Information, Institutions and, Constitutional Arrangements

INFORMATION, INSTITUTIONS AND CONSTITUTIONAL ARRANGEMENTS

ABHINAY MUTHOO AND KENNETH A. SHEPSLE

ABSTRACT. This paper develops a theory of optimal institutional structure for staggered-term (overlapping generations) organizations such as legislative bodies. Our model is a simple stochastic game of multi-principal, multi-agent dynamic relationships. Our results emphasize two key features that are determined by legislative founders at the "constitutional moment". First, they will agree to institute a mechanism that endows (imperfectly informed) legislators with information about the history of play. Second, we provide conditions in which legislative founders will be indifferent to the structure of legislative procedures.
JEL Classification Numbers: D72, D78, H11.

1. INTRODUCTION

1.1. **Motivation and overview.** Imagine two agents, each employees of separate divisions of a firm, who interact each period to allocate a surplus to be shared by their respective divisions. The subordinates are hired by their respective divisions for two periods at a fixed salary. Each subordinate's performance in the surplus allocation task in the two periods is then assessed, and he is either reappointed or fired. One subordinate, however, was hired in period $t-1$ and the other in period t. The former, therefore, comes up for renewal at the end of period t, while the latter does not face a renewal decision until the end of period $t+1$. Thus, the agents are of different "types" — one EARLY (and thus not up for renewal this period) and one LATE (up for renewal this period). The types are determined by time-varying characteristics, not immutable properties of the individual agents.

Many organizations possess this *staggered-term* feature. In the present paper we will use as a running example a prominent class of political organization — staggered-term legislatures. Upper chambers of many national and provincial legislatures

Date: July 28, 2009.
Acknowledgements: For their helpful comments and suggestions, we are grateful to the editor, William F. Shughart II, an associate editor and an anonymous referee. We also thank for their comments Daron Acemoglu, David Baron, Tim Besley, V. Bhaskar, Ernesto Dal Bo, Rui de Figueiredo, Avner Greif, Elhanan Helpman, Keith Krehbiel, Roger Myerson, Torsten Persson, Robert Powell, Ken Shotts, Guido Tabellini, Barry Weingast, Muhamet Yildiz, and seminar participants at Berkeley, Stanford, Harvard, and the Canadian Institute for Advanced Research. Shepsle thanks the Hoover Institution and the National Institute of Aging (RO1 AG021181) for support.

possess this feature. The United States Senate, for example, consists of 100 senators, each elected for six years and then eligible for reelection. They are partitioned into three approximately equal classes, with one class of senators coming up for (re)election every two years.

Strategic interaction among the agents is enriched in interesting ways by staggered terms. One complication that we emphasize is related to information. Agents come and go, as some fail to have their employment renewed, obscuring exactly who knows what about the past history of the organization and, in particular, about past patterns of strategic interaction. The ability of contemporaneous agents to condition their actions on this history is confounded as a result. Another complication has to do with the manner in which assessments are carried out by principals. If a supervisor or an electoral constituency assesses past performance in time-dependent ways — for example, weighing recent performance more heavily than performance farther in the past — then agents will have type-dependent motivations each period; the incentives of EARLY types will differ from those of LATE types.

This paper explores dynamic strategic interaction in principal-agent relationships. Equilibrium behavior is affected by the staggered timing of agent assessment, the time-dependent way in which principals assess performance, and the amount of information that agents in any period have about the history of play. After exploring the game among agents, we step back and analyze how the principals design the game in the first instance. The world we study is one in which neither principals nor agents can commit ex ante: principals cannot promise to reappoint agents when the latter come up for renewal, and agents cannot commit to one another to behave in particular ways over time.

In the next three sections we develop a baseline dynamic model involving two agents engaged each period in a divide-the-cake exercise. Each agent has the same fixed term length and the same compensation per term, but a distinct start date and hence a distinct renewal date. Principals decide whether to renew or terminate their respective agents probabilistically on the basis of past performance in delivering cake to them. We examine two informational regimes: the default informational regime in which agents in each period have imperfect information about the history of play, and a richer informational regime (made possible by the creation of an appropriate institutionalized mechanism) through which agents are endowed with information about all the important bits of the history of play. The strategic interaction — the game form so to speak — is taken as exogenously fixed. Then, in section 5, we relax this restriction, inquiring how principals (the founding fathers), in a "constitutional moment," would determine the institutional structure for subsequent strategic interaction. We also explore the robustness of such constitutional decisions to renegotiation ("constitutional amendments").

In section 6 we show how our baseline model of two principals and their respective agents extends to multiple principals each with multiple agents and, in section 7, we briefly develop an analytical description of the US Senate as a quintessential staggered-term organization. We conclude in section 8 with a discussion of some of the main assumptions and features of our model, and a brief discussion of some directions for future research. Intuitions for our theoretical propositions are provided in the body of the paper, but proofs are placed in the appendix.

1.2. **Our contribution.** This paper develops a theory of optimal institutional structure for staggered-term, or overlapping generations (OLG), organizations such as legislative bodies (e.g., US Senate and Indian Rajya Sabha). In the process of doing so, we establish several results concerning the properties of equilibrium outcomes of certain kinds of multi-principal, multi-agent dynamic relationships. Our two main results characterize for legislatures (but also other organizations) the institutional structure that maximizes the joint expected payoffs of the principals (one from each electoral district) at the "constitutional moment" when the legislature is founded.

First, the principals will agree to institute a mechanism that endows (imperfectly informed) legislators in each period with all the information required about the history of play in the legislature. Transparency of agent-actions *to agents* (in order to enable agents to hold *each other* to account) is a key and robust feature of the principal-optimal institutional structure. This result and the model from which it is derived shows that transparency as a solution to moral hazard problems — the conventional interpretation in the literature — is not the only purpose it serves. In our model, transparency is the means by which long-term relationships among *agents* are sustained; indeed, it may not even matter whether strategic interaction among agents is transparent to their principals. Principals will institute a mechanism that endows legislators with information about the history of play, because otherwise (in the default informational regime of imperfect information) the equilibrium behaviour of agents is detrimental to the principals.[1]

[1]Formally, we show that with imperfect information about past actions, equilibrium behaviour is necessarily in Markov strategies. Given this key result, we then show that the unique equilibrium in such a world generates extremal consumption paths. For principals with concave preferences, however, relatively smoother consumption paths dominate. These are sustainable in equilibrium when a mechanism provides legislators with sufficient information about past actions to sustain intertemporal cooperation credibly. We show that this result holds for *any* allocations of agenda power, which establishes the central importance to the principals of instituting a mechanism that provides agents with transparency about the history of play.

Second, we show circumstances in which the principals are indifferent to particular details of institutional procedure. At the constitutional moment principals are concerned primarily with ensuring that legislators can hold each other to account, no more no less; they do not care about the sequence of bargaining, the allocation of proposal power, etc. There are, however, some circumstances in which principal-optimality entails a particular set of procedural rules in combination with the institutionalized mechanism that endows agents with information about history.

Another of our main results derives from the manner in which principals judge the performance of their respective agents, in particular a form of recency bias affecting this judgment. Equilibrium outcomes in the kind of multi-principal, multi-agent dynamic relationship that underlies our model display a strict "back loading" of cake which arises due to the principal's query, "What have you done for me lately?" The result implies an equilibrium electoral cycle.

We contribute to two main literatures, namely, the political agency and the legislative policy-making literatures (for a review of these literatures, see Persson and Tabellini 2000). The former was initiated by Barro (1973) and Ferejohn (1986), while the latter by Baron and Ferejohn (1989). Both literatures are now fairly large and still growing.[2]

2. THE BASELINE MODEL

2.1. **The structure.** We consider an infinitely lived legislative body consisting of two legislators, each elected from a separate electoral district. A legislator's term in office consists of two periods followed by the possibility of reelection — there are no term limits. Furthermore, legislators from the two districts have staggered terms of office. This means that at the end of each period only one legislator comes up for reelection.

The legislature is founded in period -1, when the "founding fathers" (two principals, one from each district) jointly determine the institutional structure of the legislature (such as its procedural rules and informational features) as described in subsection 2.5. At this constitutional moment, there are no legislators (agents) present.[3]

The legislature starts operating from period 0 onwards. In each period $t \in \{1, 2, 3, \ldots\}$, the two legislators are different "types". One of the legislators was reelected, or elected for the first time, at the beginning of this period. This legislator is therefore

[2] A more recent and comprehensive exposition and discussion of the political agency literature can be found in Besley (2006). For a recent model that represents the current state-of-art in legislative policy-making, see Coate and Battagalini (2007).

[3] For a discussion of the choice between staggered-term and simultaneous-term legislative bodies, see Muthoo and Shepsle (2008).

in the first period of his two-period term in office. The other legislator was reelected, or elected for the first time, at the beginning of period $t-1$. He is currently in the second (and last) period of his two-period term in office. For expositional convenience, we denote the former type of legislator by EARLY and the latter type by LATE.[4]

In period 0, matters are a little different as this is the first period of operation of the legislature. In this period, a legislator from each district is elected. Given that the terms of office are staggered, one of them is randomly selected to be the EARLY type (and thus begins a two-period term in office), while the other is selected to be the LATE type (and thus comes up for reelection at the end of this very period having served only a one-period term in office).[5]

The period-t EARLY legislator is the period-$(t+1)$ LATE legislator. At the end of period t, the period-t LATE legislator faces his electorate (his period-t principal). He either is reelected or replaced by a challenger.[6] If reelected, then he becomes the period-$(t+1)$ EARLY legislator; otherwise the challenger, a newly minted legislator, is the period-$(t+1)$ EARLY legislator. Our model of elections is described in subsection 2.2. Although the legislators are not in different generations as such, as is the case in standard OLG models, our structure is similar in a few important respects to such models, especially in terms of the relative incentives of the two types of legislators.

The policy context in each period concerns the sharing of an economic surplus. We stylize this as the allocation of cake (or pork) between the two districts. In each period t, the period-t EARLY and LATE legislators negotiate over the partition of a cake (the pork barrel). Note that the legislative task is exclusively one of distribution. There are no public goods in this model, and the surplus is treated as exogenous; we defer discussion of this point to section 8. The bargaining procedure (which in particular embodies the distribution of legislative proposal power) is described in subsection 2.3. If an agreement is struck, then the agreed shares of the cake flow to the districts. The legislators receive no direct benefit from any portion

[4]Note that "types" refer to period-dependent characteristics of legislators, and not, as is standard in economic theory, to some immutable characteristic.

[5]The US Senate operated exactly like this. In its opening session in 1790, the method of lots was employed to distribute senators in this staggered-term legislature across types, subject to the constraint that both of a state's senators could not be of the same type. Repeated randomizations were held as new states were admitted to the Union. A description of these randomizations and their results over time is found at `http://www.thegreenpapers.com/Hx/SenateClasses.html`

[6]Challengers are not modelled as players in our framework. It is implicitly assumed that a challenger exists and that he has not previously served in the legislature. This means that a legislator who fails to get reelected cannot be a future challenger; he withdraws from legislative politics in that eventuality. Apart from these differences in legislative experience, there are no other differences between a challenger and a legislator seeking reelection.

of this cake. A legislator simply receives a fixed payoff (salary) $b > 0$ in each term he serves in office. Any share of the cake that he negotiates for his district, however, may help his reelection prospects.

The structure and timing is summarized in Figure 1. We now turn to a description of elections, bargaining, information and the founding fathers' problem (where we also discuss the "renegotiation" issue mentioned in Figure 1).

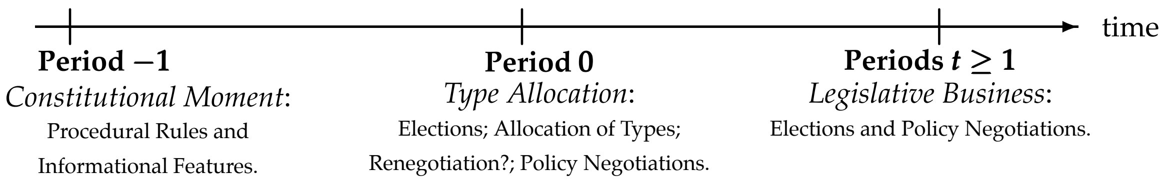

FIGURE 1. Structure and timing.

2.2. **Elections.** Let Π denote the probability that an arbitrary legislator (in an arbitrary period) is reelected. We explicitly capture two key ideas about this reelection probability. Our first idea is that voters care about the legislator's past performance in office when deciding whether or not to reelect him. We formalize this idea by positing that Π depends on the amounts of cake he obtained for his constituents during his most recent two-period term in office. With a slight abuse of notation, we write this as $\Pi(x_E, x_L)$, where x_E and x_L are the amounts of cake obtained by the legislator when EARLY and LATE, respectively, in his most recent two-period term of office.[7]

It is natural to assume that receiving more cake does not make the voters worse off, and thus does not decrease a legislator's chances of getting reelected. However, it may be that for some increases, the chances are unaffected. Hence:

Assumption 1 (Weak monotonicity)**.** *The probability Π that a legislator is reelected is non-decreasing in each of its two arguments.*

The second central idea we adopt about the election outcome is the notion that voters engage in a particular form of retrospective assessment. Known in the psychological literature as the recency effect, voters ask "What have you done for me lately?" which we abbreviate with the acronym *WHYDFML* (pronounced whid'fiml). We adopt an especially weak form of recency effect: the probability of reelection is higher if a legislator receives the entire surplus LATE rather than EARLY. Hence:

[7]Note that for the period-0 LATE legislator, $x_E = 0$, by definition. In fact, the probability of reelection function for the period-0 LATE legislator could, in principle, differ from $\Pi(0, x_L)$. However, because it does not affect our results, for expositional convenience we ignore the difference.

Assumption 2 (Weak recency effect)**.** *The probability* Π *that a legislator is reelected satisfies* $\Pi(0,1) \geq \Pi(1,0)$.

For some of our results, we need to impose a restriction involving the maximum and minimum feasible reelection probabilities, requiring that $\Pi(0,0)$ not be too small unless $\Pi(1,1)$ is large:

Assumption 3. *The probability of reelection* Π *satisfies* $\Pi(0,0) \geq 1 - \Pi(1,1)$.

Note that Assumptions 1 and 3 imply that $\Pi(1,1) \geq 0.5$. In summary, our model of elections is characterized entirely by the probability-of-reelection function Π satisfying Assumptions 1-3 (A1-A3 henceforth). Thus, the probability-of-reelection function is exogenously given (i.e., in particular the voting rule and the voters' behaviour are not explicitly modelled).

2.3. **Bargaining.** The procedural rules that influence the determination of the negotiated partition of the unit-size cake are a key part of the institutional structure of the legislature, pinning down the allocation of power (proposal power in particular) between the two legislators. Our framework abstracts from many of the details of real institutions through which power is derived (such as committees), capturing the allocation of power in a simple manner.

We posit a random proposer, "take-it-or-leave-it-offer" format. With probability $\theta \in [0,1]$ the EARLY legislator makes an offer of a partition of the unit-size cake to the LATE legislator, and with the complementary probability $1-\theta$ it is the LATE legislator who makes an offer to the EARLY legislator. If the offer is accepted, agreement is struck. But if the offer is rejected, then bargaining terminates, no agreement is reached, and no cake is obtained (in the period in question) by either district.[8]

We adopt the convention that an offer designates the share going to the proposer. It is therefore sometimes convenient to use the word "demand" rather than "offer" (Morelli 1999). The probability θ captures the relative proposal power of the legislators. If it equals one-half, then power is not type-contingent; otherwise it is. If $\theta = 0$ then the LATE legislator has all the power, while the exact opposite is the case if $\theta = 1$. Each type of legislator has some power if $0 < \theta < 1$. As θ increases more power is vested in the EARLY legislator. We adopt the following regularity assumptions:

Assumption 4 (Tie-breaking)**.**
(i) When indifferent between accepting or rejecting an offer, a legislator accepts it.

[8] In section 8 we briefly comment on the robustness of our main results to alternative bargaining procedures (such as alternating-offers procedures), and on the consequences of allowing proposal power to depend on legislators' experience or seniority.

(ii) When indifferent between making one of several offers, a legislator selects the offer which allocates the largest share of the cake to him.

For future reference, it may be noted that the expected payoff to a legislator who is reelected on each occasion with a constant probability $\pi \in [0, 1)$ equals $b/(1-\pi)$. Notice that, without much loss of generality, we do not endow legislators with a discount factor.[9]

2.4. **Information.** How much information does any legislator have in any given period about the history of play? The issue is especially pertinent here since every two periods a legislator faces reelection, and with positive probability he is replaced by a newly minted legislator. While the legislature is an infinitely lived body, operating over an indefinite number of periods, legislators come and go. As such a legislator may not know all of the important or relevant bits of the history of play at any given period. Indeed, the *default informational regime* would be one in which legislators have imperfect information about the history of play. Such a default regime could however be altered by purposeful design: a mechanism could be instituted that provides information about the important bits of the history of play. This could be instituted by the founding fathers at the constitutional moment (if of course doing so serves their joint interests).[10]

In what follows, we will therefore be interested in studying the properties of the equilibrium of our model under two alternative informational regimes: the default regime in which legislators have imperfect information about history, and a richer informational regime in which legislators (through the aid of an institutionalized mechanism) have all the information about history that they need. In order to simplify the equilibrium analysis, but without affecting our main results, we formally identify the latter informational regime with one in which legislators have perfect information, that is:

[9]To be precise, there is a potential but minor loss of generality. By not entertaining discounting, we need to assume that the reelection probability does not take the value of one. While such an assumption seems quite plausible, it does however rule out the cut-off voting rules used in the political agency literature in which a legislator is reelected with probability one if he performs sufficiently well (and fails to get reelected otherwise). The reelection probability function Π can of course approximate such a cut-off rule. We have chosen to proceed as we have in order to avoid carrying around an extra parameter (a discount factor for the legislators).

[10]The U.S. Constitution requires each chamber of the legislature to "keep a Journal of its Proceedings, and from time to time publish the same..." (Article I, Section 5). While we will return to this point, we mention this here as an example of an institutionalized mechanism providing (relatively) "untainted" information about history distinct from that which might be self-servingly reported contemporaneously by self-interested agents. That is, privately collected historical information (e.g., a private diary) transmitted to a new agent is not a substitute for untainted public information because it is unverifiable and therefore subject to misrepresentation.

Assumption 5 (Perfect information)**.** *For any t, the actions taken then are known by legislators in each and every subsequent period $s > t$. That is, any legislator in any period has full knowledge of the entire history of play.*

We define the default informational regime of imperfect information as follows:

Assumption 6 (Imperfect information)**.** *For any t there exists a finite $T > t$ such that legislators in period T and onwards do not know of the actions taken by the legislators in periods $s \leq t$.*

This formalization of imperfect information is implied, for example, by agents with finite memory, the length of which could vary across legislators. A6 implies that information about a past action is lost for sure some finite number of periods in the future.[11]

An altogether different kind of information concerns what a legislator knows about the game form, the payoffs and various parameters:

Assumption 7 (Complete information)**.** *There is common knowledge amongst all legislators about the game itself, including whether A5 or A6 holds.*

2.5. **Founding fathers' problem.** At the constitutional moment in period -1, the founding fathers select the institutional structure of the legislature. In particular, they jointly choose (a) the allocation of proposal power (value of θ), and (b) between keeping the default informational regime of imperfect information (A6 holds) or instituting a mechanism that would enable legislators to have information about all the important history (A5 holds).

The choices are made so as to optimize over the founding fathers' joint interests. We assume that the founding fathers respectively represent the interests of the two districts, and that for each district, the voters across time have the same preferences. We can therefore identify one infinitely lived principal per district. Let $u_i(c)$ denote the per-period utility obtained by the principal from district i ($i = 1, 2$) when her consumption is c in the period in question, and let $\delta_i < 1$ denote her per-period discount factor. We assume that u_i is strictly increasing and strictly concave in c.

The principals' (or founding fathers') joint expected payoffs take into account that in period 0 it will be randomly determined (with equal probability) as to which district's legislator is to be the EARLY type and which the LATE type. Furthermore,

[11]The formalization of imperfect information defined in A6 is adapted from Bhaskar (1998) who studies a version of Samuelson's OLG model with imperfect information. We discuss the game-theoretic OLG literature, including Bhaskar (1998), in section 8; we also entertain alternative formulations of imperfect information there. It may however be noted here that with the exception of Bhaskar's work, the literature assumes, without question, that players have perfect information about history.

these payoffs depend on the equilibrium outcome of the game once the legislature starts operating at the start of period 0, which in general vary according to the selected procedural rules and information structure. These equilibria are derived in sections 3 and 4 according to whether there is perfect information or imperfect information. Section 5 then studies the principals' period -1 problem. We will also ask whether the selected institutions (relating to procedures and information) are renegotiation-proof at the beginning of period 0, after the veil is lifted as to which district's legislator is EARLY and which is LATE.

This completes the description of our baseline model, which is a stochastic game with a countably infinite number of agents, but only two agents are active in any one period, and the number of periods for which an agent is active is determined endogenously.[12]

3. Intertemporal cooperation under perfect information

In this section we study the subgame perfect equilibria (SPE henceforth) of our baseline model with Assumption 5 and for an arbitrary value of EARLY's proposal probability, θ. That is, we suppose that at the constitutional moment in period -1 the principals selected some value for θ and instituted a mechanism that endows legislators with relevant information about history. Given this informationally rich environment, our analysis focuses on addressing the following question: Can *agent-optimal* outcome paths be sustained in *any* SPE? Roughly speaking (a precise definition is stated below), an agent-optimal outcome path is a path of play of our stochastic game that maximizes the legislators' joint expected payoffs. The presumption is that while there are a multiplicity of SPE in our stochastic game with perfect information, legislators will cooperate to maximize their joint payoffs.

We establish that an agent-optimal outcome path is a SPE path for any allocation of bargaining power and for any probability of reelection function satisfying our three mild assumptions, A1–A3. Legislators credibly sustain agent-optimal outcome paths via intertemporal cooperation, and with the credible threat of inflicting *maximal* punishments on deviators. It is because of the latter that the allocation of bargaining power is irrelevant, as is the magnitude of the reelection probability associated with the agent-optimal outcome path (subject to A3).

The analysis in this section rests crucially on the assumption that legislators have perfect information about the history of play, since the equilibria constructed here require that to be the case. In summary, then, the key message of this section can

[12]Our stochastic game falls outside of the classes of stochastic games studied in the current literature (see, for example, Friedman 1986, Fudenberg and Tirole 1991, and Dutta 1995). Thus, we cannot appeal to or apply results from that literature. However, some of our main results are derived using methods and ideas borrowed from that literature and from the theory of infinitely repeated games.

be put as follows: *When legislators possess all the information they need about history (which is made possible via the institutionalized information-providing mechanism), procedural rules and proposal power are irrelevant, as is the nature of the reelection probability function* (subject to A1–A3).

Our analysis is based on the insightful methodology laid down by Abreu (1988). The key to it is to derive the "optimal penal code", that is, the *worst* SPE payoffs to each and every legislator. Section 3.1 is devoted to their derivation. Our main result is then established in section 3.2. It is derived on the fundamental observation, first made by Abreu, that an outcome path is a SPE path if and only if it is sustained in a SPE in which any unilateral deviation from it by a player immediately moves play into that player's worst SPE.

3.1. **Worst punishments.** In any Nash equilibrium, each legislator's expected payoff is no less that his minmax payoff. This is the worst possible expected payoff that all other legislators can hold him down to. It involves them rejecting all offers from him when he has proposal power and demanding the whole cake when they have proposal power. Hence, a legislator's minmax payoff is $b/[1-\Pi(0,0)]$, which, given the symmetric nature of our stochastic game, is the same for every legislator (see Lemma 1).

The worst SPE for an arbitrary legislator can be conveniently defined by two paths Q^1 and Q^2, and two transition rules T^1 and T^2. If the initial path is Q^i ($i = 1, 2$), then any legislator from district i is held to his minmax payoff.

The path Q^i: Each legislator from district i always (i.e., in any period, for any history, and whether he is EARLY or LATE) offers the whole cake to the legislator from district j ($j \neq i$), and accepts all offers. Each legislator from district j always demands the whole cake, and only accepts an offer that allocates to him the whole of it.[13]

The transition rule T^i: If, when play is on path Q^i, a legislator from district j accepts an offer in which he is allocated less than the whole cake, then immediately (from the start of the next period onwards) play switches to path Q^j. For any other deviation on path Q^i, play remains on this path.

Lemma 1 (The worst SPE)**.** *Given Assumptions 1–3, 5 and 7, the strategies implicitly defined by the pair of paths* (Q^1, Q^2) *and the pair of transition rules* (T^1, T^2) *are subgame perfect. If the initial path is* Q^i*, then the expected payoff in this SPE to any legislator from district i (*$i = 1, 2$*) is his minmax payoff,* $b/[1-\Pi(0,0)]$.

[13]When we say "each" legislator from district i, it should be understood that at any time t there is only one such legislator actively playing in the game, but the claim applies as well to any legislator that may replace him.

Proof. In the appendix. □

The proof of Lemma 1 involves checking that no legislator can undertake a profitable deviation from either of the two paths. For example, Assumptions 2 and 3 respectively help ensure that an EARLY legislator and a LATE legislator from district i cannot profitably deviate from path Q^j when offered almost all (but not the whole) of the cake.

Lemma 1 has defined two extremal SPE according to whether the initial path is Q^1 or Q^2. The former is the worst SPE path for any legislator from district 1, while the latter is the worst SPE path for any legislator from district 2. Notice that when a legislator from district i is meted out maximal punishment (he never gets any cake), the legislator from the other district (who is doing the punishing) likes it as he obtains the whole cake. Interestingly, these extremal equilibria exist for any feasible value of θ; i.e., the worst punishment paths are sustainable as SPE paths irrespective of the structure of the procedural rules (given, of course, an institutionalized mechanism that endows legislators with enough information about history to enable them to credibly sustain intertemporal cooperation).

3.2. **Incentive-compatible, agent-optimal outcomes.** Given the underlying symmetric and stationary structure of our stochastic game, the legislators' joint expected payoffs are maximized with an outcome path in which in each period the partition of the unit-size cake is contingent on at most the type of the legislator who is randomly selected to make the offer (but is otherwise independent of time and history). Fix, therefore, such an arbitrary outcome path, $Q(k_E, k_L)$: in each period, the legislator who is selected to propose demands a share $k_E \in [0, 1]$ if he is EARLY, and a share $k_L \in [0, 1]$ if he is LATE, and the demand is accepted. The expected payoff $P(k_E, k_L)$ to an arbitrary legislator at the beginning of any period when he is EARLY from this outcome path is $P(k_E, k_L) = b/[1 - \Omega(k_E, k_L)]$, where[14]

$$\Omega(k_E, k_L) = \theta^2\Pi(k_E, 1-k_E) + \theta(1-\theta)\Pi(k_E, k_L) + (1-\theta)\theta\Pi(1-k_L, 1-k_E) + (1-\theta)^2\Pi(1-k_L, k_L). \tag{1}$$

[14]The expression for this expected probability is made up of four terms corresponding to four possible outcomes. Each such outcome is determined by the realizations in each of the two periods of who the proposer is. For example, with probability θ^2, in each of the two periods it is the EARLY legislator who is selected. That means that the legislator in question gets to propose when EARLY but not when LATE, and so in this eventuality his probability of reelection is $\Pi(k_E, 1-k_E)$.

An *agent-optimal* outcome path, $Q(k_E^*, k_L^*)$, is characterized by a pair of numbers (k_E^*, k_L^*) which maximizes $P(k_E, k_L)$ (or, equivalently, the expected probability of reelection, $\Omega(k_E, k_L)$) over the set of all $(k_E, k_L) \in [0,1] \times [0,1]$.[15]

We now establish that any agent-optimal outcome path is a SPE path. Following Abreu (1988), this result is achieved by constructing a SPE in which any unilateral deviation from the (initial) path $Q(k_E^*, k_L^*)$ by a legislator from district i immediately moves play onto the path Q^i (which is the worst SPE for such a legislator).

Proposition 1 (Agent-optimal SPE). *Given Assumptions 1–3, 5 and 7, the agent-optimal outcome path $Q(k_E^*, k_L^*)$, defined above, is a SPE path. The expected payoff to any legislator in any agent-optimal outcome path is*

$$P(k_E^*, k_L^*) = \frac{b}{1 - \Omega(k_E^*, k_L^*)} \quad \textit{with} \quad \Omega(k_E^*, k_L^*) = \max_{(k_E, k_L) \in [0,1] \times [0,1]} \Omega(k_E, k_L)$$

where $\Omega(k_E, k_L)$ is defined in (1).

Proof. In the appendix. □

Proposition 1 establishes that an agent-optimal outcome path is a SPE path for any allocation of proposal power and for any reelection probability function satisfying the three mild assumptions, A1–A3. The *sustainability* of this path as a SPE does not hinge either on the structure of the procedural rules (θ) or on the magnitude of the reelection probability associated with it (Π), subject only to the implication of A3 that $\Omega(k_E^*, k_L^*)$ cannot be too small unless $\Pi(1,1)$ is large. All this is made possible because the agent-optimal outcome path is being sustained by the optimal penal code (i.e., the worst SPE).

The existence of these equilibria requires legislators to possess perfect information about the history of play (A5). In short, then, *neither the structure of procedural rules nor the nature of the reelection probability function (subject to A1-A3) is relevant for the existence of an agent-optimal SPE when there exists an institutionalized mechanism that endows legislators with all the information they need about history*.

It should however be noted that since $\Omega(k_E, k_L)$ depends on θ and Π, so in general will the agent-optimal outcome (k_E^*, k_L^*) and the legislator's agent-optimal expected payoff $P(k_E^*, k_L^*)$. But without imposing additional structure on the probability of reelection function Π, it is not possible in general to establish any results concerning that dependence. We are able, however, to establish two results about the agent-optimal outcome when Π is concave. We first show for concave Π that the agent-optimal outcome and the legislator's expected payoff are independent of θ:

[15]Existence is guaranteed since Π is bounded and the feasible set is compact. Depending on the properties of Π, there may however exist more than one agent-optimal outcome path, but they will all, by definition, generate the same reelection probability and expected payoff.

Corollary 1. *If Π is concave, then the agent-optimal outcome $(k_E^*, k_L^*) = (x^*, 1 - x^*)$, where x^* maximizes $\Pi(x, 1-x)$ over $x \in [0,1]$, and the expected payoff to any legislator in any agent-optimal outcome path is $b/[1 - \Pi(x^*, 1-x^*)]$.*

Proof. In the appendix. □

Thus, if Π is concave, then the agent-optimal outcome path involves the legislators allocating, in each period, x^* to EARLY and $1 - x^*$ to LATE irrespective of which type makes the offer (hence the irrelevance of the value of θ). We now show that with a slightly stronger recency effect than what is captured in A2, it follows that $x^* < 0.5$:

Corollary 2. *The agent-optimal outcome, defined by x^* in Corollary 1, is strictly less than one-half if $\Pi(0,1) > \Pi(0.5, 0.5)$.*

Proof. In the appendix. □

The inequality in Corollary 2 entails a strengthening of recency bias. Together with concavity of Π, it implies A2. In words it says not only is it better to get the whole cake when LATE rather than EARLY (A2); it is also better to get the whole cake when LATE than half a cake each period. When this stronger condition of recency holds, then there will be a strict "back loading" of the cake in order to answer the principal's query, "What have you done for me lately?"

A main, general message that we have established in this section is that when legislators possess all the information they need about history, they can engage in intertemporal cooperation: agent-optimal outcome paths can be sustained in a subgame-perfect equilibrium. It was moreover shown that such cooperation does not depend on the details of the institutional structure such as the allocation of proposal power.

4. Procedural relevance under imperfect information

We now consider the baseline model with the default informational regime (A6) in which legislators have imperfect information about the history of play in any period. We establish, as the main result of this section, that *in any pure-strategy equilibrium, legislators use Markov strategies*. Thus, none of the equilibria described in section 3 hold here, and intertemporal cooperation is unsustainable in equilibrium. We then characterize the unique pure-strategy equilibrium (necessarily in Markov strategies) and establish the following key message: *with imperfect information, the structure of procedural rules matters, as does the nature of the reelection probability function*. This conclusion is to be contrasted with the opposite conclusion when legislators possess perfect information.

There are no proper subgames in the baseline model with A6. As such we cannot use the SPE concept. But, as is now well-established, it is desirable to work nonetheless with a solution concept that embodies the general notion of sequential rationality, which is the central element of the SPE concept. In the context of our stochastic game, the sequential rationality concept requires that in any period t and for any *observed* history, each legislator's actions are ex-post optimal (i.e., they maximize his expected payoff *from that period onwards*). We define a sequentially rational, symmetric pure strategy equilibrium (henceforth equilibrium) to be a pure strategy, adopted by all legislators, which is sequentially rational.[16] We now state the main result of this section:

Proposition 2 (Structure of equilibria with imperfect information)**.** *Given Assumptions 4, 6 and 7, any pure-strategy equilibrium is a Markov pure strategy.*

Proof. In the appendix. It may be noted that the proof is a straightforward adaptation of Bhaskar's (1998) proof of his Proposition 1 (which parallels this result). □

This remarkable and unexpected result implies that with imperfect information about the history of play, equilibria in which a legislator uses a non-Markov (history dependent) strategy cannot exist; that is, any strategy in which a legislator conditions his current actions on payoff-irrelevant past actions cannot be part of an equilibrium. This means that none of the equilibria described in section 3 can hold here — intertemporal cooperation is unsustainable in equilibrium when legislators have imperfect information.

We have formalized the notion of imperfect information about history in a particular manner, as defined in A6. As noted earlier, this would be satisfied if, for example, legislators have finite memory. The method of proof of Proposition 2 relies crucially on the implied feature that information about an action in period t is lost for sure after a finite number of periods; this allows us to deploy a backward induction argument to establish that equilibrium actions in any period after $t+1$ cannot be conditioned on period-t actions.

It may also be noted that the result of Proposition 2 does not use A1-A3, so it carries over to richer environments (e.g., to multi-person legislatures as we show in

[16]To simplify the formal analysis, we assume that the legislators in period t know the amount of cake the period-t LATE legislator obtained in period $t-1$ (which comprises the payoff-relevant bits of the history at the beginning of period t); note this means that T in A6 is strictly greater than $t+1$. Given this, we do not need to invoke any beliefs regarding past actions in defining and implementing this equilibrium concept. For example, we do not need to employ the relatively more complex sequential equilibrium concept. Our adopted solution concept is essentially the same as used in Bhaskar (1998).

section 6). Finally, note that any refinement of our equilibrium concept will *not*, by definition, sustain non-Markov equilibria involving intertemporal cooperation.[17]

Given Proposition 2, the set of pure-strategy equilibria of our baseline model with imperfect information is identical to the set of pure-strategy equilibria in Markov strategies:

Proposition 3 (Unique Markov equilibrium, ME)**.** *If the probability of reelection* Π *satisfies Assumptions 1, 4 and 7, then the following strategy, adopted by all legislators, is the unique pure-strategy ME. A legislator accepts any offer when EARLY and any offer when LATE. When making an offer, either when EARLY or when LATE, he demands the whole unit-size cake. The expected payoff to a legislator when EARLY associated with this equilibrium is*

$$P(1,1) = \frac{b}{1-\Omega(1,1)},$$

where $\Omega(1,1)$ *is obtained by setting* $k_E = k_L = 1$ *in* (1).

Proof. In the appendix. □

Notice that the unique equilibrium expected payoff to a legislator $P(1,1)$ depends on θ (through $\Omega(1,1)$). The following corollary provides a characterization of the value of θ which maximizes $P(1,1)$:

Corollary 3. *Assume that* Π *satisfies Assumption 2. Let* $\widehat{\theta}$ *denote the value of* θ *which maximizes the expected payoff* $P(1,1)$ *of a legislator in the unique ME. If* $\Pi(0,1) \geq [\Pi(1,1)+\Pi(0,0)]/2$ *then* $\widehat{\theta} = 0$*; Otherwise* $0 < \widehat{\theta} < 0.5$*, with* $\widehat{\theta}$ *decreasing in the difference* $\Pi(0,1) - \Pi(1,0)$.

Proof. In the appendix. □

Thus, if legislators have imperfect information about the history of play, then they would like the procedural rules to be such that a relatively greater amount of bargaining power is allocated to LATE legislators (and in some cases all the bargaining power). With perfect information, legislators are less concerned with procedural structure but that is far from the case with imperfect information. This point can be seen quite starkly in the case when Π is concave and $\Pi(0,1) > \Pi(0.5,0.5)$: With perfect information, legislators are indifferent to the value of θ (Corollary 1), but

[17]While finite memory is a reasonable assumption, it would be interesting to know whether the conclusion of Proposition 2 is robust to alternative formalizations of imperfect information, such as when information is lost gradually and stochastically (for example, because each legislator knows the full history from the point at which he is first elected into the legislature). In subsection 8.4 we address this issue and show that our results are indeed robust to such an alternative formalization of imperfect information.

with imperfect information, they would like to allocate all power to LATE legislators (Corollary 3; under the assumptions on Π in this case, the inequality stated in the corollary is satisfied and hence $\widehat{\theta} = 0$).

The take-home points of this section are that legislators, in contexts of imperfect information, are restricted to Markov strategies in equilibrium, that they cannot sustain intertemporal cooperation, that they are no longer indifferent to procedural arrangements (θ), and that they mutually prefer procedures that advantage the legislator closest to his contract-renewal date. A fascinating consequence of these conclusions is the expectation of greater distributive volatility in imperfect-information settings than when agents can condition on history. Agents obtain either the whole cake or none of it each period, outcomes that alternate as a function of recognition probabilities.

5. Constitutional arrangements

Given the results derived in sections 3 and 4, we can now resolve the founding fathers' problem (the principals in period -1) of selecting the procedural rule (value of θ) and choosing between the default informational regime of imperfect information (A6 holds) and instituting a mechanism to endow legislators with information about history (A5 holds). We will establish two main results.

First, the founding fathers will institute a mechanism that endows legislators (agents) with enough information about the history of actions taken in the legislature to facilitate intertemporal cooperation. Transparency of agent actions *amongst the agents* is all important for the founding fathers because it enables agents to hold each other to account. This key insight is shown to be quite robust (including renegotiation-proof).

Second, in some circumstances the founding fathers are indifferent to the structure of the procedural rules. Given that agents will be endowed with perfect information about the history of play (the first result in this section), the founding fathers do not care how proposal power is allocated between agents. But there are conditions under which a specific procedural rule (combined with agents possessing perfect information about history) would deliver the best outcome for the founding fathers.

As explained in section 2.5, we identify one infinitely lived principal per district, and so in what follows we use the terms founding fathers and principals interchangeably.

5.1. **The first-best.** The *first-best* outcome path for the principals is the one that maximizes their joint interests. The first-best serves as the ideal from the principals' perspective. Let $U^i_E \equiv U^i_E(x, y)$ and $U^i_L \equiv U^i_L(x, y)$ respectively denote the present discounted expected payoffs to the principal from district i depending on whether

her period-0 legislator is EARLY or LATE. In each period $t \geq 0$, the EARLY legislator demands x if making a proposal (with probability θ) and the LATE legislator demands y when proposing (with probability $(1-\theta)$. For each $i = 1, 2$, U_E^i and U_L^i respectively satisfy the following Bellman equations:

(2)

$$U_E^i = \left[\theta u_i(x) + (1-\theta)u_i(1-y)\right] + \delta_i\left[\theta u_i(1-x) + (1-\theta)u_i(y)\right] + \delta_i^2 U_E^i, \quad \text{and}$$

(3)

$$U_L^i = \left[\theta u_i(1-x) + (1-\theta)u_i(y)\right] + \delta_i\left[\theta u_i(x) + (1-\theta)u_i(1-y)\right] + \delta_i^2 U_L^i.$$

Straightforward computations show that the expected value, $U_P^i \equiv U_P^i(x, y)$, is:

$$U_P^i \equiv \frac{U_E^i + U_L^i}{2} = \frac{\theta\left[u_i(x) + u_i(1-x)\right] + (1-\theta)\left[u_i(y) + u_i(1-y)\right]}{2(1-\delta_i)}. \tag{4}$$

Notice that since u_i is strictly concave, U_P^i is maximized at $(x, y) = (0.5, 0.5)$. This makes sense: since each principal has strictly concave preferences, she prefers to smooth out her consumption over time. Alas, the partition of the cakes is not determined directly by principals, but rather by legislators in the legislative body. They may, however, be constrained constitutionally in various ways. Indeed, while the principals do not choose the allocations of the cakes, we can ask which institutional structures would maximize their joint expected payoffs subject to the constraint that once these arrangements are chosen, the amounts of cake that they respectively receive are determined by the legislators' equilibrium behaviour (as described in Propositions 1–3). We now proceed to answer this question.

5.2. **Principal-optimality behind the veil.** We study the choices in period -1 that are made by principals in ignorance of which district's period-0 legislator will be EARLY and which will be LATE. These constitutional choices determine the allocation of proposal power, $\theta \in [0, 1]$, and the information environment (a selection between staying with the default informational regime of imperfect information – A6 – and instituting a mechanism to endow legislators with enough information to sustain cooperation – A5). It follows from Propositions 1–3 that the equilibrium ex ante (behind the veil) expected payoffs to principal i with imperfect information is $U_P^i(1, 1)$, and with perfect information is $U_P^i(k_E^*, k_L^*)$. Since u_i is strictly concave, the latter strictly exceeds the former. Hence we have our first main result of this section:

Proposition 4 (Agent transparency). *Given Assumptions 1–4 and 7, and with constitutional choices made behind the veil, the founding fathers (or principals from both districts) strictly prefer to institute a mechanism that would endow legislators with perfect information about history (rather than retain the default informational regime of imperfect information).*

This means that the principals will at the constitutional moment institute mechanisms to ensure that the history of actions taken in the legislature are always available for inspection by any *legislator*. Whether they are also available for inspection by other parties (e.g., by the principals themselves, or the media) is not an issue of any relevance. Here, what matters is *transparency of actions amongst the legislators*. This notion of transparency differs from the standard interpretation in principal-agent relationships in which the agent's action is made known to the principals or other third parties in order to enable them to hold the agent to account.[18] Our result (relevant, of course, only in multi-agent environments) concerns the notion of *agents holding each other to account*.

Next we turn to the procedural rules (proposal power) as parameterized by θ that maximize the joint ex ante expected payoffs of the principals. Given Proposition 4, this would be the value of θ that solves the following maximization problem:

$$U_P^* \equiv \max_{\theta \in [0,1]} \left[U_P^1(k_E^*, k_L^*) + U_P^2(k_E^*, k_L^*) \right]. \tag{5}$$

Without additional assumptions on Π (which determines k_E^* and k_L^*), it is in general not possible to solve this problem. In general, $U_P^i(k_E^*, k_L^*)$ is potentially sensitive to θ, and hence one or both principals may have a strict preference for a specific value of θ. The set of solutions of (5) may be a proper subset of $[0, 1]$, and possibly there may be a unique θ which maximizes the principals' joint ex ante expected payoffs. In that case, principals not only will want to ensure that legislators can hold each other to account (Proposition 4), but also will design the legislature in such a way that proposal power is allocated appropriately.

However, when Π is concave it is easy to solve this problem. In this case, k_E^* is independent of θ, and $k_L^* = 1 - k_E^*$ (cf. Corollary 1). Hence, it can be verified that this implies for both $i = 1, 2$, $U_P^i(k_E^*, k_L^*)$ is independent of θ. Consequently, we have:

Proposition 5 (Procedural irrelevance)**.** *Given Assumptions 1–4 and 7, Π concave, and constitutional choices made behind the veil, the founding fathers do not care about procedural rules; that is, any $\theta \in [0, 1]$ maximizes their joint ex ante expected payoffs.*

Thus, combining Propositions 4 and 5, we have established that if Π is concave, then principals want to ensure that legislators can hold each other to account, no more no less; they do not care about the allocation of proposal power.[19] Taking

[18] See, for example, Besley (2006) and Prat (2005).

[19] We noted earlier that the U.S. Constitution (Article I, Section 5) requires record-keeping by each chamber. It is also relevant to observe that it does not prescribe other features of legislative organization. In particular, also in Article I, Section 5, it states that "Each House may determine the Rules of its Proceedings, punish its Members for disorderly Behaviour, and with the Concurrence of two thirds, expel a Member."

the principle of delegation as fundamental — that the allocations of the cakes are decided by legislators in the legislative body — the best that the principals can do in terms of maximizing their joint ex ante expected payoffs (when Π is concave) is to institute a mechanism that endows legislators with all the information they need to sustain intertemporal agent cooperation.

5.3. **Renegotiation-proofness and ex post optimality.** In this subsection, we address the following two different but related questions concerning legislative institutional structures that maximize the principals' joint *ex post* expected payoffs. "Ex post" means *after the veil is lifted* (i.e., immediately after it is randomly determined which district's period-0 legislator is EARLY and which is LATE):

• Renegotiation-Proofness. Can the two principals mutually benefit from *amending* the institutional structure of the legislative body that was determined behind the veil during the constitutional moment in period -1?

• Ex Post Optimality. If the constitutional choices are determined immediately after the veil is lifted (and not behind the veil), then what values would be selected by the two principals?

Letting district i's period-0 legislator be the one randomly selected to be EARLY and district j's LATE, it follows that the principals' joint ex post expected payoffs with imperfect information and perfect information are respectively $U^i_E(1,1) + U^j_L(1,1)$ and $U^i_E(k^*_E, k^*_L) + U^j_L(k^*_E, k^*_L)$. The latter joint expected payoff is strictly greater than the former (since u_1 and u_2 are strictly concave), and hence Proposition 4 is robust:

Proposition 6. *Given Assumptions 1–4 and 7:*
(i) *If the constitutional choices are made after the veil is lifted, principals from both districts strictly prefer to institute a mechanism that would endow legislators with perfect information about history.*
(ii) *If the constitutional choices are made behind the veil, then the mechanism instituted to endow legislators with perfect information is renegotiation-proof (i.e., the principals cannot mutually benefit from discarding it once the veil is lifted).*

Given Proposition 6(i), the ex post, principal-optimal allocation of proposer power is the value of θ that solves the following maximization problem:

$$\max_{\theta \in [0,1]} \left[U^i_E(k^*_E, k^*_L) + U^j_L(k^*_E, k^*_L) \right], \tag{6}$$

where district i's period-0 legislator is the one randomly selected to be EARLY and district j's LATE. Using (2)–(4), it is straightforward to verify that if the two principals have identical preferences, then the solutions of the two maximizations problems, (5) and (6), are identical, and hence the ex ante optimal and ex post optimal procedural rules are the same. But if the players' preferences differ (either with respect to their per-period utility functions and/or their rates of time preference), then the ex ante optimal and ex post optimal allocations of proposer power can differ. Of course, there will be circumstances under which they do not (e.g., when Π is concave; cf. Proposition 5, which carries over to the ex post optimality case). Similarly, the ex ante optimal allocation of proposal power is in general vulnerable to mutually beneficial renegotiation except when the principals' preferences are identical and/or Π is concave.

6. MULTI-PERSON LEGISLATURES

6.1. **Preliminaries.** We have developed a baseline model of staggered-term principal-agent relationships in the previous four sections. This baseline model examines the simplest case — two agents who come up for renewal at different dates based on retrospective assessment of their performance by their respective principals. We derived equilibrium results under two informational regimes, and then stepped back to ask how principals would structure interactions in the first instance.

Our results extend naturally to more general circumstances as we demonstrate in the present section. The baseline model partitioned agents in each period into two classes, EARLY and LATE. Now we consider the general case of M classes.[20] In period 0 each agent is assigned randomly a class. The agent in class i serves $M - i + 1$ periods ($i = 1, 2, 3, \ldots, M$). Each renewed or newly appointed agent then serves a full M-period term. If an agent is in class $i < M$ in period t, then he is in class $i + 1$ in period $t + 1$. An agent in class M in period t is in the final period of his term; hence if he is reelected at the end of period t, then he is in class 1 in period $t + 1$.

As in the baseline model, agent renewal is based on performance during the periods of his service. The bargaining is, as before, over a unit-size cake each period. However, now we assume a rule according to which a decision is taken to accept a proposal on the table that may range from simple majority rule up to unanimity.

Somewhat counterintuitively, we first consider four or more agents, showing how our main results from the baseline model extend to these cases. We separate out the three-agent case which we take up last. The reason for this treatment has to do with the issue of *pivotalness*. We had shown in the two-agent case with perfect information that an agent-optimal outcome can be sustained as a SPE through an optimal penal code (worst SPE). We develop a similar result in the case of four or more agents. To illustrate, consider four agents, perfect information, and simple majority

[20]We assume one agent per class, but then comment on how our results extend to n_i agents in the ith class.

rule (hence the support of three agents is required to accept a proposed division of the cake). Suppose behavior is on the agent-optimal path when a unilateral deviation occurs by agent i, moving play onto his worst punishment path, Q^i. Suppose also that i, suffering under this worst-SPE punishment, is randomly recognized and makes a deviant offer especially attractive to the agent in his last period. This is tempting to that agent. But since — given the SPE concept which requires equilibrium actions to be immune to profitable *unilateral* deviations — all other agents are expected to continue playing their equilibrium actions associated with the path Q^i (which involves them rejecting the deviant offer), the tempted agent will not be pivotal. So succumbing to the temptation is not profitable. It is because of this non-pivotalness that establishing the existence of worst SPE punishment paths for the case of four or more agents is straightforward. The three-agent case requires separate treatment precisely because, on the punishment path Q^i, if i makes an attractive proposal to another agent, that agent *would* be pivotal under simple majority rule, and so he might well be tempted to defect, making the existence of worst SPE punishment paths a more challenging exercise. These distinctions will become clear in the development below.

The structure and timing of our extended baseline model to multi-person legislatures is as in the baseline model, illustrated in Figure 1. In the next subsection, we study the extended baseline model with M principals and M agents, where each agent (legislator) represents a distinct principal (electoral district) and is in a class on his own; a full term of office is M periods. This perfectly symmetric set-up allows us to focus attention on the novel element of pivotalness (or otherwise) that arises when $M > 2$. The asymmetric set-up in which each principal i ($i = 1, 2, \ldots, M$) can have n_i agents represent it in the legislative body and for them to be in potentially different classes raises further issues (such as whether or not principal i can distinguish the performances of his n_i agents) and as such we take up this more general case in subsection 6.3.

6.2. **The perfectly symmetric extended framework.** At the beginning of period 0, M legislators are elected, one from each of the separate electoral districts, and randomly allocated into the M classes. The probability that a legislator is reelected is $\Pi(x_1, x_2, \ldots, x_M)$ (or simply $\Pi(\underline{x})$), where $x_t \in [0, 1]$ is the amount of cake that he delivered to his principal in period t of his most recent term in office, $t = 1, 2, \ldots, M$).[21] For the time being we make no assumptions on Π as these will differ according to whether $M \geq 4$ or $M = 3$. Let θ_i denote the probability with which a legislator from class i is selected to make an offer of the unit-size cake, where $\theta_i \in [0, 1]$ and $\sum_{i=1}^{M} \theta_i = 1$. An offer is a partition of the cake amongst the M legislators. If it

[21]Note that for a period-0 legislator put in class $i \geq 2$, $x_t = 0$ for $t = 1, \ldots, i - 1$, by definition.

is accepted by q or more legislators, then the cake is partitioned according to the proposed offer; otherwise no principal receives any cake in the period in question (where $q \geq (M/2)+1$ if M is even, and $q \geq (M+1)/2$ if M is odd).[22]

As in the baseline model, each agent receives a fixed payoff of $b > 0$ for each period of office, and principal i's utility from consumption c is $u_i(c)$, where u_i is strictly increasing and strictly concave.

We first show that when legislators have perfect information about the history of play, then any agent-optimal outcome path can be sustained in a SPE for any class-contingent distribution of bargaining power. Given the symmetric and stationary structure of this extended baseline model, an agent-optimal outcome path is one in which in each period, the legislator who is randomly selected to propose, offers a partition $\widehat{\underline{x}}^i$ if he belongs to class i ($i = 1, 2, \ldots, M$), where these M vectors maximize a legislator's expected probability of reelection.[23]

Proposition 7 (Perfect information in multi-person legislatures)**.** *Suppose that at the constitutional moment in period -1, the principals have instituted a mechanism that endows legislators with perfect information about history in the perfectly symmetric, multi-person legislative body with M legislators. Furthermore, assume that Π is non-decreasing in each of its M arguments.*
(a) If $M \geq 4$, then the agent-optimal outcome is a SPE.
(b) If $M = 3$, then the agent-optimal outcome is a SPE provided Π satisfies the following three inequalities: $\Pi(0, 0.5, 0.5) \geq \Pi(1, 0, 0)$, $\Pi(0.5, 0, 0.5) \geq \Pi(0.5, 1, 0)$ *and*

$$\frac{\Pi(0.5, 0.5, 0)}{\Pi(0.5, 0.5, 1)} \geq \frac{1 - \Pi(0.5, 0.5, 0.5)}{1 - \Pi(0, 0, 0)}.$$

Proof. In the appendix. □

The key aspect of the proof involves the construction of M worst SPE punishment paths, which extend those used in section 3 for the two-agent case. Notice that when $M \geq 4$, the existence of the agent-optimal SPE is obtained under extremely mild conditions on Π. We only require that Π be non-decreasing (which extends A1). But when $M = 3$, we require Π also to satisfy three inequalities (as stated in Proposition 7); the first two capture recency effects while the third has implications for the magnitudes of the reelection probabilities. The reason for this non-trivial difference has already been noted above in subsection 6.1 (arising due to the issue

[22] It may be noted that the parameter q — the exact quota in the multi-member legislature — does not play any significant role in the analysis. As such, allowing it to be chosen by the founding fathers at the constitutional moment in period -1 would not generate anything of interest.

[23] If Π is concave, then (like in the two-agent case; cf. Corollary 1) an agent-optimal outcome is characterized by a single, class-independent, partition of the unit-size cake, denote it by $\widehat{\underline{x}}$, which maximizes the probability of reelection $\Pi(\underline{x})$.

of pivotalness), and can be seen formally in the proof of this proposition. We note that in either case the agent-optimal outcome is being sustained as a SPE for *any* class-contingent distribution of bargaining power (just like in the two-agent case), i.e., for any feasible $\underline{\theta} = (\theta_1, \theta_2, \ldots, \theta_M)$.

We now turn attention to the default informational regime when agents have imperfect information. In this case, the results of section 4 extend straightforwardly for any $M \geq 3$ (it is no longer necessary to treat the $M = 3$ case distinctly).[24]

Proposition 8 (Imperfect information with multi-person legislators)**.** *Given Assumptions 4, 6, and 7, and assuming that* Π *is nondecreasing, the following strategy, adopted by all legislators, is the unique equilibrium. A legislator, irrespective of the class he is in, always accepts any offer and always demands the whole unit-size cake.*

Proof. In the appendix. □

Propositions 7 and 8 extend the arguments and the results of section 5 to the M-principal, M-agent case. The assumption that each principal has strictly concave preferences plays, as before, a key role. The first-best, or ideal solution for each principal, is the same, and it involves splitting each period's unit-size cake equally amongst them (i.e., in each period, each principal is allocated a share $1/M$ of the cake). But this is not directly achievable since cake allocations are determined by the legislators. The main results are as follows.

The principal-optimal outcome behind the veil has the fundamental feature that principals strictly prefer to institute a mechanism that endows legislators with information about history (rather than have them operate in the default informational regime of imperfect information). This feature is renegotiation-proof and ex post optimal. These two results extend Propositions 4 and 6. Of course the assumptions required for them differ according to whether $M = 3$ or $M \geq 4$. Besides the assumptions required for Propositions 7 and 8 above, we require that principals' have strictly concave preferences. With respect to the principal-optimal procedural structure, just like for the two-principal, two-agent case, it is in general not possible to say much without additional assumptions. For example, if Π is concave, then the conclusion of Proposition 5 carries over.

6.3. **Asymmetric multi-person legislatures.** We now briefly discuss the more general dynamic multi-principal, multi-agent framework in which principal i is represented in the legislature by n_i legislative agents who are potentially in different

[24] As in the two-agent case, imperfect information is formally defined as in A6. Furthermore, to simplify the formal analysis we assume that legislators in any period t know the actions taken in the preceding M periods.

classes. One novel issue that this raises is whether or not the principal can distinguish the per-period performances of her $n_i > 1$ agents. If all that the principal observes is the aggregate amount of cake delivered to her district in any period, but does not observe the contributions of each of her agents, then this complicates the analysis. It brings into play free-rider and team considerations and the logic of "credit-claiming". These issues are outside the scope of this paper, and so we adopt the assumption that the principal can observe exactly what each legislator delivered in each period, a reliable credit-claiming technology. This assumption may be applicable in some contexts but not in others. With this assumption, the probability of reelection of any legislator can, once again, be conditioned on his performance during his term in office.

There are some other issues that this asymmetric framework raises that are absent from the analysis conducted thus far. For example, although legislators are assumed to be identical (have identical preferences and an identical probability of reelection function Π), the sizes of the M classes would differ in each period, and this would mean that the outcome path that maximizes the legislators' joint payoffs would be asymmetric and relatively more complex than in the symmetric case. If the size of each class is the same, then the arguments and conclusions reported in subsection 6.2 carry over with minor modifications to the assumptions required for them to hold. But with unequal class sizes the arguments would have to be somewhat altered.

7. An application to the U.S. Senate

The United States Senate, like many of the world's upper chambers, is a staggered-term legislature. Each state sends two senators to the Senate (by the selection of the state legislature until the beginning of the twentieth century and by popular election since then). Beginning with the ten initial states which selected senators in May 1789 (the three remaining of the original 13 sending senators shortly thereafter), random assignment was employed to establish the staggered-term arrangement. Each of the ten states was randomly assigned to two of three groups, with the proviso that no state could be twice assigned to the same group. Then each of these groups was randomly assigned a class (= I, II, II). A designated senator from each state in class I must vacate his seat after two years, a senator in class II after four years, and those in class III after six years. A vacated seat is then filled by the state's (s)electorate, so that a third of the seats were filled every two years and each newly filled seat had a term of six years. For example, Maryland may have been assigned to the second and third groups. If the second group were randomly assigned class I and the third class II, then Maryland's two senators would begin with terms of two years and four years, respectively. The original 20 senators did not divide evenly into the three classes — one class was a senator short. When the next admitted state sent two senators, there would be a double-random drawing, the first to determine which of the two senators would be assigned to the "short" class, and the second to determine which class the other senator would join. Now there are two "short" classes, so that the next newest state's two senators would simply be randomized between these two classes. This assignment sequence would repeat as new states entered the union. In this manner a staggered-term legislative chamber was implemented in accord with constitutional provision, with each class approximately equal. Today, fifty states supply two senators each with approximately one-third facing reelection every two years.[25]

In the modern era politicians are held accountable in the electoral arena by their respective constituents. They are judged on criteria as diverse as their personal characteristics (some of which are observable — race, gender, charisma, wealth — and some of which are not — competence, honesty, intelligence), their ideology (as measured by their voting records, their speeches, their association with legislative products or special interests), their party, and their position in the legislature (seniority, committee assignments, chairmanships, party and institutional leadership posts). In our model, we abstract from these features, treating senators as essentially identical agents, distinguished only by their location in the electoral cycle and their success in the divide-the-cake distributive politics game.

Despite abstracting from the empirical richness of a real legislative body like the US Senate, we believe we have captured some of its essential features, as the following stylized facts suggest:

- Aside from the various appropriations that are mandated by permanent legislation or that accompany new legislation, about 15% of each of the thirteen separate appropriations bills that is passed each fiscal year consists of expenditures devoted to *earmarks*. These are specific line items designating a particular project and an expenditure amount in a particular location (e.g., $3.5 million to expand the visitors center in Yosemite National Park in California). An appropriations subcommittee (the Interior subcommittee, for example, has jurisdiction over expenditures in national parks) will be inundated with as many as 2500 or 3000 such requests from senators in a fiscal year, each request a claim on a small piece of the earmarks budget. (The roughly

[25] Upper chambers in most national and subnational legislatures employ four-year terms with half the legislators up for election every two years. However, India's upper house, the Rajya Sabha, is a staggered-term legislative body, with one-third of its members elected every two years. A term of office is six years. It differs from the contemporary US Senate in two ways – members are elected from Indian states by their respective state legislatures (like the pre-twentieth century US Senate), and the number of members from a state depends on state population (like most lower houses around the world).

$10 billion Interior appropriation will have approximately $1.5 billion worth of earmarks, for example). Thus, in the context of each appropriations subcommittee bill, there is an active "divide-the-earmarks-budget" game. From conversations with subcommittee staff (personal interviews by one of the authors), some clear criteria emerge by which winners and losers in a given fiscal year are determined: Priority is given first to the requests of senators on the subcommittee, then senators on the full committee, and finally senators "in cycle" (by which is meant senators in the last two years of their six-year term). Since committee memberships are proxies for agenda power, those not on the (sub)committee, but in cycle, are effectively given special treatment. The practice of tilting toward those in cycle is a recognition, if not an institutionalization, of the *WHYDFML* logic.[26]

- The allocation of earmark funding is consistent with the equilibrium outcomes we would expect in a world with recency effects. That politicians believe they are subject to retrospective assessment, and that this assessment is indeed biased toward recent performance, is an established stylized fact in the legislative scholarship field. It is exemplified by the (probably apocryphal) story of the Kentucky senator who is shocked to learn that one of his reliable constituents might not support him in the upcoming election. In response to the surprise expressed by the senator, the constituent concedes, "Sure you got my brother that agricultural subsidy. And you got my grandson admitted to the US Military Academy. And you even got my factory a small military contract. But what have you done for me lately?"
- Because earmarks are thought to be wasteful, often not even passing a simple benefit-cost test, there are always "reformers" in the legislature who seek to eliminate them from the appropriations process. Indeed, most of the professional staffs of appropriations subcommittees dislike earmarks for this reason (plus the fact that they are a time-intensive nuisance). For all these reasons, plus the fact it reduces their discretion, bureaucrats are not keen about earmarks either. In the 1970s a bold attempt was made by Senator James Buckley of New York. When an appropriations bill came to the floor of the Senate, Buckely introduced amendments to strike 50 projects, one from each state. Although these amendments were intended to symbolize the profligacy and waste of earmarking, only the amendment striking the project from New York passed, causing the *New York Times* to ask "Why is New York State handicapped by being the only state with but one U.S. senator?" We would

[26]Empirical evidence for the distinct treatment given senators who are in-cycle is found in Shepsle, Van Houweling, Abrams, and Hanson (2008).

characterize Buckley's attempt as an off-the-equilibrium path deviation that elicited a punishment.

These stylized facts portray a U.S. Senate engaging in distributive politics along the lines we have described in our theoretical development — overlapping terms, retrospective assessment, recency bias, and punishment of those seeking to overturn equilibrium practices. In addition to these facts, we would call attention to two key features of the U.S. Senate which, we claim, are consistent with our main results. The Constitution requires, first, the keeping and publishing of a journal, a daily record of Senate proceedings that records bills and motions introduced, the speeches and other deliberative acts of the senators, and finally their votes (Proposition 4). Increasingly, information about activities off the floor of the Senate, in its committee and subcommittee rooms, is part of the official record. Second, and perhaps more remarkable, the Constitution permits senators to make their own procedural arrangements (Proposition 5). Thus, the constitutional founders imposed a record-keeping requirement to enhance the availability and reliability of the historical record, but imposed no other constraints on the operation of the legislature. That is, they encouraged transparency among agents but expressed little interest in fine-tuning procedural parameters (like θ).

8. Discussion

Our approach departs from several conventions in the literature, so we first, in sections 8.1-8.3, provide further elaboration and justification on three issues — probabilistic election, retrospective assessment, and recency bias — while at the same time relating our approach to some of the existing literature. We then, in section 8.4, show that our main results are robust to an alternative formalization of imperfect information, one in which information is lost gradually and stochastically. Section 8.5 concludes with a brief discussion of areas for future research.

8.1. **Probabilistic election.** In our approach agent reappointment is probabilistic. Exemplars of the agency-theoretic approach to legislative organization — Barro (1973), Ferejohn (1986, 1999) — utilize uncertainty in a very different way. For them, an uncertain state of the world affects the productivity of an agent's effort for his principal's welfare. Thus, if x measures agent effort and ξ is the state (with "good" states having larger values), then the payoff to the principal is weakly monotonic in $x\xi$. The state (ξ) is unknown ex ante to both principal and agent, but comes to be known to the agent by the time he must make his (costly) effort decision (x). Although neither the state nor agent effort is observable to the principal (voter), she nevertheless makes a *deterministic* decision about whether to renew the agent's contract based on past performance. Employing a *cut-off rule*, she retains the agent if $x\xi$

exceeds a threshold (optimally determined, and committed to, ex ante in light of the commonly known probability distribution over ξ), and replaces him otherwise. She cannot distinguish the agent's effort from the state of the world because these elements are pooled into an undifferentiated performance result. The optimal rule set by the principal is deterministic: reappoint if performance exceeds a cut-off value; replace otherwise. Therefore, at the time the agent chooses his effort, he knows with certainty whether it is worthwhile for him to bear the cost of enough effort so that, pooled with the state of the world (then known to him), the performance it yields will exceed the principal's cut-off requirement. If he concludes that it is not worthwhile, then he will do nothing and not be reappointed. Uncertainty in these models enters only in the determination of the principal's optimal cut-off rule. Once the cut-off threshold is set, the principal's choice on agent retention is deterministic and this is fully appreciated by the agent at the time he selects his effort level.

In our model, in contrast, the legislative agent is not able to resolve uncertainty surrounding his reappointment prospects entirely. At the end of his two-period term, his reelection depends upon past performance but in no completely discernable way (apart from weak monotonicity and recency). Many factors confound the relationship between performance and reappointment. For example, voters may participate probabilistically. The choices of those who do participate may be stochastic. The characteristics of an opponent (unmodeled in most approaches in this literature) may not be known ex ante. Put differently, the agent does not know the identity of the decisive voter who will determine a renewal decision. There are, in short, myriad reasons why an agent enters his reappointment phase with some randomness in his fate unresolved. In our model the best he can do is take actions that increase the likelihood of reappointment.

Thus, we characterize the agent's optimization problem in terms of a probability-of-reelection function, Π. We have made relatively weak assumptions about this function (A1-A3). A1, it should be noted, is compatible with the notion of performance thresholds so that the accumulation of cake beyond some fixed amount need not strictly improve reelection probabilities. In the limit, when $\Pi = 1$ for performance exceeding an upper threshold, our formulation approaches the spirit of cut-off formulations. Indeed, cut-off formulations are special cases of our probabilistic formulation.[27] As is evident in the development in earlier sections, many existence results depend on nothing stronger than the three assumptions we make. In order to provide substantive characterizations, however, we often must restrict ourselves

[27] A slightly different kind of cut-off rule is the *threshold contract* described in Gersbach and Liessem (2005). A threshold contract stipulates the minimum performance level an agent must attain in order to be eligible for renewal. This is also a special case of our probabilistic formulation.

to more specific classes of Π, e.g., a probability-of-reelection function concave in its arguments.

By not subscripting Π we have implicitly assumed that agents share identical probability-of-reelection functions. In effect, this means that each agent believes that his principal draws her reappointment choice from a common probability distribution. This is unrealistically restrictive in contexts in which wealth and tastes are likely to vary across districts. We have acknowledged this in indexing the utility function and discount parameter of principals. This and the preceding observation emphasize the need to elaborate more detailed micro-foundations for Π.[28]

8.2. **Retrospective assessment.** As in most of the literature, our approach depends upon retrospective assessment of agents by principals. Although this contrasts with Downsian approaches to elections in which the promise of future performance is the coin of the realm (prospective voting), retrospective voting models are now the conventional approach to this subject. There is both an empirical and a theoretical literature on elections in which retrospective assessment is taken as unexceptional. In addition to Barro and Ferejohn mentioned above, theoretical contributions by Austen-Smith and Banks (1989), Banks and Sundaram (1998), Dixit (1995), and Maskin and Tirole (2004) adopt, without comment, a retrospective point of view as axiomatic. Theirs are primarily moral hazard frameworks in which retrospective assessments by principals provide ex ante incentives for agents to perform.[29] Fearon (1999) in contrast raises selection issues, suggesting that elections are chiefly about selecting agent qualities — for example, preferences concordant with those of principals or immutable qualities like competence, ability, and honesty. Retrospective assessment for Fearon involves the aquisition of information by principals about agent type, allowing principals to update beliefs and reappoint those agents more likely than potential replacements to possess appropriate attributes. Our model is more in the moral hazard tradition in which the reelection probability reflects incentives for agents to perform in a manner desired by principals. Departures from the wishes of principals may arise, and this is one of the main points of our analysis, because of institutional features of elections — particularly staggered terms that provide agents with time-dependent considerations at odds with the objectives of principals.

[28]We have conceptualized Π as an agent's beliefs about how he will be retrospectively assessed by his principal, and have assumed that these beliefs are common among agents (as just noted) and commonly known by both principal and agent. Thus, Π is stipulated exogenously. One way to move beyond this would be to specify a deterministic assessment by principals subject to generic and idiosyncratic shocks with known probability distributions. We thank Torsten Persson for this suggestion.

[29]A recent model of retrospective voting, formalizing earlier arguments of the American political scientist V.O. Key, may be found in Bendor, Kumar, and Siegel (2005).

There is a large empirical literature in political science on retrospective assessment. The *locus classicus* of this literature is Fiorina (1981). Kramer (1971) was one of the earliest to demonstrate a retrospective effect empirically, showing for the US that stronger economic performance in the year of an election translated into a greater share of the national vote for the congressional candidates of the incumbent president's party. Tufte (1978) and others experiment with weighted averages of multiple years worth of past economic performance to operationalize a retrospective effect. Kiewiet and Udall (1998), on the basis of new data, re-estimate a number of different specifications, demonstrating a very robust retrospective effect. As in the theoretical literature, there is very little attention paid to justifying or explaining retrospective assessments; researchers are content to demonstrate the existence of such effects. As Kahneman, Wakker, and Sarin (1997: 389) observe, "...retrospective evaluation of outcomes is a cognitive activity in which people routinely engage, much as they engage in grammatical speech or in deductive reasoning." Our approach fits right into this literature.

8.3. **Recency effects.** Recency effects in the context of retrospective assessment are commonly noted in the literature. Levitt (1996), for example, finds that "...[US] senators give twice as much weight to (median) constituent preferences in the year before elections as compared to four years or more before elections." We assume a very weak version of recency bias in Assumption 2. (We discuss other versions shortly.)

In our model principals with concave preferences for the consumption of cake prefer, subject to discounting, a relatively smooth distribution over the two periods of an agent's term. If, ex post, a principal's actions display a recency bias, as is evident in the empirical literature, then it is conceivable that there is some incentive rationale for her acting this way. Perhaps a principal elicits better responses from an agent when she acts as if she gives greater weight to more recent performance relative to more distant performance. Our results cast doubt on this belief since (if Π is concave) a recency bias will encourage agents to prefer extraordinary performance (getting more cake) in later periods, even if it means foregoing performance success earlier. This attenuates the smooth intertemporal provision of cake, contrary to what is preferred by principals.

An alternative consideration, suggested to one of the authors by David Laibson (personal communication), emphasizes, in the spirit of selection models, information extraction as the foundation for recency bias. Recent information is viewed, rightly or wrongly, as more informative about agent type. Related to this, recent information may be more easily remembered (whether more reliably informative or not) or may be given more prominence by the media. Sarafidis (2007) presents a model of memory (of a principal) from which it may be deduced that particular patterns of information transmission from agent to principal — sometimes reflecting a recency effect, sometimes not — are optimal for the agent.

Perhaps the most famous psychological experiments on recency bias are Kahneman's colonoscopy studies (Kahneman, Fredrickson, Schreiber, and Redelmeier 1993, Redelmeier and Kahneman 1996, Redelmeier, Katz, and Kahneman 2003). There it was discovered that patient memories of unpleasant medical procedures are highly correlated with peak periods of unpleasantness on the one hand, and the experience in the most recent periods on the other. In particular, they found that a number of periods of relatively less unpleasant experience tacked on to the end of the procedure reduced patient retrospective assessment of the unpleasantness, even if these additional less unpleasant experiences actually lengthened the time of the procedure.

Thus, while we do not have a good micro-theoretical rationale for recency bias, there are several useful conjectures and considerable empirical and experimental evidence for the bias. One of the interesting effects of recency bias is that discounting by voter-principals is effectively the reverse of ordinary economic discounting. At the outset of an agent's two periods of incumbency, for example, he can anticipate that principals, at the time of a renewal decision, will place at least as much weight on the agent's second period performance as on his first.[30]

Finally, we note that there are a variety of ways to specify a recency effect theoretically. Assumption 2 is, as noted, a very weak requirement of a recency bias. Another version, mentioned in the discussion of Corollary 3, has $\Pi(0, 1) > \Pi(0.5, 0.5)$ — in effect, moving half the cake in a smooth distribution from the early period to the late increases the renewal likelihood. Perhaps the strongest version would be one which requires that $\Pi(x_E - \epsilon, x_L + \epsilon) > \Pi(x_E, x_L)$ for any (x_E, x_L) and any $0 < \epsilon < x_E$. This strong version of recency bias is one in which the probability of reappointment is strictly increasing in *any* reallocation of cake from an early period to a late period. Clearly, repeated application of this definition yields Assumption 2. It turns out that this requirement imposes strong restrictions on Π.[31]

[30]This suggests that the combination of ordinary economic discounting and recency bias produces inconsistent time preferences: principals have a bias for recent periods over the past. This parallels the hyperbolic discounting literature (Laibson 1997) in which players have dynamically inconsistent time preferences because of self-control problems: a principal has a bias for current periods over the future.

[31]To illustrate this point suppose that Π is an increasing function of $\beta v(x_E) + v(x_L)$, where $\beta < 1$, and v is differentiable and increasing. The strong version of recency bias holds if and only if for any (x_E, x_L), $v'(x_L) > \beta v'(x_E)$. This, in turn, holds only if $\min_x v'(x) > \beta \max_x v'(x)$. But this means that either v is linear, or if non-linear then β is sufficiently small (i.e., the premium on cake obtained when LATE is sufficiently large).

8.4. **Robustness to stochastic information loss.** A fundamental element of our model is that the default informational regime is one in which legislators have imperfect information about history. We have conceptualized and formalized "imperfect information" in a particular manner (cf. A6). As noted earlier, finite memory implies it. But A6 would not hold if we were to assume that a legislator knows the full history since his first election but has little information about history before he entered the legislature. Such a conceptualization of imperfect information would imply that information about actions taken in any period is lost gradually and stochastically. The main issue for our argument and results to hold is whether the conclusion of Proposition 2 carries over. This is because a key element that underlies our argument and main results is that with imperfect information, intertemporal cooperation is unsustainable in equilibrium (any equilibrium is necessarily Markovian). We now show that the result stated in Proposition 2 carries over to an alternative formalization of imperfect information, one in which information is lost gradually and stochastically.

Before proceeding further, we note that with the exception of Bhaskar (1998), the game-theoretic OLG literature has assumed that players have perfect information about the history of play. It may also be noted that this literature (including Bhaskar's work) is characterized by the feature that each player operates for a finite and exogenously given number of periods; in contrast, in our multi-principal, multi-agent model, the number of periods in which an agent operates is endogenously determined. Furthermore, the focus of this literature has been on establishing folk theorem type results when players have perfect information in an overlapping-generations structure. Bhaskar extends this literature by considering the consequences of imperfect information (as formalized by an assumption similar to our A6) and establishes a result similar to our Proposition 2.

Turning to stochastic information loss, we now assume that:

Assumption 8 (Stochastic information loss)**.** *For any t and for any $T > t$ there exists a $p(T;t) \in [0,1]$ such that legislators in period T do not know of the actions taken by the legislators in periods $s \leq t$ with probability $p(T;t)$.*

Note that when $p(T;t) = 0$ (for all t and $T > t$), A8 collapses to A5, and when $p(T;t) = 1$ (for all t and $T > t$), it collapses to A6. Indeed, A8 encapsulates both the perfect information setting and the deterministic loss of information scenario as formally captured by the conceptualization of imperfect information adopted by Bhaskar (1998) and in this paper (A6).

Our objective now is to in particular establish the robustness of Proposition 2 (and hence of our main results) to small perturbations in the information structure (and as such we will show that Proposition 2 carries over when A8 holds with $p(T;t)$ sufficiently close to one). The desired result is as follows:

Proposition 9. *Assume that A4 and A8 hold, and there is common knowledge amongst all legislators about the game itself.*
(i) If for any t, $p(T;t) \to 1$ as $T \to \infty$, then any pure-strategy equilibrium is a Markov pure strategy.
(ii) If for any t and any $T > t$, $p(T;t)$ is sufficiently close to zero, then agent-optimal outcomes are sustainable in a sequentially rational equilibrium.

Proof. In the Appendix. □

Proposition 9 implies that the main results established in this paper are robust to small perturbations in the information structure. More precisely, and especially, that if instead of information being lost deterministically (for sure) in finite time (A6), it is lost gradually and stochastically but for sure in infinite time, then the result of Proposition 2 carries over. As such this is useful check on the robustness of our main insights. Having said this, it would be interesting to explore in future research the best possible sustainable equilibrium payoffs for arbitrary probability, $p(.;,)$, functions. The key idea behind the argument leading to Proposition 2 (which underlies the argument for Proposition 9 as well) is that the sets of legislators operating between any consecutive periods have different information about history with a relatively high probability, and it is this kind of feature which ensures that pure strategy equilibria will necessarily be Markovian.

8.5. **Directions for future research.** We now briefly discuss some possible extensions of our model. First, a key element of our model concerns the modelling of elections. The previous three subsections above have discussed this issue at some length, and a central extension would be to provide microfoundations for our probability of reelection function Π.

Second, the economic environment in our model is restrictive, especially since the per-period surplus is fixed and exogenously given. It would be interesting to consider not just the partition of a fixed surplus, but also the choice of taxes (hence the surplus would be endogenously determined) and the allocation of such tax revenue between pork (as in our model), national public goods, and possibly rents for the legislators (as in Persson and Tabellini, 2000). Such an extension would deliver results and insights about issues (such as the size of government) which are outside of the model studied in this paper. Our main results (principal-optimality of agent-transparency being a key one) will carry over to some richer economic environments, but the opportunity for agents to extract rents (that is, consume some of the cake themselves) puts them in a more adversarial relationship with their respective principals and with one another. Transparency may allow principals to keep an eye on their agents, but it will also enable agents to cooperate at their principals' expense.

Third, although we adopted a simple bargaining format, it can be established that our main results are robust to alternative bargaining procedures such as those that allow legislators to make offers and counter-offers as in Rubinstein (1982) and Baron and Ferejohn (1989). Of great interest is to make proposal power dependent on the relative experiences of the legislators. Although this would imply that the allocation of power is history-dependent (hence significantly complicating the analysis), it has much appeal. This is especially relevant if legislative experience affects the size of the pork barrel, reelection probabilities, or both.

Fourth, extending the model by allowing for heterogenous agents is obviously an important direction for future research. Legislative experience is an endogenously determined source of heterogeneity, which we have suppressed, but clearly plays an important role in real-world legislatures. There are of course other sources of exogenously determined heterogeneity such as ability, competence and utility (or ego-rents) from holding office. Interestingly, since b, the fixed payoff received by a legislator in each term of office (which could be interpreted as the sum of salary and ego-rent), plays no critical role in the analysis and hence in our results, it can be shown that our main results are robust to allowing legislators to have differing ego-rents (and hence differing payoffs for holding office).

Fifth, in our model, there are conflicts of interests on the one hand amongst agents, and on the other hand amongst principals. But the interests of each principal and his agents are highly correlated in that each agent wants to deliver as much cake as possible to his principal (in order to maximize his prospects for reappointment), although agents do exploit the recency bias displayed by principals. It would be interesting to enrich our model by introducing the standard kinds of conflict between a principal and his agent, while at the same time maintaining the other conflicts in this multi-principal, multi-agent environment — for example, by entertaining the prospect that the size of the surplus depends on the efforts of agents and/or that agents enjoy some of the cake themselves (as mentioned above).

Last, but not least, it would be interesting and important to extend our normative analysis of the choices made by the founding fathers at the constitutional moment in period -1 by allowing for them to select other institutional features such as the choice of whether to institute staggered or non-staggered terms (see Muthoo and Shepsle 2008, who study this issue and the choice between a unicameral legislature, as is implicitly assumed in the current paper, and a bicameral legislature).

APPENDIX

Proof of Lemma 1. To establish that the proposed strategies comprise a SPE, we need only to check that no legislator can benefit from any one-shot, unilateral deviation from path Q^i ($i = 1, 2$). It is straightforward to see that any one-shot, unilateral deviation from the path Q^i by any legislator from district i does not increase his expected payoff.

Now consider a one-shot, unilateral deviation by a legislator from district j ($j \neq i$) from path Q^i. If, in any period, he either demands less than the whole cake, or rejects the offer of the whole cake, then he is worse off in that period and there is no effect on his continuation expected payoff. We now come to the final, but critical deviation: Suppose, in some period, a district j legislator considers, while on path Q^i, accepting an offer which gives him a share $x < 1$. His expected payoffs from accepting (i.e., deviating) and from rejecting (i.e., conforming) such an offer depend on whether in the period in question he is EARLY or LATE. If he is EARLY, then he will reject the offer if and only if his payoff from rejecting is greater than or equal to his payoff from accepting, i.e.,

$$b + \Pi(0,1)\left[\frac{b}{1-\Pi(1,1)}\right] \geq b + \Pi(x,0)\left[\frac{b}{1-\Pi(0,0)}\right].$$

Assumptions 1 and 2 imply that this inequality is satisfied. If he is LATE in period $t \geq 1$, then he will reject the offer if and only if

$$b + \Pi(1,0)\left[\frac{b}{1-\Pi(1,1)}\right] \geq b + \Pi(1,x)\left[\frac{b}{1-\Pi(0,0)}\right].$$

This inequality is satisfied for any possible $x < 1$ if and only if

$$\frac{\Pi(1,0)}{\Pi(1,1)} \geq \frac{1-\Pi(1,1)}{1-\Pi(0,0)}.$$

Assumptions 1 and 3 imply that this inequality is satisfied. A3 implies (making use of A1) that $\Pi(1,1)^2 - \Pi(0,0)^2 \geq \Pi(1,1) - \Pi(0,0)$. This, in turn, implies that $\Pi(0,0)/\Pi(1,1) \geq [1-\Pi(1,1)]/[1-\Pi(0,0)]$. The desired conclusion follows since (by A1) $\Pi(1,0) \geq \Pi(0,0)$.[32]

Proof of Proposition 1. Consider the strategies implicitly defined by the following three paths and three transition rules. The initial path is an agent-optimal path $Q(k_E^*, k_L^*)$. The other two paths are Q^1 and Q^2, and two of the transition rules are T^1 and T^2, all of which are associated with the worst SPE (cf. Lemma 1). The third transition rule, denoted by $T(k_E^*, k_L^*)$, concerns transitions from $Q(k_E^*, k_L^*)$: If, when on path $Q(k_E^*, k_L^*)$, a legislator from district i ($i = 1, 2$) either makes or accepts a deviant offer, then immediately (before the next decision node) play moves on to path Q^i.[33]

[32]If he is LATE in period 0, then he will reject the offer if and only if $\Pi(0,0)/\Pi(0,1) \geq [1-\Pi(1,1)]/[1-\Pi(0,0)]$. This is slightly different, but it is satisfied by essentially the same argument.

[33]A deviant offer made by EARLY is a demand $x \neq k_E^*$; and a deviant offer made by LATE is a demand $x \neq k_L^*$.

Given Lemma 1, the proposition follows once we establish that no legislator can benefit by a one-shot, unilateral deviation from the agent-optimal path $Q(k_E^*, k_L^*)$. The following four steps, which establish that, can be easily verified. First, given Assumption 1, each legislator's payoff from accepting the demand associated with the path $Q(k_E^*, k_L^*)$ is no less than rejecting it. Second, given Assumption 1, each legislator cannot profit from making a deviant demand (either when he is EARLY or when he is LATE). Third, given Assumptions 1 and 2, when a deviant offer is received by EARLY, he cannot profit from accepting it. Fourth, the same is true when a deviant offer is received by LATE if and only if

$$\frac{\Pi(z,0)}{\Pi(z,1)} \geq \frac{1-\Pi(1,1)}{1-\Pi(0,0)},$$

where $z \in [0,1]$ denotes the amount of cake he obtained in the preceding period, when he was EARLY. Assumptions 1 and 3 imply that this inequality is satisfied. As shown in the proof of Lemma 1, A3 implies (making use of A1) that $\Pi(0,0)/\Pi(1,1) \geq [1-\Pi(1,1)]/[1-\Pi(0,0)]$. The desired conclusion follows since (by A1) $\Pi(z,0)/\Pi(z,1) \geq \Pi(0,0)/\Pi(1,1)$.

Proof of Corollary 1. Fix an arbitrary pair $(k_E, k_L) \in [0,1]\times[0,1]$. Rewrite $\Omega(k_E, k_L)$ as follows:

$$\Omega(k_E,k_L) = \theta\big[\theta\Pi(k_E,1-k_E) + (1-\theta)\Pi(k_E,k_L)\big] + (1-\theta)\big[\theta\Pi(1-k_L,1-k_E) + (1-\theta)\Pi(1-k_L,k_L)\big].$$

Since (by the hypothesis of the corollary) Π is concave, it follows that

$$\Omega(k_E,k_L) \leq \theta\Pi\big(k_E, \theta(1-k_E) + (1-\theta)k_L\big) + (1-\theta)\Pi\big(1-k_L, \theta(1-k_E) + (1-\theta)k_L\big).$$

Using concavity again, it follows that $\Omega(k_E,k_L) \leq \Pi(\widehat{x}, 1-\widehat{x})$, where $\widehat{x} = \theta k_E + (1-\theta)(1-k_L)$. We have thus established that for any $(k_E,k_L) \in [0,1]\times[0,1]$ there exists an $x \in [0,1]$ such that $\Pi(x,1-x) \geq \Omega(k_E,k_L)$. Hence, it immediately follows that

$$\max_{0\leq x\leq 1} \Pi(x,1-x) \geq \max_{(k_E,k_L)\in[0,1]\times[0,1]} \Omega(k_E,k_L),$$

which establishes the corollary.

Proof of Corollary 2. We argue by contradiction. Thus suppose that $x^* \geq 0.5$. Since Π is concave,

$$\Pi(0.5,0.5) \geq \left[\frac{1}{2x^*}\right]\Pi(x^*,1-x^*) + \left[1-\frac{1}{2x^*}\right]\Pi(0,1).$$

Since (by definition) $\Pi(x^*,1-x^*) \geq \Pi(0.5,0.5)$, and (by the hypothesis of the corollary) $\Pi(0,1) > \Pi(0.5,0.5)$, it follows that $\Pi(0.5,0.5) > \Pi(0.5,0.5)$, a contradiction.

Proof of Proposition 2. Fix an arbitrary pure-strategy equilibrium, and fix an arbitrary period $t \geq 1$. We will show that the equilibrium actions in period t are conditioned on at most z_t (the amount of cake obtained by the period-t LATE legislator in period $t-1$), but on no other bits of observed history, which then establishes the proposition. The argument involves induction.

First, note that A6(i) implies that there exists a $T \geq t+2$ such that the equilibrium actions in any period from and including period T onwards cannot be conditioned on the actions taken in any period before and including $t-1$. Second, we establish the following inductive step:

Fix an arbitrary period s, where $s \geq t+1$. If the equilibrium actions in any period from and including period $s+1$ onwards are not conditioned on the actions taken in any period before and including period $t-1$, then the same is true of the equilibrium actions in period s.

Proof of inductive step. Since $s \geq t+1$, none of the actions in any period before and including period $t-1$ directly affects the payoffs of any legislator in period s. Given this and the hypothesis of the inductive step, it follows that the equilibrium expected payoff to a legislator from period s onwards does not depend on the actions in any period before and including $t-1$. Let h_{t-1} and h'_{t-1} denote two different histories till the end of period $t-1$ that are observable to an arbitrary legislator in period s. Furthermore, let h denote a history of actions observed by the arbitrary legislator between and including periods t and $s-1$. Hence, two different observed histories at the beginning of period s are (h_{t-1}, h) and (h'_{t-1}, h). The equilibrium expected payoffs to this arbitrary legislator from period s onwards will be the same following either observed history (for any set of period s actions and given the equilibrium pure-strategy). Hence, given Assumption 4, the legislator's equilibrium actions in period s following these two observed histories are the same. This completes the proof of the inductive step. It should be noted that the above argument needs a little elaboration. The role of our particular stage game is crucial, in that it is sequential. To be precise, one first considers the above argument w.r.t. to the responder's behaviour, and then one rolls back to consider and show that the proposer would, in equilibrium, make the same offer in period s following the two observed histories.

Hence, it now follows from the principle of mathematical induction that the equilibrium actions in any period from and including period $t+1$ are not conditioned on the actions taken in any period before and including period $t-1$. The desired conclusion follows immediately.

Proof of Proposition 3. A Markov pure strategy for a legislator is made up of two numbers, k_E and k_L, and two functions, f_E and f_L: k_i denotes the legislator's demand when he is type i, and $f_i : [0,1] \to \{\text{“Accept”}, \text{“Reject”}\}$ such that $f_i(x)$ denotes whether the legislator accepts or rejects the demand x when he is type i, where $i = E, L$ (E stands for EARLY and L stands for LATE). Fix an arbitrary Markov equilibrium, and let W denote the expected payoff associated with this equilibrium to the legislator when he is EARLY at the beginning of any period (before the proposer is randomly selected). We first establish the following result:

Claim 1. If the probability of reelection Π satisfies Assumption 1, then a legislator accepts any offer when EARLY and any offer when LATE.

Proof of Claim 1. To establish this claim, we need to show that the legislator, when EARLY and when LATE, respectively, accepts any demand $x \in [0,1]$ made by the proposer. It follows from the *One-Shot Deviation* Principle that the legislator, when EARLY, accepts a demand

$x \in [0,1]$ if and only if $H_E(x) \geq H_E(1)$, where $H_E(x) = b + [\theta\Pi(1-x, y_E) + (1-\theta)\Pi(1-x, y_L)]W$, where

$$y_E = \begin{cases} 1 - k_E & \text{if } f_L(k_E) = \text{“Accept”} \\ 0 & \text{if } f_L(k_E) = \text{“Reject”} \end{cases} \quad \text{and} \quad y_L = \begin{cases} k_L & \text{if } f_E(k_L) = \text{“Accept”} \\ 0 & \text{if } f_E(k_L) = \text{“Reject”}. \end{cases}$$

Assumption 1 implies that for any $x \in [0,1]$, $H_E(x) \geq H_E(1)$. Hence, this means that $f_E(x) =$“Accept” for all $x \in [0,1]$. A legislator, when LATE, accepts an offer $x \in [0,1]$ if and only if $H_L(x) \geq H_L(1)$, where $H_L(x) = b + \Pi(z, 1-x)W$, and z is the amount of cake received by LATE in the previous period. Assumption 1 implies that for any $x \in [0,1]$, $H_L(x) \geq H_L(1)$. Hence, this means that $f_L(x) =$“Accept” for all $x \in [0,1]$. This completes the proof of Claim 1.

Given Claim 1, it follows from the One-Shot Deviation Principle that the pair (k_E, k_L) satisfy the following conditions:

$$k_E \in \arg\max_{x\in[0,1]} \left[b + [\theta\Pi(x, 1-k_E) + (1-\theta)\Pi(x, k_L)]W \right] \text{ and } k_L \in \arg\max_{x\in[0,1]} \left[b + \Pi(z, x)W \right],$$

where z is the amount of cake earned by the LATE legislator a period earlier. That is,

$$k_E \in \arg\max_{x\in[0,1]} \left[\theta\Pi(x, 1-k_E) + (1-\theta)\Pi(x, k_L) \right] \text{ and } k_L \in \arg\max_{x\in[0,1]} \left[\Pi(z, x) \right].$$

Assumptions 1 and 4(b) thus imply that $(k_E, k_L) = (1,1)$ is the unique solution.

Proof of Corollary 3. Maximizing $P(1,1)$ over θ is equivalent to maximizing $\Omega(1,1)$ over θ. Differentiating $\Omega(1,1)$ *w.r.t.* θ, we obtain:

$$\frac{\partial\Omega(1,1)}{\partial\theta} = 2\alpha\theta + \beta, \quad \text{where}$$

$$\alpha = \Pi(1,0) + \Pi(0,1) - \Pi(1,1) - \Pi(0,0) \text{ and } \beta = \Pi(1,1) + \Pi(0,0) - 2\Pi(0,1).$$

First, we consider the case in which $\beta > 0$. Since $\alpha + \beta = \Pi(1,0) - \Pi(0,1)$ is strictly negative (by Assumption 2), it follows that $\alpha < 0$. Now note that $2\alpha + \beta = -\beta + 2[\Pi(1,0) - \Pi(0,1)]$, which is strictly negative (by Assumption 2 and since, by hypothesis, $\beta > 0$). Finally note that since $\alpha < 0$, it follows that $\Omega(1,1)$ is strictly concave in θ. Putting these results together, it follows that $\Omega(1,1)$ is increasing in θ over the interval $[0, \widehat{\theta})$, decreasing over the interval $(\widehat{\theta}, 1]$ and achieves a maximum at $\theta = \widehat{\theta}$, where $\widehat{\theta} = -\beta/2\alpha$. The second part of the corollary follows immediately since $\widehat{\theta} \in (0, 0.5)$, and since $\widehat{\theta}$ is increasing in $\Pi(1,0)$ and decreasing in $\Pi(0,1)$.

Now consider the case in which $\beta \leq 0$. We break our argument into three cases. First suppose that $\alpha < 0$. This immediately implies that $\Omega(1,1)$ is maximized at $\theta = 0$ (since $\Omega(1,1)$ is in this case decreasing and strictly concave in θ). Now suppose that $0 < \alpha < -\beta/2$. In this case $\Omega(1,1)$ is also maximized at $\theta = 0$ (since $\Omega(1,1)$ in this case is decreasing but strictly convex in θ). Finally suppose that $\alpha > -\beta/2$. In this case $\Omega(1,1)$ is strictly convex in θ, decreasing in θ over the interval $(0, \tilde{\theta})$, achieves a minimum at $\theta = \tilde{\theta}$ and is increasing over the interval $(\tilde{\theta}, 1]$, where $\tilde{\theta} = -\beta/2\alpha$. This implies that $\Omega(1,1)$ achieves a maximum either at $\theta = 0$ or at $\theta = 1$. The desired conclusion follows immediately, since (by Assumption 2) $\Omega(1,1)$ takes a higher value at $\theta = 0$ than at $\theta = 1$.

Proof of Proposition 7. Given the symmetric and stationary structure of the extended baseline model, the legislators' joint expected payoffs are maximized with an outcome path in which in each period the partition of the unit-size cake is contingent on at most the class of the legislator who is randomly selected to make an offer. Fix such an arbitrary outcome path, $Q(x)$, where $x = (\underline{x}^i)_{i=1}^M$ are M partitions of the unit size cake: in each period, with probability θ_i the legislator from class i ($i = 1, 2, \ldots, M$), who is selected to propose, offers $\underline{x}^i$, which is accepted by all the other legislators. The expected payoff to an arbitrary legislator at the beginning of his M-period term in office from this outcome path is $P(x) = b/[1 - \Omega(x)]$, where

$$\Omega(x) = \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \cdots \sum_{i_M=1}^{M} \left[\theta_{i_1}\theta_{i_2}\ldots\theta_{i_M}\right] \Pi(x_1^{i_1}, x_2^{i_2}, \ldots, x_M^{i_M})$$

where for each $t \in \{1, 2, \ldots, M\}$, the symbol $i_t \in \{1, 2, \ldots, M\}$ denotes the class of the legislator who is selected to propose in the t-th period of the legislator's term in office. An agent-optimal outcome path, $Q(\widehat{x})$, is characterized by the M, $(M-1)$-dimensional vector $\widehat{x}$ which maximizes $P(x)$ (or equivalently $\Omega(x)$). We now proceed to establish (by construction) the existence of a SPE that sustains the agent-optimal outcome path. As in the two-agent case (and following Abreu (1988)), we construct a SPE defined by an initial path (which is the agent-optimal outcome path $Q(\widehat{x})$), M district-contingent worst punishment paths, and $M+1$ transition rules (one for each of the $M+1$ paths):

Path Q^i ($i = 1, 2, \ldots, M$): Each and every legislator from district i always (i.e., in any period, for any history, and whatever his class) offers y_i and accepts any offer, where y_i is the partition of the unit-size cake in which he gets no cake, and every other legislator gets a share $1/(M-1)$. Every other legislator always makes the offer y_i and only accepts the offer y_i.

Transition rule T^i ($i = 1, 2, \ldots, M$): If, when play is on path Q^i, a legislator from district j either makes a deviant offer (i.e., does not offer y_i) or accepts a deviant offer (i.e., accepts an offer that differs from y_i), then immediately play switches to path Q^j. For any other deviation from path Q^i, play remains on this path.

Transition rule $\widehat{T}$: If, when on path $Q(\widehat{x})$, a legislator from district i ($i = 1, 2, \ldots, M$) either makes or accepts a deviant offer, then immediately play moves on to path Q^i.

We first establish that no legislator can make a profitable unilateral, one-shot deviation from path Q^i ($i = 1, 2, \ldots, M$). It is straightforward to see that any unilateral, one-shot deviation from Q^i by a legislator from district i does not increase his expected payoff. Now consider a one-shot, unilateral deviation by a legislator from district j ($j \neq i$) from path Q^i. If, in any period, he either makes a deviant offer, or rejects the offer y_i, then he is worse off in that period and his continuation expected payoff decreases (as play will have moved onto path Q^j).

We now come to the final, but critical deviation: Suppose, in some period, a district j legislator considers, while on path Q^i, accepting a deviant offer $y \neq y_i$. We divide the argument according to whether $M \geq 4$ or $M = 3$. First suppose $M \geq 4$. In that case, since he is not pivotal (the deviant offer will be rejected in equilibrium by all the other players), whether he unilaterally deviates (accepts the deviant offer) or conforms (and rejects it), the amount of cake he obtains in this period is the same (i.e., zero) but his continuation payoffs differ since in the former case play moves onto path Q^j while in the latter case play stays on path Q^i. The assumption that Π is nondecreasing in each of its arguments implies that the legislator in question is strictly better off conforming and rejecting the deviant offer.

Now suppose $M = 3$. In this case he would be pivotal, and he may have an incentive to accept the deviant offer (if it is sufficiently attractive). There will be three incentive-compatibility conditions that need to hold (corresponding to the three possible classes to which the legislator belongs when he considers the deviant offer) in order for it to be the case that he would reject the deviant offer. Since the most attractive deviant offer is one in which he is offered the whole cake, the three conditions corresponding to when he is in class 1 (EARLY), class 2 (MIDDLE), or class 3 (LATE) are respectively

$$b + \Pi(0, 0.5, 0.5)\left[\frac{b}{1 - \Pi(0.5, 0.5, 0.5)}\right] \geq b + \Pi(1, 0, 0)\left[\frac{b}{1 - \Pi(0, 0, 0)}\right]$$

$$b + \Pi(0.5, 0, 0.5)\left[\frac{b}{1 - \Pi(0.5, 0.5, 0.5)}\right] \geq b + \Pi(0.5, 1, 0)\left[\frac{b}{1 - \Pi(0, 0, 0)}\right]$$

$$b + \Pi(0.5, 0.5, 0)\left[\frac{b}{1 - \Pi(0.5, 0.5, 0.5)}\right] \geq b + \Pi(0.5, 0.5, 1)\left[\frac{b}{1 - \Pi(0, 0, 0)}\right].$$

The conditions on Π stated in the proposition imply that these three incentive constraints are satisfied. Finally, using similar arguments to those used here to establish that the punishment path Q^i is subgame perfect, it is easy to verify that the agent-optimal path $Q(\widehat{x})$ is subgame perfect. This then concludes the proof of the proposition.

Proof of Proposition 8. It is trivial to verify that the stationary strategy described in the proposition, when adopted by all legislators is an equilibrium. It is straightforward to extend the arguments in the proofs of Propositions 2 and 4 and establish that there does not exist any other equilibrium (by first establishing that in any pure-strategy equilibrium, a legislator uses a Markov pure strategy, and then establishing that the only Markov equilibrium is the one described in the proposition).

Proof of Proposition 9. We provide an informal sketch of the main elements of the argument for part (i); the key elements for the argument of part (ii) are similar. Fix an arbitrary pure-strategy equilibrium, and fix an arbitrary time t. We will show that the equilibrium actions in period t are conditioned on at most z_t (the amount of cake obtained by the period-t LATE legislator in period $t - 1$), but on no other bits of observed history, which then establishes the desired result.

The central part of the argument involves showing that there exists a period $T > t$ such that the legislators in period T will choose, *in equilibrium*, not to condition their period-T actions on the actions taken in any period before and including $t - 1$. Having established that (which is essentially establishing that A6 emerges in equilibrium), the remainder of the argument is a backward induction argument exactly along the lines of the proof of Proposition 2.

Given A8, in general it is the case that any legislator in any period $T > t$ will, with some probability, know actions taken in period $s < t$. As such, legislators can, with such probability, condition their equilibrium actions on payoff-irrelevant bits of the observed history (since there is a positive probability that such history is common knowledge especially amongst the sets of legislators operating between any two consecutive periods). Any gain for an arbitrary legislator from doing so, in some arbitrary period $T > t$, is secured in a subsequent period (since the whole point of doing so is to influence the behaviour of future legislators). However, using A4, it follows that there is unique short-run best action for the legislator in period T. This, in turn, implies that deviation from such action is costly to the legislator in period T. Consequently, when $p(T;t)$ is sufficiently large, the *expected* (future) gain to a legislator in period T from conditioning his period-T action on payoff-irrelevant bits of history can be made smaller than the loss incurred in period T from such conditioning. [The intuition for this argument is formally equivalent to having a sufficiently small discount rate; as is well-known, in that case only Markov strategies can be part of a subgame perfect equilibrium in infinitely-repeated games.] The desired conclusion is immediate from the hypothesis of part (i) since it implies that for the appropriate, sufficiently large $p > 0$ there exists a $T' > t$ such that for any $T > T'$, $p(T;t) > p$.

References

Abreu, D. (1988). On the theory of infinitely repeated games with discounting. *Econometrica* 56, 383-396.

Austen-Smith, D. and J. Banks (1989). Electoral acountability and incumbency. In P.C. Ordeshook (ed.) *Models of Strategic Choice in Politics*, 121-148. Ann Arbor: University of Michigan Press.

Banks, J. and R. Sundaram (1998). Optimal retention in agency problems. *Journal of Economic Theory*, 82, 293-323.

Baron, D. and J. Ferejohn (1989). Bargaining in legislatures. *American Political Science Review*, 83, 1181-1206.

Barro, R. (1973). The control of politicians: an economic model. *Public Choice*, 14, 19-42.

Bendor, J., S. Kumar, and D. Siegel (2005). V. O. Key Formalized: Retrospective voting as Adaptive Behavior. American Political Science Association, Washington D.C.

Besley, T. (2006). *Principled Agents? The Political Economy of Good Government*. Oxford: Oxford University Press.

Bhaskar, V. (1998). Informational constraints and the Overlapping generations model: folk and anti-folk theorems. *Review of Economic Studies*, 65, 135-149.

Coate, S. and M. Battagalini (2007). Inefficiency in legislative policy-making: a dynamic analysis. *American Economic Review*, 97, 118-149.

Dixit, A. (1995). *The Making of Economic Policy: A Transaction Costs Approach*. Cambridge: MIT Press.

Dutta, P. (1995). A folk theorem for stochastic games. *Journal of Economic Theory* 66, 1-32.

Fearon, J. (1999). Electoral accountability and the control of politicians: selecting good types versus sanctioning poor performance. In A. Przeworski, S. Stokes, and B. Manin (eds.), *Democracy, Accountability, and Representation*, 55-98. New York: Cambridge University Press.

Ferejohn, J. (1986). Incumbent performance and electoral control. *Public Choice*, 50, 5-25.

Ferejohn, J. (1999). Accountability and authority: toward a political theory of electoral accountability." In A. Przeworski, S. Stokes, and B. Manin (eds.), *Democracy, Accountability, and Representation*, 131-154. New York: Cambridge University Press.

Fiorina, M. (1981). *Retrospective Voting in American National Elections.* New Haven: Yale University Press.

Friedman, J. (1986). *Game Theory with Applications to Economics*. Oxford: Oxford University Press.

Fudenberg, D. and J. Tirole (1991). *Game Theory*. Cambridge, Massachusetts: MIT Press.

Gersbach, H. and V. Liessem (2005). Re-election threshold contracts in politics. CEPR Discussion Paper No. 5175.

Kahneman, D., P. Wakker, and R. Sarin (1997). Back to Bentham? explorations of experienced utility. *Quarterly Journal of Economics*, 112, 375-405.

Kiewiet, R. and M. Udall (1998). Twenty-Five Years after Kramer: An assessment of economic retrospective voting based upon improved estimates of income and employment. *Economics and Politics*, 10, 219-248.

Kramer, G. (1971). Short-term fluctuations in U.S. voting behavior, 1896-1964. *American Political Science Review*, 65, 131-143.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics*, 62, 443-477.

Levitt, S. (1996). How do senators vote? disentangling the role of voter preferences, party affiliation, and senator ideology. *American Economic Review*, 86, 425-41.

Maskin, E. and J. Tirole (2004). The Politician and the Judge: Accountability in Government. *American Economic Review*, 94, 1034-1054.

Morelli, M. (1999). Demand Competition and Policy Comprise in Legislative Bargaining. *American Political Science Review*, 93, 809-820.

Muthoo, A. and K. Shepsle (2008). The constitutional choice of bicameralism," in *Institutions and Economic Performance*, 249-292. edited by Elhanan Helpman, Cambridge, MA: Harvard University Press.

Persson, T. and G. Tabellini. (2000). *Political Economics: Explaining Economic Policy*. Cambridge, Mass: MIT Press.

Prat, A. (2005). The Wrong Kind of Transparency. *American Economic Review*, 95, 862-877.

Redelmeier, D. and D. Kahneman (1996). 'Patients' Memories of Painful Medical Treatments: Real-Time and Retrospective Evaluations of Two Minimally Invasive Procedures. *Pain*, 66, 3-8.

Redelmeier, D., J. Katz, and D. Kahneman (2003). Memories of colonoscopy: a randomized trial. *Pain*, 104, 187-194.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, 50, 97-110.

Sarafidis, Y. (2007). What have you done for me lately?' Release of Information and Strategic Manipulation of Memories. *Economic Journal* 117: 307-326.

Shepsle, K., R. Van Houweling, S. Abrams and P. Hanson (2009). The senate electoral cycle and bicameral appropriations politics. *American Journal of Political Science*, 53, 343-359.

Tufte, E. (1978). *Political Control of the Economy*. Princeton: Princeton University Press.

DEPARTMENT OF ECONOMICS, UNIVERSITY OF WARWICK, COVENTRY CV4 7AL, UK.
E-mail address: A.Muthoo@warwick.ac.uk

DEPARTMENT OF GOVERNMENT, HARVARD UNIVERSITY, CAMBRIDGE, MA 02138, USA.
E-mail address: kshepsle@iq.harvard.edu