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Bribing in First-Price Auctions: Corrigendum

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Abstract

We clarify the sufficient condition for a trivial equilibrium to exist in the model of Rachmilevitch (2013).

Rachmilevitch (2013), henceforth R13, studies the following game. Two ex ante identical players are about to participate in an independent-private-value first-price, sealed bid auction for one indivisible object. After the risk-neutral players learn their valuations but prior to the actual auction, player 1 can offer a take-it-or-leave-it (TIOLI) bribe to his opponent in exchange for the opponent dropping out of the contest. If the offer is accepted, player 1 is the only bidder and obtains the item for free; otherwise, both players compete non-cooperatively in the auction as usual. This is called the *first-price TIOLI game*.[1] R13 shows that under the restriction to continuous and monotonic bribing strategies for player 1, any equilibrium of this game must be *trivial*—the equilibrium bribing function employed by player 1, if it is continuous and non-decreasing, must be identically zero. In this note, we clarify the sufficient conditions under which a trivial equilibrium exists. These are less stringent than originally proposed.

Let F denote the cumulative distribution function of players' types (valuations). F is atomless, has full support on $[0,1]$, and its density is f. The following is Theorem 2 from R13.

Theorem 2. *Suppose that F is differentiable and that it satisfies $2F(t) + tf(t) \geq 1$ for all $t \in (0,1]$, where $f = F'$. Then, the first-price TIOLI game has a trivial equilibrium.*

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[1] See R13 for a detailed description of the game. The inspiration for this game comes from Eső and Schummer (2004), who study a *second-price* auction preceded by a bribing stage.

An unfortunate fact regarding this theorem is that it is vacuously true.

Claim 1. There does not exist a distribution F such that $2F(t) + tf(t) \geq 1$ for all $t \in (0, 1]$.

Proof. Suppose the contrary and choose $0 < \epsilon < 1/2$. Since F is continuous and $F(0) = 0$, there exists $\bar{t} > 0$ such that for all $t < \bar{t}$, $F(t) < \epsilon$. Thus, for all $0 < t < \bar{t}$, $2\epsilon + tf(t) \geq 1$. However, this implies that for $t \in (0, \bar{t})$, $F(t) = \int_0^t f(x)dx \geq \int_0^t \frac{1-2\epsilon}{x}dx = \infty$, which is a contradiction. □

Fortunately, Theorem 2's conclusion is true under a relatively weak alternative condition. All that is required is that F is concave. The intuition is that when there is a high probability that bidders have low valuations, player 1 does not find it worthwhile to bribe player 2. This is the same intuition as initially proposed by R13.

Theorem 2′. *If F is concave, the first-price TIOLI game has a trivial equilibrium.*

To prove this theorem we first establish a useful lemma using a geometric argument.

Lemma 1. *Suppose b and x are two positive numbers such that $b + x \leq 1$. Then*

$$F(b)x + [F(b + x) - F(b)]\, b \leq \int_0^{b+x} F(t)dt. \tag{1}$$

Proof. We consider two cases. In case 1, suppose $0 \leq x \leq b$. We make our argument with reference to Figure 1a. In the figure, $\int_0^{b+x} F(t)dt$ is the region below the thick curve, $F(t)$, to the left of $b+x$. The left-hand side of (1), $F(b)x + [F(b + x) - F(b)]\, b$, equals the shaded region, or $A+B+C+D$. Since F is concave and therefore $F(x) \geq F(b+x) - F(b)$, it easily follows that $Y \geq B$. Thus, it is sufficient to show that $X \geq C$. By concavity of F, $D \geq C$. Finally,

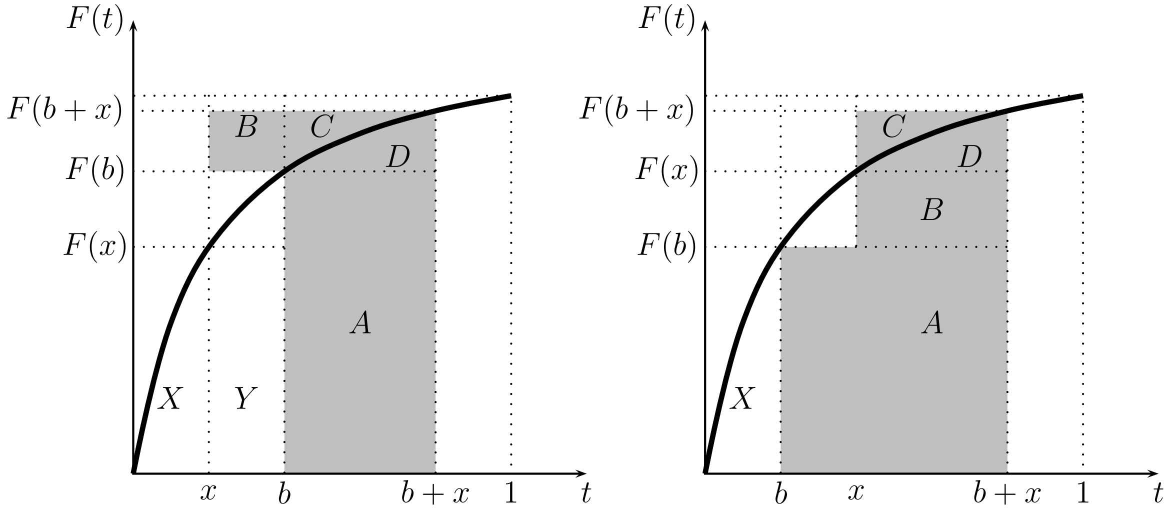
$$D = \int_b^{b+x} F(t) - F(b)dt = \int_0^x F(t + b) - F(b)dt \leq \int_0^x F(t)dt = X.$$

Hence, $X \geq D \geq C$ as required.

For case 2, suppose $0 \leq b \leq x$. The situation is as in Figure 1b. Again it is sufficient to show that $D \leq X$. This inequality follows since

$$D = \int_x^{b+x} F(t) - F(x)dt = \int_0^b F(x + t) - F(x)dt \leq \int_0^b F(t)dt = X.$$

□



(a) Case 1.

(b) Case 2.

Figure 1: The geometric argument in Lemma 1. Figures not to scale.

We can now prove Theorem 2′ by adapting the argument from the proof of Theorem 2 in R13.

Proof of Theorem 2′. Consider the following strategy profile. Player 1 offers a bribe of zero independent of his type. If this bribe is rejected, he bids as in the one-shot, symmetric Bayesian Nash equilibrium (BNE) of the first-price auction. Irrespective of type, player 2 rejects a bribe of zero and post-rejection bids as in the symmetric BNE of the auction. Player 2 accepts the bribe of $b > 0$ if and only if his valuation $\theta_2 \leq b$. If player 2 rejects the bribe $b > 0$, he believes that player 1 is bidding $(\theta_2 - b)$, and he bids $(\theta_2 - b)^+$.[2] In this case, player 1 is prescribed his optimal bid in this post-rejection-of-b information set (it is easy to show that such a best-response exists).

It is sufficient to verify that player 1 does not have a profitable deviation to a strictly positive bribe. Let $b > 0$ be the bribe offered by player 1 and let x be player 1's bid in the auction following the (possible) rejection of b by player 2. Obviously, we can assume that $x \leq 1 - b$.[3] Given the prescribed (off-equilibrium path) behavior of player 2, the expected

[2]The bid r^+ is identical to the bid r, except that it wins for sure if the competing bid is $r' \leq r$. See R13 for the details.

[3]If player 2 rejects the sure payoff b, then optimality dictates that he does not bid more than $\theta_2 - b$ in the auction; therefore, player 1 has no reason to bid strictly above $1 - b$.

payoff of player 1 is

$$\Pi(b, x|\theta_1) = F(b)(\theta_1 - b) + \left[F(b+x) - F(b)\right](\theta_1 - x). \tag{2}$$

On the equilibrium path, the expected payoff of bidder 1 of type θ_1 is $\pi(\theta_1) = \int_0^{\theta_1} F(t)dt$. It is sufficient to verify that for all θ_1 and for all $0 < b \leq \theta_1$ and $0 \leq x \leq 1 - b$, it is the case that $\Pi(b, x|\theta_1) \leq \pi(\theta_1)$. Let $\psi(\theta_1) \equiv \pi(\theta_1) - \Pi(b, x|\theta_1)$. Note that $\psi'(\theta_1) = F(\theta_1) - F(b+x)$, so ψ has a minimum at $\theta_1 = b + x$. Also, $\Pi(b, x|b + x) = F(b)x + \left[F(b+x) - F(b)\right] b$. By Lemma 1, $\psi(b + x) \geq 0$. Therefore, $\psi \geq 0$. Put differently, $\pi(\theta_1) - \Pi(b, x|\theta_1) \geq 0$ for all θ_1. □

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