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# Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools



**The Institute of Education Sciences (IES) publishes practice guides in education** to bring the best available evidence and expertise to bear on the types of systemic challenges that cannot currently be addressed by single interventions or programs. Authors of practice guides seldom conduct the types of systematic literature searches that are the backbone of a meta-analysis, although they take advantage of such work when it is already published. Instead, authors use their expertise to identify the most important research with respect to their recommendations, augmented by a search of recent publications to ensure that research citations are up-to-date.

Unique to IES-sponsored practice guides is that they are subjected to rigorous external peer review through the same office that is responsible for independent review of other IES publications. A critical task for peer reviewers of a practice guide is to determine whether the evidence cited in support of particular recommendations is up-to-date and that studies of similar or better quality that point in a different direction have not been ignored. Because practice guides depend on the expertise of their authors and their group decisionmaking, the content of a practice guide is not and should not be viewed as a set of recommendations that in every case depends on and flows inevitably from scientific research.

The goal of this practice guide is to formulate specific and coherent evidence-based recommendations for use by educators addressing the challenge of reducing the number of children who struggle with mathematics by using “response to intervention” (RtI) as a means of both identifying students who need more help and providing these students with high-quality interventions. The guide provides practical, clear information on critical topics related to RtI and is based on the best available evidence as judged by the panel. Recommendations in this guide should not be construed to imply that no further research is warranted on the effectiveness of particular strategies used in RtI for students struggling with mathematics.

# Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

**April 2009**

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# Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

## Contents

Introduction	1
The What Works Clearinghouse standards and their relevance to this guide	3
Overview	4
Summary of the Recommendations	5
Scope of the practice guide	9
Checklist for carrying out the recommendations	11
Recommendation 1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	13
Recommendation 2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	18
Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	21
Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures.	26
Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	30
Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	37
Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	41
Recommendation 8. Include motivational strategies in tier 2 and tier 3 interventions.	44
Glossary of terms as used in this report	48
Appendix A. Postscript from the Institute of Education Sciences	52
Appendix B. About the authors	55
Appendix C. Disclosure of potential conflicts of interest	59
Appendix D. Technical information on the studies	61
References	91

## List of tables

Table 1. Institute of Education Sciences levels of evidence for practice guides	2
Table 2. Recommendations and corresponding levels of evidence	6
Table 3. Sensitivity and specificity	16
Table D1. Studies of interventions that included explicit instruction and met WWC Standards (with and without reservations)	69
Table D2. Studies of interventions that taught students to discriminate problem types that met WWC standards (with or without reservations)	73
Table D3. Studies of interventions that used visual representations that met WWC standards (with and without reservations)	77–78
Table D4. Studies of interventions that included fact fluency practices that met WWC standards (with and without reservations)	83

## List of examples

Example 1. Change problems	27
Example 2. Compare problems	28
Example 3. Solving different problems with the same strategy	29
Example 4. Representation of the counting on strategy using a number line	33
Example 5. Using visual representations for multidigit addition	34
Example 6. Strip diagrams can help students make sense of fractions	34
Example 7. Manipulatives can help students understand that four multiplied by six means four groups of six, which means 24 total objects	35
Example 8. A set of matched concrete, visual, and abstract representations to teach solving single-variable equations	35
Example 9: Commutative property of multiplication	48
Example 10: Make-a-10 strategy	49
Example 11: Distributive property	50
Example 12: Number decomposition	51

# Introduction

Students struggling with mathematics may benefit from early interventions aimed at improving their mathematics ability and ultimately preventing subsequent failure. This guide provides eight specific recommendations intended to help teachers, principals, and school administrators use Response to Intervention (RtI) to identify students who need assistance in mathematics and to address the needs of these students through focused interventions. The guide provides suggestions on how to carry out each recommendation and explains how educators can overcome potential roadblocks to implementing the recommendations.

The recommendations were developed by a panel of researchers and practitioners with expertise in various dimensions of this topic. The panel includes a research mathematician active in issues related to K–8 mathematics education, two professors of mathematics education, several special educators, and a mathematics coach currently providing professional development in mathematics in schools. The panel members worked collaboratively to develop recommendations based on the best available research evidence and our expertise in mathematics, special education, research, and practice.

The body of evidence we considered in developing these recommendations included evaluations of mathematics interventions for low-performing students and students with learning disabilities. The panel considered high-quality experimental and quasi-experimental studies, such as those meeting the criteria of the What Works Clearinghouse (<http://www.whatworks.ed.gov>), to provide the strongest evidence of effectiveness. We also examined studies of the technical adequacy of batteries of screening and progress monitoring measures for recommendations relating to assessment.

In some cases, recommendations reflect evidence-based practices that have been demonstrated as effective through rigorous research. In other cases, when such evidence is not available, the recommendations reflect what this panel believes are best practices. Throughout the guide, we clearly indicate the quality of the evidence that supports each recommendation.

Each recommendation receives a rating based on the strength of the research evidence that has shown the effectiveness of a recommendation (table 1). These ratings—*strong*, *moderate*, or *low*—have been defined as follows:

*Strong* refers to consistent and generalizable evidence that an intervention program causes better outcomes.<sup>1</sup>

*Moderate* refers either to evidence from studies that allow strong causal conclusions but cannot be generalized with assurance to the population on which a recommendation is focused (perhaps because the findings have not been widely replicated)—or to evidence from studies that are generalizable but have more causal ambiguity than offered by experimental designs (such as statistical models of correlational data or group comparison designs for which the equivalence of the groups at pretest is uncertain).

*Low* refers to expert opinion based on reasonable extrapolations from research and theory on other topics and evidence from studies that do not meet the standards for moderate or strong evidence.

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1. Following WWC guidelines, we consider a positive, statistically significant effect or large effect size (i.e., greater than 0.25) as an indicator of positive effects.



**Table 1. Institute of Education Sciences levels of evidence for practice guides**

<b>Strong</b>	<p>In general, characterization of the evidence for a recommendation as strong requires both studies with high internal validity (i.e., studies whose designs can support causal conclusions) and studies with high external validity (i.e., studies that in total include enough of the range of participants and settings on which the recommendation is focused to support the conclusion that the results can be generalized to those participants and settings). Strong evidence for this practice guide is operationalized as:</p> <ul style="list-style-type: none"> <li>• A systematic review of research that generally meets the standards of the What Works Clearinghouse (WWC) (see <a href="http://ies.ed.gov/ncee/wwc/">http://ies.ed.gov/ncee/wwc/</a>) and supports the effectiveness of a program, practice, or approach with no contradictory evidence of similar quality; OR</li> <li>• Several well-designed, randomized controlled trials or well-designed quasi-experiments that generally meet the standards of WWC and support the effectiveness of a program, practice, or approach, with no contradictory evidence of similar quality; OR</li> <li>• One large, well-designed, randomized controlled, multisite trial that meets WWC standards and supports the effectiveness of a program, practice, or approach, with no contradictory evidence of similar quality; OR</li> <li>• For assessments, evidence of reliability and validity that meets the Standards for Educational and Psychological Testing.<sup>a</sup></li> </ul>
<b>Moderate</b>	<p>In general, characterization of the evidence for a recommendation as moderate requires studies with high internal validity but moderate external validity, or studies with high external validity but moderate internal validity. In other words, moderate evidence is derived from studies that support strong causal conclusions but when generalization is uncertain, or studies that support the generality of a relationship but when the causality is uncertain. Moderate evidence for this practice guide is operationalized as:</p> <ul style="list-style-type: none"> <li>• Experiments or quasi-experiments generally meeting the standards of WWC and supporting the effectiveness of a program, practice, or approach with small sample sizes and/or other conditions of implementation or analysis that limit generalizability and no contrary evidence; OR</li> <li>• Comparison group studies that do not demonstrate equivalence of groups at pretest and therefore do not meet the standards of WWC but that (a) consistently show enhanced outcomes for participants experiencing a particular program, practice, or approach and (b) have no major flaws related to internal validity other than lack of demonstrated equivalence at pretest (e.g., only one teacher or one class per condition, unequal amounts of instructional time, highly biased outcome measures); OR</li> <li>• Correlational research with strong statistical controls for selection bias and for discerning influence of endogenous factors and no contrary evidence; OR</li> <li>• For assessments, evidence of reliability that meets the Standards for Educational and Psychological Testing<sup>b</sup> but with evidence of validity from samples not adequately representative of the population on which the recommendation is focused.</li> </ul>
<b>Low</b>	<p>In general, characterization of the evidence for a recommendation as low means that the recommendation is based on expert opinion derived from strong findings or theories in related areas and/or expert opinion buttressed by direct evidence that does not rise to the moderate or strong levels. Low evidence is operationalized as evidence not meeting the standards for the moderate or high levels.</p>

a. American Educational Research Association, American Psychological Association, and National Council on Measurement in Education (1999).

b. Ibid.

## The What Works Clearinghouse standards and their relevance to this guide

The panel relied on WWC evidence standards to assess the quality of evidence supporting mathematics intervention programs and practices. The WWC addresses evidence for the causal validity of instructional programs and practices according to WWC standards. Information about these standards is available at <http://ies.ed.gov/ncee/wwc/references/standards/>. The technical quality of each study is rated and placed into one of three categories:

- *Meets Evidence Standards*—for randomized controlled trials and regression discontinuity studies that provide the strongest evidence of causal validity.
- *Meets Evidence Standards with Reservations*—for all quasi-experimental studies with no design flaws and randomized controlled trials that have problems with randomization, attrition, or disruption.
- *Does Not Meet Evidence Screens*—for studies that do not provide strong evidence of causal validity.

Following the recommendations and suggestions for carrying out the recommendations, Appendix D presents information on the research evidence to support the recommendations.

The panel would like to thank Kelly Haymond for her contributions to the analysis, the WWC reviewers for their contribution to the project, and Jo Ellen Kerr and Jamila Henderson for their support of the intricate logistics of the project. We also would like to thank Scott Cody for his oversight of the overall progress of the practice guide.

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# Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

## Overview

Response to Intervention (RtI) is an early detection, prevention, and support system that identifies struggling students and assists them before they fall behind. In the 2004 reauthorization of the Individuals with Disabilities Education Act (PL 108-446), states were encouraged to use RtI to accurately identify students with learning disabilities and encouraged to provide additional supports for students with academic difficulties regardless of disability classification. Although many states have already begun to implement RtI in the area of reading, RtI initiatives for mathematics are relatively new.

Students' low achievement in mathematics is a matter of national concern. The recent National Mathematics Advisory Panel Report released in 2008 summarized the poor showing of students in the United States on international comparisons of mathematics performance such as the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA).<sup>2</sup> A recent survey of algebra teachers associated with the report identified key deficiencies of students entering algebra, including aspects of whole number arithmetic, fractions, ratios, and proportions.<sup>3</sup> The National Mathematics Advisory Panel

concluded that all students should receive preparation from an early age to ensure their later success in algebra. In particular, the report emphasized the need for mathematics interventions that mitigate and prevent mathematics difficulties.

This panel believes that schools can use an RtI framework to help struggling students prepare for later success in mathematics. To date, little research has been conducted to identify the most effective ways to initiate and implement RtI frameworks for mathematics. However, there is a rich body of research on effective mathematics interventions implemented outside an RtI framework. Our goal in this practice guide is to provide suggestions for assessing students' mathematics abilities and implementing mathematics interventions within an RtI framework, in a way that reflects the best evidence on effective practices in mathematics interventions.

RtI begins with high-quality instruction and universal screening for all students. Whereas high-quality instruction seeks to prevent mathematics difficulties, screening allows for early detection of difficulties if they emerge. Intensive interventions are then provided to support students in need of assistance with mathematics learning.<sup>4</sup> Student responses to intervention are measured to determine whether they have made adequate progress and (1) no longer need intervention, (2) continue to need some intervention, or (3) need more intensive intervention. The levels of intervention are conventionally referred to as "tiers." RtI is typically thought of as having three tiers.<sup>5</sup> Within a three-tiered RtI model, each tier is defined by specific characteristics.

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2. See, for example, National Mathematics Advisory Panel (2008) and Schmidt and Houang (2007). For more information on the TIMSS, see <http://nces.ed.gov/timss/>. For more information on PISA, see <http://www.oecd.org>.

3. National Mathematics Advisory Panel (2008).

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4. Fuchs, Fuchs, Craddock et al. (2008).

5. Fuchs, Fuchs, and Vaughn (2008) make the case for a three-tier RtI model. Note, however, that some states and school districts have implemented multitier intervention systems with more than three tiers.

- Tier 1 is the mathematics instruction that all students in a classroom receive. It entails universal screening of all students, regardless of mathematics proficiency, using valid measures to identify students at risk for future academic failure—so that they can receive early intervention.<sup>6</sup> There is no clear consensus on the characteristics of instruction other than that it is “high quality.”<sup>7</sup>
- In tier 2 interventions, schools provide additional assistance to students who demonstrate difficulties on screening measures or who demonstrate weak progress.<sup>8</sup> Tier 2 students receive supplemental small group mathematics instruction aimed at building targeted mathematics proficiencies.<sup>9</sup> These interventions are typically provided for 20 to 40 minutes, four to five times each week.<sup>10</sup> Student progress is monitored throughout the intervention.<sup>11</sup>
- Tier 3 interventions are provided to students who are not benefiting from tier 2 and require more intensive assistance.<sup>12</sup> Tier 3 usually entails one-on-one tutoring along with an appropriate mix of instructional interventions. In some cases, special education services are included in tier 3, and in others special education is considered an additional tier.<sup>13</sup> Ongoing analysis of

student performance data is critical in this tier. Typically, specialized personnel, such as special education teachers and school psychologists, are involved in tier 3 and special education services.<sup>14</sup> However, students often receive relevant mathematics interventions from a wide array of school personnel, including their classroom teacher.

## Summary of the Recommendations

This practice guide offers eight recommendations for identifying and supporting students struggling in mathematics (table 2). The recommendations are intended to be implemented within an RtI framework (typically three-tiered). The panel chose to limit its discussion of tier 1 to universal screening practices (i.e., the guide does not make recommendations for general classroom mathematics instruction). Recommendation 1 provides specific suggestions for conducting universal screening effectively. For RtI tiers 2 and 3, recommendations 2 through 8 focus on the most effective content and pedagogical practices that can be included in mathematics interventions.

Throughout this guide, we use the term “interventionist” to refer to those teaching the intervention. At a given school, the interventionist may be the general classroom teacher, a mathematics coach, a special education instructor, other certified school personnel, or an instructional assistant. The panel recognizes that schools rely on different personnel to fill these roles depending on state policy, school resources, and preferences.

Recommendation 1 addresses the type of screening measures that should be used in tier 1. We note that there is more research on valid screening measures for students in

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6. For reviews see Jiban and Deno (2007); Fuchs, Fuchs, Compton et al. (2007); Gersten, Jordan, and Flojo (2005).

7. National Mathematics Advisory Panel (2008); National Research Council (2001).

8. Fuchs, Fuchs, Craddock et al. (2008); National Joint Committee on Learning Disabilities (2005).

9. Fuchs, Fuchs, Craddock et al. (2008).

10. For example, see Jitendra et al. (1998) and Fuchs, Fuchs, Craddock et al. (2008).

11. National Joint Committee on Learning Disabilities (2005).

12. Fuchs, Fuchs, Craddock et al. (2008).

13. Fuchs, Fuchs, Craddock et al. (2008); National Joint Committee on Learning Disabilities (2005).

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14. National Joint Committee on Learning Disabilities (2005).

**Table 2. Recommendations and corresponding levels of evidence**

Recommendation	Level of evidence
Tier 1	
1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	<b>Moderate</b>
Tiers 2 and 3	
2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	<b>Low</b>
3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	<b>Strong</b>
4. Interventions should include instruction on solving word problems that is based on common underlying structures.	<b>Strong</b>
5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	<b>Moderate</b>
6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	<b>Moderate</b>
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	<b>Low</b>
8. Include motivational strategies in tier 2 and tier 3 interventions.	<b>Low</b>

*Source:* Authors' compilation based on analysis described in text.

kindergarten through grade 2,<sup>15</sup> but there are also reasonable strategies to use for students in more advanced grades.<sup>16</sup> We stress that no one screening measure is perfect and that schools need to monitor the progress of students who score slightly above or slightly below any screening cutoff score.

Recommendations 2 through 6 address the content of tier 2 and tier 3 interventions and the types of instructional strategies that should be used. In recommendation 2, we translate the guidance by the National Mathematics Advisory Panel (2008) and the National Council of Teachers of Mathematics Curriculum Focal Points (2006) into suggestions for the content of intervention curricula. We argue that the mathematical focus and the in-depth coverage advocated for proficient students are also necessary for students with mathematics difficulties. For most students, the content of interventions will include foundational concepts and skills introduced earlier in the student's career but not fully understood and mastered. Whenever possible, links should be made between foundational mathematical concepts in the intervention and grade-level material.

At the center of the intervention recommendations is that instruction should be systematic and explicit (recommendation 3). This is a recurrent theme in the body of valid scientific research.<sup>17</sup> We explore the multiple meanings of explicit instruction and indicate which components of explicit instruction appear to be most related to improved student outcomes. We believe this information is important for districts and state departments to have as they consider selecting materials and

providing professional development for interventionists.

Next, we highlight several areas of research that have produced promising findings in mathematics interventions. These include systematically teaching students about the problem types associated with a given operation and its inverse (such as problem types that indicate addition and subtraction) (recommendation 4).<sup>18</sup> We also recommend practices to help students translate abstract symbols and numbers into meaningful visual representations (recommendation 5).<sup>19</sup> Another feature that we identify as crucial for long-term success is systematic instruction to build quick retrieval of basic arithmetic facts (recommendation 6). Some evidence exists supporting the allocation of time in the intervention to practice fact retrieval using flash cards or computer software.<sup>20</sup> There is also evidence that systematic work with properties of operations and counting strategies (for younger students) is likely to promote growth in other areas of mathematics beyond fact retrieval.<sup>21</sup>

The final two recommendations address other considerations in implementing tier 2 and tier 3 interventions. Recommendation 7 addresses the importance of monitoring the progress of students receiving

15. Gersten, Jordan, and Flojo (2005); Gersten, Clarke, and Jordan (2007).

16. Jiban and Deno (2007); Foegen, Jiban, and Deno (2007).

17. Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Jitendra et al. (1998); Schunk and Cox (1986); Tournaki (2003); Wilson and Sindelar (1991).

18. Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, and Finelli (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

19. Artus and Dyrek (1989); Butler et al. (2003); Darch, Carnine, and Gersten (1984); Fuchs et al. (2005); Fuchs, Seethaler et al. (2008); Fuchs, Powell et al. (2008); Fuchs, Fuchs, Craddock et al. (2008); Jitendra et al. (1998); Walker and Poeteet (1989); Wilson and Sindelar (1991); Witzel (2005); Witzel, Mercer, and Miller (2003); Woodward (2006).

20. Bernie-Smith (1991); Fuchs, Seethaler et al. (2008); Fuchs et al. (2005); Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Powell et al. (2008).

21. Tournaki (2003); Woodward (2006).

interventions. Specific types of formative assessment approaches and measures are described. We argue for two types of ongoing assessment. One is the use of curriculum-embedded assessments that gauge how well students have learned the material in that day's or week's lesson(s). The panel believes this information is critical for interventionists to determine whether they need to spend additional time on a topic. It also provides the interventionist and other school personnel with information that can be used to place students in groups within tiers. In addition, we recommend that schools regularly monitor the progress of students receiving interventions

and those with scores slightly above or below the cutoff score on screening measures with broader measures of mathematics proficiency. This information provides the school with a sense of how the overall mathematics program (including tier 1, tier 2, and tier 3) is affecting a given student.

Recommendation 8 addresses the important issue of motivation. Because many of the students struggling with mathematics have experienced failure and frustration by the time they receive an intervention, we suggest tools that can encourage active engagement of students and acknowledge student accomplishments.

## Scope of the practice guide

Our goal is to provide evidence-based suggestions for screening students for mathematics difficulties, providing interventions to students who are struggling, and monitoring student responses to the interventions. Rtl intentionally cuts across the borders of special and general education and involves school-wide collaboration. Therefore, our target audience for this guide includes teachers, special educators, school psychologists and counselors, as well as administrators. Descriptions of the materials and instructional content in tier 2 and tier 3 interventions may be especially useful to school administrators selecting interventions, while recommendations that relate to content and pedagogy will be most useful to interventionists.<sup>22</sup>

The focus of this guide is on providing Rtl interventions in mathematics for students in kindergarten through grade 8. This broad grade range is in part a response to the recent report of the National Mathematics Advisory Panel (2008), which emphasized a unified progressive approach to promoting mathematics proficiency for elementary *and* middle schools. Moreover, given the growing number of initiatives aimed at supporting students to succeed in algebra, the panel believes it essential to provide tier 2 and tier 3 interventions to struggling students in grades 4 through 8. Because the bulk of research on mathematics interventions has focused on students in kindergarten through grade 4, some recommendations for students in older grades are extrapolated from this research.

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22. Interventionists may be any number of school personnel, including classroom teachers, special educators, school psychologists, paraprofessionals, and mathematics coaches and specialists. The panel does not specify the interventionist.

The scope of this guide does not include recommendations for special education referrals. Although enhancing the validity of special education referrals remains important and an issue of ongoing discussion<sup>23</sup> and research,<sup>24</sup> we do not address it in this practice guide, in part because empirical evidence is lacking.

The discussion of tier 1 in this guide revolves only around effective screening, because recommendations for general classroom mathematics instruction were beyond the scope of this guide. For this reason, studies of effective general mathematics instruction practices were not included in the evidence base for this guide.<sup>25</sup>

The studies reviewed for this guide included two types of comparisons among groups. First, several studies of tier 2 interventions compare students receiving multicomponent tier 2 interventions with students receiving only routine classroom instruction.<sup>26</sup> This type of study provides evidence of the effectiveness of providing tier 2 interventions but does not permit conclusions about which component is most effective. The reason is that it is not possible to identify whether one particular component or a combination of components within a multicomponent intervention produced an effect. Second, several

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23. Kavale and Spaulding (2008); Fuchs, Fuchs, and Vaughn (2008); VanDerHeyden, Witt, and Gilbertson (2007).

24. Fuchs, Fuchs, Compton et al. (2006).

25. There were a few exceptions in which general mathematics instruction studies were included in the evidence base. When the effects of a general mathematics instruction program were specified for low-achieving or disabled students and the intervention itself appeared applicable to teaching tier 2 or tier 3 (e.g., teaching a specific operational strategy), we included them in this study. Note that disabled students were predominantly learning disabled.

26. For example, Fuchs, Seethaler et al. (2008) examined the effects of providing supplemental tutoring (i.e., a tier 2 intervention) relative to regular classroom instruction (i.e., tier 1).



other studies examined the effects of two methods of tier 2 or tier 3 instruction.<sup>27</sup> This type of study offers evidence for the effectiveness of one approach to teaching within a tier relative to another approach and assists with identifying the most beneficial approaches for this population.

The panel reviewed only studies for practices that sought to improve student mathematics outcomes. The panel did not consider interventions that improved other academic or behavioral outcomes. Instead, the panel focused on practices that addressed the following areas of mathematics proficiency: *operations* (either computation

or estimation), *concepts* (knowledge of properties of operations, concepts involving rational numbers, prealgebra concepts), *problem solving* (word problems), and measures of *general mathematics achievement*. Measures of *fact fluency* were also included because quick retrieval of basic arithmetic facts is essential for success in mathematics and a persistent problem for students with difficulties in mathematics.<sup>28</sup>

Technical terms related to mathematics and technical aspects of assessments (psychometrics) are defined in a glossary at the end of the recommendations.

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27. For example, Tournaki (2003) examined the effects of providing supplemental tutoring in an operations strategy (a tier 2 intervention) relative to supplemental tutoring with a drill and practice approach (also a tier 2 intervention).

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28. Geary (2004); Jordan, Hanich, and Kaplan (2003).

## Checklist for carrying out the recommendations

### Recommendation 1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

- As a district or school sets up a screening system, have a team evaluate potential screening measures. The team should select measures that are efficient and reasonably reliable and that demonstrate predictive validity. Screening should occur in the beginning and middle of the year.
- Select screening measures based on the content they cover, with an emphasis on critical instructional objectives for each grade.
- In grades 4 through 8, use screening data in combination with state testing results.
- Use the same screening tool across a district to enable analyzing results across schools.

### Recommendation 2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.

- For students in kindergarten through grade 5, tier 2 and tier 3 interventions should focus almost exclusively on properties of whole numbers and operations. Some older students struggling with whole numbers and operations would also benefit from in-depth coverage of these topics.
- For tier 2 and tier 3 students in grades 4 through 8, interventions should focus on

in-depth coverage of rational numbers as well as advanced topics in whole number arithmetic (such as long division).

- Districts should appoint committees, including experts in mathematics instruction and mathematicians with knowledge of elementary and middle school mathematics curricula, to ensure that specific criteria are covered in-depth in the curriculum they adopt.

### Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

- Ensure that instructional materials are systematic and explicit. In particular, they should include numerous clear models of easy and difficult problems, with accompanying teacher think-alouds.
- Provide students with opportunities to solve problems in a group and communicate problem-solving strategies.
- Ensure that instructional materials include cumulative review in each session.

### Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures.

- Teach students about the structure of various problem types, how to categorize problems based on structure, and how to determine appropriate solutions for each problem type.
- Teach students to recognize the common underlying structure between familiar and unfamiliar problems and to transfer known solution methods from familiar to unfamiliar problems.

**Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.**

- Use visual representations such as number lines, arrays, and strip diagrams.
- If visuals are not sufficient for developing accurate abstract thought and answers, use concrete manipulatives first. Although this can also be done with students in upper elementary and middle school grades, use of manipulatives with older students should be expeditious because the goal is to move toward understanding of—and facility with—visual representations, and finally, to the abstract.

**Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.**

- Provide about 10 minutes per session of instruction to build quick retrieval of basic arithmetic facts. Consider using technology, flash cards, and other materials for extensive practice to facilitate automatic retrieval.
- For students in kindergarten through grade 2, explicitly teach strategies for efficient counting to improve the retrieval of mathematics facts.

- Teach students in grades 2 through 8 how to use their knowledge of properties, such as commutative, associative, and distributive law, to derive facts in their heads.

**Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.**

- Monitor the progress of tier 2, tier 3, and borderline tier 1 students at least once a month using grade-appropriate general outcome measures.
- Use curriculum-embedded assessments in interventions to determine whether students are learning from the intervention. These measures can be used as often as every day or as infrequently as once every other week.
- Use progress monitoring data to re-group students when necessary.

**Recommendation 8. Include motivational strategies in tier 2 and tier 3 interventions.**

- Reinforce or praise students for their effort and for attending to and being engaged in the lesson.
- Consider rewarding student accomplishments.
- Allow students to chart their progress and to set goals for improvement.

## **Recommendation 1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.**

The panel recommends that schools and districts systematically use universal screening to screen all students to determine which students have mathematics difficulties and require research-based interventions. Schools should evaluate and select screening measures based on their reliability and predictive validity, with particular emphasis on the measures' specificity and sensitivity. Schools should also consider the efficiency of the measure to enable screening many students in a short time.

### **Level of evidence: Moderate**

The panel judged the level of evidence supporting this recommendation to be *moderate*. This recommendation is based on a series of high-quality correlational studies with replicated findings that show the ability of measures to predict performance in mathematics one year after administration (and in some cases two years).<sup>29</sup>

### **Brief summary of evidence to support the recommendation**

A growing body of evidence suggests that there are several valid and reliable approaches for screening students in the primary grades. All these approaches target

aspects of what is often referred to as number sense.<sup>30</sup> They assess various aspects of knowledge of whole numbers—properties, basic arithmetic operations, understanding of magnitude, and applying mathematical knowledge to word problems. Some measures contain only one aspect of number sense (such as magnitude comparison) and others assess four to eight aspects of number sense. The single-component approaches with the best ability to predict students' subsequent mathematics performance include screening measures of students' knowledge of magnitude comparison and/or strategic counting.<sup>31</sup> The broader, multicomponent measures seem to predict with slightly greater accuracy than single-component measures.<sup>32</sup>

Effective approaches to screening vary in efficiency, with some taking as little as 5 minutes to administer and others as long as 20 minutes. Multicomponent measures, which by their nature take longer to administer, tend to be time-consuming for administering to an entire school population. Timed screening measures<sup>33</sup> and untimed screening measures<sup>34</sup> have been shown to be valid and reliable.

For the upper elementary grades and middle school, we were able to locate fewer studies. They suggest that brief early screening measures that take about 10 minutes and cover a proportional sampling of grade-level objectives are reasonable and provide sufficient evidence of reliability.<sup>35</sup> At the current time, this research area is underdeveloped.

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29. For reviews see Jiban and Deno (2007); Fuchs, Fuchs, Compton et al. (2007); Gersten, Jordan, and Flojo (2005).

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30. Berch (2005); Dehaene (1999); Okamoto and Case (1996); Gersten and Chard (1999).

31. Gersten, Jordan, and Flojo (2005).

32. Fuchs, Fuchs, Compton et al. (2007).

33. For example, Clarke and Shinn (2004).

34. For example, Okamoto and Case (1996).

35. Jiban and Deno (2007); Foegen, Jiban, and Deno (2007).

## How to carry out this recommendation

1. As a district or school sets up a screening system, have a team evaluate potential screening measures. The team should select measures that are efficient and reasonably reliable and that demonstrate predictive validity. Screening should occur in the beginning and middle of the year.

The team that selects the measures should include individuals with expertise in measurement (such as a school psychologist or a member of the district research and evaluation division) and those with expertise in mathematics instruction. In the opinion of the panel, districts should evaluate screening measures on three dimensions.

- *Predictive validity* is an index of how well a score on a screening measure earlier in the year predicts a student's later mathematics achievement. Greater predictive validity means that schools can be more confident that decisions based on screening data are accurate. In general, we recommend that schools and districts employ measures with predictive validity coefficients of at least .60 within a school year.<sup>36</sup>
- *Reliability* is an index of the consistency and precision of a measure. We recommend measures with reliability coefficients of .80 or higher.<sup>37</sup>
- *Efficiency* is how quickly the universal screening measure can be administered, scored, and analyzed for all the students. As a general rule, we suggest that a screening measure require no

36. A coefficient of .0 indicates that there is no relation between the early and later scores, and a coefficient of 1.0 indicates a perfect positive relation between the scores.

37. A coefficient of .0 indicates that there is no relation between the two scores, and a coefficient of 1.0 indicates a perfect positive relation between the scores.

more than 20 minutes to administer, which enables collecting a substantial amount of information in a reasonable time frame. Note that many screening measures take five minutes or less.<sup>38</sup> We recommend that schools select screening measures that have greater efficiency if their technical adequacy (predictive validity, reliability, sensitivity, and specificity) is roughly equivalent to less efficient measures. Remember that screening measures are intended for administration to all students in a school, and it may be better to invest more time in diagnostic assessment of students who perform poorly on the universal screening measure.

Keep in mind that screening is just a means of determining which students are likely to need help. If a student scores poorly on a screening measure or screening battery—especially if the score is at or near a cut point, the panel recommends monitoring her or his progress carefully to discern whether extra instruction is necessary.

Developers of screening systems recommend that screening occur at least twice a year (e.g., fall, winter, and/or spring).<sup>39</sup> This panel recommends that schools alleviate concern about students just above or below the cut score by screening students twice during the year. The second screening in the middle of the year allows another check on these students and also serves to identify any students who may have been at risk and grown substantially in their mathematics achievement—or those who were on-track at the beginning of the year but have not shown sufficient growth. The panel considers these two universal screenings to determine student proficiency as distinct from progress monitoring (Recommendation 7), which occurs on a more frequent

38. Foegen, Jiban, and Deno (2007); Fuchs, Fuchs, Compton et al. (2007); Gersten, Clarke, and Jordan (2007).

39. Kaminski et al. (2008); Shinn (1989).

basis (e.g., weekly or monthly) with a select group of intervention students in order to monitor response to intervention.

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### 2. Select screening measures based on the content they cover, with an emphasis on critical instructional objectives for each grade.

The panel believes that content covered in a screening measure should reflect the instructional objectives for a student's grade level, with an emphasis on the most critical content for the grade level. The National Council of Teachers of Mathematics (2006) released a set of focal points for each grade level designed to focus instruction on critical concepts for students to master within a specific grade. Similarly, the National Mathematics Advisory Panel (2008) detailed a route to preparing all students to be successful in algebra. In the lower elementary grades, the core focus of instruction is on building student understanding of whole numbers. As students establish an understanding of whole numbers, rational numbers become the focus of instruction in the upper elementary grades. Accordingly, screening measures used in the lower and upper elementary grades should have items designed to assess student's understanding of whole and rational number concepts—as well as computational proficiency.

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### 3. In grades 4 through 8, use screening data in combination with state testing results.

In the panel's opinion, one viable option that schools and districts can pursue is to use results from the previous year's state testing as a first stage of screening. Students who score below or only slightly above a benchmark would be considered for subsequent screening and/or diagnostic or placement testing. The use of state testing results would allow districts and schools to combine a broader measure that covers more content with a screening measure that is narrower but more focused. Because of the lack of available screening measures at

these grade levels, districts, county offices, or state departments may need to develop additional screening and diagnostic measures or rely on placement tests provided by developers of intervention curricula.

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### 4. Use the same screening tool across a district to enable analyzing results across schools.

The panel recommends that all schools within a district use the same screening measure and procedures to ensure objective comparisons across schools and within a district. Districts can use results from screening to inform instructional decisions at the district level. For example, one school in a district may consistently have more students identified as at risk, and the district could provide extra resources or professional development to that school. The panel recommends that districts use their research and evaluation staff to reevaluate screening measures annually or biannually. This entails examining how screening scores predict state testing results and considering resetting cut scores or other data points linked to instructional decisionmaking.

## Potential roadblocks and solutions

**Roadblock 1.1.** *Districts and school personnel may face resistance in allocating time resources to the collection of screening data.*

**Suggested Approach.** The issue of time and personnel is likely to be the most significant obstacle that districts and schools must overcome to collect screening data. Collecting data on all students will require structuring the data collection process to be efficient and streamlined.

The panel notes that a common pitfall is a long, drawn-out data collection process, with teachers collecting data in their classrooms “when time permits.” If schools are allocating resources (such as providing an intervention to students with the 20 lowest scores in grade 1), they must wait until

all the data have been collected across classrooms, thus delaying the delivery of needed services to students. Furthermore, because many screening measures are sensitive to instruction, a wide gap between when one class is assessed and another is assessed means that many students in the second class will have higher scores than those in the first because they were assessed later.

One way to avoid these pitfalls is to use data collection teams to screen students in a short period of time. The teams can consist of teachers, special education staff including such specialists as school psychologists, Title I staff, principals, trained instructional assistants, trained older students, and/or local college students studying child development or school psychology.

**Roadblock 1.2.** *Implementing universal screening is likely to raise questions such as, “Why are we testing students who are doing fine?”*

**Suggested Approach.** Collecting data on all students is new for many districts and schools (this may not be the case for elementary schools, many of which use screening assessments in reading).<sup>40</sup> But screening allows schools to ensure that all students who are on track stay on track and collective screening allows schools to evaluate the impact of their instruction on groups of students (such as all grade 2 students). When schools screen all students, a distribution of achievement from high to low is created. If students considered not at risk were not screened, the distribution of screened students would consist only of at-risk students. This could create a situation where some students at the “top” of the distribution are in reality at risk but not identified as such. For upper-grade students whose scores were

high on the previous spring’s state assessment, additional screening typically is not required.

**Roadblock 1.3.** *Screening measures may identify students who do not need services and not identify students who do need services.*

**Suggested Approach.** All screening measures will misidentify some students as either needing assistance when they do not (false positive) or not needing assistance when they do (false negative). When screening students, educators will want to maximize both the number of students correctly identified as at risk—a measure’s *sensitivity*—and the number of students correctly identified as not at risk—a measure’s *specificity*. As illustrated in table 3, screening students to determine risk can result in four possible categories indicated by the letters A, B, C, and D. Using these categories, sensitivity is equal to  $A/(A + C)$  and specificity is equal to  $D/(B + D)$ .

**Table 3. Sensitivity and specificity**

		STUDENTS ACTUALLY AT RISK	
		Yes	No
STUDENTS IDENTIFIED AS BEING AT RISK	Yes	<b>A</b> (true positives)	<b>B</b> (false positives)
	No	<b>C</b> (false negatives)	<b>D</b> (true negatives)

The sensitivity and specificity of a measure depend on the cut score to classify children at risk.<sup>41</sup> If a cut score is high (where all students below the cut score are considered at risk), the measure will have a high degree of sensitivity because most students who truly need assistance will be

40. U.S. Department of Education, Office of Planning, Evaluation and Policy Development, Policy and Program Studies Service (2006).

41. Sensitivity and specificity are also influenced by the discriminant validity of the measure and its individual items. Measures with strong item discrimination are more likely to correctly identify students’ risk status.

identified as at risk. But the measure will have low specificity since many students who do not need assistance will also be identified as at risk. Similarly, if a cut score is low, the sensitivity will be lower (some students in need of assistance may not be identified as at risk), whereas the specificity will be higher (most students who do not need assistance will not be identified as at risk).

Schools need to be aware of this tradeoff between sensitivity and specificity, and the team selecting measures should be aware that decisions on cut scores can be somewhat arbitrary. Schools that set a cut score too high run the risk of spending resources on students who do not need help, and schools that set a cut score too low run the risk of not providing interventions to students who are at risk and need extra instruction. If a school or district consistently finds that students receiving intervention do not need it, the measurement team should consider lowering the cut score.

**Roadblock 1.4.** *Screening data may identify large numbers of students who are at risk and schools may not immediately have the resources to support all at-risk students. This will be a particularly severe problem in low-performing Title I schools.*

**Suggested Approach.** Districts and schools need to consider the amount of resources available and the allocation of

those resources when using screening data to make instructional decisions. Districts may find that on a nationally normed screening measure, a large percentage of their students (such as 60 percent) will be classified as at risk. Districts will have to determine the resources they have to provide interventions and the number of students they can serve with their resources. This may mean not providing interventions at certain grade levels or providing interventions only to students with the lowest scores, at least in the first year of implementation.

There may also be cases when schools identify large numbers of students at risk in a particular area and decide to provide instruction to all students. One particularly salient example is in the area of fractions. Multiple national assessments show many students lack proficiency in fractions,<sup>42</sup> so a school may decide that, rather than deliver interventions at the individual child level, they will provide a school-wide intervention to all students. A school-wide intervention can range from a supplemental fractions program to professional development involving fractions.

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42. National Mathematics Advisory Panel (2008); Lee, Grigg, and Dion (2007).



## **Recommendation 2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.**

The panel recommends that individuals knowledgeable in instruction and mathematics look for interventions that focus on whole numbers extensively in kindergarten through grade 5 and on rational numbers extensively in grades 4 through 8. In all cases, the specific content of the interventions will be centered on building the student's foundational proficiencies. In making this recommendation, the panel is drawing on consensus documents developed by experts from mathematics education and research mathematicians that emphasized the importance of these topics for students in general.<sup>43</sup> We conclude that the coverage of fewer topics in more depth, and with coherence, is as important, and probably more important, for students who struggle with mathematics.

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43. National Council of Teachers of Mathematics (2006); National Mathematics Advisory Panel (2008).

### **Level of evidence: Low**

The panel judged the level of evidence supporting this recommendation to be *low*. This recommendation is based on the professional opinion of the panel and several recent consensus documents that reflect input from mathematics educators and research mathematicians involved in issues related to kindergarten through grade 12 mathematics education.<sup>44</sup>

### **Brief summary of evidence to support the recommendation**

The documents reviewed demonstrate a growing professional consensus that coverage of fewer mathematics topics in more depth and with coherence is important for all students.<sup>45</sup> Milgram and Wu (2005) suggested that an intervention curriculum for at-risk students should not be oversimplified and that in-depth coverage of key topics and concepts involving whole numbers and then rational numbers is critical for future success in mathematics. The National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points (2006) called for the end of brief ventures into many topics in the course of a school year and also suggested heavy emphasis on instruction in whole numbers and rational numbers. This position was reinforced by the 2008 report of the National Mathematics Advisory Panel (NMAP), which provided detailed benchmarks and again emphasized in-depth coverage of key topics involving whole numbers and rational numbers as crucial for all students. Although the latter two documents addressed the needs of all students, the panel concludes that the in-depth coverage of key topics is especially

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44. National Council of Teachers of Mathematics (2006); National Mathematics Advisory Panel (2008); Milgram and Wu (2005).

45. National Mathematics Advisory Panel (2008); Schmidt and Houang (2007); Milgram and Wu (2005); National Council of Teachers of Mathematics (2006).

important for students who struggle with mathematics.

### How to carry out this recommendation

1. For students in kindergarten through grade 5, tier 2 and tier 3 interventions should focus almost exclusively on properties of whole numbers<sup>46</sup> and operations. Some older students struggling with whole numbers and operations would also benefit from in-depth coverage of these topics.

In the panel's opinion, districts should review the interventions they are considering to ensure that they cover whole numbers in depth. The goal is proficiency and mastery, so in-depth coverage with extensive review is essential and has been articulated in the NCTM Curriculum Focal Points (2006) and the benchmarks determined by the National Mathematics Advisory Panel (2008). Readers are recommended to review these documents.<sup>47</sup>

Specific choices for the content of interventions will depend on the grade level and proficiency of the student, but the focus for struggling students should be on whole numbers. For example, in kindergarten through grade 2, intervention materials would typically include significant attention to counting (e.g., counting up), number composition, and number decomposition (to understand place-value multidigit operations). Interventions should cover the meaning of addition and subtraction and

46. Properties of numbers, including the associative, commutative, and distributive properties.

47. More information on the National Mathematics Advisory Panel (2008) report is available at [www.ed.gov/about/bdscomm/list/mathpanel/index.html](http://www.ed.gov/about/bdscomm/list/mathpanel/index.html). More information on the National Council of Teachers of Mathematics Curriculum Focal Points is available at [www.nctm.org/focalpoints](http://www.nctm.org/focalpoints). Documents elaborating the National Council of Teachers of Mathematics Curriculum Focal Points are also available (see Beckmann et al., 2009). For a discussion of why this content is most relevant, see Milgram and Wu (2005).

the reasoning that underlies algorithms for addition and subtraction of whole numbers, as well as solving problems involving whole numbers. This focus should include understanding of the base-10 system (place value).

Interventions should also include materials to build fluent retrieval of basic arithmetic facts (see recommendation 6). Materials should extensively use—and ask students to use—visual representations of whole numbers, including both concrete and visual base-10 representations, as well as number paths and number lines (more information on visual representations is in recommendation 5).

2. For tier 2 and tier 3 students in grades 4 through 8, interventions should focus on in-depth coverage of rational numbers as well as advanced topics in whole number arithmetic (such as long division).

The panel believes that districts should review the interventions they are considering to ensure that they cover concepts involving rational numbers in depth. The focus on rational numbers should include understanding the meaning of fractions, decimals, ratios, and percents, using visual representations (including placing fractions and decimals on number lines,<sup>48</sup> see recommendation 5), and solving problems with fractions, decimals, ratios, and percents.

In the view of the panel, students in grades 4 through 8 will also require additional work to build fluent retrieval of basic arithmetic facts (see recommendation 6), and some will require additional work involving basic whole number topics, especially for students in tier 3. In the opinion of the panel, accurate and fluent

48. When using number lines to teach rational numbers for students who have difficulties, it is important to emphasize that the focus is on the length of the segments between the whole number marks (rather than counting the marks).

arithmetic with whole numbers is necessary before understanding fractions. The panel acknowledges that there will be periods when both whole numbers and rational numbers should be addressed in interventions. In these cases, the balance of concepts should be determined by the student's need for support.

**3. Districts should appoint committees, including experts in mathematics instruction and mathematicians with knowledge of elementary and middle school mathematics curriculum, to ensure that specific criteria (described below) are covered in depth in the curricula they adopt.**

In the panel's view, intervention materials should be reviewed by individuals with knowledge of mathematics instruction and by mathematicians knowledgeable in elementary and middle school mathematics. They can often be experts within the district, such as mathematics coaches, mathematics teachers, or department heads. Some districts may also be able to draw on the expertise of local university mathematicians.

Reviewers should assess how well intervention materials meet four criteria. First, the materials integrate computation with solving problems and pictorial representations rather than teaching computation apart from problem-solving. Second, the materials stress the reasoning underlying calculation methods and focus student attention on making sense of the mathematics. Third, the materials ensure that students build algorithmic proficiency. Fourth, the materials include frequent review for both consolidating and understanding the links of the mathematical principles. Also in the

panel's view, the intervention program should include an assessment to assist in placing students appropriately in the intervention curriculum.

### **Potential roadblocks and solutions**

**Roadblock 2.1.** *Some interventionists may worry if the intervention program is not aligned with the core classroom instruction.*

**Suggested Approach.** The panel believes that alignment with the core curriculum is not as critical as ensuring that instruction builds students' foundational proficiencies. Tier 2 and tier 3 instruction focuses on foundational and often prerequisite skills that are determined by the students' rate of progress. So, in the opinion of the panel, acquiring these skills will be necessary for future achievement. Additionally, because tier 2 and tier 3 are supplemental, students will still be receiving core classroom instruction aligned to a school or district curriculum (tier 1).

**Roadblock 2.2.** *Intervention materials may cover topics that are not essential to building basic competencies, such as data analysis, measurement, and time.*

**Suggested Approach.** In the panel's opinion, it is not necessary to cover every topic in the intervention materials. Students will gain exposure to many supplemental topics (such as data analysis, measurement, and time) in general classroom instruction (tier 1). Depending on the student's age and proficiency, it is most important to focus on whole and rational numbers in the interventions.

### **Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.**

The National Mathematics Advisory Panel defines *explicit instruction* as follows (2008, p. 23):

- “Teachers provide clear models for solving a problem type using an array of examples.”
- “Students receive extensive practice in use of newly learned strategies and skills.”
- “Students are provided with opportunities to think aloud (i.e., talk through the decisions they make and the steps they take).”
- “Students are provided with extensive feedback.”

The NMAP notes that this does not mean that all mathematics instruction should be explicit. But it does recommend that struggling students receive some explicit instruction regularly and that some of the explicit instruction ensure that students possess the foundational skills and conceptual knowledge necessary for understanding their grade-level

mathematics.<sup>49</sup> Our panel supports this recommendation and believes that districts and schools should select materials for interventions that reflect this orientation. In addition, professional development for interventionists should contain guidance on these components of explicit instruction.

#### **Level of evidence: Strong**

Our panel judged the level of evidence supporting this recommendation to be *strong*. This recommendation is based on six randomized controlled trials that met WWC standards or met standards with reservations and that examined the effectiveness of explicit and systematic instruction in mathematics interventions.<sup>50</sup> These studies have shown that explicit and systematic instruction can significantly improve proficiency in word problem solving<sup>51</sup> and operations<sup>52</sup> across grade levels and diverse student populations.

#### **Brief summary of evidence to support the recommendation**

The results of six randomized controlled trials of mathematics interventions show extensive support for various combinations of the following components of explicit and systematic instruction: teacher demonstration,<sup>53</sup> student verbalization,<sup>54</sup>

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49. National Mathematics Advisory Panel (2008).

50. Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Jitendra et al. (1998); Schunk and Cox (1986); Tournaki (2003); Wilson and Sindelar (1991).

51. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Fuchs et al. (2003a); Wilson and Sindelar (1991).

52. Schunk and Cox (1986); Tournaki (2003).

53. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Fuchs et al. (2003a); Schunk and Cox (1986); Tournaki (2003); Wilson and Sindelar (1991).

54. Jitendra et al. (1998); Fuchs et al. (2003a); Schunk and Cox (1986); Tournaki (2003).

guided practice,<sup>55</sup> and corrective feedback.<sup>56</sup> All six studies examined interventions that included teacher demonstrations early in the lessons.<sup>57</sup> For example, three studies included instruction that began with the teacher verbalizing aloud the steps to solve sample mathematics problems.<sup>58</sup> The effects of this component of explicit instruction cannot be evaluated from these studies because the demonstration procedure was used in instruction for students in both treatment and comparison groups.

Scaffolded practice, a transfer of control of problem solving from the teacher to the student, was a component in four of the six studies.<sup>59</sup> Although it is not possible to parse the effects of scaffolded instruction from the other components of instruction, the intervention groups in each study demonstrated significant positive gains on word problem proficiencies or accuracy measures.

Three of the six studies included opportunities for students to verbalize the steps to solve a problem.<sup>60</sup> Again, although effects of the interventions were statistically significant and positive on measures of word problems, operations, or accuracy, the effects cannot be attributed to a single component of these multicomponent interventions.

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55. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Fuchs et al. (2003a); Tournaki (2003).

56. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Schunk and Cox (1986); Tournaki (2003).

57. Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Jitendra et al. (1998); Schunk and Cox (1986); Tournaki (2003); Wilson and Sindelar (1991).

58. Schunk and Cox (1986); Jitendra et al. (1998); Darch, Carnine, and Gersten (1984).

59. Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Jitendra et al. (1998); Tournaki (2003).

60. Schunk and Cox (1986); Jitendra et al. (1998); Tournaki (2003).

Similarly, four of the six studies included immediate corrective feedback,<sup>61</sup> and the effects of these interventions were positive and significant on word problems and measures of operations skills, but the effects of the corrective feedback component cannot be isolated from the effects of other components in three cases.<sup>62</sup>

With only one study in the pool of six including cumulative review as part of the intervention,<sup>63</sup> the support for this component of explicit instruction is not as strong as it is for the other components. But this study did have statistically significant positive effects in favor of the instructional group that received explicit instruction in strategies for solving word problems, including cumulative review.

### **How to carry out this recommendation**

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1. Ensure that instructional materials are systematic and explicit. In particular, they should include numerous clear models of easy and difficult problems, with accompanying teacher think-alouds.

To be considered systematic, mathematics instruction should gradually build proficiency by introducing concepts in a logical order and by providing students with numerous applications of each concept. For example, a systematic curriculum builds student understanding of place value in an array of contexts before teaching procedures for adding and subtracting two-digit numbers with regrouping.

Explicit instruction typically begins with a clear unambiguous exposition of concepts and step-by-step models of how

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61. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Tournaki (2003); Schunk and Cox (1986).

62. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Tournaki (2003).

63. Fuchs et al. (2003a).

to perform operations and reasons for the procedures.<sup>64</sup> Interventionists should think aloud (make their thinking processes public) as they model each step of the process.<sup>65,66</sup> They should not only tell students about the steps and procedures they are performing, but also allude to the reasoning behind them (link to the underlying mathematics).

The panel suggests that districts select instructional materials that provide interventionists with sample think-alouds or possible scenarios for explaining concepts and working through operations. A criterion for selecting intervention curricula materials should be whether or not they provide materials that help interventionists model or think through difficult and easy examples.

In the panel's view, a major flaw in many instructional materials is that teachers are asked to provide only one or two models of how to approach a problem and that most of these models are for easy-to-solve problems. Ideally, the materials will also assist teachers in explaining the reasoning behind the procedures and problem-solving methods.

## 2. Provide students with opportunities to solve problems in a group and communicate problem-solving strategies.

For students to become proficient in performing mathematical processes, explicit instruction should include scaffolded practice, where the teacher plays an active role and gradually transfers the work to

the students.<sup>67</sup> This phase of explicit instruction begins with the teacher and the students solving problems together. As this phase of instruction continues, students should gradually complete more steps of the problem with decreasing guidance from the teacher. Students should proceed to independent practice when they can solve the problem with little or no support from the teacher.

During guided practice, the teacher should ask students to communicate the strategies they are using to complete each step of the process and provide reasons for their decisions.<sup>68</sup> In addition, the panel recommends that teachers ask students to explain their solutions.<sup>69</sup> Note that not only interventionists—but fellow students—can and should communicate how they think through solving problems to the interventionist and the rest of the group. This can facilitate the development of a shared language for talking about mathematical problem solving.<sup>70</sup>

Teachers should give specific feedback that clarifies what students did correctly and what they need to improve.<sup>71</sup> They should provide opportunities for students to correct their errors. For example, if a student has difficulty solving a word problem or solving an equation, the teacher should ask simple questions that guide the student to solving the problem correctly. Corrective feedback can also include re-teaching or clarifying instructions when students are not able to respond to questions or their responses are incorrect.

64. For example, Jitendra et al. (1998); Darch, Carnine, and Gersten (1984); Woodward (2006).

65. See an example in the summary of Tournaki (2003) in appendix D.

66. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Fuchs et al. (2003a); Schunk and Cox (1986); Tournaki (2003); Wilson and Sindelar (1991).

67. Tournaki (2003); Jitendra et al. (1998); Darch, Carnine, and Gersten (1984).

68. For example, Schunk and Cox (1986).

69. Schunk and Cox (1986); Tournaki (2003).

70. For example, Jitendra et al. (1998); Darch, Carnine, and Gersten (1984).

71. Tournaki (2003); Jitendra et al. (1998); Darch, Carnine, and Gersten (1984).

### 3. Ensure that instructional materials include cumulative review in each session.

Cumulative reviews provide students with an opportunity to practice topics previously covered in depth. For example, when students are working with fractions, a cumulative review activity could provide them with an opportunity to solve some problems involving multiplication and division of whole numbers. In the panel's opinion, this review can ensure that the knowledge is maintained over time and helps students see connections between various mathematical ideas.

#### Potential roadblocks and solutions

**Roadblock 3.1.** *Interventionists may be unfamiliar with how to implement an intervention that uses explicit instruction, and some may underestimate the amount of practice necessary for students in tiers 2 and 3 to master the material being taught.*

**Suggested Approach.** Districts and schools should set up professional development sessions for interventionists to observe and discuss sample lessons. The panel believes that it is important for professional development participants to observe the intervention first hand. Watching a DVD or video of the intervention being used with students can give the participants a model of how the program should be implemented.

Interventionists should also have hands-on experience, teaching the lessons to each other and practicing with students. Role-playing can give interventionists practice with modeling and think-alouds, since it is important for them to stop and reflect before formulating an explanation for their thinking processes. The trainers can observe these activities, provide feedback on what participants did well, and offer explicit suggestions for improving instruction.

As a part of professional development, be sure to convey the benefits that extended practice (not only worksheets) and cumulative review can have for student performance. If professional development is not an option, teachers can also work with mathematics coaches to learn how to implement the intervention.

**Roadblock 3.2.** *Interventionists may not be expert with the underlying mathematics content.*

**Suggested Approach.** For interventionists to explain a mathematical process accurately and develop a logical think-aloud, it is important for them to understand the underlying mathematics concept and the mathematical reasoning for the process. Professional development should provide participants with in-depth knowledge of the mathematics content in the intervention, including the mathematical reasoning underlying procedures, formulas, and problem-solving methods.<sup>72</sup> The panel believes that when interventionists convey their knowledge of the content, student understanding will increase, misconceptions will decrease, and the chances that students solve problems by rote memory will be reduced.

**Roadblock 3.3.** *The intervention materials may not incorporate enough models, think-alouds, practice, and cumulative review.*

**Suggested Approach.** Intervention programs might not incorporate enough models, think-alouds, practice, or cumulative review to improve students' mathematics performance.<sup>73</sup>

Consider using a mathematics coach or specialist to develop a template listing the essential parts of an effective lesson,

72. National Mathematics Advisory Panel (2008); Wu (2005) <http://math.berkeley.edu/~wu/Northridge2004a2.pdf>.

73. Jitendra et al. (1996); Carnine et al. (1997).

RECOMMENDATION 3. INSTRUCTION DURING THE INTERVENTION SHOULD BE EXPLICIT AND SYSTEMATIC

including the number of models, accompanying think-alouds, and practice and cumulative review items students need to understand, learn, and master the content.

A team of teachers, guided by the mathematics coach/specialist, can determine the components that should be added to the program.



## **Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures.**

Students who have difficulties in mathematics typically experience severe difficulties in solving word problems related to the mathematics concepts and operations they are learning.<sup>74</sup> This is a major impediment for future success in any math-related discipline.<sup>75</sup>

Based on the importance of building proficiency and the convergent findings from a body of high-quality research, the panel recommends that interventions include systematic explicit instruction on solving word problems, using the problems' underlying structure. Simple word problems give meaning to mathematical operations such as subtraction or multiplication. When students are taught the underlying structure of a word problem, they not only have greater success in problem solving but can also gain insight into the deeper mathematical ideas in word problems.<sup>76</sup> The panel also recommends systematic instruction on the structural connections between known, familiar word problems and unfamiliar, new problems. By making explicit the underlying structural connections between familiar and unfamiliar problems, students will know when to apply the solution methods they have learned.<sup>77</sup>

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74. Geary (2003); Hanich et al. (2001).

75. National Mathematics Advisory Panel (2008); McCloskey (2007).

76. Peterson, Fennema, and Carpenter (1989).

77. Fuchs, Fuchs, Finelli et al. (2004).

### **Level of evidence: Strong**

The panel judged the level of evidence supporting this recommendation to be *strong*. This recommendation is based on nine randomized controlled trials that met WWC standards or met standards with reservations and that examined the effectiveness of word problem-solving strategies.<sup>78</sup> Interventions that teach students the structure of problem types<sup>79</sup>—and how to discriminate superficial from substantive information to know when to apply the solution methods they have learned<sup>80</sup>—positively and marginally or significantly affect proficiency in solving word problems.

### **Brief summary of evidence to support the recommendation**

Research demonstrates that instruction on solving word problems based on underlying problem structure leads to statistically significant positive effects on measures of word problem solving.<sup>81</sup> Three randomized controlled trials isolated this practice. In these studies, interventionists taught students to identify problems of a given type by focusing on the problem structure and then to design and execute appropriate solution strategies for each problem. These techniques typically led to significant and positive effects on word-problem outcomes for students

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78. Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

79. Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Darch, Carnine, and Gersten (1984).

80. Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

81. Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Darch, Carnine, and Gersten (1984).

experiencing difficulties in mathematics across grade levels.<sup>82</sup>

Six other randomized controlled trials took the instructional intervention on problem structure a step further. They demonstrated that teaching students to distinguish superficial from substantive information in problems also leads to marginally or statistically significant positive effects on measures of word problem solving.<sup>83</sup> After students were explicitly taught the pertinent structural features and problem-solution methods for different problem types, they were taught superficial problem features that can change a problem without altering its underlying structure. They were taught to distinguish substantive information from superficial information in order to solve problems that appear new but really fit into one of the categories of problems they already know how to solve. They were also taught that the same underlying problem structures can be applied to problems that are presented in graphic form (for example, with tables or maps). These are precisely the issues that often confuse and derail students with difficulties in mathematics. These six studies consistently demonstrated marginally or statistically significant positive effects on an array of word problem-solving proficiencies for students experiencing difficulties in mathematics.<sup>84</sup>

82. Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Darch, Carnine, and Gersten (1984).

83. Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

84. Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

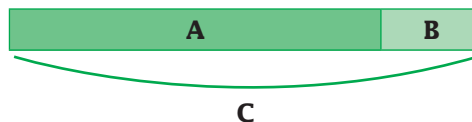
## How to carry out this recommendation

1. Teach students about the structure of various problem types, how to categorize problems based on structure, and how to determine appropriate solutions for each problem type.

Students should be explicitly taught about the salient underlying structural features of each problem type.<sup>85</sup> Problem types are groups of problems with similar mathematical structures. For example, change problems describe situations in which a quantity (such as children or pencils) is either increased or decreased (example 1). Change problems always include a time element. For these problems, students determine whether to add or subtract by determining whether the change in the quantity is more or less.

### Example 1. Change problems

The two problems here are addition and subtraction problems that students may be tempted to solve using an incorrect operation. In each case, students can draw a simple diagram like the one shown below, record the known quantities (two of three of A, B, and C) and then use the diagram to decide whether addition or subtraction is the correct operation to use to determine the unknown quantity.



**Problem 1.** Brad has a bottlecap collection. After Madhavi gave Brad 28 more bottlecaps, Brad had 111 bottlecaps. How many bottlecaps did Brad have before Madhavi gave him more?

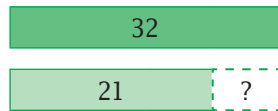
**Problem 2.** Brad has a bottlecap collection. After Brad gave 28 of his bottlecaps to Madhavi, he had 83 bottlecaps left. How many bottlecaps did Brad have before he gave Madhavi some?

85. Xin, Jitendra, and Deatline-Buchman (2005).

In contrast, *compare* problems have no time element (example 2). They focus on comparisons between two different types of items in two different sets (pears and apples, boys and girls, hot and cold items). Students add or subtract by determining whether they need to calculate the unknown difference (subtract), unknown compared amount (add), or unknown referent amount (subtract).

### Example 2. Compare problems

There are 21 hamsters and 32 kittens at the pet store. How many more kittens are at the pet store than hamsters?



Although these problem types seem simple and intuitive to adults and mathematically precocious students, they are not necessarily obvious for students requiring mathematics interventions. To build understanding of each problem type, we recommend initially teaching solution rules (or guiding questions that lead to a solution equation) for each problem type through fully and partially worked examples, followed by student practice in pairs.<sup>86</sup>

Visual representations such as those in example 2 can be effective for teaching students how to categorize problems based on their structure and determine a solution method appropriate for the underlying structure (see recommendation 5 for more information on visual representations).<sup>87</sup> Teachers can present stories with unknown information and work with students in using diagrams to identify the problem type and transform the information in the diagram into a mathematics equation to solve for the unknown quantity.

86. Fuchs, Fuchs, Finelli et al. (2004).

87. Xin, Jitendra, and Deatline-Buchman (2005).

### 2. Teach students to recognize the common underlying structure between familiar and unfamiliar problems and to transfer known solution methods from familiar to unfamiliar problems.

A known familiar problem often appears as a new and unfamiliar problem to a student because of such superficial changes as format changes (whether it is written in traditional paragraph form or as an advertisement for a brochure), key vocabulary changes (half, one-half,  $\frac{1}{2}$ ), or the inclusion of irrelevant information (additional story elements such as the number of buttons on a child's shirt or the size of a storage container for a compare problem).<sup>88</sup> These superficial changes are irrelevant to understanding the mathematical demands of a problem. But while focusing on these irrelevant superficial changes, students can find it difficult to discern the critical common underlying structure between the new and the old problems and to apply the solution that is part of their repertoire to the new unfamiliar problem.

To facilitate the transfer of the known solution from the familiar to the unfamiliar problem, students should first be shown explicitly that not all pieces of information in the problem are relevant to discerning the underlying problem structure.<sup>89</sup> Teachers should explain these irrelevant superficial features explicitly and systematically, as described in recommendation 3.<sup>90</sup> This instruction may be facilitated by the use of a poster displayed in the classroom that lists the ways familiar problems can become unfamiliar because of new wording or situations (such as information displayed in

88. Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

89. Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

90. Fuchs, Fuchs, Finelli et al. (2004).

chart versus paragraph form) or the ways relevant to problem type. The students must also be provided with opportunities to explain why a piece of information is relevant or irrelevant.<sup>91</sup>

We suggest that students practice sets of problems with varied superficial features and cover stories. Students who know how to recognize and solve a “change” problem type with whole numbers should know that they can apply the same strategy to a structurally similar word problem that looks different because of changes in wording and the presence of additional story elements (example 3).<sup>92</sup>

### Example 3. Solving different problems with the same strategy

- Mike wants to buy 1 pencil for each of his friends. Each packet of pencils contains 12 pencils. How many packets does Mike have to buy to give 1 pencil to each of his 13 friends?
- Mike wants to buy 1 pencil for each of his friends. Sally wants to buy 10 pencils. Each box of pencils contains 12 pencils. How many boxes does Mike have to buy to give 1 pencil to each of his 13 friends?

91. Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Prentice et al. (2004).

92. Fuchs, Fuchs, Finelli et al. (2004).

## Potential roadblocks and solutions

**Roadblock 4.1.** *In the opinion of the panel, the curricular material may not classify problems into problem types.*

**Suggested Approach.** The interventionist may need the help of a mathematics coach, a mathematics specialist, or a district or state curriculum guide in determining the problem types and an instructional sequence for teaching them to students. The key issue is that students are taught to understand a set of problem structures related to the mathematics they are learning in their intervention.

**Roadblock 4.2.** *As problems get complex, so will the problem types and the task of discriminating among them.*

**Suggested Approach.** As problems get more intricate (such as multistep problems), it becomes more difficult for students to determine the problem type, a critical step that leads to solving the problem correctly. It is important to explicitly and systematically teach students how to differentiate one problem type from another.

Interventionists will need high-quality professional development to ensure that they convey the information clearly and accurately. The professional development program should include opportunities for participants to determine problem types, justify their responses, and practice explaining and modeling problem types to peers and children. Trainers should provide constructive feedback during the practice sessions by telling participants both what they did well and what aspects of their instruction need improvement.

## **Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.**

A major problem for students who struggle with mathematics is weak understanding of the relationships between the abstract symbols of mathematics and the various visual representations.<sup>93</sup> Student understanding of these relationships can be strengthened through the use of visual representations of mathematical concepts such as solving equations, fraction equivalence, and the commutative property of addition and multiplication (see the glossary). Such representations may include number lines, graphs, simple drawings of concrete objects such as blocks or cups, or simplified drawings such as ovals to represent birds.

In the view of the panel, the ability to express mathematical ideas using visual representations and to convert visual representations into symbols is critical for success in mathematics. A major goal of interventions should be to systematically teach students how to develop visual representations and how to transition these representations to standard symbolic representations

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93. Hecht, Vagi, and Torgesen (2007).

used in problem solving. Occasional and unsystematic exposure (the norm in many classrooms) is insufficient and does not facilitate understanding of the relationship between the abstract symbols of mathematics and various visual representations.

### **Level of evidence: Moderate**

The panel judged the level of evidence supporting this recommendation to be *moderate*. This recommendation is based on 13 randomized controlled trials that met WWC standards or met standards with reservations.<sup>94</sup> These studies provide support for the systematic use of visual representations or manipulatives to improve achievement in general mathematics,<sup>95</sup> prealgebra concepts,<sup>96</sup> word problems,<sup>97</sup> and operations.<sup>98</sup> But these representations were part of a complex multicomponent intervention in each of the studies. So, it is difficult to judge the impact of the representation component alone, and the panel believes that a *moderate* designation is appropriate for the level of evidence for this recommendation.

### **Brief summary of evidence to support the recommendation**

Research shows that the systematic use of visual representations and manipulatives may lead to statistically significant or substantively important positive gains in math

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94. Artus and Dyrek (1989); Butler et al. (2003); Darch, Carnine, and Gertsen (1984); Fuchs et al. (2005); Fuchs, Seethaler et al. (2008); Fuchs, Powell et al. (2008); Fuchs, Fuchs, Craddock et al. (2008); Jitendra et al. (1998); Walker and Poteet (1989); Wilson and Sindelar (1991); Witzel (2005); Witzel, Mercer, and Miller (2003); Woodward (2006).

95. Artus and Dyrek (1989); Fuchs et al. (2005).

96. Witzel, Mercer, and Miller (2003).

97. Darch, Carnine, and Gersten (1984); Fuchs et al. (2005); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008); Jitendra et al. (1998); Wilson and Sindelar (1991).

98. Woodward (2006).

achievement.<sup>99</sup> Four studies used visual representations to help pave the way for students to understand the abstract version of the representation.<sup>100</sup> For example, one of the studies taught students to use visual representations such as number lines to understand mathematics facts.<sup>101</sup> The four studies demonstrated gains in mathematics facts and operations<sup>102</sup> and word problem proficiencies,<sup>103</sup> and may provide evidence that using visual representations in interventions is an effective technique.

Three of the studies used manipulatives in the early stages of instruction to reinforce understanding of basic concepts and operations.<sup>104</sup> One used concrete models such as groups of boxes to teach rules for multiplication problems.<sup>105</sup> The three studies largely showed significant and positive effects and provide evidence that using manipulatives may be helpful in the initial stages of an intervention to improve proficiency in word problem solving.<sup>106</sup>

In six of the studies, both concrete and visual representations were used, and overall these studies show that using some combination of manipulatives and visual representations may promote mathematical understanding.<sup>107</sup> In two of the six, instruction did not include fading of the

manipulatives and visual representations to promote understanding of math at a more abstract level.<sup>108</sup> One of these interventions positively affected general math achievement,<sup>109</sup> but the other had no effect on outcome measures tested.<sup>110</sup> In the other four studies, manipulatives and visual representations were presented to the students sequentially to promote understanding at a more abstract level.<sup>111</sup> One intervention that used this method for teaching fractions did not show much promise,<sup>112</sup> but the other three did result in positive gains.<sup>113</sup> One of them taught 1st graders basic math concepts and operations,<sup>114</sup> and the other two taught prealgebra concepts to low-achieving students.<sup>115</sup>

### How to carry out this recommendation

#### 1. Use visual representations such as number lines, arrays, and strip diagrams.

In the panel's view, visual representations such as number lines, number paths, strip diagrams, drawings, and other forms of pictorial representations help scaffold learning and pave the way for understanding the abstract version of the representation. We recommend that interventionists use such abstract visual representations extensively and consistently. We also recommend that interventionists explicitly link visual representations with the standard symbolic representations used in mathematics.

99. Following WWC guidelines, an effect size greater than 0.25 is considered substantively important.

100. Jitendra et al. (1998); Walker and Poteet (1989); Wilson and Sindelar (1991); Woodward (2006).

101. Woodward (2006).

102. Woodward (2006).

103. Jitendra et al. (1998); Walker and Poteet (1989); Wilson and Sindelar (1991).

104. Darch (1989); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).

105. Darch (1989).

106. Darch (1989); Fuchs, Sethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).

107. Artus and Dyrek (1989); Butler et al. (2003); Fuchs, Powell et al. (2008); Fuchs et al. (2005); Witzel (2005); Witzel, Mercer, and Miller (2003).

108. Artus and Dyrek (1989); Fuchs, Powell et al. (2008).

109. Artus and Dyrek (1989).

110. Fuchs, Powell et al. (2008).

111. Fuchs et al. (2005); Butler et al. (2003); Witzel et al. (2003); Witzel (2005).

112. Butler et al. (2003).

113. Fuchs et al. (2005); Witzel, Mercer, and Miller (2003); Witzel (2005).

114. Fuchs et al. (2005).

115. Witzel, Mercer, and Miller (2003); Witzel (2005).

In early grades, number lines, number paths, and other pictorial representations are often used to teach students foundational concepts and procedural operations of addition and subtraction. Although number lines or number paths may not be a suitable initial representation in some situations (as when working with multiplication and division), they can help conceptually and procedurally with other types of problems. Conceptually, number lines and number paths show magnitude and allow for explicit instruction on magnitude comparisons. Procedurally, they help teach principles of addition and subtraction operations such as “counting down,” “counting up,” and “counting down from.”

The figure in example 4 shows how a number line may be used to assist with counting strategies. The top arrows show how a child learns to count on. He adds  $2 + 5 = \underline{\quad}$ . To start, he places his finger on 2. Then, he jumps five times to the right and lands on 7. The arrows under the number line show how a child subtracts using a counting down strategy. For  $10 - 3 = \underline{\quad}$ , she starts with her finger on the 10. Then, she jumps three times to the left on the number line, where she finishes on 7.

The goal of using a number line should be for students to create a mental number line and establish rules for movement along the line according to the more or less marking arrows placed along the line. Such rules and procedures should be directly tied to the explicit instruction that guided the students through the use of the visual representation.<sup>116</sup>

Pictorial representations of objects such as birds and cups are also often used to teach basic addition and subtraction, and simple drawings can help students understand place value and multidigit addition

and subtraction. Example 5 (p. 34) shows how a student can draw a picture to solve a multidigit addition problem. In the figure, circles represent one unit and lines represent units of 10.

In upper grades, diagrams and pictorial representations used to teach fractions also help students make sense of the basic structure underlying word problems. Strip diagrams (also called model diagrams and bar diagrams) are one type of diagram that can be used. Strip diagrams are drawings of narrow rectangles that show relationships among quantities. Students can use strip diagrams to help them reason about and solve a wide variety of word problems about related quantities. In example 6 (p. 34), the full rectangle (consisting of all three equal parts joined together) represents Shauntay’s money before she bought the book. Since she spent  $\frac{2}{3}$  of her money on the book, two of the three equal parts represent the \$26 she spent on the book. Students can then reason that if two parts stand for \$26, then each part stands for \$13, so three parts stand for \$39. So, Shauntay had \$39 before she bought the book.

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2. If visuals are not sufficient for developing accurate abstract thought and answers, use concrete manipulatives first. Although this can also be done with students in upper elementary and middle school grades, use of manipulatives with older students should be expeditious because the goal is to move toward understanding of—and facility with—visual representations, and finally, to the abstract.

Manipulatives are usually used in lower grades in the initial stages of learning as teachers introduce basic concepts with whole numbers. This exposure to concrete objects is often fleeting and transitory. The use of manipulatives in upper elementary school grades is virtually nonexistent.<sup>117</sup>

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116. Manalo, Bunnell, and Stillman (2000). Note that this study was not eligible for review because it was conducted outside the United States.

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117. Howard, Perry, and Lindsay (1996); Howard, Perry, and Conroy (1995).

The panel suggests that the interventionist use concrete objects in two ways.

First, in lower elementary grades, use concrete objects more extensively in the initial stages of learning to reinforce the understanding of basic concepts and operations.<sup>118</sup>

Concrete models are routinely used to teach basic foundational concepts such as place value.<sup>119</sup> They are also useful in teaching other aspects of mathematics such as multiplication facts. When a multiplication fact is memorized by question and answer alone, a student may believe that numbers are to be memorized rather than understood. For example,  $4 \times 6$  equals 24. When shown using manipulatives (as in example 7, p. 35),  $4 \times 6$  means 4 groups of 6, which total as 24 objects.

Second, in the upper grades, use concrete objects when visual representations do not seem sufficient in helping students understand mathematics at the more abstract level.

Use manipulatives expeditiously, and focus on fading them away systematically to reach the abstract level.<sup>120</sup> In other words, explicitly teach students the concepts and operations when students are at the concrete level and consistently repeat the instructional procedures at the visual and abstract levels. Using consistent language across representational systems (manipulatives, visual representations, and abstract symbols) has been an important component in several research studies.<sup>121</sup> Example 8 (p. 35) shows a set of matched concrete, visual, and abstract representations of a concept involving solving single-variable equations.

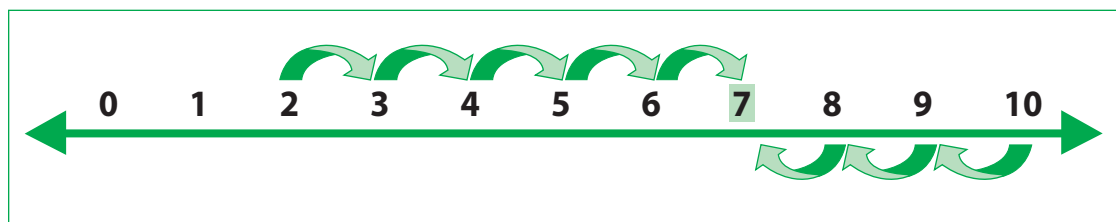
118. Darch (1989); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).

119. Fuchs et al. (2005); Fuchs, Seethaler et al. (2008); Fuchs, Powell et al. (2008).

120. Fuchs et al. (2005); Witzel (2005); Witzel, Mercer, and Miller (2003).

121. Fuchs et al. (2005); Butler et al. (2003); Witzel (2005); Witzel, Mercer, and Miller (2003).

#### Example 4. Representation of the counting on strategy using a number line





**Example 5. Using visual representations for multidigit addition**

A group of ten can be drawn with a long line to indicate that ten ones are joined to form one ten:

Simple drawings help make sense of two-digit addition with regrouping:

36  
+27  
—  
63

**Example 6. Strip diagrams can help students make sense of fractions**

Shauntay spent  $\frac{2}{3}$  of the money she had on a book that cost \$26. How much money did Shauntay have before she bought the book?

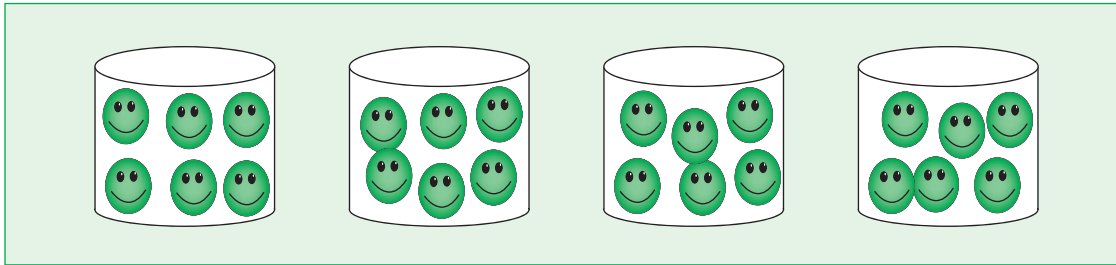
Shauntay's money at first

\$26 book

2 parts → \$26  
1 part →  $\$26 \div 2 = \$13$   
3 parts →  $3 \times \$13 = \$39$

Shauntay's had \$39

**Example 7. Manipulatives can help students understand that four multiplied by six means four groups of six, which means 24 total objects**



**Example 8. A set of matched concrete, visual, and abstract representations to teach solving single-variable equations**

$$3 + X = 7$$

Solving the Equation with Concrete Manipulatives (Cups and Sticks)	Solving the Equation with Visual Representations of Cups and Sticks	Solving the Equation with Abstract Symbols
A		$3 + 1X = 7$
B		$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$
C		$\frac{1X}{1} = \frac{4}{1}$
D		$\frac{1X}{1} = \frac{4}{1}$
E		$X = 4$

**Concrete Steps**

- A. 3 sticks plus one group of X equals 7 sticks
- B. Subtract 3 sticks from each side of the equation
- C. The equation now reads as one group of X equals 4 sticks
- D. Divide each side of the equation by one group
- E. One group of X is equal to four sticks (i.e.,  $1X/\text{group} = 4 \text{ sticks}/\text{group}$ ;  $1X = 4 \text{ sticks}$ )

## Potential roadblocks and solutions

**Roadblock 5.1.** *In the opinion of the panel, many intervention materials provide very few examples of the use of visual representations.*

**Suggested Approach.** Because many curricular materials do not include sufficient examples of visual representations, the interventionist may need the help of the mathematics coach or other teachers in developing the visuals. District staff can also arrange for the development of these materials for use throughout the district.

**Roadblock 5.2.** *Some teachers or interventionists believe that instruction in concrete manipulatives requires too much time.*

**Suggested Approach.** Expedient use of manipulatives cannot be overemphasized. Since tiered interventions often rely on foundational concepts and procedures, the use of instruction at the concrete level allows for reinforcing and making explicit the foundational concepts and operations. Note that overemphasis on manipulatives can be counterproductive, because students manipulating only concrete objects may not be learning to do math at an abstract level.<sup>122</sup> The interventionist should

use manipulatives in the initial stages strategically and then scaffold instruction to the abstract level. So, although it takes time to use manipulatives, this is not a major concern since concrete instruction will happen only rarely and expeditiously.

**Roadblock 5.3.** *Some interventionists may not fully understand the mathematical ideas that underlie some of the representations. This is likely to be particularly true for topics involving negative numbers, proportional reasoning, and interpretations of fractions.*

**Suggested Approach.** If interventionists do not fully understand the mathematical ideas behind the material, they are unlikely to be able to teach it to struggling students.<sup>123</sup> It is perfectly reasonable for districts to work with a local university faculty member, high school mathematics instructor, or mathematics specialist to provide relevant mathematics instruction to interventionists so that they feel comfortable with the concepts. This can be coupled with professional development that addresses ways to explain these concepts in terms their students will understand.

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122. Witzel, Mercer, and Miller (2003).

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123. Hill, Rowan, and Ball (2005); Stigler and Hiebert (1999).

## **Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.**

Quick retrieval of basic arithmetic facts is critical for success in mathematics.<sup>124</sup> Yet research has found that many students with difficulties in mathematics are not fluent in such facts.<sup>125</sup> Weak ability to retrieve arithmetic facts is likely to impede understanding of concepts students encounter with rational numbers since teachers and texts often assume automatic retrieval of facts such as  $3 \times 9 = \underline{\quad}$  and  $11 - 7 = \underline{\quad}$  as they explain concepts such as equivalence and the commutative property.<sup>126</sup> For that reason, we recommend that about 10 minutes be devoted to building this proficiency during each intervention session. Acknowledging that time may be short, we recommend a minimum of 5 minutes a session.

### **Level of evidence: Moderate**

The panel judged the level of evidence supporting this recommendation to be *moderate*. This recommendation is based on seven randomized controlled trials that met WWC standards or met standards with reservations and that included fact fluency

instruction in the intervention.<sup>127</sup> These studies reveal a series of small but positive effects on measures of fact fluency<sup>128</sup> and procedural knowledge for diverse student populations in the elementary grades.<sup>129</sup> In some cases, fact fluency instruction was one of several components in the intervention, and it is difficult to judge the impact of the fact fluency component alone.<sup>130</sup> However, because numerous research teams independently produced similar findings, we consider this practice worthy of serious consideration. Although the research is limited to the elementary school grades, in the panel's view, building fact fluency is also important for middle school students when used appropriately.

### **Brief summary of evidence to support the recommendation**

The evidence demonstrates small positive effects on fact fluency and operations for the elementary grades and thus provides support for including fact fluency activities as either stand-alone interventions or components of larger tier 2 interventions.<sup>131</sup> These positive effects did not, however, consistently reach statistical significance, and the findings cannot be extrapolated to areas of mathematics outside of fact fluency and operations.

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124. National Mathematics Advisory Panel (2008).

125. Geary (2004); Jordan, Hanich, and Kaplan (2003); Goldman, Pellegrino, and Mertz (1988).

126. Gersten and Chard (1999); Woodward (2006); Jitendra et al. (1996).

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127. Bernie-Smith (1991); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Hamlett et al. (2006); Fuchs et al. (2005); Fuchs, Powell et al. (2008); Tournaki (2003); Woodward (2006).

128. Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Hamlet et al. (2006); Fuchs et al. (2005); Fuchs, Powell et al. (2008); Tournaki (2003); Woodward (2006).

129. Bernie-Smith (1991); Fuchs, Seethaler et al. (2008); Fuchs et al. (2005); Tournaki (2003); Woodward (2006).

130. Fuchs, Seethaler et al. (2008); Fuchs et al. (2005).

131. Bernie-Smith (1991); Fuchs et al. (2005); Fuchs, Fuchs, Hamlet et al. (2006); Fuchs, Seethaler et al. (2008); Fuchs, Powell et al. (2008); Tournaki (2003); Woodward (2006).

Two studies examined the effects of being taught mathematics facts relative to the effects of being taught spelling or word identification using similar methods.<sup>132</sup> In both studies, the mathematics facts group demonstrated positive gains in fact fluency relative to the comparison group, but the effects were significant in only one of the studies.<sup>133</sup>

Another two interventions included a facts fluency component in combination with a larger tier 2 intervention.<sup>134</sup> For example, in the Fuchs et al. (2005) study, the final 10 minutes of a 40 minute intervention session were dedicated to practice with addition and subtraction facts. In both studies, tier 2 interventions were compared against typical tier 1 classroom instruction. In each study, the effects on mathematics facts were small and not significant, though the effects were generally positive in favor of groups that received the intervention. Significant positive effects were detected in both studies in the domain of operations, and the fact fluency component may have been a factor in improving students' operational abilities.

Many of the studies in the evidence base included one or more of a variety of components such as teaching the relationships among facts,<sup>135</sup> making use of a variety of materials such as flash cards and computer-assisted instruction,<sup>136</sup> and teaching math facts for a minimum of 10 minutes

per session.<sup>137</sup> Since these components were typically not independent variables in the studies, it is difficult to attribute any positive effects to the component itself. There is evidence, however, that strategy-based instruction for fact fluency (such as teaching the counting-on procedure) is superior to rote memorization.<sup>138</sup>

### How to carry out this recommendation

1. Provide about 10 minutes per session of instruction to build quick retrieval of basic arithmetic facts. Consider using technology, flash cards, and other materials for extensive practice to facilitate automatic retrieval.

The panel recommends providing about 10 minutes each session for practice to help students become automatic in retrieving basic arithmetic facts, beginning in grade 2. The goal is quick retrieval of facts using the digits 0 to 9 without any access to pencil and paper or manipulatives.

Presenting facts in number families (such as  $7 \times 8 = 56$ ,  $8 \times 7 = 56$ ,  $56/7 = 8$ , and  $56/8 = 7$ ) shows promise for improving student fluency.<sup>139</sup> In the panel's view, one advantage of this approach is that students simultaneously learn about the nature of inverse operations.

In the opinion of the panel, cumulative review is critical if students are to maintain fluency and proficiency with mathematics facts. An efficient way to achieve this is to integrate previously learned facts into practice activities. To reduce frustration and provide enough extended practice so that retrieval becomes automatic (even for

132. Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Powell et al. (2008).

133. In Fuchs, Fuchs, Hamlett et al. (2006), the effects on addition fluency were statistically significant and positive while there was no effect on subtraction fluency.

134. Fuchs, Seethaler et al. (2008); Fuchs et al. (2005).

135. Bernie-Smith (1991); Fuchs et al. (2005); Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Seethaler et al. (2008); Woodward (2006).

136. Bernie-Smith (1991); Fuchs et al. (2005); Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Seethaler et al. (2008); Fuchs, Powell et al. (2008).

137. Bernie-Smith (1991); Fuchs et al. (2005); Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Seethaler et al. (2008); Fuchs, Powell et al. (2008); Tournaki (2003); Woodward (2006).

138. Bernie-Smith (1991); Tournaki (2003); Woodward (2006).

139. Fuchs et al. (2005); Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Seethaler et al. (2008).

those who tend to have limited capacity to remember and retrieve abstract material), interventionists can individualize practice sets so students learn one or two new facts, practice several recently acquired facts, and review previously learned facts.<sup>140</sup> If students are proficient in grade-level mathematics facts, then the panel acknowledges that students might not need to practice each session, although periodic cumulative review is encouraged.

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**2. For students in kindergarten through grade 2, explicitly teach strategies for efficient counting to improve the retrieval of mathematics facts.**

It is important to provide students in kindergarten through grade 2 with strategies for efficiently solving mathematics facts as a step toward automatic, fluent retrieval. The counting-up strategy has been used to increase students' fluency in addition facts.<sup>141</sup> This is a simple, effective strategy that the majority of students teach themselves, sometimes as early as age 4.<sup>142</sup> But students with difficulties in mathematics tend not to develop this strategy on their own, even by grade 2.<sup>143</sup> There is evidence that systematic and explicit instruction in this strategy is effective.<sup>144</sup>

Students can be explicitly taught to find the smaller number in the mathematics fact, put up the corresponding number of fingers, and count up that number of fingers from the larger number. For example, to solve  $3 + 5 = \underline{\quad}$ , the teacher identifies the smaller number (3) and puts up three fingers. The teacher simultaneously says and points to the larger number before counting three fingers, 6, 7, 8.

Note that learning the counting-up strategy not only improves students' fact fluency<sup>145</sup> but also immerses students in the commutative property of addition. For example, students learn that when the larger number is presented second ( $3 + 5 = \underline{\quad}$ ), they can rearrange the order and start counting up from 5. In the view of the panel, this linkage is an important part of intervention. After this type of instruction, follow-up practice with flash cards might help students make the new learning automatic.

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**3. Teach students in grades 2 through 8 how to use their knowledge of properties, such as commutative, associative, and distributive law, to derive facts in their heads.**

Some researchers have argued that rather than solely relying on rote memorization and drill and practice, students should use properties of arithmetic to solve complex facts involving multiplication and division.<sup>146</sup> These researchers believe that by teaching the use of composition and decomposition, and applying the distributive property to situations involving multiplication, students can increasingly learn how to quickly (if not automatically) retrieve facts. For example, to understand and quickly produce the seemingly difficult multiplication fact  $13 \times 7 = \underline{\quad}$ , students are reminded that  $13 = 10 + 3$ , something they should have been taught consistently during their elementary career. Then, since  $13 \times 7 = (10 + 3) \times 7 = 10 \times 7 + 3 \times 7$ , the fact is parsed into easier, known problems  $10 \times 7 = \underline{\quad}$  and  $3 \times 7 = \underline{\quad}$  by applying of the distributive property. Students can then rely on the two simpler multiplication facts (which they had already acquired) to quickly produce an answer mentally.

The panel recommends serious consideration of this approach as an option for students who struggle with acquisition of

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140. Hasselbring, Bransford, and Goin (1988). Note that there was not sufficient information to do a WWC review.

141. Bernie-Smith (1991); Tournaki (2003).

142. Siegler and Jenkins (1989).

143. Tournaki (2003).

144. Tournaki (2003).

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145. Tournaki (2003).

146. Robinson, Menchetti, and Torgesen (2002); Woodward (2006).

facts in grades 2 through 8. When choosing an intervention curriculum, consider one that teaches this approach to students in this age range. Note, however, that the panel believes students should also spend time after instruction with extensive practice on quick retrieval of facts through the use of materials such as flash cards or technology.

### **Roadblocks and solutions**

**Roadblock 6.1.** *Students may find fluency practice tedious and boring.*

**Suggested Approach.** Games that provide students with the opportunity to practice new facts and review previously learned facts by encouraging them to beat their previous high score can help the

practice be less tedious.<sup>147</sup> Players may be motivated when their scores rise and the challenge increases. Further recommendations for motivating students are in recommendation 8.

**Roadblock 6.2.** *Curricula may not include enough fact practice or may not have materials that lend themselves to teaching strategies.*

**Suggested Approach.** Some contemporary curricula deemphasize fact practice, so this is a real concern. In this case, we recommend using a supplemental program, either flash card or technology based.

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147. Fuchs, Seethaler et al. (2008).

## Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.

Assess the progress of tier 2 and tier 3 students regularly with general outcome measures and curriculum embedded measures. Also monitor regularly the progress of tier 1 students who perform just above the cutoff score for general outcome measures so they can be moved to tier 2 if they begin to fall behind.

In addition, use progress monitoring data to determine when instructional changes are needed. This includes regrouping students who need continuing instructional support within tier 2 or tier 3, or moving students who have met benchmarks out of intervention groups and back to tier 1.

Information about specific progress monitoring measures is available in Appendix D. A list of online resources is in the text below.

### Level of evidence: **Low**

The panel judged the level of evidence supporting this recommendation to be *low*. No studies that met WWC standards supported this recommendation.<sup>148</sup> Instead, the recommendation is based on the panel's expert opinion as well as con-

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148. The technical adequacy studies of mathematics progress monitoring measures were not experimental; the researchers typically used correlation techniques to evaluate the reliability and criterion validity of the measures and regression methods to examine sensitivity to growth.

sideration of the standards for measurement established by a joint committee of national organizations.<sup>149</sup>

### Brief summary of evidence to support the recommendation

Although we found no studies that addressed the use of valid measures for struggling students within an RtI framework, nonexperimental studies demonstrate the technical adequacy of various progress monitoring measures.<sup>150</sup> Measures for the primary grades typically reflect aspects of number sense, including strategic counting, numeral identification, and magnitude comparisons.<sup>151</sup> Studies investigating measures for the elementary grades focus mostly on the characteristics of general outcome measures that represent grade-level mathematics curricula in computation and in mathematics concepts and applications.<sup>152</sup> Widely used, these measures are recommended by the National Center for Student Progress Monitoring.<sup>153</sup> Less evidence is available to support progress monitoring in middle school.<sup>154</sup> But research teams have developed measures focusing on math concepts typically taught in middle school,<sup>155</sup> basic facts,<sup>156</sup> and estimation.<sup>157</sup>

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149. The American Psychological Association, the American Educational Research Association, and the National Council on Measurement in Education (1999).

150. For example, Clarke et al. (2008); Foegen and Deno (2001); Fuchs et al. (1993); Fuchs, Fuchs, Hamlett, Thompson et al. (1994); Leh et al. (2007); Lembke et al. (2008).

151. For example, Clarke et al. (2008); Lembke et al. (2008); Bryant, Bryant, Gersten, Scammacca, and Chavez (2008).

152. Fuchs and Fuchs (1998); Fuchs et al. (1999).

153. [www.studentprogress.org](http://www.studentprogress.org).

154. Foegen (2008).

155. Helwig, Anderson, and Tindal (2002).

156. Espin et al. (1989).

157. Foegen and Deno (2001); Foegen (2000).



## How to carry out this recommendation

### 1. Monitor the progress of tier 2, tier 3, and borderline tier 1 students at least once a month using grade-appropriate general outcome measures.

General outcome measures typically take 5 to 10 minutes to administer and should be used at least monthly to monitor tier 2 and tier 3 students. General outcome measures use a sample of items from the array of concepts covered over one year to assess student progress. They provide a broad perspective on student proficiency in mathematics. They target concepts such as magnitude comparison, counting ability, and knowledge of place value for students in kindergarten and grade 1, and increasingly complex aspects of place value and proficiency with operations for students in grades 2 through 6. Examining student performance on these measures allows teachers to determine whether students are integrating and generalizing the concepts, skills, and strategies they are learning in the core curriculum and the intervention.<sup>158</sup>

In addition to monitoring the progress of tier 1 and tier 2 students, the panel recommends monitoring the progress of borderline tier 1 students with general outcome measures on a monthly basis. Since these students scored just above the cut score, they were not selected for supplemental instruction. The panel suggests using one standard error of measurement (a statistic available in the technical information for the measures) above the cut score to define the range of scores for borderline students. Using this approach, teachers can continue to monitor the progress of students whose scores fell just above the cut score and determine whether they should receive supplemental instruction.

Choose progress monitoring measures with evidence supporting their reliability, validity, and ability to identify growth. This will require input from individuals with expertise in these areas, typically school psychologists or members of district research departments. Consider whether the measure produces consistent results (reliability) and provides information that correlates with other measures of mathematics achievement (criterion validity). Ability to identify growth helps interventionists ensure that students are learning and making progress toward an annual goal through the full array of services they are receiving.

In some cases, general outcome measures may also be used for screening, as described in recommendation 1. Resources that teachers can turn to for identifying appropriate measures include the National Center on Student Progress Monitoring's review of available tools (<http://www.studentprogress.org/>) and the Research Institute on Progress Monitoring (<http://www.progressmonitoring.org/>).

### 2. Use curriculum-embedded assessments in interventions to determine whether students are learning from the intervention. These measures can be used as often as every day<sup>159</sup> or as infrequently as once every other week.<sup>160</sup>

Many tier 2 and tier 3 intervention programs (commercially developed, researcher developed, or district developed) include curriculum-embedded assessments (sometimes called unit tests, mastery tests, or daily probes). The results of these assessments can be used to determine which concepts need to be reviewed, which need to be re-taught, and which have been mastered. Curriculum-embedded assessments are often administered daily

158. Fuchs, Fuchs, and Zumeta (2008).

159. Bryant, Bryant, Gersten, Scammacca, and Chavez (2008).

160. Jitendra (2007).

for students in kindergarten<sup>161</sup> and grade 1 and biweekly for students in grades 2 through 6.<sup>162</sup> These assessments usually do not possess the same high technical characteristics of the general outcome measures. Curriculum-embedded assessments often result in very useful information for interventionists because they can detect changes in student performance in the concepts and skills being taught at the time. Interventionists need to be cautious about assuming that mastery of individual skills and concepts will translate into improvements in overall proficiency. As a result, the panel recommends using both general outcome measures and curriculum-embedded assessments for students receiving interventions.

If the intervention program does not include curriculum-embedded assessments, use efficient, reliable, and valid screening measures, which can also be used as progress monitoring measures (see recommendation 1).

### 3. Use progress monitoring data to regroup students when necessary.

Since student skill levels change over time and in varying degrees, the panel

161. For example, one tier 2 intervention program for 1st and 2nd grade students reported by Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) included daily activity-level progress monitoring that consisted of four oral or written problems drawn from the content focus for that day. Teachers were instructed that a majority of the students in the group had to complete at least three of the four problems correctly to consider the daily lesson successful.

162. A parallel example in grades 3 and beyond can be found in Jitendra's *Solving math word problems* instructional materials on teaching word problems (2007). There are many other examples in available commercial programs.

recommends using progress monitoring data to regroup students within tiers so that the small groups used in tier 2 interventions are as homogeneous as possible. If a student does not fit into any of the intervention groups from his or her class, consider putting the child in an intervention group from another class if the schedule permits.

## Roadblocks and solutions

**Roadblock 7.1.** *Students within classes are at very different levels. This can make it difficult to group students into appropriate tier 2 and tier 3 intervention groups.*

**Suggested Approach.** If students within a class are at such diverse levels that appropriate tier 2 and tier 3 intervention groups cannot be made, consider grouping students across classes. This will facilitate clustering students with similar needs. For example, teachers of upper elementary students may find that students who have not yet mastered basic concepts in a particular area (fractions) are spread across several classrooms. Putting these students in a single tier 2 intervention group would be the most efficient means of meeting their needs, rather than trying to provide one or two students in each class with services duplicated across classrooms. In such a case, a math specialist, paraprofessional, or other school personnel who have received training can conduct the intervention.

**Roadblock 7.2.** *There is insufficient time for teachers to implement progress monitoring.*

**Suggested Approach.** If teachers are too busy to assess student progress with monitoring measures, consider training paraprofessionals or other school staff to do so.

## Recommendation 8. Include motivational strategies in tier 2 and tier 3 interventions.

Adults can sometimes forget how challenging so-called “basic” arithmetic is for students in tier 2 and tier 3 interventions. Many of these students have had experiences of failure and frustration with mathematics by the time they receive an intervention. They may also have a particularly difficult time storing and easily retrieving information in their memories.<sup>163</sup> Therefore, it seems particularly important to provide additional motivation for these students.<sup>164</sup>

Praising students for their effort and for being engaged as they work through mathematics problems is a powerful motivational tool that can be effective in increasing students’ academic achievement.<sup>165</sup> Tier 2 and tier 3 interventions should include components that promote student effort (*engagement-contingent rewards*), persistence (*completion-contingent rewards*), and achievement (*performance-contingent rewards*). These components can include praise and rewards. Even a well-designed intervention curriculum may falter without such behavioral supports.

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163. Geary (2003).

164. The scope of this practice guide limited the motivational strategies reviewed to strategies used in studies of students struggling with mathematics. For a wider review of effective motivational strategies used in classrooms, see Epstein et al. (2008) and Halpern et al. (2007).

165. Schunk and Cox (1986); Fuchs et al. (2005). Fuchs, Fuchs, Craddock et al. (2008).

### Level of evidence: **Low**

The panel judged the level of evidence supporting this recommendation to be *low*. This recommendation is based on the professional opinion of the panel, and on nine studies that met WWC standards or met standards with reservations that included motivational strategies in the intervention.<sup>166</sup> Although one of these studies demonstrated that praising struggling students for their effort significantly improved their ability to solve subtraction problems with regrouping,<sup>167</sup> other studies included a motivational component as one of several components of the intervention. In the opinion of the panel, these studies did not show that a motivational component is essential but suggest that it may be useful for improving mathematics achievement.<sup>168</sup>

### Brief summary of evidence to support the recommendation

One study that met WWC standards examined the effects of a motivational component by comparing the performance of students who received praise for their effort during subtraction instruction with those who did not receive praise.<sup>169</sup> This study found significant positive effects on student subtraction scores in favor of providing effort feedback.<sup>170</sup> Although this study provides some evidence of the effectiveness of a motivational strategy, it is the only study that

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166. Fuchs et al. (2005); Fuchs, Fuchs, Craddock et al. (2008); Schunk and Cox (1986); Fuchs, Seethaler et al. (2008); Heller and Fantuzzo (1993); Artus and Dyrek (1989); Fuchs, Fuchs et al. (2003b); Fuchs, Fuchs, Hamlett, Phillips et al. (1994); Fuchs, Fuchs, Finelli et al. (2006).

167. Schunk and Cox (1986).

168. There is an extensive literature on motivational strategies outside the scope of this practice guide. For more information on motivational strategies see Epstein et al. (2008) and Halpern et al. (2007).

169. Schunk and Cox (1986).

170. Schunk and Cox (1986).

explicitly tested the effects of motivational strategies on mathematics outcomes.

In two studies, students received points for engagement and attentiveness,<sup>171</sup> and in three studies, students were provided with prizes as tangible reinforcers for accurate mathematics problem-solving.<sup>172</sup> However, in each of these studies, it was not possible to isolate the effects of reinforcing attentiveness and accuracy. For example, in two of the studies, students in tier 2 tutoring earned prizes for accuracy.<sup>173</sup> Although in both studies, the tier 2 intervention group demonstrated substantively important positive and sometimes significant gains on a variety of mathematics measures relative to the students who remained in tier 1, it is not possible to isolate the effects of the reinforcers from the provision of tier 2 tutoring. Another study examined the impact of parental involvement on students' mathematics achievement and found statistically significant positive effects on operations and general math achievement.<sup>174</sup> However, because the parental involvement component was multifaceted, it is not possible to attribute the positive effects to rewards alone.

Five studies in the evidence base included interventions in which students graphed their progress and in some cases set goals for improvement on future assessments.<sup>175</sup> One experimental study examined the effects of student graphing and goal setting as an independent variable and found substantively important positive effects on measures of word problems in favor

of students who graphed and set goals.<sup>176</sup> The other four studies did not isolate the effects of graphing progress.<sup>177</sup> Because this recommendation is based primarily on the opinion of the panel, the level of evidence is identified as *low*.

### How to carry out this recommendation

#### 1. Reinforce or praise students for their effort and for attending to and being engaged in the lesson.

Verbally praise students for their effort<sup>178</sup> and for listening carefully and following the lesson in a systematic fashion (*engagement-contingent rewards*).<sup>179</sup> The panel believes that praise should be immediate and specific to highlight student effort and engagement. But we also believe that it is ineffective to offer generic and empty praise ("good job!" or "keep up the good work!") that is not related to actual effort. Instead, praise is most effective when it points to specific progress that students are making and recognizes students' actual effort.<sup>180</sup> Systematically praising students for their effort and engagement may encourage them to remain focused on the completion of their work.

#### 2. Consider rewarding student accomplishments.

Consider using rewards to acknowledge completion of math tasks (*completion-contingent rewards*) and accurate work (*performance-contingent rewards*). This can be done by applauding or verbally

171. Fuchs et al. (2005); Fuchs, Fuchs, Craddock et al. (2008).

172. Fuchs et al. (2005); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).

173. Fuchs et al. (2005); Fuchs, Seethaler et al. (2008).

174. Heller and Fantuzzo (1993).

175. Artus and Dyrek (1989); Fuchs, Seethaler et al. (2008); Fuchs et al. (2003b); Fuchs, Fuchs, Hamlett, Phillips et al. (1994); Fuchs, Fuchs, Finelli et al. (2006).

176. Fuchs et al. (2003b).

177. Artus and Dyrek (1989); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Hamlett, Phillips et al. (1994); Fuchs, Fuchs, Finelli et al. (2006).

178. Schunk and Cox (1986).

179. For example, Fuchs et al. (2005); Fuchs, Fuchs, Craddock et al. (2008).

180. See Bangert-Drowns et al. (1991) and Halpern et al. (2007) for a review.

praising students for actual accomplishments, such as finishing assignments, improving their score from 70 percent to 80 percent correct, or giving students points or tokens each time they answer a problem correctly, which they can use to “buy” tangible rewards at a later time.<sup>181</sup> Again, praise should be specific rather than generic.<sup>182</sup> Consider notifying the student’s parents to inform them of their child’s successes in mathematics by phone or email or in a note sent home with the student.<sup>183</sup> Remember that parents of these students are likely to receive notification of problems rather than successes, and some evidence suggests that this specific positive attention might support achievement growth.<sup>184</sup>

### 3. Allow students to chart their progress and to set goals for improvement.

Several of the interventions in the evidence base for this practice guide had students graph their progress on charts<sup>185</sup> and set goals for improving their assessment scores.<sup>186</sup> For example, students might graph their scores on a chart showing a series of thermometers, one for each session of the intervention.<sup>187</sup> At the beginning of each session, students can examine their charts and set a goal to beat their previous score or to receive the maximum score. This type of goal setting is believed to help students develop self-regulated learning because students take independent responsibility for setting and achieving goals.<sup>188</sup>

181. For example, Fuchs et al. (2005); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).

182. Halpern et al. (2007).

183. For example, Heller and Fantuzzo (1993).

184. For example, Heller and Fantuzzo (1993).

185. Artus and Dyrek (1989); Fuchs, Fuchs, Hamlett, Phillips et al. (1994); Fuchs, Fuchs, Finelli et al. (2006); Fuchs, Seethaler et al. (2008).

186. Fuchs et al. (2003b).

187. See the procedure in Fuchs et al. (2003b).

188. Fuchs et al. (1997).

## Roadblocks and suggested approaches

**Roadblock 8.1.** *Rewards can reduce genuine interest in mathematics by directing student attention to gathering rewards rather than learning math.*

**Suggested Approach.** It is important to inform interventionists that research in other content areas has demonstrated that rewards and praise increase the likelihood of students’ academic success without diminishing their interest in learning.<sup>189</sup> Given the frequent history of failure for many of these students, at least in the elementary grades, we suggest using rewards and praise to encourage effort, engagement, and achievement. As students learn and succeed more often in mathematics, interventionists can gradually fade the use of rewards because student success will become an intrinsic reward. The WWC *Reducing Behavior Problems in the Elementary School Classroom Practice Guide*<sup>190</sup> is a good reference for more information on the use of rewards and praise.

**Roadblock 8.2.** *It is difficult to determine appropriate rewards for individual students.*

**Suggested Approach.** Consider each student’s interests before choosing an appropriate reward. Also consider using opportunities to engage in activities students are interested in as rewards to reinforce effort, engagement, and accurate work. Parents may also have ideas for rewards that will help motivate their children. Schools can engage parents in rewarding students and coordinate efforts to reward children at home as well.<sup>191</sup>

189. Epstein et al. (2008).

190. Epstein et al. (2008).

191. For example, Heller and Fantuzzo (1993).

**Roadblock 8.3.** *Providing feedback and rewarding achievement detracts from classroom instructional time. It is difficult to fit it into the classroom schedule.*

**Suggested Approach.** Verbally praising students for their effort individually and their engagement in small group lessons requires very little time. Awarding points or tokens for correct responses

can be done when the teacher grades the student's work. To reduce the amount of time it takes for students to "buy" prizes with accumulated points or tokens, ask students to choose which prize they want before they "buy" it. The prizes can then be distributed quickly at the end of the day, so that students are not distracted by the items throughout the school day.

## Glossary of terms as used in this report

The **associative property of addition** states that  $(A + B) + C = A + (B + C)$  for all numbers A, B, C. This property allows for flexibility in calculating sums. For example, to calculate  $85 + 97 + 3$ , we do not have to add 85 and 97 first but may instead calculate the easier sum  $85 + (97 + 3)$ , which is  $85 + 100$ , which equals 185. The associative property is also used in deriving basic addition facts from other basic facts and therefore helps with learning these basic facts. For example, to add  $8 + 5$ , a child can think of breaking the 5 into 2 + 3, combining the 2 with the 8 to make a 10, and then adding on the 3 to make 13. In an equation, this can be recorded as:  $8 + 5 = 8 + (2 + 3) = (8 + 2) + 3 = 10 + 3 = 13$ .

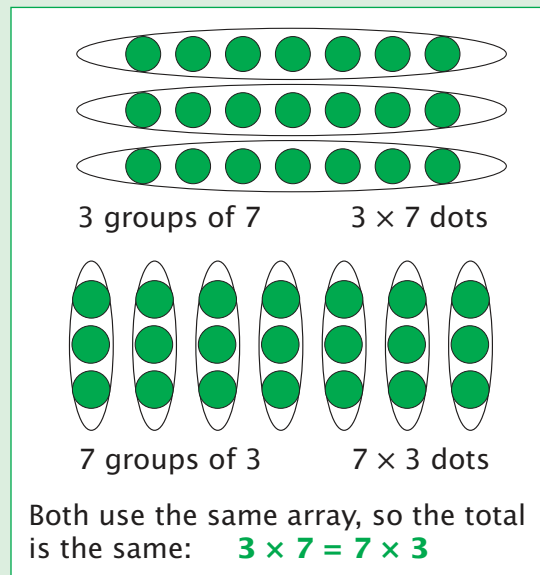
The **associative property of multiplication** states that  $(A \times B) \times C = A \times (B \times C)$  for all numbers A, B, C. This property allows for flexibility in calculating products. For example, to calculate  $87 \times 25 \times 4$ , we do not have to multiply 87 and 25 first but may instead calculate the easier product  $87 \times (25 \times 4)$ , which is  $87 \times 100$ , which equals 8,700. The associative property is also used in deriving basic multiplication facts from other basic facts and therefore helps with learning these facts. For example, to calculate  $7 \times 6$ , a child who already knows  $7 \times 3 = 21$  can double the 21 to calculate that  $7 \times 6 = 42$ . In an equation, this can be recorded as:  $7 \times 6 = 7 \times (3 \times 2) = (7 \times 3) \times 2 = 21 \times 2 = 42$ .

The **commutative property of addition** states that  $A + B = B + A$  for all numbers A, B. This property allows for flexibility in calculating sums and helps lighten the load of learning the basic addition facts. For example, to add  $2 + 9$ , a child might want to count on 9 from 2 by counting “3, 4, 5, 6, . . . 11,” which is cumbersome. By applying the commutative property, a child can instead add  $9 + 2$  by counting on 2 from 9, by saying “10, 11,” which is much easier (this is the

“minimum addend strategy” or the strategy of “counting on from larger”).

The **commutative property of multiplication** states that  $A \times B = B \times A$  for all numbers A, B. This property allows for flexibility in calculating products and helps lighten the load of learning the basic multiplication facts. As shown in example 9, once a child has learned the multiplication fact  $3 \times 7 = 21$ , the child will also know the fact  $7 \times 3 = 21$  if the child understands the commutative property.

### Example 9: Commutative property of multiplication



**Concurrent validity** refers to the correlation between the assessment that is being investigated and a similar assessment when the assessments are completed at the same point in time. Correlation coefficients range from -1 to 1. A correlation coefficient close to 1 indicates a strong overlap between the assessments.

**Counting up/on** is a strategy that young children can use to solve addition (and subtraction) problems. To calculate  $8 + 3$  by counting on, the child starts with 8 and then “counts on” 3 more, saying “9, 10, 11.” The child may use fingers in order to determine when to stop counting.

**Counting up/on from a larger addend** is a strategy in which children apply counting on to solve an addition problem, but first apply the commutative property, if necessary, in order to count on from the larger number. For example, to calculate  $3 + 9$ , a child first changes the problem to  $9 + 3$  and then counts on 3 from 9.

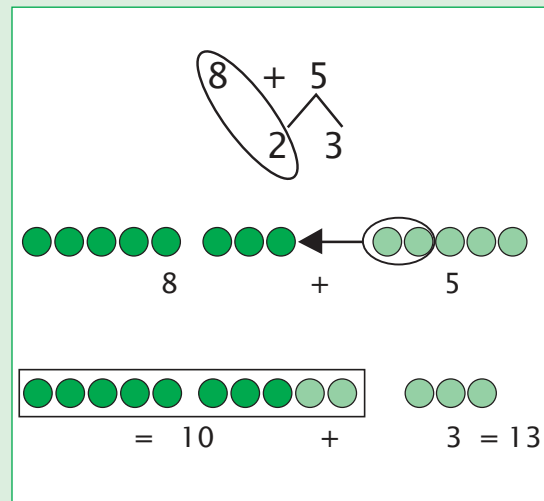
**Counting up/on to solve unknown addend and subtraction problems.** Counting on can be used to solve unknown addend problems such as  $11 + ? = 15$ . To solve this problem, the child can count on from 11 to 15, counting 12, 13, 14, 15 and raising one finger for each number. Since 4 fingers were raised,  $11 + 4 = 15$ . To solve a subtraction problem such as  $15 \text{ minus } 11 = ?$  using counting on, the child must first understand that the problem can be reformulated as  $11 + ? = 15$ . Then the child can use counting on as described previously. Note too that solving this subtraction problem using the counting on strategy is much easier than counting down 11. Note too that the reformulation of a subtraction problem as an unknown addend problem is important in its own right because it connects subtraction with addition.

In mathematics assessment, **riterion-related validity** means that student scores on an assessment should correspond to their scores or performance on other indicators of mathematics competence, such as teacher ratings, course grades, or standardized test scores.

**Derived fact strategies in addition and subtraction** are strategies in which children use addition facts they already know to find related facts. Especially important among derived fact strategies are the make-a-10 methods because they emphasize base 10 structure. As shown in example 10, to add  $8 + 5$ , a child can think of breaking the 5 into  $2 + 3$ , combining the 2 with the 8 to make a 10, and then adding on the 3 to make 13 (see the associative property of addition). Note that to use this make-a-10

strategy, children must know the “10 partner” (number that can be added to make 10) for each number from 1 to 9 and must also know how to break each number into a sum of two (positive whole) numbers in all possible ways. Furthermore, the child must understand all the “teen” numbers (from 11 to 19) as a 10 and some ones (for example, 15 is 10 and 5 ones).

### Example 10: Make-a-10 strategy



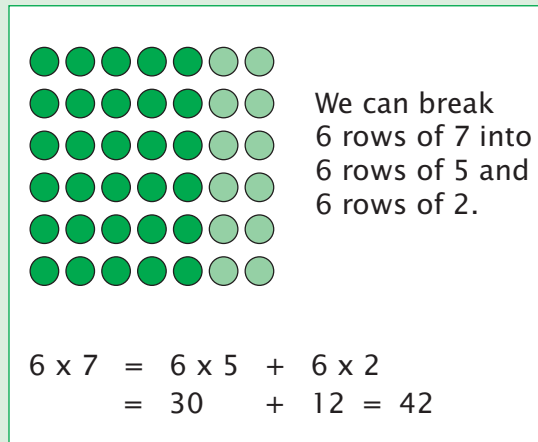
**Derived fact strategies in multiplication and division** are strategies in which children use multiplication facts they already know to find related facts. For example  $5 \times 8$  is half of  $10 \times 8$ , and similarly for all the “5 times” facts (these are examples of applying the associative property). Also,  $9 \times 8$  is 8 less than  $10 \times 8$ , and similarly for all the “9 times” facts (these are examples of applying the distributive property). To calculate  $4 \times 7$ , we can **double the double** of 7, that is, the double of 7 is 14 and the double of 14 is 28, which is 4 times 7. All the “4 times” facts can be derived by doubling the double (these are examples of applying the associative property).

The **distributive property** relates addition and multiplication. It states that  $A \times (B + C) = (A \times B) + (A \times C)$  for all numbers A, B, C. This property allows for flexibility in calculating products. For example, to calculate  $7 \times 13$ , we can break 13 apart by place value as  $10 + 3$  and calculate



$7 \times 10 = 70$  and  $7 \times 3 = 21$  and add these two results to find  $7 \times 13 = 91$ . In an equation, this can be recorded as:  $7 \times 13 = 7 \times (10 + 3) = (7 \times 10) + (7 \times 3) = 70 + 21 = 91$ . This strategy of breaking numbers apart by place value and applying the distributive property is the basis for the common method of longhand multiplication. The distributive property is also used in deriving basic multiplication facts from other basic facts and therefore helps in learning these facts. As shown in example 11, to calculate  $6 \times 7$ , a child who already knows  $6 \times 5 = 30$  and  $6 \times 2 = 12$  can add the 30 and 12 to calculate that  $6 \times 7 = 30 + 12 = 42$ . In an equation, this can be recorded as:  $6 \times 7 = 6 \times (5 + 2) = (6 \times 5) + (6 \times 2) = 30 + 12 = 42$ .

### Example 11: Distributive property



We can break  
6 rows of 7 into  
6 rows of 5 and  
6 rows of 2.

$$6 \times 7 = 6 \times 5 + 6 \times 2$$

$$= 30 + 12 = 42$$

**Efficiency** is how quickly the universal screening measure can be administered, scored, and analyzed for all the students tested.

**False positives** and **false negatives** are technical terms used to describe the misidentification of students. The numbers of false positives and false negatives are related to sensitivity and specificity. As depicted in table 3 (p. 16) of this guide, sensitivity is equal to the number of true positives (students properly identified as

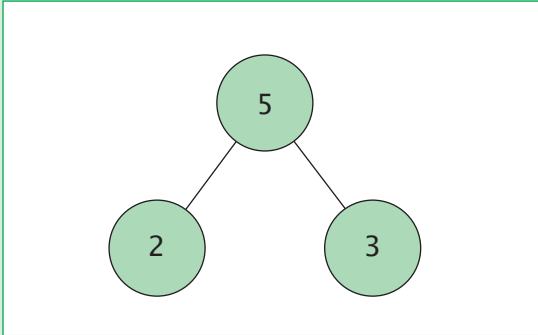
needing help in mathematics) divided by the sum of this value and the number of false negatives, while specificity is equal to the number of true negatives divided by the sum of this value and the number of false positives (students misidentified during screening).

A **general outcome measure** refers to a measure of specific proficiencies within a broader academic domain. These proficiencies are related to broader outcomes. For example, a measure of oral reading fluency serves as a general outcome measure of performance in the area of reading. The measures can be used to monitor student progress over time.

**Interventionist** refers to the person teaching the intervention. The interventionist might be a classroom teacher, instructional assistant, or other certified school personnel.

The **magnitude** of a quantity or number is its size, so a **magnitude comparison** is a comparison of size. The term *magnitude* is generally used when considering size in an approximate sense. In this case, we often describe the size of a quantity or number very roughly by its order of magnitude, which is the power of ten (namely 1, 10, 100, 1000, . . . or 0.1, 0.01, 0.001, . . .) that the quantity or number is closest to.

**Number composition** and **number decomposition** are not formal mathematical terms but are used to describe putting numbers together, as in putting 2 and 3 together to make 5, and breaking numbers apart, as in breaking 5 into 2 and 3. For young children, a visual representation like the one shown on the next page is often used before introducing the traditional mathematical notation  $2 + 3 = 5$  and  $5 = 2 + 3$  for number composition and decomposition.

**Example 12: Number decomposition**

A **number line** is a line on which locations for 0 and 1 have been chosen (1 is to the right of 0, traditionally). Using the distance between 0 and 1 as a unit, a positive real number,  $N$ , is located  $N$  units to the right of 0 and a negative real number,  $-N$ , is located  $N$  units to the left of 0. In this way, every real number has a location on the number line and every location on the number line corresponds to a real number.

A **number path** is an informal precursor to a number line. It is a path of consecutively numbered “steps,” such as the paths found on many children’s board games along which game pieces are moved. Determining locations on number paths only requires counting, whereas determining locations on number lines requires the notion of distance.

**Reliability** refers to the degree to which an assessment yields consistency over time (how likely are scores to be similar if students take the test a week or so later?) and across testers (do scores change when different individuals administer the test?). Alternate form reliability tells us the extent to which an educator can expect similar results across comparable forms or versions of an assessment.

**Predictive validity** is the extent to which a test can predict how well students will

do in mathematics a year or even two or three years later.

**Response to Intervention** (RtI) is an early detection, prevention, and support system in education that identifies struggling students and assists them before they fall behind.

**Sensitivity** indicates how accurately a screening measure predicts which students are at risk. Sensitivity is calculated by determining the number of students who end up having difficulty in mathematics and then examining the percentage of those students predicted to be at risk on the screening measure. A screening measure with high sensitivity would have a high degree of accuracy. In general, sensitivity and specificity are related (as one increases the other usually decreases).

**Specificity** indicates how accurately a screening measure predicts which students are not at risk. Specificity is calculated by determining the number of students who do not have a deficit in mathematics and then examining the percentage of those students predicted to not be at risk on the screening measure. A screening measure with high specificity would have a high degree of accuracy. In general, sensitivity and specificity are related (as one increases the other usually decreases).

**Strip diagrams** (also called model diagrams and bar diagrams) are drawings of narrow rectangles that show relationships among quantities.

A **validity coefficient** serves as an index of the relation between two measures and can range from  $-1.0$  to  $1.0$ , with a coefficient of  $.0$  meaning there is no relation between the two scores and increasing positive scores indicating a stronger positive relation.

## Appendix A. Postscript from the Institute of Education Sciences

### What is a practice guide?

The health care professions have embraced a mechanism for assembling and communicating evidence-based advice to practitioners about care for specific clinical conditions. Various called practice guidelines, treatment protocols, critical pathways, best practice guides, or simply practice guides, these documents are systematically developed recommendations about the course of care for frequently encountered problems, ranging from physical conditions, such as foot ulcers, to psychosocial conditions, such as adolescent development.<sup>192</sup>

Practice guides are similar to the products of typical expert consensus panels in reflecting the views of those serving on the panel and the social decisions that come into play as the positions of individual panel members are forged into statements that all panel members are willing to endorse. Practice guides, however, are generated under three constraints that do not typically apply to consensus panels. The first is that a practice guide consists of a list of discrete recommendations that are actionable. The second is that those recommendations taken together are intended to be a coherent approach to a multifaceted problem. The third, which is most important, is that each recommendation is explicitly connected to the level of evidence supporting it, with the level represented by a grade (strong, moderate, or low).

The levels of evidence, or grades, are usually constructed around the value of

particular types of studies for drawing causal conclusions about what works. Thus, one typically finds that a strong level of evidence is drawn from a body of randomized controlled trials, the moderate level from well-designed studies that do not involve randomization, and the low level from the opinions of respected authorities (see table 1, p. 2). Levels of evidence also can be constructed around the value of particular types of studies for other goals, such as the reliability and validity of assessments.

Practice guides also can be distinguished from systematic reviews or meta-analyses such as What Works Clearinghouse (WWC) intervention reviews or statistical meta-analyses, which employ statistical methods to summarize the results of studies obtained from a rule-based search of the literature. Authors of practice guides seldom conduct the types of systematic literature searches that are the backbone of a meta-analysis, although they take advantage of such work when it is already published. Instead, authors use their expertise to identify the most important research with respect to their recommendations, augmented by a search of recent publications to ensure that the research citations are up-to-date. Furthermore, the characterization of the quality and direction of the evidence underlying a recommendation in a practice guide relies less on a tight set of rules and statistical algorithms and more on the judgment of the authors than would be the case in a high-quality meta-analysis. Another distinction is that a practice guide, because it aims for a comprehensive and coherent approach, operates with more numerous and more contextualized statements of what works than does a typical meta-analysis.

Thus, practice guides sit somewhere between consensus reports and meta-analyses in the degree to which systematic processes are used for locating relevant research and characterizing its meaning. Practice guides

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192. Field and Lohr (1990).

are more like consensus panel reports than meta-analyses in the breadth and complexity of the topic that is addressed. Practice guides are different from both consensus reports and meta-analyses in providing advice at the level of specific action steps along a pathway that represents a more-or-less coherent and comprehensive approach to a multifaceted problem.

### **Practice guides in education at the Institute of Education Sciences**

IES publishes practice guides in education to bring the best available evidence and expertise to bear on the types of systemic challenges that cannot currently be addressed by single interventions or programs. Although IES has taken advantage of the history of practice guides in health care to provide models of how to proceed in education, education is different from health care in ways that may require that practice guides in education have somewhat different designs. Even within health care, where practice guides now number in the thousands, there is no single template in use. Rather, one finds descriptions of general design features that permit substantial variation in the realization of practice guides across subspecialties and panels of experts.<sup>193</sup> Accordingly, the templates for IES practice guides may vary across practice guides and change over time and with experience.

The steps involved in producing an IES-sponsored practice guide are first to select a topic, which is informed by formal surveys of practitioners and requests. Next, a panel chair is recruited who has a national reputation and up-to-date expertise in the topic. Third, the chair, working in collaboration with IES, selects a small number of panelists to co-author the practice guide. These are people the chair believes can work well together and have the requisite

expertise to be a convincing source of recommendations. IES recommends that at least one of the panelists be a practitioner with experience relevant to the topic being addressed. The chair and the panelists are provided a general template for a practice guide along the lines of the information provided in this appendix. They are also provided with examples of practice guides. The practice guide panel works under a short deadline of six to nine months to produce a draft document. The expert panel members interact with and receive feedback from staff at IES during the development of the practice guide, but they understand that they are the authors and, thus, responsible for the final product.

One unique feature of IES-sponsored practice guides is that they are subjected to rigorous external peer review through the same office that is responsible for independent review of other IES publications. A critical task of the peer reviewers of a practice guide is to determine whether the evidence cited in support of particular recommendations is up-to-date and that studies of similar or better quality that point in a different direction have not been ignored. Peer reviewers also are asked to evaluate whether the evidence grade assigned to particular recommendations by the practice guide authors is appropriate. A practice guide is revised as necessary to meet the concerns of external peer reviews and gain the approval of the standards and review staff at IES. The process of external peer review is carried out independent of the office and staff within IES that instigated the practice guide.

Because practice guides depend on the expertise of their authors and their group decisionmaking, the content of a practice guide is not and should not be viewed as a set of recommendations that in every case depends on and flows inevitably from scientific research. It is not only possible but also likely that two teams of recognized experts working independently to produce

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193. American Psychological Association (2002).

a practice guide on the same topic would generate products that differ in important respects. Thus, consumers of practice guides need to understand that they are, in effect, getting the advice of consultants. These consultants should, on average, provide substantially better advice than an

individual school district might obtain on its own because the authors are national authorities who have to reach agreement among themselves, justify their recommendations in terms of supporting evidence, and undergo rigorous independent peer review of their product.

**Institute of Education Sciences**

## Appendix B. About the authors

### Panel

**Russell Gersten, Ph.D.**, is the director of the Instructional Research Group in Los Alamitos, California, as well as professor emeritus in the College for Education at the University of Oregon. Dr. Gersten recently served on the National Mathematics Advisory Panel, where he co-chaired the Work Group on Instructional Practices. He has conducted meta-analyses and research syntheses on instructional approaches for teaching students with difficulties in mathematics, early screening in mathematics, RtI in mathematics, and research on number sense. Dr. Gersten has conducted numerous randomized trials, many of them published in major education journals. He has either directed or co-directed 42 applied research grants addressing a wide array of issues in education and has been a recipient of many federal and non-federal grants (more than \$17.5 million). He has more than 150 publications and serves on the editorial boards of 10 prestigious journals in the field. He is the director of the Math Strand for the Center on Instruction (which provides technical assistance to the states in terms of implementation of No Child Left Behind) and the director of research for the Southwest Regional Educational Laboratory.

**Sybilla Beckmann, Ph.D.**, is a professor of mathematics at the University of Georgia. Prior to arriving at the University of Georgia, Dr. Beckmann taught at Yale University as a J. W. Gibbs Instructor of Mathematics. Dr. Beckmann has done research in arithmetic geometry, but her current main interests are the mathematical education of teachers and mathematics content for students at all levels, but especially for Pre-K through the middle grades. Dr. Beckmann developed three mathematics content courses for prospective

elementary school teachers at the University of Georgia and wrote a book for such courses, *Mathematics for Elementary Teachers*, published by Addison-Wesley, now in a second edition. She is especially interested in helping college faculty learn to teach mathematics content courses for elementary and middle grade teachers, and she works with graduate students and postdoctoral fellows toward that end. As part of this effort, Dr. Beckmann directs the *Mathematicians Educating Future Teachers* component of the University of Georgia Mathematics Department's VIGRE II grant. Dr. Beckmann was a member of the writing team of the National Council of Teachers of Mathematics Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics, is a member of the Committee on Early Childhood Mathematics of the National Research Council, and has worked on the development of several state mathematics standards. Recently, Dr. Beckmann taught an average grade 6 mathematics class every day at a local public school in order to better understand school mathematics teaching. She has won several teaching awards, including the General Sandy Beaver Teaching Professorship, awarded by the College of Arts and Sciences at the University of Georgia.

**Benjamin Clarke, Ph.D.**, is a research associate at the Instructional Research Group and Pacific Institutes for Research. He serves as a co-principal investigator on three federally funded research grants in mathematics instructions and assessment. His current research includes testing the efficacy of a kindergarten mathematics curriculum, evaluating the effectiveness of a grade 1 mathematics intervention program for at-risk students and examining the effects of a computer software program to build student understanding of and fluency with computational procedures. He also serves as the deputy director of the Center on Instruction Mathematics. Dr. Clarke was a 2002 graduate of the University of Oregon School Psychology

program. He was the recipient of AERA Special Education Dissertation Award for his work in early mathematics assessment. He has continued to investigate and publish articles and materials in this area and has presented his work at national conferences.

**Anne Foegen, Ph.D.**, is an associate professor in the Department of Curriculum and Instruction at Iowa State University. Her research focuses on the mathematics development of students with disabilities, including efforts to develop measures that will allow teachers to monitor the progress of secondary students in mathematics. Dr. Foegen has also been involved in examining the mathematics performance of students with disabilities on large-scale assessments, such as the National Assessment of Educational Progress. Her current work in progress monitoring extends research in curriculum-based measurement (CBM) in mathematics from kindergarten through grade 12. Her particular focus is on studies at the middle school and high school levels. In a related project, Dr. Foegen developed CBM measures and conducted technical adequacy research on the use of the measures to track secondary students' progress in algebra and prealgebra.

**Laurel Marsh, M.S.E. and M.A.T.**, is a professional education instructor who serves as a math coach at Swansfield Elementary School, Howard County School District, in Columbia, Maryland. Also an instructor for the University of Maryland Baltimore County and Johns Hopkins University, she has served as an elementary teacher at multiple levels. Ms. Marsh received a Master of Arts in Teaching from Johns Hopkins University, with a concentration in both Early Childhood Education and Elementary Education, as well as a Master of Science in Education from Johns Hopkins University, with a concentration in School Administration and Supervision. As a math support teacher, she provides

professional development to teachers of kindergarten through grade 5 for multiple schools in Howard County. She works with both general educators and special educators through demonstration lessons, co-teaching situations, and school-based professional development. She also oversees and coordinates interventions for students struggling in mathematics.

**Jon R. Star, Ph.D.**, is an assistant professor of education at Harvard University's Graduate School of Education. Dr. Star is an educational psychologist who studies children's learning of mathematics in middle and high school, particularly algebra. His current research explores the development of flexibility in mathematical problem solving, with flexibility defined as knowledge of multiple strategies for solving mathematics problems and the ability to adaptively choose among known strategies on a particular problem. He also investigates instructional and curricular interventions that may promote the development of mathematical understanding. Dr. Star's most recent work is supported by grants from the Institute of Education Sciences at the U.S. Department of Education and the National Science Foundation. In addition, he is interested in the preservice preparation of middle and secondary mathematics teachers. Before his graduate studies, he spent six years teaching middle and high school mathematics.

**Bradley Witzel, Ph.D.**, is an associate professor and coordinator of special education at Winthrop University in Rock Hill, South Carolina. He has experience in the classroom as an inclusive and self-contained teacher of students with higher incidence disabilities as well as a classroom assistant and classroom teacher of students with low incidence disabilities. Dr. Witzel has taught undergraduate and graduate courses in educational methods for students with disabilities and secondary students with disabilities coupled with the needs of English language learning. He has supervised

interns in elementary, secondary, and special education certification tracks as well as inclusion practices. He has given numerous professional presentations including strategic math, algebra instruction, word problem solving, parent involvement, and motivational classroom management. He has published research and practitioner articles in algebra and math education as well as positive behavior interventions for students with and without learning disabilities. He has also written several books and book chapters on mathematics education and interventions. Overall, he is concerned with the development of special education teachers and works to provide researched-validated practices and interventions to professionals and preprofessionals.

## Staff

**Joseph A. Dimino, Ph.D.**, is a senior research associate at the Instructional Research Group in Los Alamitos, California, where he is the coordinator of a national research project investigating the impact of teacher study groups as a means to enhance the quality of reading instruction for 1st graders in high-poverty schools and co-principal investigator for a study assessing the impact of collaborative strategic reading on the comprehension and vocabulary skills of English language learners and English-speaking 5th graders. Dr. Dimino has 36 years of experience as a general education teacher, special education teacher, administrator, behavior specialist, and researcher. He has extensive experience working with teachers, parents, administrators, and instructional assistants in instruction, early literacy, reading comprehension strategies, and classroom and behavior management in urban, suburban, and rural communities. He has published in numerous scholarly journals and co-authored books in reading comprehension and early reading intervention. Dr. Dimino has delivered papers at various state, national, and international conferences, including the American

Educational Research Association, Society for the Scientific Study of Reading, Council for Exceptional Children, International Reading Association, and Association for Supervision and Curriculum Development. He consults nationally in early literacy and reading comprehension instruction.

**Madhavi Jayanthi, Ph.D.**, is a research associate at Instructional Research Group, Los Alamitos, California. Dr. Jayanthi has more than 10 years experience in working on grants by the Office of Special Education Programs and Institute of Education Sciences. She has published extensively in peer-reviewed journals such as *Journal of Learning Disabilities*, *Remedial and Special Education*, and *Exceptional Children*. Her research interests include effective instructional techniques for students with disabilities and at-risk learners, both in the areas of reading and mathematics.

**Shannon Monahan, Ph.D.**, is a survey researcher at Mathematica Policy Research. She has served as a reviewer for the What Works Clearinghouse for the *Reducing Behavior Problems in the Elementary School Classroom* practice guide and the early childhood interventions topic area, and she coordinated the reviews for this practice guide. Dr. Monahan has worked extensively on the development and evaluation of mathematics curricula for low-income children. Currently, she contributes to several projects that evaluate programs intended to influence child development. Her current interests include early childhood program evaluation, emergent numeracy, culturally competent pedagogy, measures development, and research design.

**Rebecca A. Newman-Gonchar, Ph.D.**, is a research associate with the Instructional Research Group. She has experience in project management, study design and implementation, and quantitative and qualitative analysis. Dr. Newman-Gonchar has worked extensively on the development of observational measures for



beginning and expository reading instruction for two major IES-funded studies of reading interventions for Title I students. She currently serves as a reviewer for the What Works Clearinghouse for reading and mathematics interventions and Response to Intervention. Her scholarly contributions include conceptual, descriptive, and quantitative publications on a range of topics. Her current interests include Response to Intervention, observation measure development for reading and mathematics instruction, and teacher study groups. She has served as the technical editor for several publications and is a reviewer for *Learning Disability Quarterly*.

**Libby Scott, M.P.P.**, is a research analyst at Mathematica Policy Research and a former classroom educator. She has experience providing research support and conducting data analysis for various projects on topics related to out-of-school time programs, disconnected youth, home schooling households, and item development for a teacher survey. She also has experience evaluating an out-of-school time program for middle school students. Ms. Scott used her background in classroom teaching and education-related research to support the panel in translating research findings into practitioner friendly text.

## Appendix C. Disclosure of potential conflicts of interest

Practice guide panels are composed of individuals who are nationally recognized experts on the topics about which they are rendering recommendations. The Institute of Education Sciences (IES) expects that such experts will be involved professionally in a variety of matters that relate to their work as a panel. Panel members are asked to disclose their professional involvements and to institute deliberative processes that encourage critical examination of the views of panel members as they relate to the content of the practice guide. The potential influence of panel members' professional engagements is further muted by the requirement that they ground their recommendations in evidence documented in the practice guide. In addition, the practice guide undergoes independent external peer review prior to publication, with particular focus on whether the evidence related to the recommendations in the practice guide has been appropriately presented.

The professional engagements reported by each panel member that appear most closely associated with the panel recommendations are noted below.

**Russell Gersten** has written articles on issues related to assessment and screening of young children with potential difficulties in learning mathematics and is currently revising a manuscript on this topic for the scholarly journal, *Exceptional Children*. However, there is no fiscal reward for this or other publications on the topic. He is a royalty author for what may become the Texas or national edition of the forthcoming (2010/11) Houghton Mifflin reading series, *Journeys*. At the time of publication, Houghton Mifflin has not determined whether they will ever release

this series. Dr. Gersten provided guidance on the product as it relates to struggling and English language learner students. This topic is not covered in this practice guide. The panel never discussed the Houghton Mifflin series. Russell Gersten has no financial stake in any program or practice mentioned in the practice guide.

**Sybilla Beckmann** receives royalties on her textbook, *Mathematics for Elementary Teachers*, published by Addison-Wesley, a division of Pearson Education. This textbook, used in college mathematics courses for prospective elementary teachers, is not within the scope of the review of this practice guide.

**Ben Clarke** has developed and authored journal articles about early numeracy measures that are referenced in the RtI-Mathematics practice guide. Dr. Clarke does not have a current financial stake in any company or publishing of the measures.

**Anne Foegen** conducts research and has developed progress monitoring assessments that are referred to in the RtI-Mathematics practice guide. Currently, she has no financial interests in these products, which are not available commercially. The algebra measures are disseminated through Iowa State University on a fee-based schedule to cover the costs of personnel for training and materials (not for profit). Dr. Foegen also has published papers that describe the measures and received a grant that is supporting a portion of the research. She occasionally does private consulting related to research on use of the mathematics progress monitoring measures.

**Jon R. Star** consults for a company owned by Scholastic, which produces mathematics educational software. Scholastic may produce other curricula related to mathematics, but the panel makes no recommendations for selecting specific curricula.

**Bradley Witzel** wrote the book, *Computation of Fractions*, and is currently writing *Computation of Integers* and *Solving Equations*, through Pearson with Dr. Paul Riccomini. He is also writing the book, *RtI Mathematics*, through Corwin Press with Dr. Riccomini. Additionally, Dr. Witzel has

delivered workshop presentations on the structure of RtI (not associated with the RtI-Mathematics practice guide). The work on his books is separate from that of the RtI-Mathematics practice guide panel, and he does not share his work from the panel with the books' co-authors.

## Appendix D. Technical information on the studies

### **Recommendation 1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.**

#### **Level of evidence: Moderate**

The panel examined reviews of the technical adequacy of screening measures for students identified as at risk when making this recommendation. The panel rated the level of evidence for recommendation 1 as *moderate* because several reviews were available for evidence on screening measures for younger students. However, there was less evidence available on these measures for older students. The panel relied on the standards of the American Psychological Association, the American Educational Research Association, and the National Council on Measurement in Education<sup>194</sup> for valid screening instruments along with expert judgment to evaluate the quality of the individual studies and to determine the overall level of evidence for this recommendation.

Relevant studies were drawn from recent comprehensive literature reviews and reports<sup>195</sup> as well as literature searches of databases using key terms (such as “formative assessment”). Journal articles summarizing research studies on screening in mathematics,<sup>196</sup> along with summary information provided by the Research In-

stitute on Progress Monitoring<sup>197</sup> and the National Center on Progress Monitoring were also used.<sup>198</sup>

The studies of screening measures all used appropriate correlational designs.<sup>199</sup> In many cases, the criterion variable was some type of standardized assessment, often a nationally normed test (such as the Stanford Achievement Test) or a state assessment. In a few cases, however, the criterion measure was also tightly aligned with the screening measure.<sup>200</sup> The latter set is considered much weaker evidence of validity.

Studies also addressed inter-tester reliability,<sup>201</sup> internal consistency,<sup>202</sup> test-retest reliability,<sup>203</sup> and alternate form reliability.<sup>204</sup> Many researchers discussed the content validity of the measure.<sup>205</sup> A few even discussed the consequential validity<sup>206</sup>—the consequences of using screening data as a tool for determining what requires intervention.<sup>207</sup> However, these studies all used standardized achievement measures as the screening measure.

In recent years, a number of studies of screening measures have also begun to

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194. American Educational Research Association, American Psychological Association, and National Council on Measurement in Education (1999).

195. For example, the National Mathematics Advisory Panel (2008).

196. Gersten et al. (2005); Fuchs, Fuchs, Compton et al. (2007); Foegen et al. (2007).

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197. <http://www.progressmonitoring.net/>.

198. [www.studentprogress.org](http://www.studentprogress.org).

199. Correlational studies are not eligible for WWC review.

200. For example, Bryant, Bryant, Gersten, Scamacca, and Chavez (2008).

201. For example, Fuchs et al. (2003a).

202. For example, Jitendra et al. (2005).

203. For example, VanDerHeyden, Witt, and Gilbertson (2003).

204. For example, Thurber, Shinn, and Smolkowski (2002).

205. For example, Clarke and Shinn (2004); Gersten and Chard (1999); Foegen, Jiban, and Deno (2007).

206. Messick (1988); Gersten, Keating, and Irvin (1995).

207. For example, Compton, Fuchs, and Fuchs (2007).

report sensitivity and specificity data.<sup>208</sup> Because sensitivity and specificity provide information on the false positive and false negative rates respectively, they are critical in determining the utility of a measure used in screening decisions linked to resource allocation. Note that work on sensitivity and specificity in educational screening is in its infancy and no clear standards have been developed.

The remainder of this section presents evidence in support of the recommendation. We discuss the evidence for measures used in both the early elementary and upper elementary grades and conclude with a more in-depth example of a screening study to illustrate critical variables to consider when evaluating a measure.

### Summary of evidence

In the early elementary grades, measures examined included general outcome measures reflecting a sampling of objectives for a grade level that focused on whole numbers and number sense. These included areas of operations and procedures, number combinations or basic facts, concepts, and applications.<sup>209</sup> Measures to assess different facets of number sense—including measures of rote and strategic counting, identification of numerals, writing numerals, recognizing quantities, and magnitude comparisons—were also prevalent.<sup>210</sup> Some research teams developed measures focused on a single aspect of number sense (such as strategic counting),<sup>211</sup> and others developed batteries to create a composite score from single

proficiency measures.<sup>212</sup> Still others developed a broader measure that assessed multiple proficiencies in their screening.<sup>213</sup> An example of a single proficiency embedded in a broader measure is having students compare magnitudes of numbers. As an individual measure, magnitude comparison has predictive validity in the .50 to .60 range,<sup>214</sup> but having students make magnitude comparisons is also included in broader measures. For example, the Number Knowledge Test (NKT)<sup>215</sup> requires students to name the greater of two verbally presented numbers and includes problems assessing strategic counting, simple addition and subtraction, and word problems. The broader content in the NKT provided stronger evidence of predictive validity<sup>216</sup> than did single proficiency measures.

Further information on the characteristics and technical adequacy of curriculum-based measures (CBM) for use in screening in the elementary grades was summarized by Foegen, Jiban, and Deno (2007). They explained that measures primarily assessed the objectives of operations or the concepts and applications standards for a specific grade level. A smaller number of measures assessed fluency in basic facts, problem solving, or word problems. Measures were timed and administration time varied between 2 and 6 minutes for operations probes and 2 to 8 minutes for concepts and applications. Reliability evidence included test-retest, alternate form, internal consistency, and inter-scorer, with most reliabilities falling between .80 and .90, meeting acceptable standards for educational decisionmaking. Similar evidence was found for validity with most

208. Locuniak and Jordan (2008); VanDerHeyden et al. (2001); Fuchs, Fuchs, Compton et al. (2007).

209. Fuchs, Fuchs, Compton et al. (2007).

210. Gersten, Clarke, and Jordan (2007); Fuchs, Fuchs, and Compton et al. (2007).

211. Clarke and Shinn (2004).

212. Bryant, Bryant, Gersten, Scammacca, and Chavez (2008).

213. Okamoto and Case (1996).

214. Lembke et al. (2008); Clarke and Shinn (2004); Bryant, Bryant, Gersten, Scammacca, and Chavez (2008).

215. Okamoto and Case (1996).

216. Chard et al. (2005).

concurrent validity coefficients in the .50 to .70 range. Lower coefficients were found for basic fact measures ranging from .30 to .60. Researchers have also begun to develop measures that validly assess magnitude comparison, estimation, and prealgebra proficiencies.<sup>217</sup>

### A study of evaluating a mathematics screening instrument—Locuniak and Jordan (2008)

A recent study by Locuniak and Jordan (2008) illustrates factors that districts should consider when evaluating and selecting measures for use in screening. The researchers examined early mathematics screening measures from the middle of kindergarten to the end of second grade. The two-year period differs from many of the other screening studies in the area by extending the interval from within a school year (fall to spring) to across several school years. This is critical because the panel believes the longer the interval between when a screening measure and a criterion measure are administered, the more schools can have confidence that students identified have a significant deficit in mathematics that requires intervention. The Locuniak and Jordan (2008) study also went beyond examining traditional indices of validity to examine specificity and sensitivity. Greater sensitivity and specificity of a measure ensures that schools provide resources to those students truly at risk and not to students misidentified

The various measures studied by Locuniak and Jordan (2008) also reflected mathematics content that researchers consider critical in the development of a child's mathematical thinking and that many researchers have devised screening

measures to assess. Included were number sense measures that assessed knowledge of counting, number combinations, non-verbal calculation, story problems, number knowledge, and short and working memory. The authors used block regression to examine the added value of the math measures in predicting achievement above and beyond measures of cognition, age, and reading ability (block 1), which accounted for 26 percent of the variance on 2nd grade calculation fluency. Adding the number sense measures (block 2) increased the variance explained to 42 percent. Although the research team found strong evidence for the measures assessing working memory (digit span), number knowledge, and number combinations, the array of measures investigated is indicative that the field is still attempting to understand which critical variables (mathematical concepts) best predict future difficulty in mathematics. A similar process has occurred in screening for reading difficulties where a number of variables (such as alphabetic principle) are consistently used to screen students for reading difficulty. Using the kindergarten measures with the strongest correlations to grade 2 mathematics achievement (number knowledge and number combinations), the researchers found rates of .52 for sensitivity and .84 for specificity.

Another feature that schools will need to consider when evaluating and selecting measures is whether the measure is timed. The measures studied by Locuniak and Jordan (2008) did not include a timing component. In contrast, general outcome measures include a timing component.<sup>218</sup> No studies were found by the panel that examined a timed and untimed version of the same measure.

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217. Foegen et al. (2007).

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218. Deno (1985).

**Recommendation 2.  
Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.**

**Level of evidence: Low**

The panel based this recommendation on professional opinion; therefore, the evidence rating is *low*. The professional opinion included not only the views of the panel members, but also several recent consensus documents that reflect input from mathematics educators and research mathematicians involved in issues related to K–12 mathematics education.<sup>219</sup> Each of these documents was influenced to some extent by comparisons of curricula standards developed by the 50 states in the United States with nations with high scores on international tests of mathematics performance, such as the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) (including the Czech Republic, Flemish Belgium, Korea, and Singapore).<sup>220</sup> We note, however, that these international comparisons are merely descriptive and thus do not allow for causal inferences. In other words, we do not know whether their more focused curricula or other factors contribute to higher performance.

We note that some of the other reports we describe here do not directly address the needs of students who receive interventions to boost their knowledge of

foundational concepts and procedures. However, we concluded that the focus on coverage of fewer topics in more depth, and with coherence, advocated for general education students is as important, and probably more important, for students who struggle with mathematics. We could not locate any experimental research that supported our belief, however. Therefore, we indicate clearly that we are reflecting a growing consensus of professional opinion, not a convergent body of scientific evidence—and conclude that the level of evidence is *low*.

**Summary of evidence**

Three seminal publications were consulted in forming our opinion.<sup>221</sup> Milgram and Wu (2005) were among the first to suggest that an intervention curriculum for at-risk students should not be oversimplified and that in-depth coverage of key topics and concepts involving whole numbers and then rational numbers was critical for future success in mathematics. They stressed that mastery of this material was critical, regardless of how long it takes. Many before had argued about the importance of mastery of units before proceeding forward.<sup>222</sup> Milgram and Wu argued that stress on precise definitions and abstract reasoning was “even more critical for at-risk students” (p. 2). They acknowledged this would entail extensive practice with feedback and considerable instructional time.

*The National Council of Teachers of Mathematics Curriculum Focal Points* (2006) made a powerful statement about reform of mathematics curriculum for all students by calling for the end of brief ventures into many topics in the course of a school year.

219. Milgram and Wu (2005); National Council of Teachers of Mathematics (2006); National Mathematics Advisory Panel (2008).

220. For more information on the TIMSS, see <http://nces.ed.gov/timss/>. For more information on PISA, see [www.oecd.org](http://www.oecd.org).

221. Milgram and Wu (2005); National Council of Teachers of Mathematics (2006); National Mathematics Advisory Panel (2008).

222. For example, Bloom (1980); Guskey (1984); Silbert, Carnine, and Stein (1989).

The topics it suggests emphasize whole numbers (properties, operations, problem solving) and especially fractions and related topics involving rational numbers (proportion, ratio, decimals). The report is equally clear that algorithmic proficiency is critical for understanding properties of operations and related concepts and that algorithmic proficiency, quick retrieval of mathematics facts, and in-depth knowledge of such concepts as place value and properties of whole numbers are all equally important instructional goals. This position was reinforced by the report of the National Mathematics Advisory Panel (2008) two years later, which provided detailed benchmarks and again emphasized in-depth coverage of key topics involving whole numbers and rational numbers as crucial for all students.

In the view of the panel, students in intervention programs need to master material on whole numbers and rational numbers, and they must ultimately work with these concepts and principles at an abstract level. We feel that it is less important for 4th graders in an intervention program to cover the entire scope and sequence of topics from the year before. Instead, the aim is to cover the key benchmarks articulated in the National Mathematics Advisory Panel report, involving whole numbers and rational numbers that students do not fully grasp, and build proficiencies they lack.

**Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.**

**Level of evidence: Strong**

The panel judged the level of evidence supporting the recommendation to be *strong*.

The panel found six studies<sup>223</sup> conducted with low achieving or learning disabled students<sup>224</sup> between 2nd and 8th grades that met WWC standards or met standards with reservations and included components of explicit and systematic instruction.<sup>225</sup> Appendix table D1 (p. 69) provides an overview of the components of explicit instruction in each intervention, including the use of teacher demonstration (such as verbalization during demonstration and the use of multiple examples), student verbalization (either as a procedural requirement or as a response to teacher questions), scaffolded practice, cumulative review, and corrective feedback. The relevant treatment and comparison groups compared in each study and the outcomes found for each domain are included in the table, as are grade-level, typical session length, and duration of the intervention.

Because of the number of high-quality randomized and quasi-experimental design studies using explicit and systematic mathematics instruction across grade levels and diverse student populations, the frequency of significant positive effects, and the fact that numerous research teams independently produced similar findings, the panel concluded that there is strong evidence to support the recommendation to provide explicit and systematic instruction in tier 2 mathematics interventions.

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223. Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Jitendra et al. (1998); Tournaki (2003); Schunk and Cox (1986); Wilson and Sindelar (1991).

224. These students specifically had difficulties with mathematics.

225. For this practice guide, the components of explicit and systematic mathematics instruction are identified as providing models of proficient problem solving, verbalizing teacher and student thought processes, scaffolded practice, cumulative review, and corrective feedback.



## Summary of evidence

*Teacher demonstrations and think-alouds.* The panel suggests that teachers verbalize the solution to problems as they model problem solving for students. Tournaki (2003) assessed this approach by comparing a group of students whose teachers had demonstrated and verbalized an addition strategy (the Strategy group) against a group of students whose teacher did not verbalize a strategy (the Drill and Practice group). As depicted in appendix table D1, the effects on an assessment of single-digit addition were significant, positive, and substantial in favor of the students whose teacher had verbalized a strategy.<sup>226</sup>

All six studies examined interventions that included teacher demonstrations early in the mathematics lessons.<sup>227</sup> For example, Schunk and Cox (1986), Jitendra et al. (1998), and Darch, Carnine, and Gersten (1984) all conducted studies in which instruction began with the teacher verbalizing the steps to solve sample mathematics problems. Because this demonstration procedure was used to instruct students in both treatment and comparison groups, the effects of this component of explicit instruction cannot be evaluated from these studies. However, the widespread use of teacher demonstration in interventions that include other components of explicit instruction supports the panel's contention that this is a critical component of explicit instructional practice.

For teacher demonstration, the panel specifically recommends that teachers provide

numerous models of solving easy and hard problems proficiently. Demonstration with easy and hard problems and the use of numerous examples were not assessed as independent variables in the studies reviewed. However, Wilson and Sindelar (1991) did use numerous examples in instruction for both groups evaluated. The key difference between the groups was that students in the treatment group were explicitly taught problem-solving strategies through verbal and visual demonstrations while students in the comparison group were not taught these strategies. This study demonstrated substantively important positive effects with marginal significance in favor of the treatment group.<sup>228</sup>

*Scaffolded practice.* Scaffolded practice, a transfer of control of problem solving from the teacher to the student, was a component of mathematics interventions in four of the six studies.<sup>229</sup> In each study, the intervention groups that included scaffolded practice demonstrated significant positive effects; however, it is not possible to parse the effects of scaffolded instruction from the other components of explicit instruction in these multicomponent interventions.

*Student verbalization.* Three of the six studies<sup>230</sup> included student verbalization of problem-solution steps in the interventions. For example, Schunk and Cox (1986) assessed the effect of having students verbalize their subtraction problem-solving steps versus solving problems silently. There were significant and substantial positive effects in favor of the group that

226. Note that during the intervention, students in the Strategy condition were also encouraged to verbalize the problem-solving steps and that this may also be a factor in the success of the intervention. The Tournaki (2003) study is described in more detail below.

227. Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Jitendra et al. (1998); Tournaki (2003); Schunk and Cox (1986); Wilson and Sindelar (1991).

228. For this guide, the panel defined marginally significant as a p-value in the range of .05 to .10. Following WWC guidelines, an effect size greater than 0.25 is considered substantively important.

229. Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Jitendra et al. (1998); Tournaki (2003).

230. Schunk and Cox (1986); Jitendra et al. (1998); Tournaki (2003).

verbalized steps. The Tournaki (2003) intervention also included student verbalization among other components and had significant positive effects. Among other intervention components, Jitendra et al. (1998) included student verbalization through student responses to a teacher's facilitative questions. Again, the effects were substantively important or statistically significant and positive, but they cannot be attributed to a single component in this multi component intervention.

*Corrective feedback.* Four of the six studies included immediate corrective feedback in the mathematics interventions.<sup>231</sup> For example, in the Darch, Carnine, and Gersten (1984) study, when a student made an error, teachers in the treatment group would first model the appropriate response, then prompt the students with questions to correct the response, then reinforce the problem-solving strategy steps again. In three of the studies,<sup>232</sup> the effects of the corrective feedback component cannot be isolated from the effects of the other instructional components; however, the effects of the interventions including corrective feedback were positive and significant.

*Cumulative review.* The panel's assertion that cumulative review is an important component of explicit instruction is based primarily on expert opinion because only one study in the evidence base included cumulative review as a component of the intervention.<sup>233</sup> This study had positive significant effects in favor of the instructional group that received explicit instruction in strategies for solving word problems.

In summary, the components of explicit and systematic instruction are consistently associated with significant positive effects on mathematics competency, most often when these components are delivered in combination. An example of a study that examines the effects of a combination of these components is described here.

### A study of explicit and systematic instruction—Tournaki (2003)

Explicit and systematic instruction is a multicomponent approach, and an intervention examined in Tournaki (2003) exemplifies several components in combination. This study was conducted with 42 students in grade 2 special education classrooms.<sup>234</sup> The students, between 8 and 10 years old, were classified as learning disabled with weaknesses in both reading and mathematics. Twenty-nine were boys, and 13 were girls.

Prior to the intervention, the students completed a pretest assessment consisting of 20 single-digit addition problems (such as  $6 + 3 = \underline{\quad}$ ). Internal consistency of the assessment was high (Cronbach's alpha of .91). Student performance on the assessment was scored for accuracy and latency (the time it took each student to complete the entire assessment). The accuracy score is a measure of student ability to perform mathematical operations, and the latency score is an indication of student fluency with single-digit addition facts. After the intervention, students completed a posttest assessment that was identical to the pretest.

Students were randomly assigned to one of three groups (two instruction groups and

231. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Tournaki (2003); Schunk and Cox (1986).

232. Darch, Carnine, and Gersten (1984); Jitendra et al. (1998); Tournaki (2003).

233. Fuchs et al. (2003a).

234. The sample also included 42 grade 2 students from general education classrooms, but only the results for the special education students are presented as relevant to this practice guide.

a comparison group).<sup>235</sup> Students in the two instruction groups met individually with a graduate assistant for a maximum of eight 15-minute supplemental mathematics sessions on consecutive school days. Instructional materials for both groups consisted of individual worksheets for each lesson with a group of 20 single-digit addition problems covering the range from  $2 + 2 = \underline{\quad}$  to  $9 + 9 = \underline{\quad}$ .

The Strategy instruction group received explicit and systematic instruction to improve fact fluency. The instruction began with the teacher modeling the minimum addend strategy for the students and thinking aloud. This strategy is an efficient approach for solving single-digit addition problems (such as  $5 + 3 = \underline{\quad}$ ). The teacher began by saying, “When I get a problem, what do I do? I read the problem: 5 plus 3 equals how many? Then I find the smaller number.” Pointing to the number, the teacher says, “Three. Now I count fingers. How many fingers am I going to count? Three.” The teacher counts three fingers, points to the larger number and says, “Now, starting from the larger number, I will count the fingers.” The teacher points to the 5, then touches each finger

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235. Students in the comparison group received only the pretest and posttest without any supplemental mathematics instruction outside their classroom. Because the scope of the practice guide is examining the effects of methods of teaching mathematics for low-achieving students, the comparison group findings are not included here.

as she says, “5, 6, 7, 8. How many did I end up with? Eight. I’ll write 8 to the right of the equal sign.” After writing the number, the teacher finishes modeling by saying, “I’ll read the whole problem: 5 plus 3 equals 8.”

The teacher and student solved two problems together through demonstration and structured probing. The student was then asked to solve a third problem independently while verbalizing the strategy steps aloud. When a student made an error, the teacher gave corrective feedback. The student was asked to solve the remaining problems without verbalization and to work as fast as possible, but when an error occurred, the teacher interrupted the lesson and reviewed the steps in the strategy.

Students in the Drill and Practice group were asked to solve the problems as quickly as possible. At the completion of each lesson, the teacher marked the student’s errors and asked the student to recompute. If the error persisted, the teacher told the student the correct answer. Results indicate significant and substantial positive effects in favor of the Strategy group, which received explicit and systematic instruction, relative to Drill and Practice group, which received a more traditional approach. In this study, the combination of teacher demonstration, student verbalization, and corrective feedback was successful in teaching students with mathematics difficulties to accurately complete single-digit addition problems.

**Table D1. Studies of interventions that included explicit instruction and met WWC Standards (with and without reservations)**

Study	Comparison	Components of explicit instruction included in the intervention					Grade level	Duration	Domain	Outcomes <sup>a</sup>
		Teacher demonstration	Student verbalizations	Guided practice	Corrective feedback	Cumulative review				
Darch, Carnine, and Gersten (1984)	Explicit strategy instruction versus traditional basal instruction	✓		✓	✓		4	30 minutes/session; 11 sessions	Word problems	1.79*
Jitendra et al. (1998)	Explicit visual strategy instruction versus traditional basal instruction	✓	✓	✓	✓		2-5	40-45 minutes/session; 17-20 sessions	Word problems Transfer	.56 (n.s.) 1.01*
Schunk and Cox (1986)	Continuous verbalizations by students versus no student verbalizations	✓	✓		✓		6-8	45 minutes/session; 6 sessions	Operations	1.01*
Tournaki (2003)	Counting-on strategy instruction versus drill and practice	✓	✓	✓	✓		2	15 minutes/session; up to 8 sessions	Operations Transfer	2.21* 1.10*
Wilson and Sindelar (1991)	Strategy instruction versus sequencing of practice problems	✓					2-4	30 minutes/session; 14 sessions	Word problems	.82~
Fuchs et al. (2003a)	Instruction on solving word problems that is based on common underlying structures versus traditional basal instruction	✓	✓	✓			3	25-40 minutes/session; 36 sessions	Word problems	2.09*

a. Outcomes are reported as effect sizes. For a p-value < .05, the effect size is significant (\*); for a p-value < .10, the effect size is marginally significant (~); for a p-value ≥ .10, the effect size is not significant (n.s.).

Source: Authors' analysis based on studies in table.

## **Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures.**

### **Level of evidence: Strong**

The panel rated the level of evidence as *strong*. We located nine studies that met the standards of the WWC or met the standards with reservations and demonstrated support for the practice of teaching students to solve word problems based on their underlying structures.<sup>236</sup> Appendix table D2 (p. 73) provides an overview of each of the interventions examined in these nine studies.

In all nine interventions, students were taught to recognize the structure of a word problem in order to solve it, and they were taught how to solve each problem type.<sup>237</sup> Six of the studies took the instruction on problem structure a step further. Students were taught to distinguish superficial from substantive information in word problems in order to transfer solution methods from familiar problems they already knew how to solve to problems that appeared unfamiliar.<sup>238</sup> Because of the large number of high-quality randomized studies conducted that examined this practice and because most of the interventions examined led to significant and positive effects on word problem outcomes for children designated as low achieving and/or learning disabled,

we conclude that there is *strong* evidence to support this recommendation.

### **Summary of evidence**

*Teach the structure of problem types.* In three of the studies, students were taught to identify problems of a given type and then to design and execute appropriate solution strategies for each type.<sup>239</sup> In one of these interventions, students learned to represent the problem using a schematic diagram.<sup>240</sup> Once students learned to identify the key problem features and map the information onto the diagram, they learned to solve for unknown quantities in word problems while still representing the problem using a schematic diagram. This intervention had significant and positive effects on a word problem outcome based on a test of problems similar to those taught during the intervention.

In another intervention that also led to a significant and positive effect on a word problem outcome, students were taught to discriminate between multiplication and addition problems, and between multiplication and division problems.<sup>241</sup> To discriminate multiplication from addition problems, students were taught that if a problem asks them to use the same number multiple times (sometimes signaled by the words “each” and “every”) to obtain the total number, the problem requires multiplication. If the problem does not ask the student to use the same number multiple times to obtain the total number, the problem requires addition. Next, after students learned the relationship between multiplication and division through the concept of number families, they learned to multiply when the two smaller numbers are given without the big number and to divide when the big number is given.

236. Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).

237. Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Darch, Carnine, and Gersten (1984); Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).

238. Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

239. Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Darch, Carnine, and Gersten (1984).

240. Xin, Jitendra, and Deatline-Buchman (2005).

241. Darch, Carnine, and Gersten (1984).

*Transfer solution methods from familiar problem types to problems that appear unfamiliar.* In addition to teaching students to recognize and solve different problem types, six of these studies taught students how to transfer solution methods to problems that appear different but really require the same solution methods as those they already know how to solve.<sup>242</sup> In each of these interventions, students were first taught the pertinent structural features and problem-solution methods for different problem types. Next, they were taught about superficial problem features that can change a problem without altering its structure or solution (for example, different format, different key vocabulary, additional or different question, irrelevant information) and how to solve problems with varied cover stories and superficial features.

In all six studies, word problem outcome measures ranged from those where the only source of novelty was the cover story (immediate transfer), to those that varied one or more superficial features (near or far transfer). In five cases<sup>243</sup> the average impact of the intervention on these outcome measures was positive and significant for the samples designated as low achieving and/or learning disabled, and in one case,<sup>244</sup> the impact was marginally significant. These studies show that instruction on problem structure and transferring known solution methods to unfamiliar problems is consistently associated with marginally or statistically significant positive effects on word problem solving proficiencies for students experiencing mathematics difficulties.

242. Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008).

243. Fuchs et al. (2003a); Fuchs et al. (2003b); Fuchs, Fuchs, Prentice et al. (2004); Fuchs, Fuchs, Finelli et al. (2004); Fuchs, Fuchs, Craddock et al. (2008).

244. Fuchs, Seethaler et al. (2008).

### A study of teaching students to transfer solution methods—Fuchs, Fuchs, Finelli, Courey, and Hamlett (2004)

Fuchs and colleagues (2004) conducted a study that investigated the effects of teaching students how to transfer known solution methods to problems that are only superficially different from those they already know how to solve.<sup>245</sup> The authors randomly assigned 24 teachers to three groups: 1) transfer instruction, 2) expanded transfer instruction, and 3) regular basal instruction (comparison group).<sup>246</sup> The 351 students in these 24 classes that were present for each of the pretests and posttests were participants in the study.

The intervention included 25- to 40-minute lessons, approximately twice per week for 17 weeks.<sup>247</sup> Students in the expanded transfer condition learned basic math problem-solving strategies in the first unit of instruction (six sessions over three weeks). They were taught to verify that their answers make sense; line up numbers from text to perform math operations; check operations; and label their work with words, monetary signs, and mathematical symbols.

The remaining units each focused on one of four problem types: 1) shopping list problems (buying multiple quantities of items, each at a different price); 2) buying bag problems (determining how many bags containing a specified number of objects are needed to come up with a desired total number of

245. Fuchs, Fuchs, Finelli et al. (2004).

246. Since the comparison between the expanded transfer condition and the control condition (regular basal instruction) is most relevant to this practice guide, we do not discuss the transfer instruction condition here.

247. Although this intervention was taught in a whole-class format, the authors reported separate effects for students classified as low achieving and for students classified as learning disabled; therefore, the results are relevant to this practice guide.

objects); 3) half problems (determining what half of some total amount is); and 4) pictograph problems (summing two addends, one derived from a pictograph). There were seven sessions within each unit.

In sessions one through four, with the help of a poster listing the steps, students learned problem-solution rules for solving the type of problem being taught in that particular unit. In the first session, teachers discussed the underlying concepts related to the problem type, presented a worked example, and explained how each step of the solution method was applied in the example. After presenting several worked examples, the teachers presented partially worked examples while the students applied the steps of the solution method. Students then completed one to four problems in pairs. Sessions two through four were similar, but more time was spent on partially worked examples and practice, and at the end of each session, students completed a problem independently.

In sessions five and six, teachers taught students how to transfer the solution methods using problems that varied cover stories, quantities, and one transfer feature per problem. In session five, the teachers began by explaining that transfer means to move and presented examples of how students transfer skills. Then, teachers taught three transfer features that change a problem without changing its type or solution, including formatting, unfamiliar vocabulary, and posing a different question. These lessons were facilitated by a poster displayed in the classroom about the three ways problems change. Again, teachers presented the information and worked examples, and moved gradually to partially worked examples and practice in pairs. Session six was similar to session five, but the students spent more time working in pairs, and they completed a transfer problem independently.

In the seventh session, teachers instructed students on three additional superficial problem features including irrelevant information, combining problem types, and mixing superficial problem features. Teachers taught this lesson by discussing how problems encountered in “real life” incorporate more information than most problems that the students know how to solve. They used a poster called Real-Life Situations to illustrate each of these superficial problem features with a worked example. Next, students worked in pairs to solve problems that varied real-life superficial problem features and then completed a problem independently.

The authors used four measures to determine the results of their intervention on word problem-solving proficiencies. The first measure used novel problems structured the same way as problems used in the intervention. The second incorporated novel problems that varied from those used in instruction in terms of the look or the vocabulary or question asked. The third incorporated novel problems that varied by the three additional transfer features taught in session seven. The fourth was a measure designed to approximate real-life problem solving. Although this intervention was taught in a whole-class format, the authors separated results for students classified as low performing<sup>248</sup> and for students classified as learning disabled. The average impacts on these four outcome measures were positive and significant for both the sample designated as low performing and the sample designated as learning disabled. It is notable that the intervention had a positive and significant impact on the far transfer measure (the measure that approximated real-life problem solving). This study demonstrates a successful approach for instructing students with mathematics difficulties on solving word problems and transferring solution methods to novel problems.

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248. Using pretest scores on the first transfer problem-solving measure, the authors designated each student as low performing, average performing, or high performing.

**Table D2. Studies of interventions that taught students to discriminate problem types that met WWC standards (with or without reservations)**

Study	Comparison	Grade level	Duration	Learning disabled/ Low achieving	Domain	Outcomes <sup>a</sup>
Darch, Carnine, and Gersten (1984)	Instruction on solving word problems that is based on common underlying structures versus traditional basal instruction	4	30 minutes/ session; 11 sessions	Low achieving	Word problems	1.79*
Fuchs, Fuchs, Prentice et al. (2004)	Instruction on solving word problems that is based on common underlying structures versus traditional basal instruction	3	25–40 minutes/ session; 32 sessions	Low achieving	Word problems	4.75*
Fuchs, Fuchs, Finelli et al. (2004)	Instruction on solving word problems that is based on common underlying structures versus traditional basal instruction	3	25–40 minutes/ session; 34 sessions	Learning disabled <sup>b</sup>	Word problems	1.10*
Fuchs, Seethaler et al. (2008)	Instruction on solving word problems that is based on common underlying structures versus nonrelevant instruction	3	20–30 minutes/ session; 36 sessions	Low achieving	Word problems	3.08*
Fuchs, Fuchs, Craddock et al. (2008)	Instruction on solving word problems that is based on common underlying structures versus nonrelevant instruction	3	20–30 minutes/ session; 38 sessions	Learning disabled <sup>c</sup>	Word problems	1.23*
Jitendra et al. (1998)	Instruction on solving word problems that is based on common underlying structures versus traditional basal instruction	2–5	40–45 minutes/ session; 17–20 sessions	Low achieving	Word problems	.66~
Xin, Jitendra, and Deatline-Buchman (2005)	Instruction on solving word problems based on common underlying structures versus general strategy instruction	6–8	60 minutes/ session; 12 sessions	Learning disabled <sup>d</sup> and low achieving combined	Concepts	.60 (n.s.)
Fuchs et al. (2003a)	Instruction on solving word problems that is based on common underlying structures versus traditional basal instruction	3	25–40 minutes/ session; 36 sessions	Low achieving	Word problems	.73*
Fuchs et al. (2003b)	Instruction on solving word problems that is based on common underlying structures versus traditional basal instruction	3	Number of minutes not reported; 32 sessions	Learning disabled <sup>e</sup>	Word problems	.56 (n.s.)
				Low achieving	Transfer	1.01*
				Learning disabled <sup>f</sup>	Word problems	1.87*
				Low achieving	Word problems	2.09*
				Learning disabled <sup>f</sup>	Word problems	2.05*
				Low achieving	Word problems	.86*

a. Outcomes are reported as effect sizes. For a p-value < .05, the effect size is significant (\*); for a p-value < .10, the effect size is marginally significant (~); for a p-value ≥ .10, the effect size is not significant (n.s.).

b. Thirteen students in this sample were classified as having a learning disability, one as having mental retardation, eight as having a speech disorder, and two as having attention-deficit/hyperactivity disorder.

c. Fifteen students in this sample were classified as having a learning disability and five were classified as having an “other” disability.

d. Seventeen students in this sample were classified as having a learning disability, five as being educable mentally retarded, and three as being seriously emotionally disturbed.

e. Eighteen students in this sample were classified as having a learning disability, one as being seriously emotionally disturbed, and three were not labeled.

f. Twenty-two students in this sample were classified as having a learning disability, one as being mildly mentally retarded, one as having a behavior disorder, and three as having speech delay.

Source: Authors’ analysis based on studies in table.



**Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.**

**Level of evidence: Moderate**

The panel judged the level of evidence for this recommendation to be *moderate*. We found 13 studies conducted with students classified as learning disabled or low achieving that met WWC standards or met standards with reservations.<sup>249</sup> Four in particular examined the impact of tier 2 interventions against regular tier 1 instruction.<sup>250</sup> Appendix table D3 (p. 77) provides an overview of these 13 studies. Note that in an attempt to acknowledge meaningful effects regardless of sample size, the panel followed WWC guidelines and considered a positive statistically significant effect, or an effect size greater than 0.25, as an indicator of positive effects.<sup>251</sup>

**Summary of evidence**

The representations in 11 of the 13 studies were used mainly to teach word problems and concepts (fractions and prealgebra).<sup>252</sup>

In one study, visual representations were used to teach mathematics facts.<sup>253</sup> In all 13 studies, representations were used to understand the information presented in the problem. Specifically, the representations helped answer such questions as what type of problem it is and what operation is required. In all 13 studies, visual representations were part of a complex multicomponent instructional intervention. Therefore, it is not possible to ascertain the role and impact of the representation component.

Of the 13 studies, 4 used visual representations, such as drawings or other forms of pictorial representations, to scaffold learning and pave the way for the understanding of the abstract version of the representation.<sup>254</sup> Jitendra et al. (1998) examined the differential effects of two instructional strategies, an explicit strategy using visual representations and a traditional basal strategy. Students were taught explicitly to identify and differentiate among word problems types and map the features of the problem onto the given diagrams specific to each problem type. The intervention demonstrated a nonsignificant substantively important positive effect. Wilson and Sindelar (1991) used a diagram to teach students the “big number” rule (e.g., when a big number is given, subtract) (ES = .82~). Woodward (2006) explored the use of visuals such as a number line to help students understand what an abstract fact such as  $6 \times 7 = \underline{\quad}$  meant. The study yielded a substantively important positive effect on mathematics facts, and a positive and marginally significant average effect on operations.

249. Artus and Dyrek (1989); Darch, Carnine, and Gersten (1984); Fuchs, Powell et al. (2008); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008); Fuchs et al. (2005); Jitendra et al. (1998); Butler et al. (2003); Walker and Poteet (1989); Wilson and Sindelar (1991); Witzel, Mercer, and Miller (2003); Witzel (2005); Woodward (2006).

250. Fuchs, Powell et al. (2008); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008); Fuchs et al. (2005).

251. For more details on WWC guidelines for substantively important effects, see the *What Works Clearinghouse Procedures and Standards Handbook* (WWC, 2008).

252. Jitendra et al. (1998); Butler et al. (2003); Witzel (2005); Darch, Carnine, and Gersten (1984); Fuchs, Powell et al. (2008); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008);

Fuchs et al. (2005); Walker and Poteet (1989); Wilson and Sindelar (1991); Witzel, Mercer, and Miller (2003).

253. Woodward (2006).

254. Jitendra et al. (1998); Walker and Poteet (1989); Wilson and Sindelar (1991); Woodward (2006).

Three studies used manipulatives in the early stages of instruction to reinforce understanding of basic concepts and operations.<sup>255</sup> For example, Darch et al. (1984) used concrete models such as groups of boxes to teach rules for multiplication problems. Similarly, Fuchs, Fuchs, Craddock et al. (2008) used manipulatives in their tutoring sessions to target and teach the most difficult concepts observed in the classroom. In another study, Fuchs, Seethaler et al. (2008) used concrete materials and role playing to help students understand the underlying mathematical structure of each problem type. In all these studies, manipulatives were one aspect of a complex instructional package. The studies resulted in mostly significant positive domain average effect sizes in the range of .60 to 1.79.

In six studies, both concrete and visual representations were used to promote mathematical understanding.<sup>256</sup> For example, Artus and Dyrek (1989) used concrete objects (toy cars, empty food wrappers) and visuals (drawings) to help students understand the story content, action, and operation in the word problems ( $ES = .87$ ). Likewise, Fuchs, Powell et al. (2008) used manipulatives in the initial stages and later pictorial representations of ones and tens in their software program ( $ES = .55$ , n.s.). However, in both studies, concrete objects and visual representations were not part of an instructional format that promoted systematic scaffolded learning. In other words, instruction did not include fading the manipulatives and visual representations to promote understanding of math at the more typical abstract level.

In the remaining four studies, manipulatives and visual representations were

presented to the students sequentially to promote scaffolded instruction.<sup>257</sup> This model of instruction, with its underpinning in Bruner's (1966) work, is referred to as a concrete to representation to abstract (CRA) method of instruction. The CRA method is a process by which students learn through the manipulation of concrete objects, then through visual representations of the concrete objects, and then by solving problems using abstract notation.<sup>258</sup> Fuchs et al. (2005) taught 1st grade students basic math concepts (e.g., place value) and operations initially using concrete objects, followed by pictorial representations of blocks, and finally at the abstract level (e.g.,  $2 + 3 = \underline{\quad}$ ) without the use of manipulatives or representations. Butler et al. (2003) examined the differential impact of using two types of scaffolded instruction for teaching fractions, one that initiated scaffolding at the concrete level (concrete-representation-abstract) and the other that started at the representation level (representation-abstract). Neither variation resulted in significant differences.

Witzel (2005) and Witzel, Mercer, and Miller (2003) investigated the effectiveness of the scaffolded instruction using the CRA method to teach prealgebra (e.g.,  $X - 4 = 6$ ) to low-achieving students and students with disabilities. Using an explicit instructional format, Witzel taught students initially using manipulatives such as cups and sticks. These were replaced with drawings of the same objects and finally faded to typical abstract problems using Arabic symbols (as seen in most textbooks and standardized exams). Both studies resulted in statistically significant or substantively important positive gains (Witzel, Mercer, and Miller, 2003 and  $ES = .83^*$ ; Witzel, 2005 and  $ES = .54$ , n.s.). One of these studies is described in more detail here.

255. Darch et al. (1984); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).

256. Artus and Dyrek (1989); Butler et al. (2003); Fuchs, Powell et al. (2008); Fuchs et al. (2005); Witzel (2005); Witzel, Mercer, and Miller (2003).

257. Butler et al. (2003); Fuchs et al. (2005); Witzel (2005); Witzel, Mercer, and Miller (2003).

258. Witzel (2005).

### A study of CRA instruction—Witzel, Mercer, and Miller (2003)

In 2003, Witzel, Mercer, and Miller published a study that investigated the effects of using the CRA method to teach prealgebra.<sup>259</sup> The participants in the study were teachers and students in 12 grade 6 and 7 classrooms in a southeastern urban county. Each teacher taught one of two math classes using CRA instruction (treatment group) and the other using abstract-only traditional methods (traditional instruction group). Of those participating, 34 students with disabilities<sup>260</sup> or at risk for algebra difficulty<sup>261</sup> in the treatment group were matched with 34 students with similar characteristics across the same teacher's classes in the traditional instruction group.

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259. Witzel, Mercer, and Miller (2003).

260. These students were identified through school services as those who needed additional support, had a 1.5 standard deviation discrepancy between ability and achievement, and had math goals listed in their individualized education plans.

261. These students met three criteria: performed below average in the classroom according to the teacher, scored below the 50th percentile in mathematics on the most recent statewide achievement test, and had socioeconomically disadvantaged backgrounds.

The students in both groups were taught to transform equations with single variables using a five-step 19-lesson sequence of algebra equations. In each session, the teacher introduced the lesson, modeled the new procedure, guided students through procedures, and began to have students working independently. For the treatment group, these four steps were used for instruction at the concrete, representational, and abstract stages of each concept. Teachers taught the concrete lessons using manipulative objects such as cups and sticks, the representational lessons using drawings of the same objects, and the abstract lessons using Arabic symbols. For the traditional instruction group, the teachers covered the same content for the same length of time (50 minutes), but the teachers used repeated abstract lessons rather than concrete objects and pictorial representations.

A 27-item test to measure knowledge on single-variable equations and solving for a single variable in multiple-variable equations was administered to the students one week before treatment (pretest), after the last day of the treatment (posttest), and three weeks after treatment ended (follow-up). The CRA intervention had a positive and significant effect on knowledge of the prealgebra concepts assessed.

**Table D3. Studies of interventions that used visual representations that met WWC standards (with and without reservations)**

Study	Comparison <sup>a</sup>	Grade level	Duration	Domain	Outcomes <sup>b</sup>
Artus and Dyrek (1989)	Instructional intervention using concrete objects and representational drawings versus traditional lecture format	4–6	90 minutes/ session; 6 sessions	Math general achievement	.87~
Butler et al. (2003)	Instructional intervention using concrete objects and representational drawings versus representational drawings only	6–8	45 minutes/ session; 10 sessions	Concepts	-.14 (n.s.)
Darch, Carmine, and Gersten (1984)	Instructional intervention using concrete objects versus traditional basal instruction	4	30 minutes/ session; 11 sessions	Word problems	.07 (n.s.)
Fuchs et al. (2005)	Instructional intervention using concrete objects and representational drawings versus no instruction condition	1	40 minutes/ session; 48 sessions	Word problems	1.79*
Fuchs, Seethaler et al. (2008)	Instructional intervention using concrete materials and role playing versus no instruction condition	3	20–30 minutes/ session; 36 sessions	Math general achievement	.34~
Fuchs, Powell et al. (2008)	Instructional intervention using concrete objects and pictorial representations versus no instruction condition	3	15–18 minutes/ session; 45 sessions	Word problems	.66~
Fuchs, Fuchs, Craddock et al. (2008)	Instructional intervention using concrete objects versus no instruction condition	3	20–30 minutes/ session; 38 sessions	Concepts	.60 (n.s.)
Jitendra et al. (1998)	Instructional intervention using diagrammatic representations versus traditional basal instruction	2–5	40–45 minutes/ session; 17–20 sessions	Operations	.55 (n.s.)
Walker and Poteet (1989)	Instructional intervention using diagrammatic representations versus traditional instruction	6–8	30 minutes/ session; 17 sessions	Transfer—story problems	-.07 (n.s.)
				Transfer—math concepts	.12 (n.s.)
				Word problems	.95*
				Transfer—word problems	.30*
				Word problems	.56 (n.s.)
				Transfer—word problems	1.01*
				Word problems	.35 (n.s.)

*(continued)*

**Table D3. Studies of interventions that used visual representations that met WWC standards (with and without reservations) (continued)**

Study	Comparison <sup>a</sup>	Grade level	Duration	Domain	Outcomes <sup>b</sup>
Wilson and Sindelar (1991)	Instructional intervention using diagrammatic representations versus instruction without diagrams	2-4	30 minutes/ session; 14 sessions	Word problems	.82~
Witzel (2005)	Instructional intervention using concrete objects and pictorial representations versus traditional instruction	6,7	50 minutes/ session; 19 sessions	Concepts (prealgebra)	.54 (n.s.)
Witzel, Mercer, and Miller (2003)	Instructional intervention using concrete objects and pictorial representations versus traditional instruction	6,7	50 minutes/ session; 19 sessions	Concepts (prealgebra)	.83*
Woodward (2006)	Instructional intervention using pictorial representations versus an intervention not using representations	4	25 minutes/ session; 20 sessions	Math facts	.55 (n.s.)
				Operations	.11~

a. Instructional interventions in all the studies listed were multicomponent in nature, with visuals being one of those components.

b. Outcomes are reported as effect sizes. For a p-value < .05, the effect size is significant (\*), for a p-value < .10, the effect size is marginally significant (~); for a p-value ≥ .10, the effect size is not significant (n.s.).

Source: Authors' analysis based on studies in table.

**Recommendation 6.  
Interventions at all grade levels  
should devote about 10 minutes  
in each session to building fluent  
retrieval of basic arithmetic facts.**

**Level of evidence: Moderate**

The panel judged the level of evidence supporting the recommendation to be *moderate*. We found seven studies conducted with low-achieving or learning disabled students between grades 1 and 4 that met WWC standards or met standards with reservations and included fact fluency instruction in an intervention.<sup>262</sup> Appendix table D4 (p. 83) provides an overview of the studies and indicates whether fact fluency was the core content of the intervention or a component of a larger intervention. The relevant treatment and comparison groups in each study and the outcomes for each domain are included in the table. Grade level, typical session length, and duration of the intervention are also in the table.

Given the number of high-quality randomized and quasi-experimental design studies conducted across grade levels and diverse student populations that include instruction in fact fluency as either an intervention or a component of an intervention, the frequency of small but substantively important or significant positive effects on measures of fact fluency and mathematical operations (effect sizes ranged from .11 to 2.21), and the fact that numerous research teams independently produced similar findings, the panel concluded that there is *moderate* evidence to support the recommendation to provide instruction in fact fluency for both tier 2 and tier 3 mathematics interventions across grade levels. The panel

acknowledges that the broader implication that general mathematics proficiency will improve when fact fluency improves is the opinion of the panel.

**Summary of evidence**

The panel recognizes the importance of knowledge of basic facts (addition, subtraction, multiplication, and division) for students in kindergarten through grade 4 and beyond. Two studies examined the effects of teaching mathematics facts relative to the effects of teaching spelling or word identification using similar methods.<sup>263</sup> In both studies, the mathematics facts group demonstrated substantively important or statistically significant positive gains in facts fluency relative to the comparison group, although the effects were significant in only one of these two studies.<sup>264</sup>

Another two interventions included a facts fluency component in combination with a larger tier 2 intervention.<sup>265</sup> For example, in the Fuchs et al. (2005) study, the final 10 minutes of a 40-minute intervention session were dedicated to practice with addition and subtraction facts. In both studies, tier 2 interventions were compared against typical tier 1 classroom instruction. In each study, the effects on mathematics facts were not significant. Significant positive effects were detected in both studies in the domain of operations, and the fact fluency component may have been a factor in improving students' operational abilities.

*Relationships among facts.* The panel suggests emphasizing relationships among basic facts, and five of the studies examined

262. Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Seethaler et al. (2008); Fuchs et al. (2005). Bernie-Smith (1991); Fuchs, Powell et al. (2008); Tournaki (2003); Woodward (2006).

263. Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Powell et al. (2008).

264. In Fuchs, Fuchs, Hamlett et al. (2006), the effects on addition fluency were positive while there was no effect on subtraction fluency.

265. Fuchs, Seethaler et al. (2008); Fuchs et al. (2005).

exemplify this practice.<sup>266</sup> In Woodward (2006), the Integrated Strategies group was specifically taught the connection between single-digit facts (e.g.,  $4 \times 2 = \underline{\quad}$ ) and extended facts ( $40 \times 2 = \underline{\quad}$ ). In Fuchs et al. (2005, 2006c, 2008e), mathematics facts are presented in number families (e.g.,  $1 + 2 = 3$  and  $3 - 2 = 1$ ). Bernie-Smith (1991) examined the effects of a counting up/on procedure that highlighted the relationship between facts versus a rote memorization method that did not highlight this relationship. There was a substantively important nonsignificant positive effect in favor of the group that was taught the relationship between facts. Note that fact relationships were not isolated as independent variables in this study.

*Materials to teach math facts.* The studies used a variety of materials to teach mathematics facts. Woodward (2006) used worksheets, number lines, and arrays of blocks projected on overheads to help students visualize fact strategies. Tournaki (2003) also used worksheets. Three studies included flash cards.<sup>267</sup> For example, the Fuchs, Seethaler et al. (2008) intervention included flash cards with individual addition and subtraction problems on each card. Students had up to two minutes to respond to as many cards as they could, and they were provided with corrective feedback on up to five errors each session.

Three studies included computer assisted instruction to teach mathematics facts.<sup>268</sup> In all three interventions, students used a computer program designed to teach addition and subtraction facts. In this program, a mathematics fact was presented briefly on the computer screen. When the fact dis-

appeared, the student typed the fact. If the student made an error, the correct fact was displayed with accompanying audio, and the student had the opportunity to type the fact again. For two of the studies, the duration of the presentation on the screen was tailored to the student's performance (with less time as the student gained proficiency) and the difficulty of facts increased as competency increased.<sup>269</sup>

*Time.* The panel advocates dedicating about 10 minutes a session to building fact fluency in addition to the time dedicated to tier 2 and tier 3 interventions. The seven studies supporting this recommendation dedicated a minimum of 10 minutes a session to fact fluency activities.

*Explicit teaching strategies for building fact fluency.* Another three studies in the evidence base address methods for teaching basic facts to students by comparing instructional approaches.<sup>270</sup> Both Bernie-Smith (1991) and Tournaki (2003) investigated the effects of being taught a counting up/on strategy relative to a rote memorization procedure for promoting fact fluency. In the Bernie-Smith (1991) study, perhaps not surprisingly as both interventions were taught to enhance fact fluency, the addition facts competency of students in both groups improved. However, there was a substantively important nonsignificant positive effect in favor of the counting-on group when the two groups were compared. In the Tournaki (2003) study, the latency of responses on a fact fluency posttest decreased<sup>271</sup> while the accuracy of posttest responses significantly increased

266. Bernie-Smith (1991); Fuchs et al. (2005); Fuchs, Fuchs, Hamlett et al. (2006), Fuchs, Seethaler et al. (2008); Woodward (2006).

267. Bernie-Smith (1991); Fuchs, Seethaler et al. (2008); Fuchs, Powell et al. (2008).

268. Fuchs et al. (2005); Fuchs, Fuchs, Hamlett et al. (2006); Fuchs, Powell et al. (2008).

269. Fuchs et al. (2005); Fuchs, Fuchs, Hamlett et al. (2006).

270. Bernie-Smith (1991); Tournaki (2003); Woodward (2006).

271. The latency decrease was marginally significant. A decrease in latency indicates that students in the counting-on group were answering fact problems more quickly than students in the rote memorization group.

for the counting-on group relative to the rote memorization group.

Similarly, Woodward (2006) examined an integrated approach that combined instruction in strategies, visual representations, and timed practice drills (the Integrated Strategies group) versus a traditional timed drill and practice approach for building multiplication fact fluency (the Timed-Practice Only group). In the Integrated Strategies group, difficult facts were taught through derived fact strategies or doubling and doubling again strategies. When WWC multiple comparison adjustments were applied to outcomes, none of the multiplication fact outcomes were significant, though effects were substantively important and positive in favor of the integrated approach.<sup>272</sup> The operations domain showed mixed effects with approximation scores in favor of the integrated approach and operations scores in favor of the Timed-Practice Only group.

In summary, the evidence demonstrates substantively important or statistically significant positive effects for including fact fluency activities as either stand-alone interventions or components of larger tier 2 interventions. However, because these effects did not consistently reach statistical significance, the panel is cautious and acknowledges that the level of evidence for this recommendation is moderate. There is also evidence that strategy-based instruction for fact fluency (e.g., teaching the counting-on procedure) is a superior approach over rote memorization. Further, many of the studies included here taught the relationships among facts, used a variety of materials such as flash cards and computer assisted instruction, and taught math facts for a minimum of 10 minutes a

272. When a study examines many outcomes or findings simultaneously, the statistical significance of findings may be overstated. The WWC makes a multiple comparison adjustment to prevent drawing false conclusions about the number of statistically significant effects (WWC, 2008).

session. Although these components of the interventions were most often not independent variables in the studies, they are all advocated by the panel. An example of a study investigating the effects of a fact fluency intervention is detailed here.

#### A study of a fact fluency intervention—[Fuchs, Powell, Hamlett, and Fuchs \(2008\)](#).

This study was conducted with 127 students in grade 3 classrooms in Tennessee and Texas.<sup>273</sup> The students were all identified as having either math difficulties or math and reading difficulties.

Before the intervention, the students completed several pretest assessments. The assessment that related to fact retrieval consisted of one subtest with three sets of 25 addition fact problems and a second subtest with three sets of 25 subtraction fact problems. Students had one minute to write answers for each set within a subtest. Internal consistency for the sets ranged between .88 and .93. Scores on sets of items were combined into a single fact retrieval score. After the intervention, students completed this same fact retrieval assessment among a battery of posttests.

Students were randomly assigned to one of four groups (three mathematics instruction groups and a reading instruction comparison group). For this recommendation, we report only on the comparison between the Fact Retrieval group ( $n = 32$ ) and the Word Identification comparison group ( $n = 35$ ).<sup>274</sup> Students in both groups met in-

273. This study met standards with reservations because of high attrition. The sample initially included 165 students randomized to the conditions and 127 in the postattrition sample. The authors did demonstrate baseline equivalence of the postattrition sample.

274. The third group was Procedural/Estimation Tutoring, which targeted computation of two-digit numbers. The fourth group was a combination of Procedural/Estimation Tutoring and Fact Retrieval.



dividually with a tutor<sup>275</sup> for 15 to 18 minutes during three sessions each week for 15 weeks.

Sessions for the Fact Retrieval instruction group consisted of three activities. First, the students received computer assisted instruction (CAI). In the computer program, an addition or subtraction mathematics fact appeared on the screen for 1.3 seconds. When the fact disappeared, the student typed the fact using short-term memory. A number line illustrated the mathematics fact on the screen with colored boxes as the student typed. If the student typed the fact correctly, applause was heard, and the student was awarded points. Each time the student accumulated five points, animated prizes (e.g., a picture of a puppy) appeared in the student's animated "treasure chest." If the student typed the mathematics fact incorrectly, the fact reappeared and the student was prompted to type it again.

The second instructional activity, flash card practice, began after 7.5 minutes of CAI. Flash card practice with corrective feedback included two types of flash cards. The first set of flash cards depicted written facts without answers. Students were encouraged to answer as many problems

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275. There were 22 tutors. Some were masters or doctoral students. Most had teaching or tutoring experience.

as possible in two minutes. After three consecutive sessions with a minimum of 35 correct responses, the student was presented with a second set of flash cards that contained a number line similar to the CAI number line. The student was asked to respond with the appropriate mathematics facts to accompany the number line for as many cards as possible within the time frame. Corrective feedback was provided for a maximum of five errors per flash card activity. The third activity during Fact Retrieval instruction focused on cumulative review. Students were allotted two minutes to complete 15 mathematics fact problems using paper and pencil.

Students in the Word Identification comparison group received computer assisted instruction and participated in repeated reading with corrective feedback during their sessions. The content was tailored to the student's reading competency level as determined by a pretest.

Results indicated significant positive effects on fact fluency in favor of the group that received fact retrieval instruction relative to the comparison group that received instruction in word identification. These results suggest that it is possible to teach struggling students mathematics facts in as small an amount of time as 45 minutes of instruction a week when using flash cards and CAI.

**Table D4. Studies of interventions that included fact fluency practices that met WWC standards (with and without reservations)**

Study	Comparison <sup>a</sup>	Intervention or component <sup>a</sup>	Grade level	Duration	Domain	Outcomes <sup>b</sup> Effect size
Bernie-Smith (1991)	Counting on method for learning facts versus rote memorization of facts	Intervention	1–5	30 minutes/session; 5 times/week; 4 weeks	Operations	.26 (n.s.)
Fuchs, Seethaler et al. (2008)	Multicomponent intervention (with one component on math facts) versus no instruction condition	Component	3	20–minutes/session (2 minutes on math facts); 3 times/week; 12 weeks	Fact fluency Operations	.33 (n.s.) .68*
Fuchs, Fuchs, Hamlett et al. (2006)	Intervention in math facts versus non relevant instruction	Intervention	1	10 minutes/session; 3 sessions/week; 18 weeks	Fact fluency	.34 (n.s.)
Fuchs et al. (2005)	Multicomponent intervention (with one component on math facts) versus no instruction condition	Component	1	40 minutes/session (10 minutes on math facts); 3 times/week; 16 weeks	Operations Fact fluency	.40* .19 (n.s.)
Fuchs, Powell et al. (2008)	Intervention in math facts versus nonrelevant instruction	Intervention	3	15–18 minutes/session; 3 sessions/week; 15 weeks	Fact fluency	.60*
Tournaki (2003)	Counting-on strategy for learning math facts versus drill and practice of math facts	Intervention	2	15 minutes/session, 8 sessions over consecutive school days	Fact fluency Operations Transfer	.71~ 2.21* 1.10*
Woodward (2006)	Strategies for learning facts plus timed math fact drills versus timed math fact drills only	Intervention	4	25 minutes/session, 5 sessions/week; 4 weeks	Fact fluency Operations	.55 (n.s.) .11~

a. Intervention means that the entire intervention was on math facts. Component means that math facts were just one part of the intervention.

b. Outcomes are reported as effect sizes. For a p-value < .05, the effect size is significant (\*); for a p-value < .10, the effect size is marginally significant (~); for a p-value ≥ .10, the effect size is not significant (n.s.).

Source: Authors' analysis based on studies in table.

## **Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.**

### **Level of evidence: Low**

The panel rated the level of evidence for this recommendation as *low*. The panel relied on the standards of the American Psychological Association, the American Educational Research Association, and the National Council on Measurement Education<sup>276</sup> for valid assessment instruments, along with expert judgment, to evaluate the quality of the individual studies and to determine the overall level of evidence for this recommendation.

Evidence for the recommendation included research studies on mathematics progress monitoring,<sup>277</sup> summary reviews of mathematics progress monitoring research,<sup>278</sup> and summary information provided by the Research Institute on Progress Monitoring<sup>279</sup> and the National Center on Progress Monitoring.<sup>280</sup> Very little research evidence specifically addresses the use of mathematics progress monitoring data within the context of RtI.

Most research on mathematics progress monitoring measures falls into two categories. One group of studies examines the technical adequacy of the measures, including their reliability, validity, and sensitivity to growth. The second investigates teacher use of the measures to modify instruction for individual students in order to enhance achievement; the bulk of

this second body of research has been conducted primarily in special education settings and therefore is less relevant to the RtI focus of this practice guide. As a result, we focus on the technical adequacy studies in this appendix. Note that because similar and often identical measures are used for screening and progress monitoring, many of the studies reviewed here overlap with those discussed for recommendation 1 on screening. The same measure may be used as both a screening measure and a progress monitoring measure; however, the psychometric properties of these measures are more firmly established when used as screening measures with fewer researchers investigating the function of the measures for modeling growth when used for progress monitoring. This disparity in the research base leads to the panel assigning a *moderate* level of evidence to Recommendation 1 and a *low* level of evidence to Recommendation 7.

The technical adequacy studies of mathematics progress monitoring measures were not experimental; the researchers typically used correlational techniques to evaluate the reliability and criterion validity of the measures and regression methods to examine sensitivity to growth. If progress monitoring measures are to be deemed trustworthy, relevant empirical evidence includes data on reliability, concurrent criterion validity, and sensitivity to growth. Evidence of reliability generally includes data on inter-scorer agreement,<sup>281</sup> internal consistency,<sup>282</sup> test-retest reliability,<sup>283</sup> and alternate form reliability.<sup>284</sup> Evidence of concurrent criterion validity is gathered by examining relations between scores on the progress monitoring measures and other indica-

276. American Educational Research Association, American Psychological Association, and National Council on Measurement in Education (1999).

277. Clarke et al. (2008); Foegen and Deno (2001); Fuchs et al. (1993); Fuchs, Fuchs, Hamlett, Thompson et al. (1994); Leh et al. (2007); Lembke et al. (2008).

278. Foegen, Jiban, and Deno (2007).

279. [www.progressmonitoring.net/](http://www.progressmonitoring.net/).

280. [www.studentprogress.org/](http://www.studentprogress.org/).

281. For example, Fuchs, Fuchs, Hamlett, Thompson et al. (1994).

282. For example, Jitendra, Sczesniak, and Deatline-Buchman (2005).

283. For example, Clarke and Shinn (2004).

284. VanDerHeyden et al. (2001).

tors of proficiency in mathematics. Common criterion measures include scores on group and individual standardized tests of mathematics, course grades, and teacher ratings.<sup>285</sup>

Although issues of reliability and criterion validity are common to both screening and progress monitoring measures, a critical feature specific to progress monitoring is that the measures be sensitive to growth. If teachers are to use progress monitoring measures to evaluate the effects of instruction on student learning, researchers must provide evidence that student scores on the measures change over time, thus providing an indication of their learning. Most research studies have examined sensitivity to growth by administering parallel forms of a measure across a period of several weeks or months. In some studies, students receive typical instruction, and in others, teachers adapt and refine the instruction in response to the progress monitoring data (often in special education contexts). In either case, evidence of sensitivity to growth typically involves computing regression equations to determine slopes of improvement and reporting these as mean weekly growth rates for a group of students. As an example, if a progress monitoring measure has a mean weekly growth rate of .5, teachers could expect that, on average, a student's score would increase by 1 point every two weeks. Growth rates reported in the literature vary considerably across measures and grade levels; no established standards exist for acceptable rates of student growth under typical instruction.

We discuss the evidence for measures used across the elementary and middle school grades and conclude with a more in-depth example of a technical adequacy study of mathematics progress monitoring measures.

285. For example, Foegen and Deno (2001); Fuchs et al. (2003a); Chard et al. (2005).

## Summary of evidence

*Progress monitoring in the primary grades.* Measures for the primary grades typically reflect aspects of number sense, including strategic counting, numeral identification, and magnitude comparison. Among the studies examining sensitivity to growth in the early grades, researchers have relied on separate measures for each of the different aspects of numeracy.<sup>286</sup> Other researchers have combined individual measures to create composite scores<sup>287</sup> or used more comprehensive multiskill measures.<sup>288</sup> But so far, the focus of these studies has been on screening rather than on progress monitoring. Reliability coefficients for these measures generally exceed .85. Concurrent criterion validity coefficients with standardized achievement tests are generally in the .5 to .7 range.<sup>289</sup> Mean rates of weekly growth reported in the literature vary widely, ranging from .1 to .3<sup>290</sup> problems a week to .2 to more than 1.0 problems.<sup>291</sup>

*Progress monitoring in the elementary grades.* Two types of measures have been investigated for monitoring the mathematics learning of students in the elementary grades. The bulk of the research, conducted by a research group led by Dr. Lynn Fuchs, investigates the characteristics of general outcome measures that represent grade-level mathematics curricula in computation and in mathematics concepts and applications.<sup>292</sup> These measures were developed in the late 1980s and early 1990s,

286. For example, Clarke et al. (2008); Lembke et al. (2008).

287. Bryant, Bryant, Gersten, Scammacca, and Chavez (2008).

288. Fuchs, Fuchs, Compton et al. (2007).

289. Chard et al. (2005); Clarke and Shinn (2004); Clarke et al. (2008); Lembke et al. (2008).

290. Lembke et al. (2008).

291. Chard et al. (2005).

292. Fuchs and Fuchs (1998); Fuchs, Hamlett, and Fuchs (1998).

reflecting the Tennessee state elementary mathematics curriculum of that time. The measures continue to be widely used and are recommended by the National Center for Student Progress Monitoring.<sup>293</sup> Teachers should carefully examine the content of the measures to ensure that they are representative of the existing mathematics curricula in their states and districts.

The second type of measure is not broadly representative of the instructional curriculum as a whole, but instead serves as an indicator of general proficiency in mathematics. Examples of such measures include basic facts (number combinations)<sup>294</sup> and word problem solving.<sup>295</sup> Because the general outcome measures are representative of the broader curriculum, they offer teachers more diagnostic information about student performance in multiple aspects of mathematics competence; this advantage is often gained by using measures that are longer and require more administration time. The indicator measures are more efficient to administer for regular progress monitoring but may be as useful for diagnostic purposes.

Evidence of the reliability of the measures is generally strong, with correlation coefficients above .8, except for the word problem-solving measures developed by Jitendra's research team, which are slightly lower.<sup>296</sup> Concurrent criterion validity measures have included group and individual achievement tests. Validity correlation coefficients range widely across measure types and grade levels. At the lower end, Espin et al. (1989) found correlations between the Wide Range Achievement Test and basic fact measures in the .3 to .5 range for basic

facts. In contrast, Fuchs, Hamlett, and Fuchs (1998) found correlations between the Stanford Achievement Test Math Computation subtest<sup>297</sup> and general outcome measures of computation to range from .5 to .9 across grades 2 through 5. In general, concurrent criterion validity coefficients for elementary mathematics progress monitoring measures are in the .5 to .6 range.

Evidence of sensitivity to growth for elementary measures exists for the computation and concepts/applications measures developed by Fuchs and for the word problem-solving measures developed by Jitendra. Mean growth rates for the Fuchs measures range from .25 to .70. A study by Shapiro and colleagues,<sup>298</sup> using the same measures for students with disabilities, resulted in mean growth rates of .38 points per week for both types of measures. Mean weekly growth rates for the Jitendra measures were .24 points per week.

*Progress monitoring in middle school.* Less evidence is available to support progress monitoring in middle school.<sup>299</sup> Research teams have developed measures focusing on math concepts typically taught in middle school,<sup>300</sup> basic facts<sup>301</sup> and estimation.<sup>302</sup> Criterion validity across the types of measures varies, but the majority of correlations coefficients fall in the .4 to .5 range. Helwig and colleagues<sup>303</sup> found higher correlation coefficients with high-stakes state tests in the range of .6 to .8. Reliability estimates including alternate form, inter-rater, and test-retest were all of sufficient quality. Greater rates of growth were found for the non-concept-based measures with rates around .25 units per

293. [www.studentprogress.org](http://www.studentprogress.org).

294. Espin et al. (1989); VanDerHeyden, Witt, and Naquin (2003).

295. Jitendra, Sczesniak, and Deatline-Buchman (2005); Leh et al. (2007).

296. Jitendra, Sczesniak, and Deatline-Buchman (2005); Leh et al. (2007).

297. Gardner, Rudman, Karlsen, and Merwin (1982).

298. Shapiro, Edwards, and Zigmond (2005).

299. Foegen (2008).

300. Helwig, Anderson, and Tindal (2002).

301. Espin et al. (1989).

302. Foegen and Deno (2001); Foegen (2000).

303. Helwig, Anderson, and Tindal (2002).

week. A recent study<sup>304</sup> compared these measure types with two grade 6 measures<sup>305</sup> similar to the measures described above assessing student understanding of operations and concepts for their grade level. In this case, middle school students in grades 6, 7, and 8 were assessed using multiple measures. Evidence was found that even into grades 7 and 8, using grade 6 measures focusing on operations and mathematical concepts still shows reliability, validity, and sensitivity to growth.

### An example of a study of the technical adequacy of mathematics progress monitoring measures—Fuchs, Fuchs, Hamlett, Thompson, Roberts, Kupek, and Stecker (1994)

A study conducted by Fuchs, Fuchs, Hamlett, Thompson, Roberts, Kupek, and Stecker (1994) illustrates the type of technical adequacy evidence that education professionals can use when evaluating and selecting mathematics progress monitoring measures. The research team examined the technical features of grade-level general outcome measures of mathematics concepts and applications. The measures were developed by analyzing the Tennessee mathematics curriculum at grades 2 through 6 to identify critical objectives essential for mathematics proficiency at each grade level. The researchers created 30 alternate forms at each grade level and conducted numerous pilot tests to refine the items and determine appropriate time limits for administration.

A total of 140 students in grades 2 through 4 participated in the study, completing weekly versions of the measures for 20 weeks. All students were enrolled in general education classrooms; about 8 percent of the students had been identified as having learning disabilities. The students' general education

teachers administered the measures using standardized procedures, including administration time limits of 6 to 8 minutes, depending on grade level.

The results of the study illustrate the types of data educators should consider to determine if a mathematics progress monitoring measure is trustworthy. The authors report evidence of the reliability of the concepts and application measures by describing internal consistency coefficients for students at each grade level (which ranged from .94 to .98). Concurrent criterion validity was examined by computing correlations between student scores on the concepts and applications general outcome measures and their scores on three subscales of the Comprehensive Test of Basic Skills (Computation, Concepts and Applications, and Total Math Battery). Results are reported for each subscale at each of the three grade levels, with coefficients ranging from .63 to .81. Considering these results in the general context of mathematics progress monitoring measures summarized above, teachers could feel confident that the concepts and applications measures demonstrated strong levels of reliability and criterion validity in this study.

A final consideration is the degree to which the measures are sensitive to student growth. To explore this feature, the researchers completed a least-squares regression analysis between calendar days and scores on the progress monitoring measures; the scores were then converted to represent weekly rates of improvement. The results ranged from an average increase of .40 problems per week in grade 2 to .69 in grade 4. Together with the evidence of reliability and criterion validity, the mean growth rate data suggest that teachers can have confidence that students will show improvements in their scores on the measures as their mathematics learning progresses.

304. Foegen (2008).

305. Fuchs, Hamlett, and Fuchs (1998); Fuchs et al. (1999).

One factor not evident in the technical adequacy data in this study, but critical for teachers to consider, is the alignment between the general outcome measure, the instructional curriculum, and expected learning outcomes. This study produced strong technical adequacy data when it was conducted in the early 1990s. Teachers considering alternative mathematics progress monitoring measures to represent the instructional curriculum are advised to review the content of these measures in light of current learning expectations for students at each grade level. Given changes in mathematics curricula over the past 10 to 15 years, it is important to evaluate the degree to which the measures continue to represent important mathematics outcomes.

**Recommendation 8. Include motivational strategies in tier 2 and tier 3 interventions.**

**Level of evidence: Low**

The panel judged the level of evidence supporting this recommendation to be *low*. The panel found nine studies<sup>306</sup> conducted with low-achieving or learning disabled students between grades 1 and 8 that met WWC standards or met standards with reservations and included motivational strategies in an intervention. However, because only two of these studies investigated a motivational strategy in a tier 2 or tier 3 mathematics intervention as an independent variable, the panel concluded that there is *low* evidence to support the recommendation. The panel recommends this practice for students in tier 2 and tier 3 based both on our opinion

and on the limited evidence base.<sup>307</sup> The evidence base is described below.

**Summary of evidence**

*Reinforce effort.* The panel advocates reinforcing or praising students for their *effort*. Schunk and Cox (1986) examined the effects of providing effort-attributational feedback (e.g., “You’ve been working hard”) during subtraction instruction versus no effort feedback and found significant positive effects on subtraction posttests in favor of providing effort feedback. This study, described in greater detail below, was one of two studies in the evidence base that examined a motivational strategy as an independent variable.

*Reinforce engagement.* The panel also recommends reinforcing students for attending to and being engaged in lessons. In two of the studies, students received “points” for engagement and attentiveness as well as for accuracy.<sup>308</sup> Accumulated points could be applied toward “purchasing” tangible reinforcers. It is not possible to isolate the effects of reinforcing attentiveness in the studies. In Fuchs et al. (2005), both the treatment and comparison groups received reinforcement, and in Fuchs, Fuchs, Craddock et al. (2008), the contrast between reinforcement and no reinforcement was not reported. But the presence of reinforcers for attention and engagement in these two studies echoes the panel’s contention that providing reinforcement for attention is particularly important for students who are struggling.

306. Fuchs et al. (2005); Fuchs, Fuchs, Craddock et al. (2008); Fuchs, Seethaler et al. (2008); Artus and Dyrek (1989); Fuchs et al. (2003b); Fuchs, Fuchs, Hamlett, Phillips et al. (1994); Fuchs, Fuchs, Finelli et al. (2006); Heller and Fantuzzo (1993); Schunk and Cox (1986).

307. The scope of this practice guide limited the evidence base for this recommendation to studies that investigated mathematics interventions for students with mathematics difficulties and included motivational components. There is an extensive literature on motivational strategies outside the scope of this practice guide, and the panel acknowledges that there is considerable debate in that literature on the use of rewards as reinforcers.

308. Fuchs et al. (2005); Fuchs, Fuchs, Craddock et al. (2008).

*Consider rewarding accomplishments.* The panel recommends that interventionists consider using rewards to acknowledge accurate work and possibly notifying parents when students demonstrate gains. In three of the studies, students were provided prizes as tangible reinforcers for accurate mathematics problem solving.<sup>309</sup> In both Fuchs et al. (2005) and Fuchs, Seethaler et al. (2008), students in tier 2 tutoring earned prizes for accuracy. In both studies, the tier 2 intervention group demonstrated substantively important positive and sometimes significant gains relative to the students who remained in tier 1. But it is not possible to isolate the effects of the reinforcers from the provision of tier 2 tutoring. In Fuchs, Fuchs, Craddock et al. (2008), the authors note that the provision of “dollars” that could be exchanged for prizes was more effective than rewarding students with stickers alone. Because this was not the primary purpose of the study, the reporting of the evidence for that finding was not complete; therefore, a WWC review was not conducted for that finding.

In a fourth study, Heller and Fantuzzo (1993) examined the impacts of a parental involvement supplement to a mathematics intervention. The parental involvement component included parents providing rewards for student success as well as parental involvement in the classroom. The performance of students who received the parental involvement component in addition to the school-based intervention significantly exceeded the performance of students in only the school-based intervention. Because the parental involvement component was multifaceted, it is not possible to attribute the statistically significant positive effects to rewards alone.

*Allow students to chart their progress and to set goals for improvement.* Five studies included interventions in which students

graphed their progress and in some cases set goals for improvement on future assessments.<sup>310</sup> One experimental study examined the effects of student graphing and goal setting as an independent variable and found substantively important positive nonsignificant effects in favor of students who graphed and set goals.<sup>311</sup> In two studies, the interventions included graphing in both groups being compared; therefore, it was not possible to isolate the effects of this practice.<sup>312</sup> In another two studies, students in the treatment groups graphed their progress as one component of multicomponent interventions.<sup>313</sup> Although it is not possible to discern the effect of graphing alone, in Artus and Dyrek (1989), the treatment group made marginally significant gains over the comparison group on a general mathematics assessment, and in Fuchs, Seethaler et al. (2008), there were substantively important positive non-significant effects on fact retrieval in favor of the treatment group.

In summary, the evidence base for motivational components in studies of students struggling with mathematics is limited. One study that met evidence standards demonstrated benefits for praising struggling students for their effort. Other studies included reinforcement for attention, engagement, and accuracy. Because the effects of these practices were not examined as independent variables, no inferences can be drawn about effectiveness based on these studies. Because this recommendation is based primarily on the opinion of the panel, the level of evidence is identified as *low*.

310. Artus and Dyrek (1989); Fuchs, Seethaler et al. (2008); Fuchs et al. (2003b); Fuchs, Fuchs, Hamlett, Phillips et al. (1994); Fuchs, Fuchs, Finelli et al. (2006).

311. Fuchs et al. (2003b).

312. Fuchs, Fuchs, Hamlett, Phillips et al. (1994); Fuchs, Fuchs, Finelli et al. (2006).

313. Artus and Dyrek (1989); Fuchs, Seethaler et al. (2008).

309. Fuchs et al. (2005); Fuchs, Seethaler et al. (2008); Fuchs, Fuchs, Craddock et al. (2008).



### An example of a study that investigated a motivational component—Schunk and Cox (1986).

This study was conducted with 90 students in grades 6 through 8 classrooms in six schools in Texas. The mean age of the students was 13 years and 7 months, and all students were identified as having learning disabilities in mathematics.

Before the intervention, the students completed a pretest assessment that consisted of 25 subtraction problems that required regrouping operations. After the intervention, a similar assessment of 25 subtraction problems was completed as a post-test. A separate reliability assessment demonstrated that the two forms of the subtraction assessment were highly correlated ( $r = .82$ ).

Students were stratified by gender and school and then randomly assigned to one of nine experimental groups. In all groups, the students received instruction for solving subtraction problems in 45-minute sessions conducted over six consecutive school days. For this recommendation,

we report only on the comparison between three groups. One group ( $n = 30$ ) received effort feedback in addition to performance feedback during the first three sessions. Another group ( $n = 30$ ) received effort feedback in addition to performance feedback during the last three sessions. A third group ( $n = 30$ ) did not receive effort feedback (received only performance feedback).<sup>314</sup> Effort feedback consisted of the proctor commenting to the student, “You’ve been working hard.” Students in both effort feedback groups received 15 statements of effort feedback across the entire intervention.

Results indicated significant positive effects for effort feedback relative to the comparison group regardless of when the student received the effort feedback. These results suggest that effort feedback is beneficial for learning disabled students who may not otherwise recognize the causal link between effort and outcomes.

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314. Other group distinctions were related to student verbalization and are described in the discussion of recommendation 3 (explicit instruction).

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