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Event Count Models for International Relations: Generalizations and Applications

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International relations theorists tend to think in terms of continuous processes. Yet we observe only discrete events, such as wars or alliances, and summarize them in terms of the frequency of occurrence. As such, most empirical analyses in international relations are based on event count variables. Unfortunately, analysts have generally relied on statistical techniques that were designed for continuous data. This mismatch between theory and method has caused bias, inefficiency, and numerous inconsistencies in both theoretical arguments and empirical findings throughout the literature. This article develops a much more powerful approach to modeling and statistical analysis based explicitly on estimating continuous processes from observed event counts. To demonstrate this class of models, I present several new statistical techniques developed for and applied to different areas of international relations. These include the influence of international alliances on the outbreak of war, the contagious process of multilateral economic sanctions, and reciprocity in superpower conflict. I also show how one can extract considerably more information from existing data and relate substantive theory to empirical analyses more explicitly with this approach.

Introduction

Whereas most theories in the social sciences attempt to explain underlying continuous *processes*, we generally observe only finite numbers of discrete *events*. Bertram M. Gross writes: “[T]he world or my part of it is seen as an ongoing stream of events in time . . . Facts and process are separated into discrete elements only by human analysis . . . Change—whether rapid or slow, hidden or open—is continuous” (Gross, 1968:262). For example, influence among political actors, the continuing allocation of resources, constituency representation, and other aspects of politics can all be described as unobserved continuous processes that generate observed discrete events. A legislator probably represents constituents in varying degrees continually in all aspects of his or her work, but most observers cannot record much more than roll call votes and compare them to occasional polls of constituent opinion. The constant trade flows between nations are important features of economic cooperation, but an

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analyst might only have a list of major treaties or quarterly summaries of economic activities. In the U.S., presidential-congressional relations continuously advance and decline, but presidential vetoes and congressional overrides only occur at discrete points.

Since these continuous processes are generally of primary interest but are usually unobservable, scholars study the events produced by these processes. Among all the fields in political science, international relations is most closely identified with this approach: "Events are at least as important in international behavior as measures such as power, development, and status" (Schrodt and Mintz, 1988:217). The most obvious consequence of this approach is the creation of a number of large international events data bases. The Conflict and Peace Data Bank (COPDAB), for example, records the number of international events each day from 1945 to the present for each actor-target pair of nations, issue area, and level of conflict or cooperation. Other well known international events data collections include the World Events Interaction Survey (WEIS), the Comparative Research on the Events of Nations Project (CREON), and the Dimensionality of Nations project (DON).¹ Large data sets such as these are only the most conspicuous international event collections. Throughout the literature of international relations, many smaller data sets have been created by individual researchers. For example, scholars have studied formal and informal alliances (Russett, 1971; McGowan and Rood, 1975), economic sanctions (Hufbauer and Schott, 1983), and international crises. More than forty such collections exist in the ICPSR data archive (Vincent, 1983). Even much of the data on the U.S. presidency in international affairs is in the form of counts of events such as executive orders, international treaties, and executive agreements (King and Ragsdale, 1988:Chapter 3). International events data have been used to study a wide variety of empirical questions and have been used for forecasting and policy making (Azar et al., 1977; Rummel, 1979).

Because events and events data play such an important role in the fields of international relations, political scientists should have access to empirical methods especially suited to this type of data. Ideally, one ought to be able to theorize about the unobservable continuous processes in international politics and estimate their features with existing data on discrete events. Unfortunately, with few exceptions, scholars in international relations have neither designed nor exploited such methods. The most frequently used statistical model in this area, linear regression, makes the incorrect assumption that underlying continuous processes generate observations that are also continuous.

In this paper, I argue for a new strategy of modeling and data analysis in this field. Toward this end, I present several new but related statistical models developed for and applied to different types of existing international relations data sets. I also show how one can extract considerably more information by this explicit modeling strategy. Computer programs written to estimate all the statistical models presented here are easily accessible.²

The first section of this paper provides a brief discussion of the existing methodology used in the study of international relations. The elements of a more sophisticated methodology exist in this literature, but only extremely simplistic

¹ For studies of the quality of these data, see Azar and Ben Dak (1975), Kegley et al. (1975), and *International Studies Quarterly* (1983). See also McClelland and Hoggard (1969), Azar, Brody, and McClelland (1972), Burgess and Lawton (1972), and Azar (1982), for definitions, coding rules, and theoretical perspectives.

² I have written an easy-to-use computer program called COUNT to estimate the models presented here and many other models for event count data. The program works with the Gauss statistical package and is available either from me or from the makers of Gauss, Aptech Systems, Inc., 26250 196th Place South East, Kent, Washington 98042; 206-631-6679.

models have been used. The second section briefly outlines previous methodological work from other fields relevant to improving methodology in international relations. The third section builds a “hurdle” event count model of international alliances. The fourth section introduces a truncated event count regression model and a variance function estimation for data on international economic sanctions. The fifth section analyzes conflictual events between the U.S. and the Soviet Union in a new joint model that enables one to distinguish U.S. → Soviet conflict, Soviet → U.S. conflict, and tit-for-tat behavior. The final section concludes.

Empirical Methods in International Relations

An exhaustive classification and analysis of the various methodologies used in international relations research would be a valuable contribution, but it is not something I intend to do here. For present purposes, one can place empirical methods in this field in three basic categories.

First, many analyses use essentially descriptive statistics such as graphs, percentages, annual frequencies, and cross-tabulations. These are the basic tools of statistical description, highlighting what is to be explained by theory. As such, descriptive statistics are essential, but they are not relevant to statistical *inference*—estimating parameters from existing data. Thus, for example, descriptive statistics do not enable researchers to properly apply observed events data to the unobserved continuous processes of international conflict and cooperation.

Second, among the inferential methods used in this field, statistical techniques designed for continuous, interval level dependent variables are by far the most common. Linear regression analysis and bivariate correlations are the primary examples, but factor analysis, structural equation models, and most other commonly used techniques also belong in this category.

Consistent with the assumptions of regression analysis and other techniques designed for continuous variables, international politics can be thought of as inherently continuous. For example, imagine international cooperation between the U.S. and the Soviet Union as moving down a real number line representing time. At any instant, denoted by t and corresponding to a point on the line, this process has a continuous, interval level value λ_t , indicating how warm relations are between the two superpowers. If λ_t were observed all along the line (i.e., for every t), one would have a perfect dependent variable with which to analyze the ebb and flow of superpower cooperation and conflict.

Although conceptualizing U.S.–Soviet relations as a continuous process is intuitively reasonable, the values of this process are not observed at any point. Fortunately, the process does have observable consequences. For example, when U.S.–Soviet relations warm significantly (i.e., λ_t is large), a discrete event such as a summit conference or treaty signing might occur. These discrete events can be thought of as dots on the real number line. Many other events are also observable, such as visits of lower officials, verbal accusations, letters of protest, or troop mobilizations. Each of these events occurs with higher or lower probability as λ_t increases or decreases. The fundamental assumption underlying international events data sets is that by observing only these discrete events (the dots on the line), an observer still has a sense of the unobserved continuous variable, “U.S.–Soviet relations.”

Whenever analysts in international relations construct operational versions of their dependent variables from the lists of observable events, these variables will not be continuous, interval-level measures, as with many measures in political economy, for example. (They also tend not to be ordinal or nominal categorical variables, as is common with survey data in American politics). Instead, most international relations

data are “noncategorical discrete variables” (Maddala, 1983:51)—that is, analysts mark arbitrary divisions on the real number line corresponding to months, years, or some other convenient decision rule. The number of events in each time period are then recorded and used as a dependent variable in empirical analyses.³ Variables measuring the number of times a particular event occurs are called *event counts*. All such dependent variables take on values of zero or some positive integer. The number of conflictual events directed from all nations toward the U.S. in a year, the number of cooperative events directed from the U.S. to the Soviet Union, the number of international alliances, the number of nations involved in an economic sanction, and the number of coups d’etat in African states are a few examples of event counts.⁴

What happens when event count data are analyzed by linear regression and related techniques? The usual procedure is to conceptualize something like $E(Y_t) = \lambda_t > 0$ as the expected number of events or the rate of event occurrence at time t . The realized number of events y_t (for a finite number of points, $t = 1, \dots, n$) is the dependent variable. Researchers then typically regress y_t on a set of explanatory variables. This procedure has been shown to yield surprisingly large inefficiencies and nonsensical results (see King, 1988).

To get around the severe heteroskedasticity and other problems associated with this procedure, some have taken the natural log of y_t and regressed it on the same explanatory variables. The log of zero is not defined, however, so ad hoc procedures are used. The most common of these is to add a small constant to y_t before taking logs, but this seemingly innocuous procedure introduces arbitrarily large biases into the analysis. King (1988) showed that by making small adjustments in the value of this constant, one could make the parameter estimates biased by almost any amount in any direction. However, no general procedure exists to avoid these biases in the context of the logged regression model.

Some empirical analyses in international relations should probably be disregarded entirely, but in many instances they simply fail to extract all potential information from the data. In some cases, scholars have probably missed substantial patterns and relationships that could have been found in their data with more powerful techniques. This problem is particularly serious in international relations because the data tend to be especially noisy, with very large amounts of measurement error. For example, Howell (1983) showed that the COPDAB and WEIS data sets disagree on the direction of change in levels of U.S.–Soviet cooperation and conflict in as much as 29 percent of years examined. More troubling is Vincent’s (1983) finding that many of the inconsistencies between these two data sets can be accounted for by systematic rather than random variation. In data with such a low signal-to-noise ratio, more powerful statistical methods tuned to the special nature of these data can be more valuable than decades of new data collection projects.

The third category of statistical methods used in international relations are Poisson process models (see Richardson, 1944; McGowan and Rood, 1975). These methods are more closely applicable to the special nature of event count data, but they have been used in only very simple ways. An understanding of this research begins with two principles of the process generating a series of event counts: *independence* and *homogeneity*. The principle of independence holds that the probability of an event

³ Methods exist to use the events without this last stage of aggregation, but data on explanatory variables in political science tend not to be known at such a detailed level for each time t . See Allison (1984) and Tuma and Hannan (1984) on event history analysis.

⁴ When events are weighted in the usual way with non-negative integers, such as with the COPDAB scheme, the weighted event counts are also appropriately analyzed with the class of methods described here.

occurring at time $t + 1$, given what has occurred up to time t , is independent of all previous history within a single observation period. The principle of homogeneity holds that the rate of event occurrence, λ_t , is constant over period t . Under the independence principle, for example, wars are not contagious—that is, the occurrence of some wars do not increase the probability of future wars. Under the homogeneity principle, the rate of war outbreak is constant within entire observation periods (but may change between them). It is easy to see how tests of these assumptions are critically important to international relations research.

During an observation period, the rate of event occurrence remains unobserved but the count of events at the end of the observation is observed. From these first principles about the process generating a single event count, and a few regularity conditions, one can derive a formal probability distribution describing the probability that any number of events will occur. This is the Poisson distribution (see King, 1988:Appendix 1, for a derivation):

$$Y_t \sim f_p(y_t | \lambda_t, T_t) = \frac{e^{-T_t \lambda_t} (T_t \lambda_t)^{y_t}}{y_t!} \quad (1)$$

where T_t is the length of time in which events were counted during observation t . Since all the observation periods are usually the same length (years, months, etc.), this variable is set to one and the distribution is rewritten as follows:

$$Y_t \sim f_p(y_t | \lambda_t) = \frac{e^{-\lambda_t} \lambda_t^{y_t}}{y_t!} \quad (2)$$

Since at least Richardson (1944), scholars in international relations have often fit their data to a Poisson distribution. When the fit to the event count is good, they conclude that the two first principles about the underlying process are true. The virtue of this approach is that it enables users to analyze the observed events data but still make generalizations about the underlying process of interest.

Unfortunately, other sets of first principles can lead to the identical aggregate Poisson distribution of events, making some of these backward deductions to first principles indeterminate (see Houweling and Kune, 1984). For example, suppose one were analyzing the outbreak of war but the rate of outbreak λ_t was heterogeneous (i.e., varied over the years). If we merely assume that the realizations of the process (the events) do not influence the expected rate (λ_t), a Poisson distribution of the counts would still fit the data, and one might falsely conclude that λ_t was constant over t . The reason for this is explained by Cramer's theorem: the sum of two independent Poisson random variables is itself a Poisson random variable. All backward deductions to first principles are not invalid, but we must pay much closer attention to probability theory in attempting to make such generalizations.

Another problem with this third methodology in international relations is that it can only address very narrow questions about randomness or deviations from randomness. For example, in part of their analysis McGowan and Rood (1975) use one period of one hundred years to study the pattern of alliance formation and its fit to a Poisson distribution. Cramer's theorem essentially allows them to partial out and then ignore variation in the expected number of events over the years. However, this information is arguably among the most interesting parts of the research problem. Indeed, such uses of the Poisson process models are analogous to performing a regression analysis, discarding the parameter estimates, and reporting only a test for normality! McGowan and Rood did explore variation in the expected number of events, but existing methodology limited them to the ad hoc procedures of breaking up the periods for further analysis. More appropriate methods (not available at the

time of their article) would provide them with a way to model explicitly the systematic variation in the expected number of events.⁵

Although these Poisson process models are on the right track by explicitly modeling the underlying process and using available observed events for empirical estimation, considerably more information exists in events data than one can hope to extract with such methods.

Event Count Regression Models

One significant improvement in the methods used to study international politics is to combine the regression concepts from the second category of methods in international relations with the Poisson process models from the third. The basic form of the solution to this problem was analyzed by King (In press-b, 1988). I briefly summarize the results here.

The unobserved nonrandom variable of interest is λ_t and refers, for example, to the degree of international conflict over time. If this were measurable, it would be included directly in the analysis. Because, instead, only a count of events from this process is observed at the end of each period, the basic procedure is to assume that the process being analyzed within *each* observation period (year, month, or other) may be characterized by the two first principles, above. Then this count of events occurring within observation period t , Y_t , may be described by a Poisson distribution with mean $E(Y_t) = \lambda_t$ (see Equation 2).

Finally, we specify the way this continuous underlying process varies as a function of measured explanatory variables:

$$E(Y_t) = \lambda_t = \exp(x_t\beta) \quad (3)$$

where x_t is a vector of k explanatory variables and β is a $k \times 1$ parameter vector indicating the influence of each explanatory variable on λ_t . x_t can include continuous, dichotomous, or any other type of meaningful explanatory variables. The exponential is the functional form chosen because λ_t must always be positive, and for other theoretical reasons detailed by King.

Although the left-hand side of Equation 3 is completely unobservable, the model does make it possible to estimate β , the effect of the explanatory variables on the dependent variable, using the method of maximum likelihood (see King, In press-a). The likelihood function may be written as

$$L(\beta|y) = \prod_{i=1}^n f(y_i|\lambda_i) = \prod_{i=1}^n \frac{e^{-\lambda_i}(\lambda_i)^{y_i}}{y_i!} \quad (4)$$

with $\lambda_i = \exp(x_i\beta)$.

The basic idea of maximum likelihood turns on the concept of a probability distribution. A probability distribution is used to calculate the uncertainty involved in the outcome of an experiment (e.g., a coin landing heads for three flips in a row), given some parameter ($p = 0.5$ if the coin is fair). Given a discrete probability distribution like this, one can easily calculate the absolute uncertainty associated with any outcome by plugging in the values for the parameters and the outcome of interest into the probability distribution; the resulting measure of absolute uncertainty is called a "probability," which ranges between zero and one. Inference, on the other hand, requires an almost exactly opposite calculation. Here the goal is to assume knowledge of the outcome of the experiment (three flips were tossed, all landed heads)

⁵ Essentially the same mistake was made by scholars in three separate disciplines over five decades in explaining the frequency of appointments to the U.S. Supreme Court (see King, 1987).

and to calculate the uncertainty associated with a particular parameter value ($p = ?$). The likelihood function [$L(p|y)$], which is assumed to be proportional to the probability distribution [$Pr(y|p)$], enables one to calculate the relative probabilities of different values of the parameter (p) having produced the data we actually observed. In this case, by fixing y at the observed data, one may substitute in hypothetical values of the parameters β and watch the value of the likelihood go up or down. The values of the parameters for which the likelihood is maximized have the highest relative likelihood of having generated the observed data (see King, In press-a).

Since we are primarily interested in finding the maximum, any representation of the likelihood function that preserves ordinal rankings of $L(\beta|y)$ with respect to β may be used. A representation that is particularly convenient mathematically is the *log-likelihood*. In the case of the Poisson regression model, the log-likelihood is as follows:

$$\ln L(\beta|y) = \sum_{i=1}^n \{y_i(x_i\beta) - \exp(x_i\beta) - \ln(y_i!)\} \tag{5}$$

Since $\ln(y_i!)$ does not vary with test values of β , it can be dropped during estimation. Standard numerical maximization methods easily can be applied to this globally concave function by using one of many available computer programs (see King' 1988:Appendix 2) that produce maximum likelihood estimates and corresponding standard errors for each parameter estimate.

King (In press-b) then relaxes the assumption that events within an observation period must be independent and homogeneous by deriving a new probability distribution with parameters λ_t and σ^2 . If $\sigma^2 = 1$, the distribution reduces to the Poisson distribution and the assumption of independence. $\sigma^2 > 1$, when the data are overdispersed, is evidence of either contagion or heterogeneity; $\sigma^2 < 1$ is evidence of negative contagion.

This new probability distribution is called the generalized event count (GEC) distribution and may be written as follows:

$$Pr(Y_t = y_t | \lambda_t, \sigma^2) = f_{grec}(y_t | \lambda_t, \sigma^2) = \begin{cases} f_{grec}(0 | \lambda_t, \sigma^2) \prod_{j=1}^{y_t} \left[\frac{\lambda_t + (\sigma^2 - 1)(j - 1)}{\sigma^2 j} \right] & \text{for } y_t = 1, 2, \dots \\ e^{-\lambda_t} & \text{for } y_t = 0 \text{ and } \sigma^2 = 1 \\ (\sigma^2)^{-\lambda_t / (\sigma^2 - 1)} & \text{for } y_t = 0 \text{ and } \sigma^2 > 1 \\ (\sigma^2)^{-\lambda_t / (\sigma^2 - 1)} D_t^{-1} & \text{for } y_t = 0 \text{ and } 0 < \sigma^2 < 1 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where if $\sigma^2 < 1$ (indicating negative contagion) the probability of Y_t being greater than $[-\lambda_t / \sigma^2 - 1] + 1$ is zero, and where D_t is a binomial distribution summed from 0 to $[-\lambda_t / \sigma^2 - 1] + 1$.

This more general distribution then may be used to derive a more general estimator, enabling a researcher to estimate rather than assume independence or homogeneity of the underlying process. Substituting $\exp(x_i\beta)$ for λ_t , the log-likelihood—reduced to sufficient statistics—may be written as follows:

$$\ln L(\beta, \sigma^2|y) = \sum_{i=1}^n \left\{ C_i - y_i \ln(\sigma^2) + \sum_{j=1}^{y_i} \ln [\exp(x_i\beta) + (\sigma^2 - 1)(j - 1)] \right\} \tag{7}$$

where

$$C_i = \begin{cases} -\exp(x_i\beta) & \text{for } \sigma^2 = 1 \\ -\exp(x_i\beta) \ln(\sigma^2)(\sigma^2 - 1)^{-1} & \text{for } \sigma^2 > 1 \\ -\exp(x_i\beta) \ln(\sigma^2)(\sigma^2 - 1)^{-1} - \ln(D_i) & \text{for } 0 < \sigma^2 < 1 \end{cases} \tag{8}$$

The maximum of this function gives the values of β and σ^2 that have the highest relative likelihood of having generated the data, given the model. This equation is more complicated than that for the Poisson log-likelihood, but it has only one additional parameter. The likelihood is being maximized here with respect to β and σ^2 rather than just β .

The following three sections generalize these basic results to produce more sophisticated statistical models directly relevant to the study of different areas of international politics. Although the specific models presented below seem likely to be useful in other areas of the field, the general approach, more than any individual model, is of primary concern. Thus, these three should be considered illustrations of how one can model the underlying continuous processes of international relations and estimate features of these processes with only aggregate event counts.

Hurdle Modes of Instability and International Alliances

Do international alliances affect the rate at which nations enter into war? Deriving statistical models for the analysis of this question is the subject of this section. The data for the analysis come from the classic studies of Singer and Small (1966, 1968, 1969, 1972). The observed dependent variable is the number of nations who entered into a war each year from 1816 through 1965 (excluding data from the world wars, 1915–1919 and 1940–1945). The key explanatory variable is the percent of nations in the system involved in formal international alliances.

Since the purpose of this paper is to introduce new methodologies to international relations research, in this section and the two that follow I use the simplest possible specifications. These help display the essential features of the data and methods but do not attempt to address every sophisticated substantive argument in the literature. For a sampling of studies on international alliances, see Singer and Small (1968) Wallace (1973), McGowan and Rood (1975), Siverson and Tennefoss (1984) McDonald and Rosecrance (1985), and Walt (1985).

To begin the analysis, imagine a continuous unobserved nonrandom variable λ_t representing the instability of the international system, as indicated by the rate of war occurrence at time t . λ_t is always a positive number, since there is always some small chance of a war breaking out. Thus a larger value of λ_t increases the probability of an event, but at no point does it guarantee the occurrence or nonoccurrence of one.

The theoretical question of interest is whether the international system becomes more or less unstable when additional nations enter into formal international alliances. Of course, instability in the international system (λ_t) is unobservable at any of the infinite number of time points t in the process, but, the lists of events and the count of the number of events during each year is available.

Thus, let Y_t denote a random variable representing the number of nations that got involved in wars in year t . Y_t is assumed to have expectation $E(Y_t) \equiv \lambda_t$. By making the two plausible assumptions described above about the underlying process, we are led to the conclusion that Y_t is a Poisson random variable with mean (and variance) λ_t . This Poisson assumption is made all the more plausible by all the studies showing a reasonable fit to this distribution.⁶

⁶ Further analyses indicate that these data are slightly overdispersed, probably as a result of war being somewhat contagious. A reasonable correction, in this particular case only, is merely to double the estimated standard errors. The coefficients presented are consistent, and only marginally inefficient, in the face of modest overdispersion (Gourieroux, Monfort, and Trognon, 1984a, 1984b). One could improve the efficiency of these estimates and arrive at correct standard errors by moving to the GEC distribution. Because of the modest degree of overdispersion, I avoid this complication in this example.

TABLE 1. Poisson regression of nations in war.

<i>Variable</i>	<i>Estimate</i>	<i>Std. Error</i>	<i>t-Stat</i>
Constant	0.848	0.059	
Alliances	0.007	0.001	6.454

Log-likelihood = 49.749

Observations: 139

The systematic component of this model is specified as follows:

$$\lambda_t = \exp(\beta_0 + \beta_1 \text{Alliances}_t) \quad (9)$$

To estimate the parameters β_0 and β_1 , the right-hand side of this equation is substituted into the Poisson distribution in Equation 2, logs are taken and summed over all n observations, and the resulting log-likelihood function in Equation 5 is maximized. Estimates of this model appear in Table 1.

The focus of attention should be on the coefficient of the "Alliances" variable. To interpret its effects, note that the derivative of λ_t with respect to x_t is $\lambda_t \beta_1$. To make the interpretation more concrete, note that the empirical range of Y is from zero to eighteen nations entering into war, with a mean of 3.029. The percent of nations involved in alliances ranges from zero to eighty. Thus, consider the effect of a fifty percentage point increase in the number of nations involved in alliances in the typical year (that is, with about three nations entering into wars). This increase in alliances leads to about $0.007 \times 3.029 \times 50 = 1.06$ more nations expected to enter into war.

Since alliances establish peace among their signatories, it may seem odd that β_1 is positive. Alliances are also mutual defense pacts, however, sometimes formed to allow a nation to go to war. The standard error on this coefficient is quite small, and the t -statistic would surely lead one to conclude that this coefficient is significantly greater than zero. But do alliances really destabilize the international system, causing more nations to go to war? If alliances are made in part to ensure a nation's security (Waltz, 1979), this finding is surely questionable.

Further consideration of these results might lead one to specify a more sophisticated model of the underlying process. One possibility more consistent with international relations theory is to imagine that λ_t really represents two values, λ_{0t} and λ_{+t} . λ_{0t} is the rate at which the first additional nation gets involved in a war, or, in other words, the rate at which the international system switches from a constant number of participants in war to one additional participant. λ_{+t} , then, is the rate at which other nations get involved, given that at least one additional nation has become involved during the year. These unobserved processes may very well occur simultaneously. The substantive hypothesis is that the percent of alliances has a small or even negative effect on the probability of any additional nations being involved in wars. Once the first additional nation commits to an international conflict, however, the existence of additional international alliances will drag new nations into the fray. Vasquez (1987:121), for example, concludes that "alliances not only fail to prevent wars, but make it likely that wars that do occur will expand."

Modeling the onset of war separately from its escalation requires a two-part model. Mullahy's (1986) work on hurdle Poisson regression models represents the state of the art in this area; the discussion in this section draws, in part, on his work. I first define the general form of the hurdle regression model and then derive an estimable model as a special case.

Begin by defining a dummy variable d_t which takes on the value 0 when $y_t = 0$ and 1 otherwise (for $t = 0, \dots, n$). Then a Bernoulli distribution may be used to

describe the “hurdle” that the system goes through between no additional nations and some additional nations getting involved in international wars:

$$f_B(d_t|p_t) = Pr(Y_t = d_t|p_t) = p_t^{d_t}(1 - p_t)^{1-d_t} = \begin{cases} 1 - p_t & \text{for } y_t = 0 \\ p_t & \text{for } y_t \geq 1 \end{cases} \quad (10)$$

where the p_t parameter stands for the probability that $Y_t > 0$ according to a separate stochastic process, f_0 for the probability that $Y_t = 0$:

$$\begin{aligned} p_t &= Pr(d_t = 1) \\ &= Pr(Y_t > 0) \\ &= 1 - f_0(0|\lambda_{0t}) \end{aligned} \quad (11)$$

Conditional on at least one additional nation getting involved in a war, the distribution of Y_t is written as a truncated event count distribution. The method of deriving a truncated distribution is based directly on the basic conditional probability rule:

$$Pr(A|B) = \frac{Pr(AB)}{Pr(B)} \quad (12)$$

If f represents some event count distribution defined on the nonnegative integers, f_+ represents a corresponding truncated-at-zero event count distribution for the positive integers only:

$$f_+(y_t|\lambda_{+t}, y_t > 0) = \frac{f(y_t|\lambda_t)}{Pr(Y_t > 0)} \quad (13)$$

or, equivalently,

$$f_+(y_t|\lambda_{+t}) = \frac{f(y_t|\lambda_t)}{1 - f(0|\lambda_t)} \quad (14)$$

for $y_t \in \{1, 2, \dots\}$ and zero otherwise.

Note that f_0 and f_+ define the full stochastic nature of this process. In the standard Poisson regression model, for example, f_0 and f_+ have the same distribution with the same mean. In hurdle event count models, however, they may differ completely or merely because of different parameters. Following Mullahy (1986), I restrict attention to the case where f_0 and f_+ are both Poisson distributions, but where λ_{0t} may differ from λ_{-t} .

To construct the likelihood function, observations with $y_t = 0$ must be treated differently than observations with $y_t > 0$. The two parts appear in separate brackets in the likelihood function:

$$\begin{aligned} L(\lambda_{0t}, \lambda_{+t}) &= \left[\prod_{y_t=0} (1 - p_t) \right] \left[\prod_{y_t>0} p_t f_+(y_t|\lambda_{+t}) \right] \\ &= \left[\prod_{y_t=0} f_0(0|\lambda_{0t}) \right] \left[\prod_{y_t>0} [1 - f_0(0|\lambda_{0t})] f_+(y_t|\lambda_{+t}) \right] \end{aligned} \quad (15)$$

This likelihood function specifies $f_0(0|\lambda_{0t})$ for the observations with zeros, the probability of zero nations getting involved in war. For those years where at least one nation took up arms, the probability of a particular number of nations getting involved in war is equal to $1 - f_0(0|\lambda_{0t})$ multiplied by a truncated event count distribution with its own parameter.

A special case of this model is the Poisson hurdle regression model. Here we assume that both f_0 and f_+ are generated by nonidentically distributed Poisson processes (with means λ_{0t} and λ_{+t} respectively). To derive this model, we first calculate the probability of zero events with Equation 2 and simplify:

$$\begin{aligned}
 f(0|\lambda_{0t}) &= \frac{e^{\lambda_{0t}} \lambda_{0t}^0}{0!} \\
 &= e^{-\lambda_{0t}}
 \end{aligned}
 \tag{16}$$

We then derive the truncated Poisson distribution for the positive integers by calculating the probability that $Y_t = 0$ under a Poisson distribution with a different parameter:

$$f(0|\lambda_{+t}) = e^{-\lambda_{+t}} \tag{17}$$

and then substituting this result into Equation 14:

$$f_{+}(y_t|\lambda_{+t}) = \frac{\lambda_{+t}^{y_t}}{(e^{\lambda_{+t}} - 1)y_t!} \tag{18}$$

Only λ_{0t} and λ_{+t} separate the process governing the hurdle crossing from the process governing the number of nations involved in an existing war. These two parameters each vary in some unobserved way over the same time period. In general, we let each vary separately as functions of (possibly different) measured explanatory variables (x_{0t} and x_{+t}):

$$\begin{aligned}
 \lambda_{0t} &= \exp(x_{0t}\beta_0) \\
 \lambda_{+t} &= \exp(x_{+t}\beta_+)
 \end{aligned}
 \tag{19}$$

Reduced to sufficient statistics, the full Poisson hurdle regression model log-likelihood function may then be written as follows:

$$\begin{aligned}
 \ln L(\beta_0, \beta_+|y) &= - \sum_{y=0} \exp(x_{0t}\beta_0) + \sum_{y>0} \{\ln [1 - \exp(-e^{x_{0t}\beta_0})] \\
 &\quad + y(x_{+t}\beta_+) - \ln[\exp(e^{x_{+t}\beta_+}) - 1]\}
 \end{aligned}
 \tag{20}$$

which is easily maximized with respect to β_0 and β_+ . Indeed, since β_0 and β_+ appear in separate terms in the log-likelihood function, these terms may be maximized separately. In my experience, however, even simultaneous estimations converge very quickly. Note that if $X_0 = X_+$ and $\beta_0 = \beta_+$,

$$\ln\{1 - \exp[-\exp(x_t\beta)]\} - \ln\{\exp[\exp(x_t\beta)] - 1\} = -\exp(x_t\beta) \tag{21}$$

and in this special case the Poisson hurdle specification reduces directly to the basic Poisson regression model. In empirical examples, of course, the effect parameters are not necessarily equal and the explanatory variables need not be the same.

Another point of interest about this model is the implied parameterization of the probability that the hurdle is crossed:

$$Pr(Y_t > 0|\lambda_{0t}) = 1 - \exp(-\lambda_{0t}) = 1 - \exp[-\exp(x_{0t}\beta_0)] \tag{22}$$

If we were not deriving a statistical model from basis assumptions made about the deeper underlying process, as we are here, but instead were putting together a data-based model, the first choice would probably be a logit:

$$Pr(Y_t > 0|\lambda_{0t}) = [1 - \exp(-x_{0t}\beta_0)]^{-1} \tag{23}$$

Because its justification is derived from first principles much closer to international relations theory, Equation 22 should be more satisfying than the ad hoc specification in Equation 23. At the same time, however, researchers are probably more familiar with the logistic specification.

A reasonable question, then, is how the two curves differ. Figure 1 provides an

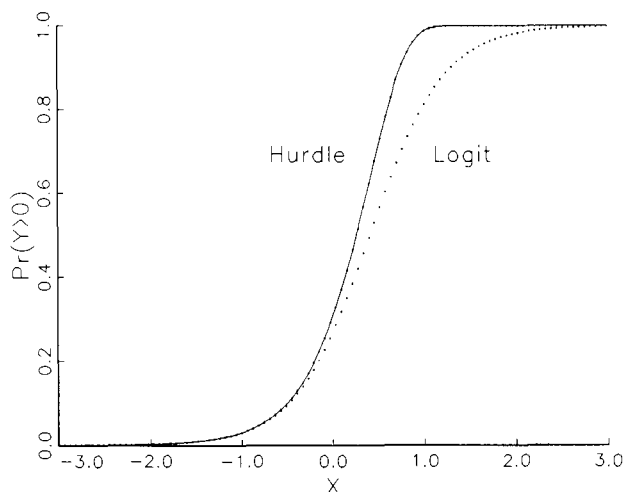


FIG. 1. Functional forms.

intuitive answer, showing that the logistic curve is symmetric while the one we derived is not. The two curves coincide near the bottom as the lower bound of zero “bends” the line up. At the top, the upper bound has an effect much later in our curve than in the logit. This asymmetric shape is quite plausible here since in the hurdle model the probability of crossing the hurdle might arbitrarily approach 1.0 when the expected number of nations initiating conflict (λ_{+t}) is high. The existence of the other process, represented by λ_{+t} , thus serves to release some of the pressure near the top of the curve and creates the small asymmetry.

Without a large number of observations, empirically distinguishing between the fit of these two alternative specifications would be difficult. Nevertheless, having derived this form from deeper principles about the theoretical process being analyzed, discovering such subtle sophistications gives one further confidence in the first principles and the derived model.

Consider again the data on the effects of formal international alliances. Table 2 provides estimates of the Poisson hurdle regression model with a constant and the percent of nations involved in alliances in each equation. The key result is the difference between the coefficients on the alliance variable in the two parts of the model. The existence of international alliances has no noticeable effect on the presence of war (see the small coefficient and the near zero t -statistic). However, once

TABLE 2. Poisson hurdle regression of nations in war.

Variable	Estimate	Std. Error	t -Stat
Constant ₀	0.511	0.241	
Alliances	0.001	0.006	0.239
Constant ₁	1.010	0.068	
Alliances	0.007	0.001	5.688

Log-likelihood = 68.43

Observations: 139

TABLE 3. Poisson hurdle regression of nations in war, an alternative specification.

<i>Variable</i>	<i>Estimate</i>	<i>Std. Error</i>	<i>t-Stat</i>
Constant ₀	0.141	0.290	
Alliances	0.003	0.006	0.430
Nations _{t-1}	0.143	0.031	4.587
Constant ₁	1.010	0.068	
Alliances	0.007	0.001	5.688

Log-likelihood = 75.859

Observations: 139

a new war has begun (i.e., once the hurdle has been crossed), alliances have essentially the same effect as they were estimated to have in the standard Poisson regression model.

Aside from these important substantive differences between the two models, a likelihood ratio test of the improvement provided by the hurdle model can easily be calculated by taking advantage of the fact that the standard model is nested within it. With two degrees of freedom, the chi-square statistic is $2(68.43 - 49.74) = 37.36$; so one can reject with considerable confidence the hypothesis that no improvement occurred with the hurdle model.

Further analysis into this substantive problem would require the inclusion of appropriate control variables and the testing of a number of specifications to check for sensitivity in the empirical results. Although this is obviously not my purpose here, I do present one additional estimation in Table 3. This estimation includes a lag of the dependent variable in the first but not the second part of the model.

Several features of this alternative specification are worthy of note. First, this model emphasizes that the parameterizations of λ_{0t} and λ_{+t} need not be the same. Second, the significant positive coefficient for Nations_{t-1} indicates that when more nations are involved in war one year, the probability increases that the hurdle will be crossed again in the following year (i.e., at least one more nation will be involved). The coefficient on alliances does increase somewhat, but it is still only half the size of its standard error, so the substantive interpretation does not change. Third, although I do not provide a full complement of event count models that account for autocorrelation in this paper, this specification is one example of how to model time dependence in λ_{0t} and λ_{+t} . For further analysis, one could include lags in the model for λ_{+t} (instead of y_t) or include additional lags in either portion of the model. Fourth, note that the two coefficients and standard errors in the model for λ_{+t} are identical in Tables 2 and 3. This is a consequence of β_0 and β_+ falling in separate terms in the likelihood function.

Finally, from one perspective, including a lag of the dependent variable makes the model internally inconsistent. The first principles required to derive the Poisson distribution for this model included the assumption that events within each year are independent. However, using the lag of the dependent variable as an explanatory variable implies dependence across years. The inconsistency can be resolved in two ways. We could relax the assumption of independence within observations by using the generalized event count distribution. This can and does work fine, but in this empirical case with only modest over-dispersion, I find that it has no substantial effect on the results. Alternatively, one could insist that war is not contagious in the short term but that broad aggregate patterns in the rate of the onset of war are dependent. Just how plausible this assertion is depends on the features of one's

empirical question and on the length of the observation periods. In fact, the only study that attempts explicitly to model the time series properties in event count models makes this assumption (see Holden, 1987).

Truncated and Variance Function Models of Multilateral Economic Sanctions

What are the conditions under which nations are able and willing to cooperate to achieve political objectives (see Oye, 1986)? I analyze one important example of this situation: international cooperation in imposing economic sanctions on a target country. In an interdependent world, unilateral economic sanctions are seldom successful. Without sufficient cooperation in imposing a sanction, target countries can often switch to alternative markets at little cost. Thus, to achieve political objectives through economic sanctions, securing international cooperation is usually essential.

Data to help analyze this and many other related questions about economic sanctions have been collected by Martin (In progress) and Hufbauer and Schott (1983). The data to be used here involve seventy-eight incidents of economic sanctions since the economic blockade of Germany in World War I. An economic sanction is defined as "the deliberate government-inspired withdrawal, or threat of withdrawal, of 'customary' trade or financial relations" (Hufbauer and Schott, 1983:2).

The continuous unobserved nonrandom variable in this problem is the level of international cooperation in economic sanctioning behavior, λ_t . In principle, λ_t exists at all points in time, since economic incentives of all kinds are ubiquitous tools in foreign policy. If λ_t were observable and measurable, one could use it as an optimal dependent variable; but optimal variables rarely exist in political science. In its place as the dependent variable, however, we have the number of nations participating in each of seventy-five instances of economic sanctions.⁷ Thus the observed dependent variable is again an event count. However, this particular event count has at least two interesting features worthy of future study. I now analyze this model in the traditional way, and then complicate the model in two ways to exploit these features of the data.

Consider, first, a simple model of the systematic component for λ_t , the expected number of nations cooperating:

$$E(Y_t) \equiv \lambda_t = \exp(\beta_0 + \beta_1 \text{Stability}_t + \beta_2 \text{Cost}_t) \quad (24)$$

where the variable Stability_t is a measure of the target country's overall economic health and political stability during the sanctions episode, abstracting from the effects of the sanctions. Stability_t is coded on a scale from 1 (distressed) to 3 (strong and stable). The hypothesis is that more nations will join the sanction if the target country is weak and, therefore, the effort is likely to be successful. Cost_t is a measure of the effect of the sanction on the sanctioning (or "sender") country. Hufbauer and Schott's (1983:84) analysis implies that a more costly sanction will encourage the sender country to obtain cooperation from other nations. Another possibility is that a sanctioning country willing to bear high cost is also a country with strong resolve; other nations might be more likely to participate in this situation. Cost is coded from 1 (net gain to sender) to 4 (major loss to sender).

Appealing to the two principles required to derive the Poisson distribution in this case seems quite unreasonable. Indeed, a key feature of this substantive problem is

⁷ I deleted three outliers. Whereas the mean number of senders is 3.4 in the sample of seventy-five, the three omitted sanctions included primarily U.N. sponsored activities where a large proportion of the nations of the world joined the effort.

TABLE 4. Negative binomial regression of nations sanctioning.

<i>Variable</i>	<i>Estimate</i>	<i>Std. Error</i>	<i>t-Stat</i>
β_0	0.707	0.409	1.726
Stability	-0.217	0.145	-1.496
Cost	0.510	0.108	4.711
$\hat{\gamma}$	0.607	0.334	1.814

Log-likelihood = 124.769

Observations: 75

Mean Number of Nations Participating = 3.4

the contagion among nations. In many cases, the sending nation intentionally tries to get others to join the effort against the target. Hence, the assumption of independence is not only implausible, but it would seem to strip the problem of one of its most interesting features.

Before complicating the model, then, I move from the basic Poisson process to the generalized event count (GEC) distribution (Equation 6). Since over-dispersion (resulting in this case from contagion) is almost certain to be present, σ^2 will be greater than one. To simplify later analysis, I use the negative binomial distribution, a special case of the GEC when $\sigma^2 > 1$. Also for later simplification, I reparameterize this distribution so that $\sigma^2 = 1 + \theta$ and $\theta = \exp(\gamma)$. Thus, the expected value is still

$$E(Y_t) \equiv \lambda_t \quad (25)$$

but the variance is now

$$V(Y_t) \equiv (1 + \theta)\lambda_t = (1 + e^\gamma)\lambda_t \quad (26)$$

The full distribution is then written for a single observation as follows:

$$f_{nb}(y_t | \lambda_t, \theta) = \frac{\Gamma(\lambda_t/\theta + y_t)}{y_t! \Gamma(\lambda_t/\theta)} \theta^{y_t} (1 + \theta)^{-(\lambda_t/\theta + y_t)} \quad (27)$$

where $\theta = \exp(\gamma)$. Note that this reparameterization has no substantive effect on the present analysis, but it will make calculations easier in the models developed below.⁸ Larger values of γ mean that more overdispersion (and therefore contagion) is present in these data.

By substituting the right-hand side of Equation 24 into the probability distribution in Equation 27, taking logs, and summing over observations, the log-likelihood may be derived. Maximizing this function gives the maximum likelihood estimates of β_0 , β_1 , β_2 , and γ . Empirical results appear in Table 4.

First note the level of dispersion, $\hat{\gamma}$. If nations chose to institute economic sanctions unilaterally, the variance of Y_t would equal its mean, λ_t . In this case, however, the variance is $[1 + \exp(0.607)] = 2.83$ times greater than its mean, indicating moderate contagion in sanctioning decisions (but see the next model, below).

Both of the explanatory variables appear to have modest effects. The stability of a target country decreases international participation by about $-0.217\lambda_t$ more nations. Thus, for the typical sanction with about 3.4 nations participating, an increase on the stability scale from a distressed nation to a strong and stable nation decreases participation by about $-0.217 \times 2 \times 3.4 = -1.48$ more nations, although this effect

⁸ The other advantage of this parameterization is that all parameters now vary between negative and positive infinity. This is an advantage because the theory and practice of numerical optimization have not yet dealt adequately with bounded parameter spaces.

is not quite significant by conventional standards. For each unit increase in the four-point cost-to-sender scale, an additional $3 \times 0.510 = 1.53$ more nations are convinced to join in the sanction.

In virtually all substantive problems in international relations, one has only a sample of realized values from the process governed by λ_t . In many problems, however, the sample is either roughly random or periodic, as in monthly or yearly observations. For the number of nations entering into wars, analyzed in Section 4, the sample was based on annual data; but, in the present example realizations of the process are only observed when a major sanction takes place. Thus, these data likely have two types of *selection bias*. The first is that if zero nations cooperate in an economic sanction, the observation never appears in the data set. As a result, the observed data y_t are always greater than zero. Second, the cases included in the analysis are "somewhat biased toward the big case" (Hufbauer and Schott, 1983:2). Thus, some cases of economic sanctions with relatively few nations participating did not come to the attention of the coders.

I now construct a model that takes into account the truncation-at-zero problem. This model will not directly address the second type of selection bias, where the truncation threshold is greater than zero and probably stochastic, but this problem seems less severe in this substantive example (see Achen [1986] for insight into selection bias problems in the context of linear models, Maddala [1983] for a review of limited dependent variable problems in general, and Cohen [1960] and especially Grogger and Carson [1988] for studies of truncated count data models).

The key problem with sample selection appears when the rule for selecting observations into the analysis is correlated with the dependent variable. Selection on an explanatory variable causes no particular problems. The present example is extreme since an international economic sanction is observed and included in the data set only if $y_t > 0$, so the selection rule is deterministically related to the dependent variable. What effect do sample truncation problems have on empirical results? An intuitive way to think of the problem is that sample truncation causes the regression line to be artificially bounded (in this case from below). The more dramatic the truncation the flatter the regression line is estimated to be. Thus, *truncation causes effect parameters to be biased toward zero*. The estimates in Table 4 are probably too small. Estimating these parameters from a model that explicitly takes into account the truncation should yield larger estimates.

A truncated-at-zero data distribution can be derived from the parent negative binomial distribution just as it was for the Poisson in Section 4. Equation 14 provides the necessary formula. The probability of a zero under a negative binomial distribution is derived by substituting $y_t = 0$ into Equation 27:

$$f_{nb}(0|\lambda_t, \theta) = (1 + \theta)^{-\lambda_t/\theta} \quad (28)$$

Thus, the full truncated-at-zero negative binomial probability distribution may be written as follows:

$$f_{nb}(y_t|\lambda_t, \theta) = \frac{\Gamma(\lambda_t/\theta + y_t)}{y_t! \Gamma(\lambda_t/\theta) [1 - (1 + \theta)^{-\lambda_t/\theta}]^{\theta y_t}} \theta^{y_t} (1 + \theta)^{-(\lambda_t/\theta + y_t)} \quad (29)$$

The bracketed term in the denominator is the only difference between the negative binomial and this truncated negative binomial distribution. If negative contagion seems a reasonable possibility, one could generalize this to a truncated-at-zero GEC distribution. But since countries such as South Africa that are likely to create negative contagion are not in the habit of imposing economic sanctions on other nations, this generalization seems unnecessary in the present example.

TABLE 5. Truncated negative binomial regression of nations sanctioning.

<i>Variable</i>	<i>Estimate</i>	<i>Std. Error</i>	<i>t-Stat</i>
Constant ₀	-0.785	1.623	-0.483
Stability	-0.869	0.453	-1.918
Cost	1.265	0.619	2.045
$\hat{\gamma}$	1.531	0.428	3.578

Log-likelihood = 155.571

Observations: 75

The log-likelihood is then derived directly from this distribution:

$$\ln L(\lambda_t, \theta|y) = \sum_{i=1}^n \left\{ \ln \Gamma \left(\frac{\lambda_t}{\theta} + y_t \right) - \ln \Gamma \left(\frac{\lambda_t}{\theta} \right) + y_t \ln(\theta) - \left(\frac{\lambda_t}{\theta} + y_t \right) \ln(1 + \theta) - \ln[1 - (1 + \theta)^{-\lambda_t/\theta}] \right\} \quad (30)$$

with λ_t defined in Equation 24 and $\theta = \exp(\gamma)$. The maximum likelihood estimates based on this model appear in Table 5.

Note that the log likelihood has increased significantly, indicating that this model is more likely to be the true one than the untruncated model estimated in Table 4. The key substantive result here is that by explicitly taking into account the truncation, the effect parameters are now estimated to be more than three times as large. Thus, if about 3.4 nations could be expected to participate in an economic sanction on average, a two-point increase on the stability scale (from 1 to 3) would decrease the number of nations participating in the sanction by about $-0.869 \times 2 \times 3 = -5.21$ more nations (compared to an estimated effect of -1.48 nations from the untruncated negative binomial model). The t-statistic has also increased. For each unit increase in the four point cost-to-sender variable, an additional $3.4 \times 1.265 = 4.3$ more nations are convinced to join in the sanction (compared to only 1.53 under the negative binomial). In addition, the truncated model allows a better estimate of contagion among nations in sanction participation: the variance is now $1 + \exp(1.531) = 5.62$ times greater than the mean, reflecting a considerable amount of contagion. The fundamental lesson here is that explicit modeling of the underlying continuous process and its relationship to the observed data dramatically improves empirical results.

I now complicate this truncated model further with a more explicit examination of the contagious process by which nations convince other nations to participate in economic sanctions. For all the models presented until now, I have assumed that the variance of Y_t was proportional to its mean. Thus, in the present parameterization,

$$V(Y_t) = \lambda_t(1 + \theta) \quad (31)$$

Both the mean and the variance are assumed to vary over the different observations, but the two are closely tied together. In the present substantive example, however, θ is not a nuisance parameter. It indicates the degree to which participation in economic sanctions is contagious, a fundamental part of the research problem. Whereas theory usually causes us to focus on how the mean λ_t varies as a function of a set of explanatory variables, the present substantive example causes us to focus on θ as well.

The general problem is called *variance function estimation* and, although some work has been done in the area (see Davidian and Carroll, 1987), it has heretofore not been extended to event count models. I derive this new model by first adding the subscript t to θ so that it can vary over the observations. θ_t is then conceptualized as the degree of contagion among nations at time t . Like λ_t , θ_t is not observed at any point in time, but something like it certainly does exist in theory. We can use the same events data to estimate the influence of specified explanatory variables on θ_t .

Since θ_t has some of the same formal characteristics as λ_t , we use the same functional form. Hence,

$$\theta_t = \exp(z_t\gamma) \quad (32)$$

where z_t is a vector of k_1 explanatory variables and γ is now a $k_1 \times 1$ parameter vector. The variables selected to comprise z_t can be the same as or different than the ones in the mean function, x_t .

The log-likelihood function is derived for this model by substituting Equations 32 and 24 into the truncated negative binomial probability distribution and taking logs. One could create a simpler version of this model by substituting into the untruncated negative binomial distribution, but the truncated distribution is most appropriate for the present case. Thus,

$$\begin{aligned} \ln L(\beta, \gamma|y) = \sum_{t=1}^n \left\{ \ln \Gamma \left[\frac{\exp(x_t\beta)}{\exp(z_t\gamma)} + y_t \right] - \ln \Gamma \left[\frac{\exp(x_t\beta)}{\exp(z_t\gamma)} \right] \right. \\ \left. + y_t(x_t\beta) - \left[\frac{\exp(x_t\beta)}{\exp(z_t\gamma)} + y_t \right] \ln [1 + \exp(x_t\beta)] \right. \\ \left. - \ln \left[1 - [1 + \exp(z_t\gamma)]^{-\frac{\exp(x_t\beta)}{\exp(z_t\gamma)}} \right] \right\} \end{aligned} \quad (33)$$

For present exploratory purposes, I let z_t contain just one variable, US, which is coded as 1 if the U.S. is the major sender and 0 otherwise. Empirical estimates appear in Table 6.

Note first that the log-likelihood for this model is considerably higher than that for the standard negative binomial model and the truncated negative binomial generalization. The advantages of this truncated variance-function negative binomial model over the standard models are apparent. First, this more realistic model of how contagion varies allows better estimates of the β parameters; both turn out to be larger and more precisely estimated here than in the previous negative binomial model. Second, this model enables one to extract considerably more information from the same data. For example, the negative coefficient on the US variable says

TABLE 6. Truncated negative binomial regression with variance function of nations sanctioning.

Variable	Estimate	Std. Error	t-Stat
$\hat{\beta}_0$	-0.868	1.232	-0.705
Stability	-0.938	0.451	-2.070
Cost	1.417	0.517	2.740
$\hat{\gamma}_0$	2.328	0.830	2.806
US	-1.441	0.859	-1.678

Log-likelihood = 172.608

Observations: 75

TABLE 7. Independent poisson regressions of U.S.–Soviet Conflict.

Variable	U.S. → Sov.			Sov. → U.S.		
	Est.	Std. Err.	t-Stat	Est.	Std. Err.	t-Stat
Constant	3.888	0.030		3.843	0.095	
Military\$	-0.003	0.001	-5.050	-0.002	0.002	-1.042
President	0.120	0.018	6.576	0.468	0.017	28.323

U.S. → Sov. Log-likelihood = -223.72

Sov. → U.S. Log-likelihood = -241.80

Mean number of events U.S. → Sov. = 44.53

Mean number of events Sov. → U.S. = 52.64

Observations: 28

nothing about how many more or fewer nations will participate when the U.S. is the leading sender, but it does indicate that international participation in sanctioning targets chosen by the U.S. is less contagious than when other nations are the leading senders. This effect reflects the fact that the U.S. tends to make economic sanctioning decisions without receiving prior support from its allies. In these cases, then, decisions to participate by other nations tend to be more isolated.

Unobserved Poisson Variable Models of Reciprocity in U.S.–Soviet Relations

Does military spending by one superpower deter or provoke the other superpower? The observed dependent variable is from the COPDAB event data archive. For 1951 to 1978, the number of conflictual actions the U.S. directed at the Soviet Union and the number the Soviets directed at the U.S. are recorded as annual event counts.

I again begin by focusing on the underlying continuous processes of interest. Let θ_{1t} be a nonrandom variable representing the degree of conflict originating with the U.S. and directed at the Soviet Union. Similarly, let θ_{2t} be the degree of conflict originating with the Soviet Union and directed at the U.S.⁹ To explain conflict originated by the U.S., I include a measure of Soviet military expenditures (in constant 1970 billions of U.S. dollars) and a dummy variable for the party of the U.S. president (coded 1 for Democratic presidents and 0 for Republicans). U.S. military expenditures (also measured in constant 1970 billions of U.S. dollars) and the same president variable are included to explain conflict originated by the Soviet Union (military expenditure data are from Ward, 1984:311).

I begin the empirical estimation with two independent Poisson regression models, allowing θ_{1t} and θ_{2t} to be exponential linear functions of military spending of the other superpower and the dummy variable for the party of the U.S. president. Empirical results appear in Table 7.

The variable Military\$_t\$ refers to Soviet domestic military spending in the first equation and U.S. spending in the second. In both models, this coefficient is moderately negative, indicating that military spending by a superpower deters conflict directed at it by the other superpower. In the first equation, explaining the conflict directed from the U.S. toward the Soviets, a ten billion dollar increase in the Soviet defense budget yields about $-0.003 \times 10 \times 44.53 = -1.3$ fewer hostile acts directed at the Soviets per year. The coefficient is smaller in the Soviet → U.S. equation, with a larger standard error. Nevertheless, by running only these two

⁹ θ_{1t} and θ_{2t} could be called λ_{1t} and λ_{2t} to be consistent with previous usage. I introduce this alternative notation here in order to be consistent in the more sophisticated model to be developed below.

regressions, an analyst might reasonably conclude that deterrence works: the level of conflictual actions directed at a nation appears to drop if that nation increases its defense budget.¹⁰

Although these results seem intuitive from one perspective, a further analysis yields considerably different insights. The most critical problem in the current setup is defining more precisely what θ_{1t} and θ_{2t} should mean. As it is, they represent the overall level of hostility directed from one superpower to the other. However, in theory at least, one can separate out at least two types of conflictual dyadic behavior in each of these nonrandom variables. For example, some of the aggregate level of U.S. \rightarrow Soviet conflict is surely domestically generated. No matter how good relations are, the U.S. will probably always object to what it views as Soviet human rights abuses. Similarly, the Soviets are unlikely to stop complaining about effects of U.S.-style imperialism. On the other hand, some of the conflictual behavior between these two nations is merely a response to each others' conflictual actions. For example, the U.S. claims to have caught a Soviet spy and expels a half-dozen members of the Soviet embassy. In response, the Soviets expel a dozen members of the U.S. embassy in Moscow. This tit-for-tat conflictual behavior or specific reciprocity may continue for several more iterations until one side eventually stops. In theory, pure conflict directed toward another superpower and tit-for-tat behavior are fundamentally different types of superpower relations. For different substantive purposes either or both might be of interest. For example, in a study of the domestic sources of international conflict, tit-for-tat behavior should probably be excluded or analyzed separately.

In international relations theory, reciprocity is of considerable interest. Under certain conditions, it can lead to cooperation or even a semipermanent "feud" between nations (see Axelrod, 1984; Axelrod and Keohane, 1985; Keohane, 1986). Whereas λ_{1t} and λ_{2t} are the degrees of conflict originated solely by the U.S. or solely by the Soviet Union, respectively, I define a separate variable, λ_{3t} , for the degree of tit-for-tat conflictual behavior. This specification assumes that superpower responses to each other occur at roughly the same level and intensity; if they did not, one might think about including a separate tit-for-tat variable for each country.

Thus, let $\theta_{1t} \equiv \lambda_{1t} + \lambda_{3t}$ be the total degree of conflict directed from the U.S. to the Soviet Union, with λ_{1t} as the domestically originated portion of this conflict. Similarly, let $\theta_{2t} \equiv \lambda_{2t} + \lambda_{3t}$ be total conflict directed from the Soviet Union at the U.S., with λ_{2t} as the domestically originated portion. These three variables are each unobserved, nonrandom, and theoretically distinct. The goal is to derive explanatory models for λ_{1t} , λ_{2t} , and λ_{3t} . Just as with the models in the previous sections, we could easily specify:

$$\lambda_{1t} = \exp(x_t\beta) \quad (34)$$

$$\lambda_{2t} = \exp(w_t\gamma) \quad (35)$$

$$\lambda_{3t} = \exp(z_t\delta) \quad (36)$$

where x_t , w_t , and z_t are vectors of explanatory variables and β , γ , and δ are effect parameter vectors. However, not only are λ_{1t} , λ_{2t} , and λ_{3t} unobserved, as they were in the previous models, but we have no obvious empirical measure of any of the three. The COPDAB data set records total conflictual events with an actor and target, but none of the variables distinguish domestically originated from tit-for-tat behavior. With the model I derive below, existing data *can* be used to estimate β , γ , and δ . This case is an interesting example of my approach: deriving coding rules for distin-

¹⁰ Note that the two log-likelihoods cannot be compared with each other because they are calculated from different data sets. They are included here for later reference.

guishing between the types of international conflict seems difficult, if not impossible. Thus, existing data combined with this new model are almost sure to reveal more useful information than would an expensive new data collection effort.

I begin by assuming that Y_{1t}^* , Y_{2t}^* , and Y_{3t}^* are unobserved random Poisson variables representing international conflict of the U.S. directed toward the Soviet Union, the Soviet Union toward the U.S., and tit-for-tat actions directed toward each other, respectively. Thus,

$$Y_{1t}^* \sim f_p(y_{1t}^* | \lambda_{1t}) \quad (37)$$

$$Y_{2t}^* \sim f_p(y_{2t}^* | \lambda_{2t})$$

$$Y_{3t}^* \sim f_p(y_{3t}^* | \lambda_{3t})$$

where $E(Y_{jt}^*) = \lambda_{jt}$ for $j = 1, 2, 3$, and, conditioning on these expected values, the three variables are assumed independent. Thus, the expected length of the tit-for-tat behavior, once initiated, may depend on the true levels of U.S. \rightarrow Soviet and Soviet \rightarrow U.S. conflict, but the random variation of these three variables around their own expected values are stochastically independent. If the realizations of these random variables, y_{jt}^* (for $j = 1, 2, 3$), were each observed, this analysis could proceed just as with the standard Poisson regression model in Section 2. Although they are not observed, we do observe realizations of random variables that are two functions of these three variables:

$$Y_{1t} = Y_{1t}^* + Y_{3t}^* \quad (38)$$

$$Y_{2t} = Y_{2t}^* + Y_{3t}^*$$

and, because of Cramer's theorem—the sums of independent Poisson distributed random variables are also Poisson distributed—we can write:

$$Y_{1t} \sim f_p(y_{1t} | \theta_{1t}) = f_p(y_{1t} | \lambda_{1t} + \lambda_{3t}) \quad (39)$$

$$Y_{2t} \sim f_p(y_{2t} | \theta_{2t}) = f_p(y_{2t} | \lambda_{2t} + \lambda_{3t})$$

But, this is still insufficient to derive an estimable model, since the two terms in the expected value in each distribution cause them each to be separately unidentified.

Fortunately, thanks to Holgate (1964:241; see also Johnson and Kotz, 1969:297–98; and King, In press-c), we can generalize a result to solve this problem. The solution is based on a proof that, given conditions equivalent to those stated above, Y_{1t} and Y_{2t} are distributed as bivariate Poisson variables:

$$(Y_{1t}, Y_{2t}) \sim f_{bp}(y_{1t}, y_{2t} | \lambda_{1t}, \lambda_{2t}, \lambda_{3t}) \quad (40)$$

$$= \exp(-\lambda_{1t} - \lambda_{2t} - \lambda_{3t}) \sum_{j=0}^{\min(y_{1t}, y_{2t})} \frac{\lambda_{1t}^{(y_{1t}-j)}}{(y_{1t}-j)!} \frac{\lambda_{2t}^{(y_{2t}-j)}}{(y_{2t}-j)!} \frac{\lambda_{3t}^j}{j!}$$

Because of the special properties of this distribution, if no tit-for-tat behavior exists and therefore λ_{3t} turns out to be zero for all t , then the bivariate Poisson factors into the product of two independent Poisson variates. As such, this distribution is a straightforward generalization of the univariate Poisson, and since the covariance of Y_{1t} and Y_{2t} is λ_{3t} , this setup is a generalization of the Seemingly Unrelated Poisson REgression Model Estimator (SUPREME) developed by King (In press-c). Since the unobserved variables Y_{1t}^* , Y_{2t}^* , and Y_{3t}^* remain the primary focus of the analysis, this variant of the SUPREME model can also be conceptualized as a discrete factor model. Thus, a test for λ_{3t} equaling zero is equivalent to a test for whether this model is extracting more information from the data than two independent Poisson regression models applied to y_{1t} and y_{2t} separately. To the extent that tit-for-tat behavior exists and λ_{3t} is different from zero, separate Poisson models produce

estimates of β and γ that are not only statistically inefficient but are biased and inconsistent as well. And in addition to improving the properties of existing estimators, this model also enables one to estimate the explanatory variables' effect on and the raw extent of tit-for-tat behavior—answering key substantive questions one could not hope even to address with standard methods.

To estimate β , γ , and δ , I substitute the right hand sides of Equations 34, 35, and 36 into Equation 40, take logs, and sums over the n observations. The result is the log likelihood function:

$$L(\beta, \gamma, \delta | y_{1t}, y_{2t}) = \sum_{t=1}^n \left\{ -\exp(x_t\beta) - \exp(z_t\gamma) - \exp(w_t\delta) \right. \tag{41}$$

$$\left. + \ln \left[\sum_{j=0}^{\min(y_{1t}, y_{2t})} \frac{\exp(x_t\beta)^{(y_{1t}-j)} \exp(z_t\gamma)^{(y_{2t}-j)} \exp(w_t\delta)^j}{(y_{1t}-j)! (y_{2t}-j)! j!} \right] \right\}$$

This well-behaved function is then easily optimized to yield the maximum likelihood estimates of the effect parameters, β , γ , and δ . No identifying restrictions need be put on the three sets of explanatory variables. They may be identical or may differ, depending on theory.

Consider now a joint estimation with the new model developed above. This model now permits the degree of tit-for-tat behavior between the superpowers to be estimated rather than assumed. To estimate this model, I leave the specifications for λ_{1t} and λ_{2t} as functions of military spending of the other superpower and the party of the U.S. president. These two nonrandom variables are interpreted as international conflict, stripped of any tit-for-tat behavior. In addition, I specify a model for λ_{3t} , as in Equation 36. Although one could develop a long list of explanatory variables, I keep the specification simple by assuming that tit-for-tat behavior is an exponential function only of the average superpower military spending (AvgMilitary $\$_t$) and the party of the U.S. president. The empirical results appear in Table 8.

The overall improvement in moving from the two Poisson regression models to this joint estimation can be judged by examining the log-likelihoods. Since the two models in Table 7 were estimated independently, the log-likelihoods may be summed to arrive at a total log-likelihood, $-223.72 = 241.80 = -465.52$. This number can be compared to the likelihood from Table 8 to derive a test statistic. The likelihood ratio test statistic in this case is $2(-397.04 + 465.52) = 136.96$. This is a chi-square statistic with 2 degrees of freedom. Thus, the hypothesis of no difference between the models is comfortably rejected.

TABLE 8. SUPREME2 model of U.S.–Soviet conflict.

<i>Variable</i>	<i>Estimate</i>	<i>Std. Error</i>	<i>t-Stat</i>
$\hat{\beta}_0$	2.673	0.119	
SovMilitary\$	0.011	0.001	8.095
President	0.438	0.055	7.947
$\hat{\gamma}_0$	2.590	0.170	
USMilitary\$	0.010	0.002	5.108
President	0.794	0.062	12.753
$\hat{\delta}_0$	8.598	0.745	11.534
AvgMilitary\$	-0.103	0.016	-6.555
President	-0.674	0.202	-3.334

Log-likelihood = -397.04
 Observations: 28

The most surprising result is the parameter estimates for military spending. It appears that deterrence does not work as it seemed to work from the results of the independent Poisson regression models. Instead, U.S. military spending seems very clearly to provoke hostile Soviet action toward the U.S. Indeed, Soviet military spending also provokes U.S. conflictual actions at almost exactly the same rate. Whereas the independent Poisson models explained total U.S. and Soviet actions, this model extracts tit-for-tat behavior as a separate variable. Indeed, this more sophisticated model shows that higher levels of average superpower military spending reduce tit-for-tat behavior, presumably because such superfluous conflictual behavior becomes more dangerous with bigger military budgets. Military spending appears to provoke serious hostile actions but to deter superfluous ones.

Since the typical value of average military spending is 61.44 billion U.S. dollars, the typical value of $\hat{\lambda}_{3t}$ may be calculated as:

$$\begin{aligned}\hat{\lambda}_{3t} &= \exp(\delta_0 + \delta_1 \text{AvgMilitary}\$_t + \delta_2 \text{President}_t) & (42) \\ &= \exp(8.598 - 0.103 \times 61.44 - 0.674 \times 0.5) \\ &= 6.9\end{aligned}$$

Thus, of all the conflictual acts between the U.S. and the Soviet Union, an average of 6.9 of these events a year are merely tit-for-tat behavior. This represents 15.5% of the typical year of U.S. \rightarrow Soviet acts and 13.1% of Soviet \rightarrow U.S. acts. Only with this new model can these three types of behavior be extracted from the two existing data series.

Goldstein and Freeman (1988) show that virtually all previous studies based on annual data are unable to find evidence of reciprocity, and nearly all based on less aggregated data find substantial evidence. This analysis, which uses annual data but nevertheless finds clear evidence of reciprocity, dramatically demonstrates how the considerably more powerful models introduced here can extract far more information than the commonly used techniques.

The independent Poisson models were biased primarily by the contamination from tit-for-tat behavior. Once this behavior is separated out analytically, empirical results become much clearer and substantive findings significantly different.

Conclusions

I have introduced several related statistical models designed explicitly for the theoretical perspectives and existing data in the field of international relations. Scholars in this field often think in terms of the continuous but unobserved processes of international conflict and cooperation, while their data consist primarily of noncategorical discrete event count variables. The methods introduced here permit researchers in international relations to connect theory with empirical analyses more explicitly by estimating features of these continuous processes of international politics with existing collections of event count data.

The models developed were illustrated with three applications from international relations data—the influence of international alliances on the outbreak of war, the contagious process of economic sanctions, and an analysis of dyadic superpower conflict. If only the specific models I present here are applied to future research, this paper will have made its contribution. But I also hope that scholars will begin to think about political methodology somewhat more creatively. Too often we choose our methods because they exist in our local computer package. Imagine how silly the field would look if we chose theoretical arguments in a similar manner. The class of statistical techniques developed here offers solutions to several specific problems in empirical research in international relations. But it also offers a new and more

flexible approach to quantitative methodology. Both the specific techniques and the more general methodology should be exploited.

The field of international relations is more than a list of facts and theories about international cooperation and conflict; it is a way of understanding world affairs. At its most ambitious, the field attempts to develop methods by which nations with conflicting interests can survive in an interdependent world. International relations is as fundamentally a *methodological* discipline as it is a theoretical or empirical one. As the field progresses, we need to pay more attention to these methodological foundations and to develop new statistical models that enable us to estimate more directly new features of theoretically interesting processes and to find new ways of extracting information from the enormous body of existing data.

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