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Political Economy of Committee Voting and Its Application

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Political Economy of Committee Voting and its Application

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ABSTRACT

This dissertation consists of three essays on information aggregation in committees and its application.

The first essay analyzes how the distribution of votes affects the accuracy of group decisions. In a weighted voting system, votes are typically assigned based on the criteria that are unrelated to the voters' ability to make a correct judgment. I introduce an information aggregation model in which voters are identical except for voting shares. If the information is free, the optimal weight distribution is equal weighting. When acquiring information is costly, by contrast, I show that the accuracy of group decisions may be higher under some weighted majority rules than under unweighted majority rule. I characterize the equilibrium and find the optimal weight distribution to maximize the accuracy of group decisions. Asymmetric weight distributions may be optimal when the cost of improving signal is moderately high.

The second essay analyzes how intergenerational family transfers can be sustained. Why are generous transfers from the younger to the older generations made in some families and not in others? My paper argues that differences in intergenerational dependence are due to variation in community networks. My analysis of the sustainability of intergenerational transfers posits game theoretical models of overlapping generations in which breadwinners make transfers to their parents and children. A novel feature of my models is that there is a local community that may supply information about its members past behaviors. I demonstrate that an efficient level of intergenerational transfers can be sustained if neighbors gossip about each other.

The third essay, co-authored with Fuhito Kojima, investigates a jury decision when hung juries and retrials are possible. When jurors in subsequent trials know that previous trials resulted in hung juries, informative voting can be an equilibrium if and only if the accuracy of signals for innocence and guilt are exactly identical. Moreover, if jurors are informed of numerical split of votes in previous trials, informative voting is not an equilibrium regardless of signal accuracy.

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1. WEIGHTED VOTING AND INFORMATION ACQUISITION IN COMMITTEES

1.1 Introduction

Collective decision-making frequently involves situations in which actors have different numbers of votes. Some institutions assign unequal voting weights explicitly. Examples include the Council of the European Union, the U.S. Electoral College, shareholder meetings, the International Monetary Fund and the World Bank, and the International Energy Agency. In addition to the cases in which weighted voting is used as a formal rule, there are also cases that can be interpreted as weighted voting. For example, parties in parliamentary systems and factions within the party are generally characterized as highly unified. Thus, each party and faction can be seen as a weighted voter. Also, seniority arrangements may be seen as a weighted voting rule. In a legislative party, senior group members often have greater influence over the group decisions, which can be interpreted as weighted voting rule where senior members have higher weight than junior members.[1] How does the distribution of votes affect the accuracy of group decisions? Several scholars have considered a setting in which voters have common interests and the only purpose of voting is information aggregation. **?** and **?** show that the optimal collective decision rule assigns greater weights to the voters with higher ability to make a correct decision.[2]

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[1] In addition to those examples, decision making in regulatory organizations can be weighted voting. Bureaucratic and regulatory organizations make numerous errors. For example, the FDA may approve a faulty drug or reject a good drug. How do the errors of regulators depend upon the administrative structures? More specifically, those organizations may be hierarchical in the sense that some agents' opinions are more respected than others. For example, in Japanese bureaucracy, officials have a significantly greater say than the younger officials. Also, it has been reported that opinion of the chairman of the Federal Reserve Board is more influential than other members of the board.

[2] If p_j is the probability that voter j is correct in any given judgment, and if the judgments are independent, then the maximum likelihood rule for two alternatives is to use weighted majority rule, where the weight on individual j's vote is $log\frac{p_j}{1-p_j}$ (**?**, **?**).

Their main result implies that equally weighted majority rule is optimal when voters are identical.

However, in the real world, the criteria for the weight distribution vary and are typically unrelated to the voters' ability to make a correct judgement, i.e., voters with greater weights are not necessarily a priori more likely than others to make correct judgements. For example, votes are assigned based on population of each member country at the E.U. Council, the number of shares that each shareholder owns at shareholder meetings, and financial contribution of each member county at the IMF and the World Bank.[3] These systems may be fair, but do they sacrifice the accuracy of group decision? The aim of this paper is to analyze the influence of heterogeneous voting shares on the accuracy of group decisions.

To this effect, I set up a model with the following features: Voters are identical except for voting shares[4]; Voters have common interests and the only purpose of the voting is information aggregation; Information is a public good in the sense that the social benefits of one voter acquiring information exceed the private benefits. The designer chooses the distribution of weights so that the mechanism not only aggregates information efficiently, but also induces the voters to acquire information.

My main result is that the group decisions may be more likely to be correct under heterogeneous voting shares compared to the case in which every voter has one vote. This is because of the improvement of information possessed by the group: When information acquisition is costly, the voters with greater weights have higher incentives to invest in information than they would have under equal voting shares, and more investment means that more accurate information is aggregated, which makes the group decision more accurate.

To put it another way, an unequal distribution of voting power can sometimes be a solution to the problem of under-provision of a public good, i.e., the information. This

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[3] One exception may be the plural voting system that John Stuart Mill proposed. Mill argued that more educated citizens should be given more votes than uneducated citizens (**?**).

[4] Since the weight distribution is a parameter chosen by the designer, by voters being identical I mean that voters are identical except for weight shares in the this paper.

may give a justification to delegating authority to someone even if she is no more capable than others when information acquisition is costly. More generally, I characterize the equilibrium and find the optimal weight distribution to maximize the accuracy of group decisions. Asymmetric weight distributions may be optimal when the cost of improving signal is moderately high.

1.2 Related Literature

Weighted Voting

The literature of the weighted voting can be classified into two categories: the studies that assume common interest among agents and those that assume conflicting interests. First, consider a setting in which voters have common interests and the only purpose of the voting is information aggregation, which is perhaps best known as the setting of the Condorcet Jury Theorem model. In research on weighted voting in this common-interest framework, **?** and **?**, assuming heterogeneous abilities to make a correct judgement, show that the optimal collective decision rule assigns greater weights to the voters with higher ability to make a correct decision.[5] Their main result implies that equally weighted majority rule is optimal when voters are identical.

Those results are based on the assumption that voters are non-strategic and little is known about the strategic behavior in this setting. The current paper, to my knowledge, is the first study to analyze the strategic aspect of weighted voting in the common-interests setting. In contrast to the previous work, this paper shows that heterogeneous voting shares may be optimal even when voters are identical.

The reason why my result is different from those of the previous literature is as follows. The previous literature is only concerned with the negative effect of heterogeneous voting shares: Heterogeneity causes inefficiency in aggregating the information. By introducing the strategic behavior and costly information, the current paper is concerned with both

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[5] If p_j is the probability that voter j is correct in any given judgment, and if the judgments are independent, then the maximum likelihood rule for two alternatives is to use weighted majority rule, where the weight on individual j's vote is $\ln \frac{p_j}{1-p_j}$ (**?**, **?**).

negative and positive effects: Heterogeneity causes inefficiency in information aggregation but it also gives highly weighted voters incentives to acquire information. I show that sometimes the loss in efficiency is more than compensated by the greater amount of information acquired in equilibrium, relative to the case of equal distribution of voting shares.

The other category in weighted voting is the studies that assume conflicting interests among voters. As in the first category, some studies assume non-strategic voters and others assume strategic voters. The former's central question has been how to measure the power distribution among voters. A voter's ability to affect the group decision is not always proportional to her weight. To measure the power, scholars employ power indices such as the Shapley-Shubik index and the Banzhaf index (?, ?, ?).[6][7] As applications, there is a large amount of literature employing power indices to study the voting weights in the Council of the European Union, the IMF and the World Bank.[8]

The latter, the studies that assume strategic voters, typically analyze legislative bargaining as a game of weighted voting. Most of them employs proposal-based bargaining models, developed by ?.[9] ? provide an existence result for a generalized Baron and Ferejohn model that encompasses weighted voting. ? and ? study variants of the Baron-Ferejohn model with veto players. ? characterize the generalized Baron and Ferejohn model under weighted voting. Others employ demand-based bargaining models (?) and two-stage proposal-based bargaining models (?). By focusing on the aspect of conflicting

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[6] For a review, see Roth (1988) and Felsenthal and Machover (1998).

[7] Other cooperative solution concepts applied to weighted voting games include bargaining sets, bargaining aspirations, the kernel, and the competitive solution. See Schofield ?, ?, ?, McKelvey et al. (1978), Bennett (1983), Holler (1987), and Morelli and Montero (2003).

[8] For the Council of the European Union, see ?, ?, ?, BramsandAffuso(1985),Bergetal.(1993),Kaisa Herne and Hannu Nurmi (1993), ?, ?. Mika Widgren (1994, 2000), R. J. Johnston(1995), Jan-Erik, Lane-and, Reinert, Maeland (1995, 1996), Laneet al. (1995, 1996), Matthias Bruckner and Torsten Peters (1996), Anthony L. Teasdale (1996), George Tsebelis and Geoffrey Garrett(1996), Ulrich Bindseil and Cordula Hantke(1997), Dan S. Felsenthal and Moshe Machover (1997, 2000, 2001), Annick Laruelle and Widgren (1998), Konig and Brauninger (1998), Garrett and Tsebelis (1999a,1999b,2001), Holler and Widgren (1999). For the IMF, see Dreyer and Schotter (1980), (?), Alonso-Meijide and Bowles (2005), and Strand and Rapkin (2006), ?, ?.

[9] Power indices are based on the idea that all coalitions are equally likely to form, regardless of how expensive they are. Under the competitive bargaining, cheap coalitions will form more often than expensive ones.

interests among voters, those studies of conflicting interests analyze distributional politics, i.e., how the distribution of votes affects who gets what. In contrast, the current paper, by focusing on the aspect of the common-interest among voters, studies the likelihood that the group make a correct decision, i.e., how the distribution of votes affects the accuracy of group decisions.

Condorcet Jury Theorem

In addition to the literature of weighted voting, this paper contributes to the literature of Condorcet Jury Theorem. The framework within which this paper addresses the collective choice problems is a variant of the Condorcet Jury Theorem model, in which voters have common interests and the only purpose of the voting is information aggregation (**?**).[10]

The Condorcet Jury Theorem model with strategic voting is pioneered by **?**. They question Condorcet's assumption of sincere voting by showing that sincere voting do not constitute a Nash equilibrium in general. In response, **?** and **?** demonstrate that allowing mixed strategies sustains Condorcet's argument that groups are more likely to make correct decisions than individuals. **?** support Condorce's argument on the group size. They show that the accuracy of a group decision becomes higher as the group grows larger under non-unanimity rules while not under unanimity rule. Under unanimity rule, the probabilities that a group makes a wrong decision stay bounded away from zero regardless of the size of jury. Moreover, increasing the size of the group does not help and actually may increase the probability of convicting an innocent defendant. On the other hand, under nonunanimous rule, both types of mistakes converge to zero as the jury grows large.

The current paper falls into the Condorcet Jury models with costly information ac-

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[10] **?** claims that majority rule is superior to dictatorship for a society even when the dictator has common interests with others. He considers the situation in which there are two alternatives on an epistemic issue and a society is deciding which one to select. Key feature are (1) that all individuals have common interests to the extent that all agree on the alternative when the true state of the world is revealed and (2) that each individual receives a signal about true state of the world. The resulting claim that a group of voters using majority is more likely to choose the right action than an arbitrary single voter is known as the Condorcet Jury Theorem. Condorcet also argues that the decisions by majorities get better as the size of the groups becomes larger, which some scholars consider part of the Condorcet Jury Theorem.

quisition, in which agents privately gather costly information, and then aggregate it to produce a collective decision. Because information is a public good and it causes the collective action problem among agents, the information will be under-provided relative to the social optimum.[11] **?** shows that a larger committee may actually make poorer decisions because of the collective action problem. He assumes majority rule and focuses on symmetric mixed strategy equilibria. **?** studies a case in which agents acquire their policy preferences and information structures, which are captured by normal random variables. He shows that when information cost is high, preference heterogeneity can provide agents additional incentives to gather information.

In research on the jury model with costly information, a few papers analyze a problem similar to the current paper. **?** focuses on the rules that are symmetric, i.e., voters are treated equally in aggregating information, while the current paper allows for asymmetric rules.[12] In his set up, the ex ante optimal threshold rule is ex post efficient, i.e., it is efficient at aggregating information reported from a statistical point of view. In stark contrast, in my setup the ex ante optimal rule may be ex post inefficient. **?** allow for a broader class of voting rules than **?** but their analysis is also restricted to symmetric rules.[13] Gerardi and Yariv yield the same insight as the current paper that the optimal rule may be ex post inefficient. They characterize the equilibrium only for extreme values of a parameter while the current paper does so for more general range of parameters.

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[11] **?** considers a similar setting to Condorcet Jury model. He analyses a setting in which a fixed number of jurors each purchase the precision of a noisy signal, which is public information. It is shown that there is an incentive to distort the rule away from the rule that would be optimal if information was exogenously given.

[12] Also, Persico focuses on general threshold rules by which one of the alternatives is selected if and only if a certain number of voters support that alternative while the current paper focuses on simple majority rule by which one of the alternatives is selected if and only if more than half of the total votes are cast for that alternative.

[13] More specifically, **?** allow for other rules than threshold rules but focus on symmetric rules. In contrast, the current paper focuses on simple majority rule by which one of the alternatives is selected if and only if more than half of the total votes are cast for that alternative but allows for asymmetric rules.

This paper is related to the literature on the hegemonic stability theory. The central idea behind hegemonic stability theory is that the world needs a single dominant state, a hegemonic state, to create and enforce the rules of fee trade among the most important members of the system (See Gilpin 1981, 1994; Grunberg 1990; Kennedy 1987, Keohane1984; kindleberger 1973; Krasner 1976; Strange 1987). That is, a hegemon provides to the international economy in the form of public goods and other states free ride on the benefits. In international economic affairs, for example, an open trading system, well-defined property rights, common standards of measures including international money, consistent macroeconomic policies, proper action in case of economic crisis, and stable exchange rates, are said to be public goods. The problem is an under-provision of those public goods in the absence of external enforcement. The hegemonic stability theory is based on the basic idea that this collective action problem is solved by the unequal distribution of benefits. Whereas the hegemonic stability theory explains that a hegemon has an incentive to provide public goods because it is the largest beneficiary, this paper explains that highly weighted voters have an incentive to do so because they have greater influence in collective decisions.

1.3   Basic Model: Costless Information

Following the convention, I use a jury analogy to explain the model. In the political context, deciding whether to convict the defendant would be choosing between two candidate for office, whether to build a nuclear power plant, whether to launch a space shuttle, whether to approve the drug, deciding whether to send troops to Iraq, and so on.

A finite set of jurors $N = \{1, 2, \ldots, n\}$ (n odd) is to make a collective decision $d \in \{A, C\}$ where A and C correspond to acquittal or conviction, respectively. The unknown state is $\omega \in \{G, I\}$: the defendant is either guilty (G) or innocent (I), with prior distribution $\Pr(G) = \Pr(I) = \frac{1}{2}$.

All the jurors and the designer have identical preferences over the choice $d \in \{A, C\}$

and state ω.[14] The common utility is given by

$$
u(d, \omega) = \begin{cases} -\frac{1}{2} & \text{if } (d, \omega) = (C, I) \\ -\frac{1}{2} & \text{if } (d, \omega) = (A, G) \\ 0 & \text{otherwise} \end{cases}
$$

where $q \in (0, 1)$. Both jurors and the designer maximize expected utility.[15]

Given the state of the world $\omega \in \{G, I\}$, each juror j simultaneously receives a private signal $s_j \in \{g, i\}$. Conditional on the state, signals are independent across jurors. Let $p \in (1/2, 1)$ represent the probability that each juror observes the correct signal.[16]

Once jurors receive the signals, the group decision is made as follows. Let $\mathbf{w} = (w_1, \ldots, w_n)$ be a vector of non-negative weights with $w_j \geq 1$. Throughout the paper, I focus on the simple majority rule with weight $\mathbf{w}$ such that $w_1 = \ldots = w_m = w$ and $w_{m+1} = \ldots = w_n = 1$. That is, jurors $1, \ldots, m$ are weighted by $w \geq 1$ and jurors $m+1 \ldots n$ are not weighted.[17] Formally, the group decision rule is defined as $f^{\mathbf{w}} : \{g, i\}^n \to \{A, C\}$ such that

$$
f^{\mathbf{w}}(s_1, \ldots, s_n) = \begin{cases} C & \text{if } \sum_{j=1}^n w_j I_{s_j=g} \geq \frac{\sum w_j + 1}{2} = \frac{n + m(w-1) + 1}{2} \\ A & \text{otherwise.}^{18} \end{cases}
$$

I define a mechanism as the following game:

Stage 1 The mechanism designer chooses the weight distribution $\mathbf{w}$, i.e., the number of weighted jurors m and their weight w, which becomes common knowledge among

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[14] I assume jurors are all identical except for their weights in order to see if weighted rule can be better than unweighted rule for some parameters even if jurors are all identical.

[15] Juror j prefers conviction to acquittal if and only if she places at least probability q that the defendant is guilty. We say that the outcome of the trial is correct if either the defendant is guilty and convicted or he is innocent and acquitted.

[16] For the sake of simplicity, I assume that the probability that each juror observes the correct signal when the true state is G is equal to the one when the true state is I.

[17] Note that (m, w) is well-defined so that $n + m(w - 1)$ is odd.

[18] For the sake of simplicity, I exclude the possibility $\sum_{j=1}^n w_j v_j = \frac{n + m(w-1)}{2}$.

the jurors.

Stage 2 Each juror independently receives signal.

Stage 3 If $\frac{n+m(w-1)+1}{2}$ or more weighted average of the jurors receive guilty signal, the defendant is convicted. Otherwise, acquitted.

As a corollary of **?**'s result, the aggregation rule that maximizes the probabilities of convicting a guilty defendant and acquitting an innocent defendant is to distribute the weights equally.[19]

In Section 1.3, it is assumed that jurors receive information for free. In Section 1.4, I incorporate the stage in which jurors decides whether to purchase highly accurate information or receive less accurate information for free. The goal of the both sections is to find the optimal weight distribution $\mathbf{w}$, i.e., (m, w), to maximize the designer's expected utility of the collective decision.

1.4 Costly Information

The model with costless information illustrates that the weighted rule is inefficient at aggregating reported information compared to unweighted rule. In this section, I incorporate the stage where the jurors decide whether or not to invest in information before voting. Because information is a public good, information is under provided relative to the social optimum. When the information is costless, the mechanism designer needs to care only about the efficiency of aggregating information. By contrast, when the information is costly, he needs to care about whether a rule gives jurors incentives to acquire information, as well as whether it aggregates information efficiently.

At Stage 2, each juror j simultaneously and independently makes a decision $t_j \in \{0, 1\}$ about signal acquisition where 1 and 0 correspond to invest and not invest, respectively: She chooses whether to purchase a highly accurate signal at a cost c (> 0) or receive a

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[19] This corollary is based on the assumption that signal accuracies are identical for all jurors. Otherwise, the optimal allocation of weights is proportional to each juror's log-likelihood ratio.

low-quality signal for free. [20] The conditional probabilities of high-quality signals and low-quality signals are $P(s_j = i|I, t_j = 1) = P(s_j = g|G, t_j = 1) = p_H$ and $P(s_j = i|I, t_j = 0) = P(s_j = g|G, t_j = 0) = p_L$, respectively, where $p_H > p_L \geq \frac{1}{2}$.[21] Denote the weighted juror's signal accuracy by $p \in \{p_H, p_L\}$ and unweighted jurors' signal accuracy by $p' \in \{p_H, p_L\}$. The mechanism designer does not take into account the cost c incurred by a juror who purchases a high-quality signal.

Denote the probability that the jury makes a correct decision by

$$V(w, \mathbf{t}) := \Pr(G)\Pr(C|G) + \Pr(I)\Pr(A|I).$$

Since $\Pr(G) = \Pr(I) = \frac{1}{2}$ and $\Pr(C|G) = \Pr(A|I)$, $V(w, \mathbf{t}) = \Pr(C|G)$.[22] Also, since the cost of convicting an innocent defendant and the one of acquitting a guilty defendant are both $\frac{1}{2}$, the designer's expected utility is $-\frac{1}{2} + \frac{1}{2}\Pr(C|G)$. Therefore, in order to maximize the expected utility, the designer maximizes the probability of making a correct decision $V(w, \mathbf{t})$.

In the following, I restrict attention to the symmetric equilibria: Jurors of the same weight play the same strategy. Because all jurors with the same weight face a similar decision problem, it is natural to assume that jurors with the same weight use the same decision rule in equilibrium. We restrict the analysis to this kind of equilibrium, which I refer to as a symmetric equilibrium.

The purpose of this section is to (1) characterize the symmetric equilibria given the weight and (2) find the optimal weight for the designer. For now, I consider the case in which there is at most one weighted juror, $m = 1$. For the future research, I intend to extend the analysis to the case in which more than one jurors may be weighted, $1 < m < \frac{n+1}{w+1}$ (See Appendix A.4).

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[20] Information acquisition could be reinterpreted as information processing. In that case, the cost c captures the effort that each decision maker puts into updating his beliefs given the available information.

[21] For the sake of simplicity, I assume that the probability that each juror observes the correct signal when the true state is G is equal to the one when the true state is I.

[22] Since the signal accuracy is symmetric, i.e., $\Pr(s_j = g|\omega = G) = \Pr(s_j = i|\omega = I)$, and I focus on the simple majority rule, $\Pr(C|G) = \Pr(A|I)$.

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Note that w is well-defined. Since n is odd and $m = 1$, w needs to be odd in this case. We start with a simple example which suggests that weighting may improve the quality of the group decision and jurors' payoffs.

Let $N = \{1, 2, 3, 4, 5\}$, $\Pr(G) = 0.5$, $q = 0.5$, $c = 0.05$, $p_H = 0.8$, and $p_L = 0.6$.

First, consider the unweighted simple majority rule. Suppose that no jurors acquire high-quality signal. Since

$$EU_j[t_j = 1] - EU_j[t_j = 0] = -0.01544,$$

no jurors have an incentive to deviate. In this situation, the designer's expected utility is -0.31744.

Second, consider the weighted simple majority rule where juror 1 has three votes while each of the others has one vote. Suppose that only juror 1 acquires high-quality signal. Since

$$EU_1[t_1 = 1] - EU_1[t_1 = 0] = 0.03448 > 0$$
$$EU_j[t_j = 1] - EU_j[t_j = 0] = -0.04056 < 0 \qquad \text{for } j \neq 1,$$

no jurors have an incentive to deviate. In this situation, the designer's expected utility is -0.19456.

In this example, no jurors have incentives to invest without weighting while juror 1 does so with weight $w = 3$. As a consequence, the designer's expected utility is higher under the weighted rule.

This example demonstrates that the probability of making a correct decision (therefore the designer's expected utility) is higher under weighted aggregation rule than unweighted aggregation rule.

To see how individual weights affect the jury's decision, I first examine how individual

weights affect the probability of each juror's being pivotal. Let $\Pr(\text{piv}_j | \omega, w, \mathbf{t}_{-j})$ be the probability that juror j becomes pivotal given ω. Then, juror j invests in information if and only if $\Pr(G)\Pr(\text{piv}_j | G, w, \mathbf{t}_{-j})(p_H - p_L) \geq c$. [23]

Lemma 1.1. *Consider $w \geq n$. the weighted juror is decisive and unweighted jurors never becomes pivotal for any w. Consider $w \leq n - 2$. As w becomes larger, the weighted juror is more likely to become pivotal. As w increases, the unweighted jurors are*

(1) *less likely to be pivotal if (1) $p < \dfrac{\left(\frac{p'}{1-p'}\right)^{w+1}}{1+\left(\frac{p'}{1-p'}\right)^{w+1}}$ and $n < N(p,p',w)$ or (2) $p \geq \dfrac{\left(\frac{p'}{1-p'}\right)^{w+1}}{1+\left(\frac{p'}{1-p'}\right)^{w+1}}$.*

(2) *independent of w if $p < \dfrac{\left(\frac{p'}{1-p'}\right)^{w+1}}{1+\left(\frac{p'}{1-p'}\right)^{w+1}}$ and $n = N(p,p',w)$*

(3) *more likely to be pivotal if $p < \dfrac{\left(\frac{p'}{1-p'}\right)^{w+1}}{1+\left(\frac{p'}{1-p'}\right)^{w+1}}$ and $n > N(p,p',w)$*

for $p,p' \in \{p_L, p_H\}$ where

$$N(p,p',w) := w \cdot \frac{\left(1 + \frac{1-p'}{p'}\right)\left(\left(\frac{p'}{1-p'}\right)^{w+1} + \frac{p}{1-p}\right)}{\left(1 - \frac{1-p'}{p'}\right)\left(\left(\frac{p'}{1-p'}\right)^{w+1} - \frac{p}{1-p}\right)} + 2 \cdot \frac{\frac{p}{1-p} \cdot \frac{1-p'}{p'} + \left(\frac{p'}{1-p'}\right)^{w+1}}{\left(1 - \frac{1-p'}{p'}\right)\left(\left(\frac{p'}{1-p'}\right)^{w+1} - \frac{p}{1-p}\right)}.$$

Proof. See Appendix A.1 $\qquad\qquad\square$

It is surprising that, as well as the weighted juror, even unweighted jurors may be more likely to be pivotal as the weight increases. To understand this phenomenon, it is crutial

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[23] A juror j has an incentive to pay for the highly accurate information if and only if $\Pr(G, \text{piv}_j | w, \mathbf{t}_{-j})(p_H - p_L) \geq c$ because

$$EU_j[t_j = 1] - EU_j[t_j = 0]$$
$$= \Pr(G, \text{piv}_j | w, \mathbf{t})\{0 \cdot p_H - (1-q)(1-p_H)\} + \Pr(I, \text{piv}_j | w, \mathbf{t})\{0 \cdot p_H - q(1-p_H)\} - c$$
$$- \Pr(G, \text{piv}_j | w, \mathbf{t})\{0 \cdot p_L - (1-q)(1-p_L)\} - \Pr(I, \text{piv}_j | w, \mathbf{t})\{0 \cdot p_L - q(1-p_L)\}$$
$$= (1-q)(p_H - p_L) \cdot \Pr(G, \text{piv}_j | w, \mathbf{t}) + q(p_H - p_L) \cdot \Pr(I, \text{piv}_j | w, \mathbf{t}) - c$$
$$= (p_H - p_L) \cdot \Pr(G, \text{piv}_j | w, \mathbf{t}) - c.$$

The last equality holds because $\Pr(G, \text{piv}_j | w, \mathbf{t}) = \Pr(I, \text{piv}_j | w, \mathbf{t})$. Note that the probability of juror j's being pivotal is independent of of his investment.

to remark that unweighted jurors are more likely to be pivotal as the expected weight of correct signals becomes closer to the simple majority of the total weight.

Consider the event that the weighted juror receives a wrong signal. As w becomes larger, the expected weight of correct signals conditional on this event decreases, i.e., becomes closer to the simple majority of the total weight. This increases the chances that unweighted jurors are pivotal. On the other hand, consider the event that the weighted juror receives a correct signal. As w becomes larger, the expected weight of correct signals conditional on this event increases, i.e., becomes farther away from the simple majority of the total weight. This decreases the chances that unweighted jurors are pivotal. When the weighted and unweighted jurors receive the same quality signals and n is large, for example, the former effect is greater than the latter, which means that unweighted jurors are more likely to be pivotal as w increases.

As a corollary of Lemma 1.1, I have sufficient conditions of (p_H, p_L, w, n) for unweighted jurors being less likely to be pivotal as w increases for all $p, p' \in \{p_H, p_L\}$ as follows.

Corollary 1.1. *If* $p_H < \dfrac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$ *and* $n < N(p = p_L, p' = p_H, w)$, *unweighted jurors are less likely to be pivotal as w increases for all* $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$.

Proof. Suppose $p_H < \dfrac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$. Then, it follows that $N(p = p_H, p' = p_H, w) > N(p_L, p_L, w) > N(p = p_L, p' = p_H > N(p = p_L, p' = p_H, w) > 0$. Thus, if $p_H < \dfrac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$ and $n < N(p = p_L, p' = p_H, w)$, then $n < N(p, p', w)$ for all $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$. By Lemma 1.1, it follows that $\Pr(\text{piv}_j | G, w, \mathbf{t}_{-j})$ is decreasing in w for $j \neq 1$ for all $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$. □

By Corollary 1.1, Lemma 1.2 shows the existence and sufficient conditions of (p_H, p_L) for unweighted jurors being less likely to be pivotal as w increases for all $w = 1, 3, \ldots, n-2$, regardless of jurors' investment behavior, i.e., for all $p, p' \in \{p_H, p_L\}$.

Lemma 1.2. *Suppose* $p_H < \dfrac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$. *For every n, there exists* (p_H^*, p_L^*) *such that, if* $p_H \leq p_H^*$ *and* $p_L \leq p_L^*$, *unweighted jurors are less likely to be pivotal as w increases for all* $w = 1, 3, \ldots, w - 2$ *and for all* $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$.

Proof. Suppose $p_H < \dfrac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$. For all $w = 1, \ldots, w-2$,

$$\lim_{p_H \to \frac{1}{2}} \lim_{p_L \to \frac{1}{2}} N(p = p_L, p' = p_H, w) = \infty. \tag{1.1}$$

By (1.1), it follows that for given n there exists p_H^* such that

$$\lim_{p_L \to \frac{1}{2}} N(p = p_L, p' = p_H, w) > n + 1 \tag{1.2}$$

for every $p_H \leq p_H^*$. (1.2) implies that there exists p_L^* such that

$$N(p = p_L, p' = p_H, w) > n \tag{1.3}$$

for every $p_L \leq p_L^*$. By Corollary 1.1, (1.3) implies that unweighted jurors are less likely to be pivotal as w increases, regardless of jurors' investment behavior, i.e., for all $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$. $\qquad\square$

Lemma 1.2 shows that unweighted jurors are less likely to be pivotal as $w(\leq n-2)$ increases if p_H and p_L are sufficiently small. In the following, I focus on the cases of sufficiently small p_H and p_L, where unweighted jurors are less likely to be pivotal as $w(\leq n-2)$ increases.

Assumption 1. *The accuracy of high-quality signal p_H and the one of low-quality signal p_L are sufficiently low so that unweighted jurors are less likely to be pivotal as $w(\leq n-2)$ increases.*

Proposition 1.1 describes the equilibria given c and w. It shows that for sufficiently high but not too high cost c increasing the weighted juror's weight w may improve the jury decision.

Define $(c_1, c_2, c_3, c_4, c_5, c_6)$ as follows:

$$c_1 := f_{-1}(n - 2, (1, 1, \ldots, 1))$$

$$c_2 := f_j(1, (1, 1, \ldots, 1)) \qquad \text{for any } j \in N$$

$$c_3 := f_{-1}(n - 2, (1, 0, \ldots, 0))$$

$$c_4 := f_{-1}(1, (1, 0, \ldots, 0))$$

$$c_5 := f_1(1, (1, 0, \ldots, 0)) = f_1(1, (0, 0, \ldots, 0))$$

$$c_6 := f_1(n - 2, (1, 0, \ldots, 0)) = f_1(n - 2, (0, 0, \ldots, 0))$$

where $f_j(w, \mathbf{t}_{-j}) := (p_H - p_L) \cdot \Pr(G, \mathrm{piv}_j | w, \mathbf{t}_{-j})$ for $j \in N$.[24]

Proposition 1.1. *Consider sufficiently small p_H and p_L such that unweighted jurors are less likely to be pivotal as $w(\leq n - 2)$ increases. Then, Figure 1.1 illustrates the equilibria*
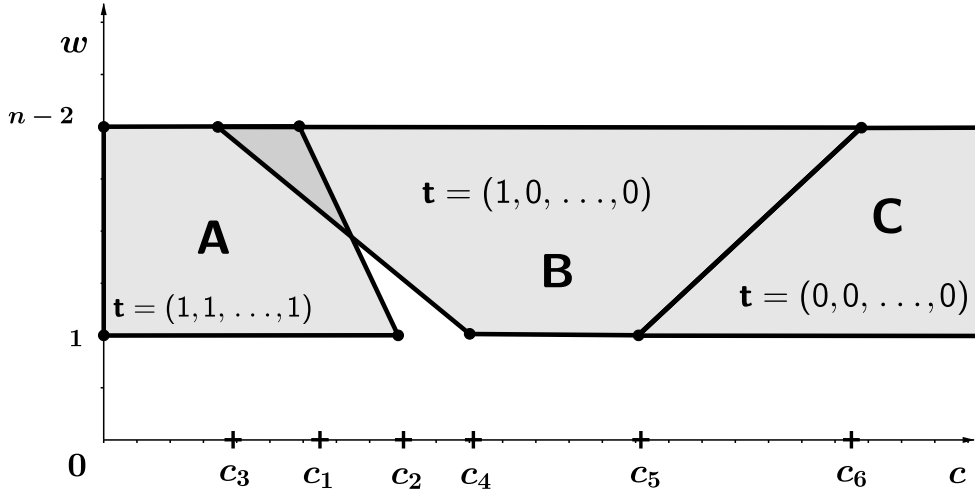
Figure 1.1: $\{(c, w) | w_1(c, \mathbf{t}) \leq w \leq w_{-1}(c, \mathbf{t})\}$ for $\mathbf{t} = (1, 1, \ldots, 1)$

where $\mathbf{t} = (1, 1, \ldots, 1), (1, 0, \ldots, 0), (0, 0, \ldots, 0)$.[25] The area A represents (c, w) for which $\mathbf{t} = (1, 1, \ldots, 1)$ is an equilibrium; the area B represents (c, w) for which $\mathbf{t} = (1, 0, \ldots, 0)$ is an equilibrium; the area C represents (c, w) for which $\mathbf{t} = (0, 0, \ldots, 0)$ is an equilibrium.

Proof. See Appendix A.2. □

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[24] $f_{-1} := f_j$ for $j \neq 1$.

[25] $w_{-1} := w_j$ for $j \neq 1$

Proposition 1.1 illustrates the equilibrium behavior, i.e, who invests in information in equilibrium, given the weight (w) and the cost of improving signal (c). Based on Proposition 1.1, we find the optimal weight for the designer given the cost of improving signal (c).

Proposition 1.2. *The optimal weight $\hat{w}$ for the designer is*

$$\hat{w} = \begin{cases} 1 & \text{if } c \in [0, c_2] \cup [c_4, c_5] \cup [c_7, \infty) \\ w^{**}(c, \mathbf{t}) & \text{if } c \in (c_2, c_4) \\ w^*(c, \mathbf{t}) & \text{if } c \in (c_5, c_7] \end{cases}$$

if $p_H \left(1 - (1 - p_L)^{n-1}\right) + (1 - p_H) p_L^{n-1} - \sum_{x \geq \frac{n+1}{2}} \binom{n}{x} p_L^x (1 - p_L)^{n-x} < 0$ (Figure 1.2), and

$$\hat{w} = \begin{cases} 1 & \text{if } c \in [0, c_2] \cup [c_4, c_5] \cup [c_6, \infty) \\ w^{**}(c, \mathbf{t}) & \text{if } c \in (c_2, c_4) \\ w^*(c, \mathbf{t}) & \text{if } c \in (c_5, c_6) \end{cases}$$

if $p_H \left(1 - (1 - p_L)^{n-1}\right) + (1 - p_H) p_L^{n-1} - \sum_{x \geq \frac{n+1}{2}} \binom{n}{x} p_L^x (1 - p_L)^{n-x} \geq 0$ (Figure 1.3).

Figure 1.2: Optimal Weights
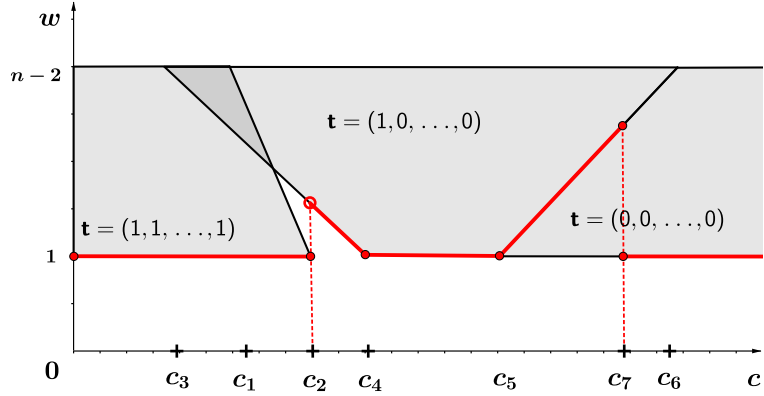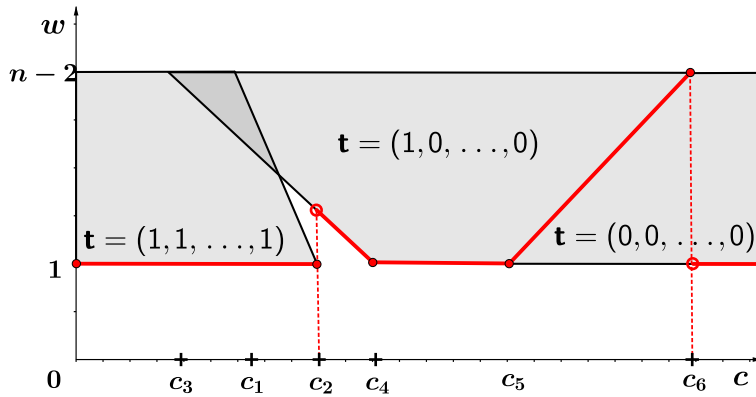

Figure 1.3: Optimal Weights

Proof. See Appendix A.3. □

Proposition 1.2 shows that if the cost of improving the signal is moderately high, the designer may maximize the probability of the jury's making a correct decision by distributing weights unequally.

1.5 Discussions

In this section, I describe the possible implications to political theory and possible future research.

1.5.1 Implications

Equal Suffrage and Weighted Voting

This paper may shed light on political philosophies. light Equal suffrage has been one of the most important concepts for political theorists. It has been commonly accepted that political equality is a central feature of a democratic system (See Barry Holden, *The Nature off Democracy*, 1974, p.19; Giovanni Sartori, *Democratic Theory*, 1965, ch 14; Ivor BVrown, *The Meaning of Democracy*, 1926, p.44, George Edwards III, *Why the Electoral College is Bad for America*, 2004, ch 2, James S. Fishkin, *Democracy and Deliberation*, p.29). Robert Dahl, for example, argues that equality in voting is a crucial part of a democratic system: "every member must have an equal and effective opportunity to vote, and all voters must be counted as equal." A constitution for democratic government, he adds, "must be in conformity with one elementary principle: that all members are to be treated (under the constitution) as if they were equally qualified to participate in the process of making decisions about the policies the association will pursue. Whatever may be the case on other matters, then, in governing this association all members are to be considered as politically *equal*." (*On Democracy*, Robert A. Dahl (2000) p.37).

On the other hand, John Stuart Mill advocated the weighted voting system whereby educated and more responsible persons would be given more votes than the uneducated. As much as this weighted system may be unfair to uneducated citizens, there is no guarantee that the educated have better sense about what is good for the society than the uneducated. This paper shows that Mill's argument may hold even if the educated are not more likely to make a correct judgement than the uneducated. Moreover, it suggests that the quality of society's decisions may be higher under the concentration of power compared to the one in democratic societies.

Correspondence with the argument of John Stuart Mill

Both the current paper and **?** consider the same environment. In particular, both assume the common-interest. For Mill it is vital that voters should vote in accordance with their ideas of the general interest; that is they should vote for whichever candidates

they feel most likely to improve the citizens and efficiently manage the affairs of the country in the interests of all. In fact, Mill uses an analogy with jury service:

> "[The citizen's] vote is not a
> thing in which he has an
> option; it has no more to do
> with his personal wishes than
> the verdict of a juryman. It is
> strictly a matter of duty; he is
> bound to give it according to
> his best and most
> conscientious opinion of the
> public good."
>
> ———————————————
> —John Stuart Mill,
> *Representative Government*,
> 299

Both the current paper and Mill address weighted voting systems but the arguments differ in the weight assignment. Mill argues that the particularly intelligent or well educated should be given two or more votes. Mill's concern is that the uneducated poor–the numerical majority–will make a terrible mistake. Mill wants to ensure that representative democracy contains certain safeguards to prevent it from being dictated to by stupidity and class interest. (*Representative Government*, 284). The current paper also supports weighted voting by demonstrating that the group may be more likely to make a correct judgement under a weighted voting system than under an equal representation. The difference in those studies is that, while Mill argued that weights should be distributed based on the education level, the current paper argues that weighted voting systems may work even if voters are all equally capable of making a judgement.

Thus, by considering the same situation as Mill's, the current paper supports the unequal representation advocated by Mill. Moreover, this paper uses the same setting whereby Condorcet supported democracy.

1.5.2 Future Research

There are many directions this framework suggests pursuing. One example is to analyze the optimal size of the committee. In the current model, the size of the committee is exogenously given and the designer cannot choose. Another possibility is adding heterogeneity amongst agents, in the form of differential preferences, may affect the optimal design. Indeed, in our model, both the designer and all of the players share the same utility parameter q.

The Size of Nations

The current model may contribute to the literature on the size of nations and the regime type. A large body of literature deals with the size of nations, the regime type, and the relationship linking these two variables (See Alesina and Spolaore (1997), Alesina et al. (1997), Alesina and Wacziarg 1998). Political theorists have argued that democracy cannot survive in a large state. In particular, Plato, Aristotle and Montesquieu worried about the political costs of large states: Plato wrote that "*the number of citizens should be sufficient to defend themselves agains the injustice of their neighbors,*" (*Laws*, Book V); Aristotle argued that a polity should be no larger than a size in which everybody knows personally everybody else because "*experience has shown that it is difficult, if not impossible, for a populous state to be run by good laws*" (*The Politics*); Montesquieu wrote that "*In a large republic, the common good is sacrificed to a thousand considerations. It is subordinated to various exceptions. It depends on accidents. In a small republic, the public good is more strongly felt, better known, and closer to each citizen.*" (*The Sprit of the Laws*).

On the other hand, Madison objected that a large size, far from being a problem, was actually an advantage for a democracy. His point was that ht enlarger territory becomes in size, the greater will be its variety of parties and interests, and hence the smaller will be the chance that "*a majority of the whole will have a common motive to invade the rights of other citizens; or if such a common motive exists, it will be more difficult for all who feel it to discover their own strength, and to act in unison with each other.*" In other

words, according to Madison, in larger states rent-seeking groups who want to *"invade the rights of other citizens"* will have a harder time to overcome problems of collective action. Moreover, according to Madison, " *the influence of factious leaders may kindle a flame within their particular States, but will be unable to spread a general conflagration through the other States.*"

The current model may support the former thinkers, the advocates for small nations. The current model may demonstrate that weighted voting becomes superior to equal suffrage as the group size becomes larger: Because incentives to free-ride becomes stronger as the group size become larger, the designer may want to giver higher weight to some voters to improve their incentives to acquire accurate information at the cost of efficiency in information aggregation. More generally, the current model gives the optimal distribution of weights for a given size of the group. That is, on the assumption that the distribution of weights can be interpreted as the distribution of the power, the model suggests the optimal regime design for a given size of the country.

<center>*Heterogeneous Signal Accuracy*</center>

The next step is to incorporate the heterogeneity of signal accuracy to the current model. The model with heterogeneous signal accuracy may have several important implications. Firstly, if one accepts the interpretation that the distribution of weights represents the distribution of the power, a variant of my model may shed new lights on one of the most important topics in political science: By incorporating the heterogeneity of signal accuracy to the current model, one may be able to find the optimal distribution of the power, i.e., the optimal regime design.

Examples of classic arguments over regime design are those by Plato, Mill, and Rousseau. Mill's aristocratic liberalism falls between Plato's guardianship and Rousseau's democratic principle: Plato asserted that only a few selected experts should rule; Mill asserted that even uneducated citizens should be enfranchised but educated citizens should be given more votes than the uneducated; Rousseau supported democratic principle that all citizens have an equal say (except for female citizens). My model may explain when Plato's

<center>21</center>

guardianship works, when Mill's aristocratic liberalism works, and when Rousseau's democratic principle works.

One way to generalize those three types of systems is to consider them to be a variant of weighted rule: In Plato's benevolent dictatorship, a few selected experts, *guardians*, have votes while others' weight is zero; In Rousseau's democratic system, everyone has an equal weight; In Mill's system, everyone has positive weight but their weights are allocated based on their education level. The question is what the optimal weighting is.[26]

Suppose that some voters inherently receive low-quality signal whose accuracy is $\frac{1}{2}$ whereas the others inherently receive high-quality signal whose accuracy is higher than $\frac{1}{2}$. Consider (1) a group of voters all of whom receive high-quality signal and (2) a group of voters some of whom receive low-quality signal and the others receive high-quality signal. Suppose that the voters with high-quality signal have more than one vote while the voters with low-quality signal have only one vote each. Under a certain range of parameters, the former group may be more likely to make a correct decision than the latter group. If that

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[26] The following problem about Mill's system may help us understand the question. Mill's critics to his proposal about plural voting argue that, if uneducated revere the educated then I need not give the latter extra votes, for the uneducated can simply seek out their opinions. But if they do not respect such opinions then they would not accept plural voting. Plural voting is either unnecessary or unjustified. In fact, Mill himself recognized this point and made the following remark.

> "I may remark, that if the
> voter acquiesces in this
> estimate of his capabilities,
> and really wishes to have the
> choice made for him by a
> person in whom he places
> reliance, there is no need of
> any constitutional provision
> for the purpose; he has only to
> ask the confidential person
> privately what candidate he
> had better vote for."
>
> ---
> —John Stuart Mill,
> *Representative Government*,
> 294

The question is whether and when the power should be concentrated on a few selected people as Plato suggested, and whether and when everyone should be enfranchised whereas the distribution of the votes are not equal as Mill suggested, and whether and when all citizens should be equally enfranchised as Rousseau suggested.

is the case, it implies that Mill's partial democracy works better than Plato's guardianship or Rousseau's democracy under such parameters. More generally, this variant of my model may provide the conditions for each of the thee regime types to be optimal.

Secondly, the model with heterogeneous signal accuracy may also examine Mill's proposal about plural voting from another perspective. According to Mill, plural voting has two benefits: By giving higher weight to highly educated people, it is efficient at aggregating information; By enfranchising uneducated citizens, it also helps poorly educated citizens educate themselves through participation. My model may demonstrate that those two benefits may contradict each other and, if so, provide conditions for plural voting to have those benefits all together.

Recall that my model has shown that there may be a trade-off between the efficiency of information aggregation and the incentives to acquire information under a certain range of parameters. Because "costly information acquisition" can be interpreted as the process for citizens to learn about the policy, it implies that there may a trade-off between the efficiency of information aggregation and the citizens' incentives to educate themselves.

Before getting to the model, note that weighted voting may reduce the performance of the group decision for two possible reasons: The inefficiency of information aggregation caused by weighting disproportionate to the voters' signal accuracy and the decrease in investment in information (education). We pay close attention to the second aspect. If some voters have higher weight than others, it may discourage other voters to acquire more accurate signal even though they would be willing to invest if all voters had an equal say.

Suppose that there are voters who inherently receive more accurate signal than the other voters. Consider (1) a group of voters all of whom are unweighted regardless of their signal accuracy and (2) a group of voters whose weight are determined based on their signal accuracy: voters who receive highly accurate signal have more than one vote while the others have only one vote each. There may be a range of parameters under which all voters in the former group invest in information while none of the unweighted voters do so

in the latter group. In this case, the quality of information possessed by the latter group is lower than the the one possessed by the former. Thus, the latter group is more likely to make a wrong decision than the former even if the inefficiency caused by the weighting is adjusted.

Therefore, under some weighted rule, the accuracy of group decision becomes lower because the weighted rule discourages unweighted voters to invest in signal (educate themselves), which decreases the quality of information possessed by the group. This implies that making good use of highly educated people's opinions (efficient information aggregation) and the positive effect of the franchise on uneducated people's learning, both of which are supported by Mill, may contradict each other.

2. LOCAL GOSSIP AND INTERGENERATIONAL FAMILY TRANSFERS: COMPARATIVE POLITICAL ECONOMY OF WELFARE PROVISION

2.1   Introduction

The level and means of welfare provision vary across societies and countries. Why are transfers within the family generous in some societies and not in others? Why does the family play a central role in care for the elderly and childcare in some countries while the government does so in others? For example, in southern European societies, the family functions as a welfare provider in terms of care for the elderly, childcare, and helping the unemployed (**?, ?, ?**); In Scandinavian countries, the government plays a central role in welfare provision.

Furthermore, East Asian welfare regimes are often characterized by their low level of state-based transfers to the elderly and high levels of family-based transfers. To explain this East Asian variant of the welfare state, scholars of the region argue that Confucian ideology plays an important role (**?, ?**). According to Confucian ethics, the family is an important source of welfare provision, with aged parents being cared for by children.

The Hong Kong special administrative region government, for example, has adopted Confucian ethics of filial piety to control social welfare costs (**?**). In fact, care for the elderly is primarily a family responsibility and the government is considered to be the last resort in Hong Kong. The family rather than the government has been regarded as the main source of welfare provision in Japan as well (Harada 1988). Thus, public welfare services play only a secondary role in the private welfare practice within the family (**?**). For example, public home care services are designated merely to support family care on the basis that the care for the elderly is their children's responsibility (**?**).

The reliance on the welfare role of the family in East Asian countries contrasts sharply

with the Western welfare system in which states play a central role. In addition to the cross-national differences, the level of family-based welfare provision varies even within the same country. For example, people in northeast Japan tend to provide better nursing care to their aged parents and invest more in their children's education than those in the southwest.

Although it has been recognized that the family plays an important role in welfare provision, scholars of the field have focused on understanding the role of the state and market. As a result, little is known about the role and mechanisms of other institutions such as civic associations and the family. In this paper, I present a theoretical framework to analyze how family-based welfare provision is achieved and the role of the local community in the familial provision.This theoretical framework may give a microfoundation to the "familialism" argument in the southern European welfare states and the "Confucian" argument in the East Asian welfare states.

In this paper, I argue that differences in the level of transfers within the family are due to demographic variation in community networks. To analyze the sustainability of intergenerational transfers, I propose game theoretical models of overlapping generations in which breadwinners make transfers to their parents and children. A novel feature of the models is that there is a local community that can supply information about its members' past behavior. I demonstrate that intergenerational transfers can be sustained if neighbors gossip about each others' reputations. My theory suggests that individuals in a close-knit community prefer lower levels of social protection.

This paper is organized as follows. In Section 2.2, I discuss the previous literature and the current paper's contribution to the literature. In Section 2.3, I present formal models and demonstrate that intergenerational transfers can be supported if neighbors gossip about each other. In Section 2.4, I discuss this paper's implications and future extension.

2.2 Related Literature

2.2.1 Literature on the Welfare State

Scholars on the welfare state have placed emphasis on the state and the market, but the family has been largely discussed in the context of gender (**?**, **?**, **?**, **?**)). However, little is known about how *inter*-generational rather than *intra*-generational redistribution works within the family. By analyzing the *inter*-generational family transfers, this paper sheds new light on the role of the family in welfare provision.

Understanding the role of the family also helps us understand the East Asian variant of the welfare state, which is marked by a low level of social spending. Two approaches are used in the literature to explain the small welfare state in the region. The first approach explains the incentives of the governments. (**?**) ascribes the small social spending in East Asia to the absence of strong leftist parties and unions while **?** argues that social spending in the region was minimized to promote economic growth. However, those authors fail to explain why the family works as a provider of welfare. They do not offer compelling accounts of the incentives to the family, such as why breadwinners are willing to provide care for their aged parents, for example.

The second approach, on the other hand, explains the incentives of family members. This group of scholars argues that heavy reliance on the family is possible because of Confucian values in East Asia (**?**, **?**). Their findings suggest the family is responsible for social protection rather than the state because of their values. This paper can be seen as falling within this second approach. While the previous literature simply assumes that people in the region have "Confucian" values and considers the values to be exogenous, this paper explains why the "Confucian" way of family transfers prevails in some regions and not in others. In this sense the present paper gives a microfoundation to the Confucian theory.

2.2.2 Theoretical Contributions

This work is part of the literature on overlapping generations models (OLG models). Pioneered by **?**, OLG models have been widely studied by game theorists (**?, ?, ?, ?, ?**). **?**, for example, studies the ability of non-market institutions, such as the government and the family, to invest optimally in forward intergenerational goods (FIGs), such as education and environment, and backward intergenerational goods (BIGs), such as social security. BIGs transfer income from one generation to earlier generations; FIGs transfer income forward to younger generations. Most of the game theoretic OLG studies cited above assume that the entire history of the play of a game is common knowledge. That is, players are assumed to be informed of past events. In the context of families, it is assumed that each generation knows even its great-great-great-grandparent's past behavior, for example. Considering that the history may include actions taken by preceding generations before the current generations are born, this assumption of perfect information becomes questionable.

Very few attempts at relaxing this assumption have been made. **?**, however, does so and demonstrates that no intergenerational cooperation can be supported in pure-strategy equilibrium if information about the history of play is limited. The intuition is as follows: The limited information makes it impossible for each generation to condition its actions on the observed history in equilibrium, a condition crucial to support intergenerational cooperation in equilibrium. If older generations have better knowledge about past events than younger generations, the older generations can manipulate the information, an act that will be interpreted and acted on by younger generations by behaving as though different past events happened.

In response to **?**, **?** incorporate institutional features of overlapping generations organizations into Bhaskar's model to solve his impossibility theorem. Considering legislative bodies such as the U.S. Senate to be overlapping generations organizations, they show that the principals (the founding fathers) will agree to institute a mechanism that provides imperfectly informed legislators with the information about the history in the legislature, enabling intergenerational cooperation. Similarly to **?**, the present paper provides another

solution to Bhaskar's impossibility theorem in the setting of families. This paper demonstrates that intergenerational cooperation can be supported if neighbors gossip about each other. There are two key institutional differences between **?** and the current paper: First, this paper assumes that the local community stores the information as a summary statistic of information of unboundedly high order, while **?** assume that the institutionalized mechanism stores the information as the entire history of play in the legislature. Second, **?** endogenize the institutionalized mechanism, whereas the local community is exogenous in the current paper.

In addition to the literature on OLG models, this work can be viewed as part of the literature on community enforcement, which is usually modeled as repeated games with random matching. The literature on community enforcement can be divided into two strands based on the assumptions about players' knowledge. The first strand assumes that players know the past plays to some extent: **?** assumes that all players know the history of all matches in the population; **?**, **?**, and **?** assume that players' knowledge is limited to the matches in which they have been directly involved; **?**, **?**, **?**, **?**, **?**, **?**, and **?** assume that players only have first-order information; that is, they have information about their partners' past play but do not know their partners' past partner's past play.

The present work falls into the second strand, which assumes that each player is labeled with a status that is observable to his partner, and that a player can condition his action on the partner's status (**?**, **?**). A player's status is updated based on the realized action profile of the stage game and the player's and the partner's status in the previous period. Because a player's status at the next period depends on his current partner's status, which, in turn, depends on the partner's previous partner's status, and so on, a status is a summary statistic of information of unboundedly high order. The equilibrium in this setting is called a norm equilibrium. This paper introduces the concept of norm equilibrium to overlapping generations models.

In terms of the role of the third party in sustaining cooperation, this paper is related to **?**, who investigate how a judge (a law merchant) serves to facilitate cooperation, while the current paper explores how a local community does so. Both this paper and that of

Milgrom et al. may be viewed as attempts to investigate the role of the third party as an information device in sustaining cooperation.

2.3   Formal Models

2.3.1   Basic Three-generations Model

This paper investigates (1) whether intergenerational transfers can be sustained when individuals are imperfectly informed about past events in their families and (2) whether and how a local community and its social norm contribute to the intergenerational family transfers. Throughout this paper, I assume all generations are selfish and are not altruistic.[1].

I propose an overlapping three-generations model to analyze both backward and forward intergenerational transfers: Examples of the former include nursing care and financial support from individuals to their aged parents, and examples of the latter include school expenses and nutrition from individuals to their children.[2] I consider a situation in which a breadwinner in a family takes care of his older dependent and younger dependent. A breadwinner decides how much to invest in backward intergenerational goods (BIGs) that benefits only the older dependent and forward intergenerational goods (FIGs) that benefits only the younger dependent.

The key assumption is that information is limited in the sense that individuals observe the behavior of several preceding generations but not the one of their distant ancestors. For example, individuals may know how their parents treated their grandparents, but may not know how their great-grandparents treated their great-great-grandparents. I first demonstrate that neither BIGs nor FIGs can be sustained under limited information, and then show that a close-knit community serves to facilitate cooperation between genera-

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[1] I do not argue that individuals are completely selfish and no transfers within families derives from altruism. Individuals may make transfers simply because they care about family members. The amount of transfers which they make only with altruistic motives alone may not, however, be as high as the one they would make with both altruistic and selfish motives in cooperative equilibrium. In this paper, I focus on the transfers made out of selfish motives and do not address those made for altruistic reasons.

[2] As a variant of this three-generations model, analogous results for the two-generations model can be obtained by essentially identical logic. See Appendix B.1 for formal analysis.

tions.

Consider an infinitely-lived family with three generations alive in each period t ($t = 1, 2, 3 \ldots$). Generation t is born to the family at period $t-1$ and lives for three periods: He is called a (dependent) child at period $t-1$, a breadwinner at t, and a (dependent) parent at $t+1$. At period 1 there are generation 2 that lives for three periods, generation 1 that lives for only two periods, as well as generation 0 that lives for only that period.

The breadwinner has positive endowment, and the dependents have endowment that is normalized to zero. At every period t, generation t decides how much to transfer to his dependents, generations $t+1$ and $t-1$. Let $b_t \in B_t$ and $f_t \in F_t$ denote the amounts transferred at period t from generation t to generations $t-1$ and $t+1$, respectively, and 0 means that there is no transfer.[3] Action spaces are common across generations, i.e., $B_t = B$ and $F_t = F$ for all t. The dependents have no choices to make.

Generation t's utility function, $u_t : F_{t-1} \times F_t \times B_t \times B_{t+1} \to \mathbb{R}$, is decreasing in b_t and f_t (the amount he transfers at period t), and increasing in b_{t+1} and f_{t-1} (the amount he receives at period $t+1$ and $t-1$), respectively. Utility function $u(\cdot)$ satisfies the condition:

Assumption 2. $\forall (f_{t-1}, f_t), (f'_{t-1}, f'_t) \in F^2$, $\forall (b_t, b_{t+1}), (b'_t, b'_{t+1}) \in B^2$,

$$u((f_{t-1}, f_t), (b_t, b_{t+1})) = u((f'_{t-1}, f'_t), (b'_t, b'_{t+1}))$$
$$\Rightarrow ((f_{t-1}, f_t), (b_t, b_{t+1})) = ((f'_{t-1}, f'_t), (b'_t, b'_{t+1})).$$

This condition ensures that $\arg\max_{(f_t, b_t) \in F \times B} u(f_{t-1}, f_t, b_t, b_{t+1})$ is unique. Note that this overlapping generations game has a unique Markov equilibrium, in which every generation makes no transfer.

Let $\mathbf{h}_t = ((f_1, b_1), \ldots, (f_{t-1}, b_{t-1}))$ denote the history of preceding actions taken up to period t and $H_t (= F^{t-1} \times B^{t-1})$ denote the set of all possible histories at t. I define the default informational environment as follows:

Definition 2.1. For any $t = 1, 2, \ldots$, generation t has **m-th order information** if he

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[3] The commodity is assumed to be infinitely divisible.

knows the actions of the last m generations, $((f_{t-m}, b_{t-m}), \ldots, (f_{t-1}, b_{t-1}))$, but not any action taken prior to $t - m$, $((f_1, b_1), \ldots, (f_{t-m-1}, b_{t-m-1}))$.

Assumption 3. *For every t, generation t has m-th order information.*

If (h_t, h_t') is any pair of histories which differ only in the actions taken by some of players $i \leq t - m - 1$, then the histories observed by generation t are identical for h_t and h_t'. For example, the first-order information is a record of the preceding generation's past play. That is, the information is limited in the sense that each generation does not know the actions taken by the generations prior to his parents.

A pure strategy for generation t is a mapping $s_t : F_{t-m} \times B_{t-m} \times \cdots \times F_{t-1} \times B_{t-1} \to F_t \times B_t$ for $t > m$ and $s_t : H_t \to F_t \times B_t$ for $t \leq m$. Thus, $s_t(f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1}) \in F_t \times B_t$ is the pair of transfers by generation t induced by the observed history $(f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1})$ when s_t is played. Let S_t be the set of generation t's pure strategies. A strategy profile is an infinite sequence $(s_t)_{t=1}^{\infty}$ where $s_t \in S_t$ for every t.

I define our equilibrium concept as follows.

Definition 2.2. A strategy profile $(s_t)_{t=1}^{\infty}$ is a sequentially rational equilibrium if
$$\forall t, \ \forall (f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1}) \in F_{t-m} \times B_{t-m} \times \cdots \times F_{t-1} \times B_{t-1}, \ \forall f_t \in F_t, \ \forall b_t \in B_t,$$

$$u(s_t, s_{t+1} | f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1}) \geq u(f_t, b_t, s_{t+1} | f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1})$$

where

$$u(f_t, b_t, s_{t+1} | f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1}) = u[f_t, b_t, s_{t+1}(f_{t-m+1}, b_{t-m+1}, \ldots, f_t, b_t)]$$

and

$$u(s_t, s_{t+1} | f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1})$$
$$= u[s_t(f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1}), s_{t+1}(f_{t-m+1}, b_{t-m+1}, \ldots, f_t, b_t)].$$

The following theorem is a variant of Theorem 1 in **?**.

Theorem 2.1. *Under Assumption 3, the overlapping generations game with m-th order information has a unique pure strategy equilibrium. In that equilibrium, no intergenerational transfer is made.*

Proof. Suppose that generation t's equilibrium strategy s_t is conditioned on action taken by generation $t - m$. Then, there exist $f_{t-m}, b_{t-m}, f'_{t-m}, b'_{t-m}$ such that $u_t(s_t, s_{t+1} | f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1}) > u_t(s_t, s_{t+1} | f'_{t-m}, b'_{t-m}, \ldots, f_{t-1}, b_{t-1})$. Because s_{t+1} is conditioned only on actions taken by generations $t - m + 1, \ldots, t$ and is independent of generation $t - m$'s action, generation t can improve his payoff by choosing $s_t(f_{t-m}, b_{t-m}, \ldots, f_{t-1}, b_{t-1})$ instead of $s_t(a'_{t-m}, \ldots, a_{t-1})$ when he observes $s_t(f'_{t-m}, b'_{t-m}, \ldots, f_{t-1}, b_{t-1})$. Hence, generation t's equilibrium strategy does not condition on $t - m$'s actions. Similarly, generation t's equilibrium strategy is not conditioned on k's action for any $k < t$. Thus, generation t's equilibrium strategy is not conditioned on history. Therefore generation t's best response is $f_t = b_t = 0$, which forms a unique pure strategy equilibrium where no intergenerational transfer is made. $\square$

Theorem 2.1 indicates that when individuals have limited information about past events, no strategy profile (that does not have to be Markov) in which players condition their behavior on the observed history, which is payoff-irrelevant, constitutes an equilibrium. Thus, again, intergenerational cooperation cannot be supported in pure-strategy equilibrium when information about past events is limited.

Intuitively, the limited information makes it impossible for each generation to condition its actions on the observed history, which is crucial to intergenerational cooperation. When the information is limited, individuals have better knowledge about the past events than their children. Thus, by behaving as though different past events happened, they can manipulate the information that will be interpreted by their children. For instance, suppose that $m = 2$ and each generation t plays the following strategy: Make no transfer if generation $t - 1$ transferred (b_{t-1}, f_{t-1}) such that $b_{t-1} < b$ or $f_{t-1} < f$ and the generation $t - 2$ transferred (b_{t-2}, f_{t-2}) such that $b_{t-2} \geq b$ and $f_{t-2} \geq f$ as a breadwinner; Otherwise, make transfers $(f_t, b_t) = (f, b)$. Suppose also that an individual knows that neither his parent nor grandparent made transfers. The above strategy tells him to take good care

of his parent. However, knowing that his child will follow the above strategy, he can deviate from the strategy profile without being punished by his child. More generally, any strategy profile that is conditioned on history fails to be an equilibrium, which prevents intergenerational cooperation.

2.3.2   Three-generations Model with a Local Community

In the basic model, it is shown that positive amount of intergenerational transfers cannot be supported by pure-strategy equilibria when information is limited. In reality, however, we do observe intergenerational family transfers. The question is whether they are all based on altruistic motives.

As a solution to this limited information problem, I modify the basic model by adding a local community that provides information about the history of play in each family through gossiping. In a close-knit community, it is natural to assume that intergenerational family transfers are observed by neighbors. For example, how an individual treats his parent may be revealed to their neighbors from the way his parent looks or because his parent grumbles about it. As a result, neighbors' gossip may provide the breadwinner with information necessary to see whether the parent's past behavior can be justified.

I consider a situation in which the family in the basic model resides in a close-knit community. In a close-knit community, the breadwinner receives a reputation based on how he treated his parent and his child, and the reputation will become known to his child through his neighbors' gossip. The transfers the breadwinner will receive from his child in old age may depend on his reputation, which subsequently determines the child's reputation. In this setting, positive amount of BIGs and FIGs can be transferred by pure-strategy equilibria. Formally, this situation is modeled as follows.

Before introducing a local community to the basic model, I define the "efficient" level of transfers. Let $\arg\max_{f \in F\, b \in B} u((f,b),(f,b)) \equiv (1,1)$ be the efficient transfers, which I assume to be different from $(0,0)$, the individually optimal action. Otherwise, it is trivial that no transfer is sustained in equilibrium. From Assumption 2,

Lemma 2.1. $u((0,0),(1,1)) > u((1,1),(1,1)) > u((0,0),(0,0)) > u((1,1),(0,0))$

Next, I introduce a local community that labels individuals based on its social norm. At the end of each period t, the breadwinner t is assigned an element x_t of a finite set $X_t = \{\text{"good", "bad"}\}$, which I refer to as the breadwinner's *status* or *status label*. A generation's status label $x_t \in X_t$ is determined through a function $\tau : X_{t-1} \times F_t \times B_t \rightarrow X_t$. τ specifies the status label of generation t in the next period, $\tau(x_{t-1}, f_t, b_t) \in X_t$, when the previous generation's status label is $x_{t-1} \in X_{t-1}$ and his current actions are $f_t \in F_t$ and $b_t \in B_t$. Because a generation's status depends on its parent's status, which in turn depends on the grandparent's status and so on, a status is a summary statistics of information of unboundedly high order. I call τ a *social norm* and a social norm is common knowledge.

A social norm τ is **family reciprocity** if

$$x_t = \tau(x_{t-1}, f_t, b_t) = \begin{cases} \text{"bad"} & \text{if} \begin{cases} x_{t-1} = \text{"good" and } b_t < 1 \\ \text{or} \\ f_t < 1 \end{cases} \\ \\ \text{"good"} & \text{if} \begin{cases} x_{t-1} = \text{"bad" or } b_t \geq 1 \\ \text{and} \\ f_t \geq 1 \end{cases} \end{cases}$$

and $x_0 = $ "good". Individuals are considered "bad" only when they did not take good care of their parents whose label is "good" or their children.

Generation t is labeled by a social norm τ and the community informs the succeeding generation $t + 1$ of t's status label x_t; that is, the status label of the elderly dependent is known to the breadwinner at period $t + 1$. In particular, the breadwinner's action choice is a function of the previous generation's status label.

The history of intergenerational transfers may not be known; it becomes known to each generation only to the extent that it is reflected in the status labels of the elderly

dependent.[4] A pure (Markov) strategy for a generation t is a function $s_t : X_{t-1} \rightarrow F_t \times B_t$ specifying a choice of action $s_t(x_{t-1})$ when the previous generation's status label is $x_{t-1} \in X_{t-1}$.[5]

To examine if the efficient amount of transfers, $(b, f) = (1, 1)$, can be sustained with a social norm, I first define equilibrium. My equilibrium definition here slightly differs from the last one in the information possessed by the decision makers. Because past actions do not directly affect current or future utility, I do not have to deal with any beliefs regarding the histories.

Definition 2.3. A strategy profile $(s_t)_{t=1}^{\infty}$ is a sequentially rational equilibrium if

$$\forall t, \ \forall x_{t-1} \in X_{t-1}, \ \forall f_t \in F, \ \forall b_t \in B, \quad u(s_t, s_{t+1}|x_{t-1}) \geq u(f_t, b_t, s_{t+1}|x_{t-1})$$

where

$$u(f_t, b_t, s_{t+1}|x_{t-1}) = u[f_t, b_t, s_{t+1}(\tau(f_t, b_t, x_{t-1}))]$$

and

$$u(s_t, s_{t+1}|x_{t-1}) = u[s_t(x_{t-1}), s_{t+1}(\tau(s_t(x_{t-1}), x_{t-1}))].$$

Next, I show that the efficient amount of transfers, $(f, b) = (1, 1)$, can be sustained in a sequentially rational equilibrium if a social norm is family reciprocity and individuals play a tit-for-tat strategy. I say that a strategy profile $(s_t)_{t=1}^{\infty}$ is a **tit-for-tat strategy** if

$$s_t(x_{t-1}) = \begin{cases} (1, 1) & \text{if } x_{t-1} = \text{``good''} \\ (0, 0) & \text{if } x_{t-1} = \text{``bad''}. \end{cases}$$

Theorem 2.2. *In a community whose social norm is family reciprocity, an efficient level of intergenerational transfers can be sustained as a sequentially rational equilibrium by a tit-for-tat strategy.*

Proof. Each generation is labeled by a social norm τ, i.e., $x_t = \tau(x_{t-1}, f_t, b_t)$ for all t.

---

[4] Individuals may not know labels of their grandparents or more distant ancestors.

[5] It is assumed that individuals cannot observe the amount of FIGs they receive.

Consider a community whose social norm is family reciprocity; that is, the breadwinner has to take care of both his parent and child to be labeled "good" if his parent is "good".

Suppose that each generation plays a tit-for-tat strategy. Take an arbitrary period t. When $x_{t-1} = $ "good", generation t's utility is

$$u(s_t, s_{t+1}|x_{t-1}) = u(1,1)$$
$$u(f_t, b_t, s_{t+1}|x_{t-1}) = u(f_t, b_t, 0) \quad \text{if } f_t \neq 1 \text{ or } b_t \neq 1$$

where $u((1,1),(1,1)) > u((f_t, b_t),(0,0))$ for all $(f_t, b_t) \neq (1,1)$. When $x_{t-1} = $ "bad", generation t's utility is

$$u(s_t, s_{t+1}|x_{t-1}) = u(0,1)$$
$$u(f_t, b_t, s_{t+1}|x_{t-1}) = u(f_t, b_t, 1) \quad \text{for all } (f_t, b_t) \neq (1,1)$$

where $u((0,0),(1,1)) \geq u((f_t, b_t),(1,1))$. Hence,

$$\forall t, \ \forall x_{t-1} \in X_{t-1}, \ \forall (f_t, b_t) \neq (1,1), \ \ u(s_t, s_{t+1}|x_{t-1}) \geq u(f, b, s_{t+1}|x_{t-1}).$$

$\square$

Theorem 2.2 establishes that in a community whose social norm is family reciprocity, an efficient level of intergenerational transfers can be sustained as a sequentially rational equilibrium by a tit-for-tat strategy. This result contrasts sharply with the case without a local community (Theorem 2.1) in which positive amount of transfers cannot be supported by pure-strategy equilibrium.

The intuition is as follows. In the basic model, intergenerational cooperation is not sustainable because of the informational advantage of the older generation over the younger generation (Theorem 2.1). A close-knit community enables intergenerational cooperation because it nullifies the informational advantage of the older generation. Because a status label is a summary statistic of information of unboundedly high orders, it fills the information gap between the older generation and the younger generation so that the older generation cannot manipulate. Thus, by incorporating a local community, this work

demonstrates that the conclusion of the impossibility of intergenerational cooperation under limited information does not persist.

By Theorems 2.1 and 2.2, intergenerational cooperation is sustained in a close-knit community. Depending on social norms, there are three types of equilibria: one type is that no intergenerational transfers are made; another is that only BIGs are made; the other is that both BIGs and FIGs are made. For FIGs to be sustainable, BIGs are needed because they give a breadwinner incentives to make transfers. Local gossip by neighbors (or extended families) serve to facilitate cooperation between generations.

2.3.3   Erroneous Labeling

Subsections 2.3.1 and 2.3.2 assume that the reputation in a local community is always correct. That is, it is assumed that neighbors (1) can see how another neighbor treats his parent and (2) truthfully report what they observed. However, in reality, one may get a good or bad reputation that he does not deserve. In this subsection, I consider the consequences of relaxing those assumptions.

Reputations can be incorrect for different reasons. One is due to errors in neighbors' observations. For example, an individual may die early despite all his child's effort. In this case, their neighbors may think his child neglected him. Or, an individual may want his neighbors to think that he has a happy life and thus pretend as if his son is dutiful. In this case, their neighbors may believe that his son is a good son. The other type of erroneous reputation is a result of misreport by neighbors. Even if neighbors observe family affairs correctly, they may not truthfully report their observation.

Consider the same community described in Subsection 2.3.2. For simplicity, I focus on BIGs and do not consider FIGs in this section. I define the social norm with noisy reputations as follows. A social norm ν is **family reciprocity with noisy reputations**

if

$$\nu(x_{t-1}, b_t) \equiv (\Pr(x_t = \text{``good''}|x_{t-1}, b_t), \Pr(x_t = \text{``bad''}|x_{t-1}, b_t))$$

$$= \begin{cases} (p, 1-p) & \text{if } x_{t-1} = \text{``bad''} \text{ or } b_t \geq 1 \\ (1-q, q) & \text{if } x_{t-1} = \text{``good''} \text{ and } b_t < 1. \end{cases}$$

and $x_0 = \text{``good''}$.

Next, I examine whether intergenerational transfers can be sustained if reputations are noisy and the social norm is family reciprocity. Suppose that each generation plays a tit-for-tat strategy $(s_t)_{t=1}^{\infty}$. At period t, if $x_{t-1} = \text{``good''}$, generation t's utility is

$$u(b_t = 1, s_{t+1}|x_{t-1}) = pu(b_t = 1, b_{t+1} = 1) + (1-p)u(b_t = 1, b_{t+1} = 0)$$
$$u(b_t = 0, s_{t+1}|x_{t-1}) = (1-q)u(b_t = 0, b_{t+1} = 1) + qu(b_t = 0, b_{t+1} = 0).$$

If $x_{t-1} = \text{``bad''}$, generation t is labeled as "good" regardless of how he treats his parent. Thus, it is optimal for him to take $b_t = 0$. Therefore, a tit-for-tat strategy is a Nash equilibrium if and only if

$$u(b_t = 1, s_{t+1}|x_{t-1} = \text{``good''}) \geq u(b_t = 0, s_{t+1}|x_{t-1} = \text{``good''})$$
$$\Leftrightarrow p[u(b_t = 1, b_{t+1} = 1) - u(b_t = 1, b_{t+1} = 0)] + q[u(b_t = 0, b_{t+1} = 1) - u(b_t = 0, b_{t+1} = 0)]$$
$$\geq u(b_t = 0, b_{t+1} = 1) - u(b_t = 1, b_{t+1} = 0).$$

In other words, the intergenerational cooperation is sustainable in pure strategy Nash equilibrium if reputations are sufficiently accurate, i.e., p and q are sufficiently high.

The results above have several implications. First, the intergenerational cooperation is sustainable in a pure strategy Nash equilibrium when reputations are sufficiently accurate. If the reputation accuracy p is low, one may think that it is not likely to be recognized if he takes good care of his parent. Thus, he decides not to take care of the parent in equilibrium. Second and more importantly, with some modification of social norms, the maximum amount of sustainable transfers increases with the reputation accuracy because the noise makes intergenerational cooperation a risky investment. This argument is demonstrated in the following case.

Define payoff functions of the breadwinner and his parent of the stage game as $v : B \rightarrow$

$\mathbb{R}$ and $w : B \to \mathbb{R}$, respectively. Functions v and w are strictly concave and continuously differentiable. Function v is decreasing in b while function w is increasing in b. Let δ denote a discount rate, which is common to all generations. Then, generation t's utility function is denoted as $u(b_t, b_{t+1}) \equiv v(b_t) + \delta w(b_{t+1})$. Consider another variant of family reciprocity. A social norm ν_{b^*} is **family reciprocity with b^*** if

$$
\begin{aligned}
\nu_{b^*}(x_{t-1}, b_t) \quad &\equiv (\Pr(x_t = \text{``good''}|x_{t-1}, b_t), \Pr(x_t = \text{``bad''}|x_{t-1}, b_t)) \\
&= \begin{cases} (p, 1-p) & \text{if } x_{t-1} = \text{``bad'' or } b_t \geq b^* \\ (1-q, q) & \text{if } x_{t-1} = \text{``good'' and } b_t < b^*. \end{cases}
\end{aligned}
$$

and $x_0 = \text{``good''}$ where b^* represents the amount of transfers which is considered to be appropriate from a commonsense perspective. For the convenience, consider the case of $b^* > 0$.

For simplicity, suppose that the probabilities of mislabeling are identical, i.e., $p = q$. Then, from (1), intergenerational cooperation is sustainable under family reciprocity with b^* if and only if

$$
p \geq \frac{u(0, b^*) - u(b^*, 0)}{[u(b^*, b^*) - u(b^*, 0)] + [u(0, b^*) - u(0, 0)]}.
$$

Lemma 2.2. $f(b^*) \equiv \frac{u(0, b^*) - u(b^*, 0)}{[u(b^*, b^*) - u(b^*, 0)] + [u(0, b^*) - u(0, 0)]}$ *is continuous and strictly increasing in b^*.*

Proof. $f'(b^*) > 0$ is equivalent to $\frac{v'_m(b^*)}{v_m(b^*) - v_m(0)} > \frac{v'_o(b^*)}{v_o(b^*) - v_o(0)}$. Since v_m is strictly concave and decreasing, the lefthand side is greater than 1. Similarly, because v_o is strictly concave and increasing, the righthand side is less than 1. Hence, $f'(b^*) > 0$ holds for all $b^* > 0$. $\square$

Lemma 2.3. *If intergenerational cooperation is sustainable under family reciprocity with b^*, it is sustainable under any family reciprocity with $b \leq b^*$.*

Proof. Take arbitrary p. Because intergenerational cooperation is sustainable under family reciprocity with b^*, $p \geq f(b^*)$. Because $f()$ is strictly increasing (Lemma 2.2), $f(b^*) > f(b)$ for all $b < b^*$. Hence, $p \geq f(a)$ for all $b < b^*$. $\square$

Theorem 2.3. *The maximum amount of sustainable transfers $\hat{b}^*$ (> 0) is increasing in p*

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Proof. Take arbitrary $p > f(0)$. Because f is strictly increasing and continuous (Lemma 2.2), there exists a unique $\hat{b}^* > 0$ such that $p = f(\hat{b}^*)$. Because of Lemma 2.3, it implies that $\hat{b}^* \equiv \arg\max_{b^*}\{f(b^*)|f(b^*) \leq p\}$. Because f is strictly increasing and continuous (Lemma 2.2), its inverse function f^{-1} is also strictly increasing and continuous. Hence, the maximum amount of sustainable transfers $\hat{b}^* = f^{-1}(p)$ is increasing in p. $\qquad\square$

Thus, Theorem 2.3 shows that the maximum amount of sustainable transfers $\hat{b}^*$ (> 0) is increasing in the accuracy of reputations.

Lastly, with sufficiently high p and q, the probability of erroneous labeling decreases with the reputation accuracy p on the equilibrium path while not with q. This result indicates that the societies in which people tend to speak well of others support higher welfare than those in which people tend to speak ill of others because a "good" person may be labeled "bad" in the latter societies but not in the former societies in equilibrium.

2.3.4   Different Social Norms

When the treatment of a "bad" parent is controversial

In some societies, children are encouraged to take care of their parents even if their parents are not good people. Suppose that the treatment of the "bad" parent is controversial and, thus, the label can be "good" or "bad" when one neglects a "bad" parent. This situation is in sharp contrast with the previous setting in which uncertainty in labeling exists regardless of the parent's label.

This community appears to encourage people to be nice to their parents so that they will surely be regarded as "good." Consider a community whose social norm is a variant of family reciprocity. Suppose that one is sometimes given a wrong label in such a way that he may be labeled "bad" when he does not treat his "bad" parents well. This is an erroneous labeling because, according to family reciprocity, one is supposed to be labeled "good" regardless of one's behavior if the parent is "bad". This community appears to encourage people to be nice to their parents so that they will surely be regarded as good.

A variant of family reciprocity in this setting is described as follows.

$$
\begin{aligned}
\nu(x_{t-1}, b_t) \quad &\equiv (\mathrm{Prob}(x_t = \text{``good''}|x_{t-1}, b_t), \mathrm{Prob}(x_t = \text{``bad''}|x_{t-1}, b_t)) \\
&= \begin{cases} (1,0) & \text{if } b_t \geq 1 \\ (p, 1-p) & \text{if } x_{t-1} = \text{``bad''} \ \& \ b_t < 1 \\ (0,1) & \text{if } x_{t-1} = \text{``good''} \ \& \ b_t < 1. \end{cases}
\end{aligned}
$$

According to the social norm ν, one is considered "good" with probability 1 when he takes care of his parents regardless of the parents' label, "good" with probability p when he does not take care of the "bad" parents, and "bad" with probability 1 when he does not take care of the "good" parents. That is, the community mistakenly regards one as "bad" with probability $1 - p$ when he does not take care of the "bad" parents.

Next, I examine whether intergenerational transfers can be sustained in a pure Nash equilibrium in this community. Suppose that each generation plays a tit-for-tat strategy. $(s_t)_{t=1}^{\infty}$. Take an arbitrary period t. If $x_{t-1} = $ "good", no uncertainty exists and it is optimal for generation t to take $b_t = 1$. If $x_{t-1} = $ "bad", generation t's utility is

$$
u(b_t = 0, s_{t+1}|x_{t-1}) = pu(b_t = 0, b_{t+1} = 1) + (1-p)u(b_t = 0, b_{t+1} = 0)
$$
$$
u(b_t = 1, s_{t+1}|x_{t-1}) = u(b_t = 1, b_{t+1} = 1).
$$

Hence, if and only if $p \geq \frac{u(1,1)-u(0,0)}{u(0,1)-u(0,0)}$, a tit-for-tat strategy is an equilibrium. intergenerational cooperation is sustainable if and only if uncertainty $1 - p$ is sufficiently small. This setting appears to encourage individuals to take good care of their parents because they will surely be labeled "good" as long as they take good care of their parents but they may be mistakenly labeled "bad" if they do not. However, it also discourages individuals from doing so because they expect that their children will be also encouraged to take care of them anyway. This result suggests that the social norms that stress that one should take care of his parent may actually discourage individuals from doing so.

Norms of Punishment

So far, I assumed a community whose social norm is family reciprocity. According to

family reciprocity, an individual does not have to punish his "bad" parents to be labeled "good". When their parents are "bad," an individual will be labeled "good" regardless of his behavior. Norms of family reciprocity require only that one should make appropriate amount of transfers when his parent is "good". By contrast, some social norms may encourage punishment but not reward. I consider the social norm that requires only that one should NOT make transfers when one's parent is "bad". That is, one does not have to make transfers to the "good" parent to be labeled "good". This social norm is described as follows.

$$
\begin{aligned}
\nu(x_{t-1}, b_t) \quad &\equiv (\text{Prob}(x_t = \text{"good"}), \text{Prob}(x_t = \text{"bad"})) \\
&= \begin{cases} (0,1) & \text{if } x_{t-1} = \text{"bad" and } b_t > 0 \\ (1,0) & \text{otherwise} \end{cases}
\end{aligned}
$$

With the norm of punishment, the only pure Nash equilibrium is such that no generation makes any intergenerational transfers. Combined with the case of family reciprocity, it suggests that the norm of rewarding is necessary while the norm of punishment is not. As a natural extension, social norms that combine family reciprocity and punishment also support intergenerational transfers in pure strategy Nash equilibrium by a tit-for-tat strategy.[6]

2.4   Conclusion

This paper investigated how the family works as a provider of social protection. More specifically, it argues that differences in the level of transfers within the family are due to demographic variation in community networks. To analyze the sustainability of intergenerational transfers, I propose game theoretical models of overlapping generations in which the breadwinner makes transfers to the parent and child. A novel feature of the models

---

[6] The combination of family reciprocity and punishment indicates that one should take care of the "good" parent and punish the "bad" parent:

$$
\begin{aligned}
\chi_t \quad &= (\text{Prob}(x_t = G | x_{t-1}, a_t), \text{Prob}(x_t = B | x_{t-1}, a_t)) \\
&= \tau(x_{t-1}, a_t) \\
&= \begin{cases} (1,0) & \text{if } (x_{t-1}, a_t) = (G, 1), (B, 0) \\ (0,1) & \text{if } (x_{t-1}, a_t) = (G, a_t) \forall a_t \neq 1, (B, a_t) \text{ for all } a_t \neq 0 \ . \end{cases}
\end{aligned}
$$

is that there is a local community that may supply information about its members' past behavior. I demonstrate that intergenerational transfers can be sustained if neighbors gossip about each other. Furthermore, I also demonstrate that the maximum amount of sustainable transfers decreases as labeling becomes noisier. As an implication, my theory suggests that individuals in a close-knit community prefer lower levels of social protection.

As a theoretical extension, it is interesting to introduce negative economic shocks and analyze its effects on robustness of equilibrium. Suppose that negative economic shocks may occur and reduce the breadwinners ability of making transfers. The effects of shocks may be different, depending on the class of strategy profiles.

First, consider the strategy profiles in which a breadwinner will not be punished if he fails to make enough transfers because of a negative economic shock. In this type of strategy profiles, the models without a local community have a problem of imperfect monitoring whereas those with a local community don't if economic shocks are correlated.

Suppose that a negative economic shock occurs and a breadwinner cannot make enough transfers to his parent or child. In the OLG model without a local community, his offspring does not know why his father does not make transfers, i.e., whether there is a negative economic shock or his father is cheating. Therefore, even if every generation knows the entire family history (how much transfers the preceding generations made), this imperfect monitoring makes it difficult for the offspring to distinguish economic shocks and cheating.

On the other hand, in OLG models with a local community, the local community may solve the issue of imperfect monitoring. Consider a local community in which multiple families reside. First, suppose that economic shocks are uncorrelated among families. In this case, the local community does not play a role for the offspring. The equilibrium with positive transfers becomes less sustainable when there are economic shocks. Second, suppose that economic shocks are correlated among families. The breadwinners in the community work in the same industry, for example. In this case, an offspring may notice if many people of his fathers generation did not pay make transfers. Hence, the effect of economic shocks may be mitigated when the shocks are correlated in the community.

Second, consider the strategy profiles in which a breadwinner's not making enough

transfers is equally punished as cheating. In this case, there is no difference between OLG models without a local community and those with a local community. In both models, the robustness depends on the strategy profiles, whether economic shocks are correlated or uncorrelated.

Suppose that a breadwinner deviates, whether due to economic shocks or cheating. In a variation of tit-for-tat strategy, the next generation can bring his family back to the equilibrium path by punishing his father. However, in a variation of grim-trigger strategy, all the following generations will punish their parent generations once someone deviates and the family cannot go back to the coordination path.

Therefore, the effects of economic shocks may be mitigated only in the former strategy profiles:In the former strategy profiles, having a local community can solve the problem of imperfect monitoring if economic shocks are correlated among families; In the latter strategy profiles, the effects of economic shocks do not depend on a local community.

3. A THEORY OF HUNG JURIES AND INFORMATIVE VOTING

3.1 Introduction

In many jurisdictions in the United States and elsewhere, unanimity among jurors is required for jury verdict. The unanimity rule is commonly believed to minimize the possibility of convicting an innocent defendant. This view is challenged by **?** in light of game-theoretic analysis of voting behavior. They show that, if jurors vote strategically, the unanimity rule may convict the innocent and acquit the guilty more often than most rules, including the simple majority rule.

? provides a counterargument to their claim in his study of jury decision with mistrials. He points out that unanimity is required for either conviction or acquittal in many jurisdictions. Otherwise a hung jury occurs and the case faces a new trial in the future with a new group of jurors.[1] In his model jurors in a subsequent trial possess the same prior belief on the defendant's guilt as those in previous trials, and he argues that informative voting is an equilibrium for a nontrivial range of parameters.

The above model, however, does not explicitly allow information transmission between trials. By contrast, information is often disclosed between trials in reality. For example, news media regularly covers trials and information about mistrials is frequently reported.

To capture this feature, this paper allows jurors to know that previous trials (if any) resulted in hung juries and use this fact to infer the likelihood of guilt or innocence of the defendant. For any voting rule, we show that informative voting is an equilibrium only in knife-edge cases where the probability that a juror receives the correct signal when the defendant is guilty is exactly the same as the one when the defendant is innocent.

---

[1] A mistrial may be declared for a number of other reasons, such as juror misconduct. However we focus on a mistrial that occurs because of a hung jury and use these terms interchangeably.

Although previous literature such as **?** has argued that mistrials facilitate informative voting, our result demonstrates that such an argument has only limited applicability.

In addition to mistrials, **?** presents another model to justfy unanimity rule. He introduces a model with deliberation and shows that informative voting is an equilibrium for a nontrivial range of parameters. **?** further assume uncertainty about juror preferences. They show that informative voting is not an equilibrium under the unanimity rule if jurors have minimal heterogeneity in preferences, while it may be an equilibrium under nonunanimous rules. Both their paper and ours question the efficacy of the unanimity rule in realistic models: deliberation in their paper and hung juries in ours.

The current work is part of the literature on jury design under strategic voting pioneered by **?**. They study the strategic aspect of jury decision and show that informative voting often fails to be an equilibrium. **?** and (**?**) show that there exists an equilibrium, which may not be informative, that aggregates jurors' information in an adequate manner. **?** and **?** consider continuous signals. **?** find experimental evidence of strategic voting in jury setting. Study of Condorcet Jury theorems has a long tradition in a more statistically oriented literature (**?**, **?**, **?**).

3.2   Model and Result

A defendant is under a jury trial on a criminal charge. There are two states of the world $\omega \in \{G, I\}$: the defendant is either guilty (denoted G) or innocent (I). The prior probabilities that the state is G and I are $r \in (0, 1)$ and $1 - r$, respectively.

At each period $t = 0, 1, 2, \ldots$, a jury composed of n jurors is formed. The jury at t makes a decision $d_t \in \{A, C, M\}$ following a voting rule specified by an integer $\hat{k} \in (n/2, n]$. At each period t, each juror j casts a vote $v_j \in \{a, c\}$, where a is a vote for acquittal and c is a vote for conviction. If at least $\hat{k}$ jurors vote for conviction, then the defendant is convicted ($d_t = C$); if at least $\hat{k}$ jurors vote for acquittal, then the defendant is acquitted ($d_t = A$); if neither of these happens, then a mistrial is declared ($d_t = M$). When a mistrial is declared, a new jury (with new members) is formed at period $t + 1$ and the same procedure is repeated. Once the jury reaches a verdict ($d_t \in \{A, C\}$) at any t, it

becomes the final decision $d_\infty = d_t \in \{A, C\}$. We denote the jury at period t by N^t. For example, the first jury is denoted by N^0, the jury after one mistrial by N^1, and so on.

Each juror j receives private signal $s_j \in \{g, i\}$ correlated with the state of the world $\omega \in \{G, I\}$. Given ω, s_j follows an i.i.d. distribution. Let p_g (respectively p_i) represent the probability that each juror observes the "correct signal" g (respectively i) when the true state is G (respectively I). We assume $p_g, p_i \in (1/2, 1)$.

The utility of juror j when the state is ω and a final decision $d_\infty \in \{C, A\}$ is made is denoted by $u_j(d_\infty, \omega)$, and defined as $u_j(C, G) = u_j(A, I) = 0$, $u_j(C, I) = -q_j$, $u_j(A, G) = -(1 - q_j)$, where $q_j \in (0, 1)$. When an infinite sequence of mistrials occurs, each juror receives some fixed utility.[2] Juror j prefers conviction to acquittal if and only if she places at least probability q_j that the defendant is guilty. We assume that $\bar{q} \equiv \sup_{j \in \bigcup_{t=0}^\infty N^t} q_j < 1$ and $\underline{q} \equiv \inf_{j \in \bigcup_{t=0}^\infty N^t} q_j > 0$. In words, while we allow for an infinite number of potential jurors, levels of "reasonable doubt" for the population is bounded away from the extreme values, 0 and 1.[3]

All jurors in the tth trial know there were $t - 1$ trials before and all of them resulted in mistrials, but they do not know any further information from previous trials. We assume that there is at least one numerical split under which a mistrial is declared. This assumption holds except for simple majority rule with n odd.

A strategy of each juror j is a function which assigns a vote v_j for each private signal s_j. We say that a strategy profile is an **informative voting** if each juror j chooses c if $s_j = g$ and a if $s_j = i$.

When the signals for guilt and innocence are equally accurate, that is, $p_g = p_i = p$ for some $p \in (1/2, 1)$, a straightforward adaptation of **?** shows that informative voting forms

---

[2] Since our analysis focuses on informative voting, infinite mistrials occur with probability zero. Hence no specific assumption on the utility for infinite mistrials is important for the analysis. Also we note that retrials may be costly and prosecutors may decide not to seek a retrial after a mistrial with some probability in reality, but our result carries over when we introduce these additional complications.

[3] This assumption is satisfied quite broadly. One example is a model in which the parameter q_j is chosen from a finite set of possible values in $(0, 1)$. A stationary case in which the utility characteristic of each jury is the same for every period is a particular case of such a specification. Even if the population of potential jurors does not satisfy the bound, jury selection procedures (often called voir dire) may render the bound plausible, since the processes prevent highly biased individuals from serving in juries.

a sequential equilibrium if and only if, for each juror j,

$$\frac{r}{1-r} \cdot \frac{1-p}{p} \leq \frac{q_j}{1-q_j} \leq \frac{r}{1-r} \cdot \frac{p}{1-p}.$$

Next, consider the case with $p_g \neq p_i$.

Theorem 3.1. *Suppose $p_g \neq p_i$. Then informative voting never forms a sequential equilibrium.*

We defer the proof to the Appendix and offer an intuition here. Suppose, for instance, that the signal of guilt is more accurate than that of innocence $(p_g > p_i)$. Then the jurors are more likely to receive mistaken signals and, as a result, fail to agree on a verdict when the defendant is innocent than when he is guilty (Lemma 1 in the Appendix). Hence, if a juror knows that there was a hung jury before the current trial, she infers that the defendant is more likely to be innocent by Bayes' law. If hung juries occur repeatedly, information from previous trials will become so strong that a juror is willing to vote for acquittal even if she observes a guilty signal. Thus informative voting fails to be an equilibrium.

Theorem 3.1 suggests that informative voting is rarely an equilibrium. Note that $p_g \neq p_i$ holds generically. Moreover, the Theorem applies to any voting rule which allows for a hung jury (i.e., the only exception is simple majority rule with n odd). Thus the defense of unanimity rule by Coughlan is inapplicable to most cases.

The conclusion of Theorem 3.1 can be strengthened if additional information is available to jurors. For example, it can be shown that informative voting is not an equilibrium regardless of probabilities of the correct signal if jurors are informed of numerical splits of votes in previous trials.[4]

---

[4] The proof is omitted, but is available from the authors upon request.

APPENDIX

A. PROOFS FOR CHAPTER 1

A.1   Proof of Lemma 1.1

Consider $w \leq n - 2$.

Part 1

We first examine the weighted juror's incentive. Let p' denote the accuracy of unweighted jurors' signals where $p' \in \{p_H, p_L\}$. Then, the probability that the weighted juror becomes pivotal when the defendant is guilty is

$$
\Pr(\text{piv}_1 | G, w, \mathbf{t}_{-1}) = \sum_{\nu = \frac{n-w}{2}}^{\frac{n+w}{2} - 1} \binom{n-1}{\nu} p'^{\nu} (1 - p')^{n-1-\nu},
$$

which is increasing in $w \in \{1, 3, 5, 7, \ldots, n-2\}$.[12] Note that this probability is independent of the weighted juror's signal accuracy. Thus, regardless of her and unweighted jurors' investment behavior, the weighted juror is more likely to be pivotal as w increases.

Part 2

Second, I examine an unweighted juror's incentive. Take an arbitrary unweighted juror j. The probability that an unweighted juror becomes pivotal decreases as w increases if and only if $\Pr(\text{piv}_j | G, w + 2) < \Pr(\text{piv}_j | G, w)$. Suppose that the weighted juror's signal

---

[1] $\sum_{\nu = \frac{n-w}{2}}^{\frac{n+w}{2} - 1} \binom{n-1}{\nu} p'^{\nu} (1 - p')^{n-1-\nu}$ is increasing in w because $\frac{n-w}{2}$ is decreasing in w and $\frac{n+w}{2} - 1$ is increasing in w.

[2] $\mathbf{t}_{-1} := \mathbf{t}_j$ for $j \neq 1$

accuracy is $p \in \{p_H, p_L\}$ and unweighted jurors' signal accuracy is $p' \in \{p_H, p_L\}$.

$\Pr(\text{piv}_j|G, w+2) < \Pr(\text{piv}_j|G, w)$

$\Leftrightarrow p\binom{n-2}{\frac{n-w}{2}-2}p'^{\frac{n-w}{2}-2}(1-p')^{\frac{n+w}{2}} + (1-p)\binom{n-2}{\frac{n+w}{2}}p'^{\frac{n+w}{2}}(1-p')^{\frac{n-w}{2}-2}$

$< p\binom{n-2}{\frac{n-w}{2}-1}p'^{\frac{n-w}{2}-1}(1-p')^{\frac{n+w}{2}-1} + (1-p)\binom{n-2}{\frac{n+w}{2}-1}p'^{\frac{n+w}{2}-1}(1-p')^{\frac{n-w}{2}-1}$

$\Leftrightarrow \binom{n-2}{\frac{n+w}{2}}\left\{ pp'^{\frac{n-w}{2}-2}(1-p')^{\frac{n+w}{2}} + (1-p)p'^{\frac{n+w}{2}}(1-p')^{\frac{n-w}{2}-2}\right\}$

$< \binom{n-2}{\frac{n+w}{2}-1}\left\{ pp'^{\frac{n-w}{2}-1}(1-p')^{\frac{n+w}{2}-1} + (1-p)p'^{\frac{n+w}{2}-1}(1-p')^{\frac{n-w}{2}-1}\right\}$

Multiply by $(1-p)^{-1}p'^{\frac{-n+w}{2}+1}(1-p')^{-\frac{n+w}{2}+1}$,

$\Leftrightarrow (n-w-2)\cdot\left\{\frac{p}{1-p}\cdot\frac{1-p'}{p'} + \left(\frac{p'}{1-p'}\right)^{w+1}\right\} < (n+w)\cdot\left\{\frac{p}{1-p} + \left(\frac{p'}{1-p'}\right)^{w}\right\}$

Let $P := \frac{p}{1-p}$, $P' := \frac{p'}{1-p'}$,

$\Leftrightarrow (n-w-2)\cdot\left(P\cdot P'^{-1} + P'^{w+1}\right) < (n+w)\cdot\left(P + P'^{w}\right)$

$\Leftrightarrow n\left(P\cdot P'^{-1} + P'^{w+1} - P - P'^{w}\right) < w\left(P + P'^{w}\right) + (w+2)\left(P\cdot P'^{-1} + P'^{w+1}\right)$

$\Leftrightarrow n(1 - P'^{-1})(P'^{w+1} - P) < w(1 + P'^{-1})(P'^{w+1} + P) + 2\left(P\cdot P'^{-1} + P'^{w+1}\right)$

$\Leftrightarrow \begin{cases} n < w\cdot\frac{(1+P'^{-1})(P'^{w+1}+P)}{(1-P'^{-1})(P'^{w+1}-P)} + 2\cdot\frac{P\cdot P'^{-1}+P'^{w+1}}{(1-P'^{-1})(P'^{w+1}-P)} & \text{if } P'^{w+1} > P \\ \text{holds for any n, w} & \text{if } P'^{w+1} \leq P \end{cases}$

By $N(P, P', w) := w\cdot\frac{(1+P'^{-1})(P'^{w+1}+P)}{(1-P'^{-1})(P'^{w+1}-P)} + 2\cdot\frac{P\cdot P'^{-1}+P'^{w+1}}{(1-P'^{-1})(P'^{w+1}-P)}$,

$\Leftrightarrow \begin{cases} n < N(P, P', w) & \text{if } P'^{w+1} > P \\ \text{holds for any n, w} & \text{if } P'^{w+1} \leq P \end{cases}$

Thus, the probabilities that unweighted jurors become pivotal are:

(1) if $P'^{w+1} > P$,

    (a) decreasing in w for $n < N(P, P', w)$

    (b) independent of w for $n = N(P, P', w)$

    (c) increasing in w for $n > N(P, P', w)$

(2) if $P'^{w+1} \leq P$, decreasing in w for all n

A.2 Proof of Proposition 1.1

Recall that a juror j has an incentive to invest in her information if and only if $c \leq f_j(w, \mathbf{t}_{-j})$. Define $w_j(c, \mathbf{t})$ as a function $w_j : C \times \mathbf{T} \to W \cup \{-\infty, \infty\}$ as follows.

$$
w_1(c, \mathbf{t}) := \begin{cases} -\infty & \text{if } c < f_1(1, \mathbf{t}) \\ w^*(c, \mathbf{t}) & \text{if } c \in [f_1(1, \mathbf{t}), f_1(n-2, \mathbf{t})] \\ \infty & \text{if } c > f_1(n-2, \mathbf{t}) \end{cases}
$$

where $w^*(c, \mathbf{t})$ is an integer such that $f_1(w^* - 2, \mathbf{t}) < c \leq f_1(w^*, \mathbf{t})$. [3]

$$
w_j(c, \mathbf{t}) := \begin{cases} \infty & \text{if } c < f_j(n-2, \mathbf{t}) \\ w^*(c, \mathbf{t}) & \text{if } c \in [f_j(n-2, \mathbf{t}), f_j(1, \mathbf{t})] \\ -\infty & \text{if } c > f_j(1, \mathbf{t}) \end{cases}
$$

for $j \neq 1$ where $w^{**}(c, \mathbf{t})$ is an integer such that $f_j(w^{**} + 2, \mathbf{t}) < c \leq f_j(w^{**}, \mathbf{t})$. [4]

The function $w_1(c, \mathbf{t})$ describes the minimum weight for the weighted juror 1 to have an incentive to invest when the cost is c and the strategy profile is $\mathbf{t}$.[5] Thus, the weighted juror 1 has an incentive to invest upon $(w, c, \mathbf{t})$ if and only if $w \geq w_1(c, \mathbf{t})$. On the other hand, $w_j(c, \mathbf{t})$ describes the maximum weight for the unweighted juror $j \neq 1$ to have an incentive to invest when the cost is c and the strategy profile is $\mathbf{t}$. Thus, the unweighted juror j has an incentive to invest upon $(w, c, \mathbf{t})$ if and only if $w \leq w_j(c, \mathbf{t})$.[6] The function

---

[3] If $c \in [f_1(1, \mathbf{t}), f_1(n-2, \mathbf{t})]$, there is always an unique integer w^* that satisfies

$$
f_1(w^* - 2, \mathbf{t}) < c \leq f_1(w^*, \mathbf{t}).
$$

[4] If $c \in [f_j(n-2, \mathbf{t}), f_j(1, \mathbf{t})]$, there is always an unique integer w^{**} that satisfies

$$
f_j(w^* + 2, \mathbf{t}) < c \leq f_j(w^{**}, \mathbf{t}).
$$

[5] Note that $w \in \{1, 3, \ldots, n-2\}$.

[6] For simplicity, I assume that jurors invest if they are indifferent between investing and not investing.

$w^*(c, \mathbf{t})$ is increasing in c and $w^{**}(c, \mathbf{t})$ is decreasing in c.[78]

There are four possible strategy profiles, $\mathbf{t} = (1, 1, \ldots, 1)$, $(1, 0, \ldots, 0)$, $(0, 1, \ldots, 1)$, $(0, 0, \ldots, 0)$. In the following, I find (c, w) for which each strategy profile forms equilibrium. For convenience, I assume $j \neq 1$, i.e., juror j is an unweighted juror in the following part of this proof.

A) A strategy profile $\mathbf{t} = (1, 1, \ldots, 1)$ is an equilibrium if

$$(c, w) \in \{(c, w) | w_1(c, \mathbf{t}) \leq w \leq w_j(c, \mathbf{t})\}$$

where

$$w_1(c, \mathbf{t}) = \begin{cases} -\infty & \text{if } c < c_2 \\ w^*(c, \mathbf{t}) & \text{if } c \in [c_2, c_3] \\ \infty & \text{if } c > c_3 \end{cases}$$

$$w_j(c, \mathbf{t}) = \begin{cases} \infty & \text{if } c < c_1 \\ w^{**}(c, \mathbf{t}) & \text{if } c \in [c_1, c_2] \\ -\infty & \text{if } c > c_2 \end{cases}$$

for $j \neq 1$ and $\mathbf{t} = (1, 1, \ldots, 1)$.

In Figure A.1 (resp. Figure A.2), the red area represents (w, c) for which the weighted (resp. unweighted) juror has an incentive to invest in her information while the blue area represents those for which she has no incentive to do so. As a result, the black trapezoid in Figure A.3 describes $\{c, w\}$ for which $\mathbf{t} = (1, 1, \ldots, 1)$ is an equilibrium.

---

[7] The function $w^{**}(c, \mathbf{t})$ is decreasing in c because I focus on the cases where unweighted jurors are less likely to be pivotal with w.

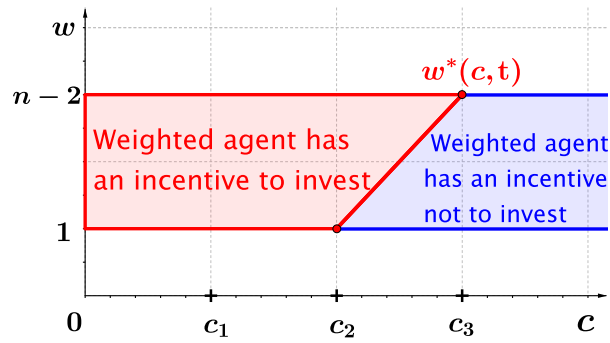[8] Note that $w^{**}(c, \mathbf{t}) \leq w^*(c, \mathbf{t})$

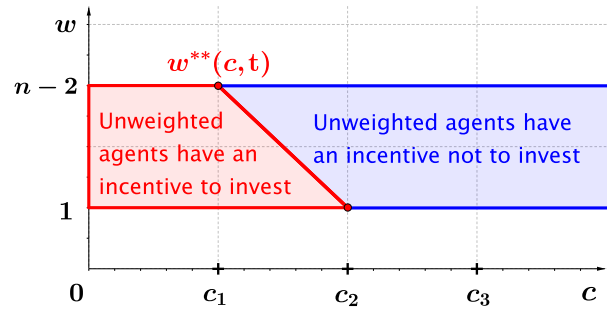*Figure A.1:* $w_1(c, \mathbf{t})$ for $\mathbf{t} = (1, 1, \ldots, 1)$

Figure A.2: $w_j(c, \mathbf{t})$ for $\mathbf{t} = (1, 1, \ldots, 1)$
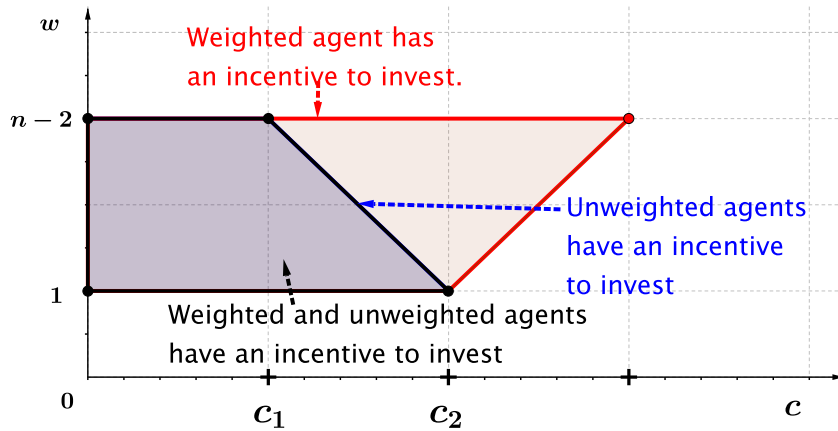

Figure A.3: $\{(c, w) | w_1(c, \mathbf{t}) \leq w \leq w_j(c, \mathbf{t})\}$ for $\mathbf{t} = (1, 1, \ldots, 1)$

B) A strategy profile $\mathbf{t} = (1, 0, \ldots, 0)$ is an equilibrium if

$$(c, w) \in \{(c, w) | w \geq w_1(c, \mathbf{t}) \;\&\; w > w_j(c, \mathbf{t})\}$$

where

$$w_1(c, \mathbf{t}) = \begin{cases} -\infty & \text{if } c < c_5 \\ w^*(c, \mathbf{t}) & \text{if } c \in [c_5, c_6] \\ \infty & \text{if } c > c_6 \end{cases}$$

$$w_j(c, \mathbf{t}) = \begin{cases} \infty & \text{if } c < c_3 \\ w^{**}(c, \mathbf{t}) & \text{if } c \in [c_3, c_4] \\ -\infty & \text{if } c > c_4 \end{cases}$$

for $j \neq 1$ and $\mathbf{t} = (1, 0, \ldots, 0)$.

In Figure A.4 (resp. Figure A.5), the red area represents (w, c) for which the weighted (resp. unweighted) juror has an incentive to invest in her information while the blue area represents those for which she has no incentive to do so and the green line represents those for which she is indifferent. As a result, the black trapezoid in Figure A.6 describes $\{c, w\}$ for which $\mathbf{t} = (1, 0, \ldots, 0)$ is an equilibrium.
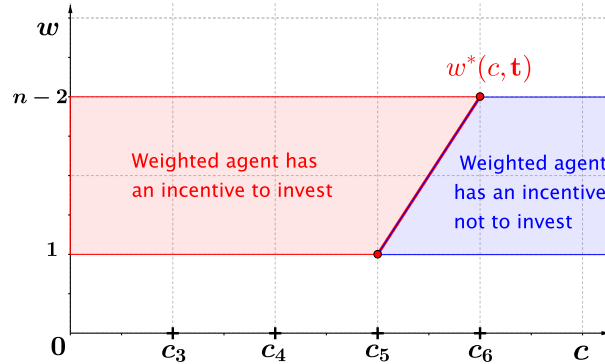
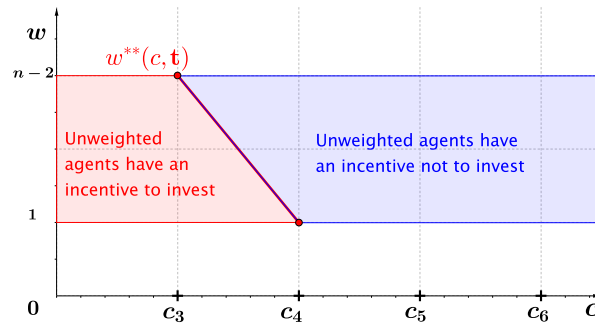*Figure A.4:* $w_1(c, \mathbf{t})$ for $\mathbf{t} = (1, 0, \ldots, 0)$

Figure A.5: $w_{-1}(c, \mathbf{t})$ for $\mathbf{t} = (1, 0, \ldots, 0)$
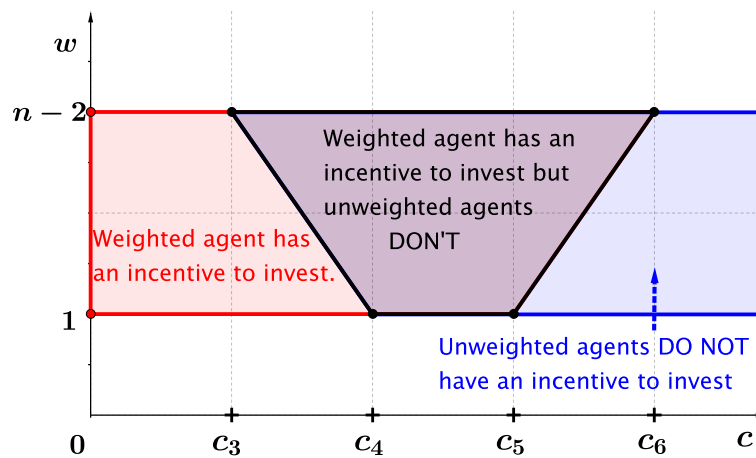

Figure A.6: $\{(c, w) | w \geq w_1(c, \mathbf{t}) \ \& \ w \geq w_j(c, \mathbf{t})\}$ for $\mathbf{t} = (1, 0, \ldots, 0)$

$C)$ A strategy profile $\mathbf{t} = (0, 0 \ldots, 0)$ is an equilibrium if and only if

$$(c, w) \in \{(c, w) | w_j(c, \mathbf{t}) < w < w_1(c, \mathbf{t})\}.$$

Similarly, the black trapezoid in Figure A.7 describes $\{c, w\}$ for which $\mathbf{t} = (0, 0, \ldots, 0)$ is an equilibrium.
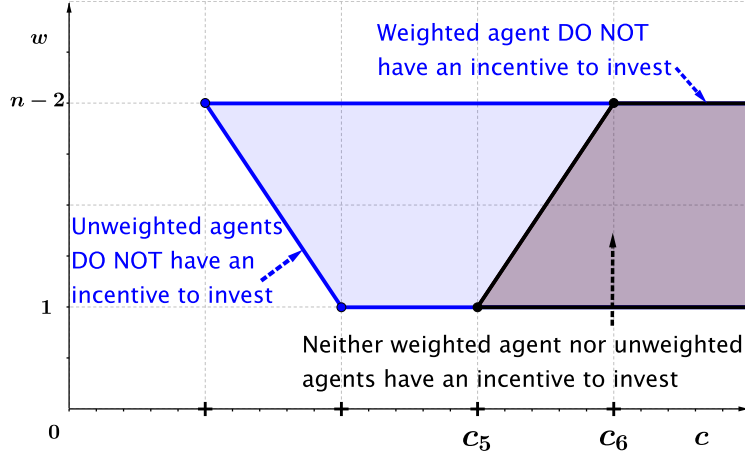

Figure A.7: $\{(c, w) | w_j(c, \mathbf{t}) \leq w \leq w_1(c, \mathbf{t})\}$ for $\mathbf{t} = (0, 0, \ldots, 0)$

$D)$ A strategy profile $\mathbf{t} = (0, 1, \ldots, 1)$ is an equilibrium if and only if

$$(c, w) \in \{(c, w) | w < w_1(c, \mathbf{t}) \ \& \ w \leq w_j(c, \mathbf{t})\},$$

which is an empty set.

Therefore, Figure A.8 represents the equilibria where $\mathbf{t} = (1, 1, \ldots, 1)$, $(1, 0, \ldots, 0)$, $(0, 0, \ldots, 0)$.

A.3 Proof of Proposition 1.2

In Step 1, I show that the probability of making a correct decision is decreasing in w and compute the optimal weight for $c \in [0, c_5] \cup (c_6, \infty)$. Next, I compute the optimal weight for $c \in (c_5, c_6]$ in Step 2.
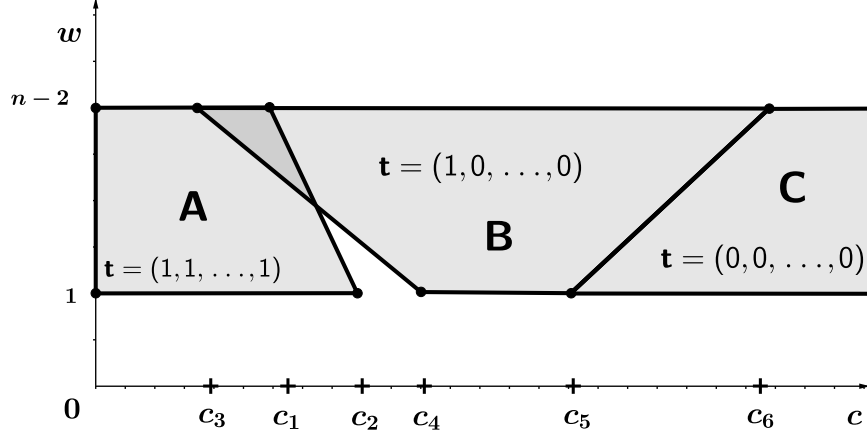
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Figure A.8: $\{(c,w)|w_1(c,\mathbf{t}) \leq w \leq w_j(c,\mathbf{t})\}$ for $\mathbf{t} = (1,1,\ldots,1)$

Step 1: $\mathbf{V(w+2,t) - V(w,t)} < 0$

First, I show that the probability of making a correct decision is decreasing in w given any investment behavior.

$$V(w+2,\mathbf{t}) - V(w,\mathbf{t})$$

$$= p \sum_{x \geq \frac{n-w}{2}-1} \binom{n-1}{x} p'^x (1-p')^{n-1-x} + (1-p) \sum_{x \geq \frac{n+w}{2}+1} \binom{n-1}{x} p'^x (1-p')^{n-1-x}$$

$$- \left(p \sum_{x \geq \frac{n-w}{2}} \binom{n-1}{x} p'^x (1-p')^{n-1-x} + (1-p) \sum_{x \geq \frac{n+w}{2}} \binom{n-1}{x} p'^x (1-p')^{n-1-x} \right)$$

$$= \binom{n-1}{\frac{n+w}{2}} p'^{\frac{n-w}{2}-1}(1-p')^{\frac{n-w}{2}-1} \left\{ p(1-p')^{w+1} - (1-p)p'^{w+1} \right\},$$

which is less than 0 for all $(p,p') \in \{p_H, p_L\} \times \{p_H, p_L\}$ because $p_H < \dfrac{\left(\frac{p_L}{1-p_L^{w+1}}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L^{w+1}}\right)^{w+1}}$.

Thus,

$$V(w+2,\mathbf{t}) - V(w,\mathbf{t}) < 0 \tag{A.1}$$

for all $\mathbf{t} \in \{(1,\ldots,1),(1,0,\ldots,0),(0,\ldots,0),(0,1\ldots,1)\}$. It follows that regardless of the qualities of signals that jurors receive, the probability of making a correct decision is decreasing in w.

Therefore, the optimal weight is $w = 1$ for $c \in [0, c_2] \cup [c_4, c_5] \cup (c_6, \infty)$ and $w = w^{**}(c, (1, 0, \ldots, 0))$ for $c \in (c_2, c_4)$. For $c \in (c_5, c_6]$, the optimal weight is either $w^*(c, (1, 0, \ldots, 0))$ where the equilibrium is $\mathbf{t} = (1, \ldots, 0)$ or 1 where the equilibrium is $\mathbf{t} = (0, 0, \ldots, 0)$.

Step 2: The optimal weight for $c \in (c_5, c_6]$

Step 1 shows the optimal weight for $c \in [0, c_5] \cup (c_6, \infty)$. Next, I compute the optimal weight for $c \in (c_5, c_6]$, which is either $w^*(c, (1, 0, \ldots, 0)$ where the equilibrium is $\mathbf{t} = (1, \ldots, 0)$ or 1 where the equilibrium is $\mathbf{t} = (0, 0, \ldots, 0)$. The optimal weight is $w^*(c, (1, 0, \ldots, 0)$ if $U(c) > 0$ and 1 if $U(c) < 0$ where

$$U(c) := V(w^*(c, (1, 0, \ldots, 0)), (1, \ldots, 0)) - V(1, (0, \ldots, 0)).$$

1) $U(c_5) > 0$

Since $w^*(c_5, (1, 0, \ldots, 0) = 1$ by construction,

$U(c_5)$

$= V(1, (1, \ldots, 0)) - V(1, (0, \ldots, 0))$

$= (p_H - p_L) \sum_{x \geq \frac{n-1}{2}} \binom{n-1}{x} p_L^x (1 - p_L)^{n-1-x} + (p_L - p_H) \sum_{x \geq \frac{n+1}{2}} \binom{n-1}{x} p_L^x (1 - p_L)^{n-1-x}$

$= (p_H - p_L) \binom{n-1}{\frac{n-1}{2}} p_L^{\frac{n-1}{2}} (1 - p_L)^{\frac{n-1}{2}} \quad > 0.$

2) $U(c)$ is decreasing in c

By (A.1), $V(w, (1, \ldots, 0))$ is decreasing in w. Since $w^*(c, (1, 0, \ldots, 0))$ is decreasing in c, $U(c)$ is also decreasing in c.

3) Optimal weight for $c \in (c_5, c_6]$

Since $w^*(c_6, (1, 0, \ldots, 0)) = n - 2$ by construction,

$$U(c_6) = V(n - 2, (1, \ldots, 0)) - V(1, (0, \ldots, 0)),$$

which may take positive or negative values, depending on the parameters, p_H, p_L, and n.[9] If $U(c_6) < 0$, there exists $c_7 \in (c_5, c_6)$ such that $U(c) = 0$. In this case, $\hat{w} = w^*(c, (1, 0, \ldots, 0))$ for $c \in (c_5, c_7]$ and $\hat{w} = 1$ for $c \in [c_7, c_6)$. If $U(c_6) \geq 0$, $\hat{w} = 1$ for $c \in [c_5, c_6)$.[10]

A.4 Future extension: more than one weighted jurors

For now, I analyzed the case in which there is at most one weighted juror, $m = 1$. For the future research, I intend to analyze the case of $m > 1$ and describe the direction here. The purpose of this future section is to characterizes the equilibria in which weighted jurors invest in information and unweighted jurors do not, and compute the optimal number of weighted jurors and their weights. Those equilibria are denoted by (m, w) and I develop the "(m, w) equilibria" where $1 < m < \frac{n+1}{w+1}$. Since I restrict our attention to symmetric equilibria and assume sincere voting, the (m, w) equilibrium is simply a pair of critical points (c_1^*, c_2^*) such that any weighted jurors invest if and only if $c < c_1^*$ and any unweighted jurors invest if and only if $c < c_2^*$.

---

[9] For example,

$$V(n - 2, (1, \ldots, 0)) - V(1, (0, \ldots, 0)) = 0.3 \quad > 0$$

for $(p_H, p_L) = (0.8, 0.5)$ and $n = 1001$ while

$$V(n - 2, (1, \ldots, 0)) - V(1, (0, \ldots, 0)) = -0.2 \quad < 0$$

for $(p_H, p_L) = (0.8, 0.7)$ and $n = 1001$.

[10]

$$U(c_6) = V(n - 2, (1, \ldots, 0)) - V(1, (0, \ldots, 0))$$

$$= p_H \left(1 - (1 - p_L)^{n-1}\right) + (1 - p_H) p_L^{n-1} - \sum_{x \geq \frac{n+1}{2}} \binom{n}{x} p_L^x (1 - p_L)^{n-x}$$

Consider the case where jurors $1, \ldots, m$ are weighted and jurors $m+1, \ldots, n$ are unweighted. A weighted juror $j \in \{1, \ldots, m\}$ has an incentive to invest conditional on the other weighted jurors investing and all the unweighted jurors not investing if and only if

$$EU_j[t_j = 1 | m, w] - EU_j[t_j = 0 | m, w] \geq 0$$

$$\Leftrightarrow \Pr(G, \mathrm{piv}_j | m, w)(p_H - p_L) - c \geq 0$$

$$\Leftrightarrow g_w(m, w) \geq c$$

where

$$g_w(m, w) := \Pr(G, \mathrm{piv}_j | m, w)(p_H - p_L) =$$

$$\frac{1}{2} \left\{ \sum_{\nu=0}^{m-1} \binom{m-1}{\nu} p_H^\nu (1-p_H)^{m-1-\nu} \sum_{\tau = \frac{n+m(w-1)+1}{2} - \nu w - w}^{\frac{n+m(w-1)+1}{2} - \nu w - 1} \binom{n-m}{\tau} p_L^\tau (1-p_L)^{n-m-\tau} \right\} (p_H - p_L)$$

for $j \in \{1, \ldots, m\}$ and $EU_j[s_j | m, w]$ is juror j's expected utility of s_j.

Similarly, an unweighted juror $j \in \{m+1, \ldots, n\}$ has an incentive not to invest if and only if

$$EU_j[s_j = 1 | m, w] - EU_j[s_j = 0 | m, w] < 0$$

$$\Leftrightarrow \Pr(G, \mathrm{piv}_j | m, w)(p_H - p_L) < c$$

$$\Leftrightarrow g_{uw}(m, w) < c$$

where

$$g_{uw}(m, w) := \Pr(G, \mathrm{piv}_j | m, w)(p_H - p_L)$$

$$= \frac{1}{2} \left\{ \sum_{\nu=0}^{m} \binom{m}{\nu} p_H^\nu (1-p_H)^{m-\nu} \binom{n-m-1}{\frac{n+m(w-1)+1}{2} - \nu w - 1} p_L^{\frac{n+m(w-1)+1}{2} - \nu w - 1} (1-p_L)^{\frac{n-m(w+1)-1}{2} + \nu w} \right\} (p_H - p_L)$$

for $j \in \{m+1, \ldots, n\}$.

We define $\bar{c}(m, w)$ by the equality $g_w(m, w) = \bar{c}(m, w)$ for $j \in \{1, \ldots, m\}$ and $\underline{c}(m, w)$ by the equality $g_{uw}(m, w) = \underline{c}(m, w)$ for $j \in \{m+1, \ldots, n\}$. An equilibrium in our model is a set of juror decision rules, $(\bar{c}, \underline{c})$ such that weighted jurors invest in information if

$c \leq \bar{c}(m, w)$ and unweighted jurors invest if $c < \underline{c}(m, w)$. Thus, the (m, w) equilibrium exists if and only if $\underline{c}(m, w) \leq c \leq \bar{c}(m, w)$. Propositions A.1 and A.2 show that jurors are less likely to be pivotal as the number of weighted jurors increases.

Proposition A.1. *Suppose that n is sufficiently large, $p_H \geq \frac{2m-1}{2m+1}$ and $p_L = \frac{1}{2}$. The probability that a weighted juror j becomes pivotal, $\Pr(G, piv_j | m, w)$, is decreasing in the number of weighted jurors, m.*

Proof. See Appendix A.5. □

Proposition A.1 gives sufficient conditions for which a weighted juror is less willing to invest in information as the number of weighted jurors increases. Similarly, Proposition A.2 is intended to give sufficient conditions for which an unweighted juror is less willing to invest in information as the number of weighted jurors increases.

Proposition A.2. *The probability that an unweighted juror j becomes pivotal, i.e., $\Pr(G, piv_j | m, w)$, is decreasing in the number of weighted jurors, m.*

Proof. The complete proof is yet to be done. Note that the following Corollary A.1 and Proposition A.3 are based on this unproven Proposition. □

As a corollary, Propositions A.1 and A.2 will give the shape of $\underline{c}(m, w)$ and $\bar{c}(m, w)$ as a function of m.

Corollary A.1. *Suppose that n is sufficiently large, $p_H \geq \frac{2m-1}{2m+1}$ and $p_L = \frac{1}{2}$. $\underline{c}(m, w)$ and $\bar{c}(m, w)$ are decreasing in m.*

Corollary A.1 will imply the range of cost c within which m investments can be made in equilibrium. The expected explanation is as follows. Weighted jurors have incentives to invest if and only if m is small enough to satisfy $\bar{c}(m, w) \geq c$. Denote the largest such m by $\bar{m}(c, w)$, i.e., $\bar{m}(c, w) := \max\{m | \bar{c}(m, w) \leq c\}$. Unweighted jurors have incentives not to invest if and only if m is large enough to satisfy $\underline{c}(m, w) \leq c$. Denote the smallest such m by $\underline{m}(c, w)$, i.e., $\underline{m}(c, w) := \min\{m | \underline{c}(m, w) \leq c\}$. These observations will lead to the following Proposition.

Proposition A.3. *Suppose that n is sufficiently large, $p_H \geq \frac{2m-1}{2m+1}$ and $p_L = \frac{1}{2}$. Given c and w, the (m, w) equilibrium exists if and only if $m \in \{\underline{m}, \dots, \overline{m}\}$.*[11]

Proof. Take arbitrary c and w. Suppose that all of m weighted jurors invest in information and all of $(n - m)$ jurors do not invest.

The weighted jurors have incentives to invest if and only if $c \leq \overline{c}(m, w)$. Since $\overline{c}(m, w)$ is decreasing in m (Corollary A.1), $\overline{c}(\overline{m}, w) \leq \overline{c}(m, w)$ for all $m \leq \overline{m}$. Hence, for given c and w, the weighted jurors have incentives to invest if and only if $m \leq \overline{m}$.

Similarly, the unweighted jurors have incentives not to invest if and only if $c \geq \underline{c}(m, w)$. Since $\underline{c}(m, w)$ is decreasing in m (Corollary A.1), $\underline{c}(\underline{m}, w) \geq \underline{c}(m, w)$ for all $m \geq \underline{m}$. Hence, for given c and w, the unweighted jurors have incentives not to invest if and only if $m \geq \underline{m}$. $\qquad\square$

In particular, if $\underline{c}(1, w) \leq c \leq \overline{c}(m^*, w)$, the (m, w) equilibrium exists for every $m \in [1, m^*]$, i.e., up to m^* investments can be made in equilibrium for given c and w.

A.5   Proof of Proposition A.1

We show that the probability that a weighted voter becomes pivotal is decreasing in m in the following. Suppose that $w \geq 2$, and let $p_L = 1/2$ for simplicity. Subsequent arguments should carry over to the other cases with $p_L \neq 1/2$ by some suitable modifications.

We derive (sufficient) conditions under which it holds that

$$\Pr\left(\mathrm{piv}_j | m + 1, w\right) < \Pr\left(\mathrm{piv}_j | m, w\right),$$

where

$$\Pr\left(\mathrm{piv}_j | m, w\right) := \left(\frac{1}{2}\right)^{n-m} (1 - p_H)^{m-1} \sum_{\nu=0}^{m-1} \binom{m-1}{\nu} \left(\frac{p_H}{1 - p_H}\right)^{\nu} \Gamma\left(\nu, m\right)$$

---

[11] (1) $\underline{c}(m, w) \leq \overline{c}(m, w)$ for any (m, w) and (2) $\underline{m}(c, w) \leq \overline{m}(c, w)$ for any (c, w).

and

$$\Gamma\left(\nu, m\right) = \Gamma_{n,w}\left(\nu, m\right) := \sum_{\tau = \frac{n+m(w-1)+1}{2}-\nu w - w}^{\frac{n+m(w-1)+1}{2}-\nu w - 1} \begin{pmatrix} n - m \\ \tau \end{pmatrix}. \tag{A.2}$$

The domain of (ν, m) should be suitably defined, which may depend on (n, w), so that the right-hand side of (A.2) is well-defined.

Step 1: $\Gamma(\nu, m)$ is strictly decreasing in m for each ν

For our purpose, I first show that $\Gamma\left(\nu, m\right)$ is strictly decreasing in m for each ν. Consider the following decomposition:

$$\Gamma\left(\nu, m+1\right) = \sum_{\tau = \frac{n+(m+1)(w-1)+1}{2}-\nu w - w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w - 1} \begin{pmatrix} n - m - 1 \\ \tau \end{pmatrix}$$

$$= \left\{ \underbrace{\sum_{\tau = \frac{n+m(w-1)+1}{2}-\nu w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w - 1}}_{(*1)} + \underbrace{\sum_{\tau = \frac{n+m(w-1)+1}{2}-\nu w - w}^{\frac{n+m(w-1)+1}{2}-\nu w - 1}}_{(*2)} - \underbrace{\sum_{\tau = \frac{n+m(w-1)+1}{2}-\nu w - w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w - w - 1}}_{(*3)} \right\} \tag{A.3}$$

$$\times \begin{pmatrix} n - m - 1 \\ \tau \end{pmatrix}. \tag{A.4}$$

Note that the number of summands is $\frac{1}{2}\left(w - 1\right)$ (each) in $(*1)$ and $(*3)$, while that in $(*2)$ is w. Note also that the sum in $(*2)$ is computed over the same values of τ as those in the sum in $\Gamma\left(\nu, m\right)$.

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Since $\binom{n-m-1}{\tau} = \underbrace{\dfrac{(n-m-\tau)}{(n-m)}}_{<1}\binom{n-m}{\tau}$, I can write (A.4) as

$$\Gamma(\nu, m+1) = \Gamma(\nu, m) + \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1} \left[\frac{n-m-\tau}{n-m} - 1\right]\binom{n-m}{\tau}$$

$$+ \left\{ \underbrace{\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-1}}_{(*1)} - \underbrace{\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-w-1}}_{(*3)} \right\}\binom{n-m-1}{\tau}$$

$$= \Gamma(\nu, m) + \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1} \frac{-\tau}{n-m}\binom{n-m}{\tau}$$

$$+ \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-w-1} \left\{\binom{n-m-1}{\tau+w} - \binom{n-m-1}{\tau}\right\}$$

$$= \Gamma(\nu, m) + \underbrace{\sum_{\tau=\frac{n+(m+1)(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1} \frac{-\tau}{n-m}\binom{n-m}{\tau}}_{<0}$$

$$+ \underbrace{\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-w-1} \left\{\binom{n-m-1}{\tau+w} - \binom{n-m-1}{\tau} + \frac{-\tau}{n-m}\binom{n-m}{\tau}\right\}}_{(\#\#)}.$$

To determine the sign of the last term on the right-hand side, look at

$$\binom{n-m}{\tau} = \binom{n-m-1}{\tau}\frac{n-m}{n-m-\tau}$$

$$\binom{n-m-1}{\tau+w} = \frac{(n-m-1-\tau)}{(\tau+w)} \times \cdots \times \frac{(n-m-1-\tau-w+1)}{(\tau+1)}\binom{n-m-1}{\tau}.$$

Therefore,

$$(\#\#) = \left\{\frac{(n-m-1-\tau)}{(\tau+w)} \times \cdots \times \frac{(n-m-1-\tau-w+1)}{(\tau+1)} - \frac{n-m}{n-m-\tau}\right\}\binom{n-m-1}{\tau}.$$

The inside of the curly bracket in the last line is negative for large n. To see this, for

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example, note that it holds that for large n,

$$\frac{(n-m-1-\tau)}{(\tau+w)} \leq \left[\frac{n-m}{(n-m-\tau)}\right]^{1/w} \iff \left[\frac{(n-m-\tau-1)}{(\tau+w)}\right]^{w}(n-m-\tau) \leq (n-m)$$

where the inequalities hold since τ takes some value between $\left(\frac{n+m(w-1)+1}{2} - \nu w - w\right)$ and $\left(\frac{n+(m+1)(w-1)+1}{2} - \nu w - w - 1\right)$. Suppose m and w are small relatively to n. Then, for n large enough,

$$\left[\frac{(n-m-\tau-1)}{(\tau+w)}\right]^{w} \sim \left[\frac{n-\tau}{\tau}\right]^{w} \sim \left[\frac{n/2}{n/2}\right]^{w} \sim 1.^{12}$$

From these arguments, I can write

$$\Gamma\left(\nu, m+1\right) = \Gamma\left(\nu, m\right) + \underbrace{R\left(v, m\right)}_{<0},$$

i.e, $\Gamma\left(\nu, m\right)$ is strictly decreasing in m (again, it needs to be confirmed that $\Gamma\left(a, b\right)$ is well-defined at $(a, b) = (m, m)$.

$$\textit{Step 2: } \Pr\left(\mathrm{piv}_{j}|m+1, w\right) - \Pr\left(\mathrm{piv}\right) < 0$$

Thanks to Step 1, I know that $\Gamma\left(\nu, m\right)$ is strictly decreasing in m. By using this, I have

$$\Pr\left(\mathrm{piv}_{j}|m+1, w\right) - \Pr\left(\mathrm{piv}_{j}|m, w\right)$$
$$= \left(\frac{1}{2}\right)^{n-m}(1-p_{H})^{m-1}\sum_{\nu=0}^{m}\left(\frac{1}{2}\right)^{-1}(1-p_{H})\binom{m}{\nu}\left(\frac{p_{H}}{1-p_{H}}\right)^{\nu}\Gamma\left(\nu, m+1\right)$$
$$- \left(\frac{1}{2}\right)^{n-m}(1-p_{H})^{m-1}\sum_{\nu=0}^{m-1}\binom{m-1}{\nu}\left(\frac{p_{H}}{1-p_{H}}\right)^{\nu}\Gamma(\nu, m)$$

$$= \left(\frac{1}{2}\right)^{n-m}(1-p_H)^{m-1}\left\{\underbrace{\sum_{\nu=0}^{m}\left(\frac{1}{2}\right)^{-1}(1-p_H)\binom{m}{\nu}\left(\frac{p_H}{1-p_H}\right)^{\nu}R(\nu,m)}_{<0}\right.$$

$$+\underbrace{\left(\frac{1}{2}\right)^{-1}(1-p_H)\binom{m}{m}\left(\frac{p_H}{1-p_H}\right)^{m}\Gamma(m,m)}_{=I}$$

$$\left.+\sum_{\nu=0}^{m-1}\underbrace{\left[\left(\frac{1}{2}\right)^{-1}(1-p_H)\binom{m}{\nu}\binom{m-1}{\nu}\right]\left(\frac{p_H}{1-p_H}\right)^{\nu}\Gamma(\nu,m)}_{=J(\nu)}\right\}. \tag{A.5}$$

The first term inside the curly bracket is strictly negative since $R(\nu,m)<0$ (Step 1).

We now consider a condition under which the sum of the 2nd and 3rd terms inside the curly bracket, $I+\sum_{\nu=0}^{m-1}J(\nu)$, is negative. To this end, I use the following result:

$$\Gamma(m,m)<\Gamma(m-1,m),\,^{13} \tag{A.6}$$

i.e., $\Gamma(a,m)$ is strictly decreasing in a, whose proof is provided below. Then, noting $\binom{m}{m}=1$,

$$I<\left(\frac{1}{2}\right)^{-1}(1-p_H)\binom{m}{m-1}\left(\frac{1}{m}\frac{p_H}{1-p_H}\right)\left(\frac{p_H}{1-p_H}\right)^{m-1}\Gamma(m-1,m).$$

---

[13] **Proof of (A.6):** $\Gamma(m,m)<\Gamma(m-1,m)$
By the definition of Γ in (A.2), I can see that

$$\Gamma(\nu-1,m):=\sum_{\tau=\frac{n+m(w-1)+1}{2}-(\nu-1)w-w}^{\frac{n+m(w-1)+1}{2}-(\nu-1)w-1}\binom{n-m}{\tau}=\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1}\binom{n-m}{\tau+w},$$

Therefore,

$$\Gamma(m,m)-\Gamma(m-1,m)=\sum_{\tau=\frac{n+m(w-1)+1}{2}-mw-w}^{\frac{n+m(w-1)+1}{2}-mw-1}\left[\binom{n-m}{\tau}-\binom{n-m}{\tau+w}\right]$$

$$=\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1}\left[\frac{(n-m)!}{\tau!(n-m-\tau)!}-\frac{(n-m)!}{(\tau+w)!(n-m-\tau-w)!}\right]$$

$$=\sum_{\tau=\frac{n+m(w-1)+1}{2}-mw-w}^{\frac{n+m(w-1)+1}{2}-mw-1}\frac{(n-m)!}{\tau!(n-m-\tau)!}\left[1-\frac{(n-m-\tau)}{(\tau+w)}\times\cdots\times\frac{(n-m-\tau-w+1)}{(\tau+1)}\right]<0,$$

Therefore,

$$I + \sum_{\nu=0}^{m-1} J(\nu) = [I + J(m-1)] + \sum_{\nu=0}^{m-2} J(\nu)$$

$$< \left[\left(\frac{1}{2} \right)^{-1} (1 - p_H) \binom{m}{m-1} \left(1 + \frac{1}{m} \frac{p_H}{1 - p_H} \right) - \binom{m-1}{m-1} \right] \left(\frac{p_H}{1 - p_H} \right)^{m-1} \Gamma(m-1, m)$$

$$+ \sum_{\nu=0}^{m-2} J(\nu)$$

$$= \underbrace{[2(m(1 - p_H) + p_H) - 1]}_{(@)} \left(\frac{p_H}{1 - p_H} \right)^{m-1} \Gamma(m-1, m)$$

$$+ \sum_{\nu=0}^{m-2} \underbrace{\left[\left(\frac{1}{2} \right)^{-1} (1 - p_H) \binom{m}{\nu} - \binom{m-1}{\nu} \right]}_{(@@)} \left(\frac{p_H}{1 - p_H} \right)^{\nu} \Gamma(\nu, m),$$

where I below derive the conditions under which (@) and (@@) are negative. As for (@), look at

$$(@) \leq 0 \iff 2m - 1 \leq (2m+1) p_H \iff \frac{2m-1}{2m+1} \leq p_H \tag{A.7}$$

For the component of (@@), I only need to consider the case where $0 \leq \nu \leq m - 2$. Since $\binom{m-1}{\nu} = \binom{m}{\nu} \frac{m - \nu}{m}$,

$$(@@) = \binom{m}{\nu} \left[\left(\frac{1}{2} \right)^{-1} (1 - p_H) - \frac{m - \nu}{m} \right] < \binom{m}{\nu} \left[2(1 - p_H) - \frac{2}{m} \right]$$

which is negative when

$$1 - \frac{1}{m} \leq p_H. \tag{A.8}$$

Since (A.7) implies (A.8), the condition (A.7) implies that the sum of the 2nd and

---

where the last inequality holds by noting that

$$(\tau + w) < (n - m - \tau)$$
$$\iff 2\tau + w + m < n$$
$$\impliedby 2 \left(\frac{n + m(w-1) + 1}{2} - mw - 1 \right) + w + m < n$$
$$\iff -mw - 1 + w < 0.$$

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3rd terms inside the curly brackets, $I + \sum_{\nu=0}^{m-1} J(\nu)$, is negative. Therefore, $\frac{2m-1}{2m+1} \leq p_H$ implies that the probability that a weighted juror j becomes pivotal is decreasing in m, i.e., $\Pr\left(\mathrm{piv}_j | m+1, w\right) < \Pr\left(\mathrm{piv}_j | m, w\right)$. $\square$

Notes for Computation 2:

In Step 2, I have derived the condition of (A.7) by showing that $I + \sum_{\nu=0}^{m-1} J(\nu)$ is negative. Recalling (A.5), I may utilize $\sum_{\nu=0}^{m} \left(\frac{1}{2}\right)^{-1} (1-p_H)\binom{m}{\nu} \left(\frac{p_H}{1-p_H}\right)^{\nu} R(\nu, m) < 0$. For example, I can work with the following decomposition:

$$I + \sum_{\nu=0}^{m-1} J(\nu) = \left[I + \sum_{\nu=\mathrm{Int}\left(\sqrt{m}\right)}^{m-1} J(\nu) \right] + \sum_{\nu=0}^{m-\mathrm{Int}\left(\sqrt{m}\right)-1} J(\nu), \tag{A.9}$$

where $\mathrm{Int}(x)$ stands for the integer part of x, and may derive the conditions under which $I + \sum_{\nu=\mathrm{Int}\left(\sqrt{m}\right)}^{m-1} J(\nu)$ and $\sum_{\nu=0}^{m-\mathrm{Int}\left(\sqrt{m}\right)-1} J(\nu)$ are negative. We consider the decomposition (A.9) in Step 2'.

Step 2': Another approach to show $\Pr\left(\mathrm{piv}_j | m+1, w\right) - \Pr\left(\mathrm{piv}\right) < 0$?

We here derive conditions under which $\Pr\left(\mathrm{piv}_j | m+1, w\right) < \Pr\left(\mathrm{piv}_j | m, w\right)$, based on the decomposition (A.9).

To work with (A.9), look at

$$\left(\frac{p_H}{1-p_H}\right)^m = \binom{m}{m-1} \left(\frac{1}{m}\frac{p_H}{1-p_H}\right)\left(\frac{p_H}{1-p_H}\right)^{m-1}$$

$$\left(\frac{p_H}{1-p_H}\right)^m = \binom{m}{m-2} \left(\binom{m}{m-2}^{-1}\left(\frac{p_H}{1-p_H}\right)^2\right)\left(\frac{p_H}{1-p_H}\right)^{m-2}$$

$$\cdots$$

$$\left(\frac{p_H}{1-p_H}\right)^m$$

$$= \binom{m}{m-\mathrm{Int}\left(\sqrt{m}\right)} \left(\binom{m}{m-\mathrm{Int}\left(\sqrt{m}\right)}^{-1}\left(\frac{p_H}{1-p_H}\right)^{\mathrm{Int}\left(\sqrt{m}\right)}\right)\left(\frac{p_H}{1-p_H}\right)^{m-\mathrm{Int}\left(\sqrt{m}\right)}.$$

Then, I can write

$$\left(\frac{p_H}{1-p_H}\right)^m \Gamma(m,m) = \sum_{\nu=m-\text{Int}(\sqrt{m})}^{m-1} \binom{m}{\nu} \left(\frac{1}{\text{Int}(\sqrt{m})}\binom{m}{\nu}^{-1}\left(\frac{p_H}{1-p_H}\right)^{m-\nu}\right)\left(\frac{p_H}{1-p_H}\right)^{\nu}$$

and therefore,

$$I < \sum_{\nu=m-\text{Int}(\sqrt{m})}^{m-1} \left(\frac{1}{2}\right)^{-1}(1-p_H)\binom{m}{\nu}\left(\frac{1}{\text{Int}(\sqrt{m})}\binom{m}{\nu}^{-1}\left(\frac{p_H}{1-p_H}\right)^{m-\nu}\right)$$
$$\times \left(\frac{p_H}{1-p_H}\right)^{\nu}\Gamma(\nu,m). \tag{A.10}$$

By (A.10),

$$\left[I + \sum_{\nu=\text{Int}(\sqrt{m})}^{m-1} J(\nu)\right] + \sum_{\nu=0}^{m-\text{Int}(\sqrt{m})-1} J(\nu)$$

$$< \sum_{\nu=m-\text{Int}(\sqrt{m})}^{m-1} \underbrace{\left[2(1-p_H)\binom{m}{\nu}\left[1+\frac{1}{\text{Int}(\sqrt{m})}\binom{m}{\nu}^{-1}\left(\frac{p_H}{1-p_H}\right)^{m-\nu}\right] - \binom{m-1}{\nu}\right]}_{(\%)}$$

$$\times \left(\frac{p_H}{1-p_H}\right)^{\nu}\Gamma(\nu,m) + \sum_{\nu=0}^{m-\text{Int}(\sqrt{m})-1} \underbrace{\left[2(1-p_H)\binom{m}{\nu} - \binom{m-1}{\nu}\right]}_{(\%\%)}\left(\frac{p_H}{1-p_H}\right)^{\nu}\Gamma(\nu,m).$$

(1) For the component of (%), $m-\text{Int}(\sqrt{m}) \leq \nu \leq m-1$. Since $\binom{m-1}{\nu} = \binom{m}{\nu}\frac{m-\nu}{m}$,

$$(\%) = \left[2(1-p_H)\binom{m}{\nu}\left[1+\frac{1}{\text{Int}(\sqrt{m})}\binom{m}{\nu}^{-1}\left(\frac{p_H}{1-p_H}\right)^{m-\nu}\right] - \binom{m-1}{\nu}\right]$$

$$= \binom{m}{\nu}\left[2(1-p_H)\left(1+\frac{1}{\text{Int}(\sqrt{m})}\binom{m}{\nu}^{-1}\left(\frac{p_H}{1-p_H}\right)^{m-\nu}\right) - \frac{m-\nu}{m}\right]$$

$$\leq \binom{m}{\nu}\left[2(1-p_H)\left(1+\frac{1}{\text{Int}(\sqrt{m})}\right) - \frac{m-\nu}{m}\right] \leq 0,$$

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where the last inequality holds when

$$2\left(1 - p_H\right)\left(1 + \frac{1}{\text{Int}\left(\sqrt{m}\right)}\right) \leq \frac{m - \nu}{m}$$

$$\Longleftarrow 2\left(1 - p_H\right)\left(1 + \frac{1}{\text{Int}\left(\sqrt{m}\right)}\right) \leq \frac{1}{m}$$

$$\Longleftrightarrow \left(p_H - 1\right)\left(1 + \frac{1}{\text{Int}\left(\sqrt{m}\right)}\right) \geq -\frac{1}{2m}$$

$$\Longleftarrow \left(p_H - 1\right)\left(1 + \frac{1}{\sqrt{m} + 1}\right) \geq -\frac{1}{2m}$$

$$\Longleftrightarrow p_H \geq 1 - \frac{1}{2m}\frac{\sqrt{m} + 1}{\sqrt{m} + 2}.$$

(2) For the component of (%%), I only need to consider the case where $0 \leq \nu \leq m - \text{Int}\left(\sqrt{m}\right) - 1$:

$$(\%\%) = \binom{m}{\nu}\left[\left(\frac{1}{2}\right)^{-1}\left(1 - p_H\right) - \frac{m - \nu}{m}\right] < \binom{m}{\nu}\left[2\left(1 - p_H\right) - \frac{\text{Int}\left(\sqrt{m}\right) + 1}{m}\right]$$

$$\leq \binom{m}{\nu}\left[2\left(1 - p_H\right) - \frac{\left(\sqrt{m} - 1\right) + 1}{m}\right] = \binom{m}{\nu}\left[2\left(1 - p_H\right) - \frac{\sqrt{m}}{m}\right],$$

which is negative when

$$p_H \geq 1 - \frac{1}{2\sqrt{m}}.$$

A.6   Notes for Computation (##) in Step 1 of Appendix A.5

In this section, I examine weaker conditions for (##) to be negative than n being large.

$$\left[\frac{n - m - \tau - 1}{\tau + w}\right]^w \left(n - m - \tau\right) \leq n - m$$

τ takes some value in the following range:

$$\tau \in \left[\frac{n + m\,(w-1) + 1}{2} - \nu w - w, \; \frac{n + (m+1)\,(w-1) + 1}{2} - \nu w - w - 1 \right].$$

Since $\nu \in [0, m-1]$, τ takes some value in the following range:

$$\tau \in \left[\frac{n - m(w+1) + 1}{2}, \; \frac{n + (m+1)\,(w-1) + 1}{2} - w - 1 \right].$$

$\left(\frac{n-m-\tau-1}{\tau+w} \right)^{w} (n - m - \tau)$ is decreasing in τ

At $\tau = \frac{n - m(w+1) + 1}{2}$ (when $\nu = m - 1$),

$$\left[\frac{n - m - \tau - 1}{\tau + w} \right]^{w} (n - m - \tau) \le n - m$$

$$\Leftrightarrow \frac{n + m(w-1) - 3}{n - m(w+1) + 1 + 2w} \cdot \frac{n + m(w-1) - 1}{2} \le n - m$$

$$\Leftrightarrow n^2 + (-4mw + 6 + 4w - 2m)n - m^2 w^2 - 6m - 3 + 4m^2 w + m^2 \ge 0 \qquad (*)$$

Define $f(n)$ as

$$f(n) := n^2 + (-4mw + 6 + 4w - 2m)n - m^2 w^2 - 6m - 3 + 4m^2 w + m^2.$$

Then,

$$
(*) \Leftrightarrow
\begin{cases}
f(n^*) \geq 0 \\[4pt]
\quad \text{or} \\[4pt]
n_2 \leq 3 \\[4pt]
\quad \text{or} \\[4pt]
n_1 \leq 3 < n_2 \text{ and } n \geq n_2 \\[4pt]
\quad \text{or} \\[4pt]
n_1 > 3 \text{ and } n \in (-\infty, n_1] \cup [n_2, \infty)
\end{cases}
$$

where n^* is s.t. $f'(n^*) = 0$ and (n_1, n_2) are s.t. $f(n_1) = f(n_2) = 0$ and $n_1 \leq n_2$.

B. ADDITIONAL RESULTS FOR CHAPTER 2

B.1 Two-generations Model: Backward Intergenerational Transfers from Children to Parents

B.1.1 Basic Two-generations Model

In this subsection, I propose an overlapping generations model, using two generations to analyze backward intergenerational transfers, such as nursing care and financial support provided by individuals to their parents. I consider a situation in which a breadwinner decides how much to invest in a backward intergenerational good (BIG) that benefits only an elderly dependent. The key informational feature is that it is assumed that children can observe several preceding generations' behavior, such as how their parents treated their grandparents, but information about their distant ancestors' behavior, such as how their great-grandparents treated their great-great-grandparent, is unavailable. I first demonstrate that intergenerational transfers cannot be sustained under limited information and then show that close-knit communities serve to facilitate cooperation between generations.

Consider an infinitely-lived family with two generations alive in each period t ($t = 1, 2, \ldots$). At every period t, a single player, called generation t, is born to the family and lives for two periods: t and $t + 1$. I call generation t the breadwinner in period t and the elderly dependent in period $t + 1$. In period 1 there are a breadwinner 1 who stays in periods 1 and 2, and an elderly dependent 0 who stays only for that period.

The breadwinner has positive endowment, and the elderly dependent has endowment that is normalized to zero. Every period t, the breadwinner t decides how much to transfer to the elderly dependent $t - 1$. Let $a_t \in A_t$ denote the amount transferred to the current elderly dependent, $t - 1$, where action spaces are common across generations, $A_t = A$ for

all t. The commodity is assumed to be infinitely divisible. The elderly dependent has no choices to make.

Generation t's utility function, $u : A_t \times A_{t+1} \to \mathbb{R}$, is decreasing in the transfer made in his middle-age and increasing in the transfer he receives in his old age. That is, a generation's lifetime utility depends only on the action taken while a breadwinner and the action that the breadwinner takes when he is old. Utility function $u(\cdot)$ satisfies the condition:

Assumption 4. $\forall \mathbf{a}, \mathbf{a}' \in A^2$, $u(\mathbf{a}) = u(\mathbf{a}') \implies \mathbf{a} = \mathbf{a}'$.

This condition ensures that $\arg\max_{a_t \in A_t} u(a_t, a_{t+1})$ is unique. Assuming that $\arg\max_{a_t \in A_t} u(a_t, a_{t+1})$ is independent of a_{t+1}, I label it $a_t = 0$, which can be interpreted as no transfers. Note that this overlapping generation game has a unique Markov equilibrium, in which every player chooses 0.

Let $h_t = (a_1, \ldots, a_{t-1})$ denote the history of preceding actions taken until period t and $H_t(= A^{t-1})$ denote the set of all possible histories at t. I define the default informational environment as follows:

Definition B.1. For any $t = 1, 2, \ldots$, generation t has **m-th order information** if he knows the actions of the last m generations, $(a_{t-1}, \ldots, a_{t-m})$, but not any action taken prior to $t - m$, $(a_{t-m-1}, \ldots, a_1)$.

Assumption 5. *There exists a natural number m such that generation t has m-th order information for all $t = 1, 2, \ldots$.*

If (h_t, h_t') is any pair of histories that differ only in the actions taken by some of players i $(\leq t - m - 1)$, then the histories observed by generation t are identical for h_t and h_t'. For example, the first-order information is a record of the preceding generation's past play. That is, the information is limited in the sense that a generation does not know the actions taken in the family prior to its parents.

A pure strategy for generation t is a function $s_t : A_{t-m} \times \cdots \times A_{t-1} \to A_t$. Let S_t be the set of t's pure strategies. A strategy profile is an infinite sequence $(s_t)_{t=1}^{\infty}$ where

$s_t \in S_t$ for all t. Thus, $s_t(a_{t-m}, \ldots, a_{t-1})$ is the element of A_t which is induced by the observed history $(a_{t-m}, \ldots, a_{t-1})$ when s_t is played.

An action by generation t, a_t, when he observes $(a_{t-m}, \ldots, a_{t-1})$, induces an observed history for $t+1$, $(a_{t-m+1}, \ldots, a_t)$. Different actions by generation t generate different information for $t+1$, and generation $t+1$ varies her actions with her observed history, $(a_{t-m+1}, \ldots, a_t)$.

Adopting the terminology of **?**, I define our equilibrium definition as follows. Because past actions do not directly affect current or future utility, I do not have to deal with any beliefs regarding the histories.

Definition B.2. A strategy profile $(s_t)_{t=1}^\infty$ is a sequentially rational equilibrium if $\forall t, \ \forall (a_{t-m}, \ldots, a_{t-1}) \in A_{t-m} \times \cdots \times A_{t-1}, \ \forall a_t \in A_t,$

$$u(s_t, s_{t+1} | a_{t-m}, \ldots, a_{t-1}) \geq u(a_t, s_{t+1} | a_{t-m}, \ldots, a_{t-1})$$

where

$$u(a_t, s_{t+1} | a_{t-m}, \ldots, a_{t-1}) = u[a_t, s_{t+1}(a_{t-m+1}, \ldots, a_t)]$$

and

$$u(s_t, s_{t+1} | a_{t-m}, \ldots, a_{t-1}) = u[s_t(a_{t-m}, \ldots, a_{t-1}), s_{t+1}(a_{t-m+1}, \ldots, a_t)].$$

The following theorem is a variant of Theorem 1 in **?**.

Theorem B.1. *Under Assumption 1, the overlapping generations game with m-th order information has a unique pure strategy equilibrium where no intergenerational transfer is made.*

Proof. Suppose that generation t's equilibrium strategy s_t is based on action taken by generation $t-m$. Then, there exist $a_{t-m}, a'_{t-m}(a_{t-m} \neq a'_{t-m})$ such that $u_t(s_t, s_{t+1} | a_{t-m}, \ldots, a_{t-1}) > u_t(s_t, s_{t+1} | a'_{t-m}, \ldots, a_{t-1})$. Since s_{t+1} conditions only on actions taken by generations $t-m+1, \ldots, t$ and is independent of generation $t-m$'s action, generation t can improve his payoff by choosing $s_t(a_{t-m}, \ldots, a_{t-1})$ instead of $s_t(a'_{t-m}, \ldots, a_{t-1})$ when he observes $(a'_{t-m}, \ldots, a_{t-1})$. Hence, generation t's equilibrium

strategy is not conditioned on $t-m$'s actions. Similarly, generation t's equilibrium strategy does not condition on k's action for any $k < t$. Thus, generation t's equilibrium strategy is not based on history. Therefore generation t's best response is $a_t = 0$, which forms a unique pure strategy equilibrium where no intergenerational transfer is made. $\qquad\square$

Theorem B.1 indicates that when individuals have limited information about past events, no strategy profile (which does not have to be Markov) in which players condition their behavior on the observed history, which is payoff-irrelevant, constitutes an equilibrium. Thus, intergenerational cooperation cannot be supported in pure-strategy equilibrium when information about past events is limited.

B.1.2 Two-generations Model with a Local Community

As in the three-generations model, I modify the basic two-generations model by adding a local community that provides information about the history of play in each family through gossiping. I consider a situation in which families in the previous basic two-generations model reside in a close-knit community. In a close-knit community, the breadwinner receives a reputation based on how he treated his parent and child, and that reputation will become known to his child through neighbors' gossip. The transfers a breadwinner will receive from his child may depend on that reputation, which subsequently determines his child's reputation. In this setting, positive amount of BIGs can be transferred by pure-strategy equilibria. Formally, this situation is modeled as follows.

Before introducing a local community to the basic model, I define the efficient level of transfers. Let $\arg\max_{a \in A} u(a, a) = 1$ be the efficient transfer, which I assume to be different from 0, the individually optimal action. Otherwise, it is trivial that no transfer is sustained in equilibrium. From Assumption 1,

Lemma B.1. $u(0, 1) > u(1, 1) > u(0, 0) > u(1, 0)$

In each period t, the breadwinner t is assigned a status label x_t of a finite set $X_t = \{$"good", "bad"$\}$. A generation's status label $x_t \in X_t$ is determined through $\tau : X_{t-1} \times$

$A_t \rightarrow X_t$. τ specifies the status label of generation t in the next period, $\tau(x_{t-1}, a_t) \in X_t$, when his previous generation's status label is $x_{t-1} \in X_{t-1}$ and his current action is $a_t \in A_t$. A social norm, τ, is common knowledge.

A social norm τ is **family reciprocity** if

$$
x_t = \tau(x_{t-1}, a_t) = \begin{cases} \text{``bad''} & \text{if } x_{t-1} = \text{``good'' \& } a_t < 1 \\ \text{``good''} & \text{otherwise} \end{cases}
$$

and $x_0 =$ "good". Individuals are considered "bad" only when they do not take care of their parents whose status label is "good".

Generation t is labeled by a social norm τ and the community informs the succeeding generation $t + 1$ of t's status label x_t; that is, the status label of the elderly dependent is known to the breadwinner at period $t + 1$. In particular, the breadwinner's action choice is typically a function of the previous generation's status label.

The history of intergenerational transfers may not be known; it becomes known to each generation only to the extent that it is reflected in the status labels of the elderly dependent. A pure (Markov) strategy for a generation t is a mapping $s_t : X_{t-1} \rightarrow A_t$ specifying a choice of action $s_t(x_{t-1})$ when the previous generation's status label is $x_{t-1} \in X_{t-1}$.

Our equilibrium definition here slightly differs from the last one in the information possessed by the decision makers. Because past actions do not directly affect current or future utility, I do not have to deal with any beliefs regarding the histories.

Definition B.3. A strategy profile $(s_t)_{t=1}^{\infty}$ is a sequentially rational equilibrium if

$$
\forall t, \ \forall x_{t-1} \in X_{t-1}, \ \forall a \in A, \ \ u(s_t, s_{t+1}|x_{t-1}) \geq u(a, s_{t+1}|x_{t-1})
$$

where
$$
u(a_t, s_{t+1}|x_{t-1}) = u[a_t, s_{t+1}(\tau(a_t, x_{t-1}))]
$$

and
$$
u(s_t, s_{t+1}|x_{t-1}) = u[s_t(x_{t-1}), s_{t+1}(\tau(s_t(x_{t-1}), x_{t-1}))].
$$

I say that a strategy profile $(s_t)_{t=1}^{\infty}$ is a **tit-for-tat strategy** if

$$s_t(x_{t-1}) = \begin{cases} 1 & \text{if } x_{t-1} = \text{``good''} \\ 0 & \text{if } x_{t-1} = \text{``bad''}. \end{cases}$$

Theorem B.2. *In a close-knit community whose social norm is family reciprocity, an efficient level of intergenerational transfers can be sustained as a sequentially rational equilibrium by a tit-for-tat strategy.*

Proof. Each generation is labeled by a social norm τ, i.e., $x_t = \tau(x_{t-1}, a_t)$ for all t. Consider a community whose social norm is family reciprocity, that is, the breadwinner has to take care of his parent to be labeled "good" the his parent is "good".

Suppose that each generation plays a tit-for-tat strategy. Take an arbitrary period t. When $x_{t-1} =$ "good", generation t's utility is

$$u(s_t, s_{t+1}|x_{t-1}) = u(1, 1)$$
$$u(a_t, s_{t+1}|x_{t-1}) = u(a_t, 0) \quad \text{if } a_t \neq 1$$

where $u(1,1) > u(a_t, 0)$ for all $a_t \neq 1$. When $x_{t-1} =$ "bad", generation t's utility is

$$u(s_t, s_{t+1}|x_{t-1}) = u(0, 1)$$
$$u(a_t, s_{t+1}|x_{t-1}) = u(a_t, 1) \quad \text{for all } a_t$$

where $u(0,1) \geq u(a_t, 1)$. Hence,

$$\forall t, \; \forall x_{t-1} \in X_{t-1}, \; \forall a \in A \;\; u(s_t, s_{t+1}|x_{t-1}) \geq u(a, s_{t+1}|x_{t-1}).$$

$\square$

Theorem B.2 establishes that in a community whose social norm is family reciprocity, an efficient level of intergenerational transfers can be sustained as a sequentially rational equilibrium by a tit-for-tat strategy. This result is in a sharp contrast with the case without a local community (Theorem 1) where BIGs cannot be supported by pure-strategy

equilibrium.

By Theorems B.1 and B.2 , the intergenerational cooperation is sustained in a close-knit community. Local gossip by neighbors (or extended families) serve to facilitate cooperation between generations.

C. PROOFS FOR CHAPTER 3

C.1 Proof of Theorem 3.1

For any $p \in [0,1]$, let

$$\pi_M(p) = \sum_{k=n-\hat{k}+1}^{\hat{k}-1} \binom{n}{k} p^k (1-p)^{n-k},$$

$$\pi_C(p) = \sum_{k=\hat{k}}^{n} \binom{n}{k} p^k (1-p)^{n-k},$$

and $\pi_A(p) = 1 - \pi_M(p) - \pi_C(p)$. For $d \in \{M, C, A\}$, $\pi_d(p)$ is the probability that a single jury composed of n jurors makes a decision d if each juror independently votes for conviction with probability p. It is clear by definition that $\pi_M(p) = \pi_M(1-p)$ for any p. The following lemma is key for our analysis.

Lemma C.1. $\pi_M(p)$ *is strictly decreasing in* $p \in (1/2, 1]$.

Proof. By differentiation,

$$
\frac{d\pi_M(p)}{dp}
$$

$$
= \sum_{k=n-\hat{k}+1}^{\hat{k}-1} \binom{n}{k} k p^{k-1}(1-p)^{n-k} - \sum_{k=n-\hat{k}+1}^{\hat{k}-1} \binom{n}{k}(n-k)p^k(1-p)^{n-k-1}
$$

$$
= \sum_{k=n-\hat{k}}^{\hat{k}-2} \binom{n}{k+1}(k+1)p^k(1-p)^{n-k-1} - \sum_{k=n-\hat{k}+1}^{\hat{k}-1} \binom{n}{k}(n-k)p^k(1-p)^{n-k-1}
$$

$$
= \sum_{k=n-\hat{k}+1}^{\hat{k}-2} \left(\binom{n}{k+1}(k+1) - \binom{n}{k}(n-k) \right) p^k(1-p)^{n-k-1}
$$

$$
+ \binom{n}{n-\hat{k}+1}(n-\hat{k}+1)p^{n-\hat{k}}(1-p)^{\hat{k}-1}
$$

$$
- \binom{n}{\hat{k}-1}(n-\hat{k}+1)p^{\hat{k}-1}(1-p)^{n-\hat{k}}. \tag{C.1}
$$

By a well-known identity $\binom{n}{k+1}(k+1) = \binom{n}{k}(n-k)$, the terms in summation in equation (C.1) cancel each other out. By this and the fact $\binom{n}{n-\hat{k}+1} = \binom{n}{\hat{k}-1}$,

$$
\frac{d\pi_M(p)}{dp}
$$

$$
= \binom{n}{\hat{k}-1}(n-\hat{k}+1)p^{n-\hat{k}}(1-p)^{\hat{k}-1} - \binom{n}{\hat{k}-1}(n-\hat{k}+1)p^{\hat{k}-1}(1-p)^{n-\hat{k}}
$$

$$
= \binom{n}{\hat{k}-1}(n-\hat{k}+1)[p^{n-\hat{k}}(1-p)^{\hat{k}-1} - p^{\hat{k}-1}(1-p)^{n-\hat{k}}]. \tag{C.2}
$$

The assumption that there is at least one numerical split under which a mistrial is declared implies $\hat{k} > (n+1)/2$ and hence $\hat{k} - 1 > n - \hat{k}$. Also, the assumption $p \in (1/2, 1]$ implies $p > 1-p$. Thus expression (C.2) is always negative. Therefore $\pi_M(p)$ is strictly decreasing in $p \in (1/2, 1]$. $\square$

Proof of Theorem 1. First consider the case with $p_g > p_i$. Let m_j^ω be the expected disutility of juror j when the jury which contains j is hung, while each juror follows an informative voting and the true state is ω.[1]

For an arbitrary j, let $\text{Prob}(\omega = \bar{\omega}, |s|_{n-1} = \hat{k} - 1|s_j, t)$ be the probability that the

---

[1] m_j^ω can be calculated by $m_j^G = (1 - q_j)\pi_A(p_g) + m_j^G \pi_M(p_g)$ and $m_j^I = q_j \pi_C(1 - p_i) + m_j^I \pi_M(1 - p_i)$.

state is $\bar{\omega}$ and, of $n-1$ jurors other than j, $\hat{k}-1$ jurors observe signal s, conditional on the event that juror $j \in N^t$ observes signal s_j and the existence of all the previous mistrials. Since jurors in N^t observe t hung juries at periods $0, \ldots, t-1$, we obtain, for example,

$$\text{Prob}(\omega = G, |g|_{n-1} = \hat{k} - 1|g, t) = \frac{r\binom{n-1}{\hat{k}-1}[\pi_M(p_g)]^t p_g^{\hat{k}}(1-p_g)^{n-\hat{k}}}{r[\pi_M(p_g)]^t p_g + (1-r)[\pi_M(1-p_i)]^t(1-p_i)}.$$

Define $EU_j(v_j|s_j)$ to be the expected utility for juror j of casting a vote v_j, conditional on j getting to vote and observing signal s_j. Juror $j \in N^t$ who observes $s_j = g$ has incentives to vote for conviction if and only if $EU_j(c|g) - EU_j(a|g) \geq 0$. The expression is

$$
\begin{aligned}
EU_j(c|g) - EU_j(a|g) &= [-q_j - (-m_j^I)]\text{Prob}(\omega = I, |g|_{n-1} = \hat{k} - 1|g, t) \\
&\quad + [-m_j^G - (-(1-q_j))]\text{Prob}(\omega = G, |i|_{n-1} = \hat{k} - 1|g, t) \\
&\quad + [-m_j^I - 0]\text{Prob}(\omega = I, |i|_{n-1} = \hat{k} - 1|g, t) \\
&\quad + [0 - (-m_j^G)]\text{Prob}(\omega = G, |g|_{n-1} = \hat{k} - 1|g, t) \\
&\leq [-\underline{q} - (-m_j^I)]\text{Prob}(\omega = I, |g|_{n-1} = \hat{k} - 1|g, t) \\
&\quad - [m_j^G - (1-q_j)]\text{Prob}(\omega = G, |i|_{n-1} = \hat{k} - 1|g, t) \\
&\quad - m_j^I\text{Prob}(\omega = I, |i|_{n-1} = \hat{k} - 1|g, t) \\
&\quad + m_j^G\text{Prob}(\omega = G, |g|_{n-1} = \hat{k} - 1|g, t).
\end{aligned}
$$

By Lemma C.1, $p_g > p_i$ implies $\pi_M(p_g) < \pi_M(p_i) = \pi_M(1-p_i)$. Therefore $\lim_{t \to \infty} \text{Prob}(\omega = G, |g|_{n-1} = \hat{k} - 1|g, t) = \lim_{t \to \infty} \text{Prob}(\omega = G, |i|_{n-1} = \hat{k} - 1|g, t) = 0$. Moreover $\lim_{t \to \infty} \text{Prob}(\omega = I, |g|_{n-1} = \hat{k} - 1|g, t) = \binom{n-1}{\hat{k}-1}(1-p_i)^{\hat{k}-1}p_i^{n-\hat{k}} < \binom{n-1}{\hat{k}-1}p_i^{\hat{k}-1}(1-p_i)^{n-\hat{k}} = \lim_{t \to \infty} \text{Prob}(\omega = I, |i|_{n-1} = \hat{k} - 1|g, t)$ by $p_i > 1/2$. Thus we obtain $\lim_{t \to \infty} EU_j(c|g) - EU_j(a|g) \leq -\underline{q}\binom{n-1}{\hat{k}-1}(1-p_i)^{\hat{k}-1}p_i^{n-\hat{k}} < 0$. Thus for any sufficiently large t, $EU_j(c|g) - EU_j(a|g) < 0$ for any juror $j \in N^t$. In other words, all jurors in N^t have strict incentives to vote for acquittal even if they observe private signal g, which establishes that informative voting is not an equilibrium.

In the case with $p_g < p_i$, an argument analogous to the above one establishes that, for any sufficiently large t, all jurors in N^t who observe private signal i have strict incentives

to vote for conviction. Hence informative voting does not form an equilibrium in this case either. $\square$

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