DIGITAL ACCESS TO SCHOLARSHIP AT HARVARD

Rapid Single-Shot Measurement of a Singlet-Triplet Qubit

Citation	Barthel, Christian, David Reilly, Charles M. Marcus, M. P. Hanson, and Arthur Gossard. 2009. Rapid single-shot measurement of a singlet-triplet qubit. Physical Review Letters 103(160503).
Published Version	doi:10.1103/PhysRevLett.103.160503
Accessed	February 18, 2015 5:45:56 PM EST
Citable Link	http://nrs.harvard.edu/urn-3:HUL.InstRepos:5110747
Terms of Use	This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Open Access Policy Articles, as set forth at http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#OAP

(Article begins on next page)

Rapid Single-Shot Measurement of a Singlet-Triplet Qubit: Supplementary Material

C. Barthel[1], D. J. Reilly[1,2], C. M. Marcus[1], M. P. Hanson[3], and A. C. Gossard[3]

[1] Department of Physics, Harvard University, 17 Oxford Street, Cambridge, Massachusetts 02138, USA
[2] School of Physics, University of Sydney, Sydney, 2006, Australia
[3] Materials Department, University of California, Santa Barbara, California 93106, USA

This supplementary note presents a derivation of an equation that appears in the main text of the Letter "Rapid Single-Shot Measurement of a Singlet-Triplet Qubit" for the ensemble-averaged triplet T_+ return probability, P_T. The equation is used to fit experimental data in Fig. 4(b) of that Letter. The analysis assumes classical Overhauser fields that are static on the time scale of electron spin evolution but fluctuate randomly on longer time scales, where ensemble statistics are measured.

In one of the experiments described in the accompanying Letter, "Rapid Single-Shot Measurement of a Singlet-Triplet Qubit", two electrons in a double quantum dot are initialized in the singlet state, then brought to the anticrossing of the lower branch of the hybridized singlet, S, and the $m = +1$ triplet, $T_+ = |\uparrow\uparrow\rangle$. The hyperfine interaction felt by the electrons due to the large number ($\sim 10^6$) of nuclei in the host material is treated as an effective classical Zeeman (Overhauser) field acting on the electrons. The Overhauser field is further assumed to be static on time scales of the electron dynamics. Near the $S - T_+$ resonance, the difference in transverse components of the Overhauser fields in the left (L) and right (R) dots, $\Delta B_{x(y)} = [B^L_{x(y)} - B^R_{x(y)}]/2$ mix singlet and triplet state, while the average longitudinal Overhauser field, $B_z = [B^L_z + B^R_z]/2$, acts as a random energy detuning. An external magnetic field, B, defines the quantization axis z. This supplementary note presents a derivation of the triplet probability, P_T, used to fit experimental data in Fig. 4(b), following Ref. [3]. The Hamiltonian in the basis (S, T_+) is [1, 2]

$$H = g^* \mu_B \begin{pmatrix} -J/(g^* \mu_B) & \cos\theta\, B_+/\sqrt{2} \\ \cos\theta\, B_-/\sqrt{2} & B + B_z \end{pmatrix}, \tag{1}$$

where $\cos\theta$ is the (1,1) component of the hybridized singlet ground state, $|S\rangle = \cos\theta\,|(1,1)S\rangle + \sin\theta\,|(0,2)S\rangle$ and $B_\pm = \Delta B_x \pm i\Delta B_y$. The effective electron g-factor is $g^* = -0.44$. An additional factor $\sqrt{2}$ in the numerator of the off-diagonal term stems from constructive interference of two electron spin flip - pathways, since there are two electrons participating [1]. The exchange, J, the external magnetic field, B, and the nuclear Overhauser z-component, B_z, are combined to the energy mismatch $\delta = -g^* \mu_B (B + B_z) - J$. In the described experiment the exchange is tuned to resonance, leaving $\delta = -g^* \mu_B B_z$. For an initial singlet $c_T(\tau_I = 0) = 0$ the solution for the probability amplitude, $c_T(\tau_I)$, is [4]

$$c_T(\tau_I) = \frac{\cos\theta(\Delta B_x + i\Delta B_y)}{\sqrt{2}\,\hbar\omega/|g^*\mu_B|} \sin(\omega\tau_I), \tag{2}$$

with the precession frequency, ω,

$$\omega = \frac{1}{2\hbar}\sqrt{\delta^2 + 2\cos^2\theta\,(g^*\mu_B)^2(\Delta B_x^2 + \Delta B_y^2)} = \frac{|g^*\mu_B|}{2\hbar}\sqrt{B_z^2 + 2\cos^2\theta\,(\Delta B_x^2 + \Delta B_y^2)}. \tag{3}$$

The nuclear Overhauser field z-component, B_z, and the gradient fields, ΔB_x and ΔB_y, are not known and constant, but distributed [1, 2] according to the distribution function $\rho(\mathbf{B}) = (2\pi B^{\mathrm{nuc}})^{-3/2} e^{-(\mathbf{B}/(B^{\mathrm{nuc}}))^2/2}$, with $\mathbf{B} = (\Delta B_x, \Delta B_y, B_z)$. The evolution of the nuclear fields is slower than the evolution of the electron spin, hence the triplet probability for an ensemble of measurements, $P_T^{\mathrm{ideal}}(\tau_I)$, can be written as the integral of all probabilities for a single constant Overhauser field, weighted by $\rho(\mathbf{B})$,

$$P_T^{\mathrm{ideal}}(\tau_I) = \int d^3\mathbf{B}\,\rho(\mathbf{B})c_T(\tau_I)c_T^*(\tau_I) = \int d^3\mathbf{B}\,\rho(\mathbf{B})\frac{\cos^2\theta\,(\Delta B_x^2 + \Delta B_y^2)}{2\,(\hbar\omega/|g^*\mu_B|)^2}\sin^2(\omega\tau_I). \tag{4}$$

The evolution occurs far detuned from the (1,1)-(0,2) charge degeneracy, therefore the hybridized singlet is approximately identical to the (1,1) singlet, and $\cos\theta \sim 1$. Furthermore allowing an offset, P_T^0, of P_T, because of imperfect preparation or miscounting of singlets as triplets and a smaller than one visiblity, V, yields the equation in the paper:

$$P_T = P_T^0 + V \int d^3\mathbf{B}\,\rho(\mathbf{B})\frac{(\Delta B_x^2 + \Delta B_y^2)^2}{2\,(\hbar\omega/|g^*\mu_B|)^2}\sin^2(\omega\tau_I), \tag{5}$$

which describes the measured triplet probability, averaged over many singleshot measurements.

––––––––––

[1] W. A. Coish and D. Loss, Phys. Rev. B,**72**, 125337,(2005).
[2] J. M. Taylor, J. R. Petta, A. C. Johnson, A. Yacoby, C. M. Marcus and M. D. Lukin, Phys. Rev. B,**76**, 035315,(2007).
[3] J. M. Taylor, Ph.D. thesis, Harvard University (2007).
[4] C. Cohen-Tannoudji, Quantum Mechanics **2**, 4.3.3 (1999).