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Maximal temperature of safe combustion in case of an autocatalytic reaction

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Abstract

We consider the problem of thermal explosion of a gas mixture in the case of an autocatalytic combustion reaction in a homogeneous medium. We determine the maximal temperature on the trajectories located in the transition region between the slow combustion regime and the explosive one.

1 Introduction

We consider the problem of thermal explosion of a gas mixture in case of an autocatalytic combustion reaction in a homogeneous medium. As a mathematical model we use the following differential system proposed in [1, 2]

$$\begin{aligned} \varepsilon \frac{d\theta}{dt} &= \eta(1-\eta)e^{\theta} - \alpha\theta, \\ \frac{d\eta}{dt} &= \eta(1-\eta)e^{\theta}. \end{aligned} \tag{1.1}$$

Here, θ denotes the temperature, η is the depth of conversion of the gas mixture, $-\alpha\theta$ describes the volumetric heat loss, and ε is a positive parameter which is small in case of a highly exothermic reaction.

The chemically relevant phase space $\mathcal{P}$ of system (1.1) is defined by $\mathcal{P} := \{(\theta, \eta) \in R^2 : \theta \geq 0, 0 \leq \eta \leq 1\}$. As ε is small, (1.1) represents a singularly perturbed system of autonomous differential equations. A qualitative investigation of this system can be found in [1, 2, 3, 4]. In what follows we recall the main results of these studies.

Proposition 1.1 *Consider system* (1.1) *in* $\mathcal{P}$ *for* $\alpha \geq 0, \varepsilon > 0$. *Then*

(i) The region $\Delta := \{(\theta, \eta) \in R^2 : 0 \leq \eta \leq 1, 0 \leq \theta \leq \eta/\varepsilon\}$ *is positively invariant.*

(ii) Δ *contains exactly two equilibria: the point* $P = (0, 1)$ *is a stable node, the origin* O *is a saddle, where one separatrix tanges the* θ*-axis at the origin, while the other one tanges the straight line* $\eta = (\alpha + \varepsilon)\theta$.

(iii) A trajectory of (1.1) *starting at any point in* Δ *different from the origin tends for increasing* t *to the equilibrium* P.

The degenerate equation to (1.1) reads

$$0 = \eta(1-\eta) - \alpha\theta e^{-\theta}. \tag{1.2}$$

Its solution set is called the slow manifold S_α of (1.1). It consists of equilibria of the associated differential equation

$$\frac{d\theta}{d\tau} = \eta(1-\eta) - \alpha\theta e^{-\theta}. \tag{1.3}$$

The following figures represent S_α for different values of α.

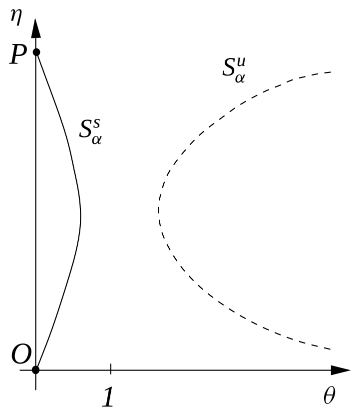

Figure 1: $\alpha > e/4$

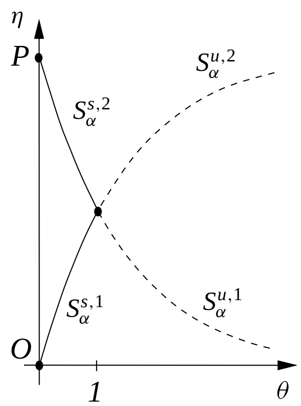

Figure 2: $\alpha = e/4$

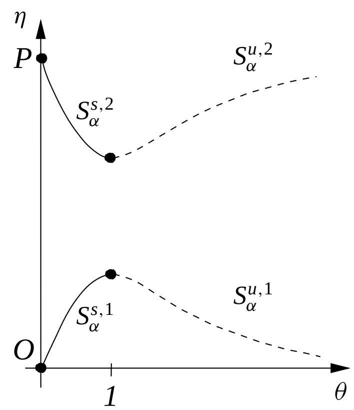

Figure 3: $\alpha < e/4$

Figure 1 shows S_α for some $\alpha > e/4$ in $\mathcal{P}$. In that case, S_α consists exactly of two different curves S_α^s and S_α^u which represent asymptotically stable and unstable equilibria of the associated equation, respectively. For ε sufficiently small, system (1.1) has an attracting invariant manifold $S_{\alpha,\varepsilon}^s$ near S_α^s and a repelling invariant manifold $S_{\alpha,\varepsilon}^u$ near S_α^u, respectively.
To study the behavior of (1.1) for small ε, we consider to (1.1) the initial value problem

$$\theta(0,\varepsilon) \ = \ 0, \ \ \eta(0,\varepsilon) \ = \ \eta_0, \quad 0 < \eta_0 < 1/2. \tag{1.4}$$

Since the initial point $(0,\eta_0)$ belongs to the basin of attraction of the set S_α^s, after some short transition period, the solution of (1.1), (1.4) follows the attracting slow invariant manifold $S_{\alpha,\varepsilon}^s$ and tends to the equilibrium $P = (0,1)$ as $t \to \infty$. We call this behavior the *slow combustion regime.*

Figure 3 shows S_α for some $\alpha < e/4$ in $\mathcal{P}$. As in the case $\alpha > e/4$, S_α consists exactly of two different curves S_α^1 and S_α^2. But different from the case considered before, each of these curves contains stable equilibria of the associated equation (1.3) (namely, for $\theta < 1$, denoted by $S_\alpha^{s,1}$ and $S_\alpha^{s,2}$) and unstable equilibria of (1.3) (for $\theta > 1$, denoted by $S_\alpha^{u,1}$ and $S_\alpha^{u,2}$). For $\alpha < e/4$ and ε sufficiently small, system (1.1) has local attracting invariant manifolds $S_{\alpha,\varepsilon}^{s,1}$ near $S_\alpha^{s,1}$ and $S_{\alpha,\varepsilon}^{s,2}$ near $S_\alpha^{s,2}$ and repelling local invariant manifolds $S_{\alpha,\varepsilon}^{u,1}$ near $S_\alpha^{u,1}$ and $S_{\alpha,\varepsilon}^{u,2}$ near $S_\alpha^{u,2}$, respectively. In that case, we have the following behavior of the solution of the initial value problem (1.1), (1.4): after some short transition period, the solution follows the component of $S_{\alpha,\varepsilon}^{s,1}$ until it reaches the value $\theta = 1$. After this moment, $\theta(t)$ will increase very fast, but the solution remains inside Δ and ultimately tends to the stable equilibrium P. This behavior characterizes the *explosive regime*.

Figure 2 depictures the special case $\alpha = e/4$. Here, the slow manifold S_α can be considered to consist of four branches $S_{e/4}^{u,i}, S_{e/4}^{s,i}, i = 1, 2$. The branch $S_{e/4}^{s,1}$ has the representation

$$S_{e/4}^{s,1} := \{(\theta,\eta) \in \mathcal{P} : \eta = \zeta_1^s(\theta) := 0.5(1 - \sqrt{1-\theta e^{1-\theta}}),\ 0 \le \theta \le 1\}. \tag{1.5}$$

If we denote by $\psi_1^s(\theta)$ the inverse function of $\zeta_1^s(\theta)$, then $S_{e/4}^{s,1}$ can be represented also in the form

$$S_{e/4}^{s,1} := \{(\theta,\eta) \in \mathcal{P} : \theta = \psi_1^s(\eta),\ 0 \le \eta \le 0.5\}. \tag{1.6}$$

Analogously, the branch $S_{e/4}^{u,2}$ can be represented as

$$\begin{aligned} S_{e/4}^{u,2} &:= \{(\theta,\eta) \in \mathcal{P} : \eta = \zeta_2^u(\theta) := 0.5(1 + \sqrt{1-\theta e^{1-\theta}}),\ \theta \ge 1\} = \\ &= \{(\theta,\eta) \in \mathcal{P} : \theta = \psi_2^u(\eta),\ \frac{1}{2} \le \eta < 1\}, \end{aligned} \tag{1.7}$$

where $\psi_2^u(\eta)$ is the inverse function of $\zeta_2^u(\theta)$.

For $\alpha = e/4$ and ε sufficiently small, system (1.1) possesses the local attracting invariant manifolds $S_{\alpha,\varepsilon}^{s,1}$ and $S_{\alpha,\varepsilon}^{s,2}$ near $S_\alpha^{s,1}$ and $S_\alpha^{s,2}$ respectively and the repelling local invariant manifolds $S_{\alpha,\varepsilon}^{u,1}$ and $S_{\alpha,\varepsilon}^{u,2}$ near $S_\alpha^{u,1}$ and $S_\alpha^{u,2}$, respectively.
If we study the initial value problem (1.1), (1.4), then to given small ε and for α exponentially near $e/4$ but less than $e/4$ we can observe the existence of canard solutions which describe the transition between the slow combustion regime and the explosive regime and which exhibits the phenomenon of delayed exchange of stabilities. That means, to given small ε, there is an exponentially small α-interval $(\alpha_e(\varepsilon), \alpha_c(\varepsilon))$ containing $\alpha^*(\varepsilon)$, where

$$\alpha^*(\varepsilon) = \alpha_0 + \alpha_1\varepsilon + \alpha_2\varepsilon^2 + O(\varepsilon^3),\ \alpha_0 = e/4,\ \alpha_1 = -e/\sqrt{2},\ \alpha_2 = 49e/36, \tag{1.8}$$

such that for $\alpha > \alpha_c(\varepsilon)$ ($\alpha < \alpha_e(\varepsilon)$) the solution of (1.4) belongs to the slow regime (explosive regime). The interval $(\alpha_e(\varepsilon), \alpha_c(\varepsilon))$ characterizes the *critical regime*, that is, for $\alpha \in (\alpha_e(\varepsilon), \alpha_c(\varepsilon))$, after some short transition time, the solution of (1.1), (1.4) follows the attracting invariant manifold $S_{\alpha,\varepsilon}^{s,1}$ until it reaches the value $\theta = 1$. After this moment, it stays near the unstable component $S_{\alpha,\varepsilon}^{u,2}$ which is located in the region $\eta > \frac{1}{2}$, up to some point J from which the solution "jumps" towards the attracting manifold $S_{\alpha,\varepsilon}^{s,2}$ and follows this manifold approaching P as $t \to \infty$ (see Figure 4).

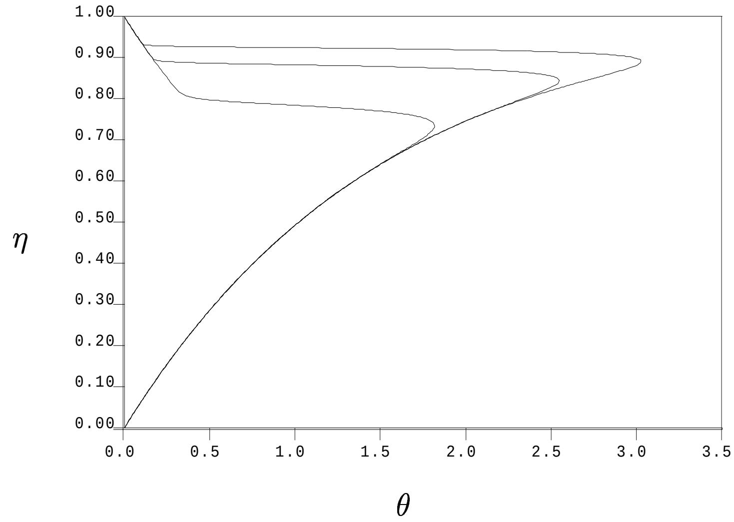

Figure 4: Canard trajectories of system (1.1) for $\varepsilon = 0.05$, $\alpha' = 0.659941603$, $\alpha'' = 0.659941646$, $\alpha''' = 0.659952218$

In the sequel, we are interested in the determination of the jump-off point J, and therefore the maximal temperature on a canard trajectory as a function of the initial point $(0, \eta_0)$. For this purpose, in the following section we present an estimate of the delay of exchange of stabilities in scalar singularly perturbed equations [6].

2 Delayed exchange of stabilities in scalar non-autonomous differential equations

We consider the scalar singularly perturbed differential equation

$$\varepsilon \frac{du}{d\eta} = g(u, \eta, \varepsilon) \tag{2.1}$$

and study the initial value problem

$$u(\eta_0, \varepsilon) = u_0, \quad \eta \in I_\eta := \{\eta \in R : \eta_0 < \eta < \eta_1\} \tag{2.2}$$

for sufficiently small ε.

If we set $\varepsilon = 0$ in (2.1) we get the degenerate equation

$$g(u, \eta, 0) = 0. \tag{2.3}$$

If this equation has a simple isolated root $u = \psi(\eta)$ which is a stable equilibrium of the associated equation to (2.1)

$$\frac{du}{d\tau} = g(u, \eta, 0), \tag{2.4}$$

and if u_0 belongs to region of attraction of $\psi(\eta_0)$, then the asymptotic behavior of the solution of the initial value problem (2.1), (2.2) is uniquely determined by the standard theory of singularly perturbed systems (see, e.g., [7]). In what follows we consider this initial value problem in case of exchange of stabilities, that is, we assume that the degenerate equation has two intersecting solutions. Here, we have to distinguish two different situations: immediate exchange of stabilities ([5]) and delayed exchange of stabilities ([6]). The last case is related to the existence of a canard trajectory. We recall now a result concerning the delayed exchange of stabilities in case of transversal bifurcation.

Let U be an open bounded interval containing the origin, I_{ε_0} is the open interval defined by $I_{\varepsilon_0} := \{\varepsilon \in R : 0 < \varepsilon < \varepsilon_0\}$, $\varepsilon_0 > 0$.

We consider the initial value problem (2.1), (2.2) under the following assumptions:

(A_1). $g : U \times I_\eta \times I_{\varepsilon_0} \to R$ is continuous and twice continuously differentiable with respect to u and ε.

(A_2). $g(0, \eta, \varepsilon) \equiv 0$ for $(\eta, \varepsilon) \in \overline{I}_\eta \times \overline{I}_{\varepsilon_0}$ ($\bar{I}$ means the closure of I).

From (A$_1$) and (A$_2$) it follows that a solution of (2.1) starting at $u = u_0$ remains positive (negative) if u_0 is positive (negative). In the sequel we restrict ourselves to the case $u_0 < 0$. We denote by U^- the set defined by $U^- := \{u \in U : u \leq 0\}$.

(A_3). The solution set of the degenerate equation $g(u, \eta, 0) = 0$ in $\overline{U^- \times I_\eta}$ consists of the two curves $u \equiv 0$ and $u = \psi_-(\eta)$ where ψ_- belongs to the class $C^1([\eta_c, \eta_1], R^-)$ and satisfies $\psi_-(\eta_c) = 0, \psi_-(\eta) < 0$ for $\eta \in (\eta_c, \eta_1]$ (see Figure 5).

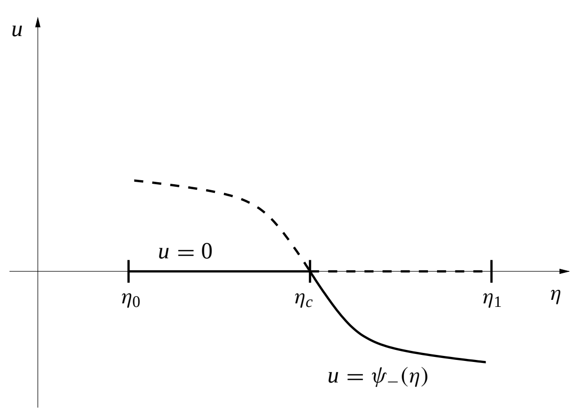

Figure 5: Solution set of $g(u, \eta, 0) = 0$

(A_4).

$$g_u(0, \eta, 0) \begin{cases} < 0 & \text{for } \eta \in [\eta_0, \eta_c), \\ > 0 & \text{for } \eta \in (\eta_c, \eta_1]. \end{cases}$$

Assumption (A_4) implies that $u = 0$ is an equilibrium point of the associated equation (2.4) which is exponentially stable for $\eta \in [\eta_0, \eta_c)$ and unstable for $\eta \in (\eta_c, \eta_1]$.

Let

$$G(\eta, \eta_0, \varepsilon) := \int_{\eta_0}^{\eta} g_u(0, s, \varepsilon) ds. \tag{2.5}$$

From assumption (A_4) we get that $G(\eta, \eta_0, 0) = 0$ has at most one root $\eta = \eta^*$ in (η_0, η_1). Therefore, we assume

(A$_5$). $G(\eta, \eta_0, 0) = 0$ has a root η^* in (η_0, η_1).

It is easy to see that η^* is such that it holds

$$\eta^* > \eta_c, \quad G'(\eta^*, \eta_0, 0) > 0. \tag{2.6}$$

The following assumption on the function g is fulfilled if the second derivative of g with respect to u at $u = 0$ is positive for all (η, ε) under consideration.

(A$_6$). There are sufficiently small positive numbers c_0 and ε_0, such that $[-c_0, c_0] \in U$ and

$$g(u, \eta, \varepsilon) \geq g_u(0, \eta, \varepsilon)u \quad \text{for} \quad \eta_0 \leq \eta \leq \eta^*, \ \varepsilon \in \overline{I}_{\varepsilon_0}, \ -c_0 \leq u \leq 0.$$

The following result about the delayed exchange of stabilities has been proved in [6].

Theorem 2.1 *Assume the hypotheses* (A$_1$) - (A$_6$) *to be valid. Then for sufficiently small* ε *and* $u_0 < 0$ *there exists a unique solution of* (2.1), (2.2) *satisfying*

$$\lim_{\varepsilon \to 0} u(\eta, \varepsilon) = \begin{cases} 0 & \text{for} \quad \eta \in [\eta_0, \eta^*), \\ \psi_-(\eta) & \text{for} \quad \eta \in (\eta^*, \eta_1]. \end{cases}$$

3 Maximal temperature of combustion

We return to the combustion model

$$\begin{aligned} \varepsilon \frac{d\theta}{dt} &= \eta(1-\eta)e^{\theta} - \alpha\theta, \\ \frac{d\eta}{dt} &= \eta(1-\eta)e^{\theta}. \end{aligned} \tag{3.1}$$

As we mentioned in the introduction, for small ε, the transition from a slow combustion regime to an explosive regime takes place when α is exponentially near $\alpha^*(\varepsilon)$ but less than $e/4$. It is characterized by the existence of a canard trajectory $\mathcal{C}_{\alpha,\varepsilon}$.

From the second equation of (3.1) it follows that we can represent any solution of system (3.1) located in the region $0 < \eta < 1$ in the form $\theta = \tilde{\varphi}(\eta, \varepsilon)$, where $\tilde{\varphi}(\eta, \varepsilon)$ satisfies the differential equation

$$\varepsilon \frac{d\theta}{d\eta} = \frac{\eta(1-\eta)e^{\theta} - \alpha\theta}{\eta(1-\eta)e^{\theta}}. \tag{3.2}$$

Our goal is for $\alpha = \alpha^*(\varepsilon)$ and sufficiently small ε to estimate the maximal temperature $\theta^{\varepsilon}_{max}$ on the canard trajectory $\mathcal{C}_\varepsilon := \{(\theta, \eta) \in R^2 : \theta = \varphi(\eta, \eta_0, \varepsilon)\}$ of system (3.2), which starts at the given initial point $(\eta = \eta_0, \theta = \theta_0 = 0)$.

It follows from (3.2) that for $0 < \eta < 1$ all extrema of any solution $\theta = \tilde{\varphi}(\eta, \varepsilon)$ of this differential equation are located on the curve S_α defined in (1.2). By (1.8) it holds $\alpha^*(\varepsilon) < e/4$ for sufficiently small ε. Since any canard solution of (3.2) is bounded, we can conclude that the maximal temperature is determined by the intersection of the canard solution with the curve $S^{u,2}_{\alpha^*(\varepsilon)}$ which is located in the region $\eta > 0.5$. At the same time we get from (3.2) that any canard solution has only one maximum in that region. This follows from the fact that if we calculate $\frac{d^2\tilde{\varphi}}{d\eta^2}$ on $S_{\alpha^*(\varepsilon)}$ we get

$$\frac{d^2\tilde{\varphi}}{d\eta^2} = \frac{(1-2\eta)e^{\tilde{\varphi}}}{\varepsilon\eta(1-\eta)e^{\tilde{\varphi}}}.$$

Hence, in the region $\eta > 0.5$ this derivative has negative sign, and we have always a maximum. Therefore, we can conclude that the jumping point J is located on the curve $S^{u,2}_{\alpha^*(\varepsilon)}$, and that $\theta^{\varepsilon}_{max}$ uniquely determines its position on that curve.

Our idea to estimate the maximal temperature on the canard trajectory $\mathcal{C}_\varepsilon$ of the differential equation

$$\varepsilon \frac{d\theta}{d\eta} = \frac{\eta(1-\eta)e^{\theta} - \alpha^*(\varepsilon)\theta}{\eta(1-\eta)e^{\theta}} \tag{3.3}$$

for sufficiently small ε is to apply Theorem 2.1. To do this we use the coordinate transformation

$$\theta = \bar{\varphi}(\eta, \varepsilon) + u, \tag{3.4}$$

where $\bar{\varphi}(\eta, \varepsilon)$ is the canard trajectory Σ_ε of (3.3) satisfying

$$\lim_{\eta \to 0} \bar{\varphi}(\eta, \varepsilon) = 0.$$

Since the origin is a saddle point of system (3.1) we can conclude that the canard trajectory describes the separatrix Σ_ε of system (3.1) entering the positive orthant for increasing t.

If ε tends to zero, then the canard trajectory Σ_ε tends to a discontinuous curve Σ_0 consisting of $S^{s,1}_{e/4}$, of the part of $S^{u,2}_{e/4}$ bounded by the jump–off point $J(\theta = \theta^*, \eta = \eta^*)$

and the point $(\theta = 1, \eta = \frac{1}{2})$ and of the part of $S^{s,2}_{e/4}$ located in the region $\eta^* < \eta \leq 1$) (see Fig. 4). In what follows we restrict our investigation on the region $0 < \eta < \eta^*$, where Σ_0 has the representation

$$\Sigma_0 = \begin{cases} \theta = \psi_1^s(\eta) & \text{for} \quad 0 < \eta \leq 0.5, \\ \theta = \psi_2^u(\eta) & \text{for} \quad 0.5 \leq \eta < \eta^*. \end{cases} \tag{3.5}$$

Applying the coordinate transformation (3.4) we get from (3.3) the differential equation

$$\varepsilon \frac{du}{d\eta} = \frac{\alpha^*(\varepsilon)}{\eta(1-\eta)e^{\bar{\varphi}(\eta,\varepsilon)}} \left(\frac{\bar{\varphi}(\eta,\varepsilon)(e^u - 1) - u}{e^u} \right) \equiv g(u,\eta,\varepsilon). \tag{3.6}$$

Now, we want to verify that $g(u,\eta,\varepsilon)$ satisfies all assumptions of Theorem 2.1 for $u \leq 0, 0 < \eta \leq \eta^*, \varepsilon$ sufficiently small.

It is easy to see that g is smooth in the region $u \in R, 0 \leq \eta < \eta^*, 0 \leq \varepsilon \leq \varepsilon_0$ and obeys $g(0,\eta,\varepsilon) \equiv 0$ for all η and ε under consideration. Thus, g satisfies the conditions (A_1), (A_2) of Theorem 2.1 for $\varepsilon > 0$.
To check hypothesis (A_3) we note that the degenerate equation $g(u,\eta,0) = 0$ is equivalent to

$$\bar{\varphi}(\eta,0)((e^u - 1) - u) = 0. \tag{3.7}$$

If we represent the solutions of the degenerate equation (1.2) in the form

$$\theta = \varphi_1(\eta), \qquad \theta = \varphi_2(\eta)$$

and if we set $\bar{\varphi}(\eta,0) = \varphi_1(\eta)$, then it can be verified that $u = \varphi_2(\eta) - \varphi_1(\eta)$ is a solution of (3.7) intersecting in $(\theta = 1, \eta = 0.5)$.

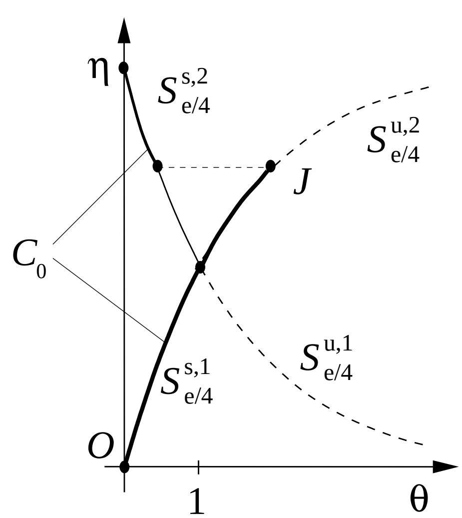

Figure 6: Limit position $\mathcal{C}_0$ of the canard trajectory $\mathcal{C}_\varepsilon$

Next we determine the stability of the branch $u = 0$ of the solution set of $g(u, \eta, 0) = 0$.

From (1.2), (1.5)–(1.7) and (3.6) we obtain

$$\begin{aligned} \frac{\partial g}{\partial u}_{|u=0,\varepsilon=0} &\equiv \frac{e(\psi_1^s(\eta)-1)}{4\eta(1-\eta)e^{\psi_1^s(\eta)}} = 1-\left(\psi_1^s\right)^{-1}(\eta) < 0 \qquad \text{for} \qquad 0<\eta<0.5, \\ \frac{\partial g}{\partial u}_{|u=0,\varepsilon=0} &\equiv \frac{e(\psi_2^u(\eta)-1)}{4\eta(1-\eta)e^{\psi_2^u(\eta)}} = 1-\left(\psi_2^u\right)^{-1}(\eta) > 0 \qquad \text{for} \qquad 0.5<\eta<\eta^*. \end{aligned} \tag{3.8}$$

That means, assumption (A_4) is satisfied.

In order to verify the assumption (A_5) we consider the function

$$G(\eta, \eta_0, 0) := \int_{\eta_0}^{\eta} g_u(0, s, 0)ds.$$

From (3.8) we get

$$G(\eta, \eta_0, 0) = \int_{\eta_0}^{0.5} \left(1-\left(\psi_1^s\right)^{-1}(s)\right) ds + \int_{0.5}^{\eta} \left(1-\left(\psi_2^u\right)^{-1}(s)\right) ds.$$

We have by (1.5)–(1.7)

$$G(\eta, \eta_0, 0) = \int_{\theta_0}^{1} (1-\sigma^{-1})(\zeta_1^s)'(\sigma)d\sigma + \int_{1}^{\theta} (1-\sigma^{-1})(\zeta_2^u)'(\sigma)d\sigma := \tilde{G}(\theta, \theta_0, 0), \tag{3.9}$$

where θ_0 is the root of the equation

$$\eta_0(1-\eta_0)e^{\theta_0} - \frac{e}{4}\theta_0 = 0.$$

Since

$$(1-\sigma^{-1})(\zeta_1^s)'(\sigma) = -\frac{(1-\sigma)^2 e^{-1}}{4\sqrt{1-\sigma e^{1-\sigma}}} < 0 \quad \text{for} \quad 0<\theta<1$$

and

$$(1-\sigma^{-1})(\zeta_1^s)'(\sigma) = \frac{(1-\sigma)^2 e^{-1}}{4\sqrt{1-\sigma e^{1-\sigma}}} > 0 \quad \text{for} \quad \theta>1,$$

there is a number $\overline{\theta}_0$, $0 < \overline{\theta}_0 < 1$ such that to $\theta_0 \in [\overline{\theta}_0, 1]$ there is a $\theta = \theta^*(\theta_0)$ such that $\tilde{G}(\theta^*, \theta_0, 0) = 0$. Thus, hypothesis (A_5) is valid.

Assumption (A_6) is fulfilled if $g_{uu}(0, \eta, \varepsilon)$ is positive for $\eta_0 \le \eta \le \eta^*$ and for sufficiently small ε. From (3.6) we get

$$g_{uu}(0, \eta, \varepsilon) = \frac{\alpha^*(\varepsilon)}{\eta(1-\eta)e^{\bar{\varphi}(\eta,\varepsilon)}}(2-\bar{\varphi}(\eta, \varepsilon)),$$

that is hypothesis (A_6) is fulfilled as long as $\bar{\varphi}(\eta, \varepsilon) < 2$.

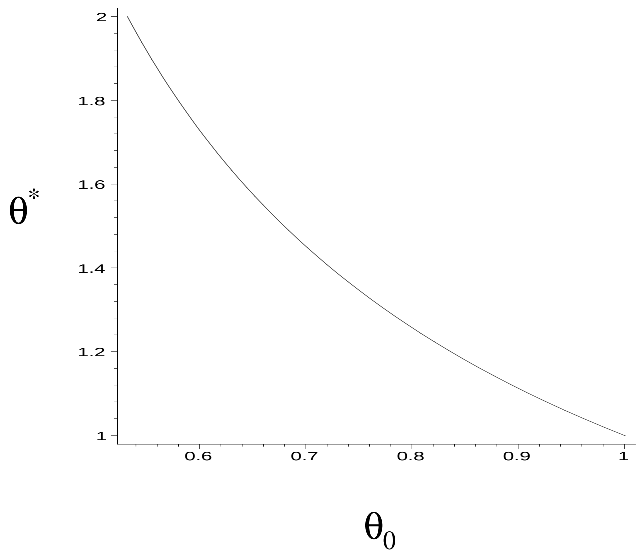

Figure 7: Dependence of maximal temperature on θ_0

From (3.9) we get

$$\begin{aligned}\tilde{G}(\theta,\theta_0,0) &= \frac{1}{2}\left(1-\frac{1}{\theta}\right)(1+\sqrt{1-\theta e^{1-\theta}})-\frac{1}{2}\left(1-\frac{1}{\theta_0}\right)(1-\sqrt{1-\theta_0 e^{1-\theta_0}})-\\ &-\frac{1}{2}\left[\int_{\theta_0}^{1}\frac{1-\sqrt{1-\sigma e^{1-\sigma}}}{\sigma^2}d\sigma+\int_{1}^{\theta}\frac{1+\sqrt{1-\sigma e^{1-\sigma}}}{\sigma^2}d\sigma\right]=0.\end{aligned}$$

Thus, the equation

$$\begin{aligned}&\frac{1}{2}\left(1-\frac{1}{\theta}\right)(1+\sqrt{1-\theta e^{1-\theta}})-\left(1-\frac{1}{\theta_0}\right)\eta_0+\frac{1}{2}\left(\frac{1}{\theta}-\frac{1}{\theta_0}\right)+\\ &+\frac{1}{2}\int_{\theta_0}^{1}\frac{\sqrt{1-\sigma e^{1-\sigma}}}{\sigma^2}d\sigma-\frac{1}{2}\int_{1}^{\theta}\frac{\sqrt{1-\sigma e^{1-\sigma}}}{\sigma^2}d\sigma=0\end{aligned}$$

determines the point (η^*,θ^*) characterizing the jumping from the slow manifold, where

$$\eta^*=\frac{1}{2}+(1+\sqrt{1-\theta^* e^{1-\theta^*}}).$$

Theorem 3.1 *Consider system (1.1) for $\alpha=\alpha^*(\varepsilon)$ and ε sufficiently small. Then the maximal temperature $\theta^{\varepsilon}_{max}$ on the canard trajectory $\mathcal{C}_\varepsilon$ starting at the point $(\eta=\eta_0,\theta=0)$ with $0<\eta_0<0.5$ satisfies*

$$\lim_{\varepsilon\to 0}\theta^{\varepsilon}_{max}=\theta^*(\theta_0).$$

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