Flavor Anarchy in a Randall-Sundrum Model with 5D Minimal Flavor Violation and a Low Kaluza-Klein Scale

Flavor from Minimal Flavor Violation & a Viable Randall-Sundrum Model

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Abstract

We present a variant of the warped extra dimension, Randall-Sundrum (RS), framework which is based on five dimensional (5D) minimal flavor violation (MFV), in which the only sources of flavor breaking are two 5D *anarchic* Yukawa matrices. The Yukawa matrices also control the bulk masses, which are responsible for the resulting flavor structure and mass hierarchy in the low energy theory. An interesting result of this set-up is that at low energies the theory flows to next to MFV model where flavor violation is dominantly coming from the third generation. Low energy flavor violation is further suppressed by a single parameter that dials the amount of violation in the up or down sector. There is therefore a sharp limit in which there is no flavor violation in the down type quark sector which, remarkably, is favored when we fit for the flavor parameters. This mechanism is used to eliminate the current RS flavor and CP problem even with a Kaluza-Klein scale as low as 2 TeV! Our construction also suggests that economic supersymmetric and non-supersymmetric, strong dynamic-based, flavor models may be built based on the same concepts.

Introduction. The standard model (SM) agrees very well with data. However, it is widely perceived to be an incomplete theory. In particular, in the SM, the hierarchy between the Planck scale and the electroweak (EW) symmetry breaking (EWSB) scale is unnatural since the Higgs mass is ultra-violet (UV) sensitive.

Solutions to the hierarchy problem therefore involve extending the SM at just above the EWSB scale which, in general, spoils the good agreement of the SM with data when trying to explain flavor as well. Given this inherent tension, it is important to identify new physics (NP) frameworks that preserve the approximate symmetries of the SM.

In this letter we consider the Randall-Sundrum scenario (RS1) [1], which potentially provides an elegant solution to the hierarchy problem. In this framework, due to warped higher-dimensional spacetime, the mass scales in an effective $4D$ description depend on location in an extra dimension: the Higgs sector is localized at the "TeV" brane where it is protected by a low warped-down fundamental scale of order a TeV while $4D$ gravity is localized near the "Planck" brane which has a Planckian fundamental scale.

In the original RS1 model, the entire SM was localized on the TeV brane. In this set-up, flavor issues are sensitive to the UV completion of the RS1 effective field theory: there is no understanding of the hierarchies in fermion masses or of smallness of flavor changing neutral currents (FCNCs) from higher-dimensional operators that would be too large if suppressed only by the warped-down cut-off $\sim$ TeV. Similar tension arises when considering the model predictions regarding EW precision tests.

Allowing the SM fermions and gauge fields to propagate in the bulk gives an opportunity to explain flavor, and makes flavor issues UV-insensitive as follows. The light fermions can be localized near the Planck brane (using a $5D$ fermion mass parameter [2, 3]) where the effective cut-off is much higher than TeV so that FCNCs from higher-dimensional operators are suppressed [3, 4]. Moreover, this results in small $4D$ Yukawa couplings to the Higgs, even if there are no small $5D$ Yukawa couplings [3, 4]. The top quark can be localized near the TeV brane to obtain a large $4D$ top Yukawa coupling. Because the fermion profiles depend exponentially on the bulk masses, this provides an understanding of the hierarchy of fermion masses (and mixing) with*out* hierarchies in fundamental ($5D$) parameters, solving the SM flavor puzzle.

However, with bulk fermions and gauge fields, calculable FCNCs from exchange of gauge Kaluza-Klein (KK) modes are induced. The couplings of light fermions to gauge KK modes are non-universal which induce FCNCs. Unlike the flat case, there is a significant protection from a built-in RS1-GIM [5] due to the approximate flatness of the KK gauge boson wavefunctions in the UV and a hierarchy in the fermion wavefunctions in the IR; nevertheless, the resulting contributions to FCNCs are non-negligible. In [6] it was shown that at low energies this class of models flows to next to MFV (NMFV); that is, flavor changing effects are generated primarily through mixing with the third generation. Generically, within the NMFV framework flavor violation occurs through NP sources, with a typical scale of $\Lambda_{\rm NMFV}$, which breaks the

SM flavor group from $U(3)_Q \times U(3)_u \times U(3)_d$ down to $U(2)_Q \times U(2)_u \times U(3)_d \times U(1)_{\rm top}$, where Q, u, d stand for quark doublets and up and down type singlets respectively. In addition the extra source is quasi-aligned with the SM sources of flavor breaking and the missalignment is at most of order the CKM matrix but new sources of CP violation (CPV) are present. Thus, transitions between the first [second] and third generation are suppressed by $\mathcal{O}(\lambda_C^3)$ [$\mathcal{O}(\lambda_C^2)$], where $\lambda_C \sim 0.23$ is the Cabibbo mixing angle. Despite these suppressions, it was recently pointed out [7] that the presence of additional, flavor violating, right handed (RH) currents would yield a stringent bound on this framework resulting with a bound of $\Lambda_{\rm NMFV} \geq 8\,$TeV. This implies a rather severe little hierarchy problem.

We present a novel variant of the above models, in which at leading order (LO) flavor violation in the down type quark sector is eliminated from the theory and at the same time leave intact the framework appealing features such as the solution of the hierarchy problem, flavor puzzle and others. The fundamental theory is also very minimal in terms of its number of parameters and contains only four flavor violating parameters, three mixing angles and one CPV phase. This implies that we also eliminated the presence of other CPV, "Majorana-like" phases, which induced an RS1 CP problem [5]. Note that unlike in the SM, in our model the flavor violating parameters are of order unity, yet no conflict is obtained with precision flavor constraints.

The model. Our set-up is very simple. Applying the MFV paradigm [8] to our case we assume that the only sources of flavor breaking are the 5D up and down Yukawa matrices, $Y_{u,d}$ to a bulk Higgs, H. However, unlike the 4D MFV case (or other extensions with trivial flavor structure, for example universal extra dimension [9]) in our framework the 5D Yukawa matrices are structureless. In other words the eigenvalues of $Y_{u,d}$ are all of the same order. Furthermore, they are totally missaligned so that the 5D "CKM" matrix $V_5^{\rm KM}$ is anarchic.

In addition, the theory contains 5D vector-like, 3×3, mass matrices $C_{Q,u,d}$ for each of the quark representations. Bulk MFV implies that the only vector-like flavor-breaking spurions for the doublets [singlets] are [10] $Y_{u,d}Y_{u,d}^\dagger$ [$Y_{u,d}^\dagger Y_{u,d}$]. We emphasize that $V_5^{\rm KM}$ is the only source of flavor and CPV in our theory. Under the global symmetry $U(3)_Q \times U(3)_u \times U(3)_d$, either Y_u or Y_d can be brought to diagonal form, and $V_5^{\rm KM}$ resides in the remaining one. According to our MFV assumption we can expand the 5D mass matrices as a power series in $Y_{u,d}$:

$$C_{u,d} = Y_{u,d}^\dagger Y_{u,d} + \ldots\,,\;\; C_Q = rY_uY_u^\dagger + Y_dY_d^\dagger + \ldots\,, \quad (1)$$

where universal terms and overall order one coefficients were omitted for simplicity and the dots stand for subdominant higher order terms (as discussed below). The relevant part of the 5D Lagrangian is given by

$$\mathcal{L}_{\rm gen} = C_{Q,u,d}\left(\bar Q, \bar u, \bar d\right)(Q,u,d) + H\; Y_{u,d}\bar Q\,(u,d)\,, \quad (2)$$

where C_i are in units of k the AdS curvature, and we will assume that the Higgs is a bulk field (see later) so that Y_i are measured in units of $1/\sqrt{k}$.

Our first result is that despite of the fact that the fundamental theory is anarchic MFV the low energy is a hierarchic one. This is since the eigenvalues the C_i matrices are sizable, which will induce geometrical separation in the extra dimension picture or the presence of sizable anomalous dimension in the dual conformal field theory (CFT) [11].

The second, maybe less trivial result, is that this theory flows to approximate NMFV with additional sources of flavor and CPV. In order to see that recall that the 4D mass matrices for the zero modes can be written as [5] $m_{u,d} \simeq 2vF_QY_{u,d}F_{u,d}$, where F_x correspond to the value of the quark zero-modes on the TeV brane. More explicitly, the eigenvalues f_{x^i} of the F_x matrices are given by [3, 5] $f_{x^i}^2 = (1/2 - c_{x^i})/(1-\epsilon^{1-2c_{x^i}})$, where c_{x^i} are the eigenvalues of the C_x matrices, $\epsilon = \exp[-k\pi r_c]$, $k\pi r_c = \log[M_{\bar{\rm Pl}}/{\rm TeV}]$, $M_{\bar{\rm Pl}}$ is the reduced Planck mass and $v \simeq 174$GeV. The f_{x^i} correspond to the amount of compositeness of the different generations. The $Y_{u,d}$ are anarchic, and therefore the corresponding mixing angles are given by ratios of the F_i eigenvalues. For instance, the form of the 4D mass matrices for the zero modes implies that the rotation to mass eigenbasis diagonalizes $(m_{u,d}^2)_{ij} = 4v^2(F_QY_{u,d}F_{u,d}F_{u,d}^\dagger Y_{u,d}^\dagger F_Q^\dagger)_{ij} \sim f_{Q^i}f_{Q^j}$. This implies that $(V_{\rm CKM})_{ij} \sim f_{Q^i}/f_{Q^j}$ and thus the c_{Q^i} eigenvalues control the CKM mixing angles. [5].

The couplings of two zero modes to the gauge KK states (which are localized near the TeV brane), have a flavor structure that is different from the 4D mass matrices. They are proportional to $F_{Q,u,d}^2$, which is not aligned with $m_{u,d}$. Thus new flavor and CPV phases are present in the low energy theory. However, the NMFV limit is realized since one eigenvalue of $(F_{u,Q,d})$ is much larger than the others, and thus an approximate $U(2)$ is preserved (so that F_Q^2 and $m_{u,d}$ are quasi-aligned) [6]. Note that the theory contains RH currents since in the mass basis the $C_{u,d}$ matrices are not diagonal.

Flavor	$c_Q,\, f_Q$	$c_u,\, f_u$	$c_d,\, f_d$
I	0.64, 0.002	0.68, $7\,10^{-4}$	0.65, $2\,10^{-3}$
II	0.59, 0.01	0.53, 0.06	0.60, 0.008
III	0.46, 0.2	- 0.06, 0.8	0.58, 0.02

TABLE I: The eigenvalues, of C_x, F_x which roughly yield the right masses and CKM elements at the TeV scale [4].

Our third result is that in the limit where r in Eq. (1) goes to zero, C_Q,C_d, and Y_d can all be simultaneously diagonalized. Therefore, flavor violation in the down sector is completely eliminated, where in this case flavor conversion (including the CKM part) is due to the up quark sector! Within our scheme and with accordance to Eq. (1) the value of r is not a free parameter but rather a function of the flavor parameters (which are in turn determined by the known masses and mixings). For concreteness, we present a numerical example that satisfies our scheme. In Table I we present the eigenvalues of C_i and F_i that yield the quark masses and mixing angles. We further need to show that there is a consistent solution to the following relation:

$$\mathrm{diag}(C_Q) = a\,\mathrm{diag}[r(V_5^{\mathrm{KM}\,\dagger}(\theta_{ij},\delta)C_u\,V_5^{\mathrm{KM}}(\theta_{ij},\delta) + C_d], \quad (3)$$

that is in accordance with the mass values in Table I, where θ_{ij} is a mixing angle between the ith and jth generations and δ is the 5D CKM phase. To see that our setup is self consistent we need to verify that the eigenvalues of the three mass matrices, $C_{Q,u,d}$ can be derived from only two anarchical matrices, $Y_{u,d}$. As an example the following numbers were found to solve the above relation, a, r, θ_{12}, θ_{23}, θ_{13}, $\delta \approx 0.8, 0.3, 115^o, 65^o, 70^o, 0.6$.

It is rather remarkable that r tends to be small. This follows since the large top quark mass favors $c_{u^3} \ll 0.5$, and thus the C_u eigenvalues differ in structure from the C_Q, C_d eigenvalues (see Table I).

To clarify this result, consider the constraint that Eq. (3) would impose in a simpler system where only the second and third generations are present. Then, the $C_{Q,u,d}$ eigenvalues split up into a trace $c^{\mathrm{tr}}_{Q,u,d} = (c_{Q^2,u^2,d^2} + c_{Q^3,u^3,d^3})/2$ and a traceless piece $c^{\mathrm{tl}}_{Q,u,d} = (c_{Q^2,u^2,d^2} - c_{Q^3,u^3,d^3})/2$. In this simpler system, Eq. (3) implies only a trace condition $c^{\mathrm{tr}}_Q = arc^{\mathrm{tr}}_u + ac^{\mathrm{tr}}_d$, and one remaining eigenvalue condition $(c^{\mathrm{tl}}_Q)^2 = a^2[r^2(c^{\mathrm{tl}}_u)^2 + (c^{\mathrm{tl}}_d)^2 + 2rc^{\mathrm{tl}}_u c^{\mathrm{tl}}_d \cos 2\theta_{23}]$. From the values in Table I, $c^{\mathrm{tl}}_u = 0.28 \gg c^{\mathrm{tl}}_Q = 0.07$, and it is straightforward to work out that these two constraints would imply $0.19 \le |r| \le 0.31$. The full three-generation system, which is necessary to obtain a CP-violating phase, is more complicated and the allowed range of $|r|$ must be found numerically; typically our numerical solutions favor $r = 0.1 - 0.4$. Thus, within our framework we find that the flavor violation in the down sector is suppressed by $\mathcal{O}(0.25)$.

In our numerical examples we have assumed that the typical size of the Yukawa matrix eigenvalues is $y \approx 3$ (slightly bigger than was used in [5] with Higgs on the brane). In theories where the Higgs is a bulk field such as the holographic composite Higgs models [12] y is within the perturbative region for at least three KK modes, N_{KK}, below the cutoff [13] , $N_{\mathrm{KK}}(2y/4\pi)^2 < 1$. As we shall see next, this choice yields a suppression (not due to a symmetry) of order $r_y \sim 2/3$ which together with a moderate value of r will completely relax the present tension with flavor and CPV precision bounds.

FCNC and electric dipole moment (EDM). Let us briefly review the status of the strongest constraints on the generic bulk RS1 models. These contributions are from $\Delta F = 2$ processes due to tree level exchange of KK gluon. In [5] it was shown that the ratio between the RS1, $(V-A)\times(V-A)$, contributions and the SM is proportional to $(F_Q^2)^2_{ij}$ (in the down quark mass basis). Using the relation $(V_{\mathrm{CKM}})_{ij} \sim f_{Q^i}/f_{Q^j}$ the ratio of contributions can be written as

$$h^{\mathrm{RS}} = \frac{M_{12}^{\mathrm{RS}}}{M_{12}^{\mathrm{SM}}} \sim 0.5 \times \left(\frac{3\mathrm{TeV}}{m_{\mathrm{KK}}}\right)^2 \left(\frac{f_{Q^3}}{0.3}\right)^4 . \quad (4)$$

The above contribution is proportional to $f^4_{Q^3}$ because to leading order all flavor violation comes through the third generation. At present, $h^{\mathrm{RS}} \lesssim 0.3$ [6, 7, 14]. However, in models where RH currents are present, the dominant contributions to ϵ_K involve operators with $(V-A)\times(V+A)$ structure [7]. In such a case the contributions are proportional to $(F_Q^2)_{12}(F_d^2)_{12} \propto m_d m_s/(vy)^2$ which apparently is smaller by a factor of $\mathcal{O}(20)$. This is not enough due to the the following two sources of enhacement, $\mathcal{O}(11)$ from chiral enhancement of the matrix element and $\mathcal{O}(7)$ from the running from the KK scale to the weak scale. These overcome the suppression and yield the largest contributions which imply that the KK masses have to be above the 8 TeV scale.

In our class of models both the $(V-A)\times(V-A)$ and the $(V-A)\times(V+A)$ contributions are suppressed by r^2. In addition, due to the larger overall scale for the Yukawa matrices the value of f_{Q^3} is smaller by factor of $r_y \sim 2/3$ than in the brane-localized-Higgs case. Due to the RS1-GIM mechanism LH flavor violation is proportional to f_{Q^3}. Thus these contributions are suppressed by $\mathcal{O}(r_y^4)$ where as in the case of $(V-A)\times(V+A)$ a suppressesion of $\mathcal{O}(r_y^2)$ is obtained. So, altogether we expect a suppression of down quark $\Delta F = 2$ currents to be of the order $(2/3)^{4,2}(0.25)^2 = \mathcal{O}(1, 3\%)$ in the $(V \mp A)\times(V-A)$ case, respectively. This allows us to lower the KK masses below the 2 TeV scale without violating any of the current constrains, significantly below the value allowed by EW precision tests [15].

Finally, we comment that (assuming a solution to the strong CP problem) our model does not suffer from a CP problem due to constraints from the neutron electric dipole moment since the contributions to this process arise only at two loops and not at one loop as occurs in the non 5D-MFV case [5]. One way to see that two loops are required is to compare the CPV sources of the generic case and our class of models. In the general framework even in the two generation case there are various CPV phases present, so that one loop is enough in order to be sensitive to these extra "Majorana" phases. However, in our case there is a single CPV phase in the fundamental theory which vanishes in the two generations case, as the theory becomes real in that limit. Thus only two loop diagrams can be sensitive to this 5D-CKM phase and the RS1 CP problem is solved. In more technical terms the spurion that generates the leading contributions can be written, without loss of generality, as $d_N \equiv Im\left[F_Q(Y_uY_u^\dagger + Y_dY_d^\dagger)Y_dF_d\right]_{11} = Im\left[F_Q(C_Q)(C_Q/ar + Y_dY_d^\dagger(1-1/r))Y_dF_d\right]_{11}$ where in the RH side we have used the relation in Eq. (1). This expression is only a function of $Y_dY_d^\dagger$ and C_Q where the missalignment between these two spurions is described by a single 5D CKM-like matrix. Thus for CPV all three generations must participate in the process which is possible only at two loops [16].

Conclusions and Outlook. We have presented a simple and economic warped extra dimension model based on the novel idea of 5D anarchic minimal flavor violation (MFV) in the quark sector. The idea carries several interesting features as follows: The low energy theory is anarchic and the model solves the flavor puzzle; however the theory is not described by MFV but rather by the next to MFV. New flavor and CP violating phases are generically present. However they dominantly induce flavor-changing currents only in the up type sector. In addition CP violation occurs only when three generations are considered. Thus the agreement with experimental constraints, both from flavor changing and flavor conserving, dipole moment experiments, is dramatically improved. Here we focused on the quark sector. It would be interesting to check whether the above mechanism can be extended to the lepton sector which also comes with its own flavor and CP problems [17]. We note that no extra structure was required in order to realize the above scenario. Rather the number of flavor parameters was reduced. This implies that to a large extent the LHC collider phenomenology is similar to what was already discussed in the context of the general framework [18]. We note that since flavor violation is suppressed in the down quark sector but to a lesser extent in the up quark sector a possible signal of this framework is top flavor violation [19]. (contributions to $D-\bar{D}$ mixing are subdominant as in [5])

In this work we focused on showing how our scheme solved the RS1 flavor and CP problems. Note that the bulk flavor parameters are protected by locality since corrections to their values have to involve the two Yukawa matrices which are localized on the TeV brane. We have not discussed how to dynamically realize the above set up but we expect that this should be rather straight forward. One possible way is through shining [20] from the TeV brane, where additional three bulk adjoint light scalars (transform under the quark sector flavor symmetry) can couple to the two TeV, bi-fundamental, Yukawa matrices. One can also understand/speculate from this set up why the universal contributions are somewhat suppressed or absent since masses in the bulk must be generated by a field that is odd under the Z_2 orbifold symmetry. The flavons, that generate the bulk masses, are odd under the orbifold symmetry, which prevents a bare, universal, mass term. It is not inconceivable that a solution to the strong CP problem can be also obtained via the above setup in the spirit of [21].

Our setup can be also understood from the 4D point of view where a single source of flavor breaking induces both the mixing between the elementary and composite fermions and setting the chiral operators anomalous dimensions but on the same time controls the structure of the purely composite Yukawa interaction between the Higgs and fermions. Thus the resulting flavor violation stems from a single source. In fact this is not completely unfamiliar since within anomaly mediation supersymmetry breaking [22] the flavor violation in the squark soft breaking sector is induced by the 4D Yukawa matrices. It would be interesting to see whether a realistic supersymmetric version of the above model, along the lines of [23] can be constructed. In such a case the anomalous dimension of the operators are proportional to the anarchic Yukawa matrices. The resulting flavor structure would be under better control even if the resulting soft masses are not degenerate.

Acknowledgements. We thank Kaustubh Agashe, Nima Arkani-Hamed, Ami Katz and Martin Schmaltz for discussions. We also thank KA for comments on the manuscript.

Note added: while this work near completion Ref. [24] was published which also deals with the RS flavor problem. However, the model of [24] requires introducing the fermion mass hierarchies by hand, whereas in our model such hierarchies are generated naturally.

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