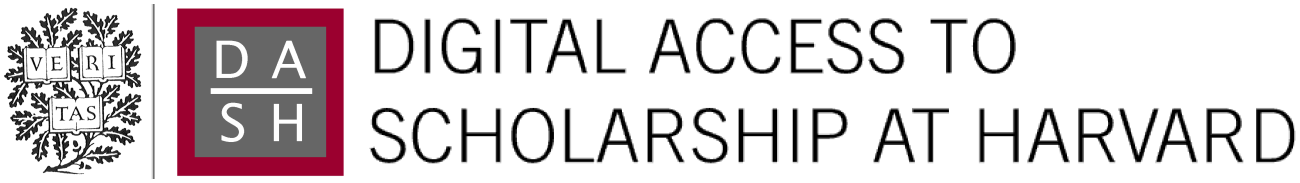

Coupling Nitrogen-Vacancy Centers in Diamond in Superconducting Flux Qubits

Coupling Nitrogen-Vacancy Centers in Diamond to Superconducting Flux Qubits

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We propose a method to achieve coherent coupling between nitrogen-vacancy (NV) centers in diamond and superconducting (SC) flux qubits. The resulting coupling can be used to create a coherent interaction between the spin states of distant NV centers mediated by the flux qubit. Furthermore, the magnetic coupling can be used to achieve a coherent transfer of quantum information between the flux qubit and an ensemble of NV centers. This enables a long-term memory for a SC quantum processor and possibly an interface between SC qubits and light.

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Among the many different approaches to quantum information processing, each has its own distinct advantages. For instance, atomic systems [1] present excellent isolation from their environment, and can be interfaced with optical photons for quantum communication. In contrast, condensed matter systems [2] offer strong interactions, and may benefit from the stability, robustness, and scalability associated with modern solid-state engineering. In recent years, much effort is being devoted to coupling atomic and solid-state qubits to form hybrid systems, combining "the best of two worlds." One attractive approach to hybrid systems involves the integration of atomic ensembles with superconducting (SC) stripline resonators. Strong coupling between SC qubits and such resonators has already been achieved [3], and approaches to extend the coupling to atomic systems have been proposed [4,5]. To achieve an appreciable coupling, these proposals often use an electric interaction, but magnetic interactions are more desirable, since long coherence times are mainly achieved in systems where spin states are used to store the information. Magnetic interactions are, however, inherently weaker, but very recently it has been proposed theoretically [5,6] and shown experimentally [7] that strong coupling to ensembles of spin systems can be achieved.

In this Letter, we propose a novel hybrid system, consisting of a SC flux qubit magnetically coupled to nitrogen-vacancy centers (NVs) in diamond. The latter system shares many of the desirable properties of atoms, such as extremely long coherence times and narrow-band optical transitions [8], but at the same time, the integration with solid-state systems can be relatively easy, as it eliminates the need for complicated trapping procedures. Additionally, much higher densities can be achieved with very limited decoherence [6,9]. As we show below, the magnetic coupling between a SC flux qubit and a single NV center can be about 3 orders of magnitude stronger than that associated with stripline resonators, thereby making the system an attractive building block for quantum information processing.

Flux qubits (FQs) form superpositions of persistent currents of hundreds of nano-Amperes, flowing clockwise and anticlockwise through micrometer-sized SC loops [10]. The magnetic field associated with this current, of the order of a μT, enables a magnetic dipole coupling to the electron spins associated with crystalline impurities such as the NV center in diamond. Of particular interest is the coincidence of energy splittings: the two states of the FQ are typically separated by a few GHz, while NV centers have a $S = 1$ ground state, with zero-field splitting $\Delta = 2\pi \times 2.87$ GHz between the $m_S = 0$ and $m_S = \pm 1$ states. By the application of a mT magnetic field, one of the spin transitions of

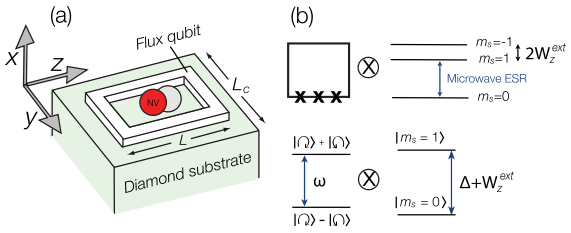

FIG. 1 (color online). (a) Schematic setup. An isotopically pure ^{12}C diamond crystal, doped with nitrogen-vacancy color centers, is located in the proximity of a flux qubit of size $L \times L$ and cross section $h \times h$. The two persistent-current quantum states of the flux qubit have different associated magnetic fields, which give rise to a state-dependent interaction with the electron spin in the NV center(s). (b) The combined system of a flux qubit and a NV center. The eigenstates of the flux qubit are superpositions of left- and right-circulating currents. An external magnetic field splits the $m_S = \pm 1$ states of the NV, resulting in a two-level system with the $m_S = 0 \leftrightarrow m_S = 1$ transition close to resonance with the flux qubit.

the NV center can be tuned into resonance with the FQ [see Fig. 1(b)]. This, together with the large magnetic moment of the FQ and the relatively long coherence times of both systems, opens the possibility of achieving coherent transfer between them.

Let us consider a *single* NV center in a diamond crystal, located near a square FQ of size L and thickness h [see Fig. 1(a)]. A static external field $\vec{B}^{\text{ext}}$ is applied, whose component perpendicular to the FQ provides half a flux quantum, and brings the qubit near the degeneracy of the clockwise and counterclockwise current states. The zero-field spin splitting of the NV center Δ sets a preferred axis of quantization to be along the axis between the nitrogen and the vacancy; thus, the field parallel to this axis sets the small additional Zeeman splitting between $m_S = \pm 1$ states, and allows us to isolate a two-level subsystem comprised by $m_S = 0, 1$.

For convenience, we denote as the z axis the crystalline axis of the NV center. The Pauli operators for the FQ system, not tied to a particular spatial axis, will be denoted by $\hat{\tau}_1$, $\hat{\tau}_2$, $\hat{\tau}_3$, with $\hat{\tau}_3$ describing the population difference between the two persistent-current states. The interaction of the total magnetic field $\vec{B}$ (external and from the FQ) with the NV center can be written as $\vec{S} \cdot \vec{W}$, where $\vec{W} \equiv g_e \mu_B \vec{B}$, g_e is the electron g-factor and μ_B is the Bohr magneton; the two persistent-current quantum states of the FQ give rise to different anti-aligned magnetic fields: $\hat{\tau}_3 \vec{W}^{\text{FQ}}$. The Hamiltonian for the system is then

$$\hat{H} = \varepsilon \hat{\tau}_3/2 + \lambda \hat{\tau}_1 + \Delta S_z^2 + W_z^{\text{ext}} S_z + \hat{\tau}_3 \vec{W}^{\text{FQ}} \cdot \vec{S}. \quad (1)$$

Here the magnitude of $\vec{W}^{\text{FQ}}$ corresponds to a Larmor frequency shift due to the circulating or counter-circulating currents in the FQ, λ is the coupling between these two current states, and ε is the bias in the two-well limit of the FQ, which depends on the external field perpendicular to the loop. If the flux qubit's plane is not perpendicular to the z axis of the NV center, both systems can be tuned on resonance by changing independently the z and, e.g., x components of $\vec{B}^{\text{ext}}$.

Because of the coupling λ in Eq. (1), the eigenstates of the FQ Hamiltonian are not left- and right- circulating current states. This means that there is a magnetic transition between the dressed states of the FQ which couples to the electronic spin of the NV. To describe this, we rotate the FQ via a unitary transformation by an angle $\cos\theta \equiv \varepsilon/2\omega$, giving two FQ dressed states with a transition frequency $\omega \equiv \sqrt{\varepsilon^2/4 + \lambda^2}$. When $\Delta + W_z^{\text{ext}} - \omega = \delta$ is small, we can transform to a rotating frame and make the rotating-wave approximation (RWA) to describe the near-resonance interaction between the NV and the FQ. Neglecting the state $m_S = -1$, due to the external field moving it far out of resonance ($|\delta| \ll |W_z^{\text{ext}}|$), the effective Hamiltonian describing the dynamics is

$$\hat{H}_{\text{RWA}} = \frac{\delta}{2}\hat{\sigma}_z + \frac{\cos\theta}{2} W_z^{\text{FQ}} \hat{\tau}_3 \hat{\sigma}_z + \frac{\sin\theta}{\sqrt{2}} W_\perp^{\text{FQ}} \hat{\tau}_- \hat{\sigma}_+ + \text{H.c.}, \quad (2)$$

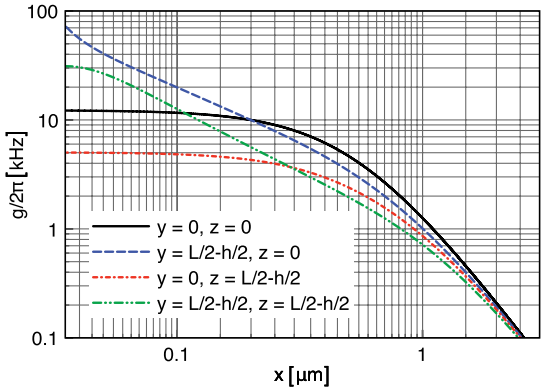

FIG. 2 (color online). Coupling g between a FQ operated at the degeneracy point ($\varepsilon = 0$) and a single NV center located at the position (x, y, z) in a reference frame with axes as in Fig. 1 but centered in the middle of the square FQ. The NV crystal axis is assumed to be parallel to one of the wires forming the flux qubit. The FQ has size $L = 1$ μm, thickness $h = 60$ nm, and a critical current of 0.5 μA.

where $\hat{\boldsymbol{\sigma}}$ are Pauli operators describing the NV $m_S = 0, 1$ electron-spin states, and $\hat{\tau}$ is in a rotated basis so that, e.g., $\hat{\tau}_3$ describes the difference in the populations of the FQ dressed states.

The coupling constant $g \equiv \sin\theta W_\perp^{\text{FQ}}/\sqrt{2}$ depends on the field perpendicular to the NV axis, and it is maximal at the degeneracy point $\varepsilon = 0$, where $\theta = \pi/2$. Furthermore, this point has a "sweet-spot" property: the energetics of the system is insensitive to small fluctuations of ε, as the eigenvalues have zero derivative with respect to ε at this point. This reduces the dephasing of the FQ due to stray magnetic fields and, e.g., paramagnetic spins in the diamond crystal [11]; it also ensures that there will be no differential shifts of the resonance frequency of the NVs due to an inhomogeneous static field from the FQ. For the remainder of this Letter we will assume that we are working at the degeneracy point. Figure 2 shows the coupling g, along four different vertical lines. We evaluate this using the magnetic field generated by a finite width square loop as given by the Biot-Savart law. For a FQ of size $L = 1$ μm and critical current 0.5 μA, the coupling reaches $g = 2\pi \times 12$ kHz for a single NV center located at the center of the loop, which is about a factor of 1000 larger than the coupling achieved with stripline resonators [6]. This coupling g is, however, too small to achieve coherent transfer, since current T_2 times in FQs are at best a few microseconds [12].

We now show that the FQ can be used as a virtual intermediary, allowing to couple coherently two or more NV centers with the same orientation and detuned δ from the FQ. For large enough detunings, $\delta \gg 1/T_2^{\text{FQ}}, g$, we can adiabatically eliminate the excited state of the FQ, and the two coupled NVs have the states $|11\rangle$ and $(|10\rangle + |01\rangle)/\sqrt{2}$ shifted by an energy $2g^2/\delta$. In contrast, the states $(|10\rangle - |01\rangle)/\sqrt{2}$ and $|00\rangle$ are not shifted by the FQ. As a result, for

a fixed time $t_X = \pi\delta/(4g^2)$ an operation resembling $\sqrt{\text{SWAP}}$—an entangling operation—between two NV centers can be implemented.

To analyze the gate fidelity, we notice that the coherence of FQs and NVs are subject to low-frequency noise. Larger coherence times are typically obtained by removing this noise during single-qubit evolution, using spin-echo sequences. Such sequences can be incorporated into the gate operation by following the prescription for composite pulses of the CORPSE family [13]. In particular, turning on and off the interaction with the FQ, e.g., by changing from a $m_S = +1, 0$ to $m_S = -1, 0$ superposition, local π pulses (described by $\hat{\sigma}_x$) allow us to realize a sequence $\text{Ex}_{\pi/4-\phi/2}\hat{\sigma}_x^1\hat{\sigma}_x^2 w\text{Ex}_\phi w\hat{\sigma}_x^1\hat{\sigma}_x^2\text{Ex}_{\pi/4-\phi/2}$, where $\phi = 2\sin^{-1}(1/\sqrt{8}) \approx 41.4°$, Ex_θ is an exchange-type interaction for a rotation θ, with $\theta = \pi$ a "SWAP", and w is a wait for a time $t_W = t_X(1/2 - 2\phi/\pi)$, which makes the time between the π pulses equal to twice the time on either end of the sequence. This approach is only sensitive to detuning errors in fourth order for both collective and individual noise on the two spins, effectively integrating a Carr-Purcell-type spin echo with the $\sqrt{\text{SWAP}}$ operation. Furthermore, the total time t_X spent interacting with the FQ during the sequence is equal to the prior case, thus inducing no additional overhead in the virtual coupling through the FQ.

With a spin echo the coherence is often limited by energy relaxation, resulting in an exponential decay $\exp(-t/T_2^{\text{FQ}})$ for the FQ [12], whereas the NVs decay as $\exp(-(t/T_2^{\text{NV}})^3)$ [9,14]. During the operation, the finite chance of exciting the detuned FQ leads to an induced decoherence rate $\gamma = 2g^2/(\delta^2 T_2^{\text{FQ}})$, resulting in a gate error $\sim((t_X + 2t_W)/T_2^{\text{NV}})^3 + \gamma t_X$. Minimizing this expression we find an optimal detuning, which gives the optimized error probability $\sim 2.2/(g^2 T_2^{\text{FQ}} T_2^{\text{NV}})^{3/4}$ [11]. For isotopically purified ^{12}C diamond [15] with an optimistic $T_2^{\text{NV}} \approx 20$ ms, $g \approx 2\pi \times 12$ kHz, and $T_2^{\text{FQ}} \approx 5$ μs, the maximum achievable fidelity of the $\sim\sqrt{\text{SWAP}}$ operation is $\gtrsim 0.98$ with an operating time $t_X + 2t_W \approx 3.3$ ms. It is thus possible to achieve a high-fidelity coherent operation between two NVs separated by micrometer distances. This coupling may even be extended to NVs separated by large distances: If two FQs are strongly coupled (directly or through resonators) with a coupling exceeding the detuning δ, two NVs residing in different FQs may be coupled through the dressed states of the two FQs. This results in a long-distance coupling between the NVs of roughly the same magnitude as derived above.

Even though the FQ coherence times are much shorter than the coupling to a single NV center, it is possible to coherently transfer the quantum-state from the FQ to an ensemble of many (N) NVs by benefiting from a $\sqrt{N}$ enhancement in the coupling constant [16]. Consider the diamond crystal depicted in Fig. 1(a), with density n of NV centers, each of them with a fixed quantization axis pointing along one of four possible crystallographic directions.

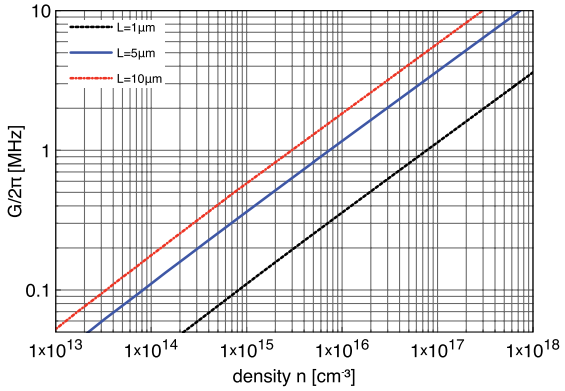

FIG. 3 (color online). Coupling between a FQ and an ensemble of NV centers, as a function of the NV density n. Results are shown for three sizes L of the square flux qubit, and are obtained by summing the inhomogeneous coupling constant, calculated as in Fig. 2, over an ensemble of NVs located in a cubic diamond crystal of size $L_C = 2L$ placed at a height $h/2$ below the FQ. In all cases, the NV crystal axis is assumed to be parallel to one of the wires forming the FQ, the width of the FQ is $h = 60$ nm, and its critical current is 0.5 μA.

If the orientations are equally distributed among the four possibilities, and the external field is homogeneous, a quarter of the centers can be made resonant with the FQ. The total coupling will then be given by the FQ—single NV interaction summed over the resonant subensemble, whose state is conveniently expressed in terms of the collective angular momentum operator $\hat{J}_+ \equiv (1/G)\sum_j g_j \hat{\sigma}_+^{(j)}$, where the sum runs over the resonant NVs, and $G \equiv (\sum_j |g_j|^2)^{1/2}$ is the collective coupling constant. At the operating temperature of the FQ (tens of mK) the NVs are near full polarization, $\hat{\sigma}_z \approx \langle\hat{\sigma}_z^{(j)}\rangle \simeq -1$, and the collective spin operators fulfill harmonic-oscillator commutation relations, thus having bosonic excitations in this limit. Tuning the system into resonance, the interaction between FQ and ensemble of NV centers is

$$\hat{H}_{\text{int}} = G\hat{\tau}_-\hat{J}_+ + \text{H.c.}, \tag{3}$$

whose dynamics can be complicated in general [17,18], but close to full polarization takes place between the states $|1\rangle_{\text{FQ}}|0^N\rangle_{\text{NV}}$ and $|0\rangle_{\text{FQ}}\hat{J}_+|0^N\rangle_{\text{NV}}$, where $|0^N\rangle_{\text{NV}}$ corresponds to all NVs being in the ground state [19]. One can thus reversibly transfer the quantum state between the FQ and the collective excitations of the NVs.

The collective coupling G as a function of the density n of NV centers, is shown in Fig. 3 for three different sized FQs. The coupling is obtained by summing the inhomogeneous coupling over NVs distributed in a crystal of size $(2L)^3$. Taking the FQ coherence time to be ~ 5 μs, we find that for a FQ with $L = 5$ μm, coherent transfer becomes possible at densities $n \gtrsim 10^{16}$ cm^{-3}, which have already been achieved in recent experiments [20]. We note that it might be possible to increase the interaction strength with the FQ by designing circuits with higher critical currents. Furthermore,

an important feature of the present approach is that strong coupling to the FQ is achieved with ensembles containing a relatively small number of spins in a few micrometers, which makes it easier to achieve fast and identical manipulation of all spins, e.g., in spin-echo approaches.

The performance of the transfer of information between FQ and collective ensemble will be limited by paramagnetic impurities present in the diamond crystal, which will interact via a dipolar coupling with the NVs encoding the collective quantum state. In the Supplementary Information we give a detailed discussion of decoherence induced by these impurities, which, due to the typically low nitrogen to NV conversion efficiency, is dominated by unpaired nitrogen electrons in the sample [11]. For a FQ of size $L = 5\ \mu$m and a density of $n = 10^{17}\ \mathrm{cm}^{-3}$ we estimate a coherence time of $T_2^* = 1.8\ \mu$s. This is sufficiently long for the infidelity induced by the paramagnetic impurities to be on the percent level, both for the transfer from the FQ to the spin wave and the nuclear storage discussed below. Furthermore the decoherence of the FQ induced by the impurities is negligible if we work close to the degeneracy point of the FQ, $\cos\theta \approx 0$.

So far we have ignored the influence of the nuclear spin. The strong hyperfine interaction with lattice ^{13}C will be detrimental to the presented schemes, but this can be overcome using isotopically purified ^{12}C diamond [15]. For the nitrogen atoms forming the centers there are no stable isotopes without nuclear spin. This spin can, however, be polarized by transferring the nuclear state to the electron spin, using a combination of radio-frequency and microwave pulses, followed by polarization of the electron spin [21], or directly through optical pumping using excited-state couplings [22]. For ^{14}N, the large quadrapolar field (~ 5 MHz) from the NV center leads to quantization of the spin-1 nucleus along the NV axis. This suppresses spin-flip terms such that the hyperfine interaction just acts as an additional parallel component to the external field. Because of the long ($\gg 1$ s) relaxation time of the ^{14}N nuclear spin, this can be accounted for by an appropriate detuning of the magnetic field so that the nuclear spin does not lead to decoherence.

The nuclear spin can actually be turned into a valuable resource for long-term storage of quantum information. The electronic spin state (m_S) can be transferred into the nuclear spin state (m_I) through a sequence of radio and microwave frequency pulses performing the evolution $\alpha|00\rangle + \beta|10\rangle \rightarrow \alpha|00\rangle + \beta|11\rangle \rightarrow \alpha|00\rangle + \beta|01\rangle$, where the states label the magnetic quantum numbers $|m_S m_I\rangle$. This allows for a long-term memory in the system while other operations are performed, e.g., while the NVs interact with light for quantum communication.

We have shown how to magnetically couple a superconducting FQ to NV centers in diamond. This may be used to achieve strong coupling of distant centers or to transfer the state of a FQ to an ensemble of NVs. The latter could be used for long-term nuclear storage, and, using the strong optical transitions of the NVs, may enable an interface between superconducting qubits and light.

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