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Investment Dynamics with Natural Expectations*

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Abstract

We study an investment model in which agents have the wrong beliefs about the dynamic properties of fundamentals. Specifically, we assume that agents under-estimate the rate of mean reversion. The model exhibits the following six properties. (1) Beliefs are excessively optimistic in good times and excessively pessimistic in bad times. (2) Asset prices are too volatile. (3) Excess returns are negatively autocorrelated. (4) High levels of corporate profits predict negative future excess returns. (5) Real economic activity is excessively volatile; the economy experiences amplified investment cycles. (6) Corporate profits are positively autocorrelated in the short-run and negatively autocorrelated in the medium run. The paper provides an illustrative model of animal spirits, amplified business cycles and excess volatility.

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1 Introduction

We study an economy in which agents have the wrong beliefs about the dynamic properties of fundamentals (cf. Friedman, 1979). The premise of our approach is that economic agents tend to make forecasts based on statistical models or mental representations that tend to underestimate the degree of long-run mean reversion in fundamentals (cf. Fuster, Laibson, and Mendel, 2010; Fuster, Hebert, and Laibson, 2011).

In particular, we analyze a standard investment q -model in which agents underestimate the degree of mean reversion.¹ An economy that features such a bias will exhibit the following six properties: (1) procyclical excess optimism, (2) excessively volatile asset prices, (3) negatively autocorrelated excess returns, (4) a negative relationship between current corporate profits and future excess returns, (5) amplified investment cycles, and (6) negatively autocorrelated corporate profits in the medium run. In summary, this paper presents an illustrative model of animal spirits, amplified business cycles and excess volatility. The model provides a formal description of investment boom-bust cycles associated with “this time is different” (Reinhart and Rogoff, 2009) or “new era” (Shiller, 2005) forecasting errors.

Studying macroeconomic models in which agents underestimate the degree of mean reversion is relevant for three inter-related reasons.

First, there are several psychological biases that lead agents to underestimate mean reversion; e.g., representativeness, anchoring, and availability bias (Kahneman and Tversky, 1973; Tversky and Kahneman, 1973, 1974). Representativeness refers to the bias of mistakenly believing that properties experienced by small samples are equally present in larger samples. Thus, a small sample of recent observations are viewed as representative of the future. Anchoring and availability bias refer to the overweighting of easily accessible information, such as the most recent observation, which leads agents to overestimate the persistence of current conditions.

¹This complements the analysis presented in Fuster et al. (2011), where we study the consequences of biased expectations for asset prices and consumption dynamics in an economy with an exogenous stock of domestic capital. In the current paper we allow the domestic capital stock to be endogenous.

Second, statistical arguments favor parsimonious models, and in practice, agents do tend to estimate and employ simple forecasting models that incorporate a small number of variables. In earlier work, we have discussed how simple univariate forecasting models underestimate the amount of mean reversion when true fundamentals follow hump-shaped dynamics. For example, in Fuster et al. (2011), we study total capital income in the U.S. NIPA accounts. We find that the estimated level of long-run persistence of shocks is very sensitive to the order of the model being estimated: models with a small number of (high-frequency) lags generate estimates of persistence around one, while models with a large number of lags generate much lower estimates of persistence. For example, Figure 1 plots the associated impulse response functions for ARIMA($p,1,0$) models with $p = 1, 10, 20, 30, 40$. For ARIMA($p,1,0$) models with $p = 1$ and 10, the estimated magnitude of persistence is greater than or equal to one. For ARIMA($p,1,0$) models with $p = 30$ and 40, the estimated level of persistence is less than or equal to 0.6. More generally, Fuster, Laibson, and Mendel (2010) show that several macroeconomic time series have persistence estimates that fall sharply with the order of the model being estimated. We refer to forecasts that are based on simple forecasting models – e.g., low order ARIMA models – as “natural expectations.”

Third, a large body of evidence is consistent with agents overweighting recent observations and underestimating mean reversion. Some of the best-known evidence comes from the asset allocation decisions and expectations of investors (e.g. Chevalier and Ellison, 1997; Benartzi, 2001; Vissing-Jorgensen, 2003) and analysts (e.g. De Bondt and Thaler, 1990; Bulkeley and Harris, 1997). It is also well-established that many features of the cross-section of stock returns can be explained with investors overweighting recent observations (e.g. De Bondt and Thaler, 1985, 1989; Lakonishok, Shleifer, and Vishny, 1994). Additional support comes from lab experiments where subjects are asked to predict financial or other time series, or to trade assets (e.g. De Bondt, 1993; Hey, 1994; Haruvy, Lahav, and Noussair, 2007).

A modeling approach related to ours assumes that agents are rational but do not initially know the relevant parameters and have to learn them over time (e.g. Friedman, 1979; Sargent,

1993; Evans and Honkapohja, 2001, 2011). Such learning, in particular if the model agents are updating is misspecified or if they discard old data, can also generate volatility and additional persistence of shocks to asset prices and the economy (e.g. Friedman and Laibson, 1989; Branch and Evans, 2007, 2010). A paper that is closely related to ours is Lansing (2009), which studies an endogenous growth model in which agents overestimate the persistence of exogenous technology shocks and explores the welfare consequences of this misperception.

The argument of the paper is organized in the following way. Section 2 presents the model and its solution. Section 3 discusses the key properties of the model and illustrates these properties by studying the impulse response functions for an illustrative calibration. Section 4 concludes and identifies directions for future research.

2 Investment Model

We study a tractable version of the continuous-time q -model (e.g. Hayashi, 1982). This is a partial equilibrium model in which agents/firms are assumed to be risk neutral and the risk free rate is fixed.

We first present the model assuming that agents have correct beliefs about the data generating process (DGP) for fundamentals. We then analyze the model's properties assuming that agents *believe* that they have the correct beliefs about the DGP but actually don't. We study the model in a deterministic setting, but this assumption is without loss of generality. Adding Brownian motion to the DGP won't change the impulse response functions that we report below.

2.1 Notation and definition of the problem for a rational agent

Let i index a fixed set of firms on the unit interval, $i \in [0, 1]$. Let $k(i, t)$ represent the level of firm i 's capital stock at time t . It therefore follows that aggregate capital is given by

$$K(t) = \int_0^1 k(i, t) di.$$

Henceforth, we assume that all firms are identical and suppress the i index. Therefore, we can write

$$K(t) = k(t).$$

Let $\pi(K, X)$ represent the instantaneous flow of revenue per unit of capital, where X is an exogenous productivity measure. We make the standard assumption that greater (industry-wide) competition reduces the flow of revenue per unit of capital (holding all else equal). In other words,

$$\frac{\partial \pi(K, X)}{\partial K} < 0.$$

By definition, $k(t) \pi(K, X)$ is the instantaneous revenue flow realized by a firm with $k(t)$ units of capital. This multiplicative structure implies that individual firms have a constant returns to scale technology.

To make the model tractable, we assume

$$\pi(K, X) = 1 - K(t) + X(t).$$

We assume that the exogenous productivity parameter X mean reverts to its long-run value of zero at rate ϕ . Specifically,

$$dX(t) = -\phi X(t)$$

where ϕ is a constant.²

Firms only have one decision to make: the flow of investment. Let $\dot{k} = I$, so I is firm-level investment. Since firms are identical and indexed on $[0,1]$ it also follows that

$$\dot{K} = \int_0^1 I(i, t) di = I(t).$$

We assume that firms pay quadratic adjustment costs

$$C(I) = \frac{\alpha}{2} I^2.$$

We assume that firms also pay a (normalized) price of one for each unit of uninstalled capital. So the total instantaneous flow cost of a flow of I units of capital is $I + C(I)$.

Finally, ρ is the discount rate, which is also the (fixed) real interest rate, r . Hence, the objective function of a firm at date t can be written:

$$\int_{s=t}^{\infty} \exp(-\rho(s-t)) [k(s)\pi(K(s), X(s)) - I(s) - C(I(s))] ds$$

subject to the dynamic accumulation equation

$$\frac{dk(t)}{dt} = I(t).$$

The optimizing firms take $K(t)$ as exogenous. In other words, their own choice of $I(t)$ does not affect the path of $K(t)$.

²Adding Brownian motion to these dynamics, e.g.,

$$dX = -\phi X + \sigma dz$$

does not affect the impulse response functions that we discuss below. Hence, we omit Brownian motion to simplify the analysis.

In equilibrium it must also be true that

$$\frac{dK(t)}{dt} = I(t).$$

2.2 Value Function, FOC, and q

The state variables for this optimization problem are k , K , and X . We include both k and K since these variables can deviate in principle, though they won't deviate in equilibrium.

The continuous-time Bellman Equation is

$$\rho V(k, K, X) = \sup_I \left\{ (k \pi(K, X) - I - C(I)) + E \left[\frac{dV}{dt} \right] \right\}.$$

Expanding $\frac{dV}{dt}$,

$$E \left[\frac{dV}{dt} \right] = \frac{\partial V}{\partial k} I + \frac{\partial V}{\partial K} I - \frac{\partial V}{\partial X} \phi X.$$

The first order condition is the standard one:

$$1 + C'(I) = \frac{\partial V}{\partial k}.$$

This equation implies that the marginal cost of acquiring and installing capital equals the marginal value of installed capital.

Alternatively, we can define the value function as the expected present value of the flow payoffs.

$$V(k(t), K(t), X(t)) = \sup_{I(s)} E_t \int_{s=t}^{\infty} \exp(-\rho(s-t)) [k(s)\pi(K(s), X(s)) - I(s) - C(I(s))] ds$$

For now, assume the firm has correct expectations about the future. Following the standard treatment of this model, define $q(t)$ as the marginal present value of a unit of

installed capital:

$$q(t) = E_t \int_t^\infty \exp(-\rho(s-t)) \pi(K(s), X(s)) ds$$

It follows that,

$$\frac{\partial V(k(t), K(t), X(t))}{\partial k(t)} = q(t)$$

To show this, note that

$$k(s) = k(t) + \int_t^s I(u) du$$

Substituting into the value function integral,

$$V(k, K, X) = \sup_{i(s)} E \left[\int_t^\infty \exp(-\rho(s-t)) \left[\left(k(t) + \int_t^s I(u) du \right) \pi(K(s), X(s)) - I(s) - C(I(s)) \right] ds \right]$$

Differentiating by $k(t)$, and applying the envelope theorem,

$$\frac{\partial V(k(s), K(s), X(s))}{\partial k(s)} = E \left[\int_{s=t}^\infty \exp(-\rho(s-t)) \pi(K(s), X(s)) ds \right] = q(s)$$

We can think of q as a value function, with a flow payoff of $\pi(K(t), X(t))$. Apply Leibniz's rule to show that

$$\rho q = \pi(K(t), X(t)) + E_t \left[\frac{dq}{dt} \right].$$

This equation has a standard asset-return interpretation. The required return on the marginal unit of capital, ρq , can be decomposed into a flow return, $\pi(K(t), X(t))$, and an anticipated instantaneous capital gain, $E_t \left[\frac{dq}{dt} \right]$.

2.3 Solving the System

From our assumption about $C(i)$,

$$\begin{aligned} C'(I) &= \alpha I \\ C'^{-1}(y) &= \frac{y}{\alpha}. \end{aligned}$$

The firm's policy is

$$I = C'^{-1}(q - 1) = \frac{1}{\alpha}(q - 1)$$

Aggregate capital evolves as

$$\frac{dK}{dt} = I = \frac{1}{\alpha}(q - 1)$$

We can now define a system of first-order differential equations. Define the state vector, z , for the differential equation system:

$$z \equiv \begin{bmatrix} q \\ K \\ X \end{bmatrix}$$

It is convenient to express the evolution of the system in terms of a vector D and a matrix B :

$$\frac{dz(t)}{dt} = D + Bz(t) = \begin{bmatrix} -1 \\ -\frac{1}{\alpha} \\ 0 \end{bmatrix} + \begin{bmatrix} \rho & 1 & -1 \\ \frac{1}{\alpha} & 0 & 0 \\ 0 & 0 & -\phi \end{bmatrix} z(t),$$

Define the vector z_∞ :

$$z_\infty \equiv \begin{bmatrix} 1 \\ 1 - \rho \\ 0 \end{bmatrix}.$$

This will turn out to be the steady state value of z . Note that,

$$Bz_\infty = -D.$$

Assuming B is invertible (which is a convergence assumption), the expectation of $z(t)$ can be expressed in terms of z_∞ , and deviation term, $\exp(B\tau)H(t)$, that vanishes as τ goes to infinity:

$$E_t[z(t + \tau)] = z_\infty + \exp(B\tau)H(t).$$

All that remains is to solve for the date- t forecasting “constant” $H(t)$.

We know the initial conditions for K and X . Combining these with a transversality condition – finite q as τ goes to infinity – allows us to eliminate one of the eigenvalues of B . The characteristic equation for B is

$$-(\phi + \lambda)(\lambda^2 - \rho\lambda - \frac{1}{\alpha}).$$

The positive eigenvalue from the right term is greater than ρ , implying infinite expected present value. Let V be the eigenvectors of B . Define a 2×3 matrix, L , as

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We define V to have the eigenvectors in the usual order, so that the first vector in V is the one associated with the largest eigenvalue (the one that should have zero weight). Define $H(t)$ as

$$H(t) = VL'A(t)$$

for some length-2 vector $A(t)$. The initial conditions for $z(t)$ satisfy

$$Lz(t) = Lz_\infty + LVL'A(t)$$

Solving,

$$A(t) = (LVL')^{-1}L(z(t) - z_\infty)$$

The $H(t)$ vector can therefore be written as the product of a matrix, M , and $z(t) - z_\infty$,

$$H(t) = M(z(t) - z_\infty) = VL'(LVL')^{-1}L(z(t) - z_\infty).$$

Hence the evolution of the system can be written,

$$z(t + \tau) = z_\infty + \exp(B\tau)M(z(t) - z_\infty). \quad (1)$$

It is also useful to note that

$$MB^kM = B^kM \quad \forall k,$$

which we use in the next subsection. This is true because M is constructed from the eigenvectors of B . We can also use M to determine how the vector $H(t)$, and therefore $q(t)$, evolve. Substituting for $\tau = 0$ into (1),

$$z(t) = z_\infty + M(z(t) - z_\infty)$$

Taking the total derivative,

$$dz(t) = M(D + Bz(t))$$

We can therefore also represent $z(t + \tau)$ as

$$z(t + \tau) = z_\infty + \exp(MB\tau)M(z(t) - z_\infty).$$

Note that this formulation is consistent with (1).

2.4 When Agents Have the Wrong Beliefs

Until this point, we have characterized a model in which agents have correct beliefs about the data generating process (DGP) for X . We now study the case in which the representative agent has incorrect beliefs. Let \hat{B} be the perceived DGP process, where the true rate of mean reversion in productivity, ϕ , is replaced by the perceived rate of mean reversion, $\hat{\phi}$. \hat{B}

has associated eigenvectors \hat{V} and related matrix \hat{M} . The initial condition is

$$z(t) = z_\infty + \hat{M}(z(t) - z_\infty).$$

As in the previous section, we can differentiate and plug in the evolution of $z(t)$. Note that

$$dz(t) = \hat{M}(D + Bz(t)),$$

where B is the true dynamics (not the perceived dynamics \hat{B}). Again solving for $z(t + \tau)$,

$$z(t + \tau) = z_\infty + \exp(\hat{M}B\tau)\hat{M}(z(t) - z_\infty) \tag{2}$$

Note that this equation simplifies to the no-mistakes solution if $\hat{M} = M$. Equation (2) characterizes the evolution of the system under the mistaken policy.

2.5 Impulse Response Function

To derive the impulse response function, we study the economy's dynamics when it is perturbed out of an initial steady state at date t . Assume for $\tau < 0$, that

$$z(t + \tau) = z_\infty = \begin{bmatrix} 1 \\ 1 - \rho \\ 0 \end{bmatrix}.$$

Note that when $z(s) = z_\infty$,

$$dz(s) = D + Bz_\infty = 0.$$

Assume that

$$Lz(t) = \begin{bmatrix} 1 - \rho \\ X(t) \end{bmatrix},$$

where $X(t)$ is the size of the initial (date t) impulse to X , the mean-reverting productivity variable. Note that $Lz(t)$ has only two elements– it does not include q . Then for all time $\tau \geq 0$,

$$z(t + \tau) = z_\infty + \exp\left(\hat{M}B\tau\right) \hat{M} (L'Lz(t) - z_\infty).$$

3 Properties of the Model

3.1 Illustrative Calibration

We now present an illustrative calibration. The model has four free parameters: ρ (discount rate), α (convex costs of capital adjustment), ϕ (rate of true mean reversion), and $\hat{\phi}$ (rate of perceived mean reversion). The qualitative properties of the model are not affected by the specific calibration decisions. Hence, the calibration characterizes the general qualitative properties of the model. We also use the calibration to illustrate the *quantitative* properties of the model at the calibrated parameter values.

We set the annual risk-free rate to 5% per year: $\rho = 0.05$. Because of the way that we've scaled adjustment costs, ρ does not play an important role in driving the model's properties. Hence, we could choose any (plausible) value for ρ and our dynamics would effectively be unchanged.

The parameter that scales capital adjustment costs is set to $\alpha = 10/(1 - \rho)$. With this calibration, a permanent 10% change in the steady state capital stock has a half-life of adjustment of slightly more than two years.

We assume that the true differential equation for X is given by $\dot{X} = -0.25 X$, so $\phi = 0.25$. However, agents perceive relatively little mean reversion: $\dot{X} = -0.05 X$, so $\hat{\phi} = 0.05$.

In Fuster et al. (2011), the DGP of fundamentals was hump-shaped and agents were assumed to get short-run dynamics (approximately) right but to overestimate long-run persistence. In the current paper, short-run dynamics and long-run dynamics are governed by the same parameter, ϕ , since we are now studying an environment in which agents believe

(correctly) that productivity dynamics follow a first-order auto-regressive process. In the current model it is therefore impossible for agents to get short-run dynamics right and long-run dynamics wrong. We study this particular productivity process merely because of its simplicity. In fact, we don't believe that agents *would* misforecast such a simple auto-regressive process. The mistake that agents are assumed to make in this model – underestimating mean reversion – is meant to be a proxy for underestimating *long-run* mean reversion in a more realistic model with more complicated dynamics in fundamentals (i.e., short-run momentum and long-run mean reversion). In a setting where short-run and long-run dynamics are different, it is plausible that agents would misforecast the long-run mean reversion, and that is what we are capturing in this calibration. The misforecast short-run dynamics are collateral damage in the current framework. Future research should pull the short- and long-run dynamics apart and isolate the misforecasts of long-run dynamics.

We study a productivity shock of $\Delta X = 0.10$. In the case of rational expectations, this would correspond to a temporary increase in the capital stock that would peak about 2% above the steady state capital stock four years after the initial impulse.

3.2 Impulse Response Functions

We first report a series of impulse response functions that characterize the behavior of the economy. For these figures we report the impulse response function for the first 20 years following the shock. In all of these figures we adopt the following conventions.

The dashed line represents the equilibrium path that would arise if agents all had rational expectations (the case $\phi = \hat{\phi} = 0.25$).

The dotted line represents the equilibrium path that would arise if agents' beliefs about the future dynamics of X were accurate ($\phi = \hat{\phi} = 0.05$). Motivated by our earlier work (Fuster et al., 2011), we call this case the “natural expectations forecast.”³ This is the

³This label is a partial misnomer in the current paper. In Fuster et al. (2011), natural expectations are associated with correct short-run forecasts but incorrect long-run forecasts (the long-run forecasts do not reflect enough mean reversion). In the current paper, the short-run forecast *and* the long-run forecast reflect

impulse response function that our agents (mistakenly) anticipate.

The solid line represents the equilibrium path that actually does arise, given the mismatch between beliefs ($\hat{\phi} = 0.05$) and reality ($\phi = 0.25$). We call this case the “natural expectations path”. This is the impulse response function that an outsider would observe. However, once noise is added to the economy, it would be difficult to accurately estimate this impulse response function unless the observer had a long time series database.

Figure 2 reports the impulse response for the productivity parameter X . In our illustrative calibration, the process decays at an annual rate of 25% (rational expectations). However, agents perceive that it decays at a rate of 5% (natural expectations forecast). Thus, they are excessively optimistic after a positive shock and excessively pessimistic after a negative shock.

Figure 3 reports the impulse response function for q , the price of a unit of installed capital. Since investment is affine in q , this figure also reports the impulse response function for investment. Under rational expectations, the price of capital should rise by 17% following the productivity impulse and then fall back to its steady state level with a small amount of overshooting on the way down. Under natural expectations, the price of capital rises by 26% following the productivity impulse and then falls back to its steady state level with more overshooting on the way down. Hence, the natural expectations case exhibits two kinds of excess volatility.⁴ The price rises far more in the first place and then overshoots more on the way back to the steady state. This overshooting arises because of the overhang of capital that needs to be decumulated as productivity falls. This capital overhang exists even when expectations are rational, however, the overhang is stronger in the natural expectations case, because agents under-estimate the degree of mean reversion in productivity (X) and therefore accumulate too much capital in the few years immediately following the impulse. Finally, note that all three plotted cases eventually return to a steady state value of 1 (though

insufficient mean reversion. That is a necessary but undesirable consequence of the simple data generating process – a first-order auto-regressive process – that we are studying in the current paper.

⁴The classic papers on excess volatility in stock markets are LeRoy and Porter (1981) and Shiller (1981).

this is not apparent on the truncated time scale in the figure).

Figure 4 reports the impulse response function for the instantaneous (annualized) excess returns (omitting the “infinite” positive rate⁵ of return when the initial impulse arrives). In the rational expectations case, which is *not* reported, there are no excess returns (the analogous rational expectations line is everywhere equal to zero). In the natural expectations *forecast*, which is also not reported, there are also no excess returns, since these agents believe that asset prices are efficient. By contrast, on the realized natural expectations *path*, there is a long trail of negative excess returns. The magnitude of these excess returns is empirically plausible. The negative excess returns begin at an annualized rate of -4% and slowly decline in absolute magnitude.⁶ After ten years, the annualized excess return is -50 basis points.

Figure 5 reports the impulse response function for the profitability of the corporate sector. Following the initial impulse profits jump up and then drift back down as (i) capital is accumulated, driving down industry profits,⁷ and (ii) productivity itself, X , reverts back toward its mean. In the rational expectations case, the convergence to the steady state level of profits is nearly monotonic, with only a modest degree of overshooting. In Figure 5 the rational expectations overshooting is nearly imperceptible.⁸ Hence, in the rational expectations case, profits are generally positively auto-correlated. In the natural expectations case,

⁵The instantaneous *rate* of return is infinite when the shock arrives, since q jumps up in value.

⁶In a more realistic model characterized by accurate short-run forecasts of X but inaccurate long-run forecasts, the negative excess returns would arise only in the long-run of the impulse response function. See e.g. Fama and French (1988a), Poterba and Summers (1988), and Cutler, Poterba, and Summers (1991) for early evidence on negative long-run autocorrelation of excess returns in the stock market. Other authors, such as Campbell and Shiller (1988a,b) and Fama and French (1988b) study earnings and dividend yields as predictors of future returns. In Fuster et al. (2011), we report that over the period 1929 to 2010, the correlation between excess returns of equity over the risk-free rate in year τ and cumulative excess returns from year $\tau + 2$ to year $\tau + 5$ was -0.22 , while the correlation between the ratio of S&P price at the end of year τ and average earnings over years $\tau - 9$ to τ and excess returns from year $\tau + 2$ to year $\tau + 5$ was -0.38 . That paper also gives an overview of statistical caveats that apply to these findings.

⁷Recall that the revenue per unit of capital function is assumed to be $1 + X - K$. As K rises, revenue per unit of capital falls.

⁸Note that the steady state value in our calibrated economy is

$$\begin{aligned} K_\infty \times \pi(K_\infty, X_\infty) &= (1 - \rho)(1 - [1 - \rho] + 0) \\ &= \rho(1 - \rho) \\ &= 0.0475 \end{aligned}$$

the overshooting is much more pronounced, since the capital overhang is much greater. The significant degree of overshooting generates intermediate-horizon negative auto-correlation in corporate profits.

Figure 6 reports the impulse response function for the level of aggregate capital. For the rational expectations case, capital follows a hump-shaped pattern that peaks about four years after the initial impulse. For the natural expectations case, capital also follows a hump-shaped pattern that peaks about four years after the initial impulse. However, in the natural expectations case, the amplitude of the capital response is 1.5 times as large as the rational expectations case. The larger hump arises because of the mistaken belief that the productivity impulse will only slowly mean-revert.

3.3 Dynamics in K - q Space

It is also useful to summarize the economy's dynamics with a figure in K - q space. Figure 7 draws out some of the key properties of the economy.

To read the figure, start in the lower left-hand corner. That point is the steady state. After a shock arrives, the path jumps vertically. Specifically, the price q jumps when the initial news arrives (the stock K is not a jump variable). The jump in q is much greater for the natural expectations case than for the rational expectations case. After the jump, the dynamics take the economy in a loop that begins by moving to the southeast and eventually returns to the (original) steady state. This loop is anticipated to be quite large (and slow) in the natural expectations forecast. The dynamics turn out to be quicker than anticipated because productivity turns out to mean-revert faster than anticipated. Nevertheless, the path that is actually observed in equilibrium – the natural expectations path – has a far larger loop than it would have had under rational expectations. Agents who under-estimate mean reversion accumulate too much capital and later come to regret it when the asset price (q) falls earlier and more than anticipated.

4 Conclusion

This paper examines a partial equilibrium investment problem in which agents underestimate the strength of mean reversion in fundamentals. This deviation from rational expectations generates the following equilibrium properties: (1) procyclical excess optimism, (2) excessively volatile asset prices, (3) negatively autocorrelated excess returns, (4) a negative relationship between current corporate profits and future excess returns, (5) excessively volatile investment cycles, and (6) negatively autocorrelated corporate profits in the medium run. The analysis that we have described provides a parsimonious and psychologically plausible explanation for a wide range of puzzling empirical patterns. The model also generates a series of falsifiable predictions of some regularities that have not yet been empirically investigated. Future work should test these predictions.

In this paper, we have assumed that the misperception of mean reversion applies to the beliefs about the “fundamental” driving process (here, productivity). However, one could argue that in reality, individuals’ and firms’ investment decisions may be influenced more directly by their perception of future price paths, which are in turn endogenous to expectations. It would be interesting to extend the model in this direction.

Another natural follow-up question is how non-rational expectations and non-fundamental asset price movements affect optimal monetary policy. While the illustrative model in this paper is too simple to allow adequate analysis of the trade-offs involved, work by Dupor (2005) and Mertens (2010) makes progress on this important question.

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Figure 1: Estimated Impulse Response Functions for Log Capital Income (NIPA) (from Fuster, Hebert, and Laibson 2011)

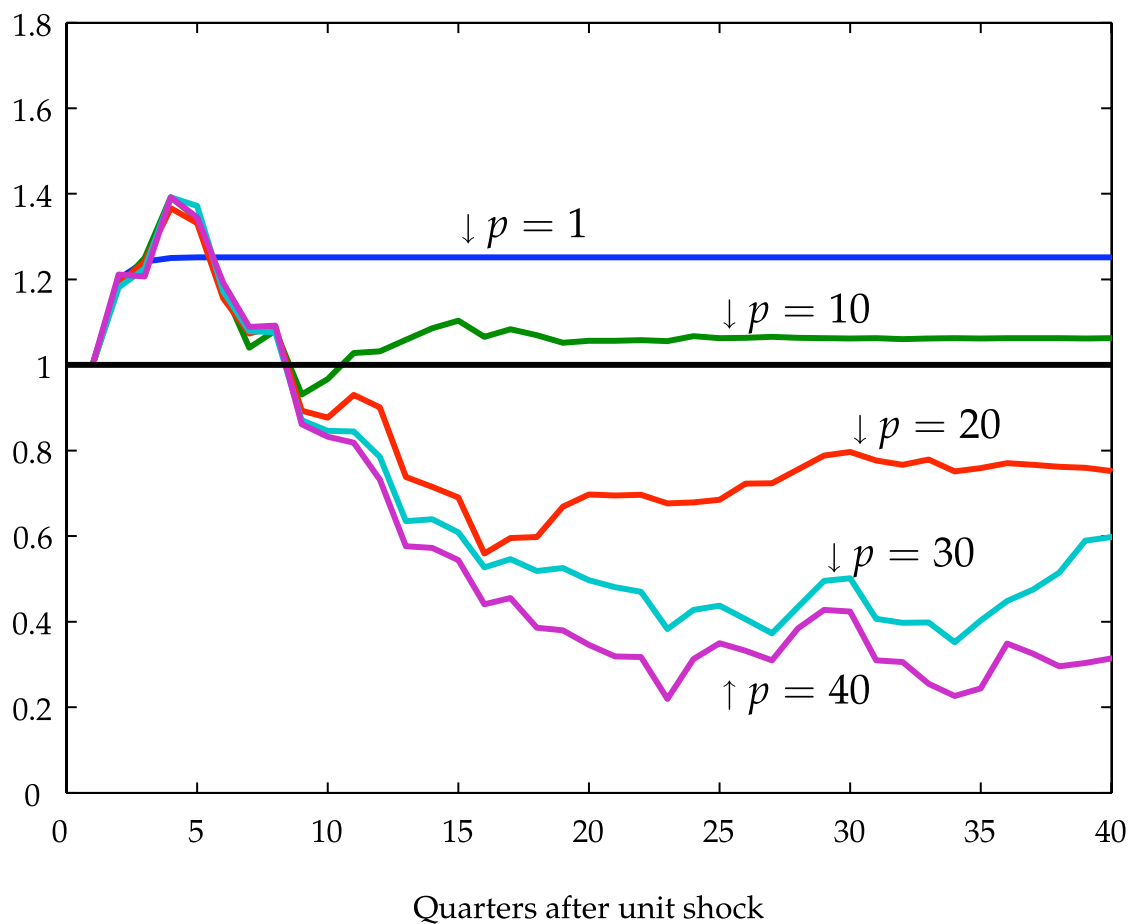


Figure 2: Impulse Response Function for Productivity

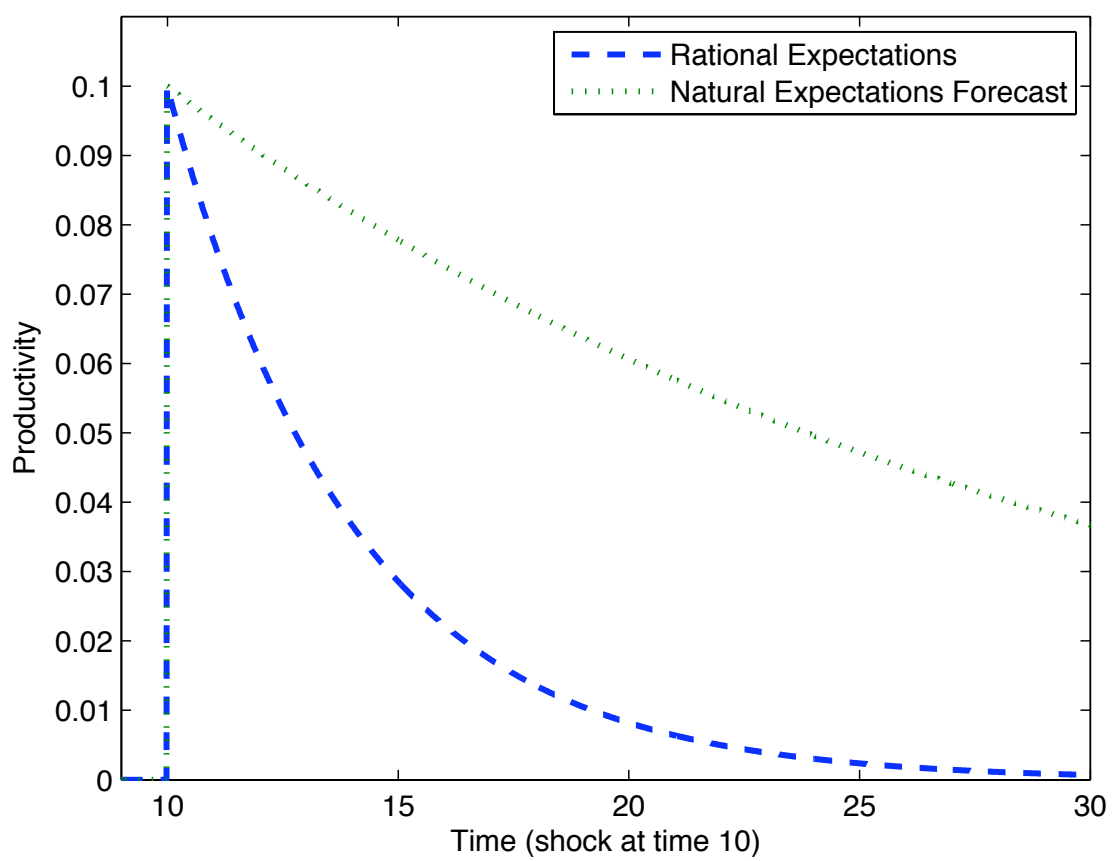


Figure 3: Impulse Response Functions for q

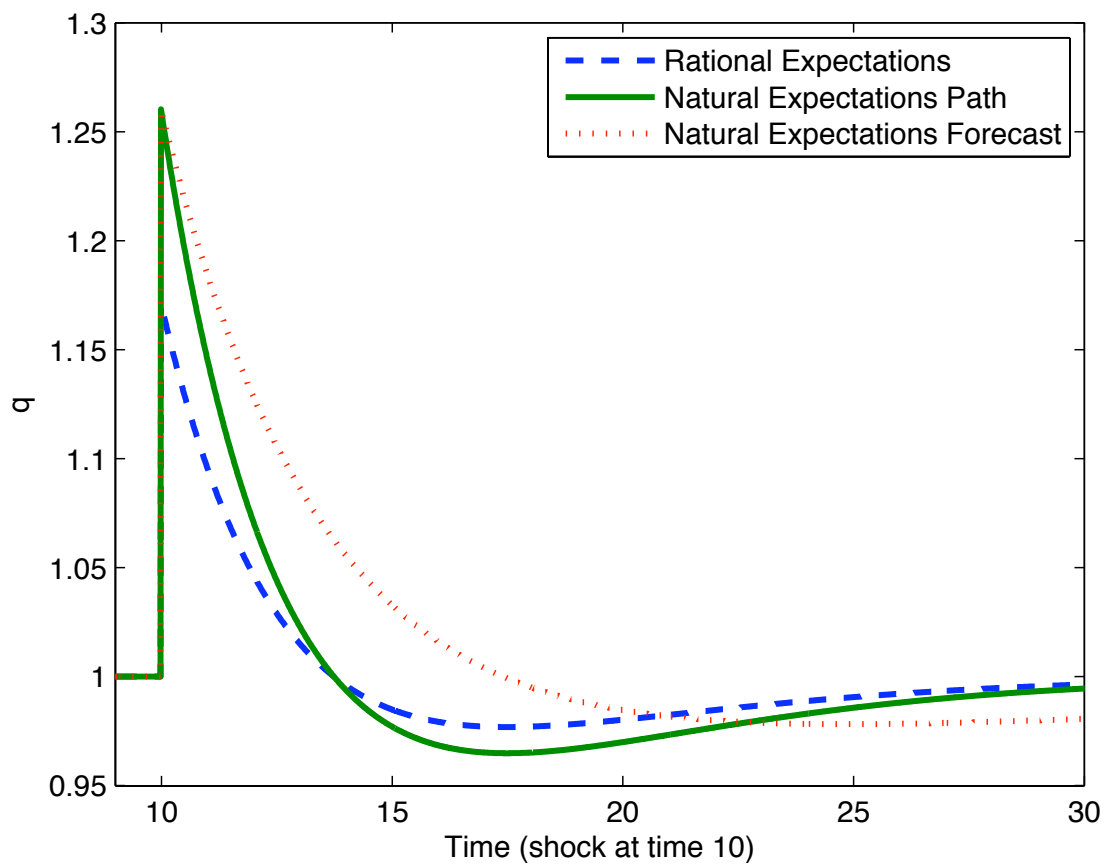


Figure 4: Impulse Response Functions for Excess Returns

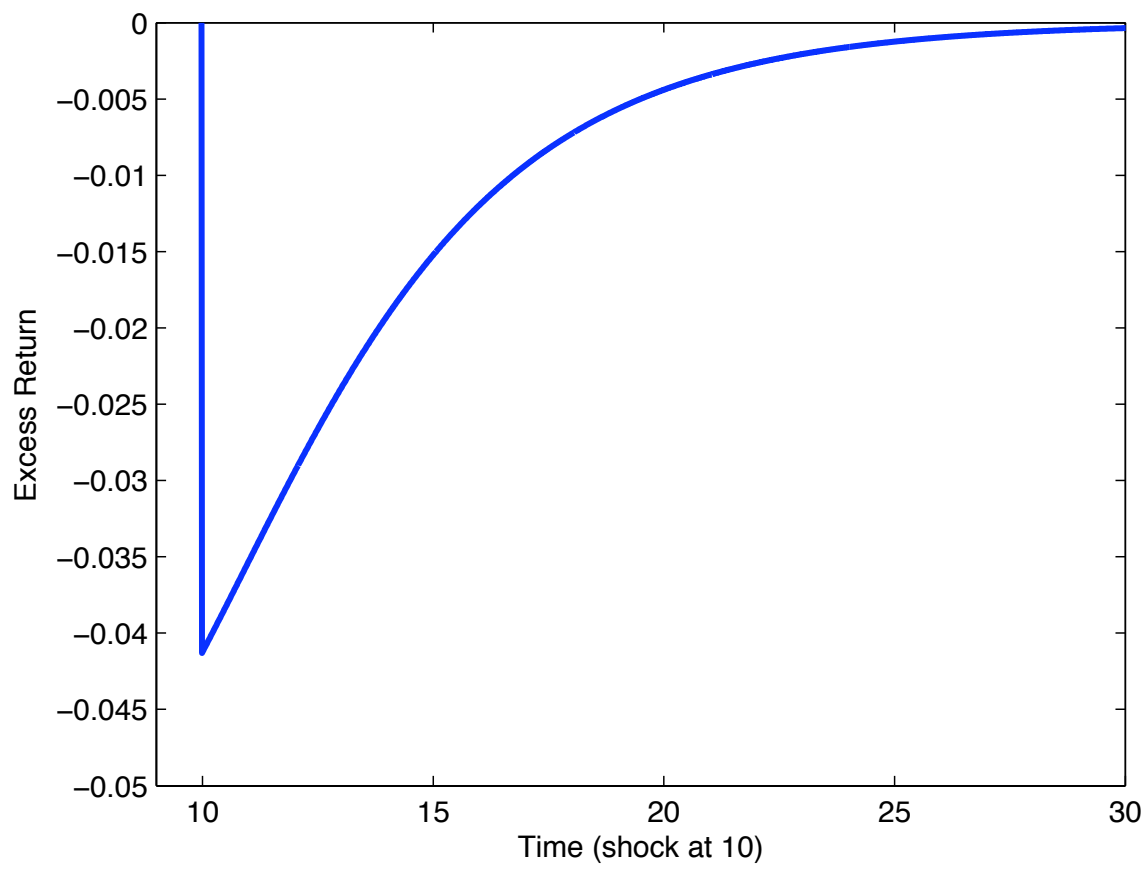


Figure 5: Impulse Response Functions for Flow Profits

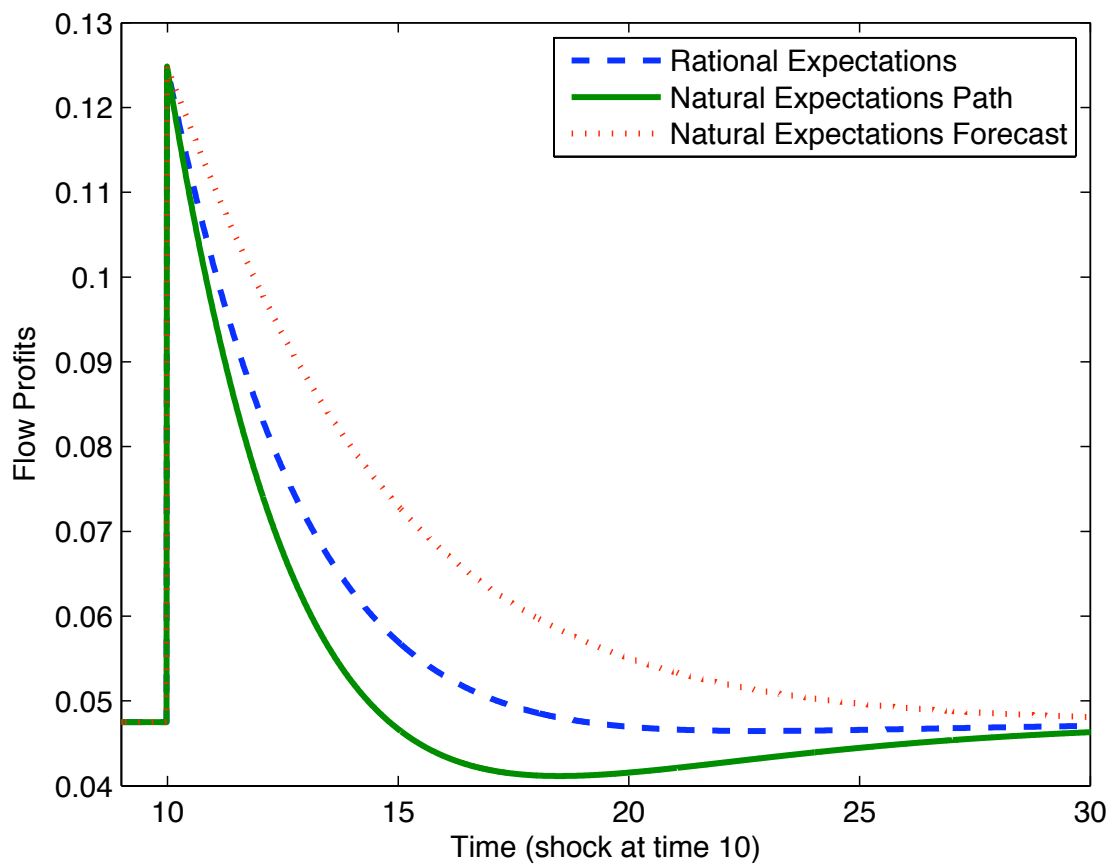


Figure 6: Impulse Response Functions for Capital

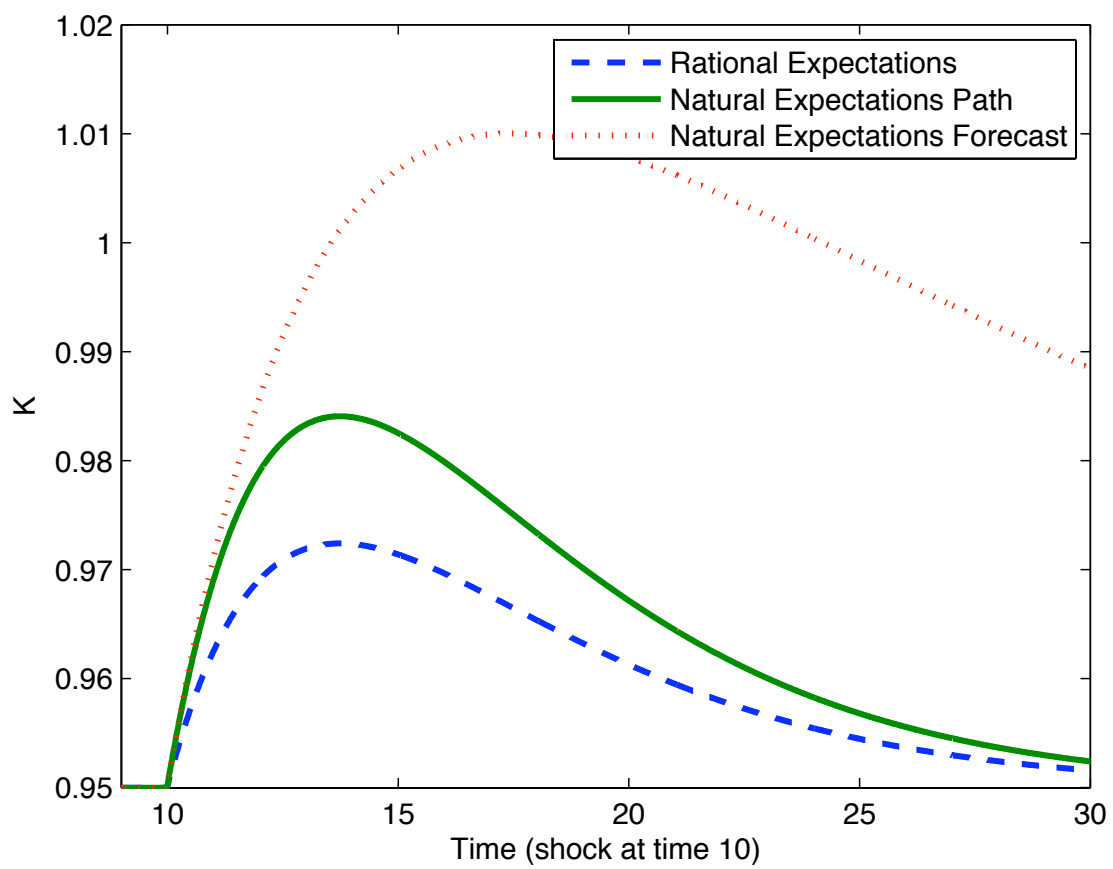


Figure 7: K-q Diagram for Impulse Response

