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Transient Cherenkov radiation from an inhomogeneous string excited by an ultrashort laser pulse at superluminal velocity

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Abstract

An optical response of one-dimensional string made of dipoles with a periodically vary-ing density excited by a spot of light moving along the string at superluminal (sub-luminal) velocity is studied. We consider in details the spectral and temporal dynamics of the Cherenkov radiation, which occurs in such system in the transient regime. We point out the resonance character of radiation and the appearance of a new Doppler-like frequency in the spectrum of the transient Cherenkov radiation. Possible applications of the effect as well as different string topologies are discussed.

1 Introduction

The problem of superluminal motion and its existence in nature attracts attention of various researchers for rather long time. At the turn of XIX-XX centuries, O. Heaviside and A. Sommerfeld considered radiation of charged particles moving in vacuum at the velocity greater than the velocity of light in vacuum c (see [1–10] and references therein). However, their works were forgotten for many years because the special theory of relativity bans such motions. Further analysis has shown that the prohibited are only those motions that involve signal (information) transfer at the superluminal velocity, this strong prohibition being related to the violation of the causality principle [1, 2, 9, 11].

If a charged particle moves faster than light in some medium so called Cherenkov radiation occurs. It appears typically in a cone with the angle depending on the ratio of the particle velocity and the speed of light in the media. Similar conical emission can appear also in nonlinear optical parametric processes [12–16]. Not only particles but also spots of light can propagate faster than the phase velocity of light in particular medium [8, 17–21]. Those can be optical pulses and solitons in fibers or filaments [22–29] as well as in other optical systems [30–38]. Cherenkov radiation in different periodically modulated media, both in space [30, 39, 40] and in time [41] where also considered. The intersection point of two wavefronts can also move at the velocity exceeding that of light [8, 42]. Similar situation occurs when a short plane-wave impulse crosses a flat screen (or the plane diffraction grating) [8, 39]. In this case, the intersection of the pulse and the screen moves along the screen at the velocity $V = c/\sin\beta > c$ (here β is the angle of wave incidence) [43, 44].

Despite the great number of different configurations studied in the context of Cherenkov radiation, in all those cases such radiation has the same nature. Namely, it is a result of interference of the secondary waves emitted by the moving "particle". The temporal shape of the radiating wave and thus the spectrum of radiation can be significantly different depending on the particular situation.

1

For instance, the spectrum of radiation induced by a charged particle moving faster than the phase velocity of light is rather unstructured [1,2]. In many cases, resonances may occur [8,39]. One important example is the so called Purcell-Smith radiation [39,45] appearing as the particle moves in the vicinity of a periodic structure. Also, a moving and at the same time oscillating dipole emits the Cherenkov radiation characterized with well defined resonance [35], similar situation is realized for optical solitons propagation [46].

In the present article, we consider in details the Cherenkov-type radiation in the case of a 1D string formed of two-level atoms with a spatially-periodic modulated number density. This system is excited at the superluminal (sub-luminal) velocity at the point of intersection of the string with a moving spot of light. This geometry is imposed by recent advances in optical technologies which allow for the reliable control of matter properties on the spatial level of the order of wavelength of light or even much smaller that allows to create quasi-1D objects (see [47, 48] and references therein). Although our consideration is rather general, we bear in mind the spatial properties of systems of atomic-size scattering elements or even nanoantenna arrays [49, 50] as well as thin microcapillaries. In particular, we consider the transient emission process following the excitation of such a system. We show that besides the resonance at the proper frequency of dipoles constituting the string, an additional frequency due to the effect similar to the Doppler one appears in the spectrum of medium response in the presence of the periodic spatial inhomogeneity. We study this effect for different string topologies, namely a linear and a circular one.

The structure of the paper is the following: In the Section II the possible physical realizations of our system are considered; Section III describes the linear and Sec. IV the circular geometry of the string. Concluding remarks are presented in section V.

2   Physical considerations

The geometry we would like to consider is illustrated in Fig. 1(a). A short spectrally broadband optical plane wave pulse is emitted by a source 1, passing through lenses 2 and 3 which makes the pulse spatially extended. The source must produce significantly broadband and flat spectrum [51–59], which includes also the resonance frequency ω_0 of the dipoles forming the string. This spatially extended short in time and in the axial direction pulse has the form of thin "sheet of light" 4, which illuminates at the angle β the string medium parallel to z-axis. Similar geometry was recently realized experimentally [20].

We also assume that the string consists of oscillators (dipoles) with the resonance frequency ω_0 and decay constant γ, which number density varies periodically along the string with the spatial period Λ_z. In any point of the string its response to the excitation pulse $\mathbf{E}(t)$ is described by the polarization $\mathbf{P}(t)$

$$\ddot{\mathbf{P}} + \gamma\dot{\mathbf{P}} + \omega_0^2\mathbf{P} = g\mathbf{E}(t), \tag{1}$$

where g is the coupling strength to the field.

We assume also, that the excitation pulse is shorter than the resonant period of oscillators, so that its spectrum not only includes ω_0 but at the same time, is significantly broad and flat [see
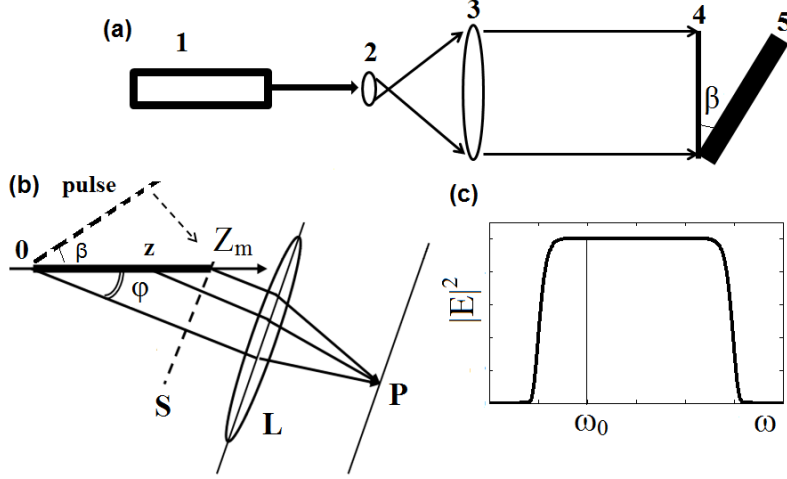
Figure 1: (a) Excitation of a string at superluminal velocity. 1 - a short spectrally broadband laser pulse source, 2,3 -lenses, 4 the plane pulse wave. The intersection of the plane pulse and the medium moves along the string at the velocity $V = c/\sin\beta > c$. (b) The observation geometry of the string of the length Z_m emission. The observer is placed far away from the string or in the focus of a lens L which collects the radiation of the string parallel to the z-axis. (c) The source must produce significantly broadband and flat spectrum [51–59], which includes also the resonance frequency ω_0 of the dipoles forming the string.

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Fig. 1(c)]. Such pulses can be obtained, for instance, in THz and MIR range using a gaseous ionization-based source pumped by a ultrashort optical pump pulse [51–59].

Under this condition, the response of the oscillators can be to a good precision described as:

$$\mathbf{P}(t) \cong \mathbf{p}_0 e^{-\gamma t}\cos(\omega_0 t)\Theta(t), \tag{2}$$

where $\mathbf{p}_0$ is the dipole polarization at the instance when the excitation burst crosses the medium, $\Theta(t)$ is the Heaviside step-function. The electric field created by this excitation observed at the remote position Q is determined by the solution of the wave equation $\Box\mathbf{E} = \mu_0\partial_{tt}\mathbf{P}$, where $\Box = \partial_{xx} + \partial_{yy} + \partial_{zz} - 1/c^2\partial_{tt}$ is the d'Alembert operator, c is the speed of light in vacuum. In particular, if the source is localized "in a single point", that is, $\mathbf{P} \propto \delta(\mathbf{r})$, the observed field at the point $\mathbf{r}'$ is:

$$\mathbf{E}(\mathbf{r}', t) \propto \mathbf{P}(\mathbf{r}, t - |\mathbf{r} - \mathbf{r}'|/c). \tag{3}$$

In the following, we normalize the later relation in such a way that the coefficient of proportionality between $\mathbf{E}$ and $\mathbf{P}$ is one (which implies that the normalization depends on the observer's position).

The physical nature of the oscillators forming the string can be very different. In particular, one can use a string of nanoantennas made of the conducting material. Such nanoantennas have indeed the resonance frequencies defined by their plasmonic resonances, which are highly flexible and are determined by the geometry and size of the structures [47–50]. Using such structures the resonance frequency ω_0 can be tuned in the wide range from THz up to the visible.

3

3 A linear string

3.1 General considerations

In this section we consider the case when our string has the form of a straight line of the length Z_m [Fig. 1(b)]. The observer is located at the very large distance from the string or in the focal point P of the lens L. The string consists of identical dipole oscillators having the resonance frequency ω_0 (λ_0 is the corresponding wavelength) and the decay rate γ. The number density of oscillators along the Oz axis varies periodically with the period Λ_z:

$$N(z) = \frac{1}{2}\left(1 + a\cos\frac{2\pi}{\Lambda_z}z\right), \tag{4}$$

where $a \leq 1$ is the amplitude of density oscillations. This equation describes a sort of 1D diffraction grating formed of particles possessing proper resonance frequency. In the following, for simplicity, we take $a = 1$.

At the initial time moment $t = 0$ the excitation point crosses the point $z = 0$ and starts to propagate at the velocity V along the string towards its other end. We suppose that the exciting pulse is linearly polarized and the polarization direction is perpendicular to the plane of Fig. 1(b). The oscillators start to emit electromagnetic radiation according to the response law Eqs. (2)-(3). We now consider this secondary radiation propagating at the angle φ to the string, which reaches the amplitude detector at the point P (Fig. 1(b)). Under those circumstances we can restrict ourself by a single linear polarization, thus obtaining a scalar problem.

The electric field emitted by the oscillator located in z (as being observed in the same point) is proportional, according to Eq. (2), to:

$$E(t,z) = \exp\left[-\frac{\gamma}{2}\left(t - \frac{z}{V}\right)\right]\cos\left[\omega_0\left(t - \frac{z}{V}\right)\right]\Theta\left[t - \frac{z}{V}\right], \tag{5}$$

where the delayed argument describes the fact that the excitation appears in the point z at the moment delayed by z/V. Instead of point P one may consider the electric field at the plane S orthogonal to the direction φ passing through the point Z_m. Propagation time from this reference plane to P is constant and will be omitted in the following analysis.

The light propagation time from the point z to the reference plane is given by $\frac{Z_m - z}{c}\cos\varphi$. Thus, the electric field emitted at the point z will have at the reference plane the value:

$$E_{ref}(t,z) = \exp\left[-\frac{\gamma}{2}f(t,z)\right]\cos\left[\omega_0 f(t,z)\right]\Theta\left[f(t,z)\right], \tag{6}$$

where $f(t,z) = t - \frac{z}{V} - \frac{Z_m - z}{c}\cos\varphi$.

The total field observed in P is obtained by the integration of Eq. (6) over the whole string:

$$E(t,\varphi) = \int_0^{Z_m} N(z)\exp\left[-\frac{\gamma}{2}f(t,z)\right]\cos\left[\omega_0 f(t,z)\right]\Theta\left[f(t,z)\right]\,dz. \tag{7}$$

The analytical solution of Eq. (7) in the case of $\gamma = 0$ is given in the Appendix. As one can see from analytical calculation in Appendix [see Eq. (20)] the response contains the resonance frequency of oscillators ω_0 together with a new component given, by the expression:

$$\Omega_1 = 2\pi \frac{V/\Lambda_z}{|\frac{V}{c}\cos\varphi - 1|}. \tag{8}$$

The inverse numerator of Eq. (8) is the time interval which the excitation spot needs to cross the single oscillation period of $N(z)$. When this time is equal to the period of the oscillations ($V/c = \Lambda_z/\lambda_0$) formula (8) leads to:

$$\Omega_{1_D} = \frac{\omega_0}{|\frac{V}{c}\cos\varphi - 1|}. \tag{9}$$

This relation formally coincides with that one for the Doppler frequency shift [43, 44], so we will call it the Doppler frequency but will keep in mind that its physical origin differs from that of the Doppler effect.

Equation Eq. (8) is valid for arbitrary V and has the same form as the one appearing in the case of Purcell-Smith radiation. The appearance of this frequency and other related questions will be studied in detail in the next section.

3.2 Temporal and spectral field properties and their dependence on parameters

Now we explore the temporal and spectral shape of the linear string response defined by Eq. (7) and its dependence on the system parameters.

We start from the numerical simulations of Eq. (7) for some "typical" parameter values. Namely, we choose the normalized parameters as: $\frac{V}{c} = 2$, $\frac{Z_m}{\Lambda_z} = 9.55$, $\frac{\Lambda_z}{\lambda_0} = 5$, $\frac{\omega_0}{\gamma} = 22.22$. The real-world values of the parameters corresponding to this set depend on the resonance frequency of the oscillators in the string. For instance, assuming $\omega_0 = 2\pi \times 10$ ps^{-1} (frequency for which the δ-function assumption from Fig. 1(c) is especially easy fulfilled), we will have $\Lambda_z = 150\mu m$, $Z_m = 1.4$ mm, and $\gamma = 2.8$ ps^{-1}. Another example is the pump at optical frequencies $\omega_0 = 2\pi \times 375$ ps^{-1}, $\Lambda_z = 4\ \mu m$, $Z_m = 40\mu m$, $\gamma = 106.04$ ps^{-1}.

The numerical solution of integral Eq. (7) and its spectrum are shown in Fig. 2 (in normalized units) for the above mentioned parameters, and assuming $\varphi = 0$ (observation point is on the same line as the string) in Fig. 2(a,b) and the Cherenkov angle $\varphi = 60$ degree in Fig. 2(c,d).

As one can see, the resonant response at $\omega = \omega_0$ dominates in both cases. Nevertheless, for $\varphi = 0$ additional frequency arises. As it is seen in Fig. 2(a), the new frequency appears in the transient process for the time interval from approx. $t_1/T_0 = 20$ to approx. $t_2/T_0 = 47$, at the moment t_1 the excitation spot reaches the end of the string. During the period t_1 to t_2, the radiation from the points $z = Z_m$ to $z = 0$ arrives to the observation plane. As the result of the interference of the incoming waves, a transition process occurs. It lasts until the moment t_2. Only the decaying emission with the frequency ω_0 remains at the later time.
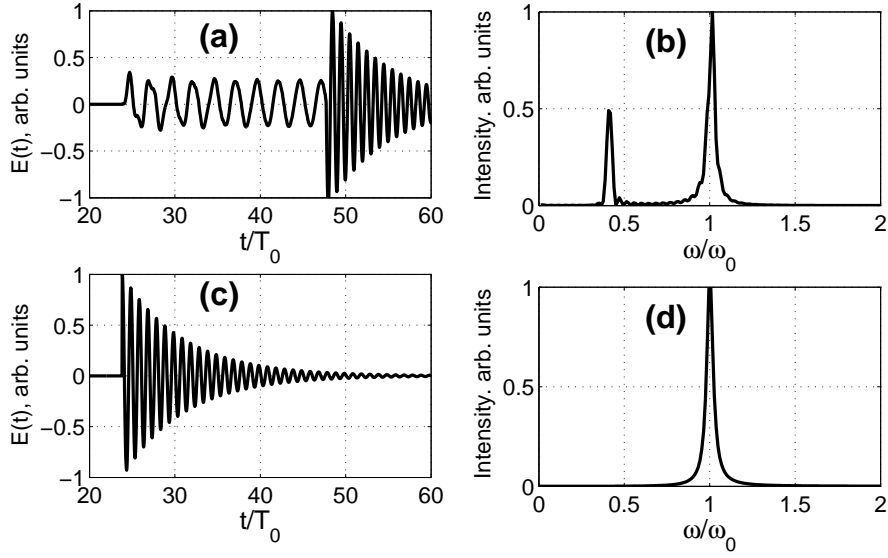
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Figure 2: Time dependence of the field $E(t)$ according to Eq. (7) (a,c) and its spectral intensity $I(\omega)$ (b,d) normalized to their maximal values vs. normalized time t/T_0 and frequency ω/ω_0, for $\frac{V}{c} = 2$, $\frac{Z_m}{\Lambda_z} = 9.55$, $\frac{\Lambda_z}{\lambda_0} = 5$, $\frac{\omega_0}{\gamma} = 22.22$ for the observation angle $\varphi = 0$ (a,b) and $\varphi = 60°$ (c,d), the latter corresponds to the Cherenkov emission angle.

For the superluminal velocity of the excitation the denominator of Eq. (8) approaches zero if:

$$\cos \varphi_0 = c/V, \tag{10}$$

which coincides with the condition for Cherenkov radiation. Fig. 2(c) corresponds to the Cherenkov emission angle. This angle corresponds also to the zeroes-order diffraction peak of the grating formed by $N(z)$. Under the parameters of Fig. 1, $\varphi_0 = 60$ degrees. When the condition Eq. (10) is fulfilled, we have $\Omega_1 = \infty$, and the radiation from all points of the grating (the resonance medium) comes to the reference plane simultaneously, thus no transient process occurs.

Analogously, +1st- and -1st diffraction orders maxima are defined by the relation:

$$\cos \varphi_{\pm 1} = \frac{\pm \lambda_0}{\Lambda_z} + \frac{c}{V}, \tag{11}$$

which for the parameters of Fig. 3(a) gives the angles $\varphi_{+1} = 45.57$ and $\varphi_{-1} = 72.54$ degree correspondingly. For those angles, we have $\omega_0 = \Omega_1$ as well. For all values of φ different from the one given by Eq. (11), the Doppler frequency Ω_1 is not equal to ω_0. It should be noted however that in this case the radiation intensity is smaller than for the Cherenkov angle.

Dependence of the Eq. (7) solution spectrum on the system parameters is presented in Fig. 3. In particular, the dependence on the observation angle φ is presented in Fig. 3(a), on the excitation speed V in Fig. 3(b) and on the grating period Λ_z (cf. Eq. (4)) in Fig. 3(c).

Dependence of the string response spectrum on V/c assuming $\varphi = 60$ degree is presented in Fig. 3(b). The other parameters are the same as in Fig. 2(c). One can clearly see the frequency
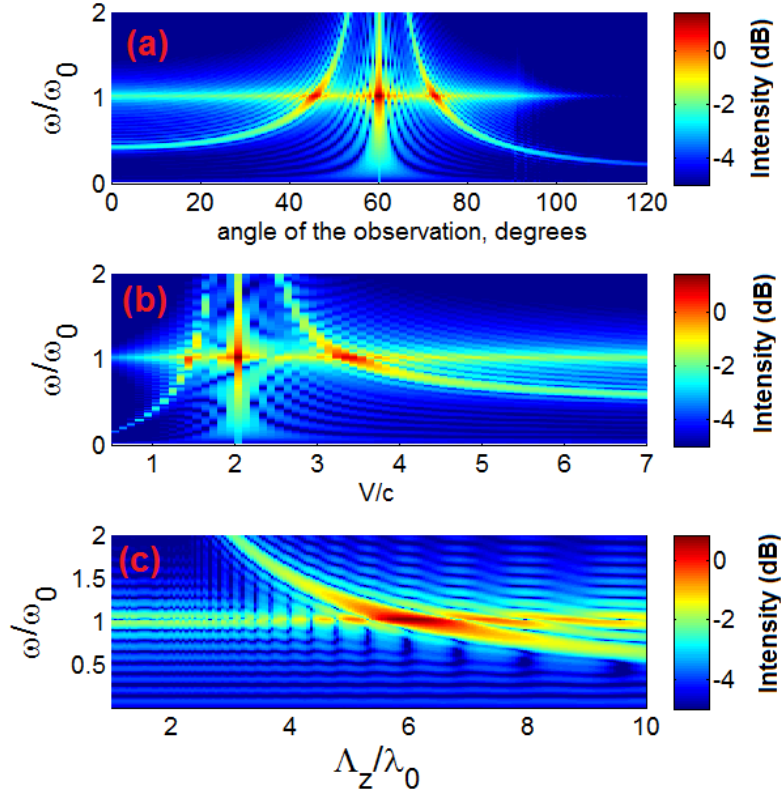
Figure 3: Dependence of the spectral intensity $I(\omega)$ of the string response according to Eq. (7) on the observation angle φ (a), the excitation velocity V (b) and on the string density modulation period Λ_z (c). The other parameters coincide with those ones given in Fig. 2(c,d). The spectral intensity is presented in the logarithmic scale.

branch corresponding to the resonance $\omega = \omega_0$, as well as the another one corresponding to the frequency shift given by Eq. (8).

According to Eq. (8) and Fig. 2(c), Ω_1 decreases with the increasing of V/c for $V/c > 2$ and increases for $V/c < 2$. From Eq. (8) it also follows that $\Omega_1 \to \infty$ for $V \to 2c$ (when $\varphi = 60$ degree). This also coincides with the typical behavior of the Doppler frequency shift.

The dependence of the string response spectrum on the modulation period Λ_z/λ_0 is presented in Fig. 3(c) for $V/c = 3, \varphi = 60$ degree. As it can be seen, Ω_1 decreases with increasing of the Λ_z/λ_0.

Up to now we have considered the case when the string is excited by a spot of light moving at the superluminal velocity. Another interesting case if the exciting spot moves at the sub-luminal velocity.

Such situation can be realized not only using the scheme in Fig. 1, but also using an electron beam moving with some velocity u at an angle ψ to the boundary of the string. In this case, the velocity of the intersection of the incident beam with the boundary of the medium is $V = u/\sin\psi$ [44].

The example of numerical solution of the integral Eq. (7) assuming $V/c = 0.7$ and $\varphi = 0$ are shown in Fig. 4. Other parameters are the same as in the Fig. 2. One can see that the additional frequency component arises during the transient process from approx. $t_1/T_0 = 47$ to appr. $t_2/T_0 = 70$. At the time moment t_1 the radiation from the point $z = 0$ reaches the end of the medium. At the time moment t_2 the radiation from the point $z = Z_m$ appears at the observation point. Later on, only decaying oscillations at the frequency ω_0 remain.

Dependence of the string response spectrum on the observation angle φ as well as on the grating period Λ_z is presented in Fig. 5. As one can see, the situation in the case $V < c$ is in many respects similar to the case of the superluminal velocity. In particular, Ω_1 decreases with the increase of φ as well as with the increase of Λ_z. On the other hand, the Cherenkov angle at which $\Omega_1 = \infty$ is never achieved.
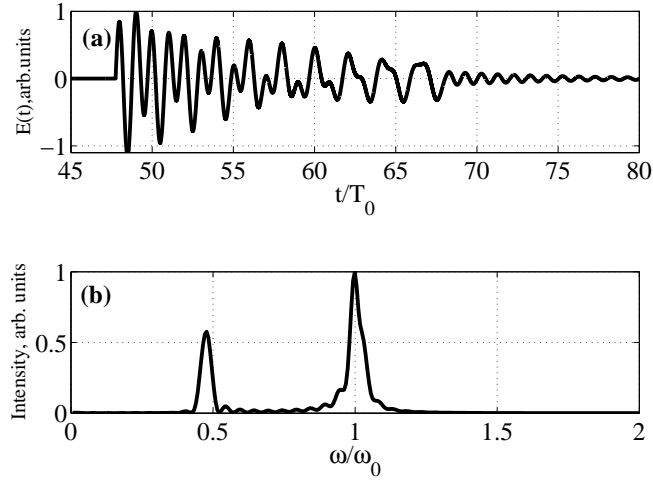

Figure 4: Time dependence (a) of the string response field $E(t)$ according to Eq. (7), and (b) its spectral intensity $I(\omega)$ normalized to the maximum values vs. normalized time t/T_0 and frequency ω/ω_0, for $\frac{V}{c} = 0.7$, $\frac{Z_m}{\Lambda_z} = 9.55$, $\frac{\Lambda_z}{\lambda_0} = 5$, $\frac{\omega_0}{\gamma} = 22.22$ and observation angle $\varphi = 0$.

4 Circular geometry of the string

4.1 General considerations

In this section we consider completely different topology of the string depictured in Fig. 6. Namely, the string made of the dipoles owing the same resonance frequency ω_0 as before is arranged along the circle of radius R. The dipole density is modulated along the string in a periodical way with the angular period Λ_ϕ as:

$$N(\phi) = \frac{1}{2}\left(1 + a\cos(\frac{2\pi}{\Lambda_\phi}\phi)\right). \tag{12}$$
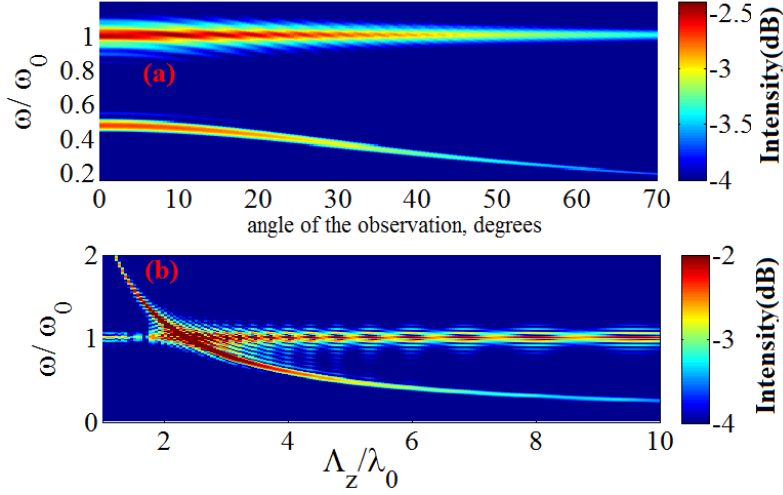
8

Figure 5: Dependence of the spectral intensity $I(\omega)$ of the string response according to Eq. (7) on the observation angle φ (a), and the string density modulation period Λ_z (b). The other parameters coincide with ones in Fig. 4. Note the logarithmic scale in the plot.

As in Eq. (4), we assume the modulation amplitude $a = 1$. In the center of the circle a source of a short spectrally broad optical pulse (see Fig. 1(c)) is located, which quickly rotates, so that the cross-section point (yellow point in Fig. 6) moves at the velocity V along the circle.

The dipoles of the string response to the excitation emitting the secondary waves. Here we will concentrate on the behavior of the string response field $E(t)$ observed in the center of circle. The electric field formed in the center of the circle by the element dE_ϕ located at the point which angular coordinate is ϕ is given by:

$$dE_\phi(t) = N(\phi) \exp\left[-\frac{\gamma}{2} f_\phi(t, \phi)\right] \cos\left[\omega_0 f_\phi(t, \phi)\right] \Theta\left[f_\phi(t, \phi)\right] d\phi,$$

where $f_\phi(t, \phi) = t - \frac{R\phi}{V} - \frac{R}{c}$. For one round pass of the excitation, the total electric field is obtained by integration (13) over ϕ:

$$E(t, \phi) = \int_0^{2\pi} N(\phi) \exp\left[-\frac{\gamma}{2} f_\phi(t, \phi)\right] \cos\left[\omega_0 f_\phi(t, \phi)\right] \Theta\left[f_\phi(t, \phi)\right] d\phi. \qquad (13)$$

The analytical solution of Eq. (13) in the case of $\gamma = 0$ is given in the Appendix. As one can see from analytical calculation in Appendix [see Eq. (22)] the response contains the resonance frequency of oscillators ω_0 together with a new component given, by the expression:

$$\Omega_2 = 2\pi \frac{V/\Lambda_\phi}{R}. \qquad (14)$$

Eq. (14) also has a simple physical meaning, namely this is the frequency at which the intersection point crosses the inhomogeneity oscillations. Under the condition

$$\frac{V}{c} = \frac{\Lambda_\phi R}{\lambda_0}, \qquad (15)$$

9

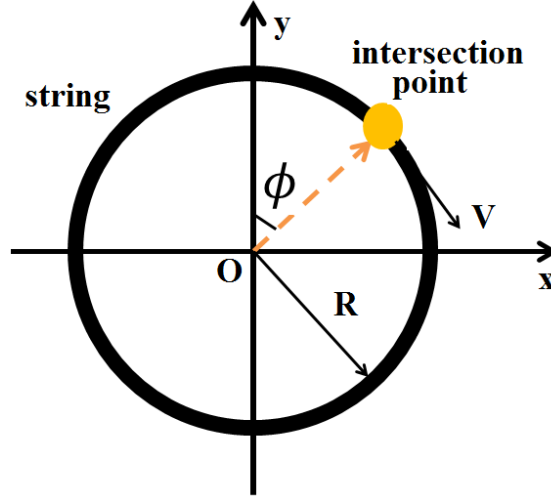the new frequency is equal to the resonance one.

Figure 6: Circular geometry of the string. The source of a short pulse with a broad spectrum (c) is located in the center of the circle and quickly rotates. The cross-section of the pulse an medium (yellow dot) moves at the velocity v along the string (black circle). As in the previous case, the string is made of dipoles characterized with resonance frequency ω_0 and the dipoles number density is modulated along the string periodically with the angular period Λ_ϕ.

Note also that Eq. (14) is valid if the observer is located anywhere on the axis passing through the center of the circle perpendicularly to its plane.

4.2   Temporal and spectral field shapes and their dependence on the parameters

We start from a typical situation in the spectrum when the frequency Ω_2 is clearly visible. Namely, we take the following parameters: $V/c = 3.75$, $\frac{\Lambda_\phi R}{\lambda_0} = 2$, $\omega_0/\gamma = 22.2$, $\omega_0/\Omega_2 = 0.53$. Assuming the same $\omega_0 = 2\pi \times 10$ ps^{-1} as in the Sec. III B and $R = 3$ cm, we obtain $\Lambda_\phi = 0.002$ rad^{-1}, $\gamma = 2.8$ ps^{-1}. The transient process for these parameters calculated using Eq. (13) is shown in Fig. 7. For these particular parameters, the frequency of oscillators practically doubles that of the transient emission, it results in the high-amplitude beatings clearly seen in Fig. 7. Once the transition process is finished, the observer at O records the ordinary decaying oscillations. This conclusion is also valid in the case when the excitation pulse moves at the sub-luminal velocity or precisely at the velocity of light. In all these cases, the radiation spectrum at the center of circle will possess a new frequency, with only exception of the resonance Eq. (15).

In order to illustrate the dependence Ω_2 on the parameters of system, we present the radiation spectrum in dependence on V (Fig. 8(a)) and R (Fig. 8(b)) whereas the other parameters are taken as in Fig. 7. As it can be easily seen from Eq. (14), the new frequency increases with the increase of V and decreases with R.
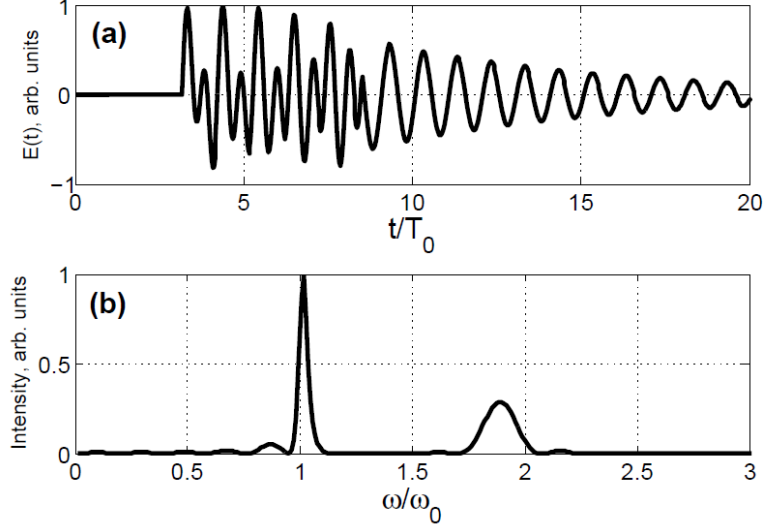
Figure 7: (a) Time dependence of the electric field $E(t)$ excited by the string and (b) the corresponding intensity spectrum $I(\omega)$ in the center of the circle for the circular scheme depictured in Fig. 6 and the parameters $V/c = 3.75$, $\frac{\Lambda_\phi R}{\lambda_0} = 2$, $\omega_0/\gamma = 22.2$, $\omega_0/\Omega_2 = 0.53$.

In the presented circular case, the role of the angle (if the excitation velocity is fixed) plays the radius of the circle. The Cherenkov resonance corresponds then to the R value defined by Eq. (15).

5 Conclusion

In this paper, the secondary radiation excited by a moving intersection of a short spectrally broadband pulse and a resonant string made of identical dipoles is discussed for the linear and circular string geometry. In such a situation, the radiation resembling the Cherenkov one appears. In contrast to many other cases where Cherenkov radiation is unstructured and has no clear frequency resonance, the presented one demonstrates obvious resonant properties. That is, the response spectrum is centered at the resonant frequency of the dipoles comprising the string. In addition, as our analysis shows, a new frequency appears in the presence of the string density oscillations, which has the meaning of a Doppler shift of the resonant frequency ω_0.

We remark that, despite of the quite complicated character, the system under study is linear. It serves therefore as a linear filter, which transmits certain frequencies and suppresses the others. To study the dynamic regimes of such a system, we assumed that the excitation has the form of sharp "kick", that is temporally short and spectrally broad pulse. Such flat broadband spectrum accomplished by a short pulse where recently produced in THz-to-MIR range using noble gas plasma ionization.

We point out also that the new Doppler frequency (Ω_1 in the case of the linear string and Ω_2 in the circular case) appears in the transient regime, when some of the secondary waves excited
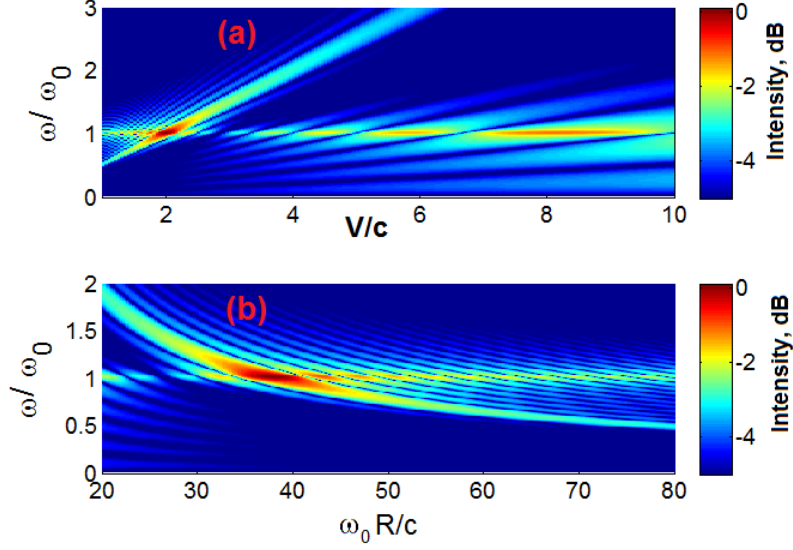
11

Figure 8: (a) Dependence of the radiation spectrum on the normalized propagation speed V of the excitation (a) and the radius of the circle R (b). Other parameters are as in the Fig. 7

by the spot have not yet reached the observation plane. The dynamics of the radiation after this moment is trivial and contains only decaying oscillations on the resonant frequency ω_0.

The behavior described there can find its application, for instance, to shape the broad spectra and short pulses in desired way using rather compact setup.

A   Analytical solutions of Eq. (7) and Eq. (13).

In this appendix we provide an analytical solutions of Eq. (7) and Eq. (13) when $\gamma = 0$. To obtain such expression we first rearrange the argument of Θ - function in Eq. (7) as $f\left(t, z\right) = t - \frac{z}{V} - \frac{Z_m - z}{c}\cos\varphi = t - z/W - \frac{Z_m}{c}\cos\varphi$, where W is the effective velocity defined as:

$$\frac{1}{W} = \frac{1}{V} - \frac{1}{c/\cos\varphi}. \tag{16}$$

One can see that W can be interpreted as the velocity of the projection of the cross-section point to the axis parallel to the observation plane in Fig. 1(b). Using this parameter we can rewrite the integral for the pulse response in the form:

$$E(t) = \int N(z)h_0\left(t' - \frac{z}{W}\right)\Theta\left(t' - \frac{z}{W}\right)dz, \tag{17}$$

where $t' = t - \frac{Z_m\cos\varphi}{c}$, and the function $h_0(t)$ denotes the response of a dipole located at $z = 0$ and excited with the excitation in the form of delta-function $\delta(t)$: $h_0(t) = \cos(\omega_0 t)\Theta(t)$.

The integral in Eq. (17) for the string placed at the interval $\left[0, Z_m\right]$ have different form in depen-

12

dence on the sign of W. Namely, it can be written as:

$$E(t) = \int_0^{Wt'} N(z)h_0\left(t' - \frac{z}{W}\right)dz \text{ for } W > 0, \qquad (18)$$

$$E(t) = \int_{Z_m}^{Wt'} N(z)h_0\left(t' - \frac{z}{W}\right)dz \text{ for } W < 0. \qquad (19)$$

If $W > 0$ the emitting element of the string moves in positive direction starting from zero as being seen by the observer. In the opposite situation, when $W < 0$, it is seen as moving in the negative direction from Z_m to 0.

Eqs. (18)-(19) can be solved separately for $0 < t < Z_m/|W|$ and for $t \geq Z_m/|W|$ assuming $\nu_z = \frac{2\pi}{\Lambda_z}$. In particular for $W > 0$ one can obtain:

$$E(t) = \frac{W}{\omega_0}\sin\left(\omega_0 t'\right) + \frac{W}{W^2\nu_z^2 - \omega_0^2}\left[\nu_z W\sin\left(\nu_z Wt'\right) - \omega_0\sin\left(\omega_0 t'\right)\right].$$

On the other hand, for $W < 0$ we have:

$$E(t) = \frac{W}{\omega_0}\sin\left(\omega_0(t' - \frac{Z_m}{W})\right) + \frac{W^2\nu_z\sin(\nu_z Wt') - \nu_z W^2\sin(\nu_z Z_m)\cos\left[\omega_0(t' - \frac{Z_m}{W})\right]}{\nu_z^2 W^2 - \omega_0^2} -$$

$$-\frac{\omega_0 W\cos(\nu_z Z_m)\sin\left[\omega_0(t' - \frac{Z_m}{W})\right]}{\nu_z^2 W^2 - \omega_0^2}. \qquad (20)$$

The last equations contain the oscillating terms with the frequencies ω_0 and $\Omega_1 = \nu_z W$ which coincides with the Eq. (8).

For $t \geq Z_m/W$, that is when the excitation impulse comes out of the string, we have:

$$E(t) = \int_0^{Z_m} N(z)\cos\left[\omega_0\left(t' - \frac{z}{W}\right)\right]dz =$$

$$= \frac{W}{\omega_0}\left[\sin(\omega_0 t') - \sin\left(\omega_0(t' - \frac{Z_m}{W})\right)\right] + \frac{\nu_z W^2\sin(\nu_z Z_m)\cos\left[\omega_0(t' - \frac{Z_m}{W})\right]}{\nu_z^2 W^2 - \omega_0^2} +$$

$$+\frac{\omega_0 W\cos(\nu_z Z_m)\sin\left[\omega_0(t' - \frac{Z_m}{W})\right] - W\omega_0\sin(\omega_0 t')}{\nu_z^2 W^2 - \omega_0^2}. \qquad (21)$$

This term describes the oscillations with the frequency ω_0 after transition process stops.

In the case of circular geometry for transient process ($\frac{R}{c} < t < \frac{2\pi R}{V} + \frac{R}{c}$ if $V > c$) one can obtain

$$E(t) = \int_{Vt''/R}^{2\pi} N(\phi)\cos\left[\omega_0\left(t'' - \frac{R\phi}{V}\right)\right]d\phi = -\frac{V}{R\omega_0}\sin\left(\omega_0(t'' - \frac{2\pi R}{V})\right) +$$

$$+\frac{V^2\nu_\phi\sin(2\pi\nu_\phi)\cos\left[\omega_0(t'' - \frac{2\pi R}{V})\right]}{\nu_\phi^2 V^2 - R^2\omega_0^2} + \frac{R\omega_0 V\cos(2\pi\nu_\phi)\sin\left[\omega_0(t'' - \frac{2\pi R}{V})\right]}{\nu_\phi^2 V^2 - R^2\omega_0^2} - \frac{V^2\nu_\phi\sin(\Omega_2 t'')}{\nu_\phi^2 V^2 - R^2\omega_0^2}.$$

$$(22)$$

13

Here $t'' = t - \frac{R}{c}$, $\nu_\phi = \frac{2\pi}{\Lambda_\phi}$. The last expression contains terms oscillating on the frequencies Ω_2 and ω_0. After transition process ends ($V > c$, $t > \frac{2\pi R}{V} + \frac{R}{c}$), we have:

$$E(t) = \int_0^{2\pi} N(\phi) \cos\left[\omega_0\left(t'' - \frac{R\phi}{V}\right)\right] d\phi = \frac{V}{R\omega_0}\left[\sin(\omega_0 t'') - \sin\left(\omega_0(t'' - \frac{2\pi R}{V})\right)\right] +$$
$$+ \frac{V^2 \nu_\phi \sin(2\pi\nu_\phi) \cos\left[\omega_0(t'' - \frac{2\pi R}{V})\right]}{\nu_\phi^2 V^2 - R^2\omega_0^2} + \frac{R\omega_0 V\left[\cos(2\pi\nu_\phi)\sin\left[\omega_0(t'' - \frac{2\pi R}{V})\right] - \sin(\omega_0 t'')\right]}{\nu_\phi^2 V^2 - R^2\omega_0^2}.$$

$$(23)$$

which contains only terms oscillating with the frequency ω_0.

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