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An exploratory study contrasting high- and low-ability students' word problem solving: The role of schema-based instruction.

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(Article begins on next page)

Abstract

This study evaluated whether schema-based instruction (SBI), a promising method for teaching students to represent and solve mathematical word problems, impacted the learning of percent word problems. Of particular interest was the extent that SBI improved high- and low-achieving students' learning and to a lesser degree on the indirect effect of SBI on transfer to novel problems, as compared to a business as usual control condition. Seventy 7th grade students in four classrooms (one high- and one low-achieving class in both the SBI and control conditions) participated in the study. Results indicate a significant treatment by achievement level interaction, such that SBI had a greater impact on high-achieving students' problem solving scores. However, findings did not support transfer effects of SBI for high-achieving students. Implications for improving the problem-solving performance of low achievers are discussed.

KEYWORDS: word problem solving, percents, middle school students, schema-based instruction

An Exploratory Study Contrasting High- and Low-Achieving Students' Percent Word Problem Solving

1. Introduction

Proportionality is an important topic that many students struggle with in middle school mathematics (Fujimura, 2001). Within the larger category of proportion reasoning, one particularly troublesome topic for students is percent. Research has identified a number of possible explanations for students' difficulties with percent word problems (Lembke & Reys, 1994; Parker & Leinhardt, 1995). Here we focus on the importance of looking beyond surface features of word problems to identify and analyze underlying mathematical relationships (Marshall, 1995). The recognition of underlying mathematical relationships is particularly challenging with percent and proportion word problems, given the many different ways that mathematically similar problems can be expressed (Lamon, 2007; Parker & Leinhardt, 1995).

More generally, the ability to look beyond surface problem features and focus on the underlying structure of problems has been found to be a defining characteristic of expert problem solvers. Studies have shown that experts, unlike novices, possess more domain-specific knowledge, relate problem solution methods to problem classification, and organize knowledge more coherently around a central set of key ideas (Chi, Feltovich, & Glaser, 1981; Lajoie, 2003; Prawat, 1989). Further, experts and novices exhibit strategic differences in solving problems. Typically, experts in a domain solve problems by using pattern recognition procedures and work forward from problem classification to solution; whereas, novices work backward by searching for the solution strategy (Lajoie, 2003; Yekovich, Thompson, & Walker, 1991). Expert problem solvers also have deep and robust knowledge of problem solving procedures, including when,

how, and why to use a broad range of methods for a given class of problems (Baroody, Feil, & Johnson, 2007; Hatano, 2003; Star, 2005, 2007).

An emphasis on deep understanding of problem structure and flexibility in strategy use is central to one instructional intervention, schema-based instruction (SBI), which has shown great promise in helping students become better word problem solvers in elementary and middle school (e.g., Fuchs, Seethaler, Powell, Fuchs, Hamlett, Fletcher, 2008; Fuchs, Zumeta, Powell, Schumacher, Hamlett, & Fuchs, 2010; Jitendra, Griffin, Haria, Leh, Adams, & Kaduvetoor, 2007; Jitendra et al., 2009; Xin, Jitendra, & Deatline-Buchman, 2005; Xin, 2008). In particular, SBI appears to be instrumental in improving students' learning of ratio and proportion problem solving (e.g., Jitendra et al., 2009). In their study, 8 classrooms of seventh-graders ($n = 148$) were randomly assigned to either a SBI or control group. Students in the SBI group received 10 days (40 min, 5 times a week for two weeks) of instruction in applying a 4-step problem solving heuristic that emphasized recognition of the underlying mathematical structure of word problems as well as incorporated multiple strategies for solving ratio and proportion problems. In contrast, the control condition received instruction from their district-adopted mathematics textbook. The results indicated significant differences favoring the SBI condition; however, the performance of students in the low-achieving SBI class was comparable to that of students in the low-achieving control class.

The evidence from the above study, although limited in the amount of time and scope (i.e., ratio and proportion content), appears to demonstrate the potential benefits of SBI, especially for high performers. What is not known is whether SBI would be effective in improving the learning of percent, given that percent word problem solving is considered to be one of the most difficult topics for many middle school students (Lembke & Reys, 1994; Parker

& Leinhardt, 1995). As such, the primary purpose of the current study was to further evaluate the effectiveness of SBI on solving percent problems by high-achieving and low-achieving students following instruction on proportional reasoning, which occurred 3 months earlier in the Jitendra et al. (2009) study. Specifically, this research addressed two questions: What are the direct effects of SBI on high and low-achieving students' mathematical (proportion and percent) problem solving performance as compared to a business as usual control condition? What are the indirect effects of SBI on high and low-achieving students' transfer to solving novel problems? We hypothesized that low-achieving students in the SBI classrooms who entered the study without a firm understanding of proportional reasoning would not benefit as much from the higher-level percent problem solving instruction as high-achieving students with greater prior knowledge (Rittle-Johnson, Star, & Durkin, 2009) and that high-achieving students in SBI classrooms would outperform high-achieving students in the control classrooms. We also hypothesized that only high-achieving students in the SBI classrooms would demonstrate transfer to solving novel problems (e.g., probability) not directly within the learned domain content, but which also involve proportional reasoning.

Note that the data for the current study were collected as part of a larger project, parts of which have been described elsewhere (Jitendra et al., 2009; Jitendra et al., in press).

2. Method

2.1. Participants

The study was conducted in four 7th grade classrooms in a large middle school in the northeastern United States. Students at this school were assigned to mathematics classrooms by achievement levels based on their grades in mathematics from the previous school year. As noted above, for the purpose of this study, we focused only on high and low-achieving classrooms of

students and their teachers. The four randomly selected classrooms included two sections each of high- and low-achieving 7th graders. Blocking by achievement level, classrooms were randomly assigned to either the SBI or control condition.

The sample for this study included 70 students (43 girls, 27 boys). The mean chronological age of students was 12.72 years (*range* = 11.42 to 14.17; *SD* = 0.47). Ethnicity across the sample was 59% European American, 20% African American, 17% Hispanic, and 4% Asian. Of the 70 participants, 36% received free or subsidized lunch, 7% were special education students, and 4% were English language learners. Demographics by condition for each achievement level status are shown in Table 1. As shown in Table 1, a two-way analysis of variance (ANOVA) on age indicated no statistically significant effect for condition, achievement level, and interaction between condition and achievement level, ($p > 0.05$). Similarly, chi-square analyses indicated no statistically significant condition or achievement level differences as a function of gender, ethnicity, free or subsidized lunch or English language learner status ($p > 0.05$). For special education status, there were achievement level differences ($p = 0.002$) indicating more special education students in the low-achieving group than in the high-achieving group.

< Insert Table 1 about here >

Four teachers (3 males, 1 female) participated in the study; teacher demographics are provided in Table 2.

< Insert Table 2 about here >

2.2. Procedure

Instruction in percent problem solving occurred during students' intact mathematics classes over nine consecutive classroom periods (40 minutes daily) delivered by their classroom

teachers. Students in both SBI intervention and control conditions were introduced to the same topics (i.e., fractions and percents; percent problem solving; percent of change).

2.2.1. SBI Intervention

The researcher-designed SBI program consisting of a series of nine lessons replaced classroom instruction on percents for the treatment students (see Table 3 for a description of the SBI program objectives). The primary focus of the SBI approach was to promote problem identification and representation critical to solving word problems. Instruction emphasized direct teacher modeling using think-alouds to illustrate the problem solving process, such as recognizing the underlying problem structure and representing information in the problem. Eventually, teachers scaffolded instruction by gradually shifting responsibility for problem solving to the students. We used a 4-step problem-solving heuristic, FOPS (Find the problem, Organize information using a diagram; Plan to solve the problem; Solve the problem), to facilitate students' problem solving behavior. See Figure 1 for an excerpt from a script illustrating teacher "Think-Aloud" to solve a percent of change problem using schematic diagrams and the FOPS problem-solving procedure.

< Insert Table 3 about here >

< Insert Figure 1 about here >

2.2.1.1. Treatment implementation fidelity

Teachers in the SBI condition were trained by project staff in one half-day session on teaching percent problem solving at their school. (Four months prior to this training, they attended one full-day professional development session conducted by project staff on teaching ratios and proportions; this prior training also introduced teachers to the SBI instructional framework).

To evaluate treatment integrity, we observed all nine lessons in the two intervention teachers' classrooms using a fidelity observation instrument that included essential components of the intervention focusing specifically on the delivery of critical information from each lesson. Across observations in the SBI classrooms, mean treatment fidelity was 87% (*range* = 64% to 100%) for Teacher 1 and 76% (*range* = 54% to 100%) for Teacher 2, indicating moderate levels of implementation. Interrater agreement of treatment fidelity averaged 98% (*range* = 87% to 100%).

2.2.2. *Control condition*

Students in the control group received instruction from their teachers. The school used *Glencoe Mathematics: Applications and Concepts: Course 2* (Bailey et al., 2004) as the core mathematics curriculum for 40 min per day. Teachers in the control condition attended one half-day training session describing the goals of the study, the problem solving content, and how to improve student performance on the state assessment. We observed and took detailed field notes in control classrooms. Observations data indicated a high degree of uniformity in the structure of the two teachers' lessons.

2.3. *Measures*

All measures were administered by teachers, who were observed by doctoral students for their adherence to standardized administration protocols.

2.3.1. *Screening measure*

In order to assess students' general mathematical knowledge and skills, the Mathematical Problem Solving subtest of the Abbreviated Stanford Achievement Test-10th Edition [SAT-10] battery (Harcourt Brace & Company, 2003) was administered in fall. This test is a norm-referenced, group administered 30-item test that assesses number theory, geometry, algebra,

statistics, and probability. Internal consistency reliability coefficient for this test (as reported in the technical manual) is above 0.80. For the present study, Cronbach's alpha was 0.79.

2.3.2. *Mathematical problem-solving (PS) test*

To evaluate participants' competence on solving proportion problems involving percents, they completed a written test prior to onset of the intervention and immediately after the intervention. The PS test consisted of 14-items derived from the 8th grade TIMSS, NAEP, and state assessments. Six of the items assessed percent word problem solving knowledge similar to the instructed content (see Table 4). The remaining 8 items were used to evaluate the indirect effect of SBI on transfer to novel problems (e.g., proportion problems involving novel content such as probability and more complex items) not employed in the treatment. We present the reliability estimates for the PS test in Table 5 and correlations between measures in Table 6. As expected, the reliability estimates for the total test are greater than for either of the subtests individually, and with the exception of the 6 items assessing percent word problem solving knowledge are within generally accepted levels. However, the reliability estimates for the 6 items assessing percent word problem solving knowledge are of concern (particularly on the pretest) and indicate that the results regarding this outcome should be interpreted with some caution.

< Insert Table 4 about here >

< Insert Table 5 about here >

< Insert Table 6 about here >

3. Results

3.1. Pretest comparisons

A 2 condition (SBI and control) x 2 achievement level (high and low) analysis of variance (ANOVA) conducted at pretest on scores for the Mathematical Problem Solving subtest of the SAT-10 and the PS indicated initial group comparability at pretest (see Table 1).

3.2. Differential percent problem solving learning as a function of treatment

We conducted a two-factor analysis of covariance (ANCOVA) with the PS pretest – percent items serving as a covariate for the PS posttest – percent items and with condition and achievement level as the between-subjects fixed factors. Results indicated that the main effect for condition was not statistically significant, $F(1, 65) = 1.05, p = .31$. However, the effect for achievement level was statistically significant, $F(1, 65) = 6.98, p < .01$. High-achieving students (adjusted means = 3.18, $SD = 1.61$) outperformed low-achieving students (adjusted means = 2.27, $SD = 1.31$) (see Table 7). The pretest score was significant, $F(1, 65) = 15.17, p < .001$. In addition, the interaction between condition and achievement level was statistically significant, $F(1, 65) = 11.46, p < .01$ (see Figure 2). Follow-up analyses indicated that the mean problem solving scores for high-achieving students were significantly greater than the mean problem solving scores of low-achieving students ($p < .000$) in the SBI condition. In contrast, the mean problem solving scores for high-achieving students were not significantly different than the mean problem solving scores of low-achieving students ($p = .85$) in the control condition. Further, scores for SBI students in the high-achieving group were significantly greater than that of high-achieving students in the control condition ($p < .001$). We computed effect size (Hedge's g) for the problem-solving test based on results from the student level ANCOVA by dividing the difference between the regressed adjusted means (i.e., adjusted for the pretest covariate) by the

pooled standard deviation. A large effect size of 0.96 was found for the high-achieving SBI students when compared to high-achieving control students. However, the scores of low-achieving SBI students were lower than the scores for low-achieving control students, but this difference was not statistically significant ($p = 0.12$; $g = -0.49$).

< Insert Table 7 about here >

< Insert Figure 2 about here >

3.3. *Transfer effect as a function of treatment*

Results of a separate ANCOVA on the PS test – transfer items only indicated that the main effect for condition was not statistically significant, $F(1, 65) = 0.08, p = .79$. In addition, the interaction between condition and achievement level was not statistically significant, $F(1, 65) = 0.02, p = .88$. However, the effect for achievement level was statistically significant, $F(1, 65) = 8.52, p < .01$. High-achieving students (adjusted means = 4.99, $SD = 1.59$) outperformed low-achieving students (adjusted means = 3.49, $SD = 1.53$) (see Table 8). The pretest score was significant, $F(1, 65) = 15.56, p < .000$.

< Insert Table 8 about here >

4. Discussion

Within the mathematics education and special education research communities, there is growing evidence about the effectiveness of SBI in supporting students' learning of word problem solving, particularly in arithmetic (e.g., Fuchs et al., 2009; Fuchs, Seethaler, et al., 2008, Fuchs et al., 2010; Fuson & Willis, 1989; Jitendra et al., 2007; Lewis, 1989; Willis & Fuson, 1988) and ratio/proportion word problem solving (e.g., Jitendra et al., 2009; Xin, 2008; Xin et al., 2005; Xin, Wiles, & Lin, 2008; Xin & Zhang, 2009). However, other studies have indicated

that the effectiveness of SBI may be more limited with low achieving students (Jitendra et al., 2009; Jitendra, Woodward, & Star, 2011).

The present study is the first to explore the effectiveness of SBI with percent word problem solving. Of particular interest was the extent that SBI improved high- and low-achieving students' learning and to a lesser degree on the indirect effect of SBI on transfer to novel problems, as compared to a business as usual control condition. Results for percent problem solving indicated a significant condition by achievement level interaction, such that SBI improved high-achieving students' problem solving as compared to a control group but failed to do so for low-achieving students. However, findings did not support transfer effects of SBI for high-achieving students as hypothesized. It is possible that the lack of transfer is due in part to not only differences between the items on the posttest and the transfer test (i.e., items such as probability maybe less sensitive to the effects of SBI), but also due to the short duration of the intervention. Despite the multiple examples in SBI that emphasized the critical features of the various problem types, nine lessons on percent were not sufficient to impact transfer, which refers to “the incremental growth, systematization, and organization of knowledge resources that only gradually extend the span of situations in which a concept is perceived as applicable” (Wagner, 2006, p. 10). Future research should explore outcomes for students when provided with longer interventions that also make explicit connections to content outside of the instructional domain.

Although encouraged by the posttest results on percent problems for high-achieving students and by the growing evidence in support of the effectiveness of SBI across a number of mathematical domains, the failure of SBI to positively impact low-achieving students in this

study is troublesome. We discuss the implications of this work for future research in improving low achievers' word problem solving.

4.1. Implications for improving problem solving instruction for low-achieving students

Based on the literature on expert/novice differences in problem solving, SBI appears to target an appropriate set of mathematical competencies that are integral to successful word problem solving. Yet some studies (including the present study) suggest that SBI may not be as effective with low achievers as it is with high achievers. Why might this be the case? The literature points to two possible explanations.

First, low achievers may need more time and support to show gains in their ability to recognize the underlying structure of word problems. Students in this study had much less time than earlier studies in which SBI interventions were implemented for about 12 weeks on average (e.g., Fuchs et al., 2004, 2008; Jitendra et al., 2007). The limited time in this study is an artifact of the school being under pressure to teach or review grade level topics that were likely to be assessed on the annual statewide test and allocating the less than optimal time for all students to learn about a topic that often requires much longer coverage, especially for struggling students. Furthermore, whereas Jitendra et al. focused on arithmetic content, percent problem solving can be characterized as a complex cognitive skill that would require considerably more time to achieve an adequate level of competence. The results for low-achieving students in the present study can be accounted for by these learners lacking task knowledge or not having mastered the previous problem schemata and splitting their attention and working memory resources between learning the new schema (percent) and integrating the new information with prior knowledge of other salient schemata that were not fully acquired. It is possible that low-achieving students, who may have working memory deficits, found it particularly challenging to remember a variety

of previously seen problems in the short duration of this study, which hindered their ability to benefit from SBI.

Second, low achievers may need more time and support to show gains in flexible knowledge of procedures for solving a wide range of problems. The importance of intensive instruction is as crucial for developing flexibility in strategy use as it is for schema acquisition and knowledge organization. Particularly relevant is scaffolding instruction so that learning multiple problem solving strategies does not overly tax the cognitive resources of some low-achieving students. SBI required teachers to spend a great deal of time teaching multiple solution strategies so that these strategies are understood and can be applied to effectively solve problems. It is quite possible that some of the low achievers in the present study lacked fluency with more basic multiplication and division strategies; as a result, these learners may have been challenged to both learn multiple solution strategies but also to determine which strategies were easier to implement on which problems (and why).

One possible solution to addressing these sources of difficulty is to consider the particular learning challenges faced by some low achievers by providing greater time and support that include intensive instruction involving longer interventions and the use of supplemental small group pull-out tutoring sessions. Prior research demonstrates that these additions can have a substantial impact on low achievers' learning (e.g., Fuchs, Powell, et al., 2009; Fuchs, Seethaler et al., 2008; Jitendra, et al, 1998; Xin et al., 2005). In our more recent work, we provided all students with greater time and support (including pull-out tutoring), with the aim of helping low-achieving students take advantage of the promise of SBI in improving word problem solving performance (Jitendra, Star, Rodriguez, Lindell, & Someki, in press).

4.2. Limitations

Limitations in the research design call for caution when interpreting the results. First, given that classrooms rather than students were randomly assigned to the treatment and control conditions, the ANCOVA model is misspecified in that it did not partial out variance due to the dependence of students within classrooms. The small number of classrooms in this study did not allow us to conduct the required nested analysis (e.g., hierarchical linear modeling [HLM] or multilevel modeling). In addition, we acknowledge that the success of this intervention (or any intervention, even in a larger sample) is highly dependent on the way it is taught by the teachers. However, we also note that our observations of treatment teachers' lessons indicate a high degree of fidelity in teachers' implementation of the intervention.

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