

DIGITAL ACCESS TO  
SCHOLARSHIP AT HARVARD

## Price Dispersion and Loss Leaders

The Harvard community has made this article openly available.  
[Please share](#) how this access benefits you. Your story matters.

<b>Citation</b>	Ambrus, Attila, and Jonathan Lewis Weinstein. 2008. Price dispersion and loss leaders. <i>Theoretical Economics</i> 3(4): 525-537.
<b>Published Version</b>	<a href="http://www.econtheory.org/ojs/index.php/te/article/view/20080525">http://www.econtheory.org/ojs/index.php/te/article/view/20080525</a>
<b>Accessed</b>	February 17, 2015 10:45:16 PM EST
<b>Citable Link</b>	<a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:4589708">http://nrs.harvard.edu/urn-3:HUL.InstRepos:4589708</a>
<b>Terms of Use</b>	This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Open Access Policy Articles, as set forth at <a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#OAP">http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#OAP</a>

*(Article begins on next page)*

# Price dispersion and loss leaders

ATTILA AMBRUS

Department of Economics, Harvard University

JONATHAN WEINSTEIN

Kellogg School of Management, Northwestern University, and  
School of Social Science, Institute for Advanced Study

Dispersion in retail prices of identical goods is inconsistent with the standard model of price competition among identical firms, which predicts that all prices will be driven down to cost. One common explanation for such dispersion is the use of a loss-leader strategy, in which a firm prices one good below cost in order to attract a higher customer volume for profitable goods. By assuming each consumer is forced to buy all desired goods at a single firm, we create the possibility of an effective loss-leader strategy. We find that such a strategy cannot occur in equilibrium if individual demands are inelastic, or if demands are diversely distributed. We further show that equilibrium loss leaders can occur (and can result in positive profits) if there are demand complementarities, but only with delicate relationships among the preferences of all consumers.

**KEYWORDS.** Price competition, price dispersion, loss leaders.

**JEL CLASSIFICATION.** D40, D43.

## 1. INTRODUCTION

Microeconomics textbooks aside, there is no shortage of evidence of price dispersion, i.e. retail firms charging different prices for identical goods. As [Varian \(1980\)](#) comments, “Economists have belatedly come to recognize that the ‘law of one price’ is no law at all.” One well-known retail strategy that results in price dispersion is the tactic of cutting prices on one good, known as a “loss leader,” in order to attract more store traffic and increase profits on other goods. Any price dispersion is, of course, contrary to the unique pure-strategy equilibrium prediction in the setting of Bertrand competition that all transactions will take place at marginal cost.<sup>1</sup> This theoretical prediction requires the following assumptions.

---

Attila Ambrus: [ambrus@fas.harvard.edu](mailto:ambrus@fas.harvard.edu)

Jonathan Weinstein: [j-weinstein@kellogg.northwestern.edu](mailto:j-weinstein@kellogg.northwestern.edu)

We thank Glenn Ellison, Ran Spiegler, Muhamet Yildiz, the coeditor, and anonymous referees for helpful comments.

<sup>1</sup>There are mixed-strategy equilibria in which prices are above cost and firms make profits. However (in the single-good case) all such equilibria involve unbounded prices and are hence eliminated if consumers have a maximum willingness to pay. In this paper, we will consider only pure-strategy equilibria.

1. All firms have identical costs.
2. Consumers have full information, at zero cost, about the prices charged by each firm.

The standard argument applies to a market for a single good. To extend it directly to a market for multiple goods, one needs another, sometimes hidden, assumption.

3. Consumers are free to buy different goods at different firms, with zero transportation costs, so that the markets for different goods are “uncoupled.”

A considerable literature has focused on whether price dispersion can result from relaxing the second assumption, i.e. introducing search costs. In this paper we keep assumption 2, but will drop assumption 3. In particular, we go to the other extreme and assume that customers are constrained to purchase their entire bundle of goods from a single firm. This abstracts the idea that it is time-consuming to do one’s shopping at multiple locations, essentially changing transportation costs from zero to infinity. Intuitively, one might expect that the resulting “coupled” markets could result in a firm being able to successfully employ a loss-leader strategy, cutting prices on good A below cost in order to attract customers who will buy the profitable good B.

In the single-good case, one proves that there is no equilibrium in which a firm makes positive profit by observing that in this case another firm could “undercut” the profitable firm, charging a slightly lower price and taking all the profit. This argument still has some validity in the multi-good case—now “undercutting” means choosing a price vector that is slightly lower for all goods. The situation is complicated, however, by the fact that when one firm undercuts another in an effort to steal its profit, it may attract a different clientele, which buys goods in different proportions. If some goods are sold below cost, this may result in losses, confounding the undercutting argument. We find, however, that under the assumptions in [Section 3](#) it is guaranteed that at least one firm can increase its profit by undercutting another. Our main assumption there is that each consumer has inelastic demand. That is, he demands a fixed vector of goods, which he buys at whichever firm prices this vector lowest. We also use a genericity assumption on the distribution of demand vectors. This allows us to avoid the technical issue of ties in consumers’ firm selection. The distributional assumption also lets us avoid a degenerate case in which two goods are always bought in a fixed proportion, which would allow one price to be raised and the other decreased with no impact on the decisions of any consumers. Under these assumptions, we find that in equilibrium no one makes any profit, and at least two firms charge exactly marginal cost for all goods, just as in the case of one-good Bertrand competition.

In [Section 4](#) we proceed to the case of more general demands. First we provide an example showing that with a particular specification of demand complementarities, there can be an equilibrium with price dispersion and positive profits. We then show that this example is a rather special case; if consumers are diverse enough, in a particular sense, we prove there can be no profits or price dispersion. The key to the more general impossibility result is a modified form of the undercutting strategy. Now a firm undercuts

another firm only on profitable goods, while it slightly increases (“overcuts”) the price of subsidized goods. We show that such a strategy steals the profitable consumers of the other firm, while leaving all unprofitable consumers (those who buy too much of the loss-leader goods) with the other firm.

Most of the literature on this topic assumes either bounded rationality, or limited information and search costs. This begs the question of why we want to consider a fully rational, complete-information model. We feel that it is an important initial step to see whether and to what extent a fully rational, complete-information model can explain the success of a loss-leader strategy. This may further elucidate the role of informational and behavioral considerations. Here, we are able to prove that with sufficient diversity of demands, loss leaders and price dispersion cannot result merely from linking markets through high transportation costs. With specific complementarities, by contrast, loss leaders can be effective in a fully rational world, although the example we use to show this assumes that the set of consumer types is restricted. A possible practical moral to be drawn from this is that (in a full-information, rational world), the loss-leader strategy is apt to work only if it attracts a specific segment of consumers, rather than being an untargeted attempt to increase volume. With limitations on information or rationality, there is more opportunity for loss leaders to succeed, as in the literature we now discuss.

## 2. LITERATURE REVIEW

Many models have been introduced in which consumers have limited information about prices, whether search costs are introduced explicitly or information is simply limited more directly. In an early paper, [Varian \(1980\)](#) describes a market divided between informed customers, who choose the lowest price, and uninformed customers who choose a store a random. This results in the existence of only mixed-strategy equilibria as firms compromise between attracting the informed customers and exploiting the uninformed.<sup>2</sup> Therefore the model predicts that price dispersion will persist, but each firm’s prices will fluctuate and no firm will consistently price lower or higher than others. This contrasts with the pure-strategy price dispersion we find in [Section 4.1](#).

[Lal and Matutes \(1994\)](#) analyze a model with advertising, in which consumers must decide where to shop knowing only the prices of goods that the stores choose to advertise, and only observe the remaining prices once at the store. The consumers have rational expectations and therefore anticipate that unadvertised goods will be overpriced. Nevertheless, in equilibrium both stores employ a loss-leader strategy; they advertise a particular good and price it below cost. More recently, [Spiegler \(2006\)](#) analyzes a model with limited information and a simplified decision rule for consumers. In this case, consumers randomly observe one price from each store and choose the store for which that one price provides the highest consumer surplus. This boundedly rational choice procedure, called  $S(1)$ , was introduced by [Osborne and Rubinstein \(1998\)](#). [Spiegler](#) finds

<sup>2</sup>In a more recent paper, [Kamenica \(forthcoming\)](#) analyses the menu choice decision of a monopolist when a fraction of consumers is uninformed about the quality of the goods, and shows that the presence of these consumers may induce the firm to introduce premium loss leaders that are unprofitable on their own but increase the demand for other goods.

that firms make positive profits in equilibrium. Also, the variance in prices increases as the number of firms increases, contrary to the usual intuition about competition. This is essentially because with a very large number of firms, the best way to get attention from  $S(1)$  consumers is to have a small number of goods with an extremely low price. In fact, if the model did not include a hard lower bound on prices, the price of the loss leaders would become arbitrarily negative as the number of firms grew. An alternative model of limited search is analyzed by [Chen et al. \(2005\)](#), who assume that consumers have limited memories for prices. In particular, the consumers divide the set of possible prices into finitely many ranges and only remember which range a price is in. They find that this limited memory enables firms to extract surplus. Finally, [Eliaz and Spiegler \(2008\)](#) study a model in which firms use costly marketing devices to influence the set of alternatives that boundedly rational consumers consider to be relevant. They identify mixed strategy equilibria in which firms offer goods that are not chosen by consumers but help attract consumers from rival stores. These goods are loss leaders because including them on the menu is costly.

The role of this paper is to examine the possibility of price dispersion in a world with full information. Since we find it can occur only in certain special circumstances, it is reasonable to think that loss leaders are more often supported by limits on information.

### 3. INELASTIC DEMAND

#### 3.1 *Model*

We have  $K > 1$  identical firms, each of which sells  $N$  goods, each with constant marginal cost, which we normalize to zero (so that prices should be interpreted throughout as the difference from marginal cost). There is a continuum of consumers of mass 1, with each consumer having inelastic demand, so that each individual's demand is characterized by a non-negative vector in  $\mathbb{R}^N$ , specifying the quantity he purchases of each good. The distribution of demands is described by a probability distribution  $P$  on  $\mathbb{R}^N$  with support in the non-negative orthant. We assume that  $P$  assigns zero mass to any hyperplane in  $\mathbb{R}^N$ . The consumers, who are able to shop at only one firm, select a firm that minimizes their cost. A profile  $(\mathbf{p}_1, \dots, \mathbf{p}_K)$  of price vectors is an equilibrium if no firm could increase its profit by changing its prices (consumers then adjusting their firm choice). As mentioned in the introduction, we consider only pure-strategy equilibria.

We are interested in whether there exists a pure-strategy equilibrium in which firms earn positive profits. Note that if  $N = 1$  we are in the case of standard Bertrand competition, and the only equilibrium outcome is for all firms to make zero profits, with at least two firms charging exactly marginal cost.

A few words are in order on the assumption that hyperplanes have zero mass. Note that a distributional assumption is clearly necessary to exclude price dispersion. In particular, if two goods were always demanded in a fixed ratio, firms could always increase one price and decrease the other without any effect on equilibrium. Also, it is clear any distribution can be slightly perturbed to give one satisfying the assumption.

### 3.2 At-cost pricing result for inelastic demands

Note that it is clearly an equilibrium for all firms to set all prices equal to zero, or even for at least two firms to do so and others to charge non-negative prices. In such a case all firms make zero profits and no one can do better. In this section we prove that under the assumptions given above, these are the only equilibria.

**THEOREM 1.** *In any equilibrium, (a) all firms make zero profits, (b) the consumers who pay exactly the marginal cost of their bundle have mass 1, (c) at least two firms have all prices equal to marginal cost, and (d) all consumers go to such firms.*

**PROOF.** First note that no firm can make negative profit in equilibrium, because it can always assure itself of non-negative profit by charging at or above cost for all goods. Now suppose we have an equilibrium  $(\mathbf{p}_1, \dots, \mathbf{p}_K)$ . Let  $S_i = \{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} < \mathbf{p}_j \cdot \mathbf{x}, \forall j \neq i\}$  be the set of consumer demands for which firm  $i$  is preferred to all other firms, and also let  $S_{i,j} = \{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} < \mathbf{p}_j \cdot \mathbf{x} < \mathbf{p}_k \cdot \mathbf{x}, \forall k \neq i, j\}$  be the set of demands for which  $i$  is preferred and  $j$  is second-best. Each firm's profit is given by  $\pi_i = \int_{S_i} \mathbf{p}_i \cdot \mathbf{x} dP(\mathbf{x})$ . Define  $\mathbf{1}$  as the vector  $(1, \dots, 1) \in \mathbb{R}^N$ . Let  $UC_{i,j,\epsilon} = \{\mathbf{x} \in \mathbb{R}^N : (\mathbf{p}_j - \epsilon \cdot \mathbf{1}) \cdot \mathbf{x} < \mathbf{p}_k \cdot \mathbf{x}, \forall k \neq i, j\}$  be the subset of demand space for which firm  $i$  is chosen after it undercuts firm  $j$  by switching to the price vector  $\mathbf{p}_j - \epsilon \cdot \mathbf{1}$ . Also define  $\pi_{i,j,\epsilon}$  as the profit of firm  $i$  if it made this undercutting deviation. Assume at least one  $\pi_i > 0$ . As discussed in the introduction, it is not automatically profitable to undercut a profitable firm. We are able to show, however, that we are not at equilibrium by showing that for *some* triple  $(i, j, \epsilon)$ ,  $\pi_{i,j,\epsilon} > \pi_i$ .

We first consider the case in which some two firms  $i$  and  $j$  offer identical price vectors. If firms  $i$  and  $j$  make positive profits, either one could double its profit by decreasing prices slightly and attracting all consumers who go to these two firms (the argument also applies if there is a tie among more than two firms). If firms  $i$  and  $j$  make zero profits and some other firm  $k$  makes positive profit, it is certainly profitable for firm  $i$  to undercut firm  $k$ —since the price vector  $\mathbf{p}_i$  is still available from firm  $j$ , firm  $i$  attracts consumers only from the set  $S_k$  and no others, so the deviation is profitable.

Henceforth we assume all price vectors are distinct. Notice that for any  $i$  and  $j$ , the set of firms for which  $i$  and  $j$  are preferred equally is the hyperplane  $\{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} = \mathbf{p}_j \cdot \mathbf{x}\}$  which is assumed to have mass zero, so we can safely ignore the issue of ties. If firm  $i$  undercuts firm  $j$ , it is then selected by all consumers who previously chose firm  $j$ . It is also chosen by those who previously chose firm  $i$  but liked firm  $j$  second best, because firm  $i$ 's old price vector is no longer available to them. That is,  $S_j \cup S_{i,j} \subset UC_{i,j,\epsilon}$ . Any  $\mathbf{x} \notin S_j \cup S_{i,j}$  satisfies  $\mathbf{p}_k \cdot \mathbf{x} \leq \mathbf{p}_j \cdot \mathbf{x}$  for some  $k \neq i, j$ . When the inequality is strict,  $\mathbf{p}_k \cdot \mathbf{x} < (\mathbf{p}_j - \epsilon \cdot \mathbf{1}) \cdot \mathbf{x}$  for small enough  $\epsilon$ , proving that  $UC_{i,j,\epsilon} - (S_j \cup S_{i,j})$  converges as  $\epsilon \rightarrow 0$  to a set where some preferences are tied. Since price vectors are distinct and hyperplanes have zero mass, this implies that  $P(UC_{i,j,\epsilon} - (S_j \cup S_{i,j})) \rightarrow 0$  as  $\epsilon \rightarrow 0$ . This in turn implies that  $\pi_{i,j,\epsilon} \rightarrow \int_{S_j \cup S_{i,j}} \mathbf{p}_j \cdot \mathbf{x} dP(\mathbf{x})$  as  $\epsilon \rightarrow 0$ .

Assume without loss of generality no firm makes smaller profit than firm 1 in our equilibrium. It suffices to show that for some  $j$ ,  $\int_{S_j \cup S_{1,j}} \mathbf{p}_j \cdot \mathbf{x} dP(\mathbf{x}) > \pi_1$ , for then there

exists an  $\epsilon$  for which  $\pi_{1,j,\epsilon} > \pi_1$ . Observe that

$$\begin{aligned} \sum_{j=2}^N \int_{S_j \cup S_{1,j}} \mathbf{p}_j \cdot \mathbf{x} dP(\mathbf{x}) &= \sum_{j=2}^N \left[ \int_{S_j} \mathbf{p}_j \cdot \mathbf{x} dP(\mathbf{x}) + \int_{S_{1,j}} \mathbf{p}_j \cdot \mathbf{x} dP(\mathbf{x}) \right] \\ &\geq \sum_{j=2}^N \left[ \pi_j + \int_{S_{1,j}} \mathbf{p}_1 \cdot \mathbf{x} dP(\mathbf{x}) \right] \\ &= \sum_{j=2}^N \pi_j + \int_{\cup_j S_{1,j}} \mathbf{p}_1 \cdot \mathbf{x} dP(\mathbf{x}) \\ &= \sum_{j=2}^N \pi_j + \int_{S_1} \mathbf{p}_1 \cdot \mathbf{x} dP(\mathbf{x}) \\ &= \sum_{j=1}^N \pi_j \end{aligned} \tag{1}$$

where the inequality comes from the revealed preference of customers in the set  $S_{1,j}$ ; we know they pay less at firm 1 than at firm  $j$ .

Because there are  $N - 1$  terms in the left-hand sum, the above inequality implies that for at least one  $j$ ,

$$\int_{S_j \cup S_{1,j}} \mathbf{p}_j \cdot \mathbf{x} dP(\mathbf{x}) \geq \sum_{j=1}^N \pi_j / (N - 1) > \sum_{j=1}^N \pi_j / N \geq \pi_1,$$

where the strict inequality is implied by at least one  $\pi_j > 0$ . Then firm 1 can improve its profit by undercutting firm  $j$ . This proves (a).

The inequality in (1) is precisely the stage of the proof at which we make use of the individually inelastic demands. In particular, this assumption implies that since firm 1's customers, in aggregate, pay a non-negative surplus to firm 1, they would continue to pay a non-negative surplus if firm 1 vanished and they had to patronize their second choice. This is effectively what happens as we consider the outcome of firm 1 undercutting each of the other firms in turn; when it undercuts firm  $j$  it attracts not only firm  $j$ 's old customers, but also its old customers who liked  $j$  second best. Our assumption on demands ensures that on *average*, across all firms it might undercut, this does not hurt firm 1.

Part (a) of the theorem leaves open the possibility of an equilibrium in which firms make profits on some customers but losses on others, but we will now see that this is not possible.

We show that if (b) is not the case, then for some  $\epsilon$ , any firm could make positive profit by switching to the price vector  $\epsilon \cdot \mathbf{1}$ . Let  $S = \{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} > \mathbf{0}, \forall i\}$  be the set of demands for which the best price available is above the cost of the bundle. Let  $S_\epsilon = \{\mathbf{x} \in \mathbb{R}^N : (\epsilon \cdot \mathbf{1}) \cdot \mathbf{x} < \mathbf{p}_i \cdot \mathbf{x}, \forall i\}$  be the set of demands for which the prices  $\epsilon \cdot \mathbf{1}$  are preferred to those currently available. Note that  $S = \cup_{n=1}^\infty S_{1/n}$ .

Note that if  $P(S_\epsilon) > 0$ , then any firm can profit by switching to the price vector  $\epsilon \cdot \mathbf{1}$ , because it would attract consumers in  $S_\epsilon$  and all customers are paying above cost. Therefore,  $P(S_\epsilon) = 0$ . Then by countable additivity,  $P(S) = P(\cup_{n=1}^{\infty} S_{1/n}) \leq \sum_{n=1}^{\infty} P(S_{1/n}) = 0$ . Therefore, the set of customers who pay at most the cost of their bundle has mass 1. Suppose a non-zero mass of customers are paying below cost. Then, some firms suffer losses, which cannot happen in equilibrium. This proves (b).

This result still leaves open the possibility that although all consumers pay the cost of their bundle, some goods are priced above cost and others below. We now exclude this possibility; the assumption that hyperplanes have zero mass is needed again here.

Note that if  $\mathbf{p}_i \neq \mathbf{0}$ , then the set  $\{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} = \mathbf{0}\}$  is a hyperplane and so has mass zero by assumption. Together with (b) this immediately gives (d). Therefore, we must have  $\mathbf{p}_i = \mathbf{0}$  for some  $i$ . If this were true only for firm  $i$ , we could repeat the argument in the proof of (b) to show that firm  $i$  could deviate and make profits by choosing an appropriate vector  $\epsilon \cdot \mathbf{1}$ , so it must hold for at least two firms, proving (c).  $\square$

#### 4. ELASTIC DEMAND

##### 4.1 *An example with positive profits*

Below we provide an example that shows that if demands are not perfectly inelastic, the game can have equilibria in which stores earn strictly positive profits. The example involves a relatively complex structure of complementarity relationships among different goods, and particular sets of consumer types. This raises the question of how relevant examples like these are in practice. In the next subsection we provide an assumption on the diversity of consumer demands that we show leads to zero profits, and therefore rules out examples like the one discussed here. The upshot is that positive profits are possible, but only with specific delicate relationships among the consumer demands.

Before we state the example formally, here is a summary of the idea. There are two stores and six goods, and in the proposed equilibrium one store ends up selling only the first three, the other store only the second three. Three types of consumers go to the first store. One type buys only the first good and has a high reservation value for it. For the second type of consumers, the first two goods are perfect complements, and they buy a unit of each. Finally, for the third type of consumers, the first three goods are perfect complements, and they buy a unit of each. Moreover, the reservation value of the second type of consumers for a pair of goods 1 and 2 is lower than the reservation value of the first type of consumers for only good 1, and the reservation value of the third type of consumers for a basket of all three goods is even lower than the former. This implies that the optimal way for the store to sell these goods is to set a high price for the first good, and subsidize (sell below marginal cost) the second and third goods. This way, a large profit can be extracted from the first type of consumers, and at the same time positive profit can be extracted from the other two types of consumers. The other store prices its goods symmetrically, and ends up selling the other three types of goods, to the other three consumer types. Furthermore, each consumer would never cross over to purchase the profitable good at the other store, but would cross over to purchase a large amount of the loss-leader good if their own firm's prices were unavailable.



The key feature of the example is that given this pricing structure, in which two of the goods are sold at a subsidized price, the stores do not want to attract the consumers of the other store. This is because these types are constructed such that they do not value the good that generates the profit to the store, but they would purchase relatively large quantities of the subsidized “loss-leader” goods. That is, stores in the equilibrium we propose could tempt over consumers from the other store, by undercutting the equilibrium prices of the goods currently sold by the rival, but they do not find it profitable, because these consumers generate more loss than gain given the optimal pricing structure for the goods already sold in the store.<sup>3</sup>

As above, let the marginal cost of all goods be normalized to 0. There are 6 goods:  $a_1, a_2, a_3, b_1, b_2, b_3$ ; 6 consumer types:  $A_1, A_2, A_3, B_1, B_2, B_3$ ; and two stores:  $I, II$ . There is a measure 2 of each of type  $A_1$  and type  $B_1$  consumers, a measure 3 of each of type  $A_2$  and type  $B_2$  consumers, and a measure 14 of each of type  $A_3$  and type  $B_3$  consumers. For type  $A_1$  consumers, the reservation value for the first unit of good  $a_1$  is 170, the reservation value for the first twenty units of  $b_2$  is  $-0.5$  (meaning that these consumers buy twenty units of  $b_2$  if the price falls below marginal cost minus 0.5 monetary units), the reservation value for the first twenty units of  $b_3$  is  $-0.5$ , and the reservation value of any other units of any good is  $-K$  (where  $K$  is a “large” number).

For type  $A_2$  consumers, the first unit of  $a_1$  and of  $a_2$  are perfect complements. In particular, the reservation value for the first units of  $a_1$  and  $a_2$  together is 164 (the reservation value for only one of these goods without the other is  $-K$ ). The reservation values for the first twenty units of  $b_2$  and  $b_3$  are  $-0.5$  each, and the reservation value for any other units of any good is  $-K$ .

For type  $A_3$  consumers, the first units of  $a_1, a_2,$  and  $a_3$  are perfect complements. In particular, the reservation value for the first units of  $a_1, a_2,$  and  $a_3$  together is 161 (the reservation value for any unit of single good or any pairs of goods without the third one is  $-K$ ). The reservation values for the first twenty units of  $b_2$  and  $b_3$  are  $-0.5$  each, and the reservation value for any other units of any good is  $-K$ .

Types  $B_1, B_2,$  and  $B_3$  have preferences symmetric to those described above, with the roles of  $a_i$  and  $b_i$  interchanged.

**CLAIM 1.** *The following profile constitutes an equilibrium. Firm I sets prices  $p(a_1) = 10, p(a_2) = -6, p(a_3) = -3,$  and  $p(b_1) = p(b_2) = p(b_3) = 300,$  while II sets prices  $q(a_1) = q(a_2) = q(a_3) = 300, q(b_1) = 10, q(b_2) = -6,$  and  $q(b_3) = -3.$  Consumers of type  $A_1$  go to I and each buys one unit of  $a_1,$  consumers of type  $A_2$  go to I and each buys one unit of  $a_1$  and one unit of  $a_2,$  and consumers of type  $A_3$  go to I and each buys one unit of each  $a_1, a_2,$  and  $a_3.$  Consumers of types  $B_1, B_2,$  and  $B_3$  go to II and behave symmetrically (buy one  $b_1,$  one  $b_1$  and one  $b_2,$  and one  $b_1, b_2,$  and  $b_3$  respectively).*

**PROOF.** First, note that each type of consumer gets a consumer surplus of 160 in the

<sup>3</sup>This construction is similar to the one presented in Ambrus and Argenziano (forthcoming) showing that in two-sided markets with network externalities Bertrand competition among platforms is consistent with positive profits in equilibrium, provided that platforms subsidize consumers on one side of the market, and generate positive profit on the other side.

profile specified in the claim, which is exactly how much they could get if switching to the other store. Hence, consumers are in optimum.

Next we check if the stores have any profitable deviation. Since the profile and the game are symmetric, without loss of generality we check for deviations by store  $I$ . Note that in the profile in the claim the profit of store  $I$  is 46 (20 from type  $A_1$  consumers, 12 from type  $A_2$  consumers, and 14 from type  $A_3$  consumers).

First, consider deviations in which firm  $I$  does not set any of the prices  $p(a_2)$ ,  $p(a_3)$ ,  $p(b_2)$ , and  $p(b_3)$  below  $-0.5$ . Then consumers of types  $A_i$  do not purchase goods  $b_j$ , and consumers of types  $B_i$  do not purchase goods  $a_j$ , and the optimal prices for  $a_j$  and  $b_j$  can be chosen separately. It is easy to see that among prices that attract only consumers of type  $A_1$  (and not  $A_2$  or  $A_3$ ) to the store, the ones generating the highest profit involve  $p(a_1) = 10$ . This generates a profit of 20. Among prices that attract only consumers of types  $A_1$  and  $A_2$ , the ones generating the highest profit involve  $p(a_1) = 4.5$  and  $p(a_2) = -0.5$ . This generates a profit of 21. Finally, among prices that attract all types  $A_i$ , the one generating the highest profit is  $p(a_1) = 2$ ,  $p(a_2) = -0.5$ , and  $p(a_3) = -0.5$ . This generates a profit of 22.5. It is easy to see that attracting different subsets of consumers among types  $A_i$  is either suboptimal or infeasible with nonnegative prices. Symmetric considerations hold for prices  $p(b_1)$ ,  $p(b_2)$ , and  $p(b_3)$  and types  $B_i$ . This implies that the highest profit store  $I$  can achieve through prices that are not below  $-0.5$  is 45.

Next, observe that any consumer going to store  $II$  could get a surplus of 160; store  $I$  is extracting the highest possible profit from consumer types  $A_i$ . Similarly, given the prices of store  $II$ , store  $I$  cannot extract a profit of more than 46 from consumer types  $B_i$ . Hence, for a profitable deviation it is necessary that the store attracts consumers both from types  $A_i$  and from types  $B_i$ .

The above observations establish that either (i)  $\min(p(a_2), p(a_3)) < -0.5$  and store  $I$  attracts some consumers among types  $B_i$ ; or (ii)  $\min(p(b_2), p(b_3)) < -0.5$  and store  $I$  attracts some consumers among types  $A_i$ . The cases are symmetric, so assume case (i). Then any consumer of type  $B_i$  who goes to store  $I$  buys 20 units of either  $a_2$  or  $a_3$ . This generates a loss of at least 10 for the store, implying that none of these consumers can generate positive profit for the store. Therefore, the deviation is not profitable, since the profit that consumer types  $A_i$  generate for the store is not higher than 46. This in turn implies that there is no profitable deviation for the store.  $\square$

#### 4.2 Zero-profit result for broad-demand case

The previous subsection demonstrates that if demand is elastic and there are complementarities among goods then there can be equilibria in which stores obtain strictly positive profits. Below we show that this cannot be the case if the set of consumer types is diverse enough, in a sense we define: the unique equilibrium in this case entails that all goods are sold exactly at marginal cost, and every store obtains zero profit. The pricing strategy that we use to show this result is not a simple undercutting strategy (that is, undercutting the rival's price for all goods), but an undercutting-overcutting strategy, which undercuts the rival at goods whose prices are above marginal cost and at the same time raises the price of subsidized goods.

In the general case consumers cannot be simply described by consumption bundles they want to purchase, as in the previous section. Instead, consumers are described by their preferences over consumption bundles. For this reason, let the consumption space be  $\mathbb{R}_+^{N+1}$ , where the first  $N$  dimensions are associated with the  $N$  goods sold by the stores, and the last dimension is associated with a numeraire good (money). We assume that all consumers have quasi-linear preferences in money. For technical convenience, we assume also that the Walrasian demand of every consumer is single-valued and continuous in prices.<sup>4</sup> That is, fixing the choice of store of a consumer, her demand is a continuous function of prices of that store. A sufficient condition for this is that the preferences of consumers are continuous and strictly quasi-concave. The example in the previous subsection does not satisfy this property, but this distinction is not the essential one. The example could be modified such that demand functions of all consumers are continuous and the equilibrium with positive profits prevails.

To summarize, our consumers have preference relations on  $\mathbb{R}_+^{N+1}$  that are quasi-linear in the numeraire good and exhibit continuous demand functions in prices. Let  $P$  denote the distribution of consumer types over this set. As before, we normalize the marginal costs of all goods to be zero. Note that in any game in this context, a profile in which all stores set all their prices to zero constitutes an equilibrium. There can be no profitable deviation, because no consumer would choose to pay above zero. Hence, existence of pure strategy equilibrium is not an issue.

Our next result, which holds without any further assumption on the distribution of consumer types, establishes that in any equilibrium, all  $K$  stores have the same profit. The basic idea is that given an arbitrary strategy profile, any store can choose a deviation strategy such that its resulting profit arbitrarily approximates any other store's profit in the previous profile. As the example in the previous section suggests, this is not always possible using a strategy that undercuts the prices of another store for all goods. In particular, while store A slightly undercutting store B's prices does imply stealing all the former consumers of store B, among the original consumers of store A there may be too many who now purchase goods that were subsidized by store B. However, below we show that there exists a slightly different deviation, which only steals the profitable consumers of store B and at the same time makes sure that only those old consumers of store A stick with the store who is profitable under the new prices. The key idea is to slightly lower the prices for goods that store B sells above marginal cost, while slightly increasing the prices for goods that store B sells below marginal cost, so that the magnitudes of these price changes are proportional to the absolute values of the prices. Note that for this infinitesimal change, those customers of store B who prefer the new set of prices to the old prices of store B are exactly those who purchase bundles that are on average profitable to the store. Similarly, among existing consumers of store A, only those who would buy profitable bundles might stick with A, while the rest of these consumers switch to either B or some other store.

---

<sup>4</sup>This requirement is stronger than is needed for our main result. It is sufficient to assume that the aggregate demand of any set of consumers with nonzero measure is a continuous function. This is a generic property of economies: see Ichiishi (1976).

**PROPOSITION.** *For any distribution of consumers  $P$ , the profits of all stores are equal in any equilibrium.*

**PROOF.** Suppose that in some equilibrium, firm  $j$ 's profit is strictly higher than firm  $i$ 's profit. Note that this in particular means that firm  $j$ 's profit is strictly positive, which in turn implies that at least one of its prices is strictly positive. Consider the deviation  $(1 - \epsilon) \cdot \mathbf{p}_j$  by firm  $i$ , for  $\epsilon > 0$ . Then, any consumer who in the candidate equilibrium profile chose firm  $j$  and purchased a bundle for strictly positive price now chooses  $i$ . To see this, note that the price of any bundle  $\mathbf{x}$  at store  $i$  after the deviation is  $(1 - \epsilon) \cdot \mathbf{p}_j \mathbf{x}$ , which is strictly less than  $\mathbf{p}_j \mathbf{x}$  if the latter is strictly positive. Hence, the previously optimal bundles for former consumers of  $j$  who spent positive amounts of money in the store are attainable at store  $i$  for strictly less money. This means that these consumers strictly prefer store  $i$  after the deviation to any of the other stores. By assumption, the aggregate demand of these consumers is continuous in prices, hence as  $\epsilon$  goes to zero, the revenue of store  $i$  from these consumers goes to the revenue of store  $j$  from the same consumers in the candidate equilibrium profile. By definition of these consumers, the latter limit revenue is weakly larger than store  $j$ 's profit in the candidate equilibrium profile. Finally, observe that any other consumer who after the deviation chooses store  $i$  spends a nonnegative amount of money. This is because if  $(1 - \epsilon) \cdot \mathbf{p}_j \mathbf{x} < 0$  then  $\mathbf{p}_j \mathbf{x} < (1 - \epsilon) \cdot \mathbf{p}_j \mathbf{x}$ , hence bundles that can be purchased for a negative amount of money can be obtained even cheaper at store  $j$ . This implies that for any  $\delta > 0$ , for small enough  $\epsilon > 0$  the above deviation yields a profit to  $i$  that is at least firm  $j$ 's profit in the candidate equilibrium profile minus  $\delta$ . This contradicts that the candidate profile is an equilibrium, since in the candidate profile firm  $i$ 's profit is strictly smaller than firm  $j$ 's profit.  $\square$

We now turn our attention to the possible existence of an equilibrium with equal and positive profits, as in the previous section. We show that in the presence of a small mass of "noise" consumers as specified below in **Assumption A**, this is not possible. Since this mass can be arbitrarily small, we can conceive its purpose as eliminating equilibria that depend on specialized, non-generic demand restrictions.

**ASSUMPTION A.** For each  $n \in \{1, \dots, N\}$ , there is a positive mass of consumers who, for any prices, buy a positive quantity of good  $n$  and none of any other good. Also, there is a positive mass of consumers who buy positive quantities of all goods at any price.

**THEOREM 2.** *Under Assumption A, in any equilibrium (a) all firms make zero profits, (b) all prices of at least two firms are equal to marginal cost, and (c) all firms set all prices weakly above marginal cost.*

**PROOF.** Consider first the case that some firm chooses a negative price in equilibrium. Let  $n$  be a good such that at least one store charges a negative price for it. By **Assumption A** there is a set of consumers with positive mass, who at all of the prices chosen by the stores in the proposed equilibrium would purchase only  $n$ . These consumers obviously choose one of the stores charging a negative price for  $n$ . Then there exists a store  $i$

that charges a negative price for  $n$ , and this store attracts a positive mass of consumers who buy only good  $n$ . Then the profit generated by the profitable consumers of store  $i$  is strictly larger than the equilibrium profit of  $i$ . Denote this set of consumers by  $S_i^*$ . Consider now the deviation  $(1-\epsilon)\mathbf{p}_i$  for  $\epsilon > 0$ , by any firm  $j \neq i$ . As argued in the proof of **Theorem 1**, this deviation steals all consumers in  $S_i^*$ , and does not attract any consumer who would spend a negative amount in the store. By assumption the aggregate demand by  $S_i^*$  is continuous in prices, hence as  $\epsilon$  converges to zero, the profit generated by  $S_i^*$  converges to the level that these consumers generate at prices  $\mathbf{p}_i$ . Therefore, for small enough  $\epsilon$  the above deviation is profitable. This proves (c).

Consider now the case that all firms charge only nonnegative prices, but profits are strictly positive. This in particular implies that there is a good for every store for which the price is strictly positive and the demand for the good by consumers of the store is strictly positive. By **Assumption A**, there is a set of consumers  $S^0$  with positive mass, who at each of the price vectors  $\mathbf{p}_1, \dots, \mathbf{p}_K$  set by the stores in the proposed equilibrium would purchase a strictly positive amount of each of the goods. For any two firms  $i$  and  $j$ , let  $S_{i,j}^0$  denote the subset of  $S^0$  containing consumers for whom the first choice at prices  $\mathbf{p}_1, \dots, \mathbf{p}_K$  is firm  $i$ , and the second choice is firm  $j$ . Since there is a finite number of firms,  $S_{i,j}^0$  has a positive mass for some firms  $i$  and  $j$ . Note that by definition consumers in  $S_{i,j}^0$  demand a positive amount of each good at prices  $\mathbf{p}_j$ , including the ones whose prices are set to be strictly positive by  $j$ . Hence, at these prices consumers in  $S_{i,j}^0$  generate a profit  $\pi_{i,j}^0 > 0$ . Consider now a deviation  $\mathbf{p}_j - \epsilon \cdot \mathbf{1}$  by firm  $i$ , for  $\epsilon > 0$ . This deviation in particular attracts all former consumers of firm  $j$ , and consumers in  $S_{i,j}^0$ . Since the aggregate demand of this combined set of consumers is continuous in prices, as  $\epsilon$  goes to zero, the profit obtained from this set of consumers after the deviation by firm  $i$  converges to the equilibrium profit of firm  $j$  plus  $\pi_{i,j}^0$ . Moreover, the profit obtained from other consumers converges to a nonnegative amount, since  $\mathbf{p}_j \geq \mathbf{0}$ . Hence, for small enough  $\epsilon$  the proposed deviation is profitable, contradicting that the profile constitutes an equilibrium. This proves (a).

Suppose now that at most one firm sets all prices to be zero. Let  $i$  be a store such that there is no other store who charges prices  $\mathbf{0}$ . By **Assumption A**, there is a positive mass of consumers who at each of the prices in the above profile would purchase a positive amount of all goods. Then there is a store  $j \neq i$ , and a subset of these consumers  $S_{-i,j}$  with positive mass whose choice among stores excluding  $i$  would be  $j$ . Store  $i$  then has a profitable deviation by setting its prices to be  $\frac{1}{2}\mathbf{p}_j$ . This deviation attracts all consumers in  $S_{-i,j}$ , each of whom purchase a positive amount of all goods. By assumption  $\mathbf{p}_j \geq \mathbf{0}$  and  $\mathbf{p}_j \neq \mathbf{0}$ , hence the above implies that the deviation yields strictly positive profit to store  $i$ , contradicting that in equilibrium all stores have zero profits. This proves (b).  $\square$

Note that the example in the previous subsection violates **Assumption A**. In particular, at the prices set by the two firms in the proposed equilibrium, no consumers would like to buy only the subsidized goods  $a_2, a_3, b_2$ , or  $b_3$ . If there were a positive mass of such consumers, no matter how small, the firms would have profitable deviations. This is because the above consumers would be unprofitable for the stores, and hence the

same undercutting-overcutting strategy that we used to show equality of profits would strictly improve a firm's profit, by stealing all the profitable consumers of the other store and getting rid of the unprofitable ones.

## REFERENCES

- Ambrus, Attila and Rosella Argenziano (forthcoming), "Asymmetric networks in two-sided markets." *American Economic Journal: Microeconomics*. [532]
- Chen, Yuxin, Ganesh Iyer, and Amit Pazgal (2005), "Limited memory and market competition." Unpublished paper, Haas School of Business, University of California, Berkeley. [528]
- Eliasz, Kfir and Ran Spiegler (2008), "Consideration sets and competitive marketing." Unpublished paper, Department of Economics, Brown University. [528]
- Ichiishi, Tatsuro (1976), "Economies with a mean demand function." *Journal of Mathematical Economics*, 3, 167–171. [534]
- Kamenica, Emir (forthcoming), "Contextual inference in markets: on the informational content of product lines." *American Economic Review*. [527]
- Lal, Rajiv and Carmen Matutes (1994), "Retail pricing and advertising strategies." *Journal of Business*, 67, 345–370. [527]
- Osborne, Martin J. and Ariel Rubinstein (1998), "Games with procedurally rational players." *American Economic Review*, 88, 834–847. [527]
- Spiegler, Ran (2006), "Competition over agents with boundedly rational expectations." *Theoretical Economics*, 1, 207–231. [527]
- Varian, Hal R. (1980), "A model of sales." *American Economic Review*, 70, 651–659. [525, 527]