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Two Higgs Models for Large $\tan\beta$ and Heavy Second Higgs

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Abstract

We study two Higgs models for large $\tan\beta$ and relatively large second Higgs mass. In this limit the second heavy Higgs should have small vev and therefore couples only weakly to two gauge bosons. Furthermore, the couplings to down type quarks can be significantly modified (so long as the second Higgs is not overly heavy). Both these facts have significant implications for search strategies at the LHC and ILC. We show how an effective theory and explicit fundamental two Higgs model approach are related and consider the additional constraints in the presence of supersymmetry or Z_2 flavor symmetries. We argue that the best tests of the two Higgs doublet potential are likely to be measurements of the light Higgs branching fractions. We show how higher dimension operators that have recently been suggested to raise the light Higgs mass are probably best measured and distinguished in this way.

1 Introduction

Two Higgs models are perhaps the simplest alternative to the Standard Model. They are particularly important because they are essential to low-energy supersymmetry but they of course can occur in other models that allow a broader parameter range. The phenomenology of the neutral Higgs sector is slightly subtle since the angles from mass mixing are not in general the same as the angle associated with the relative vevs. However we will see that they generally align to a large extent when one Higgs is somewhat heavier, greatly simplifying the analysis of the implications.

In this paper we explore the phenomenology of two Higgs doublet models for large $\tan\beta$ when the lighter Higgs h is light enough so that its decays are dominantly into bs whereas the second Higgs is somewhat heavy. We are motivated in part by the analysis of [1], which performed an operator analysis in the strongly interacting Higgs sector case to elucidate interesting effects that can occur when the light Higgs is part of a larger Higgs sector.

Similar considerations apply to two Higgs models, since the light Higgs is not exactly the eaten Goldstone boson in this case either.[1] For example, we find growth in WW scattering with energy though it corresponds to a higher order operator than in the strongly interacting Higgs models considered in Ref. [1] and is not the most significant deviation from Standard Model predictions.

The modification of the light Higgs coupling to bottom type quarks and charged leptons can be significant however for two Higgs doublet models. Although studying deviations in the light Higgs couplings from their Standard Model might not seem to be the best way to study a perturbative theory where the additional states are more likely to be kinematically accessible, we show that for a large parameter range the heavy Higgs will probably elude detection and precise measurements of light fields will be the best way to test the Higgs sector.

An operator analysis for two Higgses from a purely effective theory viewpoint was in fact completed in a recent paper [2] where it was shown that for large Yukawa coupling of the heavy Higgs to the down sector and small Yukawa of the heavy Higgs to the up sector one could find significant deviations in the Higgs partial widths for the light Higgs particle, even when the second heavy Higgs will elude direct detection. In this paper we relate the more conventional two Higgs analyses of Gunion and Haber [3] to the effective theory analysis of Mantry, Trott, and Wise [2]. We show that the conclusions reached in that paper (namely large corrections to b Yukawas and difficulties of finding a second Higgs) apply quite generally for large $\tan\beta$. We show that the assumption made there is in some sense less arbitrary than it might seem in that these characterizations apply to the Yukawa couplings for the heavy Higgs in the large $\tan\beta$ limit.

We show however that if the dimension-4 operators respect the Z_2 symmetry that guarantees a GIM mechanism that the Yukawa modifications are expected to be smaller since

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[1]Note that we use the conventional notation where the scalar particles in the Higgs sector are called Higgs particles. Purists might restrict this term for the linear combination with a vacuum expectation value but since the fields mix it is easier to call them all by this term and to distinguish the light, heavy, and charged Higgses.

they are no longer enhanced by $\tan\beta$. However, we will see that the effects can still be quite significant. We also consider the relationship between deviations in bottom and tau branching ratios in the various doublet Higgs models that preserve a Z_2 symmetry.

Finally we are motivated by recent data that point to high supersymmetry breaking or a new physics scale motivating considering a relatively heavy second Higgs and higher effective dimension operators in the Higgs sector. We find that the higher effective dimension operators of [20] can generate large deviations in Higgs branching ratios and that these deviations in Yukawas are most likely the best way to test for the new operators they suggest. Furthermore, these deviations in Yukawas could distinguish among the different possible higher dimension operators that could in principle correct the light Higgs mass.

2   General Two Higgs Analysis

We will consider a two Higgs theory in the decoupling limit where one of the Higgs is assumed to be light (in the regime in which decays to bs dominate) and the other Higgs is assumed to be relatively heavy. We will use two parameterizations below, and use both m_H and M to refer to the mass of the heavy Higgs.

Let us first parameterize the two Higgs Lagrangian with the notation of Gunion and Haber [3] (see also [4, 6]) but using the notation H_1 and H_2 for the two Higgs bosons. We have the gauge invariant scalar potential

$$
\begin{aligned}
V &= m_{11}^2 H_1^\dagger H_1 + m_{22}^2 H_2^\dagger H_2 - [m_{12}^2 H_1^\dagger H_2 + \text{h.c.}] + \frac{1}{2}\lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2 (H_2^\dagger H_2)^2 \\
&+ \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + (\frac{1}{2}\lambda_5 (H_1^\dagger H_2)^2 \\
&+ (\lambda_6 (H_1^\dagger H_1) + \lambda_7 (H_2^\dagger H_2)) H_1^\dagger H_2 + \text{h.c.})
\end{aligned}
\tag{1}
$$

We take all parameters to be real and CP-conserving for simplicity. In a supersymmetric model, these parameters take the values

$$
\lambda_1 = \lambda_2 = -\lambda_{345} = \frac{1}{4}(g^2 + g'^2), \quad \lambda_4 = -\frac{1}{2}g^2, \lambda_5 = \lambda_6 = \lambda_7 = 0
\tag{2}
$$

Where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$. Notice that the last two parameters are zero in any model that respects a Z_2 symmetry in the dimension-4 operators. We might expect this to be approximately the case in any of the standard two Higgs scenarios where an approximate Z_2 guarantees a GIM mechanism. However, breaking of the Z_2 in the dimension-4 operators above can exist while still not introducing overly-large flavor changing effects [2, 20]. As we will see such cases will lead to particularly interesting deviations from the Standard Model.

We now assume an appoximate Z_2 symmetry and follow the standard notation and define the ratio of vevs of the two fields as $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$, where H_2 is the field coupling to the top quarks and H_1 is the field coupling to the bottom quarks and charged leptons (here we are assuming a Type II model where this is the case but we will explore later other assumptions). The angle α determines the mixing angles of the Higgs fields mass eigenstates, so we have

$$H_1 = \frac{1}{\sqrt{2}}(\cos\alpha H - \sin\alpha L) \tag{3}$$

$$H_2 = \frac{1}{\sqrt{2}}(\sin\alpha H + \cos\alpha L) \tag{4}$$

where H is the heavy Higgs field and L is the light Higgs field and we are only considering the real parts of H_1 and H_2. Notice that with this parameterization the fields H and L have nonzero vevs, but this can be subtracted off as in [3].

The vevs for the two (real) Higgs fields (neglecting higher order terms in $(v/m_H)^2$ are given by

$$\langle L \rangle = v\sin(\beta - \alpha) \tag{5}$$

$$\langle H \rangle = v\cos(\beta - \alpha) \tag{6}$$

where

$$\cos(\beta - \alpha) \sim \frac{\hat{\lambda}v^2}{m_H^2} \tag{7}$$

and

$$\hat{\lambda} = \frac{1}{2}\sin 2\beta(\lambda_1\cos^2\beta - \lambda_2\sin^2\beta - \lambda_{345}\cos 2\beta) - \lambda_6\cos\beta\cos 3\beta - \lambda_7\sin\beta\sin 3\beta \tag{8}$$

In the large $\tan\beta$ limit, this reduces to

$$\hat{\lambda} = \cos\beta(-\lambda_2 + \lambda_{345}) + \lambda_7 \tag{9}$$

and in a supersymmetric theory we would have

$$\hat{\lambda} = -\frac{\cos\beta}{2}(g^2 + g'^2) \sim -0.3\cos\beta \tag{10}$$

Alternatively in the limit that $\tan\beta$ is large one can just solve for $\sin\alpha$ (using the mass matrices from [3]) (see Eq. (15) below) to find

$$\sin\alpha \sim -\cos\beta + \lambda_7(v^2/M^2). \tag{11}$$

Expanding out $\cos(\beta - \alpha)$ in Eq. (7) gives this same expression.

The equations above show that the masses and vevs are aligned up to $O(v^2/M^2)$ corrections. The heavy Higgs gets a vev through its interaction with the light Higgs field that has acquired a larger vev. This would have been more manifest with different notation. For example, the vev of H_1 is proportional to $v\cos\beta$ whereas the coefficient of L is $-\sin\alpha$, so it might have been natural when the quartic term doesn't dominate the mixing to have started with the rotated angle $-\sin\alpha \to \cos\alpha$ in the first place.

Notice that the equations above imply that $\alpha \sim \beta - \pi/2$ in the extreme decoupling limit where $\tan\beta\lambda v^2/m_H^2 < 1$. Although perhaps not as likely to be physically relevant, we also consider the opposite limit, where the off-diagonal term in the mass matrix changes sign. In

this case, the above results still hold for α and the vev of the heavy field, although when the λ_7 term dominates $\sin\alpha$ reverses sign.

The answer above suffices over the entire parameter range but for completeness we compare the result to that of [3] in this limit, noting that the intermediate results can depend on convention.[2] Ref. [3] gave $\alpha \sim \pi/2 - \beta$ [3], $\sin\alpha \sim \cos\beta$, which is the result when the initial conventions for the Lagrangian do not account for $\cos\beta > 0$. Minimizing the potential with respect to ϕ_1 (substituting in the assumed form for the H_1 and H_2 vevs) yields the equation [3]

$$m_{11}^2 = m_{12}^2 \tan\beta - \frac{1}{2}v^2 \left(\lambda_1 \cos^2\beta + \lambda_{345}\sin^2\beta + 3\lambda_6 \sin\beta\cos\beta + \lambda_7 \sin^2\beta\tan\beta\right) \quad (12)$$

When the λ_7-dependent-term dominates, one needs negative $\cos\beta$ to satisfy this equation. However, according to the [3] convention β is always between 0 and $\pi/2$. In order to maintain $\cos\beta > 0$ and $m_{11}^2 > 0$ (when $\lambda_7 > 0$), we need to change the sign of H_1. With this sign change, we can directly solve for $\sin\alpha$ (in the large $\tan\beta$ limit to find $\sin\alpha \sim \cos\beta - \lambda_7(v/M)^2$. In this case we can evaluate $\cos(\beta - \alpha)$ to find approximately $2\cos\beta$ as above, but the more useful quantity would be the quantity that appears in the H vev. Because we have changed the sign of H_1, we see that the vev of H is related to $-\cos(\alpha + \beta)$, and this again evaluates to $\lambda_7(v/M)^2$. Alternatively had we taken a convention where we also changed the sign of $\sin\alpha$, we would have obtained the answers we did for small λ_7 above. In either way of proceeding, the vev of the heavy field and $\sin\alpha$ (up to an unphysical sign) take the same form, even when $\lambda_7 \tan\beta(v/M)^2 > 1$. These are the physically relevant quantity that enter the heavy Higgs coupling to two gauge bosons and the Yukawas. So our results (6),(11) for the vev and mixing angle apply over the entire parameter range.

An alternative approach to a two Higgs model with a heavy Higgs is to take an effective theory approach as considered in [2]. Ref. [2] does not assume the existence of a Z_2 symmetry (in fact H_1 and H_2 are never mentioned) so the Yukawa couplings of the heavy Higgs can be taken as free parameters, but the parameters were chosen to be consistent with small FCNC (that is minimal flavor violation [5], assuming only a single Yukawa matrix structure for up type quarks and another for down type quarks). For simplicity in comparing to their results we will call their light Higgs H and their heavy Higgs S as in Ref. [2] (but note that H is now the *light* field and S is a doublet). Their Lagrangian is:

$$
\begin{aligned}
V(H,S) \;=\; & \frac{\lambda}{4}(H^\dagger H - \frac{v^2}{2})^2 + M^2 S^\dagger S + \frac{\lambda_S}{4}(S^\dagger S)^2 + \left[g_1(S^\dagger H)(H^\dagger H) + \text{h.c.}\right] \\
+\; & g_2(S^\dagger S)(H^\dagger H) + \left[g_{2a}(S^\dagger H)(S^\dagger S) + \text{h.c.}\right] \\
+\; & g_{2b}(S^\dagger H)(H^\dagger S) + \left[g_3(S^\dagger S)(S^\dagger H) + \text{h.c.}\right].
\end{aligned}
\quad (13)
$$

Note that g_1 and g_2 are couplings completely independent of gauge couplings; we have kept the notation of [2] for simplicity. Secondly, notice that all the same types of terms appear in the non-effective theory H_1 H_2 Lagrangian aside from the quadratic mass mixing

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[2]We thank Howie Haber for discussions on this limit.

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term. However, since H_2 and H (from [2]) are not identical, the gs would be a function of various couplings in the Lagrangian above. We can expand in terms of $\cos\beta$ to solve for one field in terms of the other.

To simply relate couplings we can consider the small $\cos\beta$ limit. In this limit, the heavy Higgs is approximately H_1 and the light Higgs is approximately H_2. In this limit we can expand to see that $g_1 = \hat\lambda$. For the more exact result, we can expand H_1 and H_2 in terms of H and L and include the additional Z_2-violating $\cos\beta$-suppressed terms to find $g_1 = \hat\lambda$. For simplicity, we concentrate on the λ_7 term below.

More relevantly for physical consequences, we can relate the vevs and in particular the heavy Higgs vev in the two pictures. Ref. [2] had [3]

$$\langle S\sqrt{2}\rangle = -\frac{g_1 v^3}{2M^2} \tag{14}$$

(again we are working to leading order in $(v/M)^2$). Notice that H and L are real fields in the first analysis so that the relevant field to compare to is $\sqrt{2}S$ (ignoring the other Higgs components, where H is the heavy Higgs in the fundamental theory).

Recall that when $\cos\beta$ is small, $\lambda_7 \sim g_1$. We see that the two values of the expectation value, though having the same parametric dependence, differ by a factor of -2. The reason for this is that the fundamental Higgs analysis uses the mass eigenstates for the full mass matrix, whereas the effective theory analysis did not use mass eigenstates once the g_1-dependent quartic term is included. The physical mass eigenstate is $S + 3g_1/2(v/M)^2 H$ and has vev that agrees with the vev for the fundamental theory when $g_1 = \hat\lambda$. Eq. (12) tells us that without the m_{12}^2 term that $\cos\beta$ would agree with Eq. (14) above. However, the g_1 quartic (or in Ref. [3] the λ_7-dependent quartic) also contributes to mass mixing, so the the heavy *physical* mass eigenstate has the vev cited in Eq. (6).

To further understand this result, it is of interest to consider the contributions of the quadratic and quartic terms to both mass mixing and vev. Had the only mixing term between H_1 and H_2 been a mass term, one could in fact simultaneously diagonalize the mass and vev. However, the relative mass squared coming from the quartic is $3/2g_1(v/M)^2$, whereas the vev contribution to the linear term is $g_1/2(v/M)^2 v$. So a piece of the quartic can be absorbed into M_A^2 as is done in [3]. That is, the mass matrix takes the form

$$M^2 = M_A^2 \begin{pmatrix} \sin^2\beta & -\sin\beta\cos\beta \\ -\sin\beta\cos\beta & \cos^2\beta \end{pmatrix} + \mathcal{B}^2 \tag{15}$$

where $M_A^2 = \frac{m_{12}^2}{\sin\beta\cos\beta} - \frac{1}{2}v^2(2\lambda_5 + \lambda_6\tan\beta^{-1} + \lambda_7\tan\beta)$ and the off-diagonal part of $\mathcal{B}^2$ contains a term $\lambda_7 v^2 \sin^2\beta$. After full diagonalization, one is left with the vev of the heavy Higgs eignenstate $\cos(\beta - \alpha) = \hat\lambda(v/M)^2$ as we found above.

Before closing this section, we remark on how small the VEV of the second field is likely to be. This makes the coupling to two Ws very suppressed, which is essentially why the heavy Higgs search is quite difficult as we will discuss further shortly. In the next section we

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[3]Note the sign correction to [2].

discuss the deviation of the *light* Higgs Yukawa from its Standard Model value. For a large range of parameters, this is the likely to be the best way to search for evidence of a second Higgs.

3 Yukawas

Given the expressions for H_1 and H_2 in terms of H and L, we can work out the Yukawas for the light and heavy fields to the up and down type quarks. In this section we will focus on precision light Higgs measurements and study the deviation of Higgs couplings to fermions from their Standard Model values. We will first consider Type II models (as in the MSSM) in which one Higgs gives mass to charged leptons and down-type quarks and the other Higgs gives mass to up-type quarks. We then have (in relation to the standard Yukawa couplings) [3,8]

$$ hD\bar{D} : -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha) \tag{16} $$

$$ hU\bar{U} : \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha) \tag{17} $$

Note that both of these are of order unity when the second Higgs is heavy and $\cos(\beta-\alpha)$ is small, as they should be in the decoupling limit. We also see that the corrections term in the down-type Yukawa can grow with $\tan\beta$ and be quite large.

Ref. [2] did not assume a Z_2 symmetry but did assume minimal flavor violation. Note that this is more general in that with Z_2 symmetry, there are only three distinct possibilities, in which either the same Higgs or orthogonal Higgses couple to up and down type quarks respectively. With only MFV, one can in principle define the Higgs that couples to up-type quarks and the one coupling to down-type quarks as H_2 and H_1, but these are not necessarily either the same or orthogonal so there is a continuum of possibilities. However, we will see that only the down-type Yukawa deviations are likely to be significant when $\tan\beta$ is large so the difference isn't necessarily significant.

The authors of Ref. [2] defined parameters η_d and η_u which when multiplied by the light Higgs Yukawas of the effective theory gave the heavy Higgs Yukawas. In terms of the quark masses (and including both the light and heavy Higgs vev contributions), the Yukawa couplings of the heavy Higgses are therefore

$$ -\eta_d\sqrt{2}\bar{d}\frac{m_d}{1+\sqrt{2}\eta_d\frac{\langle S\rangle}{v}}d \tag{18} $$

and similarly for up quarks, where this expression includes the S vev contribution to the quark masses.

Ref. [2] considered the possibility that η_d is large and η_u is small. By integrating out the heavy Higgs (and including its vacuum expectation value contribution to the quark masses),

they found a light Higgs Yukawa coupling [4]

$$\frac{1 - \frac{3}{2}g_1\eta_d\left(\frac{v}{M}\right)^2}{1 - \frac{1}{2}g_1\eta_d\left(\frac{v}{M}\right)^2} \tag{19}$$

They noted that the correction is large when η_d is big, which is clearly similar to the observation we made above for large $\tan\beta$.

We now show the similarity of these large Yukawa corrections is not a coincidence and that such a scheme is a generic prediction of large $\tan\beta$.[5] This large deviation has significant implications for the search for a second Higgs.

The heavy Higgs coupling to down type quarks (again in relation to standard Yukawas) is given by

$$\frac{\cos\alpha}{\cos\beta} = \cos(\beta - \alpha) + \tan\beta\sin(\beta - \alpha) \approx \tan\beta \tag{20}$$

whereas the coupling to up quarks is given by

$$\frac{\sin\alpha}{\sin\beta} = \cos(\beta - \alpha) - \cot\beta\sin(\beta - \alpha) << 1 \tag{21}$$

So we see that large $\tan\beta$ naturally yields a large Yukawa coupling of the heavy Higgs to down quarks and a suppressed coupling to up type quarks. We can see this directly in equation (20) noting that $\cos\alpha$ is very close to $\sin\beta$ (which follows from $\cos(\beta - \alpha)$ being small), so that the value of η_d that this model matches onto is very close to $\tan\beta$. This follows from the original Z_2 symmetry, which favors the heavy Higgs which is approximately H_1 coupling to down quarks and the light Higgs which is approximately H_2 coupling to light quarks.

For completeness and to elucidate the origin of this correction we do the matching for the heavy Higgs down-type Yukawa couplings more exactly in order to compare the two formulations. Again when comparing the results we need to take into account that the [2] analysis based on the effective theory doesn't use the fully diagonalized states. So the Yukawa for the light not quite diagonalized field in the fundamental theory would be approximately $-\sin\alpha + \frac{3}{2}\lambda_7(v/M)^2 \sim \cos\beta + \frac{1}{2}\lambda_7(v/M)^2$. So we identify

$$\eta_d = \frac{\tan\beta}{1 + \frac{\lambda_7}{2}\tan\beta\left(\frac{v}{M}\right)^2} \tag{22}$$

from which we conclude

$$\frac{1 - \frac{3}{2}g_1\eta_d(v/M)^2}{1 - \frac{1}{2}g_1\eta_d(v/M)^2} = 1 - \lambda_7\tan\beta(v/M)^2 \tag{23}$$

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[4]Notice a sign correction from Ref. [2]. This sign has physical consequences since it is the deviation from the Standard Model value.

[5]Of course the Lagrangian in [2] is more general, and $\tan\beta$ is not even defined in the absence of a Z_2 symmetry [9]. Our point is that the particularly interesting case of large $\tan\beta$ is an example of this type of parameter regime.

(where we have made the approximate identification λ_7 with g_1) which agrees with Eq. (16). Notice that when integrating out S to determine the Yukawa, one is effectively accounting for the mass mixing so in this case the results in the two formulations agree. That is, the Yukawa in Eq. (19) is really the Yukawa for the physical light Higgs. Also note that the different Z_2-violating quartic contribution to the mixing and the vev leads to the correction to the light Higgs Yukawa.

We see in either formulation that the correction can be quite large in the large $\tan\beta$ (or large η_d) regime. For large $\tan\beta$ and not overly heavy Higgs mass, we can have large corrections to the bottom and tau (in type-II models) Yukawa couplings. The sign of the correction depends on the sign of $\hat\lambda$, which is in general unknown but is determined in supersymmetric models or other models where the physics constraints determine the sign (see below).

In Ref. [2], parameters such as $g_1 \sim 2$ (note that we have changed the sign of g_1 to reflect the sign correction in the Yukawa and the S vev) and $\eta_d \sim 20$ were considered, corresponding to large $\tan\beta$ and moderate $\hat\lambda$. For these parameters the total width could change substantially, being corrected by a factor of 121 for Higgs mass of order a TeV. If, on the other hand parameters were $g_1 = -1$ and $\eta_D = -10$, the branching ratio was down by 0.008. [2] imagined that the bottom coupling was changed and the η_d-enhanced deviation from the Standard Model may or may not apply to the τ as in Type II models.

Note that for either sign of the correction, the rate of decay of the light Higgs into b quarks and hence the total width and the branching fractions into other modes (in the light Higgs regime where decays to bs dominate) will deviate from the Standard Model predictions. In Type II models where leptons and down type quarks both couple to H_1, the best measurement of this Yukawa deviation at the LHC will be the relative branching ratios of photons and taus. In other models in which the tau Yukawa is not changed directly by a large amount but only through the change in total width (as might happen for more general MFV models or in Type III models where the up-type Higgs couples to leptons), one would need to measure the absolute decay rate into τs or photons since both branching fractions change indirectly through the change in the Higgs total width. In this case, the tau rate would increase or decrease when the photon rate does, unlike Type II models where they would change in opposite directions. The ratio of photon and tau partial branching fractions is likely to be measured at the 15-30 % level [6, 14] and absolute branching fractions might also be measured at reasonable levels [14]. Of course especially for the photon loop effects from nonstandard model physics might also be significant. In addition, radiative effects involving bs might further suppress this decay [15] as we further discuss below. Whether or not radiative effects are significant, tree level effects can dominate and give rise to deviations from the expected Standard Model ratios at a potentially measurable level for the LHC and a readily attainable level for the ILC.

Notice that the results are very similar to those from [2] since the (large) corrections to the down type Yukawa coupling match. The difference would be only in the up type Yukawas, where the [2] Lagrangian has a correction η_u which is in principle independent of η_d. However, since this is small by assumption, it won't make any measurable difference.

It is also useful to note what happens to Yukawa modifications when a Z_2 symmetry is

preserved by the quartic interactions that would forbid λ_6 and λ_7. In that case, the $\tan\beta$ enhancement no longer exists, since $\hat{\lambda}$ is proportional to $\cos\beta$. This is in fact what happens in the MSSM. Although this can decouple more quickly than without $\tan\beta$ enhancement as has been noted in several places (see [3, 7] for example), and is not an enhancement that would allow the sort of large change in branching ratio that was considered in Ref. [2], it still might be measurable.

For example, from Eq. (8), we can deduce the tree-level change in Yukawa in a supersymmetric theory, which is $\frac{g^2+g'^2}{2}\frac{v^2}{m_H^2}$, which is about 0.3 for Higgs mass comparable to v. The LHC will measure couplings, even for the tau, to an accuracy of at most about 15 % [10, 11, 14]. This means that a 2 sigma measurement might just probe this deviation from the Standard Model. Our calculation would have to be performed more reliably in the limit that the second Higgs is light enough to generate a measurable deviation in Yukawas, but is probably reasonably accurate since the expansion really involves the light Higgs mass squared divided by the heavy Higgs mass squared. We leave more detailed study with light second Higgs mass for future work.

At the ILC, both b and τ partial widths will be well measured, with the b partial width particularly accurate. The anticipated experimental accuracy in the b width will be between 1 and 2.4%, that for the τ is between 4.6 and 7.1%, for the photons is between 23 and 35%, and for the c is 8.1-12.3% (see Ref. [19] and references therein). These numbers do not include the theoretical uncertainties estimated to be about 2% for the bs and 12% for the cs for example [6]. Note that the best measured mode at the ILC, the b decay mode, is most likely to have a Yukawa that deviates from its Standard Model prediction. A sufficiently accurate measurement of the total width will also probe deviations of the decay width to bs when that mode dominates. Clearly, by measuring these relative rates at the ILC one can hope to explore much higher masses indirectly through precision light Higgs studies. This could be a very interesting probe of higher-energy physics than will be directly accessible.

Notice also that the radiative corrections for very large $\tan\beta$ in supersymmetric theories can take the opposite sign to the tree level corrections we have considered here, as analyzed in [15]. If $\tan\beta$ is indeed very large these radiative corrections need to be taken into account and can end up suppressing the b branching fraction.

It is straightforward to extend our analysis to the lepton sector. We consider models that preserve a discrete Z_2 flavor symmetry so that only one type of Higgs field has a tree-level coupling to each of the different fermion types. Clearly, only in Model II, where we expect the leptons to couple to H_1 as do the down quarks, do we expect $\tan\beta$ enhancement in the lepton Yukawas. In these models we would expect the τ branching fraction and b branching fraction to change a comparable amount (up to loop effects). Radiative corrections to the bottom can be much bigger than those to the tau [12] (see also [13], but unless $\tan\beta$ is very large these are generally smaller than the tree-level corrections but eventually should be accounted for as well.

In Model I, where only a single Higgs participates in the Yukawas, we expect H_2 to couple to all fermions or else the top quark mass would be too low, which means that no fermions would get large Yukawa corrections. In Type III models as well, the leptons couple to H_2.

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In both of these latter cases, the correction is suppressed by a factor of $\cot \beta$ and will be too small to matter in the large $\tan \beta$ limit.

4 Gauge Boson Coupling

It is also interesting to consider the light Higgs to two W coupling since the growth with energy isn't fully stopped until we reach the second Higgs. This is similar to the analysis of [1] where it was argued there would be growth with energy in WW scattering until the strongly interacting scale in composite Higgs models. In practice at the LHC this will probably be a less promising way to search for evidence of a second Higgs because the $H \to WW$ won't be sufficiently precisely measured since the energy reach isn't big enough to enhance the cross section sufficiently, and because in the case of a doublet Higgs field the corrections to the scattering effectively arise from higher-dimension operators than in Ref. [1].

One way to understand the source of the correction to the Higgs WW coupling in the strong coupling case [1] is from a higher order operator of the form

$$\frac{c_H}{2f^2}\frac{v^2}{f^2}\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H) \tag{24}$$

where f is the scale of strong physics, which, after a shift in H field gives a correction

$$c_H m_W^2 \frac{h}{v}W_\mu W^\mu \tag{25}$$

where h is the light Higgs. In effect, a dimension-6 operator could arise only in the presence of a singlet or triplet to be exchanged. In our case, with only a doublet Higgs, our correction is higher order. We expect a correction to hWW of order $(v/m_H)^4$.

In practice, we know precisely the coupling of h to a pair of Ws. It is proportional to $\sin(\alpha - \beta) = 1 - \frac{\cos^2(\alpha-\beta)}{2}$. The correction to unityh is indeed suppressed by $(v/m_H)^4$ as we expected and is likely to be too small to measure.

5 Heavy Higgs Direct Searches

The heavy Higgs two vector boson coupling is suppressed by $\cos(\alpha - \beta)$, since the vev of the field is suppressed by this factor and the vev enters the single Higgs two gauge boson coupling. This means that even when the two gauge boson decay is kinematically allowed, it won't generally dominate. Similarly, heavy Higgs boson production is suppressed.[6] Notice in the coupling to two Ws there are no compensating $\tan \beta$ factors as there were for the down and potentially τ Yukawa corrections so the heavy Higgs to two gauge boson coupling is indeed small.

CMS recently (2007) [17] studied the heavy Higgs discovery reach in the MSSM with systematic uncertainties taken into account. They found for a relatively light second Higgs

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[6]Here we are neglecting the other Higgs states but these will also be difficult to find.

(CP even or odd) that to find a Higgs of 150 GeV, $\tan\beta$ must be greater than about 16 and for a Higgs of 250 GeV, $\tan\beta$ must be greater than about 35. This can be compared to the results from the Atlas TDR from 1999 [18] quoted by [10] where it was already noted that for Higgs mass of 250 GeV, $\tan\beta$ greater than 8 was necessary whereas for 500 GeV $\tan\beta$ needs to be at least 17. Clearly the situation has become worse with better understanding of the systematics and a reasonably large value of $\tan\beta$ is required to discover the heavy Higgs.

The required large value of $\tan\beta$ is readily understood from our earlier considerations. In Type II two Higgs models preserving a Z_2 symmetry, large $\tan\beta$ tells us the coupling of the heavy Higgs to bottom type quarks is enhanced whereas the coupling to top quarks and two gauge bosons is suppressed. Therefore production through bottoms is enhanced when $\tan\beta$ is large. Moreover decays to taus are enhanced in this limit as well and that is likely to be the best search mode. Note that even with the $\tan\beta$-enhanced coupling to bottom quarks, the amplitude is proportional to the bottom Yukawa as well so only when $\tan\beta$ is sufficiently sizable will the production and decay become visible.

Notice that although the analysis was done for the MSSM, the answer can be readily taken over to more general two Higgs models. The bottom and top Yukawas will be determined by $\tan\beta$ at leading order. The more model-dependent coupling is the coupling to two gauge bosons which is suppressed by the heavy Higgs vev (or equivalently $\cos(\beta-\alpha)$). Once this is sufficiently small neither production nor decay through this mode is relevant.

Note that Ref. [2] considered particular parameters in the two Higgs model to show that a heavy Higgs (of order TeV) can readily elude detection but induce large deviations from the Standard Model in the low-energy effective theory. They had large bottom Yukawa and small top Yukawa (to suppress standard Higgs production channel). Our point is that this happens automatically for large $\tan\beta$ (but not so large that the Higgs will be produced directly). Furthermore the CMS analysis shows that even a much lighter heavy Higgs than considered in [2] will not be seen unless $\tan\beta$ is sufficiently large. Of course even if $\tan\beta$ is large and the second Higgs is discovered, it will still be worthwhile to explore the types of deviations in Yukawas we have considered.

We conclude that there is a large region of parameter space where precision light Higgs decays will be the best way to search for evidence of a second Higgs. This can also be a way of distinguishing among higher-dimension operator contributions to the Higgs mass squared as we discuss in the following section.

6   Implications for Testing Higher Dimension Operators

Recently Ref. [20] suggested the existence of higher dimension operators involving Higgs fields as a way of summarizing all possible models that might raise the Higgs mass without a large stop (or A term) (see also [21, 22]) in models that didn't contain new light fields into which the Higgs could decay and escape observation (see [23–26] and references therein). In this way they hoped to address the little hierarchy problem that seems to require a large

stop mass to raise the Higgs mass adequately. It is of interest to ask how to detect such higher dimension operators.

The obvious hope would be to find and measure additional Higgs states and study the mass relations. However, as we have discussed, it will be difficult to find a second Higgs over much of the relevant parameter range and similar considerations apply to other states from the Higgs sector. This leaves the question if the light Higgs does indeed have bigger mass than expected on the basis of the MSSM, are there other ways to distinguish among different possible higher dimension operators that might be contributing to its mass? Here we show that the likely leading operator to affect the Higgs mass is also precisely the one that should be best tested in the Higgs partial widths and the Yukawa analysis above readily applies. That means that not only can studying the branching fractions test for these operators, it could help distinguish among them.

In Ref. [20], it was demonstrated that at leading order in an effective dimension expansion, only one operator contributes to the light Higgs mass in the large $\tan\beta$ (but not so large that higher order mass suppressed terms dominate over $\cot\beta$ suppressed terms) limit. This operator is

$$\frac{\lambda}{M} \int d^2\theta (H_u H_d)^2 \tag{26}$$

(For simplicity, we assume all new parameters are real. We are also retaining the notation of Ref. [20] where H_u and H_d are used for H_2 and H_1 respectively.) When combined with the supersymmetric operator $\int d^2\theta \mu H_u H_d$, we find the quartic term

$$\frac{2\lambda\mu}{M}(H_u H_d)(H_u^\dagger H_u + H_d^\dagger H_d) \tag{27}$$

Such a term can also arise from a D-term type interaction.

Defining $\epsilon_1 = \lambda\mu/M$, one finds a Higgs mass correction of order $8\epsilon_1 \cot\beta v^2$ (with the $v = 246$GeV convention we have been using) [20] The authors of Ref. [20] argued that one can get a sufficiently large correction to the Higgs mass (one that replaces the large stop contribution) for parameters such as $\tan\beta \sim 10$ and $\epsilon_1 \sim 0.06$

Notice the interesting feature of this operator. Even though the only breaking of the Z_2 symmetry in the superpotential was through the lower-dimension μ-term, it feeds into a dimension-4 Z_2-violating operator in the potential. This Z_2-breaking, characterized by μ/M, can be sizable. This will be important below.

Alternatively, $\tan\beta$ could be so big (hence $\cos\beta$ so small) that terms suppressed by more powers of $1/M$ dominate over the leading $1/M$ correction. Such an unsuppressed contribution might arise from an operator $(H_u^\dagger H_u)^2$ for example.

We can now use our previous analysis to consider the effect of such operators on a light Higgs coupling to down-type quarks and charged leptons. We see that the first operator, while suppressed by $\cot\beta$ in its impact on the Higgs mass, is in fact exactly the type of operator that gets a $\tan\beta$ *enhanced* contribution to the Yukawa coupling deviations above. That is because it arises through the Z_2-violating μ-term and contributes directly to λ_7. In particular, $\lambda_7 \sim 2\epsilon_1$. If $\tan\beta$ is large, Yukawa couplings receive corrections from $\tan\beta 2\epsilon_1(v/m_H)^2$

effects. As an example, if $m_H \sim 1.5v$, with the parameters given above, these contributions could reduce the $h \to \tau\bar{\tau}$ rate by a factor of 4, while increasing the $h \to \gamma\gamma$ rate by a similar factor (due to the decreased rate to $b\bar{b}$). Even without discovering the second Higgs, these effects could be big enough to test for the higher dimension operators indirectly.

As an aside we note that recent papers [23–25] have considered the possibility that the light Higgs does not decay into the modes that have been sought for at LEP. In those papers there were alternative beyond the minimal supersymmetric model light modes available into which the Higgs can decay. We have just seen that even without these additional light modes, the Higgs branching ratio into $b\bar{b}$ and $\tau\bar{\tau}$ can be reduced substantially. However even when the branching ratio to bs is so reduced that other modes dominate, the alternative decay modes would have been visible as well, so the Higgs mass bound would not be reduced by more than a few GeV [27] so this doesn't alter the allowed range of M significantly.

Returning to the effects on LHC branching fractions, for an operator whose contribution to the squared Higgs mass is suppressed by two powers of M but not by $\cos\beta$ such as $(H_u^\dagger H_u)^2$, the contribution to the deviation in the Yukawa will nonetheless be suppressed by $\cos\beta$. Therefore the effect on the bottom Yukawa is much smaller than for the Z_2-breaking operator we just considered.

We can readily understand the relative signs and magnitudes of Yukawa corrections from the various operators by studying the sign and $\cos\beta$ dependence contributions to both the light Higgs mass squared and to the bottom-type Yukawa couplings of the various operators in the limit that $\cos\beta$ is small.

Operator	Mass Squared Contribution	Down Yukawa Contribution
$(H_u H_d)^2$	$\cos^2\beta$	$\cos\beta$
$(H_u^\dagger H_u)(H_u H_d)$	$\cos\beta$	1
$(H_u^\dagger H_u)^2$	1	$-\cos\beta$

We see that the operators consistent with the Z_2 symmetry do indeed give $\cos\beta$-suppressed contributions to the change in the down-type Yukawas. We also see that the effect of the last operator has the opposite sign which is why it increases the branching fraction of the bottom whereas the other operators decrease it. This should be a powerful tool for distinguishing among higher dimension operators should they be present.

Therefore if a light Higgs consistent with current experimental constraints and small stop mass is discovered (assuming small A), measuring branching ratios could test which higher dimension operator is the relevant one in raising its mass. In particular the effects of the first type of operator can have significant effects on the Higgs decay rate and branching ratios which we would not expect for the higher effective dimension operators.

7   Conclusion

We conclude that is is very likely that even if there are two Higgs doublet fields and the second neutral Higgs scalar is kinematically accessible to the LHC, it is likely that the second Higgs will elude direct detection. This makes the question of indirect evidence for the full

Higgs sector very important.

We have seen that there is a large parameter range where precision measurements, in particular of the branching fraction of the light Higgs into taus vs. photons, can find indirect evidence for a second Higgs field. If there are Z_2-violating interactions in the Higgs quartic terms, there can be enormous changes to the bottom and tau branching fractions, so large that they will be reflected in the overall Higgs decay rate and will result in a significant change in the branching fraction to other modes.

We have also seen that such measurements can be a powerful way to test for higher dimension operators in a supersymmetric theory and that the operator which is perhaps most likely to affect the light Higgs mass will yield significant changes to the decay widths into bottoms and taus.

Therefore precision Higgs branching fraction measurements can be extremely important if the world does in fact contain two Higgs fields. It will be interesting to do more detailed explorations of parameters, to consider which range is most natural and for how large a parameter range the considerations of this paper apply. It will also be of interest to incorporate the effects of CP violation.

8 Acknowledgements

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