and different specifications in rows. There first two specifications use the labor word use from the current release and a four-release moving average, respectively. These results reflect the correlations shown in rows 1 and 4 of Table 3.4. The third specification includes the word use from the four most recent releases in an unconstrained manner. The table presents the sum of coefficients on all the lags along with the p-values of F-tests of these sums being greater than zero. It's apparent that these results are little changed from those for the moving average. The final three specifications repeat the first three with the addition of a time trend. It's clear that the addition of a time trend results in a more tightly identified effect of labor word use in FOMC minutes on differential sensitivity of interest rates to labor-related news while the point estimates actually increase slightly. Furthermore, the addition of a time trend greatly improves the fit as reflected in the higher adjusted R^2 values.

Table 3.6: Regressions of differential sensitivity to labor news on labor word use in FOMC minutes

	6-Month	1-Year	2-Year	5-Year	10-Year	1-Year Fwd 4 Yrs Ahead	N
Current release	0.23**	0.04	0.14	0.19*	0.18*	0.24**	129
	[2.27]	[0.37]	[1.44]	[1.89]	[1.76]	[2.39]	
time trend	no	no	no	no	no	no	
Adjusted R^2	0.05	−0.01	0.01	0.03	0.03	0.05	
4-release MA	0.28**	−0.01	0.09	0.15	0.15	0.21**	126
	[2.49]	[−0.14]	[0.94]	[1.52]	[1.55]	[2.20]	
time trend	no	no	no	no	no	no	
Adjusted R^2	0.07	−0.01	0.00	0.01	0.02	0.04	
Sum of coefficients on 4 latest releases	0.33**	−0.02	0.11	0.18	0.18	0.25**	126
F-test p-value	0.02	0.88	0.38	0.15	0.14	0.03	
time trend	no	no	no	no	no	no	
Adjusted R^2	0.05	−0.02	−0.00	0.01	0.00	0.03	
Current release	0.23**	0.05	0.16	0.21**	0.20**	0.26***	129
	[2.28]	[0.48]	[1.59]	[2.22]	[2.19]	[3.14]	
time trend	yes	yes	yes	yes	yes	yes	
Adjusted R^2	0.04	0.17	0.22	0.34	0.33	0.44	
4-release MA	0.28**	0.03	0.14	0.21**	0.21**	0.28***	126
	[2.54]	[0.30]	[1.45]	[2.36]	[2.53]	[3.69]	
time trend	yes	yes	yes	yes	yes	yes	
Adjusted R^2	0.07	0.16	0.21	0.35	0.36	0.48	
Sum of coefficients on 4 latest releases	0.33**	0.03	0.16	0.24**	0.25**	0.33***	126
F-test p-value	0.01	0.78	0.17	0.03	0.02	0.00	
time trend	yes	yes	yes	yes	yes	yes	
Adjusted R^2	0.05	0.15	0.21	0.34	0.35	0.47	

Notes: ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are in brackets. Standardized coefficients are shown for ease of interpretation.

Table 3.7 presents the corresponding results for labor word use in FOMC statements. Again, it's clear that there is a stronger relationship for interest rates of greater maturities. Unlike FOMC minutes, these results are more sensitive to the addition of a time trend since labor word use in FOMC statements is more correlated with time[14].

Table 3.7: Regressions of differential sensitivity to labor news on labor word use in FOMC statements

	6-Month	1-Year	2-Year	5-Year	10-Year	1-Year Fwd 4 Yrs Ahead	N
Current release	−0.13*	−0.08	0.09	0.20**	0.23***	0.33***	118
	[−1.92]	[−0.93]	[1.10]	[2.53]	[2.69]	[4.01]	
time trend	no	no	no	no	no	no	
Adjusted R^2	0.01	−0.00	−0.00	0.03	0.04	0.10	
4-release MA	−0.17**	−0.12	0.07	0.21***	0.26***	0.38***	115
	[−2.45]	[−1.44]	[0.89]	[2.80]	[3.28]	[5.00]	
time trend	no	no	no	no	no	no	
Adjusted R^2	0.02	0.01	−0.00	0.04	0.06	0.14	
Sum of coefficients on 4 latest releases	−0.20**	−0.14	0.08	0.26***	0.31***	0.46***	115
F-test p-value	0.02	0.16	0.37	0.01	0.00	0.00	
time trend	no	no	no	no	no	no	
Adjusted R^2	−0.01	−0.02	−0.03	0.01	0.03	0.12	
Current release	−0.08	−0.22**	−0.06	0.01	0.02	0.11	118
	[−0.81]	[−2.20]	[−0.60]	[0.14]	[0.25]	[1.28]	
time trend	yes	yes	yes	yes	yes	yes	
Adjusted R^2	0.02	0.09	0.10	0.21	0.25	0.35	
4-release MA	−0.09	−0.26**	−0.08	0.02	0.04	0.15*	115
	[−0.94]	[−2.61]	[−0.83]	[0.18]	[0.49]	[1.79]	
time trend	yes	yes	yes	yes	yes	yes	
Adjusted R^2	0.03	0.09	0.09	0.19	0.25	0.36	
Sum of coefficients on 4 latest releases	−0.11	−0.32**	−0.10	0.02	0.05	0.18*	115
F-test p-value	0.36	0.01	0.41	0.88	0.66	0.09	
time trend	yes	yes	yes	yes	yes	yes	
Adjusted R^2	0.01	0.07	0.06	0.17	0.23	0.34	

Notes: ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are in brackets. Standardized coefficients are shown for ease of interpretation.

[14] Over the sample used in Tables 3.6 and 3.7, a time trend alone explains more than 20% of the variation in labor word use in FOMC statements, but less than 0.2% for FOMC minutes.

3.5 A more parametric approach

In this section, I consider a more parametric estimation where the differential sensitivity to labor news is constrained to be a linear function of FOMC labor word use (along with a time trend in some specifications). I continue to control for time-variation in sensitivity to all news at the same frequency as the relevant FOMC text. That is, I use nonlinear least squares to estimate

$$\Delta i_t = \alpha_r + f\left(WU_r\right)\beta News_t^{labor} + \zeta_r^{all}\left(\beta News_t^{labor} + \gamma News_t^{other}\right) + \varepsilon_t \tag{3.3}$$

where the differences from (3.1) are that α and ζ^{all} are now allowed to vary at the frequency of FOMC releases. Constraints are imposed such that the values of $f\left(WU_r\right)$ and ζ_r^{all} average to 1 over the 1996-2000 period to allow for identification of β and γ.

Table 3.8 shows the results from estimating (3.3) for both indices of word use. The first specification uses the labor word use from the current release of FOMC minutes without a time trend while the second specification includes a time trend. The last two specifications are the same results for labor word use in FOMC statements. For these regressions, the word use measures are divided by their standard deviation while the interest rate changes are kept in unadjusted levels. Thus, the interpretation of the coefficients differs slightly from those in Tables 3.6 and 3.7. Magnitudes aside, these results reflect the same patterns evident in Section 3.4 which are that an increase in labor-related word use in FOMC communications are associated with larger responses of interest rates to labor-related news after controlling for general time variation in the size of responses to all news. Furthermore, the relationship appears to be slightly stronger for FOMC minutes than statements perhaps owing to the greater detail contained in FOMC minutes regarding the decision process of Committee members. Again, the table shows that these results are robust to the addition of a time-trend in $f\left(WU_r\right)$.

Table 3.8: Parametric nonlinear regressions with differential sensitivity to labor news being a linear function of labor word use in FOMC texts

	1-Year	2-Year	5-Year	10-Year	1-Year Fwd 4 Yrs Ahead	N
Minutes (current release)	0.34	1.20**	1.41*	1.86*	4.26	2093
	[0.92]	[2.24]	[1.93]	[1.76]	[1.43]	
time trend	no	no	no	no	no	
Adjusted R^2	0.16	0.16	0.14	0.11	0.10	
Minutes (current release)	0.34	1.12**	1.40**	1.91*	4.31	2093
	[0.95]	[2.15]	[2.00]	[1.78]	[1.54]	
time trend	yes	yes	yes	yes	yes	
Adjusted R^2	0.16	0.16	0.14	0.11	0.10	
Statement (current release)	0.26	0.76*	1.20**	1.04*	1.29*	2116
	[0.94]	[1.93]	[2.40]	[1.88]	[1.86]	
time trend	no	no	no	no	no	
Adjusted R^2	0.17	0.16	0.14	0.10	0.09	
Statement (current release)	0.65	1.16**	1.50**	1.09	0.93	2116
	[1.40]	[2.05]	[2.21]	[1.56]	[1.11]	
time trend	yes	yes	yes	yes	yes	
Adjusted R^2	0.17	0.16	0.14	0.10	0.09	

Notes: ***/**/* Statistically significant at 1, 5, and 10 percent, respectively. Heteroskedasticity-consistent t-statistics are in brackets.

3.6 Robustness checks

As the above tables indicate, the relationship between FOMC word use and differential sensitivity of interest rates to labor news beyond sensitivity to general news is robust to the inclusion of time trends. The analysis above was also repeated with federal funds rate surprises included in the category of other non-labor news. The addition of this piece of news helps to smooth out the rolling estimates of sensitivity to other news so that the large spikes present in the latter part of the sample are eliminated. The resulting estimates of differential sensitivity to labor news retain their positive relationship with labor word use in both FOMC minutes and statements. Repeating the parametric exercise in the previous section with federal funds rate surprises in the $News_t^{other}$ vector also yields results similar to those in Table 3.8.

The results of the parametric exercise in the previous section were also robust to controlling for time-variation in sensitivity to all news at a coarser annual frequency. Including signed squared news terms in sensitivity estimates to control for the possibility of responses being greater for larger surprises did not alter the results either.

3.7 Future work

3.7.1 Alternative text analysis methods

The measure of the extent of labor-related discussion in FOMC texts used in the above analysis sacrificed objectivity for ease of interpretation. One way to discipline this process is to use a more objective selection process to select labor-related words. One method that can be explored in future work is to measure cosine similarity (commonly used in latent semantic analysis) between FOMC texts and other labor-related documents. There are several natural choices for these external documents. To maintain objectivity, one can use the press releases pertaining to the same labor news variables used in the sensitivity estimates. Another possibility is to use speeches made by FOMC members which are about the labor market as indicated by the speech title. This latter method maintains some subjectivity in the classification of speech topics, but it could yield a better approximation to the specific language used by central bankers in discussing labor market conditions.

Another approach is to model a wider class of topics rather than restricting attention to a single topic. As discussed in the introduction, this is the approach taken by Boukus and Rosenberg (2006). Broadly speaking, they apply principal components analysis to the vectors of word use proportions that represent each document. Therefore, their analysis suffers from the usual problem that the themes (or principal components) are identified only up to a rotation and they are unknown linear combinations of the true underlying topics. To build on this method, one can attempt to extract the underlying topics by imposing restrictions on the resulting topics or by bringing in extra information from external documents like the ones discussed in the previous paragraph.

More generally, one could apply existing topic modeling methods to FOMC texts. The idea behind these methods is to model documents as being unknown combinations of underlying topics while topics themselves are modeled as parametric distributions over words. This generates a likelihood function so that the underlying parameters governing the document generating process may be estimated using either maximum likelihood or Bayesian methods. It may be possible to use information from external documents in these methods as well. One natural use for them is to generate priors on the distributions over words that represent each topic.

3.7.2 Dealing with endogeneity

The preceding analysis established a positive relationship between labor word use in FOMC texts and interest rate sensitivity to labor news. However, it does not attempt to establish causation. It may be possible that there are events driving both interest rate sensitivity to certain types of news and increased discussion of those same topics in FOMC texts. It could also be the case that the FOMC discusses some topics more because they observe financial markets exhibiting greater sensitivity to certain events.

There are several possibilities for further exploration of this issue. One method is to use additional controls that may be driving both sensitivity and word use. In terms of economic variables, a natural set of controls to use are the variables underlying the labor news measures used above. Preliminary analysis shows that the results given above are robust these additional controls. Alternatively, one could also control for word use measures from other documents that could provide a good summary of economic conditions that may be driving both interest rate sensitivity and word use in FOMC texts. These other documents may include financial news or FOMC meeting transcripts. The transcripts are the most accurate account of meeting discussions available, but are published with a five year lag. Differences in word use between FOMC minutes or statements and these other documents might better reflect a "pure communication" component. It may also be possible to exploit changes in chairmans or members of the Committee over time.

3.7.3 Decomposing sensitivity to news

One last conceptual issue that remains is the interpretation of time variation in interest rate sensitivity to news. Roughly speaking, the response of asset prices to macroeconomic news can be broken down into two components: (i) the amount by which market participants update their beliefs about underlying state variables in response to the news and (ii) the effect that these state variables have on asset prices[15]. The first component can be interpreted as the informativeness of news while, in the case of Treasury yields, the second component captures both the policy reaction function and the effect of the state variables on term premia. Hence, FOMC communication that indicates a change in the reaction function should work through this second component. Thus, it would be useful to further decompose asset price responses to macroeconomic news in order to both better understand the factors driving time-variation in sensitivity as well as to assess how changes in language used in FOMC texts affect these factors.

3.8 Conclusion

In this paper, I presented a novel measure of the extent to which FOMC texts were skewed towards labor-related language used in FOMC texts. This measure marks one of the first attempts to quantify FOMC communications along a dimension other than the intended direction of future policy rates. I then showed an interaction effect where an increase in labor-related word use in FOMC texts is positively associated with the extent to which interest

[15] Faust, Rogers, Wang, and Wright (2007) provides a succinct mathematical exposition of this idea.

rates' response to labor-related news exceeds their response to all other news. Furthermore, the relationship seems to be especially strong for interest rates of longer maturities.

In terms of policy implications, it's not yet clear whether it's desirable for FOMC communications to affect financial market variables in this way. One immediate implication is that increased central bank discussion of specific economic variables could raise financial market volatility in response to data releases. Since there is inherently noise accompanying this news, this increased sensitivity of financial market variables may not be efficient.

Going forward, it would be interesting to extend this type of analysis to more topics as well as other asset classes and news events. It would also be useful to decompose the time-variation in the sensitivity of interest rates to news into changes in beliefs about the central bank policy reaction function versus changes in the informativeness of news or the sensitivity of term premia to news. Relating these different components to FOMC communication will help to illuminate the channels through which communication impacts sensitivity.

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Appendix A

Supplement to Chapter 1

A.1 Aggregate equilibrium conditions with idiosyncratic government spending shocks

In this section, I derive equilibrium conditions for an economy where firms face idiosyncratic government spending shocks. In this environment, it is consistent for consumers and firms not to have information about current aggregate outcomes. This yields a condition for the aggregate output gap which is identical to equation (1.1) in the model in the main text. The inflation condition differs from equation (1.2) in a few ways which I outline at the end of the section.

A.1.1 Setup

The setup shares many features with Lorenzoni (2010). There is a continuum of yeoman farmer households with identical preferences and technology who produce differentiated goods and face a Calvo friction.

Each period contains three stages. In stage 1, the policymaker sees the entire history of aggregate government spending and output gap target levels $\{g^t, \bar{y}^t\}$ and sets the nominal interest rate i_t conditional on these aggregate states. In the private sector, all households have the same beginning-of-period information which contains true realizations of past state variables and the current nominal interest rate so that their Stage 1 information set is $\mathcal{I}_t^1 = \left\{i^t, g^{t-1}, \bar{y}^{t-1}\right\}$. In this stage, pre-commitments are made regarding aggregate nominal consumption.

In stage 2, each worker-firm j now realizes his firm-specific government demand shock, g_{jt}, where the idiosyncratic component of g_{jt} is iid. Firms who are able to reset prices then choose prices based their updated Stage 2 information sets $\mathcal{I}_{jt}^2 = g_{jt} \cup \mathcal{I}_t^1$. I do not include past observations of g_{jt} in these information sets since they are irrelevant for current and future payoffs once g^{t-1} is known. All firms set prices simultaneously so these decisions are made without knowledge of the current aggregate price. The household receives no further information about $\bar{y}_t$.

In stage 3, all prices are revealed and the consumer optimally allocates the pre-committed amount of nominal spending across varieties j. The revelation of prices in this stage also reveals the true aggregate states and households carry this knowledge into Stage 1 of the next period.

Prior to the realizations of $\{g_{jt}\}$, ex-ante risks are the same across households. I assume that households perfectly risk-share by trading in a complete set of contingent claims in Stage 1. These claims pay out in Stage 1 of the next period so that the amount of wealth each consumer starts the period with is the same across agents.

I assume that the idiosyncratic component of government spending is such that the resulting log-linearized total demand faced by each firm j is given by

$$\begin{aligned} y_{jt} &= \frac{C}{Y} c_t + \left(1 - \frac{C}{Y}\right) g_{jt} - \varepsilon \left(p_{jt} - p_t\right) \\ &= y_t + \left(1 - \frac{C}{Y}\right) \omega_{jt} - \varepsilon \left(p_{jt} - p_t\right) \\ \text{since } y_t &= \frac{C}{Y} c_t + \left(1 - \frac{C}{Y}\right) g_t \text{ by market clearing} \end{aligned}$$

where

$$g_{jt} = g_t + \omega_{jt}, \quad \omega_{jt} \sim \text{iid } N\left(0, \sigma_\omega^2\right) \tag{A.1}$$

Meanwhile, I continue to assume AR(1) forms for the aggregate shocks

$$\begin{aligned} g_t &= \rho_g g_{t-1} + \epsilon_{g,t}, \quad \epsilon_{g,t} \sim \text{iid } N\left(0, \sigma_g^2\right) \\ \bar{y}_t &= \rho_{\bar{y}} \bar{y}_{t-1} + \epsilon_{\bar{y},t}, \quad \epsilon_{\bar{y},t} \sim \text{iid } N\left(0, \sigma_{\bar{y}}^2\right) \end{aligned} \tag{A.2}$$

A.1.2 Consumption

Preferences are identical across households and the same as the model in the main text

$$\max E \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t\right) - V\left(L_t\right)\right], \text{ where } C_t \equiv \left[\int_0^1 C_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1$$

All households have access to the same full basket of goods in stage 3 so there's only one relevant aggregate inflation rate. Then, since all households make pre-commitments to nominal spending in Stage 1 based on the same information set and facing the same idiosyncratic risks, they all choose the same aggregate nominal consumption which yields the following Euler equation in log-linearized form

$$c_t = E\left[c_{t+1}|\mathcal{I}_t^1\right] + \frac{U_c}{U_{cc}C}\left(i_t - E\left[\pi_{t+1}|\mathcal{I}_t^1\right]\right)$$

Note that combining this consumption Euler equation with the resource constraint yields the same condition for the aggregate output gap as in equation (1.1) since I can write

$$\begin{aligned}\tilde{y}_t &= E\left[\tilde{y}_{t+1}|\mathcal{I}_t^1\right] - \frac{1}{\sigma}\left(i_t - E\left[\pi_{t+1}|\mathcal{I}_t^1\right]\right) + d_t - E\left[d_{t+1}|\mathcal{I}_t^1\right] \qquad (A.3)\\ \text{where } \tilde{y}_t \equiv y_t - y_t^n &= \frac{C}{Y}c_t + \frac{\varphi}{\sigma+\varphi}\left(1-\frac{C}{Y}\right)g_t\\ \text{and } d_t &\equiv \frac{\varphi}{\sigma+\varphi}\left(1-\frac{C}{Y}\right)g_t\end{aligned}$$

as in the main text and importantly, the information set $\mathcal{I}_t^1$ is also the same as the one used in the main text. This definition of the aggregate demand shock d_t also gives

$$\begin{aligned}d_t &= \frac{\varphi}{\sigma+\varphi}\left(1-\frac{C}{Y}\right)g_t = \rho_d d_{t-1} + \epsilon_{d,t} \qquad (A.4)\\ \text{where } \rho_d = \rho_g \quad &\text{and} \quad \epsilon_{d,t} = \frac{\varphi}{\sigma+\varphi}\left(1-\frac{C}{Y}\right)\epsilon_{g,t}\end{aligned}$$

Purchases of varieties j are made in Stage 3 after prices are revealed so that

$$c_{jt} = c_t - \varepsilon\left(p_{jt} - p_t\right)$$

A.1.3 Production and price-setting

In Stage 2, a worker-firm j learns the government portion of their demand g_{jt} so their information set is $\mathcal{I}_{jt}^2 \equiv \left\{i^t, g^{t-1}, \bar{y}^{t-1}, g_{jt}\right\}$. They face the demand function

$$y_{jt} = \frac{C}{Y}c_t + \left(1-\frac{C}{Y}\right)g_{jt} - \varepsilon\left(p_{jt} - p_t\right)$$

However, they do not see aggregate prices and so they do not know how much they'll ultimately sell for a given price p_{jt}.

Technology is again linear for each worker-firm

$$Y_{jt} = AL_{jt}$$

where the nominal cost of labor (which is a pseudo-wage) is given by the MRS multiplied by the aggregate price index which has the following log-linear form (where φ, σ retain the definitions in (1.3))

$$w_{jt} = \varphi l_{jt} + \sigma \frac{C}{Y} c_t + p_t$$

The log-linearized pricing condition for a firm is then the following

$$\begin{aligned} p_{jt}^* &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E\left[w_{j,t+k} | \mathcal{I}_{jt}^2\right] \\ &= (1-\theta\beta)\left(\sigma \frac{C}{Y} c_t + E\left[\varphi y_{jt}^* + p_t | \mathcal{I}_{jt}^2\right]\right) + \theta\beta E\left[p_{j,t+1}^* | \mathcal{I}_{jt}^2\right] \end{aligned}$$

where I use a star on y_{jt}^* to highlight that reset prices depend on output among price resetters which is different from output among non-resetters. Using the firms' demand function, this can be transformed to

$$p_{jt}^* = (1-\theta\beta)\left((\sigma+\varphi)\frac{C}{Y}c_t + \varphi\left(1-\frac{C}{Y}\right)g_{jt} - \varphi\varepsilon p_{jt}^* + (1+\varphi\varepsilon)E\left[p_t|\mathcal{I}_{jt}^2\right]\right) + \theta\beta E\left[p_{j,t+1}^*|\mathcal{I}_{jt}^2\right]$$

I assume that the Calvo shock is independent of the idiosyncratic component of government spending such that the average government spending shock among price resetters is equal to the average among all the firms. That is, I assume the following where I order firms so that the set of price resetters are those indexed by $j \in [\theta, 1]$

$$\frac{1}{1-\theta}\int_\theta^1 g_{jt} dj = g_t$$

Then, as long as p_{jt}^* is linear in the variables in $\mathcal{I}_{jt}^2$, this gives

$$\frac{1}{1-\theta}\int_\theta^1 p_{jt}^* dj = p_t^* \equiv \int_0^1 p_{jt}^* dj$$

Secondly, I note that the iid nature of the idiosyncratic component of government spending shocks along with the posited linearity of p_{jt}^* implies that

$$E\left[p_{j,t+1}^*|\mathcal{I}_{jt}^2\right] = E\left[p_{t+1}^*|\mathcal{I}_{jt}^2\right]$$

Then, the aggregate price index implies the usual log-linearized first-order dynamics

$$p_t = \theta p_{t-1} + \int_{\theta}^{1} p_{jt}^{*} dj = \theta p_{t-1} + (1-\theta) p_t^{*} \tag{A.5}$$

Then, expectations have to satisfy

$$E\left[p_t | \mathcal{I}_{jt}^2\right] = \theta p_{t-1} + (1-\theta) E\left[p_t^{*} | \mathcal{I}_{jt}^2\right]$$

The aggregate price relation also gives the following property

$$(1-\theta) E\left[p_{t+1}^{*} | \mathcal{I}_{jt}^2\right] = E\left[\pi_{t+1} | \mathcal{I}_{jt}^2\right] + (1-\theta) E\left[p_t | \mathcal{I}_{jt}^2\right]$$

Aggregating the individual reset prices over resetters $j \in [\theta, 1]$ and using these properties then gives

$$(1-\theta) p_t^{*} = \frac{(1-\theta)(1-\theta\beta)(\sigma+\varphi)}{1+(1-\theta\beta)\varepsilon\varphi} \tilde{y}_t + \frac{\theta\beta}{1+(1-\theta\beta)\varepsilon\varphi} E\left[\pi_{t+1} | \mathcal{I}_t^2\right] + (1-\theta) E\left[p_t | \mathcal{I}_t^2\right] \tag{A.6}$$

where with a slight abuse of notation, I denote aggregate expectations with

$$E\left[x | \mathcal{I}_t^2\right] \equiv \int_0^1 E\left[x | \mathcal{I}_{jt}^2\right] dj$$

This delivers the Phillips curve in this setting

$$\begin{aligned} \pi_t = &\frac{\beta}{1+(1-\theta\beta)\varepsilon\varphi} E\left[\pi_{t+1} | \mathcal{I}_t^1\right] + \frac{\kappa}{1+(1-\theta\beta)\varepsilon\varphi} \tilde{y}_t \\ &+ \frac{\beta}{1+(1-\theta\beta)\varepsilon\varphi} \left(E\left[\pi_{t+1} | \mathcal{I}_t^2\right] - E\left[\pi_{t+1} | \mathcal{I}_t^1\right]\right) + \frac{(1-\theta)^2}{\theta} \left(E\left[p_t^{*} | \mathcal{I}_t^2\right] - p_t^{*}\right) \end{aligned} \tag{A.7}$$

This aggregate inflation condition along with (A.2), (A.3), (A.4), (A.5), (A.6), and an interest rate that's linear in $\{g^t, y^t\}$ give a set of linear stochastic difference equations that define the equilibrium. Thus, it will be the case that agents' choices will be linear in the variables in their information sets as I conjectured earlier[1].

In particular, behavior of the aggregate output gap and inflation are given by (A.3) and (A.7) which are the counterparts to the key equilibrium conditions (1.1) and (1.2) from the main text. The only differences in equilibrium behavior of aggregate variables comes from the differences in the inflation equation. Looking at (A.7), it's clear that explicitly accounting for idiosyncratic shocks yields a Phillips curve that differs from

[1] Lorenzoni (2010) proves this in a model that has a similar structure.

(1.2) in the main text in two ways:

1. The coefficients are scaled down by a multiplicative factor $\frac{1}{1+(1-\theta\beta)\varepsilon\varphi} < 1$ due to the yeoman farmer decentralized labor market setup.

2. There are two new terms due specifically to the idiosyncratic shocks and information sets.

   - $E\left[\pi_{t+1}|\mathcal{I}_t^2\right] - E\left[\pi_{t+1}|\mathcal{I}_t^1\right]$ reflects the difference in aggregate beliefs that comes from individual agents having the idiosyncratic signals $\{g_{jt}\}$. $E\left[\pi_{t+1}|\mathcal{I}_t^1\right]$ will be a prior based on the histories $\left\{g^{t-1}, \bar{y}^{t-1}\right\}$ plus a term reflecting news from i_t. $E\left[\pi_{t+1}|\mathcal{I}_t^2\right]$ will be the same prior plus a term incorporating the same news from i_t as well as another term capturing news from the idiosyncratic signals whose noise averages out to zero in aggregate. Hence, the difference between these beliefs will be linear in the news terms with coefficients that are related to the informativeness of the extra signals $\{g_{jt}\}$. In equilibrium, these news terms, and hence $E\left[\pi_{t+1}|\mathcal{I}_t^2\right] - E\left[\pi_{t+1}|\mathcal{I}_t^1\right]$, are linear in $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$.

   - $E\left[p_t^*|\mathcal{I}_t^2\right] - p_t^*$ will be linear in the belief errors $E\left[g_t|\mathcal{I}_t^2\right] - g_t$ and $E\left[\bar{y}_t|\mathcal{I}_t^2\right] - \bar{y}_t$ which are themselves linear in $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$.

In summary, the inflation condition differs from the one used in the main text due to a change of coefficients and extra direct effects of the shocks $\{\epsilon_{d,t}, \epsilon_{\bar{y},t}\}$. This will change the exact expressions for the responses of endogenous variables to shocks. In addition, the government spending shock now enters into the NKPC, thus giving it properties of an additional cost-push shock which poses an inflation-output tradeoff for the policymaker. The qualitative aspects of the paper's results remain intact.

A.2 Solution under arbitrary policy coefficients

Rearranging equilibrium conditions (1.1) and (1.2) gives the following system

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} - \begin{bmatrix} 1 \\ \kappa \end{bmatrix} d_{t+1|t} + \begin{bmatrix} 1 \\ \kappa \end{bmatrix} d_t - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} i_t$$

Conjecturing that the output gap and inflation are both linear in $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$ leads to the following implied form for expectations

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} d_{t+1|t} \\ \bar{y}_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

Using this along with expression (1.4) for the interest rate then gives

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix} \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} - \begin{bmatrix} 1 \\ \kappa \end{bmatrix} \rho_d d_{t|t}$$
$$+ \begin{bmatrix} 1 \\ \kappa \end{bmatrix} d_t - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} \left(f_d d_t + f_{\bar{y}} \bar{y}_t + f_{d,b} d_{t|t} + f_{\bar{y},b} \bar{y}_{t|t} \right)$$

Using this to evaluate the one-period-ahead expectation and matching coefficients gives the solution for $\mathbf{M}$

$$\mathbf{M} = -\begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) \end{bmatrix}$$
$$\text{with } \Omega_d \equiv \frac{1}{\left(1-\rho_d\right)\left(1-\beta\rho_d\right) - \frac{\kappa}{\sigma}\rho_d} \text{ and } \Omega_{\bar{y}} \equiv \frac{1}{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{y}}}$$

This immediately gives the solution for one-period-ahead expectations and substituting this back into the above expression gives the solution for current outcomes, both as functions of current beliefs and true states

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = -\begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}} \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sigma}\Omega_d \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) - \left(1-\frac{1}{\sigma}f_d\right) \\ -\frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) - \kappa\left(1-\frac{1}{\sigma}f_d\right) \end{bmatrix} d_{t|t}$$
$$+ \begin{bmatrix} -\frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{1}{\sigma}f_{\bar{y}} \\ -\frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix} \bar{y}_{t|t} + \begin{bmatrix} 1-\frac{1}{\sigma}f_d & -\frac{1}{\sigma}f_{\bar{y}} \\ \kappa\left(1-\frac{1}{\sigma}f_d\right) & -\frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} \tag{A.8}$$

Longer horizon forecasts then evolve as

$$\begin{bmatrix} \tilde{y}_{t+h|t} \\ \pi_{t+h|t} \end{bmatrix} = -\begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d^h & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}}^h \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d^h & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}}^h \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

Setting $d_{t|t} = d_t$ and $\bar{y}_{t|t} = \bar{y}_t$ leads to the perfect information responses in Section 1.3.1.

$$\begin{bmatrix} \tilde{y}_t^{PI} \\ \pi_t^{PI} \end{bmatrix} = -\begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix}$$
$$\begin{bmatrix} \tilde{y}_{t+h|t}^{PI} \\ \pi_{t+h|t}^{PI} \end{bmatrix} = -\begin{bmatrix} \frac{1}{\sigma}\Omega_d \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d^h & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}}^h \\ \frac{\kappa}{\sigma}\Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\rho_d^h & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\rho_{\bar{y}}^h \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix}$$

Responses for $\tilde{r}_t$ can be obtained using these solutions and the definition $\tilde{r}_t \equiv i_t - \pi_{t+1|t} - \sigma\left(d_t - d_{t+1|t}\right)$.

The signs of responses depend crucially on the signs of Ω_d and $\Omega_{\bar{y}}$. In particular, these coefficients need to be positive to ensure that responses go the intuitive way (i.e., the perfect information responses of the output gap and inflation to a positive interest rate surprise are negative). I can show that Assumption 1 achieves this since for a given $\rho \in \left\{\rho_d, \rho_{\bar{y}}\right\}$ the corresponding Ω has the same sign as

$$(1-\rho)(1-\beta\rho) - \frac{\kappa}{\sigma}\rho = \beta\rho^2 - \left(1+\beta+\frac{\kappa}{\sigma}\right)\rho + 1$$

This is a U-shaped parabola with 2 real roots. The larger root is greater than one.

$$\frac{1+\beta}{2\beta} + \frac{\frac{\kappa}{\sigma} + \sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2 - 4\beta}}{2\beta} \geq 1 \text{ for } \beta \leq 1$$

Then, since $\rho_d, \rho_{\bar{y}} < 1$ must hold in order for the exogenous states to be stationary, ρ_d and $\rho_{\bar{y}}$ must be below the smaller root of the parabola for $\Omega_d, \Omega_{\bar{y}}$ to be positive. Thus, I impose

$$\begin{aligned} \rho_d, \rho_{\bar{y}} < \bar{\rho} &\equiv \frac{1+\beta+\frac{\kappa}{\sigma} - \sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2 - 4\beta}}{2\beta} \\ \text{where } \frac{\kappa}{\sigma} &= \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(1+\frac{\varphi}{\sigma}\right) \end{aligned}$$

Rearranging this shows that $\bar{\rho} = \theta$ for $\varphi = 0$. Combining this with the fact that

$$\frac{\partial\bar{\rho}}{\partial\frac{\kappa}{\sigma}} = \frac{1}{2\beta}\left[1 - \frac{1+\beta+\frac{\kappa}{\sigma}}{\sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2 - 4\beta}}\right] < 0$$

shows that $\bar{\rho} < \theta$ for $\varphi > 0$.

A.3 Proof of Proposition 1

To arrive at the results under imperfect information, I first express the interest rate surprise as a function of the policy coefficients and the relative variance

$$\begin{aligned} i_t^{surp} &\equiv i_t - E\left[x_t | \mathcal{I}_t \setminus i_t\right] \\ &= \left(1 + f_{d,b}K_{d,t} + f_{\bar{y},b}K_{\bar{y},t}\right)\left(f_d\epsilon_{d,t} + f_{\bar{y}}\epsilon_{\bar{y},t}\right) \\ &= \underbrace{\frac{f_d\left(f_d+f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}}_{\iota_d} f_d\epsilon_{d,t} + \underbrace{\frac{f_d\left(f_d+f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}}_{\iota_{\bar{y}}} f_{\bar{y}}\epsilon_{\bar{y},t} \end{aligned}$$

Then, under Assumptions 2 and 5, it's clear that

$$\frac{di_t^{surp}}{d\epsilon_{d,t}} = \iota_d > 0 > \iota_{\bar{y}} = \frac{di_t^{surp}}{d\epsilon_{\bar{y},t}}$$

From here, impulse responses for $\tilde{y}_t$ and π_t can be obtained from the equilibrium given above and belief formation which gives

$$\begin{aligned}
\frac{dd_{t|t}}{d\epsilon_{d,t}} &= f_d K_{d,t}, \quad \frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} = f_{\bar{y}} K_{d,t} \\
\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} &= f_d K_{\bar{y},t}, \quad \frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} = f_{\bar{y}} K_{\bar{y},t} \\
\text{where } K_{d,t} &= \frac{f_d \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2} \quad \text{and} \quad K_{\bar{y},t} \frac{f_{\bar{y}}}{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}
\end{aligned}$$

Putting this all together gives the following relative responses to the exogenous shocks

$$\begin{aligned}
\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} &= \frac{1}{\iota_{\bar{y}}}\left[\frac{\partial\tilde{y}_t}{\partial\bar{y}_t} + \frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial\tilde{y}_t}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}}\right] \\
&= -\frac{1}{\sigma}\frac{\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}}+f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\left[\left(f_d+f_{d,b}\right)\left(1-\beta\rho_d\right)-\kappa\rho_d\right]f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d+f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)} \\
\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} &= \frac{1}{\iota_{\bar{y}}}\left[\frac{\partial\pi_t}{\partial\bar{y}_t} + \frac{\partial\pi_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial\pi_t}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}}\right] \\
&= -\frac{\kappa}{\sigma}\frac{\Omega_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\left[f_d+f_{d,b}-\sigma\beta\rho_d\left(1-\rho_d\right)-\kappa\rho_d\right]f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d+f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)}
\end{aligned}$$

$$\begin{aligned}
\frac{d\tilde{y}_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} &= \frac{1}{\iota_d}\left[\frac{\partial\tilde{y}_t}{\partial d_t} + \frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} + \frac{\partial\tilde{y}_t}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{d,t}}\right] \\
&= \frac{1}{\sigma}\frac{1}{f_d}\frac{-\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}}+f_{\bar{y},b}\right)f_{\bar{y}}f_d + \sigma f_{\bar{y}}^2 - \Omega_d\left(1-\beta\rho_d\right)\left(f_d+f_{d,b}-\sigma\left(1-\rho_d\right)\right)f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d+f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)} \\
\frac{d\pi_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} &= \frac{1}{\iota_d}\left[\frac{\partial\pi_t}{\partial d_t} + \frac{\partial\pi_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{d,t}} + \frac{\partial\pi_t}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{d,t}}\right] \\
&= \frac{\kappa}{\sigma}\frac{1}{f_d}\frac{-\Omega_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)f_{\bar{y}}f_d + \sigma f_{\bar{y}}^2 - \Omega_d\left(f_d+f_{d,b}-\sigma\left(1-\rho_d\right)\right)f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d+f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}}+f_{\bar{y},b}\right)}
\end{aligned}$$

Assumption 1 gives $\Omega_d, \Omega_{\bar{y}} > 0$ as discussed in the previous section. For the relative responses to $\epsilon_{\bar{y},t}$, Assumption 2 ensures that the sign is opposite of the sign of the numerators. For the numerators, the same assumption ensures that the first term is positive while the second terms are negative as long as Assumption 6 holds since

$$\left(f_d + f_{d,b}\right)\left(1-\beta\rho_d\right) - \kappa\rho_d < 0 \quad \text{and} \quad f_d + f_{d,b} - \sigma\beta\rho_d\left(1-\rho_d\right) - \kappa\rho_d < 0$$
$$\Leftrightarrow f_d + f_{d,b} < \min\left\{\frac{\kappa\rho_d}{1-\beta\rho_d}, \rho_d\left(\sigma\beta\left(1-\rho_d\right)+\kappa\right)\right\} = \frac{\kappa\rho_d}{1-\beta\rho_d}$$

where the last equality comes from the fact that $\Omega_d > 0$. Meanwhile, this same fact gives

$$\frac{\kappa\rho_d}{\left(1-\rho_d\right)\left(1-\beta\rho_d\right)} - \sigma\rho_d < \frac{\kappa\rho_d}{1-\beta\rho_d}$$
$$\text{and } \frac{\kappa\rho_d}{\left(1-\rho_d\right)\left(1-\beta\rho_d\right)} \leq \sigma$$

Thus, Assumption 6 is sufficient to guarantee that these second terms in the numerators of $\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ and $\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ are negative while the last fact shows that this assumption places a tighter condition than the one in Assumption 5. Then, it's clear that $\frac{d\tilde{y}_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ and $\frac{d\pi_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$ can be positive if the second terms in the numerator are large (i.e., when $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ is large). For the relative responses to $\epsilon_{d,t}$, the first terms are negative while the last 2 terms are positive under Assumption 5. Then, it's clear that they can be positive if the last two terms in the numerator are large (i.e., when $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ is large).

The scaled covariance between an outcome x_t and the interest rate surprise is given by

$$\frac{Cov_{t-1}\left(x_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = \frac{\frac{dx_t}{d\epsilon_{d,t}}\iota_d\sigma^2_{d,t-1} + \frac{dx_t}{d\epsilon_{\bar{y},t}}\iota_{\bar{y}}\sigma^2_{\bar{y},t-1}}{\iota_d^2\sigma^2_{d,t-1} + \iota_{\bar{y}}^2\sigma^2_{\bar{y},t-1}} = \frac{dx_t/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} \frac{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2} + \frac{dx_t/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} \frac{f_{\bar{y}}^2}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}$$

so that

$$\frac{Cov_{t-1}\left(\pi_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = -\frac{\kappa}{\sigma}\frac{\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)}$$

$$\frac{Cov_{t-1}\left(\tilde{y}_t, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = -\frac{1}{\sigma}\frac{\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)}$$

Then, Assumptions 2 and 5 are sufficient to show that

$$\frac{d\frac{Cov_{t-1}(\pi_t, i_t^{surp})}{Var_{t-1}(i_t^{surp})}}{d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}} = \frac{\kappa}{\sigma}\frac{\Omega_{\bar{y}}\left(f_d + f_{d,b}\right) - \Omega_d\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)}{\left[f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\right]^2} f_d f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) > 0$$

$$\frac{d\frac{Cov_{t-1}(\tilde{y}_t, i_t^{surp})}{Var_{t-1}(i_t^{surp})}}{d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}} = \frac{1}{\sigma}\frac{\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_d + f_{d,b}\right) - \Omega_d\left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)}{\left[f_d\left(f_d + f_{d,b}\right)\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\right]^2} f_d f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) > 0$$

These 2 assumptions are also sufficient to ensure that these scaled covariances are positive for large enough $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

The responses of forecasts of horizons $h \geq 1$ and the real interest rate gap can be signed in a similar manner.

$$\begin{aligned}
\frac{d\tilde{y}_{t+h|t}}{d\epsilon_{\bar{y},t}} &= \frac{\partial\tilde{y}_{t+h|t}}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial\tilde{y}_{t+h|t}}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} \\
&= -\frac{1}{\sigma}f_{\bar{y}}\frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\rho_d^h\left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}
\end{aligned}$$

$$\frac{d\pi_{t+h|t}}{d\epsilon_{\bar{y},t}} = \frac{\partial\pi_{t+h|t}}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{d\epsilon_{\bar{y},t}} + \frac{\partial\pi_{t+h|t}}{\partial d_{t|t}}\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} = -\frac{\kappa}{\sigma}f_y\frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}} + \Omega_d\rho_d^h\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}$$

$$\frac{dx_{t+h|t}}{d\epsilon_{d,t}} = \frac{f_d}{f_{\bar{y}}}\frac{dx_{t+h|t}}{d\epsilon_{\bar{y},t}} \text{ for } x_{t+h|t} \in \left\{\tilde{y}_{t+h|t}, \pi_{t+h|t}\right\}$$

$$\begin{aligned}
\frac{d\tilde{r}_t}{d\epsilon_{d,t}} &= \frac{di_t}{d\epsilon_{d,t}} - \frac{d\pi_{t+1|t}}{d\epsilon_{d,t}} - \sigma\frac{dd_t}{d\epsilon_{d,t}} + \sigma\rho_d\frac{dd_{t|t}}{d\epsilon_{d,t}} \\
&= \frac{\Omega_{\bar{y}}\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}}f_d - \sigma f_{\bar{y}}^2 + \Omega_d\left(1-\rho_d\right)\left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2} \\
\frac{d\tilde{r}_t}{d\epsilon_{\bar{y},t}} &= \frac{di_t}{d\epsilon_{\bar{y},t}} - \frac{d\pi_{t+1|t}}{d\epsilon_{\bar{y},t}} - \sigma\frac{dd_t}{d\epsilon_{\bar{y},t}} + \sigma\rho_d\frac{dd_{t|t}}{d\epsilon_{\bar{y},t}} \\
&= \frac{\Omega_{\bar{y}}\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right)f_{\bar{y}}^2 + \left[\sigma + \Omega_d\left(1-\rho_d\right)\left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)\right]f_y f_d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2}
\end{aligned}$$

Since the responses of forecasts under the individual shocks are proportional to each other, the scaled covariance between forecasts and the interest rate surprise can be found by looking just at the relative response to the output gap target shock

$$\frac{Cov_{t-1}\left(x_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = \frac{dx_{t+h|t}/d\epsilon_{d,t}}{di_t^{surp}/d\epsilon_{d,t}} \frac{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2} + \frac{dx_{t+h|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}} \frac{f_{\bar{y}}^2}{f_d^2 \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}^2} = \frac{dx_{t+h|t}/d\epsilon_{\bar{y},t}}{di_t^{surp}/d\epsilon_{\bar{y},t}}$$

so that

$$\frac{Cov_{t-1}\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = -\frac{\kappa}{\sigma} \frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h \left(f_{\bar{y}} + f_{\bar{y},b}\right) f_{\bar{y}} + \Omega_d \rho_d^h \left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) f_d \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d + f_{d,b}\right) \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)}$$

$$\frac{Cov_{t-1}\left(\tilde{y}_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)} = -\frac{1}{\sigma} \frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h \left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) f_{\bar{y}} + \Omega_d \rho_d^h \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right) f_d \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f_d\left(f_d + f_{d,b}\right) \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)}$$

Assumptions 2 and 5 are again sufficient to ensure that these scaled covariances are positive for large enough $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ and that

$$\frac{d\frac{Cov_{t-1}\left(\pi_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}}{d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}} = \frac{\kappa}{\sigma} \frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h \left(f_d + f_{d,b}\right) - \Omega_d \rho_d^h \left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)}{\left[f_d\left(f_d + f_{d,b}\right) \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\right]^2} f_d f_{\bar{y}} \left(f_{\bar{y}} + f_{\bar{y},b}\right) > 0$$

$$\frac{d\frac{Cov_{t-1}\left(\tilde{y}_{t+h|t}, i_t^{surp}\right)}{Var_{t-1}\left(i_t^{surp}\right)}}{d\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}} = \frac{\Omega_{\bar{y}}\rho_{\bar{y}}^h \left(1-\beta\rho_{\bar{y}}\right)\left(f_d + f_{d,b}\right) - \Omega_d \rho_d^h \left(1-\beta\rho_d\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho_d\right)\right)}{\sigma\left[f_d\left(f_d + f_{d,b}\right) \frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right)\right]^2} f_d f_{\bar{y}} \left(f_{\bar{y}} + f_{\bar{y},b}\right) > 0$$

Looking back at the equilibrium solution, it's clear that setting $f_d = \sigma$ and $f_{d,b} = -\sigma\rho_d$ results in the coefficients on $d_{t|t}$ and d_t being zero. Using these parameter values in the responses immediately gives the properties presented in Section 1.3.2.

A.4 Proof of Proposition 2

Here, I repeat the equations summarizing the policymaker's problem described in Section 1.4

$$\min_{i_t^{dis}, \tilde{y}_t, \pi_t} E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2}\left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right)$$

subject to

$$\tilde{y}_t = \tilde{y}_{t+1|t} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right) + d_t - d_{t+1|t}$$
$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t$$

where

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M}\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$
$$d_{t|t} = \rho_d d_{t-1} + K_d\left(i_t^{dis} - f_d\rho_d d_{t-1} - f_{\bar{y}}\rho_{\bar{y}}\bar{y}_{t-1}\right)$$
$$\bar{y}_{t|t} = \rho_{\bar{y}}\bar{y}_{t-1} + K_{\bar{y}}\left(i_t^{dis} - f_d\rho_d d_{t-1} - f_{\bar{y}}\rho_{\bar{y}}\bar{y}_{t-1}\right)$$

with $\mathbf{M}, K_d, K_{\bar{y}}$ taken as given.

Then, I can write the output gap deviation and inflation in matrix form as the following function of current beliefs and i_t^{dis}

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix}\mathbf{M}\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} - \begin{bmatrix} \rho_d + \frac{1}{\sigma}f_{d,b} \\ \kappa\left(\rho_d + \frac{1}{\sigma}f_{d,b}\right) \end{bmatrix} d_{t|t}$$
$$- \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} f_{\bar{y},b}\bar{y}_{t|t} + \begin{bmatrix} 1 \\ \kappa \end{bmatrix} d_t - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\bar{y}_t - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} i_t^{dis} \tag{A.9}$$

By plugging in beliefs, this can be transformed into the following function of exogenous states and i_t^{dis}

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \mathbf{\Psi}\begin{bmatrix} 1 - K_d f_d & -K_d f_{\bar{y}} \\ -K_{\bar{y}} f_d & 1 - K_{\bar{y}} f_{\bar{y}} \end{bmatrix}\begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}}\bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ \kappa & 0 \end{bmatrix}\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} H_{\bar{y},i} \\ H_{\pi,i} \end{bmatrix} i_t^{dis}$$
$$\text{where } \mathbf{\Psi} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix}\mathbf{M}\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \rho_d + \frac{1}{\sigma}f_{d,b} & \frac{1}{\sigma}f_{\bar{y},b} \\ \kappa\left(\rho_d + \frac{1}{\sigma}f_{d,b}\right) & \frac{\kappa}{\sigma}f_{\bar{y},b} \end{bmatrix} \tag{A.10}$$
$$\text{and } \begin{bmatrix} H_{\bar{y},i} \\ H_{\pi,i} \end{bmatrix} \equiv \mathbf{\Psi}\begin{bmatrix} K_d \\ K_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} = \begin{bmatrix} \frac{\partial\tilde{y}_t}{\partial i_t^{dis}} + \frac{\partial\tilde{y}_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t^{dis}} + \frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t^{dis}} \\ \frac{\partial\pi_t}{\partial i_t^{dis}} + \frac{\partial\pi_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t^{dis}} + \frac{\partial\pi_t}{\partial\bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t^{dis}} \end{bmatrix}$$

In this form, it's clear that the discretionary policymaker has no control over time $t+1$ or later outcomes and the problem simplifies to

$$\min_{i_t^{dis}} \frac{1}{2}\left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right) \quad \text{subject to (A.10)}$$

which clearly gives the optimality condition

$$\left(\tilde{y}_t - \bar{y}_t\right) H_{\tilde{y},i} + \frac{\varepsilon}{\kappa}\pi_t H_{\pi,i} = 0 \Rightarrow \tilde{y}_t - \bar{y}_t = -\mathcal{R}\frac{\varepsilon}{\kappa}\pi_t$$

matching the form given in the proposition with $\mathcal{R} = \frac{H_{\pi,i}}{H_{\tilde{y},i}} = \frac{\frac{\partial \pi_t}{\partial i_t^{dis}} + \frac{\partial \pi_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t^{dis}} + \frac{\partial \pi_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t^{dis}}}{\frac{\partial \tilde{y}_t}{\partial i_t^{dis}} + \frac{\partial \tilde{y}_t}{\partial d_{t|t}}\frac{dd_{t|t}}{di_t^{dis}} + \frac{\partial \tilde{y}_t}{\partial \bar{y}_{t|t}}\frac{d\bar{y}_{t|t}}{di_t^{dis}}}$.

Solving for $\tilde{y}_t$ using this optimality condition and substituting this into the inflation condition gives

$$\pi_t = \beta\pi_{t+1|t} - \mathcal{R}\varepsilon\pi_t + \kappa\bar{y}_t$$

By restricting attention to nonnegative values of $\mathcal{R}$, I can iterate this forward while using the fact that $\bar{y}_{t+h|t} = \rho_{\bar{y}}^h \bar{y}_{t|t}$ to get a solution for π_t in terms of $\left\{\bar{y}_t, \bar{y}_{t|t}\right\}$. Substituting that expression for π_t back into the optimality condition gives the solution for $\tilde{y}_t$ in terms of the same state variables

$$\begin{aligned}
\pi_t &= \frac{\kappa}{1+\mathcal{R}\varepsilon}\bar{y}_t + \frac{\beta\rho_{\bar{y}}\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}\bar{y}_{t|t} \\
\tilde{y}_t &= \frac{1}{1+\mathcal{R}\varepsilon}\bar{y}_t - \frac{\mathcal{R}\varepsilon\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}\bar{y}_{t|t}
\end{aligned}$$

Then, this gives expressions for expectations $\tilde{y}_{t+1|t}$ and $\pi_{t+1|t}$ which immediately reveals the equilibrium value of $\mathbf{M}$ as a function of $\mathcal{R}$

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon} \\ 0 & \frac{\kappa}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon} \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

These can be used along with (1.1) to back out the implied nominal interest rate in terms of $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$

$$\begin{aligned}
i_t &= \sigma\left(d_t - d_{t+1|t}\right) + \pi_{t+1|t} + \sigma\left(\tilde{y}_{t+1|t} - \tilde{y}_t\right) \\
&= \underbrace{\sigma d_t - \sigma\rho_d d_{t|t}}_{r_t^n} \underbrace{-\sigma\frac{1}{1+\mathcal{R}\varepsilon}}_{f_{\bar{y}}^*(\mathcal{R})}\bar{y}_t + \underbrace{\sigma\left(\frac{1}{1+\mathcal{R}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\right)}_{f_{\bar{y},b}^*(\mathcal{R})}\bar{y}_{t|t}
\end{aligned} \tag{A.11}$$

Using these optimal response coefficients along with the expression for $\mathbf{M}$ gives the equilibrium condition for $\mathcal{R}$

$$\mathcal{R} = \kappa\frac{\frac{\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}K_{\bar{y}} - \frac{1}{\sigma}}{\frac{-\beta\rho_{\bar{y}}\mathcal{R}\varepsilon}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}K_{\bar{y}} - \frac{1}{\sigma}} \quad \text{since } K_{\bar{y}} = -\frac{1}{\sigma}\frac{1+\mathcal{R}\varepsilon}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1} \tag{A.12}$$

Here, it's clear that when $\beta\rho_{\bar{y}} = 0$, the terms involving $K_{\bar{y}}$ drop out of this expression and it gives $\mathcal{R} = \kappa$.

Rearranging (A.12) gives

$$0 = -\beta\rho_{\bar{y}}\left(\mathcal{R}^2\varepsilon + \kappa\right) + (\mathcal{R} - \kappa)\left(1 - \beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)\left[(1 + \mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} + 1\right] \tag{A.13}$$

To focus on equilibrium values for $\mathcal{R}$ which give finite policy response coefficients, I impose $1 + \mathcal{R}\varepsilon \neq 0$ and $1 - \beta\rho_{\bar{y}} + \mathcal{R}\varepsilon \neq 0$ which allows me to reduce this equilibrium condition to a third-order polynomial

$$\begin{aligned} 0 &= \mathcal{R}\left(1 - \beta\rho_{\bar{y}}\right) - \kappa + (\mathcal{R} - \kappa)\left(1 - \beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)(1 + \mathcal{R}\varepsilon)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \\ &= \varepsilon^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2}\mathcal{R}^3 + \varepsilon\left(2 - \beta\rho_{\bar{y}} - \varepsilon\kappa\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}\mathcal{R}^2 \\ &+ \left[\left(1 - \beta\rho_{\bar{y}}\right)\left(1 + \frac{\sigma_d^2}{\sigma_{\bar{y}}^2}(1 - \varepsilon\kappa)\right) - \varepsilon\kappa\right]\mathcal{R} - \kappa\left(1 + \left(1 - \beta\rho_{\bar{y}}\right)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}\right) \end{aligned} \tag{A.14}$$

Since the first coefficient in the polynomial is positive while the last is negative, Descartes' rule of signs says that there must be at least one positive root for any values of the middle two coefficients.

Again, attention is limited to positive solutions for $\mathcal{R}$. To see that $\mathcal{R} \geq \kappa$, note that rearranging (A.13) gives

$$\mathcal{R} - \kappa = \frac{\beta\rho_{\bar{y}}}{1 - \beta\rho_{\bar{y}} + \mathcal{R}\varepsilon}\frac{\mathcal{R}^2\varepsilon + \kappa}{(1 + \mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} + 1} \geq 0 \quad \text{for } \mathcal{R} \geq 0$$

Using the expression in (A.14) gives the upper bound $\mathcal{R} \leq \frac{\kappa}{1 - \beta\rho_{\bar{y}}}$

$$\mathcal{R}\left(1 - \beta\rho_{\bar{y}}\right) - \kappa = -(\mathcal{R} - \kappa)\left(1 - \beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)(1 + \mathcal{R}\varepsilon)\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \leq 0 \quad \text{for } \mathcal{R} \geq \kappa$$

Implicitly differentiating (A.14) gives

$$\frac{d\mathcal{R}}{d\left(\sigma_d^2/\sigma_{\bar{y}}^2\right)} = -\frac{(\mathcal{R} - \kappa)\left(1 - \beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)(1 + \mathcal{R}\varepsilon)}{1 - \beta\rho_{\bar{y}} + \left[(\mathcal{R} - \kappa)\left[\left(1 - \beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right) + (1 + \mathcal{R}\varepsilon)\right]\varepsilon + \left(1 - \beta\rho_{\bar{y}} + \mathcal{R}\varepsilon\right)(1 + \mathcal{R}\varepsilon)\right]\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}} < 0$$

Now, I look at the cases given by the limits of $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}$.

- When $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to \infty$: In this case, referring back to (A.12), it's clear that $K_{\bar{y}} \to 0$ and $\mathcal{R} = \kappa$ is the unique solution in this limit. To see that this is the solution of the perfect information case, note that the policymaker's problem in that setting is

$$\min_{i_t^{dis}} \frac{1}{2}\left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right)$$

subject to (A.10) but with $d_{t|t} = d_t$ and $\bar{y}_{t|t} = \bar{y}_t$. Then, it's clear that the optimality condition is the same as the one given in the proposition with $\mathcal{R} = \kappa$.

- When $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$: Equation (A.12) shows that

$$\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}} \quad \text{since } K_{\bar{y}} \to -\frac{1+\mathcal{R}\varepsilon}{\sigma}$$

Now, I show that this is equivalent to the case of a commitment to a rule of the form

$$i_t = r_t^n + f_{\bar{y}}^c \bar{y}_t + f_{\bar{y},b}^c \bar{y}_{t|t}$$

First, I substitute these coefficients into the solution under a given rule derived earlier in the Appendix and given in (A.8))

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}}^c + f_{\bar{y},b}^c\right) + \frac{1}{\sigma} f_{\bar{y}}^c \\ -\frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}}^c + f_{\bar{y},b}^c\right) + \frac{\kappa}{\sigma} f_{\bar{y}}^c \end{bmatrix} \bar{y}_{t|t} + \begin{bmatrix} -\frac{1}{\sigma} f_{\bar{y}}^c - 1 \\ -\frac{\kappa}{\sigma} f_{\bar{y}}^c \end{bmatrix} \bar{y}_t$$

where equilibrium beliefs in this limit are given by

$$\bar{y}_{t|t} = \bar{y}_t + \frac{\sigma}{f_{\bar{y}}^c}\epsilon_{d,t}$$

Then, the policymaker who can commit to this rule solves

$$\min_{f_{\bar{y}}^c, f_{\bar{y},b}^c} E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2}\left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right)$$

$$\text{where } \begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}}^c + f_{\bar{y},b}^c\right) - 1 \\ -\frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}}^c + f_{\bar{y},b}^c\right) \end{bmatrix} \bar{y}_t + \begin{bmatrix} -\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(1 + \frac{f_{\bar{y},b}^c}{f_{\bar{y}}^c}\right) + 1 \\ -\kappa\Omega_{\bar{y}}\left(1 + \frac{f_{\bar{y},b}^c}{f_{\bar{y}}^c}\right) + \kappa \end{bmatrix} \epsilon_{d,t}$$

Then, the two optimality conditions are given by

$$\begin{aligned} 0 &= \frac{\partial}{\partial f_{\bar{y}}^c} E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2}\left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right) \\ \Rightarrow 0 &= E_{t_0}^{CB} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\left(\tilde{y}_t - \bar{y}_t\right)\left(1-\beta\rho_{\bar{y}}\right) + \varepsilon\pi_t\right) \left[-\frac{1}{\sigma}\bar{y}_t + \frac{f_{\bar{y},b}^c}{\left(f_{\bar{y}}^c\right)^2}\epsilon_{d,t}\right] \end{aligned}$$

$$0 = \frac{\partial}{\partial f^c_{\bar{y},b}} E^{CB}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right)$$

$$\Rightarrow 0 = E^{CB}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\left(\tilde{y}_t - \bar{y}_t\right) \left(1 - \beta \rho_{\bar{y}}\right) + \varepsilon \pi_t \right) \left[-\frac{1}{\sigma} \bar{y}_t - \frac{1}{f^c_{\bar{y}}} \epsilon_{d,t} \right]$$

Both conditions are satisfied by a policy that maintains

$$\tilde{y}_t - \bar{y}_t = -\frac{\varepsilon}{1 - \beta \rho_{\bar{y}}} \pi_t \quad \forall t$$

which is equivalent to the optimality condition of the discretionary policy with $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$ in this limit.

Lastly I show that the same discretionary optimal policy condition is obtained if I start with agents who suppose that current policy responds linearly to the entire history of shocks $\{d^t, \bar{y}^t\}$. That is, I replace the supposed behavior of current policy in equation (1.12) with

$$i_t = \sum_{k=0}^{\infty} f_d^{hist}(k)\, d_{t-k} + \sum_{k=0}^{\infty} f_{\bar{y}}^{hist}(k)\, \bar{y}_{t-k} \tag{A.15}$$

(In equilibrium, a rule that also includes current and lagged private agent beliefs can be written in this form since private agent beliefs are a function of lagged and current state variables in equilibrium.)

Then, beliefs are given by a static Gaussian signal extraction problem where

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}} \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} K_d^{hist} \\ K_{\bar{y}}^{hist} \end{bmatrix} \left[i_t - E\left[i_t | \mathcal{I}_t \setminus i_t \right] \right] \tag{A.16}$$

where $E\left[i_t|\mathcal{I}_t \setminus i_t\right] = \left[f_d^{hist}(0)\,\rho_d + f_d^{hist}(1)\right] d_{t-1}$

$$+ \left[f_{\bar{y}}^{hist}(0)\,\rho_{\bar{y}} + f_{\bar{y}}^{hist}(1)\right] \bar{y}_{t-1} + \sum_{k=2}^{\infty} \left[f_d^{hist}(k)\, d_{t-k} + f_{\bar{y}}^{hist}(k)\, \bar{y}_{t-k}\right] \tag{A.17}$$

and $K_d^{hist} = \frac{f_d^{hist}(0)\,\sigma_d^2}{\left(f_d^{hist}(0)\right)^2 \sigma_d^2 + \left(f_{\bar{y}}^{hist}(0)\right)^2 \sigma_{\bar{y}}^2}, \quad K_{\bar{y}}^{hist} = \frac{f_{\bar{y}}^{hist}(0)\,\sigma_{\bar{y}}^2}{\left(f_d^{hist}(0)\right)^2 \sigma_d^2 + \left(f_{\bar{y}}^{hist}(0)\right)^2 \sigma_{\bar{y}}^2}$

To proceed, I now conjecture that the equilibrium solution for the endogenous outcomes $\tilde{y}_t$ and π_t are linear in the full history of shocks, thus resulting in expectations of the form

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M}^{hist} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} + \sum_{k=1}^{\infty} \mathbf{M}_d^{hist}(k)\, d_{t-k} + \sum_{k=1}^{\infty} \mathbf{M}_{\bar{y}}^{hist}(k)\, \bar{y}_{t-k}$$

Again, this allows me to write the output gap deviation and inflation as

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \sum_{k=0}^{\infty} H_d^{hist}(k)\, d_{t-k} + \sum_{k=0}^{\infty} H_{\bar{y}}^{hist}(k)\, \bar{y}_{t-k} + \begin{bmatrix} H_{\tilde{y},i}^{hist} \\ H_{\pi,i}^{hist} \end{bmatrix} i_t \tag{A.18}$$

$$\text{where } \begin{bmatrix} H_{\tilde{y},i}^{hist} \\ H_{\pi,i}^{hist} \end{bmatrix} \equiv \left(\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M}^{hist} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \rho_d & 0 \\ \kappa\rho_d & 0 \end{bmatrix} \right) \begin{bmatrix} K_d^{hist} \\ K_{\bar{y}}^{hist} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}$$

and $\left\{H_d^{hist}(k), H_{\bar{y}}^{hist}(k)\right\}_{k=0}^{\infty}$ are functions of $\mathbf{M}^{hist}, K_d^{hist}, K_{\bar{y}}^{hist}, \left\{f_d^{hist}(k), f_{\bar{y}}^{hist}(k), \mathbf{M}_d^{hist}(k), \mathbf{M}_{\bar{y}}^{hist}(k)\right\}_{k=0}^{\infty}$

This again reduces the discretionary policy problem to

$$\min_{i_t} \frac{1}{2}\left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa}\pi_t^2 \right) \quad \text{subject to (A.18)}$$

which gives

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}^{hist}\frac{\varepsilon}{\kappa}\pi_t \quad \text{where } \mathcal{R}^{hist} = \frac{H_{\pi,i}^{hist}}{H_{\tilde{y},i}^{hist}}$$

This is equivalent to the solution above as long as the equilibrium condition for $\mathcal{R}^{hist}$ is the same. The rest of this section proves this.

Using the equilibrium conditions gives the following expression for expectations

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} \\ 0 & \frac{\kappa}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} \end{bmatrix}}_{\mathbf{M}^{hist}} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

and an interest rate that responds only to current true states and beliefs

$$i_t^* = \sigma d_t - \sigma\rho_d d_{t|t} - \sigma\frac{1}{1+\mathcal{R}^{hist}\varepsilon}\bar{y}_t + \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} \right)\bar{y}_{t|t}$$

Combining (A.15) and (A.16) shows that equilibrium beliefs are a function only of time t and $t-1$ fundamentals

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}}\bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} K_d^{hist} \\ K_{\bar{y}}^{hist} \end{bmatrix} \left[f_d^{hist}(0)\left(d_t - \rho_d d_{t-1}\right) + f_{\bar{y}}^{hist}(0)\left(\bar{y}_t - \rho_{\bar{y}}\bar{y}_{t-1}\right) \right]$$

Then, comparing (A.15) to the optimal interest rate proves that $f_d^{hist}(k) = f_{\bar{y}}^{hist}(k) = 0$ for $k \geq 2$. Using these equilibrium beliefs in the expression for i_t^* allows me to obtain the remaining coefficients

$\left\{f_d^{hist}(0), f_d^{hist}(1), f_{\bar{y}}^{hist}(0), f_{\bar{y}}^{hist}(1)\right\}$

$$
\begin{aligned}
i_t^* &= \sigma d_t - \sigma\rho_d\left[\rho_d d_{t-1} + K_d^{hist}\left[f_d^{hist}(0)(d_t - \rho_d d_{t-1}) + f_{\bar{y}}^{hist}(0)\left(\bar{y}_t - \rho_{\bar{y}}\bar{y}_{t-1}\right)\right]\right] - \sigma\frac{1}{1+\mathcal{R}^{hist}\varepsilon}\bar{y}_t \\
&\quad + \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)\left[\rho_{\bar{y}}\bar{y}_{t-1} + K_{\bar{y}}^{hist}\left[f_d^{hist}(0)(d_t - \rho_d d_{t-1}) + f_{\bar{y}}^{hist}(0)\left(\bar{y}_t - \rho_{\bar{y}}\bar{y}_{t-1}\right)\right]\right] \\
&= \sigma\left(1 - \rho_d K_d^{hist} f_d^{hist}(0) + \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist} f_d^{hist}(0)\right)d_t \\
&\quad - \sigma\left[\rho_d\left(1 - K_d^{hist} f_d^{hist}(0)\right) - \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist} f_d^{hist}(0)\right]\rho_d d_{t-1} \\
&\quad - \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} + \rho_d K_d^{hist} f_{\bar{y}}^{hist}(0) - \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist} f_{\bar{y}}^{hist}(0)\right)\bar{y}_t \\
&\quad + \sigma\left[\rho_d K_d^{hist} f_{\bar{y}}^{hist}(0) + \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)\left[1 - K_{\bar{y}}^{hist} f_{\bar{y}}^{hist}(0)\right]\right]\rho_{\bar{y}}\bar{y}_{t-1}
\end{aligned}
$$

which gives

$$
\begin{aligned}
f_d^{hist}(0) &= \frac{\sigma}{1 + \sigma\rho_d K_d^{hist} - \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist}} \\
f_{\bar{y}}^{hist}(0) &= \frac{-\sigma\frac{1}{1+\mathcal{R}^{hist}\varepsilon}}{1 + \sigma\rho_d K_d^{hist} - \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist}}
\end{aligned}
$$

Substituting this into the expression for $K_{\bar{y}}^{hist}$ gives $\rho_d K_d^{hist}$ as a function of $K_{\bar{y}}^{hist}$.

$$
\begin{aligned}
K_{\bar{y}}^{hist} &= -\frac{1}{\sigma}\frac{\left(1+\mathcal{R}^{hist}\varepsilon\right)}{\left(1+\mathcal{R}^{hist}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1}\left[1 + \sigma\rho_d K_d^{hist} - \sigma\left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist}\right] \\
\Rightarrow \rho_d K_d^{hist} &= \left(\frac{1}{1+\mathcal{R}^{hist}\varepsilon} - \frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon} - \frac{\left(1+\mathcal{R}^{hist}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1}{1+\mathcal{R}^{hist}\varepsilon}\right)K_{\bar{y}}^{hist} - \frac{1}{\sigma}
\end{aligned}
$$

Then, using the expression for $\mathcal{R}^{hist}$ and the equilibrium expression for $\mathbf{M}^{hist}$ gives

$$
\mathcal{R}^{hist} = \kappa\frac{\frac{\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}+\frac{\kappa}{\sigma}+\beta\right)}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}K_{\bar{y}}^{hist} - \rho_d K_d^{hist} - \frac{1}{\sigma}}{\frac{\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}+\frac{\kappa}{\sigma}\right)}{1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon}K_{\bar{y}}^{hist} - \rho_d K_d^{hist} - \frac{1}{\sigma}} = \kappa\frac{\beta\rho_{\bar{y}} + \left(1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon\right)\left(\left(1+\mathcal{R}^{hist}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1\right)}{-\mathcal{R}^{hist}\varepsilon\beta\rho_{\bar{y}} + \left(1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon\right)\left(\left(1+\mathcal{R}^{hist}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1\right)}
$$

where I again restrict attention to finite interest rate coefficients by looking only for solutions where $1 + \mathcal{R}^{hist}\varepsilon \neq 0$ and $1 - \beta\rho_{\bar{y}} + \mathcal{R}^{hist}\varepsilon \neq 0$.

Rearranging this gives

$$
0 = -\beta\rho_{\bar{y}}\left(\left(\mathcal{R}^{hist}\right)^2\varepsilon + \kappa\right) + \left(\mathcal{R}^{hist} - \kappa\right)\left(1-\beta\rho_{\bar{y}}+\mathcal{R}^{hist}\varepsilon\right)\left[\left(1+\mathcal{R}^{hist}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1\right]
$$

which indeed matches equilibrium condition (A.13) derived above for $\mathcal{R}$ thus showing that the equilibrium is the same when I generalize private agents' belief about current policy to the form in (A.15).

A.4.1 Proof of Corollary 3

The proof above of Proposition 2 gave the forms of $f^*_{\bar{y}}(\mathcal{R})$ and $f^*_{\bar{y},b}(\mathcal{R})$ in (A.11). There, it was also shown that the perfect information discretionary policy optimality condition is

$$\tilde{y}_t^{PI} - \bar{y}_t = -\varepsilon \pi_t^{PI}$$

Again, using this condition along with the NKPC in equation (1.2) gives

$$\pi_t^{PI} = \frac{\kappa}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t \text{ and } \tilde{y}_t^{PI} = \frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t$$

Then, this gives expressions for expectations

$$\pi_{t+1|t}^{PI} = \frac{\kappa\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t \text{ and } \tilde{y}_{t+1|t}^{PI} = \frac{\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t$$

which can again be used along with (1.1) to back out the implied optimal nominal interest rate in terms of $\{d_t, \bar{y}_t\}$

$$i_t^{*,PI} = \underbrace{\sigma\left(1-\rho_d\right)d_t}_{r_t^n} - \sigma\frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\bar{y}_t = r_t^n + \left(f^*_{\bar{y}}(\kappa) + f^*_{\bar{y},b}(\kappa)\right)\bar{y}_t$$

Returning to the imperfect information case, I next show how the interest rate behavior can be altered to ensure determinacy so that the equilibrium in equations (1.16) and (1.17) is the unique path in this model. To do this, I add to the interest rate a term that reacts to deviations of π_t from its intended equilibrium path

$$\begin{aligned} i_t^* &= r_t^n + f^*_{\bar{y}}(\mathcal{R})\,\bar{y}_t + f^*_{\bar{y},b}(\mathcal{R})\,\bar{y}_{t|t} + \phi_\pi\left(\pi_t - \pi_t^*\right) \\ &= r_t^n + \left(f^*_{\bar{y}}(\mathcal{R}) - \phi_\pi\Gamma_{\bar{y}}\right)\bar{y}_t + \left(f^*_{\bar{y},b}(\mathcal{R}) - \phi_\pi\Gamma_{\bar{y},b}\right)\bar{y}_{t|t} + \phi_\pi\pi_t \\ \text{where } \pi_t^* &= \underbrace{\frac{\kappa}{1+\mathcal{R}\varepsilon}}_{\Gamma_{\bar{y}}}\bar{y}_t + \underbrace{\frac{\beta\rho_{\bar{y}}\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)\left(1+\mathcal{R}\varepsilon\right)}}_{\Gamma_{\bar{y},b}}\bar{y}_{t|t} \text{ is the intended equilibrium} \end{aligned}$$

Clearly, along the intended stationary equilibrium path, $\pi_t = \pi_t^*$ so that the response of i_t^* to state variables is the same as without this extra term. What this term does change are the dynamics of $[\tilde{y}_t \;\; \pi_t]'$ since the system of equilibrium conditions now becomes

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{1+\phi_\pi \frac{\kappa}{\sigma}} & \frac{1-\beta\phi_\pi}{\sigma+\phi_\pi\kappa} \\ \frac{\kappa}{1+\phi_\pi \frac{\kappa}{\sigma}} & \frac{\frac{\kappa}{\sigma}+\beta}{1+\phi_\pi \frac{\kappa}{\sigma}} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma+\phi_\pi\kappa} \\ \frac{\kappa}{\sigma+\phi_\pi\kappa} \end{bmatrix} \left(\left(f_{\bar{y}}^{*}\left(\mathcal{R}\right) - \phi_\pi \Gamma_{\bar{y}} \right) \bar{y}_t + \left(f_{\bar{y},b}^{*}\left(\mathcal{R}\right) - \phi_\pi \Gamma_{\bar{y},b} \right) \bar{y}_{t|t} \right)$$

Then, determinacy of $[\tilde{y}_t \quad \pi_t]'$ is guaranteed by the largest eigenvalue of $\mathbf{A}$ being less than one

$$\max\left\{eig\left(\mathbf{A}\right)\right\} = \frac{\frac{1+\beta+\frac{\kappa}{\sigma}}{1+\phi_\pi\frac{\kappa}{\sigma}} \pm \sqrt{\left(\frac{1+\beta+\frac{\kappa}{\sigma}}{1+\phi_\pi\frac{\kappa}{\sigma}}\right)^2 - 4\frac{\beta}{1+\phi_\pi\frac{\kappa}{\sigma}}}}{2} < 1 \Leftrightarrow \phi_\pi > 1$$

A.5 Proof of Proposition 4

Here, the equilibrium conditions in matrix form are

$$\begin{bmatrix} \tilde{y}_t^{CB} \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix} \begin{bmatrix} \tilde{y}_{t+1|t}^{CB} \\ \pi_{t+1|t} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} i_t + \begin{bmatrix} \mathbf{\Xi}_x \\ \mathbf{\Xi}_\pi \end{bmatrix} \mathbf{z}_t \tag{A.19}$$

where the shocks are given by

$$\begin{bmatrix} \mathbf{z}_{1,t} \\ \mathbf{z}_{2,t} \end{bmatrix} = \begin{bmatrix} \Upsilon_{11} & 0 \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{1,t-1} \\ \mathbf{z}_{2,t-1} \end{bmatrix} + \mathbf{e}_t,\ \mathbf{e}_t \sim \text{iid } N\left(0, \mathbf{\Sigma}\right) \text{ with } \mathbf{\Sigma} \text{ diagonal}$$

In the perfect information case, a discretionary policymaker solves

$$\min_{i_t, \tilde{y}_t^{CB}, \pi_t} \frac{1}{2}\left(\left(\tilde{y}_t^{CB}\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2 \right) \quad \text{subject to (A.19) where } \tilde{y}_{t+1|t}^{CB} \text{ and } \pi_{t+1|t} \text{ are taken as given}$$

which clearly yields

$$\tilde{y}_t^{CB} = -\varepsilon \pi_t$$

Private agents suppose that the interest rate i_t is

$$i_t = \mathbf{F}_1 \mathbf{z}_{1,t} + \mathbf{F}_2 \mathbf{z}_{2,t} + \mathbf{F}_{2,b} \mathbf{z}_{2,t|t}$$

while their information set is $\left\{i^t, \mathbf{z}_1^t, \mathbf{z}_2^{t-1}\right\}$. Again, I reframe the policymaker's problem as a choice of i_t^{dis} where implemented policy is $i_t = i_t^{dis} + \mathbf{F}_{2,b}\mathbf{z}_{2,t|t}$. Then, the same process described in Section 1.2.3 shows that beliefs are the following function of i_t^{dis} and exogenous lagged variables

$$\begin{aligned}\mathbf{z}_{2,t|t} &= \boldsymbol{\Upsilon}_{\text{row }2}\begin{bmatrix}\mathbf{z}_{1,t-1}\\ \mathbf{z}_{2,t-1}\end{bmatrix} + \mathbf{K}_z\left(i_t^{dis} - \mathbf{F}_2\boldsymbol{\Upsilon}_{\text{row }2}\begin{bmatrix}\mathbf{z}_{1,t-1}\\ \mathbf{z}_{2,t-1}\end{bmatrix}\right)\\ &= \left(\mathbf{I} - \mathbf{K}_z\mathbf{F}_2\right)\boldsymbol{\Upsilon}_{\text{row }2}\begin{bmatrix}\mathbf{z}_{1,t-1}\\ \mathbf{z}_{2,t-1}\end{bmatrix} + \mathbf{K}_z i_t^{dis}\end{aligned}$$

Then, conjecturing a linear solution for $\tilde{y}_t^{CB}$ and π_t again leads to a linear conjecture for expectations

$$\begin{bmatrix}\tilde{y}_{t+1|t}^{CB}\\ \pi_{t+1|t}\end{bmatrix} = \mathbf{M}_1\mathbf{z}_{1,t+1|t} + \mathbf{M}_2\mathbf{z}_{2,t+1|t} = \left(\mathbf{M}_1\Upsilon_{11} + \mathbf{M}_2\Upsilon_{21}\right)\mathbf{z}_{1,t} + \mathbf{M}_2\Upsilon_{22}\mathbf{z}_{2,t|t}$$

The current outcomes can then be written in terms of exogenous states and i_t^{dis}

$$\begin{aligned}\begin{bmatrix}\tilde{y}_t^{CB}\\ \pi_t\end{bmatrix} &= \begin{bmatrix}1 & \frac{1}{\sigma}\\ \kappa & \frac{\kappa}{\sigma}+\beta\end{bmatrix}\left(\mathbf{M}_1\Upsilon_{11} + \mathbf{M}_2\Upsilon_{21}\right)\mathbf{z}_{1,t} + \begin{bmatrix}\boldsymbol{\Xi}_x\\ \boldsymbol{\Xi}_\pi\end{bmatrix}\mathbf{z}_t + \boldsymbol{\Psi}\mathbf{z}_{2,t|t} - \begin{bmatrix}\frac{1}{\sigma}\\ \frac{\kappa}{\sigma}\end{bmatrix} i_t^{dis}\\ &= \begin{bmatrix}1 & \frac{1}{\sigma}\\ \kappa & \frac{\kappa}{\sigma}+\beta\end{bmatrix}\left(\mathbf{M}_1\Upsilon_{11} + \mathbf{M}_2\Upsilon_{21}\right)\mathbf{z}_{1,t} + \begin{bmatrix}\boldsymbol{\Xi}_x\\ \boldsymbol{\Xi}_\pi\end{bmatrix}\mathbf{z}_t \\ &+ \boldsymbol{\Psi}\left(\mathbf{I} - \mathbf{K}_z\mathbf{F}_2\right)\boldsymbol{\Upsilon}_{\text{row }2}\begin{bmatrix}\mathbf{z}_{1,t-1}\\ \mathbf{z}_{2,t-1}\end{bmatrix} + \begin{bmatrix}H_{\tilde{y},i}\\ H_{\pi,i}\end{bmatrix} i_t^{dis}\end{aligned} \tag{A.20}$$

$$\text{where } \boldsymbol{\Psi} \equiv \begin{bmatrix}1 & \frac{1}{\sigma}\\ \kappa & \frac{\kappa}{\sigma}+\beta\end{bmatrix}\mathbf{M}_2\Upsilon_{22} - \begin{bmatrix}\frac{1}{\sigma}\\ \frac{\kappa}{\sigma}\end{bmatrix}\mathbf{F}_{2,b} \quad \text{and} \quad \begin{bmatrix}H_{\tilde{y},i}\\ H_{\pi,i}\end{bmatrix} \equiv \boldsymbol{\Psi}\mathbf{K}_z - \begin{bmatrix}\frac{1}{\sigma}\\ \frac{\kappa}{\sigma}\end{bmatrix}$$

Then, the discretionary policy problem becomes

$$\min_{i_t^{dis}} \frac{1}{2}\left(\left(\tilde{y}_t^{CB}\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right) \quad \text{subject to (A.20)}$$

which yields the optimality condition

$$\tilde{y}_t^{CB} = -\frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\pi_t$$

I again limit attention to equilibrium solutions where $\frac{H_{\pi,i}}{H_{\tilde{y},i}} \geq 0$. Then, substituting this into the inflation equation and solving forward for π_t gives

$$\pi_t = \beta\pi_{t+1|t} - \frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon\pi_t + \boldsymbol{\Xi}_{\pi,1}\mathbf{z}_{1,t} = \frac{\boldsymbol{\Xi}_{\pi,1}}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon}\left[\mathbf{I} - \frac{\beta}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon}\Upsilon_{11}\right]^{-1}\mathbf{z}_{1,t}$$

Then, the optimality condition gives

$$\tilde{y}_t^{CB} = -\frac{H_{\pi,i}}{H_{\tilde{y},i}} \frac{\varepsilon}{\kappa} \frac{\mathbf{\Xi}_{\pi,1}}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon} \left[\mathbf{I} - \frac{\beta}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon} \Upsilon_{11} \right]^{-1} \mathbf{z}_{1,t}$$

This shows that fluctuations in the welfare-relevant outcomes $\tilde{y}_t^{CB}$ and π_t are only caused by $\mathbf{z}_{1,t}$ and changes in $\mathbf{z}_{2,t}$ and $\mathbf{z}_{2,t|t}$ do not affect these outcomes in equilibrium and so

$$\frac{d\tilde{y}_t^{CB}}{d\mathbf{z}_{2,t}} = \frac{d\pi_t}{d\mathbf{z}_{2,t}} = \frac{d\tilde{y}_t^{CB}}{d\mathbf{z}_{2,t|t}} = \frac{d\pi_t}{d\mathbf{z}_{2,t|t}} = 0$$

These expressions also reveal that $\mathbf{M}_2 = 0$ and give the equilibrium expression for $\mathbf{M}_1$ since

$$\begin{bmatrix} \tilde{y}_{t+1|t}^{CB} \\ \pi_{t+1|t} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{H_{\pi,i}}{H_{\tilde{y},i}} \frac{\varepsilon}{\kappa} \\ 1 \end{bmatrix} \frac{\mathbf{\Xi}_{\pi,1}}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon} \left[\mathbf{I} - \frac{\beta}{1+\frac{H_{\pi,i}}{H_{\tilde{y},i}}\varepsilon} \Upsilon_{11} \right]^{-1}}_{\mathbf{M}_1} \Upsilon_{11} \mathbf{z}_{1,t}$$

Then,

$$\begin{aligned} \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} &= \left(\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix} \mathbf{M}_2 \Upsilon_{22} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} \mathbf{F}_{2,b} \right) \mathbf{K}_z - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} \\ &= -\begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} (1 + \mathbf{F}_{2,b}\mathbf{K}_z) \Rightarrow \frac{H_{\pi,i}}{H_{\tilde{y},i}} = \kappa \end{aligned}$$

and the discretionary policy optimality condition is equivalent to the perfect information case.

A.6 Proof of Proposition 5

I repeat the equilibrium conditions here for convenience

$$\begin{aligned} \tilde{y}_t &= \tilde{y}_{t+1|t} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right) + d_t - d_{t+1|t} \\ \pi_t &= \beta\pi_{t+1|t} + \kappa\tilde{y}_t \end{aligned}$$

The optimal discretionary interest rate policy under perfect information implements $\tilde{y}_t^{PI} - \bar{y}_t = -\varepsilon\pi_t^{PI}$ which yields the solution

$$\begin{bmatrix} \tilde{y}_t^{PI} - \bar{y}_t \\ \pi_t^{PI} \end{bmatrix} = \begin{bmatrix} -\varepsilon\kappa \\ \kappa \end{bmatrix} \frac{1}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa} \bar{y}_t$$

The optimal discretionary interest rate policy under imperfect information implements $\tilde{y}_t - \bar{y}_t = -\mathcal{R}\frac{\varepsilon}{\kappa}\pi_t$ which yields the following solution (as shown in the proof of Proposition 2)

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} -\mathcal{R}\varepsilon \\ \kappa \end{bmatrix} \frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\left(\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{t|t}-\bar{y}_t\right)+\bar{y}_t\right)$$

The equilibrium belief error is

$$\bar{y}_{t|t} - \bar{y}_t = \left(K_{\bar{y}} f^*_{\bar{y}}\left(\mathcal{R}\right)-1\right)\epsilon_{\bar{y},t} + K_{\bar{y}}\sigma\epsilon_{d,t} = -\frac{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1}\epsilon_{\bar{y},t} - \frac{1+\mathcal{R}\varepsilon}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1}\epsilon_{d,t}$$

which gives

$$\begin{aligned} E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)^2\right] &= \frac{\left(1+\mathcal{R}\varepsilon\right)^2\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1} \quad \text{for } s>t \\ E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)\bar{y}_s\right] &= -\frac{\left(1+\mathcal{R}\varepsilon\right)^2\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1} \quad \text{for } s>t \end{aligned}$$

Thus, in equilibrium

$$\begin{aligned} l_t^{PI} &\equiv \frac{1}{2}\left[\left(\tilde{y}_t^{PI}-\bar{y}_t\right)^2+\frac{\varepsilon}{\kappa}\left(\pi_t^{PI}\right)^2\right] = \frac{1}{2}\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\bar{y}_t^2 \\ l_t &\equiv \frac{1}{2}\left[\left(\tilde{y}_t-\bar{y}_t\right)^2+\frac{\varepsilon}{\kappa}\pi_t^2\right] = \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\left(\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{t|t}-\bar{y}_t\right)+\bar{y}_t\right)^2 \end{aligned}$$

$$\begin{aligned} E_t^{CB}\mathcal{L}_{t+1}^{PI} &\equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s^{PI}-\bar{y}_s\right)^2+\frac{\varepsilon}{\kappa}\left(\pi_s^{PI}\right)^2\right) = \frac{1}{2}\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right] \\ E_t^{CB}\mathcal{L}_{t+1} &\equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s-\bar{y}_s\right)^2+\frac{\varepsilon}{\kappa}\pi_s^2\right) \\ &= \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\left(\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{s|s}-\bar{y}_s\right)+\bar{y}_s\right)^2\right] \\ &= \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\left\{\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right]-\frac{1}{1-\beta}\frac{2\left(1+\mathcal{R}\varepsilon\right)-\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\frac{\left(1+\mathcal{R}\varepsilon\right)^2\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1}\right\} \end{aligned}$$

The difference in the expected future welfare loss is then

$$\begin{aligned} E_t^{CB}\left[\mathcal{L}_{t+1}-\mathcal{L}_{t+1}^{PI}\right] &= \frac{1}{2}\left(\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}-\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right] \\ &\quad -\frac{1}{2}\frac{\beta\rho_{\bar{y}}}{1-\beta}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\frac{\left[2\left(1+\mathcal{R}\varepsilon\right)-\beta\rho_{\bar{y}}\right]\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{y}}^2}+1} \end{aligned}$$

To see that the first term is negative, note that Proposition 2 showed that $\mathcal{R}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$. Then, since

$$\begin{aligned} \frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} &= \frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2} \text{ for } \mathcal{R}=\kappa \\ \text{while } \frac{d}{d\mathcal{R}}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} &= 2\varepsilon^2\frac{\left(1-\beta\rho_{\bar{y}}\right)\mathcal{R}-\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^3}\leq 0 \text{ for } \mathcal{R}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right] \end{aligned}$$

This proves that

$$\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\leq\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2} \text{ for } \mathcal{R}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$$

The second term is clearly negative since $2\left(1+\mathcal{R}\varepsilon\right)-\beta\rho_{\bar{y}}\geq 1+2\mathcal{R}\varepsilon\geq 0$.

The difference in the current period loss is

$$\begin{aligned} l_t-l_t^{PI} &= \frac{1}{2}\left(\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}-\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\right)\bar{y}_t^2 \\ &\quad +\frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{t|t}-\bar{y}_t\right)^2+2\left(\bar{y}_{t|t}-\bar{y}_t\right)\bar{y}_t\right) \end{aligned}$$

Again, the first term is negative, but the second term may be positive and larger than the first term.

A.6.1 Proof of Corollary 6

If I exogenously impose that $\bar{y}_{s|s}=\bar{y}_s$, then this is equivalent to setting

$$E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)^2\right]=E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)\bar{y}_s\right]=0$$

which gives

$$E_t^{CB}\left[\mathcal{L}_{t+1}-\mathcal{L}_{t+1}^{PI}\right]=\frac{1}{2}\left(\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}-\frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{y}_s^2\right]$$
$$\leq 0 \ \text{ if } \mathcal{R}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right]$$

If I exogenously impose $\mathcal{R}=\kappa$, then the difference in the expected future welfare loss is then

$$\begin{aligned}
E_t^{CB}\left[\mathcal{L}_{t+1}-\mathcal{L}_{t+1}^{PI}\right]&=\frac{1}{2}\frac{\varepsilon\kappa\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\left\{\frac{\beta\rho_{\bar{y}}}{1+\varepsilon\kappa}E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)^2\right]+2E_t^{CB}\left[\left(\bar{y}_{s|s}-\bar{y}_s\right)\bar{y}_s\right]\right\}\\
&=\frac{1}{2}\frac{\varepsilon\kappa\beta\rho_{\bar{y}}}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}\frac{\beta\rho_{\bar{y}}}{1+\varepsilon\kappa}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\left(E_t^{CB}\left[\bar{y}_{s|s}^2\right]-E_t^{CB}\left[\bar{y}_s^2\right]\right)\\
&+\frac{\varepsilon\kappa\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\varepsilon\kappa}\frac{1}{1+\varepsilon\kappa}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\left(E_t^{CB}\left[\bar{y}_{s|s}\bar{y}_s\right]-E_t^{CB}\left[\bar{y}_s^2\right]\right)
\end{aligned}$$

This is clearly weakly negative if

$$E_t^{CB}\left[\bar{y}_{s|s}^2\right]\leq E_t^{CB}\left[\bar{y}_s^2\right] \quad\text{and}\quad E_t^{CB}\left[\bar{y}_{s|s}\bar{y}_s\right]\leq E_t^{CB}\left[\bar{y}_s^2\right] \quad \text{for } s>t$$

Note that this is equivalent to

$$Var_t^{CB}\left(y_{s|s}\right)\leq Var_t^{CB}\left(y_s\right) \text{ and } Cov_t^{CB}\left(\bar{y}_{s|s},\bar{y}_s\right)\leq Var_t^{CB}\left(y_s\right)$$

since $E_t^{CB}\bar{y}_{s|s}=E_t^{CB}\bar{y}_s$ for $s>t$ so that

$$\begin{aligned}
Cov_t^{CB}\left(\bar{y}_{s|s},\bar{y}_s\right)&=E_t^{CB}\left[\bar{y}_{s|s}\bar{y}_s\right]-\left(E_{t_0}^{CB}\bar{y}_s\right)^2\\
Var_t^{CB}\left(y_{s|s}\right)&=E_t^{CB}\left[\bar{y}_{s|s}^2\right]-\left(E_t^{CB}\bar{y}_s\right)^2\\
Var_t^{CB}\left(y_s\right)&=E_t^{CB}\left[\bar{y}_s^2\right]-\left(E_t^{CB}\bar{y}_s\right)^2
\end{aligned}$$

Then, another set of equivalent conditions is

$$Var_t^{CB}\left(y_{s|s}\right)\leq Var_t^{CB}\left(y_s\right) \text{ and } Corr_t^{CB}\left(\bar{y}_{s|s},\bar{y}_s\right)=\frac{Cov_t^{CB}\left(\bar{y}_{s|s},\bar{y}_s\right)}{\sqrt{Var_t^{CB}\left(y_s\right)Var_t^{CB}\left(y_{s|s}\right)}}\leq 1$$

since this gives

$$Cov_t^{CB}\left(\bar{y}_{s|s},\bar{y}_s\right)\leq\sqrt{Var_t^{CB}\left(y_s\right)Var_t^{CB}\left(y_{s|s}\right)}\leq Var_t^{CB}\left(y_s\right)$$

A.7 Proof of Proposition 7

Here, I consider the case where the central bank directly communicates d_t to private agents prior to observing i_t. Then, agents infer $\bar{y}_t$ upon observing i_t. In equilibrium, since agents know beliefs will be correct with $d_{t|t} = d_t$ and $\bar{y}_{t|t} = \bar{y}_t$. However, a key feature of this setup is that the interest rate retains its signaling effect on $\bar{y}_{t|t}$ since from the policymaker's point of view, beliefs are the following function of i_t^{dis}.

$$\bar{y}_{t|t} = \frac{1}{f_{\bar{y}}} \left(i_t^{dis} - f_d d_t \right)$$

Thus, the policymaker's choice has a marginal impact of $K_{\bar{y}} \equiv \frac{d\bar{y}_{t|t}}{di_t^{dis}} = \frac{1}{f_{\bar{y}}}$ on beliefs.

Denoting this case with superscript ^{d}, (A.12) shows that the inflation-output tradeoff is at its steepest possible value

$$\mathcal{R}^d = \frac{\kappa}{1 - \beta\rho_{\bar{y}}}$$

with the following equilibrium outcomes under the optimal discretionary interest rate policy after taking into account that beliefs are correct in equilibrium

$$\pi_t^d = \frac{\kappa \left(1 - \beta\rho_{\bar{y}}\right)}{\left(1 - \beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} \bar{y}_t \quad \text{and} \quad \tilde{y}_t^d - \bar{y}_t = -\frac{\varepsilon\kappa}{\left(1 - \beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} \bar{y}_t$$

Then, the associated welfare loss terms are

$$l_t^d \equiv \left(\tilde{y}_t^d - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa} \left(\pi_t^d\right)^2 = \frac{\varepsilon\kappa}{\left(1 - \beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} \bar{y}_t^2$$

$$E_t^{CB} \mathcal{L}_{t+1}^d \equiv E_t^{CB} \sum_{s=t+1}^{\infty} \beta^{s-(t+1)} \frac{1}{2} \left(\left(\tilde{y}_s^d - \bar{y}_s\right)^2 + \frac{\varepsilon}{\kappa} \left(\pi_s^d\right)^2 \right) = \frac{1}{2} \frac{\varepsilon\kappa}{\left(1 - \beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} \sum_{s=t+1}^{\infty} \beta^{s-(t+1)} E_t^{CB} \left[\bar{y}_s^2\right]$$

Compared to the case of full communication, communicating only d_t is strictly preferable for any realizations of the current shocks.

$$l_t^d - l_t^{PI} = \frac{1}{2} \left(\frac{\varepsilon\kappa}{\left(1 - \beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} - \frac{\varepsilon\kappa \left(1 + \varepsilon\kappa\right)}{\left(1 - \beta\rho_{\bar{y}} + \varepsilon\kappa\right)^2} \right) \bar{y}_t^2 \leq 0$$

$$E_t^{CB} \left(\mathcal{L}_{t+1}^d - \mathcal{L}_{t+1}^{PI}\right) = \frac{1}{2} \left(\frac{\varepsilon\kappa}{\left(1 - \beta\rho_{\bar{y}}\right)^2 + \varepsilon\kappa} - \frac{\varepsilon\kappa \left(1 + \varepsilon\kappa\right)}{\left(1 - \beta\rho_{\bar{y}} + \varepsilon\kappa\right)^2} \right) \frac{\rho_{\bar{y}}^2 \bar{y}_t^2 + \frac{1}{1-\beta}\sigma_{\bar{y}}^2}{1 - \beta\rho_{\bar{y}}^2} \leq 0$$

Both the current period welfare loss and expected future loss are lower in the case of communicating only

d_t since

$$\beta\rho_{\bar{y}} \geq 0 \text{ and } \varepsilon\kappa \geq 0 \Rightarrow \frac{\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2+\varepsilon\kappa} \leq \frac{\varepsilon\kappa\left(1+\varepsilon\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}$$

On the other hand, when the case of communicating only d_t is compared to the no additional communication case, neither case produces unambiguously lower losses for either current period or expected future welfare.

$$\begin{aligned} l_t^d - l_t =& \frac{1}{2}\left(\frac{\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2+\varepsilon\kappa} - \frac{\mathcal{R}^2\varepsilon+\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\right)\varepsilon\bar{y}_t^2 \\ &- \frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\frac{1}{2}\frac{\beta\rho_{\bar{y}}}{1+\mathcal{R}\varepsilon}\left(\bar{y}_{t|t}-\bar{y}_t\right)^2+\left(\bar{y}_{t|t}-\bar{y}_t\right)\bar{y}_t\right) \end{aligned}$$

$$\begin{aligned} E_t^{CB}\left(\mathcal{L}_{t+1}^d - \mathcal{L}_{t+1}\right) =& \frac{1}{2}\left(\frac{\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2+\varepsilon\kappa} - \frac{\mathcal{R}^2\varepsilon+\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\right)\varepsilon\frac{\rho_{\bar{y}}^2\bar{y}_t^2+\frac{1}{1-\beta}\sigma_{\bar{y}}^2}{1-\beta\rho_{\bar{y}}^2} \\ &+\frac{1}{2}\frac{1}{1-\beta}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2}\frac{\left(2\left(1+\varepsilon\mathcal{R}\right)-\beta\rho_{\bar{y}}\right)\beta\rho_{\bar{y}}\sigma_d^2\sigma_{\bar{y}}^2}{\left(1+\mathcal{R}\varepsilon\right)^2\sigma_d^2+\sigma_{\bar{y}}^2} \end{aligned}$$

The first term in each of these expressions is negative and reflects the benefit of maximizing the interest rate's effect on inflation expectations, thereby achieving the largest possible reduction in the stabilization bias through the signaling channel. To see that it's always negative, note the following

$$\begin{aligned} \frac{\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2+\varepsilon\kappa} &= \frac{\mathcal{R}^2\varepsilon+\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} \quad \text{for } \mathcal{R}=\frac{\kappa}{1-\beta\rho_{\bar{y}}} \\ \text{while } \frac{d}{d\mathcal{R}}\frac{\mathcal{R}^2\varepsilon+\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} &= -2\varepsilon\frac{\kappa-\left(1-\beta\rho_{\bar{y}}\right)\mathcal{R}}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^3} \leq 0 \quad \text{for } \mathcal{R}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right] \\ \text{so that } \frac{\kappa}{\left(1-\beta\rho_{\bar{y}}\right)^2+\varepsilon\kappa} &\leq \frac{\mathcal{R}^2\varepsilon+\kappa}{\left(1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon\right)^2} \quad \text{for } \mathcal{R}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right] \end{aligned}$$

The second term in $E_t^{CB}\left(\mathcal{L}_{t+1}^d - \mathcal{L}_{t+1}\right)$ is positive since $2\left(1+\mathcal{R}\varepsilon\right)-\beta\rho_{\bar{y}} \geq 1+2\mathcal{R}\varepsilon \geq 0$. This reflects the loss of the benefit of decoupling the comovement in agents' beliefs about the output gap target and its true value. Thus, whether this type of partial communication is beneficial for expected future welfare losses is ambiguous for general parameter values. Meanwhile, the second term in $l_t^d - l_t$ can always be positive for large enough negative realizations of $\left(\bar{y}_{t|t}-\bar{y}_t\right)\bar{y}_t$ so this difference stays ambiguous even for a fixed set of parameter values.

The following can be shown for special parameterizations:

- As $\sigma_d^2 \to 0$ while $\sigma_{\bar{y}}^2$ stays positive, $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$. As the demand shock becomes more negligible, so does the effect of communicating its true value. Even without any additional communication, the interest rate's signaling effect on inflation expectations is already high so the further reduction in the stabilization bias from communicating d_t disappears. Furthermore, as $\sigma_d^2 \to 0$, private agents' forecast errors regarding the output gap target become negligible and their beliefs $\bar{y}_{t|t}$ approach the true $\bar{y}_t$ so the benefit of reducing their comovement by not directly communicating also disappears.

$$\lim_{\sigma_d^2 \to 0} E_t^{CB}\mathcal{L}_{t+1} \to E_t^{CB}\mathcal{L}_{t+1}^d$$

$$\lim_{\sigma_d^2 \to 0} l_t \to l_t^d \quad \text{if } \epsilon_{d,t} = 0$$

Here, the benefit of not communicating the true value of $\bar{y}_t$ remains so that

$$\lim_{\sigma_d^2 \to 0} E_t^{CB}\mathcal{L}_{t+1} < E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \text{and} \quad \lim_{\sigma_d^2 \to 0} l_t < l_t^{PI}$$

- As $\sigma_{\bar{y}}^2 \to 0$ while σ_d^2 stays positive, $\mathcal{R} \to \kappa$. In this case, the inflation-output tradeoff disappears entirely and the economy approaches one in which the flexible price equilibrium is always efficient and is achievable regardless of the information setting.

$$\lim_{\sigma_{\bar{y}}^2 \to 0} E_t^{CB}\mathcal{L}_{t+1} \to E_t^{CB}\mathcal{L}_{t+1}^d = E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \text{if } \bar{y}_t = 0$$

$$\lim_{\sigma_{\bar{y}}^2 \to 0} l_t \to l_t^d = l_t^{PI} \quad \text{if } \epsilon_{\bar{y},t} = \bar{y}_t = 0$$

- If $\beta\rho_{\bar{y}} = 0$, then the inflation-output tradeoff is no longer affected by private agents' beliefs since inflation is driven purely by current marginal costs. Then, the information setting again becomes irrelevant.

$$E_t^{CB}\mathcal{L}_{t+1} = E_t^{CB}\mathcal{L}_{t+1}^d = E_t^{CB}\mathcal{L}_{t+1}^{PI} \quad \text{if } \beta\rho_{\bar{y}} = 0$$

$$l_t = l_t^d = l_t^{PI} \quad \text{if } \beta\rho_{\bar{y}} = 0$$

A.8 Proof of Proposition 8

I now introduce a cost-push shock that private agents are perfectly informed about (i.e., $\mathcal{I}_t = \left\{i^t, v^t, d^{t-1}, \bar{y}^{t-1}\right\}$) so that the equilibrium conditions become

$$\tilde{y}_t = \tilde{y}_{t+1|t} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right) + d_t - d_{t+1|t}$$
$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t + v_t$$

Conjecturing a solution that's linear in the expanded set of state variables $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}, v_t\right\}$ results in expectations of future outcomes of the form

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} + \mathbf{M}_v \rho_v v_t$$

Beliefs are now formed according to the supposition that

$$i_t = f_d d_t + f_{\bar{y}}\bar{y}_t + f_v v_t + f_{d,b} d_{t|t} + f_{\bar{y},b}\bar{y}_{t|t}$$

and I again define the interest rate policy problem as a choice of a discretionary component of the interest rate i_t^{dis} where the final realized nominal rate is

$$i_t = i_t^{dis} + f_{d,b} d_{t|t} + f_{\bar{y},b}\bar{y}_{t|t}$$

Beliefs can be derived using the same procedure as Section 1.2.3 which results in

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}}\bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} K_d \\ K_{\bar{y}} \end{bmatrix} \left(i_t^{dis} - f_d\rho_d d_{t-1} - f_{\bar{y}}\rho_{\bar{y}}\bar{y}_{t-1} - f_v v_t\right)$$

where $K_d = \frac{f_d \frac{\sigma_d^2}{\sigma_{\bar{y}}^2}}{f_d^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} + f_{\bar{y}}^2}$ and $K_{\bar{y}} = \frac{f_{\bar{y}}}{f_d^2 \frac{\sigma_d^2}{\sigma_{\bar{y}}^2} + f_{\bar{y}}^2}$ as before and are again taken as given constants by the discretionary policymaker.

Following the same steps as the proof of Proposition 2, I use the form of expectations and beliefs to write the output gap deviation and inflation in terms of the exogenous states and i_t^{dis} so that the discretionary policy problem becomes

$$\min_{i_t^{dis}} \frac{1}{2}\left((\tilde{y}_t-\bar{y}_t)^2+\frac{\varepsilon}{\kappa}\pi_t^2\right)$$

$$\begin{aligned}
\text{where }\begin{bmatrix}\tilde{y}_t-\bar{y}_t\\ \pi_t\end{bmatrix} &= \mathbf{\Psi}\begin{bmatrix}1-K_d f_d & -K_d f_{\bar{y}}\\ -K_{\bar{y}} f_d & 1-K_{\bar{y}} f_{\bar{y}}\end{bmatrix}\begin{bmatrix}\rho_d d_{t-1}\\ \rho_{\bar{y}}\bar{y}_{t-1}\end{bmatrix}+\begin{bmatrix}1 & -1\\ \kappa & 0\end{bmatrix}\begin{bmatrix}d_t\\ \bar{y}_t\end{bmatrix}\\
&+\left(\begin{bmatrix}1 & \frac{1}{\sigma}\\ \kappa & \frac{\kappa}{\sigma}+\beta\end{bmatrix}\mathbf{M}_v\rho_v+\begin{bmatrix}0\\ 1\end{bmatrix}-\mathbf{\Psi}\begin{bmatrix}K_d f_v\\ K_{\bar{y}} f_v\end{bmatrix}\right)v_t+\begin{bmatrix}H_{\tilde{y},i}\\ H_{\pi,i}\end{bmatrix}i_t^{dis}\\
\mathbf{\Psi} &\equiv \begin{bmatrix}1 & \frac{1}{\sigma}\\ \kappa & \frac{\kappa}{\sigma}+\beta\end{bmatrix}\mathbf{M}\begin{bmatrix}\rho_d & 0\\ 0 & \rho_{\bar{y}}\end{bmatrix}-\begin{bmatrix}\rho_d+\frac{1}{\sigma}f_{d,b} & \frac{1}{\sigma}f_{\bar{y},b}\\ \kappa\left(\rho_d+\frac{1}{\sigma}f_{d,b}\right) & \frac{\kappa}{\sigma}f_{\bar{y},b}\end{bmatrix}\\
\begin{bmatrix}H_{\tilde{y},i}\\ H_{\pi,i}\end{bmatrix} &\equiv \mathbf{\Psi}\begin{bmatrix}K_d\\ K_{\bar{y}}\end{bmatrix}-\begin{bmatrix}\frac{1}{\sigma}\\ \frac{\kappa}{\sigma}\end{bmatrix}
\end{aligned}$$

Then, clearly, the optimality condition is again

$$\tilde{y}_t-\bar{y}_t=-\mathcal{R}\frac{\varepsilon}{\kappa}\pi_t \quad \text{with } \mathcal{R}=\frac{H_{\pi,i}}{H_{\tilde{y},i}}$$

Substituting this into the equilibrium conditions and solving again for the endogenous variables as I did in the proof of Proposition 2 gives

$$\begin{aligned}
\pi_t &= \beta\pi_{t+1|t}-\mathcal{R}\varepsilon\pi_t+\kappa\bar{y}_t+v_t\\
&= \frac{\kappa}{1+\mathcal{R}\varepsilon}\bar{y}_t+\frac{\kappa}{1+\mathcal{R}\varepsilon}\frac{\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\bar{y}_{t|t}+\frac{1}{1-\beta\rho_v+\mathcal{R}\varepsilon}v_t\\
\tilde{y}_t &= \frac{1}{1+\mathcal{R}\varepsilon}\bar{y}_t-\frac{\mathcal{R}\varepsilon}{1+\mathcal{R}\varepsilon}\frac{\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\frac{H_{i,2}}{H_{i,1}}\varepsilon}\bar{y}_{t|t}-\frac{\mathcal{R}\frac{\varepsilon}{\kappa}}{1-\beta\rho_v+\mathcal{R}\varepsilon}v_t
\end{aligned}$$

and

$$\begin{bmatrix}\tilde{y}_{t+1|t}\\ \pi_{t+1|t}\end{bmatrix}=\underbrace{\begin{bmatrix}0 & \frac{1-\beta\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\\ 0 & \frac{\kappa}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\end{bmatrix}}_{\mathbf{M}}\begin{bmatrix}\rho_d & 0\\ 0 & \rho_{\bar{y}}\end{bmatrix}\begin{bmatrix}d_{t|t}\\ \bar{y}_{t|t}\end{bmatrix}+\underbrace{\begin{bmatrix}-\frac{\mathcal{R}\frac{\varepsilon}{\kappa}}{1-\beta\rho_v+\mathcal{R}\varepsilon}\\ \frac{1}{1-\beta\rho_v+\mathcal{R}\varepsilon}\end{bmatrix}}_{\mathbf{M}_v}\rho_v v_t$$

Then, this implies that the interest rate can be written in terms of $\left\{d_t, d_{t|t}, \bar{y}, \bar{y}_{t|t}, v_t\right\}$

$$\begin{aligned}
i_t^* &= \sigma\left(d_t-d_{t+1|t}\right)+\pi_{t+1|t}+\sigma\left(\tilde{y}_{t+1|t}-\tilde{y}_t\right)\\
&= \underbrace{\sigma\left(d_t-\rho_d d_{t|t}\right)}_{r_t^n}\underbrace{-\sigma\frac{1}{1+\mathcal{R}\varepsilon}}_{f_{\bar{y}}^*(\mathcal{R})}\bar{y}_t+\underbrace{\sigma\left(\frac{1}{1+\mathcal{R}\varepsilon}-\frac{1}{\Omega_{\bar{y}}}\frac{1}{1-\beta\rho_{\bar{y}}+\mathcal{R}\varepsilon}\right)}_{f_{\bar{y},b}^*(\mathcal{R})}\bar{y}_{t|t}+\underbrace{\sigma\frac{\frac{1}{\sigma}\rho_v+\mathcal{R}\frac{\varepsilon}{\kappa}(1-\rho_v)}{1-\beta\rho_v+\mathcal{R}\varepsilon}}_{f_v^*(\mathcal{R})}v_t
\end{aligned}$$

It's clear that the equilibrium conditions between $\left\{\mathbf{M}, K_{\bar{y}}, f_{\bar{y}}^{*}, f_{\bar{y},b}^{*}, \mathcal{R}\right\}$ are the same here as in the previous case without the additional cost push shock and so the equilibrium value(s) of $\mathcal{R}$ are also the same.

In the perfect information case, conjecturing a solution that's linear in state variables $\{d_t, \bar{y}_t, v_t\}$ results in expectations of future outcomes of the form

$$\begin{bmatrix} \tilde{y}_{t+1|t}^{PI} \\ \pi_{t+1|t}^{PI} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \mathbf{M}_v \rho_v v_t$$

Then, the output gap deviation and inflation written in terms of exogenous variables along with the interest rate is

$$\begin{bmatrix} \tilde{y}_t^{PI} - \bar{y}_t \\ \pi_t^{PI} \end{bmatrix} = \left(\mathbf{\Psi} + \begin{bmatrix} 1 & -1 \\ \kappa & 0 \end{bmatrix} \right) \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \left(\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M}_v \rho_v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) v_t - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} i_t^{dis}$$

Thus, the discretionary policy problem is equivalent to minimizing the current period loss subject to this condition. Then the perfect information discretionary policy optimality condition and equilibrium conditions (including the interest rate behavior) are again the same as the imperfect information case with κ in place of $\mathcal{R}$.

A.9 Overreaction to the additional cost-push shock

This section shows that when a separate cost-push shock is added to the model, the optimal interest rate under discretion no longer corresponds to the optimal commitment to a forward-looking rule in the limit where the interest rate has its largest effect on $\bar{y}_{t|t}$.

In the limit where $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$, it's still the case that $\mathcal{R} \to \frac{\kappa}{1-\beta\rho_{\bar{y}}}$ since $K_{\bar{y}} \to -\frac{1+\mathcal{R}\varepsilon}{\sigma}$. However, this is not equivalent to commitment to a rule of the form

$$i_t = r_t^n + f_{\bar{y}}^c \bar{y}_t + f_{\bar{y},b}^c \bar{y}_{t|t} + f_v^c v_t$$

The belief $\bar{y}_{t|t}$ in the limit where $\frac{\sigma_d^2}{\sigma_{\bar{y}}^2} \to 0$ is again given by

$$\bar{y}_{t|t} = \bar{y}_t + \frac{\sigma}{f_{\bar{y}}^c} \epsilon_{d,t}$$

Following the same steps given in Section A.2 to obtain a solution under a given linear interest rate rule provides me with the solution

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = -\begin{bmatrix} \frac{\Omega_{\bar{y}}}{\sigma}\left(1-\beta\rho_{\bar{y}}\right)\left(f^c_{\bar{y}} + f^c_{\bar{y},b}\right) + 1 \\ \frac{\kappa\Omega_{\bar{y}}}{\sigma}\left(f^c_{\bar{y}} + f^c_{\bar{y},b}\right) \end{bmatrix} \bar{y}_t - \begin{bmatrix} \Omega_{\bar{y}}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}} + \frac{\kappa}{\sigma}\right)\left(1 + \frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}}\right) + \frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}} \\ \kappa\Omega_{\bar{y}}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\left(1 + \frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}}\right) + \kappa\frac{f^c_{\bar{y},b}}{f^c_{\bar{y}}} \end{bmatrix} \epsilon_{d,t}$$
$$+ \begin{bmatrix} \frac{\frac{1}{\sigma}\rho_v - \frac{1}{\sigma}f^c_v(1-\beta\rho_v)}{(1-\rho_v)(1-\beta\rho_v) - \frac{\kappa}{\sigma}\rho_v} \\ \frac{1-\rho_v - \frac{\kappa}{\sigma}f^c_v}{(1-\rho_v)(1-\beta\rho_v) - \frac{\kappa}{\sigma}\rho_v} \end{bmatrix} v_t$$

Then, the optimality conditions for $f^c_{\bar{y}}$ and $f^c_{\bar{y},b}$ are the same as in the proof of Proposition 2

$$0 = E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\left(\tilde{y}_t - \bar{y}_t\right)\left(1-\beta\rho_{\bar{y}}\right) + \varepsilon\pi_t\right)\left[-\frac{1}{\sigma}\bar{y}_t + \frac{f^c_{\bar{y},b}}{\left(f^c_{\bar{y}}\right)^2}\epsilon_{d,t}\right]$$
$$\text{and } 0 = E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\left(\tilde{y}_t - \bar{y}_t\right)\left(1-\beta\rho_{\bar{y}}\right) + \varepsilon\pi_t\right)\left[-\frac{1}{\sigma}\bar{y}_t - \frac{1}{f^c_{\bar{y}}}\epsilon_{d,t}\right]$$

The new optimality condition for f^c_v is

$$0 = \frac{\partial}{\partial f^c_v}E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2\right) \Rightarrow 0 = E^{CB}_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\left(\tilde{y}_t - \bar{y}_t\right) + \frac{\varepsilon}{\kappa}\pi_t\frac{\kappa}{1-\beta\rho_v}\right)v_t$$

Now, it's clear that there's no single optimal ratio between $\tilde{y}_t - \bar{y}_t$ and π_t that could satisfy all of these.

Using the equilibrium solutions for $\tilde{y}_t - \bar{y}_t$ and π_t and evaluating expectations from an ex-ante unconditional perspective gives the following set of equations that satisfy all three optimality conditions and determine the optimal policy rule coefficients

$$0 = \left(\frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f^{*,c}_{\bar{y}} + f^{*,c}_{\bar{y},b}\right) + 1\right)\left(1-\beta\rho_{\bar{y}}\right) + \varepsilon\frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f^{*,c}_{\bar{y}} + f^{*,c}_{\bar{y},b}\right)$$
$$0 = \Omega_{\bar{y}}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}} + \frac{\kappa}{\sigma}\right)\left(1 + \frac{f^{*,c}_{\bar{y},b}}{f^{*,c}_{\bar{y}}}\right) + \frac{f^{*,c}_{\bar{y},b}}{f^{*,c}_{\bar{y}}} + \varepsilon\left(\kappa\Omega_{\bar{y}}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\left(1 + \frac{f^{*,c}_{\bar{y},b}}{f^{*,c}_{\bar{y}}}\right) + \kappa\frac{f^{*,c}_{\bar{y},b}}{f^{*,c}_{\bar{y}}}\right)$$
$$0 = \frac{\frac{1}{\sigma}\rho_v - \frac{1}{\sigma}f^{*,c}_v\left(1-\beta\rho_v\right)}{\left(1-\rho_v\right)\left(1-\beta\rho_v\right) - \frac{\kappa}{\sigma}\rho_v} + \varepsilon\frac{1-\rho_v - \frac{\kappa}{\sigma}f^{*,c}_v}{\left(1-\rho_v\right)\left(1-\beta\rho_v\right) - \frac{\kappa}{\sigma}\rho_v}$$

The resulting solutions are

$$f^{*,c}_{\bar{y}} = -\sigma\frac{1}{1 + \frac{\varepsilon\kappa}{1-\beta\rho_{\bar{y}}}}$$
$$f^{*,c}_{\bar{y},b} = \sigma\left(\frac{1}{1 + \frac{\varepsilon\kappa}{1-\beta\rho_{\bar{y}}}} - \frac{\left(1-\rho_{\bar{y}}\right)\left(1-\beta\rho_{\bar{y}}\right) - \frac{\kappa}{\sigma}\rho_{\bar{y}}}{1-\beta\rho_{\bar{y}} + \frac{\varepsilon\kappa}{1-\beta\rho_{\bar{y}}}}\right)$$
$$f^{*,c}_v = \sigma\frac{\frac{1}{\sigma}\rho_v + \frac{\varepsilon}{1-\beta\rho_v}\left(1-\rho_v\right)}{1-\beta\rho_v + \frac{\varepsilon\kappa}{1-\beta\rho_v}}$$

Then, it's clear that

$$f_v^{*,c} = f_v^*\left(\frac{\kappa}{1-\beta\rho_v}\right) \neq f_v^*\left(\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right) = \sigma\frac{\frac{1}{\sigma}\rho_v + \frac{\varepsilon}{1-\beta\rho_{\bar{y}}}(1-\rho_v)}{1-\beta\rho_v + \frac{\varepsilon\kappa}{1-\beta\rho_{\bar{y}}}}$$

and

$$f_v^{*,c} = f_v^*\left(\frac{\kappa}{1-\beta\rho_v}\right) < f_v^*\left(\frac{\kappa}{1-\beta\rho_{\bar{y}}}\right) \quad \text{iff} \quad \rho_v < \rho_{\bar{y}}$$
$$\text{since} \quad f_v^{*\prime}(\mathcal{R}) = \sigma\frac{\varepsilon}{\kappa}\frac{(1-\beta\rho_v)(1-\rho_v) - \frac{\kappa}{\sigma}\rho_v}{[1-\beta\rho_v + \mathcal{R}\varepsilon]^2} > 0 \quad \text{when} \quad \rho_v \in [0,\bar{\rho})$$

A.10 Proof of Proposition 9

I repeat the equilibrium conditions here for convenience

$$\tilde{y}_t = \tilde{y}_{t+1|t} - \frac{1}{\sigma}\left(i_t - \pi_{t+1|t}\right) + d_t - d_{t+1|t}$$
$$\pi_t = \beta\pi_{t+1|t} + \kappa\tilde{y}_t$$

The optimal discretionary interest rate policy under perfect information implements $\tilde{y}_t^{PI} = -\varepsilon\left(\pi_t^{PI} - \bar{\pi}_t\right)$ which yields the solution

$$\begin{bmatrix} \tilde{y}_t^{PI} \\ \pi_t^{PI} - \bar{\pi}_t \end{bmatrix} = \begin{bmatrix} \varepsilon \\ -1 \end{bmatrix} \frac{1-\beta\rho_{\bar{\pi}}}{1-\beta\rho_{\bar{\pi}} + \varepsilon\kappa}\bar{\pi}_t$$

The optimal discretionary interest rate policy under imperfect information implements $\tilde{y}_t = -\mathcal{R}\frac{\varepsilon}{\kappa}(\pi_t - \bar{\pi}_t)$ which yields the following solution

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t - \bar{\pi}_t \end{bmatrix} = \begin{bmatrix} -\mathcal{R}\frac{\varepsilon}{\kappa} \\ 1 \end{bmatrix} \frac{1}{1-\beta\rho_{\bar{\pi}} + \mathcal{R}\varepsilon}\left(\frac{\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}\varepsilon}\left(\bar{\pi}_{t|t} - \bar{\pi}_t\right) - (1-\beta\rho_{\bar{\pi}})\bar{\pi}_t\right)$$

with equilibrium interest rate behavior given by

$$i_t = \underbrace{\sigma d_t - \sigma\rho_d d_{t|t}}_{r_t^n} \underbrace{-\sigma\frac{\mathcal{R}\frac{\varepsilon}{\kappa}}{1+\mathcal{R}\varepsilon}}_{f_{\bar{\pi}}^*(\mathcal{R})}\bar{\pi}_t + \underbrace{\sigma\left(\frac{1}{1+\mathcal{R}\varepsilon} - \frac{1}{\Omega_{\bar{\pi}}}\frac{1}{1-\beta\rho_{\bar{\pi}} + \mathcal{R}\varepsilon}\right)\mathcal{R}\frac{\varepsilon}{\kappa}}_{f_{\bar{\pi},b}^*(\mathcal{R})}\bar{\pi}_{t|t}$$
$$\text{where } \Omega_{\bar{\pi}} \equiv \frac{1}{(1-\rho_{\bar{\pi}})(1-\beta\rho_{\bar{\pi}}) - \frac{\kappa}{\sigma}\rho_{\bar{\pi}}}$$

The equilibrium belief error is

$$\bar{\pi}_{t|t} - \bar{\pi}_t = \left(K_{\bar{\pi}} f^*_{\bar{\pi}}(\mathcal{R}) - 1\right)\epsilon_{\bar{\pi},t} + K_{\bar{\pi}}\sigma\epsilon_{d,t} = -\frac{(1+\mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}}{(1+\mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}\frac{\varepsilon}{\kappa}\right)^2}\epsilon_{\bar{\pi},t} - \frac{(1+\mathcal{R}\varepsilon)\mathcal{R}\frac{\varepsilon}{\kappa}}{(1+\mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}\frac{\varepsilon}{\kappa}\right)^2}\epsilon_{d,t}$$

which gives

$$E_t^{CB}\left[\left(\bar{\pi}_{s|s} - \bar{\pi}_s\right)^2\right] = \frac{(1+\mathcal{R}\varepsilon)^2\sigma_d^2}{(1+\mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}\frac{\varepsilon}{\kappa}\right)^2} > 0 \quad \text{for } s > t$$

$$E_t^{CB}\left[\left(\bar{\pi}_{s|s} - \bar{\pi}_s\right)\bar{\pi}_s\right] = -\frac{(1+\mathcal{R}\varepsilon)^2\sigma_d^2}{(1+\mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}\frac{\varepsilon}{\kappa}\right)^2} < 0 \quad \text{for } s > t$$

Thus, in equilibrium

$$l_t^{PI} \equiv \frac{1}{2}\left[\left(\tilde{y}_t^{PI}\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_t^{PI} - \bar{\pi}_t\right)^2\right] = \frac{1}{2}\frac{\frac{\varepsilon}{\kappa}(1+\varepsilon\kappa)(1-\beta\rho_{\bar{\pi}})^2}{(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa)^2}\bar{\pi}_t^2$$

$$l_t \equiv \frac{1}{2}\left[\tilde{y}_t^2 + \frac{\varepsilon}{\kappa}\left(\pi_t - \bar{\pi}_t\right)^2\right] = \frac{1}{2}\frac{\frac{\varepsilon}{\kappa}\left(1+\mathcal{R}^2\frac{\varepsilon}{\kappa}\right)}{(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon)^2}\left(\frac{\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}\varepsilon}\left(\bar{\pi}_{t|t} - \bar{\pi}_t\right) - (1-\beta\rho_{\bar{\pi}})\bar{\pi}_t\right)^2$$

$$\begin{aligned}
E_t^{CB}\mathcal{L}_{t+1}^{PI} &\equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s^{PI}\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_s^{PI} - \bar{\pi}_s\right)^2\right) \\
&= \frac{1}{2}\varepsilon\kappa(1+\varepsilon\kappa)\left(\frac{1-\beta\rho_{\bar{\pi}}}{\kappa(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa)}\right)^2\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right] \\
E_t^{CB}\mathcal{L}_{t+1} &\equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\tilde{y}_s^2 + \frac{\varepsilon}{\kappa}\left(\pi_s - \bar{\pi}_s\right)^2\right) \\
&= \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\kappa^2(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon)^2}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\left(\frac{\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}\varepsilon}\left(\bar{\pi}_{s|s} - \bar{\pi}_s\right) - (1-\beta\rho_{\bar{\pi}})\bar{\pi}_s\right)^2\right] \\
&= \frac{1}{2}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{\kappa^2(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon)^2}\Bigg\{(1-\beta\rho_{\bar{\pi}})^2\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right] \\
&\quad + \frac{1}{2}\frac{1}{1-\beta}\frac{\left[2(1-\beta\rho_{\bar{\pi}})(1+\mathcal{R}\varepsilon) + \mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}\right]\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}\sigma_d^2}{(1+\mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}\frac{\varepsilon}{\kappa}\right)^2}\Bigg\}
\end{aligned}$$

The difference in the expected future welfare loss is then

$$\begin{aligned}
E_t^{CB}\left[\mathcal{L}_{t+1} - \mathcal{L}_{t+1}^{PI}\right] &= \frac{1}{2}\left(\frac{1-\beta\rho_{\bar{\pi}}}{\kappa}\right)^2\left(\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon)^2} - \frac{\varepsilon\kappa(1+\varepsilon\kappa)}{(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right] \\
&\quad + \frac{1}{2}\frac{1}{1-\beta}\frac{\varepsilon\left(\mathcal{R}^2\varepsilon+\kappa\right)}{(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon)^2}\frac{\left[2(1-\beta\rho_{\bar{\pi}})(1+\mathcal{R}\varepsilon) + \mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}\right]\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}\sigma_d^2}{(1+\mathcal{R}\varepsilon)^2 \frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2} + \left(\mathcal{R}\frac{\varepsilon}{\kappa}\right)^2}
\end{aligned}$$

The proof of Proposition 5 showed that the first term is negative since $\mathcal{R} \in \left[\kappa, \frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right]$. The second term is clearly positive. Thus, the implications of full communication for expected future welfare will depend on the parameterization. Unlike the case with an output gap target, output fluctuations and deviations of inflation from target will actually be smaller when the inflation target $\bar{\pi}_{t|t}$ moves with true inflation $\bar{\pi}_t$. However, no direct communication comes with a benefit of disciplining discretionary interest rate policy so the net effect is ambiguous.

The difference in the current period loss is

$$
\begin{aligned}
l_t - l_t^{PI} = & \frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2 \left(\frac{1+\mathcal{R}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^2} - \frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\bar{\pi}_t^2 \\
& + \frac{\frac{\varepsilon}{\kappa}\left(1+\mathcal{R}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^2}\frac{\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}\varepsilon}\left(\frac{1}{2}\frac{\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}\varepsilon}\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)^2 - \left(1-\beta\rho_{\bar{\pi}}\right)\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)\bar{\pi}_t\right)
\end{aligned}
$$

Again, the first term is negative, but the second term may be positive and larger than the first term depend on the realizations of shocks even for a given set of parameter values.

Here, I consider the case where the central bank directly communicates d_t to private agents prior to observing i_t. Then, agents infer $\bar{\pi}_t$ upon observing i_t. In equilibrium, since agents know beliefs will be correct with $d_{t|t} = d_t$ and $\bar{\pi}_{t|t} = \bar{\pi}_t$. However, a key feature of this setup is that the interest rate retains its signaling effect on $\bar{\pi}_{t|t}$ since from the policymaker's point of view, beliefs are the following function of i_t^{dis}.

$$
\bar{\pi}_{t|t} = \frac{1}{f_{\bar{\pi}}}\left(i_t^{dis} - f_d d_t\right)
$$

Thus, the policymaker's choice has a marginal impact of $K_{\bar{\pi}} \equiv \frac{d\bar{\pi}_{t|t}}{di_t^{dis}} = \frac{1}{f_{\bar{\pi}}}$ on beliefs.

Denoting this case with superscript ^{d}, (A.12) shows that the inflation-output tradeoff is at its steepest possible value

$$
\mathcal{R}^d = \frac{\kappa}{1-\beta\rho_{\bar{\pi}}}
$$

with the following equilibrium outcomes under the optimal discretionary interest rate policy after taking into account that beliefs are correct in equilibrium

$$
\tilde{y}_t^d = \frac{\varepsilon\left(1-\beta\rho_{\bar{\pi}}\right)}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\bar{\pi}_t \quad \text{and} \quad \pi_t^d - \bar{\pi}_t = -\frac{\left(1-\beta\rho_{\bar{y}}\right)^2}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\bar{\pi}_t
$$

Then, the associated welfare loss terms are

$$
l_t^d \equiv \frac{1}{2}\left[\left(\tilde{y}_t^d\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_t^d - \bar{\pi}_t\right)^2\right] = \frac{1}{2}\frac{\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\bar{\pi}_t^2
$$

$$E_t^{CB}\mathcal{L}_{t+1}^d \equiv E_t^{CB}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}\frac{1}{2}\left(\left(\tilde{y}_s^d\right)^2+\frac{\varepsilon}{\kappa}\left(\pi_s^d-\bar{\pi}_s\right)^2\right)=\frac{1}{2}\frac{\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right]$$

Compared to the case of full communication, communicating only d_t is strictly preferable for any realizations of the current shocks since

$$l_t^d-l_t^{PI}=\frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}-\frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\bar{\pi}_t^2\leq 0$$
$$E_t^{CB}\left(\mathcal{L}_{t+1}^d-\mathcal{L}_{t+1}^{PI}\right)=\frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}-\frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right]\leq 0$$

while

$$\beta\rho_{\bar{\pi}}\geq 0 \text{ and } \varepsilon\kappa\geq 0\Rightarrow\frac{1}{\left(1-\beta\rho_{\bar{y}}\right)^2+\varepsilon\kappa}\leq\frac{1+\varepsilon\kappa}{\left(1-\beta\rho_{\bar{y}}+\varepsilon\kappa\right)^2}$$

Compared to the baseline no direct communication case,

$$\begin{aligned}l_t^d-l_t=&\frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}-\frac{\left(1+\mathcal{R}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\varepsilon\kappa\right)^2}\right)\bar{\pi}_t^2\\&-\frac{\frac{\varepsilon}{\kappa}\left(1+\mathcal{R}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^2}\frac{\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}\varepsilon}\left(\frac{1}{2}\frac{\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}}{1+\mathcal{R}\varepsilon}\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)^2-\left(1-\beta\rho_{\bar{\pi}}\right)\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)\bar{\pi}_t\right)\end{aligned}$$

$$\begin{aligned}E_t^{CB}\left(\mathcal{L}_{t+1}^d-\mathcal{L}_{t+1}\right)=&\frac{1}{2}\frac{\varepsilon}{\kappa}\left(1-\beta\rho_{\bar{\pi}}\right)^2\left(\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}-\frac{1+\mathcal{R}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^2}\right)\sum_{s=t+1}^{\infty}\beta^{s-(t+1)}E_t^{CB}\left[\bar{\pi}_s^2\right]\\&-\frac{1}{2}\frac{1}{1-\beta}\frac{\frac{\varepsilon}{\kappa}\left(1+\mathcal{R}^2\frac{\varepsilon}{\kappa}\right)}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^2}\frac{\left[2\left(1-\beta\rho_{\bar{\pi}}\right)\left(1+\mathcal{R}\varepsilon\right)+\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}\right]\mathcal{R}\varepsilon\beta\rho_{\bar{\pi}}\sigma_d^2}{\left(1+\mathcal{R}\varepsilon\right)^2\frac{\sigma_d^2}{\sigma_{\bar{\pi}}^2}+\left(\mathcal{R}\frac{\varepsilon}{\kappa}\right)^2}\end{aligned}$$

For general parameter values, both terms in $E_t^{CB}\left(\mathcal{L}_{t+1}^d-\mathcal{L}_{t+1}\right)$ are negative since the second term is clearly negative while

$$\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}=\frac{1+\mathcal{R}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^2}\quad\text{for } \mathcal{R}=\frac{\kappa}{1-\beta\rho_{\bar{\pi}}}$$
$$\frac{\partial}{\partial\mathcal{R}}\frac{1+\mathcal{R}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^2}=-2\frac{\varepsilon}{\kappa}\frac{\kappa-\left(1-\beta\rho_{\bar{\pi}}\right)\mathcal{R}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^3}\leq 0\quad\text{for}\quad\mathcal{R}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right]$$
$$\Rightarrow\frac{1}{\left(1-\beta\rho_{\bar{\pi}}\right)^2+\varepsilon\kappa}\leq\frac{1+\mathcal{R}^2\frac{\varepsilon}{\kappa}}{\left(1-\beta\rho_{\bar{\pi}}+\mathcal{R}\varepsilon\right)^2}\text{ for }\mathcal{R}\in\left[\kappa,\frac{\kappa}{1-\beta\rho_{\bar{\pi}}}\right]$$

Meanwhile, $l_t^d-l_t$ will again depend on realizations of shocks and can be positive for a large positive realization of $\left(\bar{\pi}_{t|t}-\bar{\pi}_t\right)\bar{\pi}_t$.

Therefore, it's clear in the case of an inflation target, partial communication of only the demand level d_t

minimizes the discounted net present value of expected future losses. In this case, the central bank is able to better achieve the inflation target if agents are perfectly aware of it in equilibrium. With only two shocks in this model, this can be achieved with direct communication by the central bank about both shocks or either one of the shocks prior to the realization of the interest rate. Revealing only d_t ex-ante gives a discretionary interest rate policy the largest incentive to reduce the stabilization bias and thus this partial communication policy is optimal for expected future welfare. However, the communication policy that minimizes the current period loss will still depend on the realizations of shocks even for a fixed set of parameter values.

A.11 Proof of Proposition 10

In the case that lagged observations are not seen perfectly, beliefs are now given by a Kalman filter. To solve for these beliefs, recall that the latent states and the interest rate signal are perceived by the private agents to be of the form

$$\begin{aligned}
d_t &= \rho_d d_{t-1} + \epsilon_{d,t}, \quad \epsilon_{d,t} \sim N\left(0, \sigma_d^2\right) \\
\bar{y}_t &= \rho_{\bar{y}} \bar{y}_{t-1} + \epsilon_{\bar{y},t}, \quad \epsilon_{\bar{y},t} \sim N\left(0, \sigma_{\bar{y}}^2\right) \\
i_t &= f_d d_t + f_{\bar{y}} \bar{y}_t + f_{d,b} d_{t|t} + f_{\bar{y},b} \bar{y}_{t|t}
\end{aligned}$$

The circularity of the signal can again be resolved by conjecturing a belief structure and then writing the problem in expectational errors defined as $x_t^{surp} \equiv x_t - x_{t|t-1}$. The conjecture I use is

$$\begin{aligned}
\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} &= \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} \left(i_t - f_d d_{t|t-1} - f_{d,b} d_{t|t} - f_{\bar{y}} \bar{y}_{t|t-1} - f_{\bar{y},b} \bar{y}_{t|t}\right) \\
&= \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} \begin{bmatrix} f_d & f_{\bar{y}} \end{bmatrix} \left(\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} - \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} \right) \quad \text{in equilibrium} \\
\text{where } \begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} &= \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix}
\end{aligned}$$

Thus, the expectational errors can be written in state-space form as

$$\begin{aligned}
\begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} &= \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} 1 - \hat{K}_d f_d & -\hat{K}_d f_d \\ -\hat{K}_{\bar{y}} f_d & -\hat{K}_{\bar{y}} f_d \end{bmatrix} \begin{bmatrix} d_{t-1}^{err} \\ \bar{y}_{t-1}^{err} \end{bmatrix} + \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \end{bmatrix} \\
i_t^{surp} &= \left(1 + \hat{K}_d f_{d,b} + \hat{K}_{\bar{y}} f_{\bar{y},b}\right) \begin{bmatrix} f_d & f_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix}
\end{aligned}$$

In this case, the steady-state Kalman filter gives

$$\begin{bmatrix} d^{err}_{t|t} \\ \bar{y}^{err}_{t|t} \end{bmatrix} = \begin{bmatrix} d^{err}_{t|t-1} \\ \bar{y}^{err}_{t|t-1} \end{bmatrix} + \tilde{K}\left(i^{surp}_t - i^{surp}_{t|t-1}\right) = \tilde{K}\left(1 + \hat{K}_d f_{d,b} + \hat{K}_{\bar{y}} f_{\bar{y},b}\right)[f_d \quad f_{\bar{y}}] \begin{bmatrix} d^{err}_t \\ \bar{y}^{err}_t \end{bmatrix}$$

$$\text{where } \tilde{K} = \tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}\left(\left(1 + \hat{K}_d f_{d,b} + \hat{K}_{\bar{y}} f_{\bar{y},b}\right)\begin{bmatrix} f_d & f_{\bar{y}} \end{bmatrix}\tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}\right)^{-1}$$

$$\tilde{P} = \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\left(\tilde{P} - \tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}\left(\begin{bmatrix} f_d & f_{\bar{y}} \end{bmatrix}\tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}\right)^{-1}\begin{bmatrix} f_d & f_{\bar{y}} \end{bmatrix}\tilde{P}\right)\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} + \begin{bmatrix} \sigma^2_d & 0 \\ 0 & \sigma^2_{\bar{y}} \end{bmatrix}$$

This fulfills our original conjecture with

$$\begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} = \tilde{P}\begin{bmatrix} f_d \\ f_{\bar{y}} \end{bmatrix}$$

and the property that $f_d\hat{K}_d + f_{\bar{y}}\hat{K}_{\bar{y}} = 1$ is maintained.

Then, I again define the interest rate as $i_t = i^{dis}_t + f_{d,b}d_{t|t} + f_{\bar{y},b}\bar{y}_{t|t}$ so that beliefs as a function of past beliefs and i^{dis}_t are

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} 1 - \hat{K}_d f_d & -\hat{K}_d f_{\bar{y}} \\ -\hat{K}_{\bar{y}} f_d & 1 - \hat{K}_{\bar{y}} f_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} i^{dis}_t \tag{A.21a}$$

$$\begin{bmatrix} d_{t+1|t} \\ \bar{y}_{t+1|t} \end{bmatrix} = \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} \tag{A.21b}$$

Then, I follow the same steps as the proof of Proposition 2 and use the linear form of expectations

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M}\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

to write $[\tilde{y}_t - \bar{y}_t \quad \pi_t]'$ as a linear function of prior beliefs, exogenous states, and i^{dis}_t

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = \mathbf{\Psi}\begin{bmatrix} 1 - \hat{K}_d f_d & -\hat{K}_d f_{\bar{y}} \\ -\hat{K}_{\bar{y}} f_d & 1 - \hat{K}_{\bar{y}} f_{\bar{y}} \end{bmatrix}\begin{bmatrix} d_{t|t-1} \\ \bar{y}_{t|t-1} \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ \kappa & 0 \end{bmatrix}\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} i^{dis}_t \tag{A.22}$$

$$\text{where } \mathbf{\Psi} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix}\mathbf{M}\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \rho_d + \frac{1}{\sigma}f_{d,b} & \frac{1}{\sigma}f_{\bar{y},b} \\ \kappa\left(\rho_d + \frac{1}{\sigma}f_{d,b}\right) & \frac{\kappa}{\sigma}f_{\bar{y},b} \end{bmatrix}$$

$$\text{and } \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} \equiv \mathbf{\Psi}\begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} \tag{A.23}$$

Now, the discretionary policymaker's problem can be written as the following Bellman recursion where his choice today now has an effect on the expected future welfare loss since today's beliefs become the prior for period $t+1$ beliefs

$$V\left(d_t, \bar{y}_t, d_{t|t-1}, \bar{y}_{t|t-1}\right) = \min_{\tilde{y}_t, \pi_t, i_t^{dis}, d_{t+1|t}, \bar{y}_{t+1|t}} \left\{ \frac{1}{2}\left(\left(\tilde{y}_t - \bar{y}_t\right)^2 + \frac{\varepsilon}{\kappa}\pi_t^2 \right) + \beta E_t^{CB} V\left(d_{t+1}, \bar{y}_{t+1}, d_{t+1|t}, \bar{y}_{t+1|t}\right) \right\}$$

subject to (A.21) and (A.22)

Then, the FOC and EC combine to give the optimality condition

$$\begin{aligned} \tilde{y}_t - \bar{y}_t + \frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\pi_t = &-\frac{\beta}{H_{\tilde{y},i}}\left(\frac{d\tilde{y}_{t+1}}{dd_{t+1|t}}\rho_d \hat{K}_d + \frac{d\tilde{y}_{t+1}}{d\bar{y}_{t+1|t}}\rho_{\bar{y}}\hat{K}_{\bar{y}} \right) E_t^{CB}\left[\tilde{y}_{t+1} - \bar{y}_{t+1}\right] \\ &- \frac{\beta}{H_{\tilde{y},i}}\left(\frac{d\pi_{t+1}}{dd_{t+1|t}}\rho_d \hat{K}_d + \frac{d\pi_{t+1}}{d\bar{y}_{t+1|t}}\rho_{\bar{y}}\hat{K}_{\bar{y}} \right)\frac{\varepsilon}{\kappa}E_t^{CB}\pi_{t+1} \end{aligned}$$

Matching coefficients gives the same equilibrium value for $\mathbf{M}$ as a function of the interest rate coefficients as the case derived in Appendix A.2 where agents could see lagged beliefs.

Then, to see if an interest rate of the form

$$i_t = r_t^n + f_{\bar{y}}\bar{y}_t + f_{\bar{y},b}\bar{y}_{t|t}$$

can satisfy this optimality condition, I use these supposed policy coefficients which gives

$$\mathbf{M} = -\begin{bmatrix} 0 & \frac{1}{\sigma}\Omega_{\bar{y}}\left(1-\beta\rho_{\bar{y}}\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) \\ 0 & \frac{\kappa}{\sigma}\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) \end{bmatrix}$$

and an equilibrium where $\tilde{y}_t - \bar{y}_t$ and π_t are linear in $\left\{\bar{y}_t, \bar{y}_{t|t}\right\}$

$$\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} = -\begin{bmatrix} 0 & \frac{1}{\sigma}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}} + \frac{\kappa}{\sigma}\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{1}{\sigma}f_{\bar{y},b} \\ 0 & \frac{\kappa}{\sigma}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{\kappa}{\sigma}f_{\bar{y},b} \end{bmatrix}\bar{y}_{t|t} + \begin{bmatrix} -1 - \frac{1}{\sigma}f_{\bar{y}} \\ -\frac{\kappa}{\sigma}f_{\bar{y}} \end{bmatrix}\bar{y}_t$$

$$\text{and} \quad \begin{bmatrix} H_{\tilde{y},i} \\ H_{\pi,i} \end{bmatrix} = -\begin{bmatrix} \frac{1}{\sigma}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}} + \frac{\kappa}{\sigma}\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{1}{\sigma}f_{\bar{y},b} \\ \frac{\kappa}{\sigma}\rho_{\bar{y}}\left(1-\beta\rho_{\bar{y}} + \frac{\kappa}{\sigma} + \beta\right)\Omega_{\bar{y}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{\kappa}{\sigma}f_{\bar{y},b} \end{bmatrix}\hat{K}_{\bar{y}} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}$$

Then, this gives

$$\begin{aligned} \frac{d\tilde{y}_{t+1}}{dd_{t+1|t}}\rho_d\hat{K}_d + \frac{d\tilde{y}_{t+1}}{d\bar{y}_{t+1|t}}\rho_{\bar{y}}\hat{K}_{\bar{y}} &= \frac{\partial\tilde{y}_{t+1}}{\partial\bar{y}_{t+1|t+1}}\left(\frac{d\bar{y}_{t+1|t+1}}{dd_{t+1|t}}\rho_d\hat{K}_d + \frac{d\bar{y}_{t+1|t+1}}{d\bar{y}_{t+1|t}}\rho_{\bar{y}}\hat{K}_{\bar{y}} \right) \\ &= \frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}\hat{K}_{\bar{y}}\hat{K}_d f_d\left(\rho_{\bar{y}} - \rho_d\right) \end{aligned}$$

and similarly for π_{t+1}. This means the policymaker's optimality condition simplifies to

$$\tilde{y}_t - \bar{y}_t + \frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\pi_t = -\frac{\beta\hat{K}_{\bar{y}}\hat{K}_d\left(\rho_{\bar{y}}-\rho_d\right)\sigma}{H_{\tilde{y},i}}\left[\frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}E_t^{CB}\left[\tilde{y}_{t+1}-\bar{y}_{t+1}\right]+\frac{\partial\pi_t}{\partial\bar{y}_{t|t}}\frac{\varepsilon}{\kappa}E_t^{CB}\pi_{t+1}\right]$$

Then, the LHS of this condition is a function of $\left\{\bar{y}_t, \bar{y}_{t|t}\right\}$ and that the term inside the expectations on the RHS is a function of $\left\{\bar{y}_{t+1}, \bar{y}_{t+1|t+1}\right\}$. However, this means that the expectation itself is a function of $\left\{d_t - d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$ since

$$\begin{aligned}
\bar{y}_{t+1|t+1} &= \rho_{\bar{y}}\bar{y}_{t|t} + \hat{K}_{\bar{y}}\left(f_d d_{t+1} + f_{\bar{y}}\bar{y}_{t+1} - f_d\rho_d d_{t|t} - f_{\bar{y}}\rho_{\bar{y}}\bar{y}_{t|t}\right)\\
\Rightarrow E_t^{CB}\bar{y}_{t+1|t+1} &= \rho_{\bar{y}}\bar{y}_{t|t} + \hat{K}_{\bar{y}}\left(f_d\rho_d\left(d_t - d_{t|t}\right) + f_{\bar{y}}\rho_{\bar{y}}\left(\bar{y}_t - \bar{y}_{t|t}\right)\right)
\end{aligned}$$

Thus, the optimality condition cannot be satisfied under the premise that $i_t = r_t^n + f_{\bar{y}}\bar{y}_t + f_{\bar{y},b}\bar{y}_{t|t}$ for general parameter values.

A.11.1 Proof of Corollary 11

Recall that the policymaker's optimality condition is

$$\tilde{y}_t - \bar{y}_t + \frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\pi_t = -\frac{\beta\hat{K}_{\bar{y}}\hat{K}_d\left(\rho_{\bar{y}}-\rho_d\right)f_d}{H_{\tilde{y},i}}\left[\frac{\partial\tilde{y}_t}{\partial\bar{y}_{t|t}}E_t^{CB}\left[\tilde{y}_{t+1}-\bar{y}_{t+1}\right]+\frac{\partial\pi_t}{\partial\bar{y}_{t|t}}\frac{\varepsilon}{\kappa}E_t^{CB}\pi_{t+1}\right]$$

In the special case where $\hat{K}_{\bar{y}}\hat{K}_d\left(\rho_{\bar{y}}-\rho_d\right) = 0$, the new terms introduced to the optimality condition drop out and it collapses to the same condition as before.

$$\tilde{y}_t - \bar{y}_t = -\frac{H_{\pi,i}}{H_{\tilde{y},i}}\frac{\varepsilon}{\kappa}\pi_t$$

Substituting this into the equilibrium conditions shows that the interest rate rule features the same responses to $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$ as in the case where agents could see lagged fundamentals. The condition $\hat{K}_{\bar{y}}\hat{K}_d\left(\rho_{\bar{y}}-\rho_d\right) = 0$ captures the case where the current policy choice no longer affects future outcomes since it no longer affects the future belief $\bar{y}_{t+1|t+1}$. This can be broken down into the following subcases:

1. $\hat{K}_{\bar{y}} = 0$ ($\Leftrightarrow \hat{K}_d = \frac{1}{f_d}$): In this case, equilibrium beliefs are given by

$$\begin{aligned}
\bar{y}_{t|t} &= \rho_{\bar{y}}\bar{y}_{t-1|t-1}\\
d_{t|t} &= \frac{1}{f_d}\left(i_t^{dis} - f_{\bar{y}}\rho_{\bar{y}}\bar{y}_{t-1|t-1}\right)
\end{aligned}$$

Then, the interest rate only affects the current belief $d_{t|t}$ and not future beliefs.

2. $\hat{K}_d = 0$ ($\Leftrightarrow \hat{K}_{\bar{y}} = \frac{1}{f_{\bar{y}}}$): In this case, equilibrium beliefs are given by

$$\begin{aligned} d_{t|t} &= \rho_d d_{t-1|t-1} \\ \bar{y}_{t|t} &= \frac{1}{f_{\bar{y}}}\left(i_t^{dis} - f_d \rho_d d_{t-1|t-1}\right) \end{aligned}$$

Again, the interest rate only affects the current belief $\bar{y}_{t|t}$ and not future beliefs.

3. $\rho_d = \rho_{\bar{y}} = \rho$: Note that it's always possible to write beliefs as a distributed lag of interest rate news

$$\begin{aligned} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} &= \begin{bmatrix} \rho_d d_{t-1|t-1} \\ \rho_{\bar{y}} \bar{y}_{t-1|t-1} \end{bmatrix} + \begin{bmatrix} \hat{K}_d \\ \hat{K}_{\bar{y}} \end{bmatrix} \left(i_t^{dis} - f_d d_{t|t-1} - f_{\bar{y}} \bar{y}_{t|t-1}\right) \\ d_{t|t} &= \hat{K}_d \sum_{j=0}^{\infty} \rho_d^j \left(i_{t-j}^{dis} - i_{t-j|t-j-1}^{dis}\right) \\ \bar{y}_{t|t} &= \hat{K}_{\bar{y}} \sum_{j=0}^{\infty} \rho_{\bar{y}}^j \left(i_{t-j}^{dis} - i_{t-j|t-j-1}^{dis}\right) \end{aligned}$$

When the autocorrelations are equal, the interest rate itself becomes AR(1) with an innovation that is the composite of the two underlying shocks

$$i_{t+1}^{dis} = f_d d_{t+1} + f_{\bar{y}} \bar{y}_{t+1} = \rho i_t^{dis} + f_d \epsilon_{d,t} + f_{\bar{y}} \epsilon_{\bar{y},t}$$

Then, beliefs collapse to a function of just today's interest rate in equilibrium.

$$\begin{aligned} d_{t|t} &= \hat{K}_d \sum_{j=0}^{\infty} \rho^j \left(i_{t-j}^{dis} - \rho i_{t-j-1}^{dis}\right) = \hat{K}_d i_t^{dis} \\ \bar{y}_{t|t} &= \hat{K}_{\bar{y}} \sum_{j=0}^{\infty} \rho^j \left(i_{t-j}^{dis} - \rho i_{t-j-1}^{dis}\right) = \hat{K}_{\bar{y}} i_t^{dis} \end{aligned}$$

In the special case where $\rho_d = 0$, equilibrium beliefs are given by

$$\begin{aligned} \bar{y}_{t+1|t+1} &= \rho_{\bar{y}} \bar{y}_{t|t} + \hat{K}_{\bar{y}} \left(f_d d_{t+1} + f_{\bar{y}} \bar{y}_{t+1} - f_{\bar{y}} \rho_{\bar{y}} \bar{y}_{t|t}\right) \\ \Rightarrow E_t^{CB} \bar{y}_{t+1|t+1} &= \rho_{\bar{y}} \bar{y}_{t|t} + \hat{K}_{\bar{y}} f_{\bar{y}} \rho_{\bar{y}} \left(\bar{y}_t - \bar{y}_{t|t}\right) \end{aligned}$$

and the RHS is now only a function of $\bar{y}_t$ and $\bar{y}_{t|t}$. Then, it's verified that the optimality condition holds with $f_d = \sigma$, $f_{d,b} = -\sigma \rho_d$. In general, the coefficients $f_{\bar{y}}$ and $f_{\bar{y},b}$ will differ from the case where lags can be seen since the coefficients in that case only set the LHS to zero.

A.12 Optimal policy under time-varying uncertainty

This section looks at optimal discretionary policy when uncertainty in the exogenous states is time-varying

$$d_t = \rho_d d_{t-1} + \epsilon_{d,t}, \quad \epsilon_{d,t} \sim N\left(0, \sigma^2_{d,t-1}\right) \text{ and is serially uncorrelated}$$

$$\bar{y}_t = \rho_{\bar{y}} \bar{y}_{t-1} + \epsilon_{\bar{y},t}, \quad \epsilon_{\bar{y},t} \sim N\left(0, \sigma^2_{\bar{y},t-1}\right) \text{ and is serially uncorrelated}$$

Private agents' information sets are $\mathcal{I}_t = \left\{i^t, d^{t-1}, \bar{y}^{t-1}, \left(\sigma^2_d\right)^t, \left(\sigma^2_{\bar{y}}\right)^t, \mathbf{f}^t\right\}$ where $\mathbf{f}_t$ denotes the vector of time t interest rate responses to the state variables $\left\{d_t, d_{t|t}, \bar{y}_t, \bar{y}_{t|t}\right\}$.

Beliefs can be derived in the same way as in Section 1.2.3. The only difference now is that the belief coefficients contain time-varying policy coefficients.

$$K_{d,t} = \frac{f_{d,t}\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}}{f^2_{d,t}\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f^2_{\bar{y},t}} \quad \text{and} \quad K_{\bar{y},t} = \frac{f_{\bar{y},t}}{f^2_{d,t}\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + f^2_{\bar{y},t}}$$

Then, if I specify implemented policy as

$$i_t = i^{dis}_t + f_{d,b,t} d_{t|t} + f_{\bar{y},b,t} \bar{y}_{t|t}$$

Beliefs are again linear in i^{dis}_t

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \begin{bmatrix} 1 - K_{d,t} f_{d,t} \\ -K_{\bar{y},t} f_{d,t} \end{bmatrix} \rho_d d_{t-1} + \begin{bmatrix} -K_{d,t} f_{\bar{y},t} \\ 1 - K_{\bar{y},t} f_{\bar{y},t} \end{bmatrix} \rho_{\bar{y}} \bar{y}_{t-1} + \begin{bmatrix} K_{d,t} \\ K_{\bar{y},t} \end{bmatrix} i^{dis}_t$$

Longer horizon forecasts will continue to be $d_{t+h|t} = \rho^h_d d_{t|t}$ and $\bar{y}_{t+h|t} = \rho^h_{\bar{y}} \bar{y}_{t|t}$.

I then posit that equilibrium expectations are linear in these beliefs with time-varying coefficients

$$\begin{bmatrix} \tilde{y}_{t+1|t} \\ \pi_{t+1|t} \end{bmatrix} = \mathbf{M}_t \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix}$$

Then, $\tilde{y}_t - \bar{y}_t$ and π_t can again be written in terms of exogenous states and i_t^{dis}

$$\begin{aligned}
\begin{bmatrix} \tilde{y}_t - \bar{y}_t \\ \pi_t \end{bmatrix} &= \mathbf{\Psi}_t \begin{bmatrix} 1 - K_{d,t} f_{d,t} & -K_{d,t} f_{\bar{y},t} \\ -K_{\bar{y},t} f_{d,t} & 1 - K_{\bar{y},t} f_{\bar{y},t} \end{bmatrix} \begin{bmatrix} \rho_d d_{t-1} \\ \rho_{\bar{y}} \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ \kappa & 0 \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} H_{\tilde{y},i,t} \\ H_{\pi,i,t} \end{bmatrix} i_t^{dis} \\
\text{where } \mathbf{\Psi}_t &\equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \mathbf{M}_t \begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \rho_d + \frac{1}{\sigma} f_{d,b,t} & \frac{1}{\sigma} f_{\bar{y},b,t} \\ \kappa \left(\rho_d + \frac{1}{\sigma} f_{d,b,t} \right) & \frac{\kappa}{\sigma} f_{\bar{y},b,t} \end{bmatrix} \\
\text{and } \begin{bmatrix} H_{\tilde{y},i,t} \\ H_{\pi,i,t} \end{bmatrix} &\equiv \mathbf{\Psi}_t \begin{bmatrix} K_d \\ K_{\bar{y}} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}
\end{aligned}$$

In this form, it's again true that the discretionary policymaker has no control over time $t+1$ or later outcomes and the problem simplifies to

$$\min_{i_t^{dis}} \frac{1}{2} \left((\tilde{y}_t - \bar{y}_t)^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right) \quad \text{subject to the preceding equation}$$

Thus, the FOC is analogous to the constant variances case but with a time-varying $\mathcal{R}_t$

$$\tilde{y}_t - \bar{y}_t = -\mathcal{R}_t \frac{\varepsilon}{\kappa} \pi_t, \ \text{ where } \mathcal{R}_t = \frac{H_{\pi,i,t}}{H_{\tilde{y},i,t}}$$

Using this FOC and the structural equations to back out the optimal equilibrium i_t, gives

$$\begin{aligned}
\pi_t &= \beta \pi_{t+1|t} - \mathcal{R}_t \varepsilon \pi_t + \kappa \bar{y}_t = \frac{\kappa}{1 + \mathcal{R}_t \varepsilon} \bar{y}_t + \frac{\kappa \beta \rho_{\bar{y}}}{1 + \mathcal{R}_t \varepsilon} E \left[\frac{1}{1 + \mathcal{R}_{t+1} \varepsilon} + \frac{\beta \rho_{\bar{y}}}{(1 + \mathcal{R}_{t+1} \varepsilon)(1 + \mathcal{R}_{t+2} \varepsilon)} + ... \middle| \mathcal{I}_t \right] \bar{y}_{t|t} \\
\tilde{y}_t &= \bar{y}_t - \mathcal{R}_t \frac{\varepsilon}{\kappa} \pi_t = \frac{1}{1 + \mathcal{R}_t \varepsilon} \bar{y}_t - \frac{\mathcal{R}_t \varepsilon \beta \rho_{\bar{y}}}{1 + \mathcal{R}_t \varepsilon} E \left[\frac{1}{1 + \mathcal{R}_{t+1} \varepsilon} + \frac{\beta \rho_{\bar{y}}}{(1 + \mathcal{R}_{t+1} \varepsilon)(1 + \mathcal{R}_{t+2} \varepsilon)} + ... \middle| \mathcal{I}_t \right] \bar{y}_{t|t}
\end{aligned}$$

when $\lim_{T \to \infty} \left(\prod_{k=0}^{T} \frac{\beta}{1 + \mathcal{R}_{t+k} \varepsilon} \right) \pi_{t+T|t} = 0$. Then, expectations are

$$\begin{aligned}
\pi_{t+1|t} &= \kappa E \left[\frac{1}{1 + \mathcal{R}_{t+1} \varepsilon} + \frac{\beta \rho_{\bar{y}}}{(1 + \mathcal{R}_{t+1} \varepsilon)(1 + \mathcal{R}_{t+2} \varepsilon)} + ... \middle| \mathcal{I}_t \right] \rho_{\bar{y}} \bar{y}_{t|t} \\
\tilde{y}_{t+1|t} &= \left\{ 1 - E \left[\mathcal{R}_{t+1} \varepsilon \left(\frac{1}{1 + \mathcal{R}_{t+1} \varepsilon} + \frac{\beta \rho_{\bar{y}}}{(1 + \mathcal{R}_{t+1} \varepsilon)(1 + \mathcal{R}_{t+2} \varepsilon)} + ... \right) \middle| \mathcal{I}_t \right] \right\} \rho_{\bar{y}} \bar{y}_{t|t}
\end{aligned}$$

By taking $\bar{y}_{t|t}$ out of the expectations, I'm assuming (and later show) that $\mathcal{R}_t$ will be a function of current and past relative variances which are not informative about future levels of the output gap target.

Then, this implies that the interest rate can be written in terms of $\left\{d_t, d_{t|t}, \bar{y}, \bar{y}_{t|t}\right\}$

$$
\begin{aligned}
i_t &= r_t^n + \pi_{t+1|t} + \sigma\left(\tilde{y}_{t+1|t} - \tilde{y}_t\right) \\
&= \sigma d_t - \sigma\rho_d d_{t|t} - \sigma\frac{1}{1+\mathcal{R}_t\varepsilon}\bar{y}_t \\
&+ \underbrace{\sigma E\left[1 + \left(\frac{\kappa}{\sigma} - \mathcal{R}_{t+1}\varepsilon + \frac{\mathcal{R}_t\varepsilon\beta}{1+\mathcal{R}_t\varepsilon}\right)\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{\left(1+\mathcal{R}_{t+1}\varepsilon\right)\left(1+\mathcal{R}_{t+2}\varepsilon\right)} + ...\right)|\mathcal{I}_t\right]\rho_{\bar{y}}\bar{y}_{t|t}}_{f^*_{\bar{y},b,t}}
\end{aligned}
$$

In addition, the above expressions for $\pi_{t+1|t}, \tilde{y}_{t+1|t}$ gives an expression for the equilibrium M_t

$$
\mathbf{M}_t = \begin{bmatrix} 0 & 1 - E\left[\mathcal{R}_{t+1}\varepsilon\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\right)\middle|\mathcal{I}_t\right] \\ 0 & \kappa E\left[\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\middle|\mathcal{I}_t\right] \end{bmatrix}
$$

Using this in the expression for $[H_{\tilde{y},i,t} \quad H_{\pi,i,t}]'$ and combining this with the expressions for $f^*_{\bar{y},b,t}$, and $K_{\bar{y},t}$ gives a non-linear stochastic difference equation implicitly relating $\mathcal{R}_t$ to future $\{\mathcal{R}_{t+k}\}_{k\geq 1}$ where the driving variable is the relative variance level $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$.

$$
\begin{aligned}
\mathcal{R}_t &= \frac{H_{\pi,i,t}}{H_{\tilde{y},i,t}} \\
\begin{bmatrix} H_{\tilde{y},i,t} \\ H_{\pi,i,t} \end{bmatrix} &= \left(\begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix}\mathbf{M}_t\begin{bmatrix} \rho_d & 0 \\ 0 & \rho_{\bar{y}} \end{bmatrix} - \begin{bmatrix} 0 & \frac{1}{\sigma}f^*_{\bar{y},b,t} \\ 0 & \frac{\kappa}{\sigma}f^*_{\bar{y},b,t} \end{bmatrix}\right)\begin{bmatrix} K_{d,t} \\ K_{\bar{y},t} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} \\
\text{where } f^*_{\bar{y},b,t} &= \sigma E\left[1 + \left(\frac{\kappa}{\sigma} + \frac{\mathcal{R}_t\varepsilon\beta}{1+\mathcal{R}_t\varepsilon} - \mathcal{R}_{t+1}\varepsilon\right)\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{\left(1+\mathcal{R}_{t+1}\varepsilon\right)\left(1+\mathcal{R}_{t+2}\varepsilon\right)} + ...\right)|\mathcal{I}_t\right]\rho_{\bar{y}} \\
\mathbf{M}_t &= \begin{bmatrix} 0 & 1 - E\left[\mathcal{R}_{t+1}\varepsilon\left(\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\right)\middle|\mathcal{I}_t\right] \\ 0 & \kappa E\left[\frac{1}{1+\mathcal{R}_{t+1}\varepsilon} + \frac{\beta\rho_{\bar{y}}}{(1+\mathcal{R}_{t+1}\varepsilon)(1+\mathcal{R}_{t+2}\varepsilon)} + ...\middle|\mathcal{I}_t\right] \end{bmatrix} \\
K_{\bar{y},t} &= -\frac{1}{\sigma}\frac{1+\mathcal{R}_t\varepsilon}{\left(1+\mathcal{R}_t\varepsilon\right)^2\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}} + 1}
\end{aligned}
$$

If the relative variance $\frac{\sigma^2_{d,t}}{\sigma^2_{\bar{y},t}}$ is Markov, then it may be possible to show that the key variable $\mathcal{R}_t$ should depend only on $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ and $\frac{\sigma^2_{d,t}}{\sigma^2_{\bar{y},t}}$. Likewise, $f^*_{\bar{y},b,t}$ would also have this property.

Appendix B

Supplement to Chapter 2

B.1 Empirical relationship from structural model

In this section, I show that giving private agents an additional signal about π_t and using a special parameterization where $\rho_d = \rho_{\bar{y}} = \rho$ allows the structural model in Tang (2014b) to produce the same key regression equation as the reduced-form empirical model. In fact, it can be shown that this parameterization allows a VAR(1) representation of the structural model (derivations available upon request). As in that paper, I continue to assume that $\rho \in [0, \bar{\rho})$ where where $\bar{\rho} \equiv \frac{1+\beta+\frac{\kappa}{\sigma}-\sqrt{\left(1+\beta+\frac{\kappa}{\sigma}\right)^2-4\beta}}{2\beta} \leq \theta$. I also continue to assume that there's a given interest rate rule

$$i_t = f_d d_t + f_{d,b} d_{t|t} + f_{\bar{y}} \bar{y}_t + f_{\bar{y},b} \bar{y}_{t|t}$$

where $f_{\bar{y}} < 0$, $f_{\bar{y}} + f_{\bar{y},b} < 0$, $f_d > 0$, and $f_d + f_{d,b} > 0$.

The Appendix of Tang (2014b) showed that the equilibrium solutions for the output gap and inflation under an interest rate of this form are

$$\begin{aligned}
\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} &= \begin{bmatrix} -\frac{1}{\sigma}\Omega\left(1-\beta\rho\right)\left(f_d + f_{d,b} - \sigma\left(1-\rho\right)\right) - \left(1 - \frac{1}{\sigma} f_d\right) \\ -\frac{\kappa}{\sigma}\Omega\left(f_d + f_{d,b} - \sigma\left(1-\rho\right)\right) - \kappa\left(1 - \frac{1}{\sigma} f_d\right) \end{bmatrix} d_{t|t} \\
&+ \begin{bmatrix} -\frac{1}{\sigma}\Omega\left(1-\beta\rho\right)\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{1}{\sigma} f_{\bar{y}} \\ -\frac{\kappa}{\sigma}\Omega\left(f_{\bar{y}} + f_{\bar{y},b}\right) + \frac{\kappa}{\sigma} f_{\bar{y}} \end{bmatrix} \bar{y}_{t|t} + \begin{bmatrix} 1 - \frac{1}{\sigma} f_d & -\frac{1}{\sigma} f_{\bar{y}} \\ \kappa\left(1 - \frac{1}{\sigma} f_d\right) & -\frac{\kappa}{\sigma} f_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix}
\end{aligned}$$

$$\text{where } \Omega_d = \Omega_{\bar{y}} = \frac{1}{\left(1-\rho\right)\left(1-\beta\rho\right) - \frac{\kappa}{\sigma}\rho}$$

Imagine now that agents receive another signal which is

$$s_t = \pi_t + \epsilon_{s,t} = \Gamma_d d_t + \Gamma_{\bar{y}}\bar{y}_t + \Gamma_{d,b} d_{t|t} + \Gamma_{\bar{y},b}\bar{y}_{t|t} + \epsilon_{s,t},\ \epsilon_{s,t} \sim N\left(0, \sigma^2_{s,t-1}\right)$$

where the Γ's are the coefficients in the solution for π_t. Then, the private agents' belief formation problem can be written in state-space form as

$$\begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} = \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \end{bmatrix}, \quad \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \end{bmatrix} \sim N\left(0, \mathbf{\Sigma}_{d,\bar{y},t-1}\right) \text{ where } \mathbf{\Sigma}_{d,\bar{y},t-1} \equiv \begin{bmatrix} \sigma^2_{d,t-1} & 0 \\ 0 & \sigma^2_{\bar{y},t-1} \end{bmatrix}$$

$$\begin{bmatrix} i_t \\ s_t \end{bmatrix} = \begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \epsilon_{s,t}$$

I follow the procedure of Svensson and Woodford (2003) to deal with the circularity involved with signals i_t and s_{jt} depending on beliefs. I conjecture a form of beliefs and then write the system in innovations. The conjecture is

$$\begin{aligned} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} &= \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} + \mathbf{K}_t \left(\begin{bmatrix} i_t \\ s_{jt} \end{bmatrix} - \begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} - \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} \right) \\ &= \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} + \mathbf{K}_t \begin{bmatrix} f_d & f_{\bar{y}} & 0 \\ \Gamma_d & \Gamma_{\bar{y}} & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \\ \epsilon_{s,t} \end{bmatrix} \end{aligned}$$

Then, writing the system in expectational errors defined as $x_t^{err} \equiv x_t - E\left[x_t | \mathcal{I}_t \setminus \{i_t, s_{jt}\}\right]$ yields

$$\begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} \equiv \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \end{bmatrix}$$

$$\begin{bmatrix} i_t^{surp} \\ s_t^{surp} \end{bmatrix} = \left(I + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \right) \left(\begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_{s,t} \end{bmatrix} \right)$$

Then, beliefs are

$$\begin{aligned}
\begin{bmatrix} d_{t|t}^{err} \\ \bar{y}_{t|t}^{err} \end{bmatrix} &= E\left[\begin{bmatrix} d_t^{err} \\ \bar{y}_t^{err} \end{bmatrix} |\mathcal{I}_t \setminus \{i_t, s_{jt}\}, i_t^{surp}, s_{jt}^{surp}\right] \\
&= \mathbf{\Sigma}_{d,\bar{y},t-1} \begin{bmatrix} f_d & \Gamma_d \\ f_{\bar{y}} & \Gamma_{\bar{y}} \end{bmatrix} \left(I + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t\right)' \mathbf{\Sigma}_{i,s,t}^{-1} \begin{bmatrix} i_t^{surp} \\ s_{jt}^{surp} \end{bmatrix} \\
\text{where } \mathbf{\Sigma}_{i,s,t} &\equiv \left(I + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t\right) \left(\begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \mathbf{\Sigma}_{d,\bar{y},t-1} \begin{bmatrix} f_d & \Gamma_d \\ f_{\bar{y}} & \Gamma_{\bar{y}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \sigma_{s,t-1}^2 \end{bmatrix}\right) \\
&\times \left(I + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t\right)'
\end{aligned}$$

and

$$\begin{bmatrix} d_{t|t} \\ \bar{y}_{t|t} \end{bmatrix} = \rho \begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} + \begin{bmatrix} d_{t|t}^{err} \\ \bar{y}_{t|t}^{err} \end{bmatrix}$$

This matches the conjecture above with

$$\begin{aligned}
\mathbf{K}_t \equiv \begin{bmatrix} K_{d,t}^i & K_{d,t}^s \\ K_{\bar{y},t}^i & K_{\bar{y},t}^s \end{bmatrix} &= \mathbf{\Sigma}_{d,\bar{y},t} \begin{bmatrix} f_d & \Gamma_d \\ f_{\bar{y}} & \Gamma_{\bar{y}} \end{bmatrix} \left(\begin{bmatrix} f_d & f_{\bar{y}} \\ \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} \mathbf{\Sigma}_{d,\bar{y},t} \begin{bmatrix} f_d & \Gamma_d \\ f_{\bar{y}} & \Gamma_{\bar{y}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \sigma_{s,t-1}^2 \end{bmatrix}\right)^{-1} \\
&= \frac{\begin{bmatrix} \left(\frac{\kappa^2}{\sigma} f_{\bar{y}}^2 \sigma_{\bar{y},t-1}^2 + f_d \sigma_{s,t-1}^2\right) \sigma_{d,t-1}^2 & \kappa f_{\bar{y}}^2 \sigma_{d,t-1}^2 \sigma_{\bar{y},t-1}^2 \\ \left(\kappa^2 \left(1 - \frac{1}{\sigma} f_d\right) \sigma_{d,t-1}^2 + \sigma_{s,t-1}^2\right) f_{\bar{y}} \sigma_{\bar{y},t-1}^2 & -\kappa f_{\bar{y}} f_d \sigma_{d,t-1}^2 \sigma_{\bar{y},t-1}^2 \end{bmatrix}}{\left(f_d^2 \sigma_{d,t-1}^2 + f_{\bar{y}}^2 \sigma_{\bar{y},t-1}^2\right) \sigma_s^2 + \kappa^2 f_{\bar{y}}^2 \sigma_{d,t-1}^2 \sigma_{\bar{y},t-1}^2}
\end{aligned}$$

$$\text{since } \begin{bmatrix} \Gamma_d & \Gamma_{\bar{y}} \end{bmatrix} = \begin{bmatrix} \kappa \left(1 - \frac{1}{\sigma} f_d\right) & -\frac{\kappa}{\sigma} f_{\bar{y}} \end{bmatrix}$$

Then, using the fact that $f_{\bar{y}} < 0 < f_d$, I obtain the following properties for fixed interest rate rule coefficients

$$K_{\bar{y},t}^i < 0 < K_{d,t}^i, K_{d,t}^s, K_{\bar{y},t}^s, \quad f_d K_{d,t}^i + f_{\bar{y}} K_{\bar{y},t}^i = 1, \quad f_d K_{d,t}^s + f_{\bar{y}} K_{\bar{y},t}^s = 0$$

Then, I can write forecast revisions and the lagged nowcast error as the following

$$\begin{aligned}
\pi_{t|t} - \pi_{t|t-1} &= -\frac{\kappa}{\sigma}\Omega \begin{bmatrix} f_d + f_{d,b} - \sigma(1-\rho) & f_{\bar{y}} + f_{\bar{y},b} \end{bmatrix} \rho \left(\begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} - \begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix} \right) \\
&\quad - \frac{\kappa}{\sigma}\Omega \begin{bmatrix} f_d + f_{d,b} - \sigma(1-\rho) & f_{\bar{y}} + f_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \begin{bmatrix} f_d & f_{\bar{y}} & 0 \\ \Gamma_d & \Gamma_{\bar{y}} & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \\ \epsilon_{s,t} \end{bmatrix} \\
\pi_{t+h|t} - \pi_{t+h|t-1} &= -\frac{\kappa}{\sigma}\Omega \begin{bmatrix} f_d + f_{d,b} - \sigma(1-\rho) & f_{\bar{y}} + f_{\bar{y},b} \end{bmatrix} \left(\begin{bmatrix} d_{t+h|t} \\ \bar{y}_{t+h|t} \end{bmatrix} - \begin{bmatrix} d_{t+h|t-1} \\ \bar{y}_{t+h|t-1} \end{bmatrix} \right) \\
&= \rho^h \left(\pi_{t|t} - \pi_{t|t-1}\right) \\
\pi_{t-1} - \pi_{t-1|t-1} &= \begin{bmatrix} \kappa\left(1 - \frac{1}{\sigma} f_d\right) & -\frac{\kappa}{\sigma} f_{\bar{y}} \end{bmatrix} \left(\begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} - \begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix} \right)
\end{aligned}$$

where

$$\begin{bmatrix} d_{t-1} \\ \bar{y}_{t-1} \end{bmatrix} - \begin{bmatrix} d_{t-1|t-1} \\ \bar{y}_{t-1|t-1} \end{bmatrix} = \begin{bmatrix} \frac{f_{\bar{y}}}{f_d} \\ -1 \end{bmatrix} \begin{bmatrix} \left[K^i_{\bar{y},t} f_d + K^s_{\bar{y},t}\kappa\left(1 - \frac{1}{\sigma} f_d\right)\right] & \left(K^i_{\bar{y},t} - \frac{\kappa}{\sigma} K^s_{\bar{y},t}\right) f_{\bar{y}} - 1 & K^s_{\bar{y},t} \end{bmatrix} \begin{bmatrix} \epsilon_{d,t-1} \\ \epsilon_{\bar{y},t-1} \\ \epsilon_{s,t-1} \end{bmatrix}$$

$$\begin{bmatrix} f_d & f_{\bar{y}} & 0 \\ \Gamma_d & \Gamma_{\bar{y}} & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{d,t} \\ \epsilon_{\bar{y},t} \\ \epsilon_{s,t} \end{bmatrix} = \left(I + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \right)^{-1} \begin{bmatrix} i_t^{surp} \\ s_t^{surp} \end{bmatrix}$$

This allows me to write forecast revisions as linear in the lagged nowcast error, the interest rate surprise, and other inflation news.

$$\begin{aligned}
\pi_{t|t} - \pi_{t|t-1} &= -\frac{1}{\sigma}\rho\Omega \left[f_d + f_{d,b} - \sigma(1-\rho) - f_d \frac{f_{\bar{y}} + f_{\bar{y},b}}{f_{\bar{y}}} \right] \left(\pi_{t-1} - \pi_{t-1|t-1}\right) \\
&\quad - \frac{\kappa}{\sigma}\Omega \begin{bmatrix} f_d + f_{d,b} - \sigma(1-\rho) & f_{\bar{y}} + f_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \left(I + \begin{bmatrix} f_{d,b} & f_{\bar{y},b} \\ \Gamma_{d,b} & \Gamma_{\bar{y},b} \end{bmatrix} \mathbf{K}_t \right)^{-1} \begin{bmatrix} i_t^{surp} \\ s_t^{surp} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\text{where } i_t^{surp} &= i_t - E\left[i_t | \mathcal{I}_t \setminus \{i_t, s_t\}\right] \\
s_t^{surp} &= \pi_t - \pi_{t|t-1} + \rho\frac{1}{\sigma}\Omega \left[f_d + f_{d,b} - \sigma(1-\rho) - \frac{f_d}{f_{\bar{y}}}\left(f_{\bar{y}} + f_{\bar{y},b}\right) \right] \left(\pi_{t-1} - \pi_{t-1|t-1}\right) + \epsilon_{s,t}
\end{aligned}$$

Further algebraic manipulation yields a relationship of the same form given by the above empirical model

$$\begin{aligned}\pi_{t+h|t} - \pi_{t+h|t-1} &= \rho^h K_t^i \left(i_t - E\left[i_t | \mathcal{I}_t \setminus \{i_t, s_t\}\right]\right) + \rho^h K_t^s \left(\pi_t - \pi_{t|t-1}\right) \\ &+ \rho^{h+1} K^{NE} \left(1 - K_t^s\right) \left(\pi_{t-1} - \pi_{t-1|t-1}\right) + \rho^h K_t^s \epsilon_{s,t} \\ \text{where } K^{NE} &= -\frac{1}{\sigma} \Omega \left[f_d + f_{d,b} - \sigma\left(1 - \rho\right) - \frac{f_d}{f_{\bar{y}}} \left(f_{\bar{y}} + f_{\bar{y},b}\right)\right] \quad \text{does not depend on variances}\end{aligned}$$

When I additionally assume that $f_d < \sigma$ and $f_d + f_{d,b} \leq \sigma\left(1 - \rho\right)$, this is sufficient (but not always necessary) to obtain the following properties:

1. K_t^i may be positive, $\quad K_t^s \geq 0, \quad K^{NE} \geq 0$
2. K_t^i increases with $\sigma^2_{s,t-1}$ for $\frac{\sigma^2_{d,t-1}}{\sigma^2_{\bar{y},t-1}}$ large enough, K_t^i decreases with $\sigma^2_{\bar{y},t-1}$ and increases with $\sigma^2_{d,t-1}$
3. K_t^s decreases with $\sigma^2_{s,t-1}$, K_t^s increases with $\sigma^2_{\bar{y},t-1}$ and $\sigma^2_{d,t-1}$

B.2 Robustness checks

Table B.1: Baseline effect of federal funds rate surprises on inflation forecasts controlling for news about real output growth

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	0.233	0.234	0.285**	0.133
	[1.14]	[1.61]	[2.25]	[1.24]
$\pi_t - \overline{\pi_{t\|t-1}}$	0.095**	0.019	0.029	0.033
	[2.31]	[0.80]	[1.31]	[1.46]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}}$	0.210***	0.150***	0.073***	0.099***
	[3.41]	[3.84]	[3.01]	[3.54]
$y_t - \overline{y_{t\|t-1}}$	0.002	0.003	0.010	0.013
	[0.07]	[0.25]	[1.01]	[1.19]
$y_{t-1} - \overline{y_{t-1\|t-1}}$	0.028	0.009	0.006	0.003
	[0.97]	[0.47]	[0.44]	[0.20]
Adjusted R^2	0.324	0.265	0.200	0.215
N	88	88	88	88

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from the GNP to GDP deflator making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table B.2: Effect of federal funds rate surprises on inflation forecasts controlling for news about real output growth with a high vs low prior uncertainty interaction

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.070	0.035	0.066	0.102
	[−0.27]	[0.22]	[0.57]	[0.83]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.689**	0.484**	0.667***	0.123
	[2.13]	[2.12]	[3.44]	[0.48]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.054	−0.022	−0.007	0.027
	[0.75]	[−0.50]	[−0.21]	[0.71]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.114**	0.037	0.046*	0.039
	[2.03]	[1.14]	[1.69]	[1.48]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.267***	0.205***	0.103***	0.115***
	[3.35]	[4.07]	[3.12]	[2.84]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.138**	0.063*	0.056*	0.079*
	[2.47]	[1.69]	[1.77]	[1.84]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.004	0.013	0.006	0.007
	[−0.14]	[0.65]	[0.50]	[0.45]
$y_t - \overline{y_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	−0.005	−0.014	0.019	0.021*
	[−0.18]	[−0.80]	[1.63]	[1.80]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.065	0.013	0.010	0.007
	[1.36]	[0.46]	[0.55]	[0.32]
$y_{t-1} - \overline{y_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	−0.001	−0.001	0.008	0.004
	[−0.03]	[−0.07]	[0.39]	[0.21]
$\overline{Std^{\pi}_{t-1}}$ high	0.152*	0.084	0.094**	0.034
	[1.83]	[1.65]	[2.28]	[0.70]
Adjusted R^2	0.340	0.297	0.265	0.171
N	88	88	88	88
P-value of F-test of difference in $i_t - \overline{i_{t\|t-1}}$ coef	0.070	0.111	0.010	0.943

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from the GNP to GDP deflator making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table B.3: Baseline effect of federal funds rate surprises on inflation forecasts controlling for news about unemployment

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}}$	0.208	0.197	0.210*	0.130
	[1.02]	[1.36]	[1.79]	[1.19]
$\pi_t - \overline{\pi_{t\|t-1}}$	0.090**	0.012	0.015	0.023
	[2.01]	[0.43]	[0.65]	[1.00]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}}$	0.198***	0.148***	0.075***	0.099***
	[3.48]	[4.07]	[3.54]	[3.73]
$U_t - \overline{U_{t\|t-1}}$	−0.084	−0.072	−0.093	−0.078
	[−0.36]	[−0.50]	[−1.12]	[−0.81]
$U_{t-1} - \overline{U_{t-1\|t-1}}$	−0.196	−0.117	−0.281*	−0.030
	[−0.62]	[−0.56]	[−1.70]	[−0.18]
Adjusted R^2	0.324	0.277	0.272	0.213
N	88	88	88	88

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from the GNP to GDP deflator making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.

Table B.4: Effect of federal funds rate surprises on inflation forecasts controlling for news about unemployment with a high vs low prior uncertainty interaction

Dependent variable: $\overline{\pi_{t+h\|t}} - \overline{\pi_{t+h\|t-1}}$				
$h =$	0	1	2	3
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.093	0.031	0.047	0.119
	[−0.42]	[0.23]	[0.47]	[1.01]
$i_t - \overline{i_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.846***	0.435**	0.548***	0.069
	[2.75]	[2.22]	[2.79]	[0.23]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.051	−0.032	−0.012	0.023
	[0.75]	[−0.83]	[−0.38]	[0.60]
$\pi_t - \overline{\pi_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.131***	0.047*	0.029	0.021
	[3.08]	[1.80]	[1.16]	[0.76]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	0.224***	0.195***	0.095***	0.110***
	[4.89]	[6.30]	[4.11]	[3.33]
$\pi_{t-1} - \overline{\pi_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.115**	0.052	0.035	0.067*
	[2.38]	[1.48]	[1.38]	[1.77]
$U_t - \overline{U_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.514**	−0.325**	−0.206**	−0.116
	[−2.05]	[−2.37]	[−2.09]	[−0.78]
$U_t - \overline{U_{t\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	0.357**	0.224**	0.033	−0.042
	[2.37]	[2.18]	[0.43]	[−0.42]
$U_{t-1} - \overline{U_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ low	−0.095	0.166	−0.014	0.079
	[−0.19]	[0.58]	[−0.06]	[0.31]
$U_{t-1} - \overline{U_{t-1\|t-1}} \times \overline{Std^{\pi}_{t-1}}$ high	−0.461	−0.482*	−0.458**	−0.172
	[−1.27]	[−1.91]	[−2.30]	[−0.69]
$\overline{Std^{\pi}_{t-1}}$ high	0.183**	0.092*	0.091**	0.023
	[2.23]	[1.82]	[2.43]	[0.46]
Adjusted R^2	0.407	0.353	0.327	0.170
N	88	88	88	88
P-value of F-test of difference in $i_t - \overline{i_{t\|t-1}}$ coef	0.015	0.097	0.026	0.880

Notes: The sample is quarterly data from 1989:Q1 to 2011:Q1 with 1992:Q1 dropped due to the switch in the SPF from the GNP to GDP deflator making the lagged forecast unavailable in that period. ***/**/* Statistically significant at 1, 5, and 10 percent, respectively.