

# 1 **A framework for assessing the value of information for health monitoring of scoured bridges**

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24 Management

25

## 26 **Abstract**

27 It is generally accepted that climate-change is leading to increased frequency of extreme weather events  
28 worldwide, and this is placing heavier demands on an already aging infrastructure-network. Bridges are  
29 particularly vulnerable infrastructure assets that are prone to damage or failure from climate-related  
30 actions. In particular, bridges over waterways can be adversely affected by flooding, specifically the  
31 washing away of foundation soils, a mechanism known as scour erosion. Scour is the leading cause of  
32 failure for bridges with foundations in water as it can rapidly compromise foundation stiffness often  
33 resulting in unacceptable movements or even collapse. There is growing interest among asset managers  
34 in applying health monitoring approaches to assess the real-time performance of bridges under  
35 damaging actions, including scour. Sensor-based approaches involve the acquisition of data such as  
36 dynamic measurements, which can be used to infer the existence of scour or other damage without the

37 laborious requirements of undertaking visual inspections. In this paper, a framework is proposed to  
38 assess the benefit obtained from health monitoring systems as compared to the scenario where no  
39 monitoring system is employed on a bridge, to ascertain how useful these systems are at assisting  
40 decision-making. Decisions typically relate to the implementation of traffic restrictions or even partial  
41 or complete bridge closure in the event of damage being detected, which has associated consequences  
42 for a network. A case study is presented to demonstrate the approach postulated in this paper.

### 43 **1. Introduction**

44 Extreme weather events are becoming more frequent as a result of climate-change and this is putting  
45 increasing pressure on built infrastructure. In tandem with this, infrastructure networks worldwide are  
46 aging, and many are approaching the end of their original design lives. These two phenomena together  
47 mean it is now more important than ever to direct attention to the maintenance and management of the  
48 aging asset stock to ensure safe, reliable transport infrastructure exists for generations to come.

49 Bridges are one of the main infrastructure assets at significant risk from climate-induced loading.  
50 Bridges with foundations in water are susceptible to scour erosion [1], whereby adverse hydraulic  
51 actions remove soil from around and under foundations compromising stability and increasing the risk  
52 of failure [2]. The occurrence of scour can cause a reduction in the stiffness and capacity of a bridge  
53 foundation [3–5] and lead to sudden failure.

54 Scour is most commonly monitored by means of visual inspections, whereby divers inspect a given  
55 bridge's foundations periodically (typically at times when flooding is not occurring). Susceptible  
56 bridges are usually rated using a scale related to the perceived severity of the scour problem affecting  
57 their foundations. The main issues with this type of approach are the subjective nature of the rating  
58 schemes adopted by respective agencies, and the fact that inspections typically occur during non-  
59 flooded conditions (when scour holes may have re-filled post flooding). It is generally not possible to  
60 inspect structures during flooding due to safety reasons, as well as the fact that flooded water conditions  
61 tend to be turbid thus obscuring the view of the foundations. Furthermore, rating-based ranking  
62 measures tend to vary between agencies responsible for the bridges (e.g. national road and railway

63 agencies) as well as from country to country. To improve on the drawbacks associated with visual-type  
64 inspections, a significant number of sensor-based systems have been developed in recent times to assist  
65 in remotely monitoring scour hole depth evolution. These systems include, among others:  
66 radar/electromagnetic systems [6–8], physical probe systems [9–11], and sound wave devices [7, 12].  
67 Interested readers are referred to Refs. [13, 14] for a comprehensive discussion on these types of  
68 systems. While these sensor-systems have varying success at monitoring scour hole depth evolution  
69 near a foundation of interest, they generally provide limited useful information on the structural  
70 condition as a result of scour hole formation. This is critical as the presence of a given scour hole may  
71 have limited or significant impact on the stability and safety of affected structures, and this will vary  
72 depending on factors such as foundation depth and type, as well as structural configuration.

73 In recent times, the application of vibration-based damage detection and health monitoring [15] to  
74 bridge scour assessment has become popular in research with many publications investigating the  
75 performance of a variety of methods at detecting and monitoring scour. The benefit of systems of this  
76 nature for scour detection is that they use actual structural response measurements to infer changes in  
77 support conditions (e.g. losses in foundation stiffness) and so can obtain a direct indication of the effect  
78 of a scour hole on a given structure. The premise underlying these damage identification methods is  
79 that changes in stiffness due to scour modify the dynamic properties of a structure, therefore measuring  
80 changes in dynamic parameters can potentially indicate the presence of scour. A variety of vibration-  
81 based scour monitoring approaches are put forward in Refs. [5, 16–26]. It should be noted, however,  
82 that the adoption and deployment of health monitoring systems of this nature on a bridge can be  
83 expensive, therefore tools and methods to assess their benefit for emergency management of bridges on  
84 a given network are needed.

85 In this paper, a framework for assessing the benefit of installing a monitoring system as a decision  
86 support tool for emergency management of scoured bridges is proposed. The framework is based on  
87 the Value of Information (VoI) from Bayesian decision theory. A case study is undertaken to  
88 demonstrate the approach. The VoI can be understood as the maximum price a bridge operator should

89 pay for the information from a Structural Health Monitoring (SHM) system: the SHM system should  
90 be installed only if the corresponding VoI is higher than the cost of the system itself. Moreover, the VoI  
91 can be considered as the money saved each time a decision maker interrogates the SHM system.  
92 Interested readers should refer to Refs. [27–36] for further details on VoI theory and applications.

93 The remainder of this paper is structured as follows. Section 2 presents the general framework for VoI  
94 analyses in the case of emergency management of structures; Section 3 presents the application of VoI  
95 analyses to scoured bridges; and Section 4 presents a case study demonstration of the approach.

## 96 **2. General framework**

97 The VoI is herein defined in the context of Bayesian decision theory, which was presented more than  
98 half a century ago by Raiffa and Schlaifer [37]. Bayesian decision theory is based on the Expected  
99 Utility Theorem by Van Morgenstern and Neumann [38] and on the Bayesian definition of probability  
100 [39] which represents a measure of the belief in the different states of a system: probabilities can be  
101 updated by means of the well-known Bayes' Theorem, when new information is obtained. Bayesian  
102 decision theory is based on the maximization of expected utility: a Bayesian decision maker associates  
103 a numerical utility to each of the possible consequences of an action, and a probability to each of the  
104 states of the system that may affect that utility. The utility expresses the desirability of a possible option  
105 in a decision scenario.

106 The classical formulation of the VoI is herein adapted to the management of civil structures in the  
107 aftermath of a disastrous event. For the purpose of the present paper, this will be specified as a severe  
108 flood affecting a given structure. According to the available information, three types of decision  
109 analyses are possible, namely prior analysis, posterior analysis and pre-posterior analysis. The terms  
110 prior and posterior refer to when an analysis is performed with respect to when information is acquired  
111 through a monitoring system. The term pre-posterior refers to when an analysis is performed before  
112 (pre) acquiring any SHM information. In this case, the analysis is carried out forecasting the information  
113 that will be acquired after (posterior) installing the monitoring system.

114 Prior decision analysis deals with decisions taken on the basis of the decision makers' prior knowledge  
 115 and uses no additional information. In relation to bridge management, the decision maker might be  
 116 concerned about the potential failure of a bridge caused by a disastrous event. Even if failure does not  
 117 occur directly because of the event, it may occur at a later time due, for example, to traffic loads, or  
 118 aftershocks in the case of earthquakes, or slowly evolving scour induced by the action of flowing water.  
 119 It is assumed that following an event of intensity measure  $IM$ , which may induce one of  $L$  discrete  
 120 damage states in a structure  $DS_l, l = 1, \dots, L$ , a choice has to be made among  $N$  actions  $A_n, n = 1, \dots, N$ .  
 121 The expected cost of action  $A_n$ , given that the state of the system is  $DS_l$ , is obtained as

$$E[c(A_n)|DS_l] = c_F(A_n)P(F|A_n, DS_l) + c_{\bar{F}}(A_n)[1 - P(F|A_n, DS_l)] \quad (1)$$

122 where  $P(F|A_n, DS_l)$  is the probability of bridge failure related to action  $A_n$  and damage state  $DS_l$ ;  
 123  $c_F(A_n)$  and  $c_{\bar{F}}(A_n)$  are the costs associated with structural failure and survival, respectively, which  
 124 change according to the action  $A_n$ . The quantity  $E[c(A_n)|DS_l]$  represents the expected cost of action  
 125  $A_n$  in the ideal case where the decision maker knows with certainty the state of the structure  $DS_l$ . In  
 126 real cases however, the knowledge of decision makers is affected by uncertainty, therefore each damage  
 127 state has a certain probability of occurrence that, when dealing with disastrous events, depends on the  
 128 intensity  $IM$  of the event. The expected cost of action  $A_n$ , given a certain  $IM$ ,  $E[c(A_n)|IM]$ , is computed  
 129 as the sum of the expected costs related to the occurrence of the possible damage states  $DS_l$ ,  
 130  $E[c(A_n)|DS_l]$ , each weighted by their probability of occurrence following the event of intensity  $IM$   
 131  $P(DS_l|IM)$ , as follows:

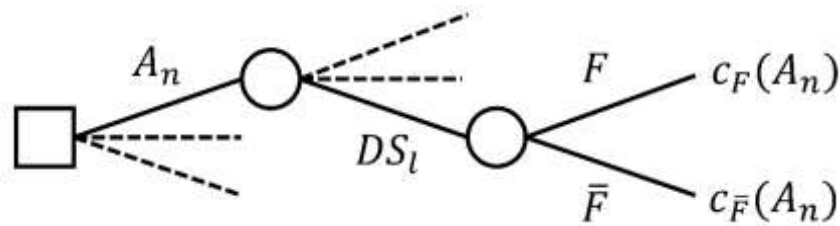
$$E[c(A_n)|IM] = \sum_{l=1}^L E[c(A_n)|DS_l]P(DS_l|IM) \quad (2)$$

132 Herein the utility is expressed as negative cost. Therefore, the prior decision is made according to the  
 133 Expected Utility Theorem by selecting the action  $\hat{A}$ , which maximizes the utility, that is the action that  
 134 corresponds to the minimum expected cost  $c_1(IM)$ , see Eq. 3 and Eq. 4.

$$\hat{A} = \hat{A}(IM) = \arg \min_n E[c(A_n)|IM] \quad (3)$$

$$c_1(IM) = E[c(\hat{A})|IM] = \sum_{l=1}^L E[c(\hat{A})|DS_l]P[DS_l|IM] \quad (4)$$

135 The described prior decision problem is represented in Fig. 1 by means of a decision tree. Round nodes  
 136 indicate a possible state of the system to which a probability of occurrence must be assigned; square  
 137 nodes indicate a decision that is made based on the minimization of costs.



138

139

Fig 1. Decision tree for prior decision analysis

140 Posterior analysis is performed when new information on the state of the structure is obtained such as  
 141 one of the possible outcomes  $O_j, j = 1, \dots, J$ , from an SHM system. This information is used to update  
 142 the prior probabilities of damage states according to Bayes' theorem, which reads

$$P(DS_l|O_j, IM) = \frac{P(O_j|DS_l)P(DS_l|IM)}{P(O_j|IM)} \quad (5)$$

143 where  $P(O_j|DS_l)$  is the probability of obtaining the outcome  $O_j$  when the state of the system is  $DS_l$ ,  
 144 which is obtained by so-called *likelihood functions*;  $P(O_j|IM)$  is the total probability given by Eq. 6.

$$P(O_j|IM) = \sum_{l=1}^L P(O_j|DS_l)P(DS_l|IM) \quad (6)$$

145 The posterior expected cost of action  $A_n$  is computed similarly to Eq. 2, but using posterior probabilities  
 146 of damage states, as follows:

$$E[c(A_n)|O_j, IM] = \sum_{l=1}^L E[c(A_n)|DS_l]P(DS_l|O_j, IM) \quad (7)$$

147 The decision is made by selecting the action  $\check{A}_{O_j}$  corresponding to the minimum expected cost

148  $E[c(\check{A}_{O_j})|O_j, IM]$ , as follows:

$$\check{A}_{O_j} = \check{A}(O_j, IM) = \arg \min_n E[c(A_n)|O_j, IM] \quad (8)$$

$$E[c(\check{A}_{O_j})|O_j, IM] = \sum_{l=1}^L E[c(\check{A}_{O_j})|DS_l]P(DS_l|O_j, IM) \quad (9)$$

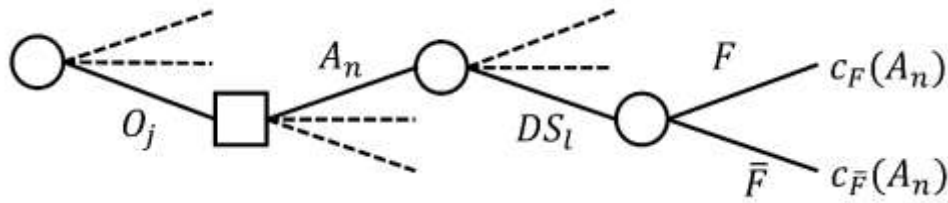
149 In this case, the optimal action and the corresponding expected cost depend on both the intensity of the  
150 event  $IM$  and the outcome  $O_j$ .

151 The pre-posterior analysis is made prior to obtaining additional information. It is used to forecast the  
152 expected cost resulting from decision making when a certain information acquisition strategy is  
153 adopted. It consists of multiple posterior analyses, where the decision maker selects the optimal action  
154 for each possible outcome  $O_j$  of the selected acquisition strategy. The expected cost  $c_0(IM)$  associated  
155 with the information acquisition strategy is computed by marginalizing the expected costs  
156  $E[c(\check{A}_{O_j})|O_j, IM]$  over the probabilities of occurrence  $P(O_j|IM)$  of each possible outcome  $O_j$ ,  
157 according to Eq. 10.

$$c_0(IM) = \sum_{j=1}^J E[c(\check{A}_{O_j})|O_j, IM]P(O_j|IM) \quad (10)$$

158 The pre-posterior decision analysis with information from SHM is represented in the decision tree in  
159 Fig. 2.

160



161

162 Fig. 2 Decision tree representing the pre-posterior decision analysis with information from SHM

163 The VoI for the decision-making process relevant to the choice of the action needed to manage the  
 164 bridge after an event of intensity  $IM$ ,  $\text{VoI}(IM)$ , is obtained as the difference between the expected cost  
 165 of the action taken without (prior) and with (pre-posterior) information, see Eq. 11.

$$\text{VoI}(IM) = c_1(IM) - c_0(IM) \quad (11)$$

166 In general, the optimal action and the corresponding cost, for both prior and pre-posterior analyses,  
 167 change according to  $IM$  that, before the occurrence of an event, is not known. The VoI over the  
 168 reference period for which it is calculated, is obtained by marginalizing over the entire range of  
 169 intensities, as follows:

$$\text{VoI} = \int_{IM} \text{VoI}(IM) f(IM) dIM \quad (12)$$

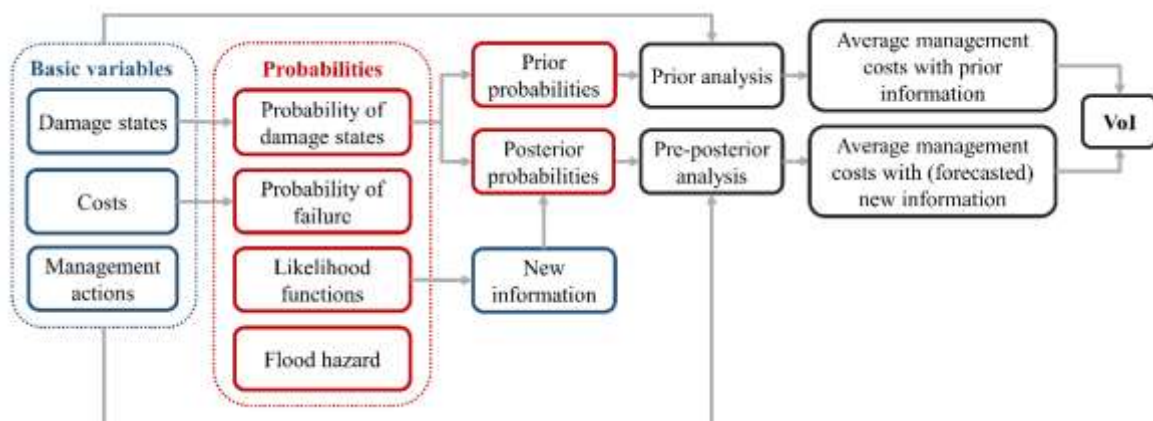
170 where  $f(IM)$  is the probability density function (PDF) of the intensity measure  $IM$  over the reference  
 171 period. The idea behind Eq. 12 is that the decision maker has at their disposal a statistical model  
 172 providing the likelihood of a disastrous event occurring in a certain geographical area in a given  
 173 reference time. Examples include seismic hazard functions for earthquakes or distributions of maximum  
 174 annual flow for rivers. In Eq. 12, the contribution to the VoI of rare events, with relatively small values  
 175 of PDF  $f(IM)$  is negligible. In turn, terms of  $\text{VoI}(IM)$  corresponding to likely events, and therefore to  
 176 high values of PDF, are dominant. In this way, accurate estimation of the VoI can be obtained before  
 177 the installation of a SHM system.



178 It can be demonstrated (see e.g. Straub [29]) that the VoI is bounded between zero and the so-called  
 179 Value of Perfect Information (*VoPI*), which is obtained, ideally, when the information acquired is not  
 180 affected by uncertainty. Nevertheless, it has been shown recently [40, 41] that the VoI could be negative.  
 181 This could occur, for example, if the person managing the bridge, i.e. the manager in charge of issuing  
 182 traffic restrictions, is not the same person making the decision on acquiring new information, i.e. the  
 183 owner who pays for the SHM system. Even if they share the same information, the perception of the  
 184 costs associated with structural failure and survival might differ between the two individuals. In the VoI  
 185 framework, this is modelled by two different utility functions that may lead to a negative VoI. In  
 186 particular, the VoI could be negative in the owner's perspective if they are forced to accept an action -  
 187 chosen by the manager - that they perceive as too risky due to their own risk averse nature. In this sense  
 188 the VoI should not be intended as the absolute benefit associated with the support to decisions provided  
 189 by the SHM system, but rather as the perception of this value on behalf of the different stakeholders  
 190 involved in the decision process.

### 191 3. Application to scoured bridges

192 In this section, the methodology for assessing the VoI of SHM in emergency management is applied to  
 193 bridges under scour hazard. Fig. 3 shows a flowchart of the general framework: the basic variables of  
 194 the decision problem are indicated in blue; the probabilities are indicated in red.



195

196

Fig. 3 Flowchart showing general methodology for assessing the VoI

197 The basic variables of the decision problem include the damage states of the system induced by scour,  
198 the possible bridge management actions, and the consequences associated with the different  
199 combinations of damage states of the system and actions. An understanding of likely damage states of  
200 the system under evolving scour is necessary, which refer to the condition of the bridge (and its  
201 elements) under various scour severities. Prior probabilities of the damage states can be calculated,  
202 which refer to the likelihood of obtaining a certain scour magnitude based on flow intensity and bridge  
203 geometrical conditions (using design formulae or otherwise). The probability of failure of the structure  
204 under various scour conditions should be calculated using assumed capacity models and estimates of  
205 demand on the system from external actions. Likelihood functions, which refer to the likely output from  
206 a SHM system (for example measurements of system frequency) under various scour scenarios, should  
207 be obtained in order to calculate the posterior probabilities of the damage states. The consequences of  
208 the actions chosen should be quantified, for example bridge closure or imposed traffic restrictions.  
209 Finally, the VoI can be obtained as a function of the hydraulic variables to ascertain the costs associated  
210 with implementing a SHM system or remaining without one. More detailed information on these  
211 elements is provided in the following subsections.

### 212 **3.1 Damage states**

213 The damage states affecting a structure, in their simplest form, can correspond to different scour depths  
214 developing at a critical pier, for example. These in turn can be related to a change in residual load  
215 bearing capacity of the given foundation. More advanced damage states including the development of  
216 cracks, differential settlement or partial collapse could also be defined, as expected to result from the  
217 development of a given scour hole. The probability that the structure is in a certain damage state depends  
218 on the scour depth produced by a flood event. In the next section the methodology to compute the prior  
219 probabilities of the different states of the bridge is described.

220 **3.2 Prior probabilities**

221 Several equations are reported in the literature for the computation of local scour depth  $y_s$  resulting  
 222 from given flow and bridge geometrical conditions. A widely used equation is the Hydraulic  
 223 Engineering Circular (HEC-18) design formula [42], which reads

$$\frac{y_s}{y_1} = 2.0\lambda K_1 K_2 K_3 K_4 \left(\frac{a}{y_1}\right)^{0.65} Fr_1^{0.43} \quad (13)$$

224 where  $y_1$  is the flow depth upstream of a pier;  $K_1$  is the correction coefficient for pier nose shape;  $K_2$  is  
 225 the correction coefficient for angle of attack of flow;  $K_3$  is the correction coefficient for bed conditions;  
 226  $K_4$  is the correction coefficient for armoring by bed material;  $a$  is pier width;  $Fr_1 = V_1/\sqrt{gy_1}$  is the  
 227 Froude Number, where  $V_1$  is the mean velocity of flow upstream of the pier;  $g$  is the acceleration due  
 228 to gravity; and  $\lambda$  is the model correction factor discussed in reference [43].

229 The quantities  $y_1$  and  $V_1$  can be computed according to the Eq. 14 and Eq. 15, respectively [44], where  
 230  $Q$  is the water flow;  $B$  is the average width of the channel;  $n$  is the Manning's coefficient;  $s$  is the slope  
 231 of the channel; and  $\lambda_Q$  is a random variable accounting for the uncertainty in the flow [43].

$$y_1 = \left(\frac{\lambda_Q Q n}{B s^{0.5}}\right)^{3/5} \quad (14)$$

$$V_1 = \frac{\lambda_Q Q}{B y_1} \quad (15)$$

232 Each damage state corresponds to a threshold  $th_l$ ,  $l = 1, \dots, L$ , for the scour depth, where  $th_1 = 0$ . The  
 233 prior probabilities of the different damage states are obtained as follows

$$\begin{aligned} P(DS_l|Q) &= P[\{y_s \geq th_l\} \cap \{y_s < th_{l+1}\}] && \text{for } l \neq L \\ P(DS_l|Q) &= P(y_s \geq th_l) && \text{for } l = L \end{aligned} \quad (16)$$

### 234 3.3 Consequences

235 The computation of the consequences of bridge management actions is a complex task, which depends  
 236 on the boundary conditions of the problem and, to some extent, on the expert judgement of the analyst  
 237 [45]. Typically, in the context of bridge management, consequences are classified into direct  
 238 consequences and indirect consequences [46]. Direct consequences are related to failures and damage  
 239 resulting from the failure of the bridge itself, such as human losses, repairs and replacements. Indirect  
 240 losses are generated by the reduced functionality of the transportation system, such as delays, re-routing  
 241 and resulting pollution. Consequences are generally expressed in monetary terms, i.e. costs. Several  
 242 equations exist in the literature to compute the consequences resulting from bridge failure, which  
 243 include both indirect and direct consequences. For instance, in reference [47] the total failure costs are  
 244 computed as the sum of rebuilding costs  $C_{RB}$ , running costs  $C_{RN}$ , costs related to time loss  $C_{TL}$ , and  
 245 costs associated with loss of life  $C_{LL}$ . Rebuilding costs and loss of life costs are generally classified as  
 246 direct, whereas running costs and time loss cost are generally considered as indirect costs.

### 247 3.4 Probabilities of failure

248 The probability of failure of a bridge under a scour hazard is a function of the capacity of the bridge (in  
 249 its given state) and the demand imposed by external actions. A limit state function, or performance  
 250 function,  $g(X)$  may be generated in the form of Eq. (17).

$$g(X) = (C - D) \begin{cases} > 0 & \text{safe state} \\ = 0 & \text{limit state} \\ < 0 & \text{failure state} \end{cases} \quad (17)$$

251 where  $C$  is the capacity of the bridge for a given scour condition and  $D$  is the demand, comprising  
 252 external actions. The capacity of the bridge can be quantified in several ways and is linked to the  
 253 assumed mode of failure of the bridge. Bridges affected by scour actions can suffer a loss in vertical  
 254 foundation capacity, therefore, a capacity distribution can be specified in terms of available vertical  
 255 foundation resistance under scour. For a case like this, simplified design codes such as the American  
 256 Petroleum Institute (API) [48] propose equations to calculate the available shaft and base resistance of

257 pile groups, whereby scour leads to a reduction in this capacity via a decrease in available pile shaft  
258 shear area. Uncertainty can be incorporated via the specification of a distribution for the soil parameters  
259 contributing to the capacity which, in the case of the API formulation, are the bulk unit weight and the  
260 angle of internal friction. For a lateral bridge failure mechanism, the lateral capacity distribution of the  
261 bridge can be defined, once again, using simplified design assumptions from codes such as API or  
262 otherwise. In this case, failure can be defined as the loss in lateral resistance and can be quantified using  
263 lateral soil reaction-displacement ( $p$ - $y$ ) curve analyses [5, 49]. Uncertainty in the operational parameters  
264 defining  $p$ - $y$  curves enables the specification of a capacity distribution. For each proposed failure  
265 mechanism, further uncertainty can be incorporated by postulating distributions for the bridge structural  
266 parameters (material and geometry) as appropriate.

267 The demand,  $D$  is a function of the externally applied actions affecting the bridge and comprises the  
268 dead load, any environmental variations, and applied traffic loading. Once the capacity and demand are  
269 defined, the performance function  $g$  can be obtained.

270 The probability of failure  $P(F)$  can be calculated from the performance function generated for a given  
271 scour condition and failure mechanism using the expression in Eq. 18.

$$P(F) = P[g(X) \leq 0] \quad (18)$$

272 The value of  $P(F)$  can be obtained by multiple reliability techniques, such as FORM, SORM and Monte  
273 Carlo simulations [50].

### 274 **3.5 Likelihood functions**

275 Likelihood functions are used to update prior probabilities of damage states as described in section 2.  
276 They describe the distribution of the outcome (indicator) provided by a monitoring system. For scour-  
277 related actions, a variety of indicators can be used to infer damage to the structure [51]. Scour causes a  
278 reduction in the stiffness of foundation elements. Therefore, a number of previous works have focussed  
279 on using changes in dynamic properties to infer scour presence. The fact that stiffness changes lead to

280 changes in modal properties was the original motivation behind using dynamic measurements for  
281 damage detection [15]. The most straightforward modal property that is influenced by scour is the  
282 frequency of vibration of the structure, which decreases with the increase of scour depth [3]. Therefore,  
283 it is sensible to suggest that observing frequency shifts could infer the presence of scour. While this is  
284 a simple concept, there exists significant uncertainty in this process, most notably due to uncertainties  
285 in operating soil conditions and stiffness at interfaces, geometrical and material properties of structural  
286 elements, and environmental influences such as temperature [52, 53]. For this reason, there exists a  
287 distribution of likely frequency values that may be retrieved from measurements obtained under a given  
288 scour condition. The likelihood function is defined as a likely distribution of frequencies that could be  
289 measured by a sensor placed on a bridge in the event of scour with a certain depth magnitude affecting  
290 the bridge. To generate likelihood functions, in the absence of real SHM information measured on a  
291 scoured bridge, finite-element models can be used accounting for the various sources of uncertainty that  
292 influence the problem (e.g. material and geometrical properties, noise in sensors, model uncertainty,  
293 environmental and operational factors, etc.).

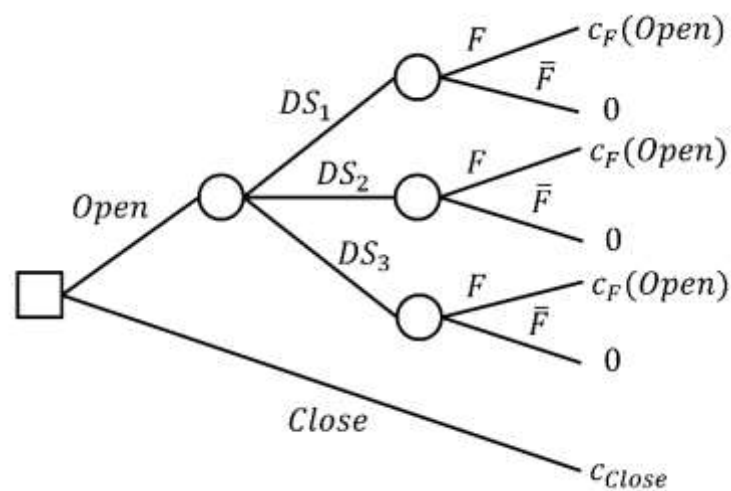
### 294 **3.6 Flood hazard**

295 As discussed in Section 2, the VoI depends on the distribution of the intensity of the event  $IM$  that, for  
296 the case of a scour hazard, can be represented by the maximum annual flow. Prior to the installation of  
297 a monitoring system, the magnitude of any future maximum annual flow is not known a priori.  
298 However, its probability distribution can be obtained by statistical inference on a sample of annual  
299 maxima. The VoI obtained by Eq. 11 as a function of  $IM$  can be integrated over the PDF of the annual  
300 maximum flow according to Eq. 12. This VoI can be interpreted as the money saved each year by using  
301 SHM information and it should be compared with the equivalent annual cost (including the annual share  
302 of the installation and decommissioning costs) of the SHM system.

## 303 **4. Demonstration of the approach**

304 The proposed framework to compute the benefit of installing a SHM system for scoured bridges is  
305 demonstrated in this section for a generic bridge. The validity of the results obtained is limited to this

306 example that has scope only to illustrate the application of the procedure. It is supposed that the operator  
 307 of a bridge is concerned about the traffic restrictions to be imposed in the aftermath of a severe flood.  
 308 Hence, they are considering the adoption of an automatic vibration-based SHM system to support  
 309 decision-making during emergency operations. In this demonstration, it is supposed that the bridge  
 310 manager and the owner are the same person. In this respect, the possibility of obtaining negative VoI is  
 311 prevented. Utility is expressed as negative cost. The decision problem in the absence of SHM  
 312 information, i.e. the prior decision problem, is represented by the decision tree in Fig. 4.



313

314

Fig. 4 Decision tree representing the prior decision analysis

315 Two possible traffic management actions are considered, namely “leave the bridge open” and “close  
 316 the bridge” indicated in the decision tree as *Open* and *Close*, respectively.

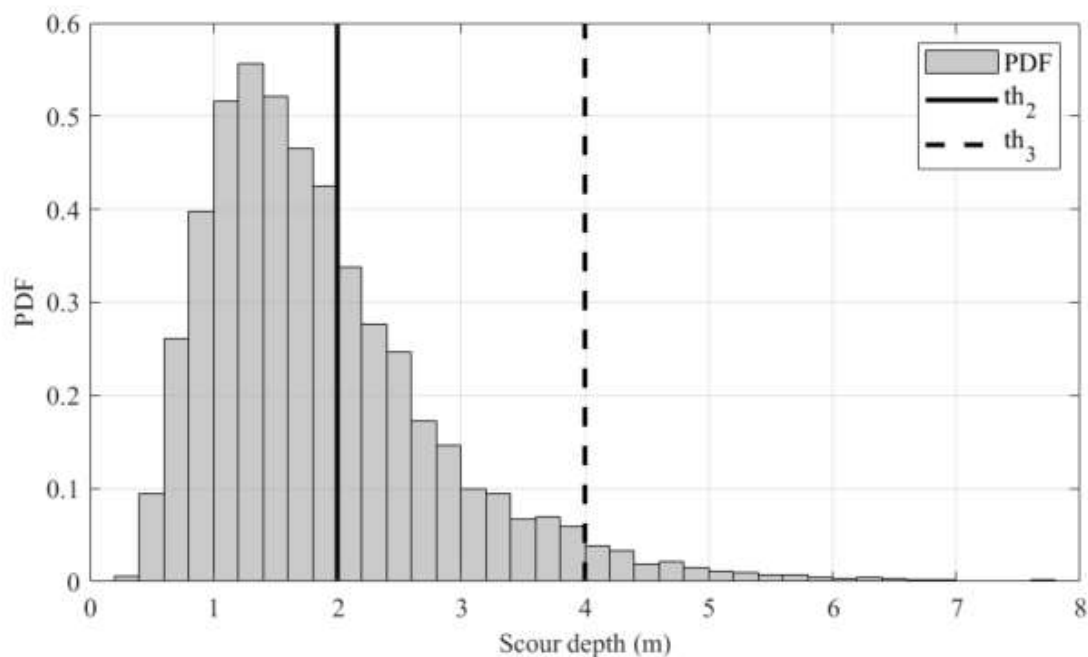
317 The damage state of the bridge due to scour is discretized into three levels: (i) no damage/minor damage,  
 318  $DS_1$ ; (ii) medium damage,  $DS_2$ ; (iii) severe damage,  $DS_3$ . The damage states correspond to different  
 319 scour depths, which in turn are related to different residual load bearing capacities of the bridge pier  
 320 foundation. The probability that the structure is in a certain damage state depends on the scour depth  $y_s$   
 321 produced by a flood event, whose intensity is represented by the flow  $Q$  (see Eq. 13 to Eq. 15). Given  
 322 the uncertainty of the parameters involved, to each value of the flow corresponds a distribution of the  
 323 scour depth. For this case study the parameters reported in Table 1 were assumed.

324

Table 1 Input variables used in the calculation of the distribution of scour depth

Variable	Unit	Distribution	Mean	CoV	Reference
$K_1$	-	Det.	1	-	-
$K_2$	-	Det.	1	-	-
$K_3$	-	Uniform	1.2	0.048	[54]
$K_4$	-	Det.	1	-	-
$a$	m	Det.	1.2	-	-
$B$	m	Lognormal	50	0.05	Assumed
$s$	-	Lognormal	0.003	0.05	Assumed
$\lambda_Q$	-	Normal	1	0.05	[43]
$\lambda$	-	Lognormal	0.412	0.646	[43]
$n$	-	Lognormal	0.035	0.28	[55]

325 Fig. 5 displays the distribution of the scour depth obtained by a Monte Carlo simulation with 10,000  
 326 random samples considering a flow  $Q=500 \text{ m}^3/\text{s}$ . Thresholds  $th_2$  and  $th_3$  refer to scour depths  
 327 corresponding to the proposed damage levels  $DS_2$  and  $DS_3$  (discussed in more detail below).



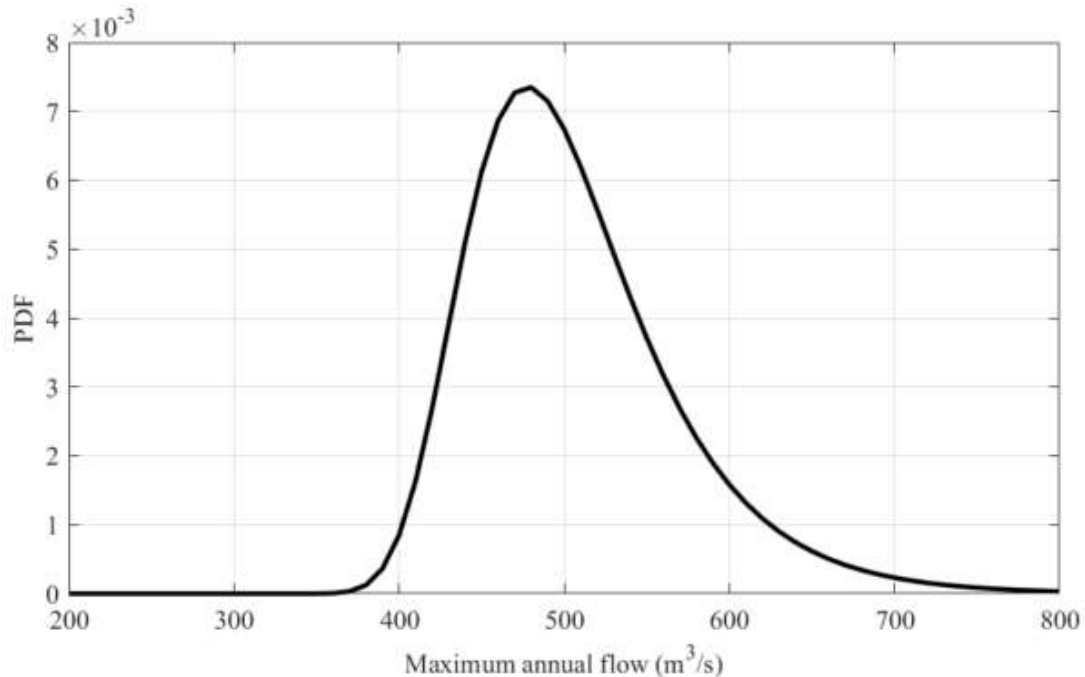
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329

Fig. 5 Distribution of the scour depth for  $Q=500 \text{ m}^3/\text{s}$



330 The Gumbel distribution is commonly employed to represent the distribution of the maximum value of  
 331 the flood flows that occur within a year [56]. Thus, the probability distribution of the maximum annual  
 332 flow is assumed as a Gumbel distribution with mean 500 m<sup>3</sup>/s and CoV of 0.10, as shown in Fig. 6.



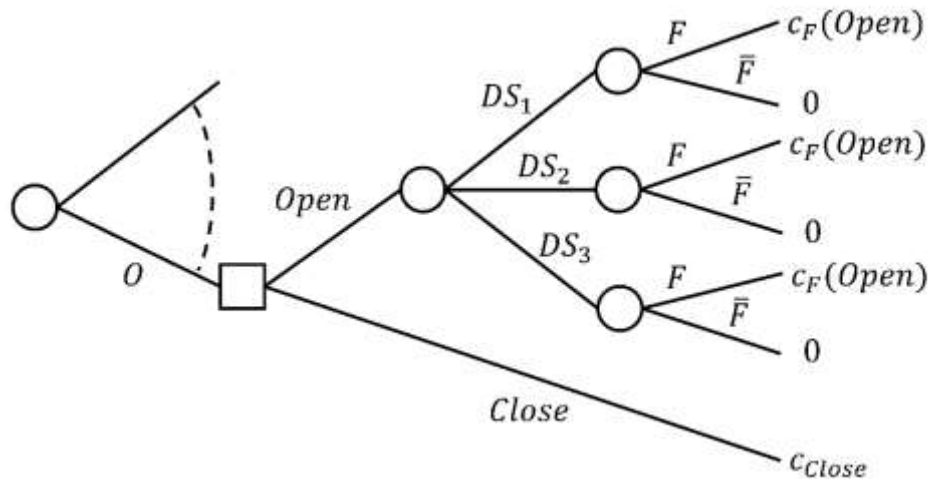
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334

Fig. 6 Distribution of annual maximum flow rate

335 Three damage levels have been considered defining three threshold values of the scour depth i.e.  $th_1 =$   
 336  $0$ ,  $th_2 = 2$  m, and  $th_3 = 4$  m. Damage state  $DS_1$  occurs for  $0 \leq y_s < 2$  m; damage state  $DS_2$  occurs  
 337 for scour depths in the interval  $2 \leq y_s < 4$  m; damage state  $DS_3$  occurs for  $y_s \geq 4$  m. The probabilities  
 338 of the damage states change according to the value of the flow. For instance, for  $Q=500$  m<sup>3</sup>/s the  
 339 probabilities of the damage states read  $P(DS_1) = 0.640$ ,  $P(DS_2) = 0.322$ , and  $P(DS_3) = 0.038$ . In  
 340 relation to the selected action and to its damage state, the bridge might fail under external actions, such  
 341 as traffic loads and/or the hydrodynamic force of flowing water. In this demonstration, the following  
 342 probabilities of failure are associated with the action *Open*:  $P(F|DS_1) = 0.0001$ ,  $P(F|DS_2) = 0.01$ ,  
 343 and  $P(F|DS_3) = 0.8$ . In a real application, a reliability analysis should be carried out to determine these  
 344 probabilities (values adopted in this case are for demonstration only).

345 The costs of bridge failure and survival for action  $A_n = Open$  are  $c_F(Open) = 1,500,000 \text{ €}$  and  
 346  $c_{\bar{F}}(Open) = 0$ , respectively. The expected cost of the action *Close*,  $c_{Close}$ , is fixed under the hypothesis  
 347 that it can generate only indirect consequences and it is taken as 55,000€. The expected costs of the two  
 348 actions *Open* and *Close* computed by means of Eq. 2 as a function of the flow are shown in Fig. 9(a).  
 349 The expected cost of action *Open* depends on the prior probabilities of the different damage states,  
 350 which in turn depend on the magnitude of the flow. As the water flow increases, the probability of the  
 351 bridge becoming damaged increases. So, the expected cost of the action *Open* increases. The upper  
 352 bound of the expected cost of the action *Open* is reached when the damage state  $DS_3$  is certain, i.e.  
 353  $P(DS_3) = 1$ , and it is computed according to Eq. 1 as  $c_F(Open) \times P(F|DS_3) = 1,500,000 \text{ €} \times 0.8 =$   
 354  $1,200,000 \text{ €}$ . The prior probabilities of damage states are obtained by Monte Carlo simulation with  
 355 10,000 samples of random variables. According to the prior analysis, rational decision makers should  
 356 close the bridge in the case where flow exceeds (approximately)  $540 \text{ m}^3/\text{s}$ , that is the value at which the  
 357 expected costs of the two actions coincide.

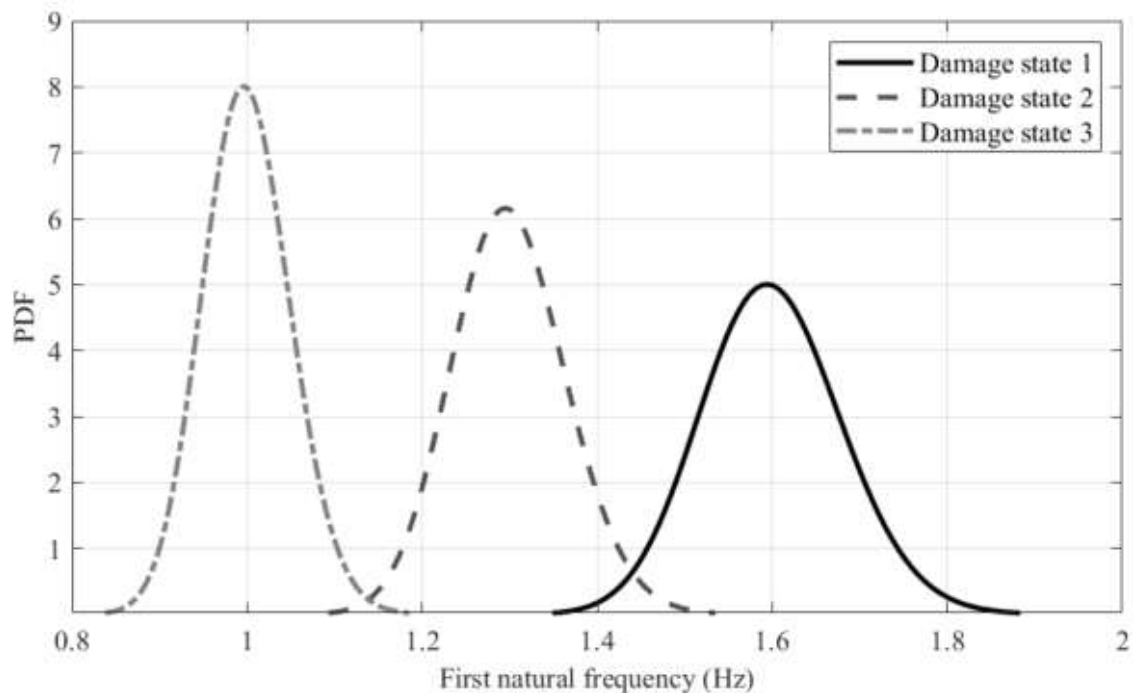


358

359 Fig. 7 Decision tree representing the decision analysis with information from SHM for the case study

360 It is now assumed that the decision maker is interested in knowing the expected costs of actions when  
 361 using SHM information, prior to installing such a system, see Fig. 7. This expected cost can be  
 362 computed by applying Eq. 10. The damage-sensitive feature used by the decision maker is the first  
 363 natural frequency of the bridge, which is expected to decrease when scour is present [20]. This

364 parameter can be estimated by means of several Operational Modal Analysis (OMA) techniques. The  
 365 estimated values of natural frequencies are typically affected by multiple sources of uncertainties. These  
 366 uncertainties are accounted for in the definition of the likelihood functions, which can be interpreted as  
 367 the probability distribution function of the first natural frequency in correspondence to the three damage  
 368 states (see section 3.5). Herein, it is assumed that the distribution of the first natural frequency  
 369 corresponding to damage states  $DS_1$ ,  $DS_2$  and  $DS_3$  of this generic bridge can be described by a  
 370 Lognormal distribution with mean value 1.6 Hz, 1.3 Hz, and 1 Hz, respectively, and 0.05 CoV, as shown  
 371 in Fig. 8. In this case, a continuous output is obtained from the SHM system and therefore the sum in  
 372 Eq. 10 is replaced by an integral.



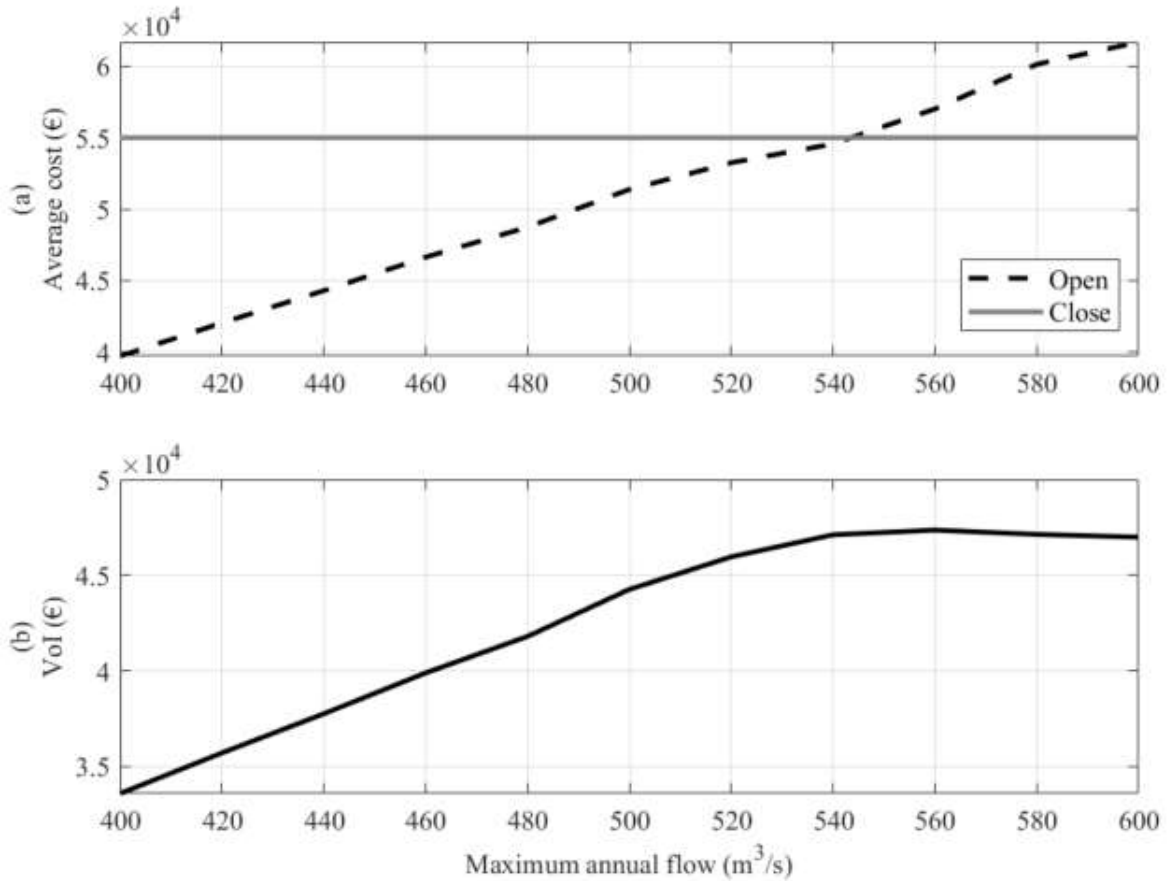
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Fig. 8 Likelihood functions

375 The VoI as a function of the flow is computed according to Eq. 11. The results are displayed in Fig.  
 376 9(b). It is observed that the VoI is maximum when the two actions *Open* and *Close* have the same  
 377 expected costs. In fact, the benefit of collecting information on the condition of the bridge is maximum  
 378 when the uncertainty on the selection of the optimal action is large, that is when alternative actions  
 379 correspond to similar expected costs. The VoI is integrated over the probability distribution of the

380 maximum annual flow to remove the dependence on the intensity measure, according to Eq. 12,  
 381 obtaining an expected cost of about 43,000 €. The SHM system should be employed if its (annual) cost  
 382 is lower than the computed VoI.



383

384 Fig. 9 (a) Prior average costs of actions; (b) VoI as a function of the flow rate

385 **5. Conclusions**

386 In this paper, a framework to assess the benefit of SHM information in the context of bridges damaged  
 387 by scour erosion is presented and a brief example (case study) is demonstrated. The framework is based  
 388 on the VoI from Bayesian decision theory, which is adapted herein to the case of emergency  
 389 management of structures in the aftermath of a flood. The purpose of the paper is to introduce the  
 390 concept of VoI in this context with a view to assisting asset managers in decision-making related to  
 391 whether to close or keep open bridges that have been damaged in the aftermath of a flood event.

392 Intermediate bridge management actions, such as imposing traffic restrictions, can also be considered  
393 in the decision problem. The framework is demonstrated and the relevant steps in the process described.

394 The elements of a VoI analysis for scour monitoring are identified and described. These include: (i)  
395 identifying the possible damage states caused by scour, which are related to different scour depths  
396 affecting a given foundation; (ii) the prior probabilities of scour occurrence; (iii) the bridge management  
397 actions that the decision maker might take after a severe flood event, i.e. imposing traffic restrictions;  
398 (iv) the costs associated with different combinations of damage states and bridge management actions;  
399 (v) the probability of failure of the scoured bridge under external actions; and (vi) the *likelihood*  
400 *functions* used to update the prior probabilities of damage states according to Bayes' theorem, which  
401 represents the probability of observing a certain scour monitoring outcome (e.g. bridge frequency),  
402 given a certain damage state (scour condition). A simple but exhaustive numerical example is presented,  
403 including all the relevant elements of a VoI analysis. In this case demonstration, the operator of a bridge  
404 is concerned about traffic management after a severe flood and for this reason they are considering the  
405 adoption of a vibration-based SHM system to facilitate emergency operations. It is observed that the  
406 expected costs of bridge management actions increase as the intensity of the water flow increases since  
407 severe damage states are more likely to occur as a result (when damage is scour development). When  
408 the expected costs of actions reach similar values, the VoI is maximum. In this situation, additional  
409 information on the actual state of the bridge is particularly useful to select the optimal action. The VoI  
410 is computed by accounting for the distributions of maximum annual flow of the river and is used by the  
411 operator of the bridge as an upper bound for a cost-effective SHM system. The presented framework  
412 will be of use to decision-makers who must make informed decisions about management of bridges  
413 during severe flood events and allows the incorporation of uncertainties associated with the measured  
414 data and the resulting consequences of a given action. The framework should inform on the benefits (or  
415 not) of installing a sensor system on a given bridge based on the VoI this provides (relative to the  
416 absence of such information).

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421           **Compliance with Ethical Standards**

422   The authors declare that they have no conflict of interest.

423           **References**

- 424   1.    Hamill L (1999) *Bridge Hydraulics*. E.& F.N. Spon, London
- 425   2.    Maddison B (2012) Scour failure of bridges. *Proc ICE - Forensic Eng* 165:39–52
- 426   3.    Prendergast LJ, Hester D, Gavin K, O’Sullivan JJ (2013) An investigation of the changes in  
427   the natural frequency of a pile affected by scour. *J Sound Vib* 332:6685–6702.  
428   <https://doi.org/http://dx.doi.org/10.1016/j.jsv.2013.08.020i>
- 429   4.    Prendergast LJ, Hester D, Gavin K (2016) Determining the presence of scour around bridge  
430   foundations using vehicle-induced vibrations. *J Bridg Eng* 21:.  
431   [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000931](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000931)
- 432   5.    Klinga J V., Alipour A (2015) Assessment of structural integrity of bridges under extreme  
433   scour conditions. *Eng Struct* 82:55–71. <https://doi.org/10.1016/j.engstruct.2014.07.021>
- 434   6.    Forde MC, McCann DM, Clark MR, et al (1999) Radar measurement of bridge scour. *NDT&E*  
435   *Int* 32:481–492
- 436   7.    Anderson NL, Ismael AM, Thitimakorn T (2007) Ground-Penetrating Radar : A Tool for  
437   Monitoring Bridge Scour. *Environ Eng Geosci* XIII:1–10
- 438   8.    Yu X (2009) Time Domain Reflectometry Automatic Bridge Scour Measurement System:  
439   Principles and Potentials. *Struct Heal Monit* 8:463–476.  
440   <https://doi.org/10.1177/1475921709340965>

- 441 9. Hunt BE (2009) NCHRP synthesis 396: Monitoring Scour Critical Bridges - A Synthesis of  
442 Highway Practice. Washington, DC
- 443 10. De Falco F, Mele R (2002) The monitoring of bridges for scour by sonar and sediment.  
444 *NDT&E Int* 35:117–123
- 445 11. Zarafshan A, Iranmanesh A, Ansari F (2012) Vibration-Based Method and Sensor for  
446 Monitoring of Bridge Scour. *J Bridg Eng* 17:829–838.  
447 [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000362](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000362).
- 448 12. Nassif H, Ertekin AO, Davis J (2002) Evaluation of Bridge Scour Monitoring Methods.  
449 Trenton, NJ
- 450 13. Prendergast LJ, Gavin K (2014) A review of bridge scour monitoring techniques. *J Rock Mech*  
451 *Geotech Eng* 6:138–149. <https://doi.org/10.1016/j.jrmge.2014.01.007>
- 452 14. Fisher M, Chowdhury MN, Khan A a., Atamturktur S (2013) An evaluation of scour  
453 measurement devices. *Flow Meas Instrum* 33:55–67.  
454 <https://doi.org/10.1016/j.flowmeasinst.2013.05.001>
- 455 15. Doebling S, Farrar C, Prime MB, Shevitz DW (1996) Damage identification and health  
456 monitoring of structural and mechanical systems from changes in their vibration  
457 characteristics: a literature review. Los Alamos, New Mexico
- 458 16. Chen C-C, Wu W-H, Shih F, Wang S-W (2014) Scour evaluation for foundation of a cable-  
459 stayed bridge based on ambient vibration measurements of superstructure. *NDT E Int* 66:16–  
460 27. <https://doi.org/10.1016/j.ndteint.2014.04.005>
- 461 17. Bao T, Andrew Swartz R, Vitton S, et al (2017) Critical insights for advanced bridge scour  
462 detection using the natural frequency. *J Sound Vib* 386:116–133.  
463 <https://doi.org/10.1016/j.jsv.2016.06.039>
- 464 18. Bao T, Liu Z (2016) Vibration-based bridge scour detection: A review. *Struct Control Heal*  
465 *Monit*
- 466 19. Foti S, Sabia D (2011) Influence of Foundation Scour on the Dynamic Response of an Existing

- 467 Bridge. *J Bridg Eng* 16:295–304. [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000146](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000146).
- 468 20. Prendergast LJ, Gavin K, Hester D (2017) Isolating the location of scour-induced stiffness loss  
469 in bridges using local modal behaviour. *J Civ Struct Heal Monit* 7:483–503.  
470 <https://doi.org/10.1007/s13349-017-0238-3>
- 471 21. Fitzgerald PC, Malekjafarian A, Cantero D, et al (2019) Drive-by scour monitoring of railway  
472 bridges using a wavelet-based approach. *Eng Struct* 191:1–11.  
473 <https://doi.org/10.1016/j.engstruct.2019.04.046>
- 474 22. Fitzgerald PC, Malekjafarian A, Bhowmik B, et al (2019) Scour Damage Detection and  
475 Structural Health Monitoring of a Laboratory-Scaled Bridge Using a Vibration Energy  
476 Harvesting Device. *Sensors* 19:
- 477 23. Xiong W, Cai CS, Kong B, et al (2017) Identification of bridge scour depth by tracing  
478 dynamic behaviors of superstructures. *KSCE J Civ Eng* 1–12. [https://doi.org/10.1007/s12205-](https://doi.org/10.1007/s12205-017-1409-9)  
479 [017-1409-9](https://doi.org/10.1007/s12205-017-1409-9)
- 480 24. Xiong W, Kong B, Tang P, Ye J (2018) Vibration-Based Identification for the Presence of  
481 Scouring of Cable-Stayed Bridges. *J Aerosp Eng* 31:.  
482 [https://doi.org/10.1061/\(ASCE\)AS.1943-5525.0000826](https://doi.org/10.1061/(ASCE)AS.1943-5525.0000826).
- 483 25. Xiong W, Cai CS, Kong B, et al (2019) Bridge Scour Identification and Field Application  
484 Based on Ambient Vibration Measurements of Superstructures. *J Mar Sci Eng* 7:1–24
- 485 26. Prendergast LJ, Hester D, Gavin K (2016) Development of a Vehicle-Bridge-Soil Dynamic  
486 Interaction Model for Scour Damage Modelling. *Shock Vib* 2016:.  
487 <https://doi.org/10.1155/2016/7871089>
- 488 27. Pozzi M, Der Kiureghian A (2011) Assessing the value of information for long-term structural  
489 health monitoring. SPIE Press, San Diego, CA
- 490 28. Pozzi M, Zonta D, Wang W, Chen G (2010) A framework for evaluating the impact of  
491 structural health monitoring on bridge management. In: *Bridge Maintenance, Safety,*  
492 *Management and Life-Cycle Optimization: Proceedings of the Fifth International IABMAS*



- 493 Conference. CRC Press, Philadelphia
- 494 29. Straub D (2014) Value of information analysis with structural reliability methods. *Struct Saf*  
495 49:75–85
- 496 30. Thons S, Faber M (2013) Assessing the value of structural health monitoring. In: ICOSAR.  
497 New York, NY
- 498 31. Zonta D, Glisic B, Adriaenssens S (2014) Value of information: impact of monitoring on  
499 decision-making. *Struct Control Heal Monit* 21:1043–1056
- 500 32. Thöns S, Stewart MG (2019) On decision optimality of terrorism risk mitigation measures for  
501 iconic bridges. *Reliab Eng Syst Saf* 188:574–583. <https://doi.org/10.1016/j.res.2019.03.049>
- 502 33. Long L, Döhler M, Thöns S (2020) Determination of structural and damage detection system  
503 influencing parameters on the value of information. *Struct Heal Monit.*  
504 <https://doi.org/10.1177/1475921719900918>
- 505 34. Skokandić D, Ivanković AM, Thöns S (2019) Quantifying the Value of B-WIM : Assessing  
506 Costs and Benefits for Value of Information Analysis. In: IABSE Symposium
- 507 35. Bayane I, Long L, Thöns S, Brühwiler E (2019) Quantification of the conditional value of  
508 SHM data for the fatigue safety evaluation of a road viaduct. In: 13th International Conference  
509 on Applications of Statistics and Probability in Civil Engineering (ICASP). Seoul, Korea
- 510 36. Iannacone L, Gardoni P, Giordano PF, Limongelli MP (2019) Decision making based on the  
511 value of information of different inspection methods. In: F-K. Chang, A. Guemes, & F.  
512 Kopsaftopoulos (Eds.), *Structural Health Monitoring 2019: Enabling Intelligent Life-Cycle*  
513 *Health Management for Industry Internet of Things (IIOT) - Proceedings of the 12th*  
514 *International Workshop on Structural Health Monitoring*. DEStech Publications Inc.
- 515 37. Raiffa H, Schlaifer R (1961) *Applied Statistical Decision Theory*. John Wiley & Sons, New  
516 York
- 517 38. Von Neumann, J., Morgenstern O (1947) *Theory of Games and Economic Behaviour*.  
518 Princeton University Press, New Jersey

- 519 39. Bayes T (1763) An essay toward solving a problem in the doctrine of chances. *Philos Trans R*  
520 *Soc London* 53:370–418
- 521 40. Bolognani D, Verzobio A, Tomelli D, et al (2017) Quantifying the benefit of SHM: what if the  
522 manager is not the owner? In: 11th International Workshop on Structural Health Monitoring
- 523 41. Verzobio A, Bolognani D, Zonta D, Quigley J (2019) Quantifying the benefit of SHM: can the  
524 VoI be negative? In: 13th International Conference on Applications of Statistics and  
525 Probability in Civil Engineering, ICASP13. Seoul, Korea
- 526 42. Richardson EV, Harrison LJ, Richardson JR, Davis SR (1993) Evaluating Scour at Bridges
- 527 43. Ghosn M, Moses F, Wang J (2003) Design of Highway Bridges for Extreme Events , National  
528 Cooperative Highway Research Program, NCHRP REPORT 489
- 529 44. BD 97/12 (2012) The assessment of scour and other hydraulic actions at highway structures.  
530 Design manual for roads and bridges, highway structures: Inspection and maintenance  
531 assessment (Vol. 3, Section 4, Part 21). London
- 532 45. Imam B, Chryssanthopoulos MK (2012) Causes and Consequences of Metallic Bridge  
533 Failures. *Struct Eng Int* 22:93–98
- 534 46. Faber MH (2008) Risk Assessment in Engineering: Principles, System Representation & Risk  
535 Criteria
- 536 47. National Academies of Sciences Engineering and Medicine (NASEM) (2007) Risk-Based  
537 Management Guidelines for Scour at Bridges with Unknown Foundations. Washington, DC
- 538 48. API (2007) RP2A: Recommended practice for planning, designing and constructing offshore  
539 platforms - Working stress design. Washington, DC
- 540 49. Chortis G, Askarinejad A, Prendergast LJ, et al (2020) Influence of scour depth and type on p  
541 – y curves for monopiles in sand under monotonic lateral loading in a geotechnical centrifuge.  
542 *Ocean Eng* (In Press): <https://doi.org/10.1016/j.oceaneng.2019.106838>
- 543 50. Ditlevsen O, Madsen H (1996) *Structural Reliability Methods*. John Wiley & Sons, Ltd,

544 Chichester

545 51. Prendergast LJ, Limongelli MP, Ademovic N, et al (2018) Structural Health Monitoring for  
546 Performance Assessment of Bridges under Flooding and Seismic Actions. *Struct Eng Int*  
547 28:296–307

548 52. Sohn H (2007) Effects of environmental and operational variability on structural health  
549 monitoring. *Philos Trans A Math Phys Eng Sci* 365:539–560.  
550 <https://doi.org/10.1098/rsta.2006.1935>

551 53. Malekjafarian A, Prendergast LJ, OBrien EJ (2019) Use of mode shape ratios for pier scour  
552 monitoring in two-span integral bridges under changing environmental conditions. *Can J Civ*  
553 *Eng In Press*:

554 54. Johnson PA, Dock DA (1998) Probabilistic Bridge Scour Estimates. *J Hydraul Eng* 124:750.  
555 [https://doi.org/10.1061/\(ASCE\)0733-9429\(1998\)124:7\(750\)](https://doi.org/10.1061/(ASCE)0733-9429(1998)124:7(750))

556 55. Hydraulic Engineering Centre (1986) Accuracy of Computed Water Surface Profiles. Davis,  
557 CA

558 56. World Meteorological Organization (2008) Guide to Hydrological Practices (WMO No. 168)  
559 Volume 2 - Chapter 5: Extreme Value Analysis. Geneva