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Self-starting stable coherent mode-locking in a two-section laser

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submitted: October 24, 2014

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No. 2030

Berlin 2014

Abstract

Coherent mode locking (CM) uses self-induced transparency soliton mechanism to achieve the pulse durations below the limit allowed by the gain line width. Despite of the great promise it has not yet been realized experimentally because of the complicated setup is required. In this article we predict that the dynamical stabilization effects allow to radically simplify the setup. We show that if the cavity length is selected properly, a very stable CM regime can be realized in an elementary two-section ring-cavity geometry, and this regime is self-developing from the non-lasing state. The stability of the pulsed regime is the result of a dynamical protection of the pulse from background fluctuations due to finite-cavity-size effects.

1 Introduction

One of the basic lasing regimes is so-called mode-locking, when a periodic train of short pulses is produced [1–6]. In the case when only a single pulse per round-trip time appears, it can be considered as a localized structure (soliton) propagating in a cavity [7,8]. Passive mode locking is very commonly studied in the approximation where the medium polarization relaxation time is much smaller than the pulse duration [1, 7, 9, 10]. Because of this, the polarization of the medium is assumed to "follow" the field. The dynamics of the pulse propagation is governed by the population inversion and thus the duration of the resulting pulse is limited by the width of the gain spectra [2, 4]. This is valid also in the case when the absorber works in the coherent regime, whereas the gain medium is still in incoherent one [11–13].

In some cases, such adiabatic approximation is not valid anymore [14, 15]. This is, for instance, the case of self-induced-transparency (SIT) solitons, or 2π-pulses [8, 16–18], which propagates through the absorbing resonant medium without loss of its shape. Such SIT solitons based on the coherent light-matter interaction may be much shorter than the phase coherence time [16–18]. In a cavity geometry, such solitonic pulse corresponds to a mode-locking, thus the term "coherent solitonic mode-locking" was coined in [8, 19]. A simple soliton in a cavity homogeneously filled with a gain medium is believed to be unstable, because the low-intensity fluctuations of the background far behind and before the soliton lead to formation of further pulses, and their interaction may break the mode-locking pulse train [8, 12, 20, 21]. To stabilize such soliton, a protection from the noise at the background before and behind the pulse is necessary. That is, the system must provide high losses for the background fluctuations, at the same time providing amplification for the pulse itself.

A stable passive coherent solitonic mode locking scheme was proposed in the seminal work [8, 19], where a "homogeneous mixture" of absorbing and amplifying media where used. If the

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Table 1: Parameter values used in the numerical simulations

Parameter	Gain ($L_a < z < L$)	Absorber ($0 < z < L_a$)
central transition wavelength	$\lambda_{12} = 0.6\,\mu m$	$\lambda_{12} = 0.6\,\mu m$
length of the medium	$L_g = 15$ cm	$L_a = 15$ cm
concentration of two-level atoms, $N_0(z) = \ldots$	$N_{0g} = 4.0 \cdot 10^{13}$ cm^{-3}	$N_{0a} = 0.8 \cdot 10^{13}$ cm^{-3}
transition dipole moment, $d_{12}(z) = \ldots$	d_{12g} = 0.5 Debye	d_{12a} = 1 Debye
population difference at equilibrium, $\Delta\rho_0(z) = \ldots$	$\Delta\rho_{0g} = -1$	$\Delta\rho_{0a} = 1$
population difference relaxation time, $T_1(z) = \ldots$	$T_{1g} = 6$ ns	$T_{1a} = 6$ ns
polarization relaxation time, $T_2(z) = \ldots$	$T_{2g} = 0.5$ ns	$T_{2a} = 0.5$ ns

matter-to-light interaction constant (dipole moment) of the absorber is two times higher than the one of the amplifier in such a "mixture", a solitonic pulse which has the pulse area π in the amplifier and, at the same time, 2π in the absorber, can propagate stably and without change of its shape [17]. Its form is self-restored even after relatively strong disturbances, for instance, reflection from a non-perfect mirror. It is also protected from the background fluctuations. Importantly, such mechanism where predicted to allow short pulses with the spectral width sufficiently exceeding the gain width in the medium, even up to single-cycle limit [8, 19–23]. Moreover, a seed pulse sent to such medium will be shortened automatically as it propagates through the medium, until it reaches its pulse duration limit.

The drawback of this approach is that the pulsed radiation can not develop spontaneously from non-lasing state. To initiate a soliton in such system, one needs a seed pump pulse injected to the laser. In addition, a "homogeneous mixture" of the amplifier and absorber, which ensures the solitonic character of the resulting mode locking, makes the practical implementation rather difficult, although practically realizable proposals exist [20, 21].

In this article, we consider radically more simple scheme when the amplified and absorber media are well spatially separated. In this case, the coherent solitonic mode locking can work equally well. We also show that, if the cavity length is selected properly, the mode-locked regime is stable, even if the non-lasing state is not. This allows to reach the pulsed regime from the non-lasing state without any need for a seed pulse. The stability of the resulting pulsed attractor is due to a dynamical protection against small perturbations taking place in the strongly-nonlinear regime due to finite-size effects in the cavity.

2   Results of numerical simulations and discussions

The ring-cavity configuration considered in this article is shown in Fig. 1a. Between the mirrors, only one of which is assumed partially reflecting with the reflection coefficient R, and the others are "ideal" for simplicity, the gain and the absorber sections are placed. Both sections consist of resonant nonlinear medium, tuned to the same frequency. The coupling to the field, namely the dipole moment of active "atoms" is different for both sections. The media and field are described

in the two-level and slowly-varying envelope approximations respectively [18]:

$$\partial_t P(z,t) = -\frac{P(z,t)}{T_2(z)} + \frac{d_{12}(z)}{2\hbar}\Delta\rho(z,t)A(z,t), \tag{1}$$

$$\partial_t \Delta\rho(z,t) = -\frac{\Delta\rho(z,t) - \Delta\rho_0(z)}{T_1(z)} - \frac{d_{12}(z)}{2\hbar}F(z,t), \tag{2}$$

$$\partial_t A(z,t) - c\partial_z A(z,t) = 4\pi\omega_{12}d_{12}(z)N_0(z)P(z,t). \tag{3}$$

Here $P(z,t)$ is the slowly-varying envelope of the non-diagonal element of the density matrix describing the two-level atom in the absorber (for $0 < z < L_a$) and gain (for $L_a < z < L \equiv L_a + L_g$) sections in rotating wave approximation; $\Delta\rho(z,t)$ is the difference of diagonal elements of the density matrix (population difference per single atom) in the gain and absorber sections, $A(z,t)$ is the lowly varying field amplitude in the gain and absorber sections. $\omega_{12} = 2\pi c/\lambda_{12}$ is the transition frequency, c is the speed of light in vacuum, and $F(z,t) = A(z,t)P(z,t)$; The other parameters, and their values in numerical simulations are given in the Table 1 and are characteristic for gases [24]. The cavity length is $L = 30$ cm (corresponding to roundtrip time $\tau = 1$ns), and the mirror reflectively is $R = 0.8$. The equations (1)-(3) are highly used in different physical situations describing resonant behavior of nonlinear media [16–18, 25–28].

Important quantity characterizing the pulse propagation dynamics in two level atoms is the pulse area Φ: $\Phi(z,t) \approx d_{1,2}/\hbar \int_{t-3\tau_p}^{t+3\tau_p} A(z,\tau)d\tau$, where τ_p is the pulse duration; that is, we integrate in the region around the pulse to get practically useful measure in the case when other pulses are present. According to the original idea of the coherent soliton mode locking [8, 19–23], the dipole moments of the gain is two times smaller than in the absorber, which ensures that fluctuations of non-lasing state decay. Such situation can be described in the following way: the stable pulsed regime exists and is well separated from the non-lasing steady state, which is also stable (see inset to Fig. 1c). The system is attracted to the regime, in which vicinity it is located initially. Other regimes may exist but they are irrelevant for our consideration. To achieve a stable pulsed regime in this situation one needs a large-intensity "perturbation", that is, a seed pulse, which makes the scheme more complicated from the practical point of view.

Although in the infinite-length "mixed" medium the soliton will be shortened until it reaches single-cycle regime, in the cavity geometry with the limited cavity length the pulse duration achieves its stationary value τ_p, which is determined by the cavity parameters. The order of magnitude of τ_p, e.g. in absorber, can be obtained from the requirement $\Phi \approx 2\pi$ leading to $A\tau_p \approx 2\pi\hbar/d$. Using also the energy flux balance in the gain section, that is, assuming that the energy flux produced by the atoms $w \approx \hbar\omega_{12}L_gN_g$ is fully transferend to the field, that is, $w \approx A^2\tau_p/8\pi c$, we will finally obtain:

$$\tau_p \approx \pi c\hbar/(d_a^2\omega_{12}N_gL_g), \tag{4}$$

which gives for the parameters we use the order of tens picoseconds. We note that Eq. (4) gives only very raw approximation of the pulse duration. Better approximation is possible using the equations given in [8, 21].

We assume the lengths of the gain and absorber sections L_g and L_a, larger than the pulse length $c\tau_p$ $L_a, L_g > c\tau_p$. Importantly, we also assume that the cavity is small enough to prevent
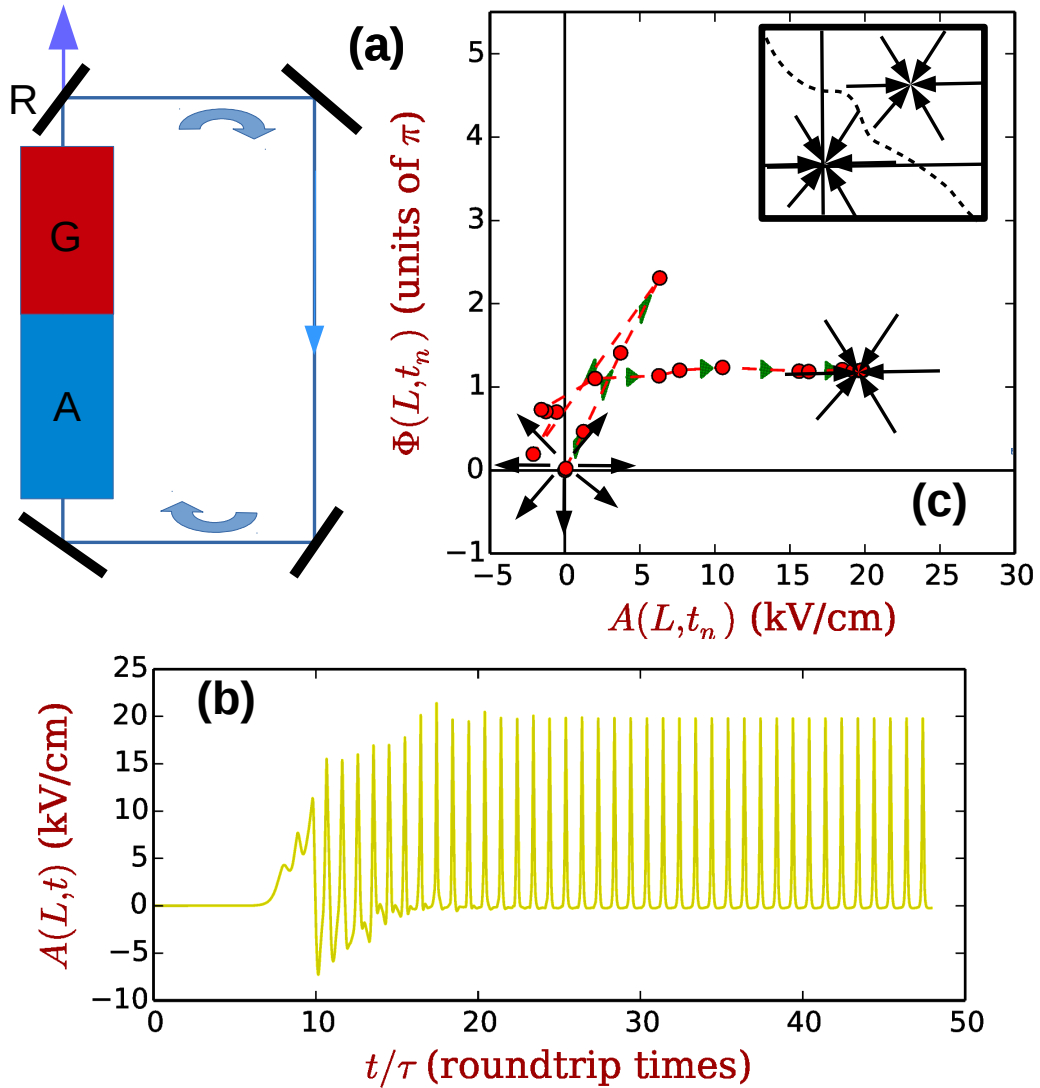
Figure 1: (a) The scheme of the ring-cavity laser with an absorber (A) and gain (G) sections. (b) The output field $A(L,t)$ evolution in dependence on time during a buildup of a stable pulsed regime from a vicinity of the non-lasing state. (c) The map in a plane of the pulse area $\Phi(L,t_n)$ and field $A(L,t_n)$ in the time moments $t_n = t_0 + n\tau$, where $n = 1, 2 \ldots$ and t_0 is a constant chosen to align t_n to the pulse maxima at large t_n. Black arrows show the attracting or repelling character of every steady-state. Green arrows show the direction of the system evolution. In the inset the corresponding dynamical picture for the configuration from [8] is shown. The laser parameters are given in Table 1.

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the population difference to relax to their equilibrium states, that is $\tau \ll T_{1g}, T_{1a}$. The last condition is crucial and makes the pulse propagating in the cavity essentially non-solitonic as we will see later.

The parameters of the system are taken such that the parameter G_0 defining the gain-loss balance for low-intensity fluctuations and thus the stability of non-lasing state:

$$G_0 = \ln \left[R \exp \left\{ \int_0^L \alpha(z)dz \right\} \right],$$

(5)

where $\alpha(z) = 2\pi d_{12}^2(z)\omega_{12}N_0(z)T_2(z)/c\hbar$ is the gain/loss coefficient for low-intensity fluctuations. We chose N_a and N_g (see Table 1) in such a way that $G_0 > 0$, facilitating the growth of low-frequency fluctuations and making the non-lazing regime unstable.

The parameters for our numerical simulations are shown in Table 1. One can see that $G_0 \approx 9.0 > 0$, that is, the non-lasing state is strongly unstable. We start the simulations from its vicinity, assuming the field close to zero, the population differences at their equilibrium values and the polarization to be zero, and track the evolution of the field in the cavity with time. An exemplary trajectory is shown in Fig. 1b, where we have started from the a short probe pulse of the the duration $\tau_{p0} = 100$ ps and initial pulse area is $\Phi_0 = 10^{-7}\pi$.

After a short but irregular transient process up to $t/\tau \sim 20$ a stable regime with the pulse duration $\tau_p = 80$ ps is born, which is significantly smaller that the relaxation time T_2, thus corroborating the coherent character of the mode-locking regime in this case.

We tested numerically the achievability and stability of the pulsed regime, preforming many simulations starting initial conditions randomly selected every time in large range of pulse shapes, with or without initial noise. All the trajectories were attracted to the single stable steady-state identical to the shown in Fig. 1b,c up to a time shift and sign of the output field.

The details of the resulting steady-state regime are shown in Fig. 2, where the pulse shapes in the absorber Fig. 2(a) and amplifier Fig. 2(b) sections are shown, together with the pulse area Φ in the cavity Fig. 2(c). As on can see, the pulse area in absorber, initially (at $z = 0$) being reduced to around 1.9π by the losses at the mirror, approaches quickly it's stable value 2π. The population difference is positive before the pulse, becomes negative at its leading edge, and at the trailing edge is restored to the initial value demonstrating dynamics which is typical for 2π-pulses.

In the gain section, the pulse area is close to π, as seen from Fig. 2(c). Typically for such pulses, the population inversion changes its sign after the pulse propagation. Importantly, the population difference before the pulse is far from its equilibrium value, and rather close to zero (although remains positive), that is, the medium is in slightly amplifying regime before the pulse. After the pulse, the gain medium is switched to slightly absorbing regime. As the relaxation time $T_{1g} \ll \tau$, the gain section has no time to achieve its stationary value after the pulse. In contrast, the population difference in the absorber remains comparable to its equilibrium value. As a result, the gain-loss balance for the small perturbations becomes negative, protecting the pulse against destroying. Namely, G_0 in the such pulsed regime (obtained by integrating everywhere outside of the pulse in Eq. 5) becomes negative, in particular for parameters in Fig. 2 $G_0 \approx -10.0$.

This dynamics is rather typical for wide range of the parameters as far as the conditions $L_a, L_g >$
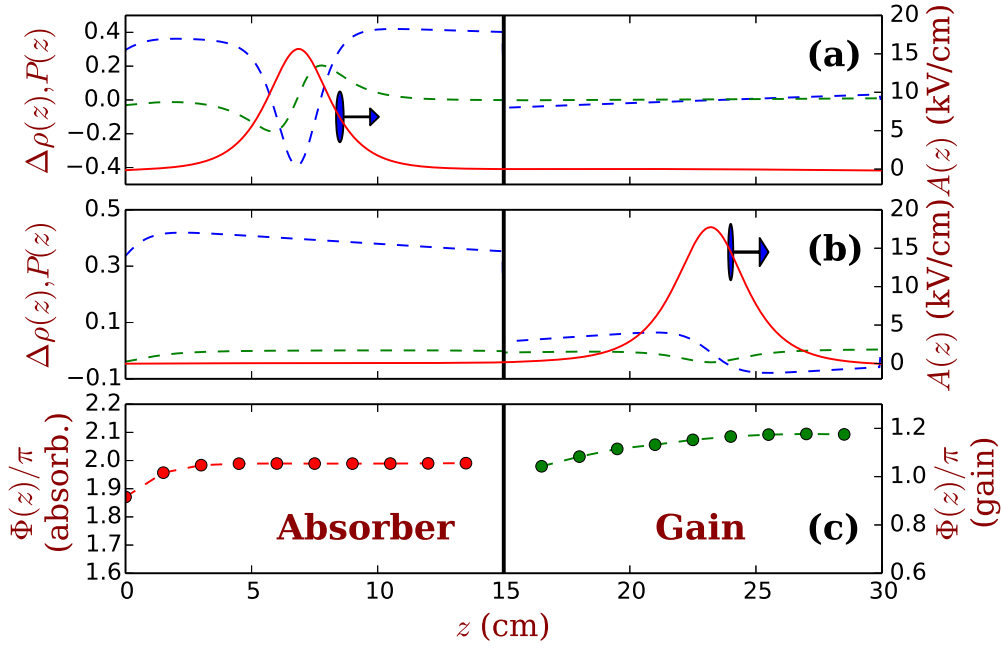
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Figure 2: Pulse shapes in the amplifier (a) and absorber (b) sections in the stationary regime. The field amplitude $A(z)$ (red line) as well as the polarization $P_s(z)$ (green dashed line) and population difference $\Delta\rho(z)$ (blue dashed line) are shown for the parameters in Table 1. In (c), the pulse area Φ in the gain and absorber sections are shown.

$c\tau_p$ and $L \ll cT_{1g}, cT_{1a}$, $G_0 > 0$ (and is not very large, $G_0 \lesssim 20$) are satisfied. For instance, in Fig. 3 the dependence of pulse duration and the peak intensity on the dipole moment of absorber d_{12a} is shown, assuming the other parameters except N_{0a} being unchanged, the later is scaled such that G_0 remains constant. The pulse durations are far from the single-cycle limit but nevertheless are significantly smaller than the phase coherence time.

Now let us briefly consider what happens if we cross these cavity length limits in various directions. If L_g approaches cT_{1g}, the population difference is able to recover before the pulse returns. In the other words, now more than one such pulse can propagate inside the cavity. In this situation, according to our simulations, the number of pulses in the cavity is indeed greater than one. The pulses typically interact with each other, leading to a complicated dynamics which we do not discuss here. In the case when L is comparable with τ_p, the soliton-like structure can not anymore exist, and (a spatially inhomogeneous) steady-state appears.

3 Conclusions

In conclusion, we have shown that the stable coherent mode locking allowing pulse durations significantly smaller than T_2 is much easier to obtain as it was thought before. No seed pulse and no complicated, difficult to realize artificial geometry is needed. Very trivial ring-cavity two-section laser with properly selected parameters makes the stable mode-locked regime being
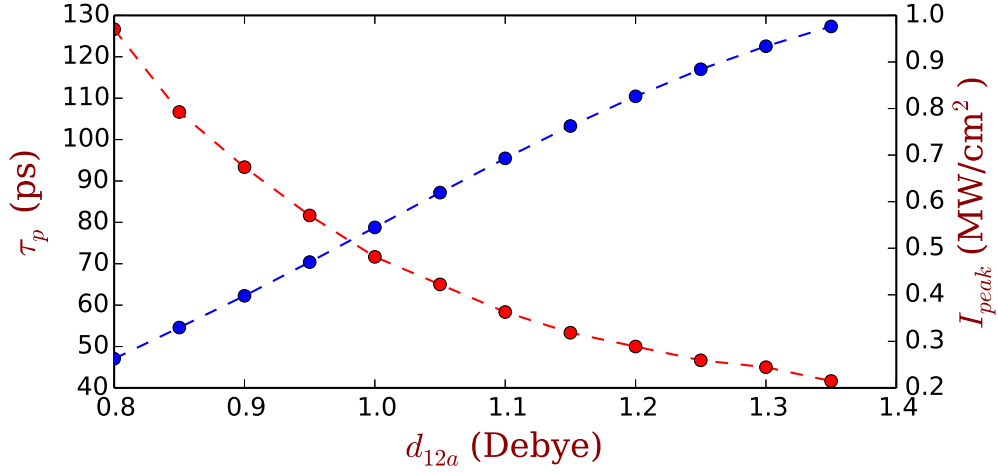
Figure 3: The pulse duration τ_p and the peak intensity I_{peak} in dependence on the d_{12a} in the stationary regime, changing the concentration in absorber section in such a way that G_0 remains the same, and keeping all other parameters as in Table 1

always automatically started from the non-lasing state. In particular, cavity round-trip time τ must be significantly less than the population difference relaxation time scale $\tau \ll T_{1g}$ but not too short to allow the pulse develop. In addition the non-lasing steady-state is to be made unstable, resulting in self-starting. Despite of such instability, the resulting pulsed regime is stable and does not depend on the initial conditions. The reason is that, although the resulting optical pulse might looks like as a cavity soliton, it is actually not. The long "tail" of the disturbed population difference persist, making the resulting pulse significantly non-local and preventing the perturbations before and behind the pulse from growing. Although here the typical case of gaseous media is considered, our simulations show that the regime exist for range number of parameters, typical for other, for instance, solid state or semiconductor media.

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