



# DIGITAL ACCESS TO SCHOLARSHIP AT HARVARD

## Learning from Comparison in Algebra

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*(Article begins on next page)*

Which is correct?

Alex and Morgan were asked to solve  $45y + 90 = 60y$

Alex's "combine like terms" way

$45y + 90 = 60y$

↓

$135y = 60y$

↓

$75y = 0$


↓

$y = 0$

I first combined like terms on the left side of the equation.

Then I subtracted both sides by  $60y$ .

Then I divided both sides by  $75$  to get the answer.



Morgan's "combine like terms" way

$45y + 90 = 60y$

↓


$90 = 15y$

↓

$6 = y$

First I subtracted  $45y$  on either side;  $60y - 45y$  is  $15y$ .

Then I divided both sides by  $15$  to get the answer.



- \* How did Alex solve the equation?
- \* How did Morgan solve the equation?
- \* Why did Alex combine the terms on the left as a first step?
- \* Why did Morgan subtract  $45y$  as a first step?
- \* Which way is correct, Alex's or Morgan's way? How do you know?
- \* Can you state a general rule about combining like terms that describes what you have learned from comparing Alex's and Morgan's ways of solving this type of problem?

Which is correct?

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Morgan's "combine like terms" way

$45y + 90 = 60y$

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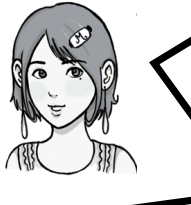
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First I subtracted  $45y$  on either side;  $60y - 45y$  is  $15y$ .

Then I divided both sides by  $15$  to get the answer.



Hey Morgan, what did we learn from comparing these right and wrong ways?

Like terms contain the same variable or group of variables raised to the same power. In order for two or more terms to be "like terms," their coefficients can be different, but the terms need to have the same variables raised to the same powers. Unlike terms cannot be combined by addition or subtraction.

- \* How did Alex solve the equation?
- \* How did Morgan solve the equation?
- \* Why did Alex combine the terms on the left as a first step?
- \* Why did Morgan subtract  $45y$  as a first step?
- \* Which way is correct, Alex's or Morgan's way? How do you know?
- \* Can you state a general rule about combining like terms that describes what you have learned from comparing Alex's and Morgan's ways of solving this type of problem?

Which is better?

### Main WEP page

Alex and Morgan were asked to simplify  $16^{\frac{3}{4}}$

Alex's "rewrite the exponent" way

Morgan's "use perfect squares to rewrite the exponent" way

I rewrote the fractional exponent as 3 times 1/4.

I expanded the expression in the parentheses.

I got 4096. Then I applied the exponent.

This is my answer.



$$\begin{array}{c}
 \checkmark \\
 16^{\frac{3}{4}} \\
 \downarrow \\
 (16^3)^{\frac{1}{4}} \\
 \downarrow \\
 (16 \cdot 16 \cdot 16)^{\frac{1}{4}} \\
 \downarrow \\
 (4096)^{\frac{1}{4}} \\
 \downarrow \\
 8
 \end{array}$$

I rewrote the fractional exponent as 1/4 times 3.

I simplified the expression in the parentheses. Since  $2^4$  is 16, I know that  $16^{1/4}$  is 2.

This is my answer.



$$\begin{array}{c}
 \downarrow \\
 16^{\frac{3}{4}} \\
 \downarrow \\
 (16^{\frac{1}{4}})^3 \\
 \downarrow \\
 (2)^3 \\
 \downarrow \\
 8
 \end{array}$$

- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Which strategy do you think is more efficient for this problem? Why?

Which is better?

### Take-away page

Alex and Morgan were asked to simplify  $16^{\frac{3}{4}}$

Alex's "rewrite the exponent" way

Morgan's "use perfect squares to rewrite the exponent" way

I rewrote the fractional exponent as 3 times 1/4.

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Hey Morgan, what did we learn from comparing these two ways?

When deciding which factors to use to rewrite the fractional exponent, be on the lookout for perfect squares.



- \* How did Alex simplify the expression?
- \* How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Which strategy do you think is more efficient for this problem? Why?

Why does it work?

### Main WEP page

Alex and Morgan were asked to simplify the expression

$$3x(5x + 2) + 4(5x + 2)$$

Alex's way

Morgan's way

$$3x(5x + 2) + 4(5x + 2)$$

$$3x(5x + 2) + 4(5x + 2)$$

$$15x^2 + 6x + 20x + 8$$

$$(3x + 4)(5x + 2)$$

$$15x^2 + 26x + 8$$

$$15x^2 + 6x + 20x + 8$$

$$15x^2 + 26x + 8$$

First I expanded the expression using the distributive property.

Then I simplified the expression.



First I factored the expression.

Then I expanded the expression.

Then I simplified the expression.



Why does it work?

### Take-away page

Alex and Morgan were asked to simplify the expression

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Alex's way

Morgan's way

$$3x(5x + 2)$$

$$+$$

$$4(5x + 2)$$

First I expanded the expression using the distributive property.

Then I simplified the expression.



Hey Alex, what did we learn from comparing these two ways?

First I factored the expression.

Then I expanded the expression.

Like expressions enclosed by grouping symbols, such as parentheses, can be combined as like terms are combined.



- \* How did Alex simplify the expression? How did Morgan simplify the expression?
- \* What are some similarities and differences between Alex's and Morgan's ways?
- \* Is Morgan's way OK to do? Why or why not?

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How do they differ?

Alex was asked to graph the equation  $y = 2x$   
and Morgan was asked to graph the equation  $y = -2x$ .

Alex's "graph  $y = 2x$ " way

Morgan's "graph  $y = -2x$ " way

I rewrote the equation in  $y = mx + b$  form.

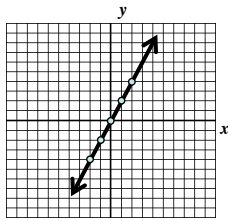
$$y = 2x$$

$$\downarrow$$

$$y = mx + b$$

$$y = 2x + 0$$

I graphed the y-intercept, (0,0) and counted up 2, right 1 and down 2, left 1 to plot other points on the line. I connected the points to draw the graph of the line.



- \* How did Alex graph the line given by his equation? How did Morgan graph the line given by her equation?
- \* Can you think of another way that Alex and Morgan could have used to find the graphs of their lines?
- \* What are some similarities and differences between Alex's and Morgan's problems?
- \* What are some similarities and differences between Alex's and Morgan's graphs?
- \* How does changing the sign of  $m$  affect the graph of a line?

How do they differ?

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Alex's "graph  $y = 2x$ " way

Morgan's "graph  $y = -2x$ " way

I rewrote the equation in  $y = mx + b$  form.

I graphed the y-intercept, (0,0) and counted up 2, right 1 and down 2, left 1 to plot other points on the line. I connected the points to draw the graph of the line.



In the slope-intercept form of a line ( $y = mx + b$ ), the coefficient of  $x$ , which is  $m$ , indicates the slope.

Changing the sign of  $m$  changes the slope, or the steepness, of the line. When a line has a positive slope, its height increases from left to right. When a line has a negative slope, its height decreases from left to right.



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