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Abstract. Unipolar is less fundamental than bipolar cognition based on truth, and composure is a restraint for truth-based worlds. Bipolarity is the most powerful phenomenon that survives when truth disappeared in a black hole due to Hawking radiation or particular / anti-particular emission. The purpose of this research study is to define few four operations, including residue product, rejection, maximal product and symmetric difference of bipolar single-valued neutrosophic graph (BSVNG) and to explore some of their related properties with examples. Bipolar single-valued neutrosophic graph (BSVNG) is the generalization of the single-valued neutrosophic graph (SVNG), intuitionistic fuzzy graph, bipolar intuitionistic fuzzy graph, bipolar fuzzy graph and fuzzy graph. BSVNG plays a significant role in the study of neural networks, daily energy issues, energy systems, and coding. Moreover, we will determine related properties like the degree of a vertex in a BSVNG or total degree of a vertex in a BSVNG. We provide examples of the vertex degree in BSVNG and the total vertex degree in BSVNG. In order to make this useful, we develop an algorithm for our useful method in steps.

Keywords: keyword 1; symmetric difference, residue product, maximal product, rejection of BSVNG, Application, algorithm.

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1. Introduction

In 1965, Zadeh [36] put forward the idea of the one-degree fuzzy set concept that determined the true membership function. Since Zadeh's pioneering work, the fuzzy set theory has been used in various disciplines such as management sciences, engineering, mathematics, social sciences, statistics, signal processing, artificial intelligence, automata theory, medical and life sciences. In the 20th century, Smarandache [31] includes the concept where uncertainty occurs

in the form of Neutrosophic set and extend the intuitionistic fuzzy set. There is also a non-membership degree that Atanassove [1] defines in an intuitionistic fuzzy set with two degrees in a set. Abdel-Basset et al. [2–6] studied many concepts on neutrosophic sets. Broumi et al. [7, 9–13, 28, 29] investigated the extension of the fuzzy graph in the form of the single-valued neutrosophic graphs, shortest path problem using bellman algorithm under neutrosophic environment, shortest path problem in fuzzy, intuitionistic fuzzy and neutrosophic environment, single valued neutrosophic coloring, and operations of single valued neutrosophic coloring.
A bipolar fuzzy theory has more scope when we compare to simply a fuzzy theory as compatibility and flexibility. Overall its model is better than the fuzzy model. Borzooei and Rashmanlou [8, 25–27] studied very well on vague graphs and bipolar fuzzy graph. Rashmanlou studied about interval-valued fuzzy graph [22–24]. The neutrosophic set has much scope in neutrosophy and the neutrosophy theory is widely used in graph theory. In this extension, Wang et el. [35] described subclass of a Neutrosophic set known as a single-valued neutrosophic set. In the fields of bio and physics, SVNG has numerous applications. In these days, its purpose evaluates incomplete and uncertainty information. BSVNG has numerous applications in the fields of geometry and operational research. It has been a useful scope in various fields of computer science.Later, Deli et al. [14] described the idea of the bipolar neutrosophic set as the extension of the Neutrosophic set. He also described the concept of the bipolar fuzzy graph with some related properties. One problem of an Fuzzy graph, Intuitionistic fuzzy graph, bipolar fuzzy graph and intuitionistic bipolar fuzzy graph found when uncertainty occurs in the relationship between two vertices. Need for the neutrosophic graph is necessary because these are not suitable properly. Many researchers [32, 33] was famous due to their research work application approach to real-world problems.
The idea of the fuzzy graph is presented by Rosenfeld [30] and [34]. Malik and Hassan [16] both described the classification of the BSVNG together. Later Malik and Naz [21] presented the operations on the SVNG. Gomathi and Keerthika [15] studied neutrosophic labeling graph. Kousik Das et al. [17] defined generalized neutrosophic competition graphs. Mordeson and Peng [18] given some operations on Fuzzy Graphs. Gani et al. [19, 20] defined order, size, and irregular fuzzy graphs. The various application of graph theory in the fields of information technology, operational research, image segmentation, social science, capturing the image, algebra. It is also applicable to bioscience, chemistry, and computer science. The fuzzy is very useful to deduce the unsolved problems in various fields like networking, clustering with a great role in the algorithm. The use of fuzzy graph by which a great extent in a few years and has a scope from 19th century [19, 20]. Neutrosophy is the type of philosophy which studies the nature and scope of neutralities. We will discuss some new properties on a BSVNG. Bipolar fuzzy set has many applications in image processing. It gives more advantages in real problems

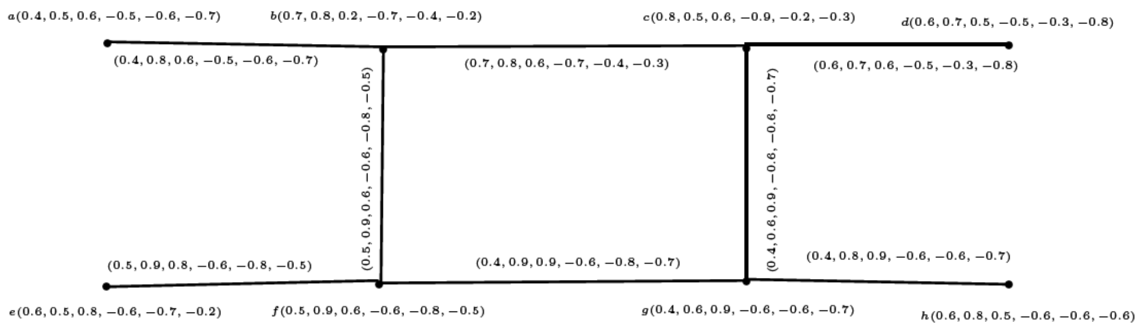

Figure 1. BSVNG

to make it in an easier form. BSVNG is the extension of an Fuzzy graph, Intuitionistic fuzzy graph, interval-valued intuitionistic fuzzy graph and SVNG. Bipolar fuzzy graphs are very useful in the fields of signal processing, computer science, and database theory. The operations we will establish are the symmetric difference and residue product in this paper. Peng [18] defined Some operations which are the join of two graphs, cartesian product of two graphs and the union of two graphs. Also, we discuss examples of these operations. We will find the degree and total degree of BSVNG. In the end, we will make an application on BSVNG with algorithm.

2. Operations on BSVNGs

In this section, we define four operations, including residue product, rejection, maximal product and symmetric difference of bipolar single-valued neutrosophic graph (BSVNG) and to explore some of their related properties with examples.

Definition 2.1. [13] A bipolar single valued neutrosophic graph is such a pair $\mathtt{G} = (X, Y)$ which is of crisp graph G=(V,E) is defined as(i) $\alpha_M : V \to [0,1]$, $\beta_M : V \to [0,1]$, $\gamma_M : V \to [0,1]$, $\delta_M : V \to [-1,0]$, $\eta_M : V \to [-1,0]$, $\theta_M : V \to [-1,0]$. (ii)

$$\alpha_N(mn) \leq \min\{\alpha_M(m), \alpha_M(n)\},\ \beta_N(mn) \geq \max\{\beta_M(m), \beta_M(n)\}$$

$$\gamma_N(mn) \geq \max\{\gamma_M(m), \gamma_M(n)\},\ \delta_N(mn) \geq \max\{\delta_M(m), \delta_M(n)\}$$

$$\eta_N(mn) \leq \min\{\eta_M(m), \eta_M(n)\},\ \theta_N(mn) \leq \min\{\theta_M(m), \theta_M(n)\}.$$

and $0 \leq \alpha_N(mn) + \beta_N(mn) + \gamma_N(mn) \leq 3$ and $-3 \leq \delta_N(mn) + \eta_N(mn) + \theta_N(mn) \leq 0$.

Example 2.2. In Figure 1, we see a graph with eight vertices {a,b,c,d,e,f,g,h} and eight edges {ab, bc, cd ,ef, fg, gh ,bf, cg} that is a bipolar single valued neutrosophic graph. It is easy to see that all conditions of Definition 2.1 is true for this example.

Definition 2.3. The height of a bipolar single valued neutrosophic set (BSVNs) (in universe discourse Y)
$Q = (\alpha_Q(y), \beta_Q(y), \gamma_Q, \delta_Q(y), \eta_Q(y), \theta_Q(y))$ is defined by:

$$h(Q) = (h_1(Q), h_2(Q), h_3(Q), h_4(Q), h_5(Q), h_6(Q))$$
$$= (Sup_{y\in Y}\alpha_Q(y), Inf_{y\in Y}\beta_Q(y), Inf_{y\in Y}\beta_Q(y), Sup_{y\in Y}\delta_Q(y), Inf_{y\in Y}\eta_Q(y), Inf_{y\in Y}\theta_Q(y))$$

Example 2.4. Take $Q = \{(a, 0.5, 0.4, 0.5, -0.2, -0.4, -0.5), (b, 0.5, 0.6, 0.4, -0.4, -0.3, -0.6), (c, 0.4, 0.6, 0.4, -0.4, -0.5, -0.3)\}$ be BSVNs then height is defined as $h(Q) = (0.5, 0.4, 0.4, 0.4, 0.3, 0.3)$.

Definition 2.5. let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ are two bipolar single valued neutrosophic fuzzy graphs defined on $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ respectively. The symmetric difference of $\mathtt{G}_1$ and $\mathtt{G}_2$ is represented by $\mathtt{G}_1 \oplus \mathtt{G}_2 = (M_1 \oplus M_2, N_1 \oplus N_2)$. Symmetric difference of $\mathtt{G}_1$ *and* $\mathtt{G}_2$ is defined as the following conditions:

(i)

$$(\alpha_{M_1} \oplus \alpha_{M_2})((m_1, m_2)) = \min\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\},\ (\beta_{M_1} \oplus \beta_{M_2})((m_1, m_2))$$
$$= \max\{\beta_{M_1}(m_1), \beta_{M_2}(m_2)\}$$
$$(\gamma_{M_1} \oplus \gamma_{M_2})((m_1, m_2)) = \max\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\},\ (\delta_{M_1} \oplus \delta_{M_2})((m_1, m_2))$$
$$= \max\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\}$$
$$(\eta_{M_1} \oplus \eta_{M_2})((m_1, m_2)) = \min\{\eta_{M_1}(m_1), \eta_{M_2}(m_2)\},\ (\theta_{M_1} \oplus \theta_{M_2})((m_1, m_2))$$
$$= \min\{\theta_{M_1}(m_1), \theta_{M_2}(m_2)\}$$

$\forall (m_1, m_2) \in (V_1 \times V_2)$

(ii)

$$(\alpha_{N_1} \oplus \alpha_{N_2})((m, m_2)(m, n_2)) = \min\{\alpha_{M_1}(m), \alpha_{N_2}(m_2n_2)\},\ (\beta_{N_1} \oplus \beta_{N_2})((m, m_2)(m, n_2))$$
$$= \max\{\beta_{M_1}(m), \beta_{N_2}(m_2n_2)\}$$
$$(\gamma_{N_1} \oplus \gamma_{N_2})((m, m_2)(m, n_2)) = \max\{\gamma_{M_1}(m), \gamma_{N_2}(m_2n_2)\},\ (\delta_{N_1} \oplus \delta_{N_2})((m, m_2)(m, n_2))$$
$$= \max\{\delta_{M_1}(m), \delta_{N_2}(m_2n_2)\}$$
$$(\eta_{N_1} \oplus \eta_{N_2})((m, m_2)(m, n_2)) = \min\{\eta_{M_1}(m), \eta_{N_2}(m_2n_2)\},\ (\theta_{N_1} \oplus \theta_{N_2})((m, m_2)(m, n_2))$$
$$= \min\{\theta_{M_1}(m), \theta_{N_2}(m_2n_2)\}$$

$\forall\ m \in V_1$ *and* $m_2n_2 \in E_2$

(iii)

$$(\alpha_{N_1} \oplus \alpha_{N_2})((m_1,m)(n_1,m)) = \min\{\alpha_{N_1}(m_1n_1), \alpha_{M_2}(m)\},\ (\beta_{N_1} \oplus \beta_{N_2})((m_1,m)(n_1,m))$$
$$= \max\{\beta_{N_1}(m_1n_1), \beta_{M_2}(m)\}$$
$$(\gamma_{N_1} \oplus \gamma_{N_2})((m_1,m)(n_1,m)) = \max\{\gamma_{N_1}(m_1n_1), \gamma_{M_2}(m)\},\ (\delta_{N_1} \oplus \delta_{N_2})((m_1,m)(n_1,m))$$
$$= \max\{\delta_{N_1}(m_1n_1), \delta_{M_2}(m)\}$$
$$(\eta_{N_1} \oplus \eta_{N_2})((m_1,m)(n_1,m)) = \min\{\eta_{N_1}(m_1n_1), \eta_{M_2}(m)\},\ (\theta_{N_1} \oplus \theta_{N_2})((m_1,m)(n_1,m))$$
$$= \min\{\theta_{N_1}(m_1n_1), \theta_{M_2}(m)\}$$

$\forall\ z \in V_2\ and\ m_1n_1 \in E_1$

$(i\vee)$

$$(\alpha_{N_1} \oplus \alpha_{N_2})((m_1,m_2)(n_1,n_2)) = \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{N_2}(m_2n_2)\}$$
$$for\ all\ m_1n_1 \notin E_1\ and\ m_2n_2 \in E_2$$
$$or$$
$$= \min\{\alpha_{M_2}(m_2), \alpha_{M_2}(n_2), \alpha_{N_1}(m_1n_1)\} for\ all\ m_1n_1 \in E_1\ and\ m_2n_2 \notin E_2$$

$$(\beta_{N_1} \oplus \beta_{N_2})((m_1,m_2)(n_1,n_2)) = \max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1), \beta_{N_2}(m_2n_2)\}$$
$$forall\ m_1n_1 \notin E_1\ and\ m_2n_2 \in E_2$$
$$or$$
$$= \max\{\beta_{M_2}(m_2), \beta_{M_2}(n_2), \beta_{N_1}(m_1n_1)\}\ forall\ m_1n_1 \in E_1\ and\ m_2n_2 \notin E_2$$

$$(\gamma_{N_1} \oplus \gamma_{N_2})((m_1,m_2)(n_1,n_2)) = \max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1), F_{N_2}(m_2n_2)\}$$
$$forall\ m_1n_1 \notin E_1\ and\ m_2n_2 \in E_2$$
$$or$$
$$= \max\{\gamma_{M_2}(m_2), \gamma_{M_2}(n_2), \gamma_{N_2}(m_1n_1)\}\ forall\ m_1n_1 \in E_1\ and\ m_2n_2 \notin E_2$$

$$(\delta_{N_1} \oplus \delta_{N_2})((m_1,m_2)(n_1,n_2)) = \max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1), \delta_{N_2}(m_2n_2)\}$$
$$forall\ m_1n_1 \notin E_1\ and\ m_2n_2 \in E_2$$
$$or$$
$$= \max\{\delta_{M_2}(m_2), \delta_{M_2}(n_2), \delta_{N_1}(m_1n_1)\}\ forall\ m_1n_1 \in E_1\ and\ m_2n_2 \notin E_2$$

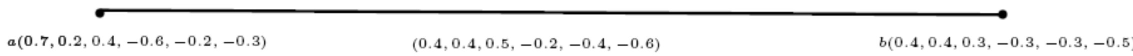

FIGURE 2. G_1

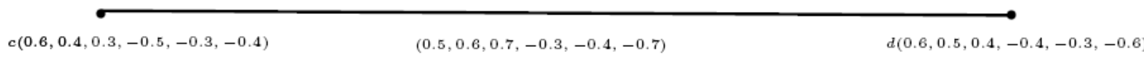

FIGURE 3. G_2

$$(\eta_{N_1} \oplus \eta_{N_2})((m_1, m_2)(n_1, n_2)) = \min\{\eta_{M_1}(m_1), \eta_{M_1}(n_1), \eta_{N_2}(m_2n_2)\}$$
$$forall\ m_1n_1 \notin E_1\ and\ m_2n_2 \in E_2$$
$$or$$
$$= \min\{\eta_{M_2}(m_2), \eta_{M_2}(n_2), \eta_{N_1}(m_1n_1)\}\ forall\ m_1n_1 \in E_1\ and\ m_2n_2 \notin E_2$$

$$(\theta_{N_1} \oplus \theta_{N_2})((m_1, m_2)(n_1, n_2)) = \min\{\theta_{M_1}(m_1), \theta_{M_1}(n_1), F_{N_2}(m_2n_2)\}$$
$$forall\ m_1n_1 \notin E_1\ and\ m_2n_2 \in E_2$$
$$or$$
$$= \min\{\theta_{M_2}(m_2), \theta_{M_2}(n_2), \theta_{N_2}(m_1n_1)\}\ forall\ m_1n_1 \in E_1\ and\ m_2n_2 \notin E_2$$

Example 2.6. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs on $V_1 = \{a, b\}$ *and* $V_2 = \{c, d\}$ respectively which shown in Figure 2 and Figure 3. Also symmetric difference shown in Figure 4.

Proposition 2.7. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively. Then the symmetric difference $\mathtt{G}_1 \oplus \mathtt{G}_2$ of $\mathtt{G}_1 = (V_1, E_1)$ and $\mathtt{G}_2 = (V_2, E_2)$ is again a BSVNG.

Proof. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively. Then the symmetric difference $\mathtt{G}_1 \oplus \mathtt{G}_2$ of $\mathtt{G}_1 = (V_1, E_1)$ and $\mathtt{G}_2 = (V_2, E_2)$ can be proved. Let $(m_1, m_2)(n_1, n_2) \in E_1 \times E_2$

(i) If $m_1 = n_1 = m$

$$\begin{aligned}(\alpha_{N_1} \oplus \alpha_{N_2})((m, m_2)(m, n_2)) &= \min\{\alpha_{M_1}(m), \alpha_{N_2}(m_2n_2)\}\\ &\leq \min\{\alpha_{M_1}(m), \min\{\alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\}\}\\ &= \min\{\min\{\{\alpha_{M_1}(m), \alpha_{M_2}(m_2)\}, \min\{\{\alpha_{M_1}(m), \alpha_{M_2}(n_2)\}\}\\ &= \min\{(\alpha_{M_1} \oplus \alpha_{M_2})(m, m_2), (\alpha_{M_1} \oplus \alpha_{M_2})(m, n_2)\}\end{aligned}$$

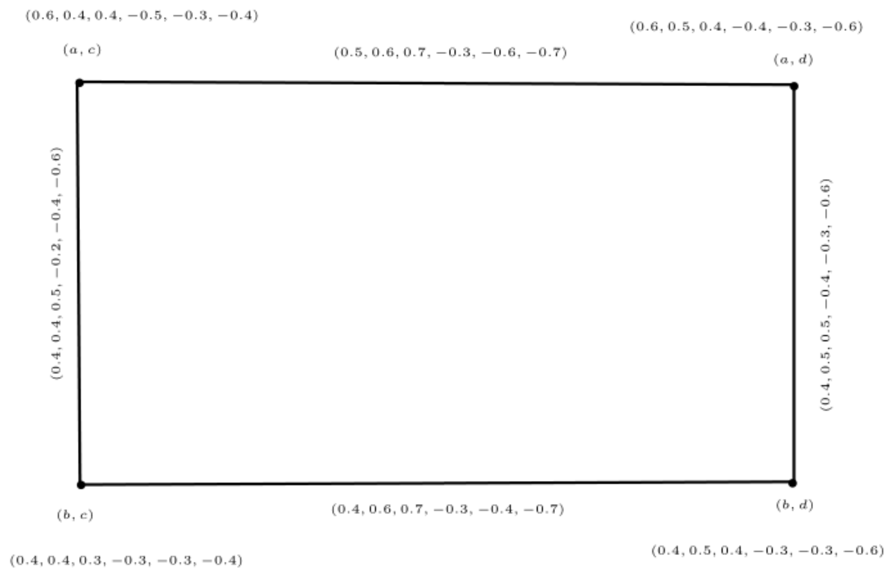

FIGURE 4. $G_1 \oplus G_2$

$$\begin{aligned}(\beta_{N_1} \oplus \beta_{N_2})((m, m_2)(m, n_2)) &= \max\{\beta_{M_1}(m), \beta_{N_2}(m_2 n_2)\}\\ &\geq \max\{\beta_{M_1}(m), \max\{\beta_{M_2}(m_2), \beta_{M_2}(n_2)\}\}\\ &= \max\{\max\{\{\beta_{M_1}(m), \beta_{M_2}(m_2)\}, \max\{\{\beta_{M_1}(m), \beta_{M_2}(n_2)\}\}\\ &= \max\{(\beta_{M_1} \oplus \beta_{M_2})(m, m_2), (\beta_{M_1} \oplus \beta_{M_2})(m, n_2)\}\end{aligned}$$

$$\begin{aligned}(\gamma_{N_1} \oplus \gamma_{N_2})((m, m_2)(m, n_2)) &= \max\{\gamma_{M_1}(m), \gamma_{N_2}(m_2 n_2)\}\\ &\geq \max\{\gamma_{M_1}(m), \max\{\gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\}\}\\ &= \max\{\max\{\{\gamma_{M_1}(m), \gamma_{M_2}(m_2)\}, \max\{\{\gamma_{M_1}(m), \gamma_{M_2}(n_2)\}\}\\ &= \max\{(\gamma_{M_1} \oplus \gamma_{M_2})(m, m_2), (\gamma_{M_1} \oplus \gamma_{M_2})(m, n_2)\}\end{aligned}$$

$$\begin{aligned}(\delta_{N_1} \oplus \delta_{N_2})((m, m_2)(m, n_2)) &= \max\{\delta_{M_1}(m), \delta_{N_2}(m_2 n_2)\}\\ &\geq \max\{\delta_{M_1}(m), \max\{\delta_{M_2}(m_2), \delta_{M_2}(n_2)\}\}\\ &= \max\{\max\{\{\delta_{M_1}(m), \delta_{M_2}(m_2)\}, \min\{\{\delta_{M_1}(m), \delta_{M_2}(n_2)\}\}\\ &= \max\{(\delta_{M_1} \oplus \delta_{M_2})(m, m_2), (\delta_{M_1} \oplus \delta_{M_2})(m, n_2)\}\end{aligned}$$

$$\begin{aligned}(\eta_{N_1} \oplus \eta_{N_2})((m, m_2)(m, n_2)) &= \min\{\eta_{M_1}(m), \eta_{N_2}(m_2 n_2)\}\\ &\leq \min\{\eta_{M_1}(m), \min\{\eta_{M_2}(m_2), \eta_{M_2}(n_2)\}\}\\ &= \min\{\min\{\{\eta_{M_1}(m), \eta_{M_2}(m_2)\}, \min\{\{\eta_{M_1}(m), \eta_{M_2}(n_2)\}\}\\ &= \min\{(\eta_{M_1} \oplus \eta_{M_2})(m, m_2), (\eta_{M_1} \oplus \eta_{M_2})(m, n_2)\}\end{aligned}$$

$$\begin{aligned}(\theta_{N_1} \oplus \theta_{N_2})((m, m_2)(m, n_2)) &= \min\{\theta_{M_1}(m), \theta_{N_2}(m_2 n_2)\}\\ &\leq \min\{\theta_{M_1}(m), \min\{\theta_{M_2}(m_2), \theta_{M_2}(n_2)\}\}\\ &= \min\{\min\{\{\theta_{M_1}(m), \theta_{M_2}(m_2)\}, \min\{\{\theta_{M_1}(m), \theta_{M_2}(n_2)\}\}\\ &= \min\{(\theta_{M_1} \oplus \theta_{M_2})(m, m_2), (\theta_{M_1} \oplus \theta_{M_2})(m, n_2)\}\end{aligned}$$

(ii) if $m_2 = n_2 = m$

$$\begin{aligned}(\alpha_{N_1} \oplus \alpha_{N_2})((m_1, m)(n_1, m)) &= \min\{\alpha_{N_1}(m_1 n_1), \alpha_{M_2}(m)\}\\ &\leq \min\{\min\{\alpha_{N_1}(m_1 n_1), \alpha_{M_2}(m)\}\\ &= \min\{\min\{\{\alpha_{M_1}(m_1), \alpha_{M_2}(m)\}, \min\{\{\alpha_{M_1}(n_1), \alpha_{M_2}(m)\}\}\\ &= \min\{(\alpha_{M_1} \oplus \alpha_{M_2})(m_1, m), (\alpha_{M_1} \oplus \alpha_{M_2})(n_1, m)\}\end{aligned}$$

$$\begin{aligned}(\beta_{N_1} \oplus \beta_{N_2})((m_1, m)(n_1, m)) &= \max\{\beta_{N_1}(m_1 n_1), \beta_{M_2}(m)\}\\ &\geq \max\{\max\{\beta_{N_1}(m_1 n_1), \beta_{M_2}(m)\}\\ &= \max\{\max\{\{\beta_{M_1}(m_1), \beta_{M_2}(m)\}, \max\{\{\beta_{M_1}(n_1), \beta_{M_2}(m)\}\}\\ &= \max\{(\beta_{M_1} \oplus \beta_{M_2})(m_1, m), (\beta_{M_1} \oplus \beta_{M_2})(n_1, m)\}\end{aligned}$$

$$\begin{aligned}(\gamma_{N_1} \oplus \gamma_{N_2})((m_1, m)(n_1, m)) &= \max\{\gamma_{N_1}(m_1 n_1), \gamma_{M_2}(m)\}\\ &\geq \max\{\max\{\gamma_{N_1}(m_1 n_1), \gamma_{M_2}(m)\}\\ &= \max\{\max\{\{\gamma_{M_1}(m_1), \gamma_{M_2}(m)\}, \max\{\{\gamma_{M_1}(n_1), \gamma_{M_2}(m)\}\}\\ &= \max\{(\gamma_{M_1} \oplus \gamma_{M_2})(m_1, m), (\gamma_{M_1} \oplus \gamma_{M_2})(n_1, m)\}\end{aligned}$$

$$
\begin{aligned}
(\delta_{N_1} \oplus \delta_{N_2})((m_1, m)(n_1, m)) &= \max\{\delta_{N_1}(m_1 n_1), \delta_{M_2}(m)\} \\
&\geq \max\{\max\{\delta_{N_1}(m_1 n_1), \delta_{M_2}(m)\} \\
&= \max\{\max\{\{\delta_{M_1}(m_1), \delta_{M_2}(m)\}, \max\{\{\delta_{M_1}(n_1), \delta_{M_2}(m)\}\} \\
&= \max\{(\delta_{M_1} \oplus \delta_{M_2})(m_1, m), (\delta_{M_1} \oplus \delta_{M_2})(n_1, m)\}
\end{aligned}
$$

$$
\begin{aligned}
(\eta_{N_1} \oplus \eta_{N_2})((m_1, m)(n_1, m)) &= \min\{\eta_{N_1}(m_1 n_1), \eta_{M_2}(m)\} \\
&\leq \min\{\min\{\eta_{N_1}(m_1 n_1), \eta_{M_2}(m)\} \\
&= \min\{\min\{\{\eta_{M_1}(m_1), \eta_{M_2}(m)\}, \min\{\{\eta_{M_1}(n_1), \eta_{M_2}(m)\}\} \\
&= \min\{(\eta_{M_1} \oplus \eta_{M_2})(m_1, m), (\eta_{M_1} \oplus \eta_{M_2})(n_1, m)\}
\end{aligned}
$$

$$
\begin{aligned}
(\theta_{N_1} \oplus \theta_{N_2})((m_1, m)(n_1, m)) &= \min\{\theta_{N_1}(m_1 n_1), \theta_{M_2}(m)\} \\
&\leq \min\{\min\{\theta_{N_1}(m_1 n_1), \theta_{M_2}(m)\} \\
&= \min\{\min\{\{\theta_{M_1}(m_1), \theta_{M_2}(m)\}, \min\{\{\theta_{M_1}(n_1), \theta_{M_2}(m)\}\} \\
&= \min\{(\theta_{M_1} \oplus \theta_{M_2})(m_1, m), (\theta_{M_1} \oplus \theta_{M_2})(n_1, m)\}
\end{aligned}
$$

(iii) If $m_1 n_1 \notin E_1$ *and* $m_2 n_2 \in E_2$

$$
\begin{aligned}
(\alpha_{N_1} \oplus \alpha_{N_2})((m_1, m_2)(n_1, n_2)) &= \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{N_2}(m_2 n_2)\} \\
&\leq \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \min\{\alpha_{M_2}(m_2)\alpha_{M_2}(n_2)\}\} \\
&= \min\{\min\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\}, \{\alpha_{M_1}(m_1), \alpha_{M_2}(n_2)\} \\
&= \min\{(\alpha_{M_1} \oplus \alpha_{M_2})(m_1, m_2), (\alpha_{M_1} \oplus \alpha_{M_2})(n_1, n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(\beta_{N_1} \oplus \beta_{N_2})((m_1, m_2)(n_1, n_2)) &= \max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1), \beta_{N_2}(m_2 n_2)\} \\
&\geq \max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1), \max\{\beta_{M_2}(m_2)\beta_{M_2}(n_2)\}\} \\
&= \max\{\max\{\beta_{M_1}(m_1), \beta_{M_2}(m_2)\}, \{\beta_{M_1}(m_1), \beta_{M_2}(n_2)\} \\
&= \max\{(\beta_{M_1} \oplus \beta_{M_2})(m_1, m_2), (\beta_{M_1} \oplus \beta_{M_2})(n_1, n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(\gamma_{N_1} \oplus \gamma_{N_2})((m_1, m_2)(n_1, n_2)) &= \max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1), \gamma_{N_2}(m_2 n_2)\} \\
&\geq \max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1), \max\{\gamma_{M_2}(m_2)\gamma_{M_2}(n_2)\}\} \\
&= \max\{\max\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\}, \{\gamma_{M_1}(m_1), \gamma_{M_2}(n_2)\} \\
&= \max\{(\gamma_{M_1} \oplus \gamma_{M_2})(m_1, m_2), (\gamma_{M_1} \oplus \gamma_{M_2})(n_1, n_2)\}
\end{aligned}
$$

$$\begin{aligned}
(\delta_{N_1} \oplus \delta_{N_2})((m_1, m_2)(n_1, n_2)) &= \max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1), \delta_{N_2}(m_2n_2)\} \\
&\geq \max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1), \max\{\delta_{M_2}(m_2)\delta_{M_2}(n_2)\}\} \\
&= \max\{\max\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\}, \{\delta_{M_1}(m_1), \delta_{M_2}(n_2)\} \\
&= \max\{(\delta_{M_1} \oplus \delta_{M_2})(m_1, m_2), (\delta_{M_1} \oplus \delta_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}
(\eta_{N_1} \oplus \eta_{N_2})((m_1, m_2)(n_1, n_2)) &= \min\{\eta_{M_1}(m_1), \eta_{M_1}(n_1), \eta_{N_2}(m_2n_2)\} \\
&\leq \min\{\eta_{M_1}(m_1), \eta_{M_1}(n_1), \min\{\eta_{M_2}(m_2)\eta_{M_2}(n_2)\}\} \\
&= \min\{\min\{\eta_{M_1}(m_1), \eta_{M_2}(m_2)\}, \{\eta_{M_1}(m_1), \eta_{M_2}(n_2)\} \\
&= \min\{(\eta_{M_1} \oplus \eta_{M_2})(m_1, m_2), (\eta_{M_1} \oplus \eta_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}
(\theta_{N_1} \oplus \theta_{N_2})((m_1, m_2)(n_1, n_2)) &= \min\{\theta_{M_1}(m_1), \theta_{M_1}(n_1), \theta_{N_2}(m_2n_2)\} \\
&\leq \min\{\theta_{M_1}(m_1), \theta_{M_1}(n_1), \min\{\theta_{M_2}(m_2)\theta_{M_2}(n_2)\}\} \\
&= \min\{\min\{\theta_{M_1}(m_1), \theta_{M_2}(m_2)\}, \{\theta_{M_1}(m_1), \theta_{M_2}(n_2)\} \\
&= \min\{(\theta_{M_1} \oplus \theta_{M_2})(m_1, m_2), (\theta_{M_1} \oplus \theta_{M_2})(n_1, n_2)\}
\end{aligned}$$

$(i\vee)$ If $m_1n_1 \in E_1$ *and* $m_2n_2 \notin E_2$

$$\begin{aligned}
(\alpha_{N_1} \oplus \alpha_{N_2})((m_1, m_2)(n_1, n_2)) &= \min\{\alpha_{M_2}(m_2), \alpha_{M_2}(n_2), \alpha_{N_1}(m_1n_1)\} \\
&\leq \min\{\alpha_{M_2}(m_2), \alpha_{M_2}(n_2), \min\{\alpha_{M_1}(m_1)\alpha_{M_1}(n_1)\}\} \\
&= \min\{\min\{\alpha_{M_2}(m_2), \alpha_{M_1}(m_1)\}, \{\alpha_{M_2}(m_2), \alpha_{M_1}(n_1)\} \\
&= \min\{(\alpha_{M_1} \oplus \alpha_{M_2})(m_1, m_2), (\alpha_{M_1} \oplus \alpha_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}
(\beta_{N_1} \oplus \beta_{N_2})((m_1, m_2)(n_1, n_2)) &= \max\{\beta_{M_2}(m_2), \beta_{M_2}(n_2), \beta_{N_1}(m_1n_1)\} \\
&\geq \max\{\beta_{M_2}(m_2), \beta_{M_2}(n_2), \max\{\beta_{M_1}(m_1)\beta_{M_1}(n_1)\}\} \\
&= \max\{\max\{\beta_{M_2}(m_2), \beta_{M_1}(m_1)\}, \{\beta_{M_2}(m_2), \beta_{M_1}(n_1)\} \\
&= \max\{(\beta_{M_1} \oplus \beta_{M_2})(m_1, m_2), (\beta_{M_1} \oplus \beta_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}
(\gamma_{N_1} \oplus \gamma_{N_2})((m_1, m_2)(n_1, n_2)) &= \max\{\gamma_{M_2}(m_2), \gamma_{M_2}(n_2), \gamma_{N_1}(m_1n_1)\} \\
&\geq \max\{\gamma_{M_2}(m_2), \gamma_{M_2}(n_2), \max\{\gamma_{M_1}(m_1)\gamma_{M_1}(n_1)\}\} \\
&= \max\{\max\{\gamma_{M_2}(m_2), \gamma_{M_1}(m_1)\}, \{\gamma_{M_2}(m_2), \gamma_{M_1}(n_1)\} \\
&= \max\{(\gamma_{M_1} \oplus \gamma_{M_2})(m_1, m_2), (\gamma_{M_1} \oplus \gamma_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}(\delta_{N_1} \oplus \delta_{N_2})((m_1, m_2)(n_1, n_2)) &= \max\{\delta_{M_2}(m_2), \delta_{M_2}(n_2), \delta_{N_1}(m_1 n_1)\} \\ &\geq \max\{\delta_{M_2}(m_2), \delta_{M_2}(n_2), \max\{\delta_{M_1}(m_1)\delta_{M_1}(n_1)\}\} \\ &= \max\{\max\{\delta_{M_2}(m_2), \delta_{M_1}(m_1)\}, \{\delta_{M_2}(m_2), \delta_{M_1}(n_1)\} \\ &= \max\{(\delta_{M_1} \oplus \delta_{M_2})(m_1, m_2), (\delta_{M_1} \oplus \delta_{M_2})(n_1, n_2)\}\end{aligned}$$

$$\begin{aligned}(\eta_{N_1} \oplus \eta_{N_2})((m_1, m_2)(n_1, n_2)) &= \min\{\eta_{M_2}(m_2), \eta_{M_2}(n_2), \eta_{N_1}(m_1 n_1)\} \\ &\leq \min\{\eta_{M_2}(m_2), \eta_{M_2}(n_2), \min\{\eta_{M_1}(m_1)\eta_{M_1}(n_1)\}\} \\ &= \min\{\min\{\eta_{M_2}(m_2), \eta_{M_1}(m_1)\}, \{\eta_{M_2}(m_2), \eta_{M_1}(n_1)\} \\ &= \min\{(\eta_{M_1} \oplus \eta_{M_2})(m_1, m_2), (\eta_{M_1} \oplus \eta_{M_2})(n_1, n_2)\}\end{aligned}$$

$$\begin{aligned}(\theta_{N_1} \oplus \theta_{N_2})((m_1, m_2)(n_1, n_2)) &= \min\{\theta_{M_2}(m_2), \theta_{M_2}(n_2), \theta_{N_1}(m_1 n_1)\} \\ &\leq \min\{\theta_{M_2}(m_2), \theta_{M_2}(n_2), \min\{\theta_{M_1}(m_1)\theta_{M_1}(n_1)\}\} \\ &= \min\{\min\{\theta_{M_2}(m_2), \theta_{M_1}(m_1)\}, \{\theta_{M_2}(m_2), \theta_{M_1}(n_1)\} \\ &= \min\{(\theta_{M_1} \oplus \theta_{M_2})(m_1, m_2), (\theta_{M_1} \oplus \theta_{M_2})(n_1, n_2)\}\end{aligned}$$

. Hence $\mathtt{G}_1 \oplus \mathtt{G}_2$ is a BSVNG. □

Definition 2.8. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, Y_2)$ be two BSVNGs. $\forall (m_1, m_2) \in \ V_1 \times V_2$

$$\begin{aligned}(d_\alpha)_{\mathtt{G}_1 \oplus \mathtt{G}_2}(m_1, m_2) = & \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\alpha_{N_1} \oplus \alpha_{N_2})((m_1, m_2)(n_1, n_2)) \\ = & \sum_{m_1 = n_1, m_2 n_2 \in E_2} \min\{\alpha_{M_1}(m_1), \alpha_{N_2}(m_2 n_2)\} \\ + & \sum_{m_1 n_1 \in E_1, m_2 = n_2} \min\{\alpha_{N_1}(m_1 n_1, \alpha_{M_2}(m_2)\} \\ + & \sum_{m_1 n_1 \notin E_1 and \ m_2 n_2 \in E_2} \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{N_2}(m_2 n_2)\} \\ + & \sum_{m_1 n_1 \in E_1 and \ m_2 n_2 \notin E_2} \min\{\alpha_{N_1}(m_1 n_1), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\}\end{aligned}$$

$$
\begin{aligned}
(d_\beta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\beta_{N_1}\oplus\beta_{N_2})((m_1,m_2)(n_1,n_2)) \\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{\beta_{M_1}(m_1),\beta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \max\{\beta_{N_1}(m_1n_1,\beta_{M_2}(m_2)\} \\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \max\{\beta_{M_1}(m_1),\beta_{M_1}(n_1),\beta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \max\{\beta_{N_1}(m_1n_1),\beta_{M_2}(m_2),\beta_{M_2}(n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(d_\gamma)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\gamma_{N_1}\oplus\beta_{N_2})((m_1,m_2)(n_1,n_2)) \\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{\gamma_{M_1}(m_1),\gamma_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \max\{\gamma_{N_1}(m_1n_1,\gamma_{M_2}(m_2)\} \\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \max\{\gamma_{M_1}(m_1),\gamma_{M_1}(n_1),\gamma_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \max\{\gamma_{N_1}(m_1n_1),\gamma_{M_2}(m_2),\gamma_{M_2}(n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(d_\delta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\delta_{N_1}\oplus\delta_{N_2})((m_1,m_2)(n_1,n_2)) \\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{\delta_{M_1}(m_1),\delta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \max\{\delta_{N_1}(m_1n_1,\delta_{M_2}(m_2)\} \\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \max\{\delta_{M_1}(m_1),\delta_{M_1}(n_1),\delta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \max\{\delta_{N_1}(m_1n_1),\delta_{M_2}(m_2),\delta_{M_2}(n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(d_\eta)_{\mathtt{G}_1 \oplus \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\eta_{N_1} \oplus \eta_{N_2})((m_1, m_2)(n_1, n_2)) \\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\eta_{M_1}(m_1), \eta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\eta_{N_1}(m_1n_1, \eta_{M_2}(m_2)\} \\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \min\{\eta_{M_1}(m_1), \eta_{M_1}(n_1), \eta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \min\{\eta_{N_1}(m_1n_1), \eta_{M_2}(m_2), \eta_{M_2}(n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(d_\theta)_{\mathtt{G}_1 \oplus \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\theta_{N_1} \oplus \eta_{N_2})((m_1, m_2)(n_1, n_2)) \\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\theta_{M_1}(m_1), \theta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\theta_{N_1}(m_1n_1, \theta_{M_2}(m_2)\} \\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \min\{\theta_{M_1}(m_1), \theta_{M_1}(n_1), \theta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \min\{\theta_{N_1}(m_1n_1), \theta_{M_2}(m_2), \theta_{M_2}(n_2)\}
\end{aligned}
$$

Theorem 2.9. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, Y_2)$ be two BSVNGs. If $\alpha_{M_1} \geq \alpha_{N_2}, \beta_{M_1} \leq \beta_{N_2}, \gamma_{M_1} \leq \gamma_{N_2}$ and $\alpha_{M_2} \geq \alpha_{N_1}, \beta_{M_2} \leq \beta_{N_1}, \gamma_{M_2} \leq \gamma_{N_1}$. Also if $\delta_{M_1} \leq \delta_{N_2}, \eta_{M_1} \geq \eta_{N_2}, \theta_{M_1} \geq \theta_{N_2}$ and $\delta_{M_2} \leq \delta_{N_1}, \eta_{M_2} \geq \eta_{N_1}, \theta_{M_2} \geq \theta_{N_1}$. Then for every $\forall(m_1, m_2) \in\ V_1 \times V_2$
$(d)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1, m_2)$ =q$(d)_{\mathtt{G}_1}(m_1) + s(d)_{\mathtt{G}_2}(m_2)$ where s=$|\ V_1\ |$ -$(d)_{G_1}(m_1)$ and q=$|\ V_2\ |$ -$(d)_{G_2}(m_2)$
.

Proof.

$$
\begin{aligned}
(d_\alpha)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\alpha_{N_1}\oplus\alpha_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\alpha_{M_1}(m_1), \alpha_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\alpha_{N_1}(m_1n_1), \alpha_{M_2}(m_2)\}\\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \min\{\alpha_{N_1}(m_1n_1), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\}\\
&= \sum_{m_2n_2\in E_2} \alpha_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1} \alpha_{N_1}(m_1n_1)\\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \alpha_{N_2}(m_2n_2)\} + \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \alpha_{N_1}(m_1n_1)\\
&= q(d_\alpha)_{\mathtt{G}_1}(m_1) + s(d_\alpha)_{\mathtt{G}_2}(m_2)
\end{aligned}
$$

$$
\begin{aligned}
(d_\theta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\theta_{N_1}\oplus\theta_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\theta_{M_1}(m_1), \theta_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\theta_{N_1}(m_1n_1), \theta_{M_2}(m_2)\}\\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \min\{\theta_{M_1}(m_1), \theta_{M_1}(n_1), \theta_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \min\{\theta_{N_1}(m_1n_1), \theta_{M_2}(m_2), \theta_{M_2}(n_2)\}\\
&= \sum_{m_2n_2\in E_2} \theta_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1} \theta_{N_1}(m_1n_1)\\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \theta_{N_2}(m_2n_2)\} + \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \theta_{N_1}(m_1n_1)\\
&= q(d_\theta)_{\mathtt{G}_1}(m_1) + s(d_\theta)_{\mathtt{G}_2}(m_2)
\end{aligned}
$$

In a similar way others four will proved obviously.

We conclude that $(d)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2)$ =q$(d)_{\mathtt{G}_1}(m_1) + s(d)_{\mathtt{G}_2}(m_2)$ where s=| V_1 | -$(d)_{G_1}(m_1)$ and q=| V_2 | -$(d)_{G_2}(m_2)$. □

Definition 2.10. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, Y_2)$ be two BSVNGs. $\forall (m_1, m_2) \in V_1 \times V_2$

$$\begin{aligned}(td_\alpha)_{\mathtt{G}_1 \oplus \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\alpha_{N_1} \oplus \alpha_{N_2})((m_1, m_2)(n_1, n_2)) + (\alpha_{M_1} \oplus \alpha_{M_2}(m_1, m_2) \\ &= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\alpha_{M_1}(m_1), \alpha_{N_2}(m_2n_2)\} \\ &+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\alpha_{N_1}(m_1n_1, \alpha_{M_2}(m_2)\} \\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{N_2}(m_2n_2)\} \\ &+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \min\{\alpha_{N_1}(m_1n_1), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\} \\ &+ \min\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\}\end{aligned}$$

$$\begin{aligned}(td_\beta)_{\mathtt{G}_1 \oplus \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\beta_{N_1} \oplus \beta_{N_2})((m_1, m_2)(n_1, n_2)) + (\beta_{M_1} \oplus \beta_{M_2}(m_1, m_2) \\ &= \sum_{m_1=n_1, m_2n_2\in E_2} \max\{\beta_{M_1}(m_1), \beta_{N_2}(m_2n_2)\} \\ &+ \sum_{m_1n_1\in E_1, m_2=n_2} \max\{\beta_{N_1}(m_1n_1, \beta_{M_2}(m_2)\} \\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1), \beta_{N_2}(m_2n_2)\} \\ &+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \max\{\beta_{N_1}(m_1n_1), \beta_{M_2}(m_2), \beta_{M_2}(n_2)\} \\ &+ \max\{\beta_{M_1}(m_1), \beta_{M_2}(m_2)\}\end{aligned}$$

$$\begin{aligned}(td_\gamma)_{\mathtt{G}_1 \oplus \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\gamma_{N_1} \oplus \gamma_{N_2})((m_1, m_2)(n_1, n_2)) + (\gamma_{M_1} \oplus \gamma_{M_2}(m_1, m_2) \\ &= \sum_{m_1=n_1, m_2n_2\in E_2} \max\{\gamma_{M_1}(m_1), \gamma_{N_2}(m_2n_2)\} \\ &+ \sum_{m_1n_1\in E_1, m_2=n_2} \max\{\gamma_{N_1}(m_1n_1, \gamma_{M_2}(m_2)\} \\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1), \gamma_{N_2}(m_2n_2)\} \\ &+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \max\{\gamma_{N_1}(m_1n_1), \gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\} \\ &+ \max\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\}\end{aligned}$$

$$(td_\delta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\delta_{N_1}\oplus\delta_{N_2})((m_1,m_2)(n_1,n_2)) + (\delta_{M_1}\oplus\delta_{M_2}(m_1,m_2)$$
$$= \sum_{m_1=n_1, m_2n_2\in E_2} \max\{\delta_{M_1}(m_1),\delta_{N_2}(m_2n_2)\}$$
$$+ \sum_{m_1n_1\in E_1, m_2=n_2} \max\{\delta_{N_1}(m_1n_1,\delta_{M_2}(m_2)\}$$
$$+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \max\{\delta_{M_1}(m_1),\delta_{M_1}(n_1),\delta_{N_2}(m_2n_2)\}$$
$$+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \max\{\delta_{N_1}(m_1n_1),\delta_{M_2}(m_2),\delta_{M_2}(n_2)\}$$
$$+ \min\{\delta_{M_1}(m_1),\delta_{M_2}(m_2)\}$$

$$(td_\eta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\eta_{N_1}\oplus\eta_{N_2})((m_1,m_2)(n_1,n_2)) + (\eta_{M_1}\oplus\eta_{M_2}(m_1,m_2)$$
$$= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\eta_{M_1}(m_1),\eta_{N_2}(m_2n_2)\}$$
$$+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\eta_{N_1}(m_1n_1,\eta_{M_2}(m_2)\}$$
$$+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \min\{\eta_{M_1}(m_1),\eta_{M_1}(n_1),\eta_{N_2}(m_2n_2)\}$$
$$+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \min\{\eta_{N_1}(m_1n_1),\eta_{M_2}(m_2),\eta_{M_2}(n_2)\}$$
$$+ \max\{\eta_{M_1}(m_1),\eta_{M_2}(m_2)\}$$

$$(td_\theta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(m_1,m_2) = \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\theta_{N_1}\oplus\theta_{N_2})((m_1,m_2)(n_1,n_2)) + (\theta_{M_1}\oplus\theta_{M_2}(m_1,m_2)$$
$$= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\theta_{M_1}(m_1),\theta_{N_2}(m_2n_2)\}$$
$$+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\theta_{N_1}(m_1n_1,\theta_{M_2}(m_2)\}$$
$$+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2} \min\{\theta_{M_1}(m_1),\theta_{M_1}(n_1),\theta_{N_2}(m_2n_2)\}$$
$$+ \sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2} \min\{\theta_{N_1}(m_1n_1),\theta_{M_2}(m_2),\theta_{M_2}(n_2)\}$$
$$+ \max\{\theta_{M_1}(m_1),\theta_{M_2}(m_2)\}$$

Theorem 2.11. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, Y_2)$ be two BSVNGs. If

(i)

$\alpha_{M_1} \geq \alpha_{N_2}$ *and* $\alpha_{M_2} \geq \alpha_{N_1}$ then $\forall (m_1, m_2) \in \ V_1 \times V_2$

$$(td_\alpha)_{G_1 \oplus G_2}(m_1, m_2) = q(td_\alpha)_{G_1}(m_1) + s(td_\alpha)_{G_2}(m_2) \\ - (q-1)T_{G_1}(m_1) - \max\{T_{G_1}(m_1), T_{G_1}(m_1)\}$$

and

$\delta_{M_1} \leq \delta_{N_2}$ *and* $\delta_{M_2} \leq \delta_{N_1}$ then $\forall (m_1, m_2) \in \ V_1 \times V_2$

$$(td_\delta)_{G_1 \oplus G_2}(m_1), m_2) = q(td_\delta)_{G_1}(m_1) + s(td_\delta)_{G_2}(m_2) \\ - (q-1)T_{G_1}(m_1) - \min\{T_{G_1}(m_1), T_{G_1}(m_1)\}$$

(ii) $\beta_{M_1} \leq \beta_{N_2}$ *and* $\beta_{M_2} \leq \beta_{N_1}$ then $\forall (m_1, m_2) \in \ V_1 \times V_2$

$$(td_\beta)_{G_1 \oplus G_2}(m_1, m_2) = q(td_\beta)_{G_1}(m_1) + s(td_\beta)_{G_2}(m_2) \\ - (q-1)I_{G_1}(m_1) - \min\{I_{G_1}(m_1), I_{G_1}(m_1\}$$

and

$\eta_{M_1} \geq \eta_{N_2}$ *and* $\eta_{M_2} \geq \eta_{N_1}$ then $\forall (m_1, m_2) \in \ V_1 \times V_2$

$$(td_\eta)_{G_1 \oplus G_2}(m_1, m_2) = q(td_\eta)_{G_1}(m_1) + s(td_\eta)_{G_2}(m_2) \\ - (q-1)I_{G_1}(m_1) - \max\{I_{G_1}(m_1), I_{G_1}(m_1)\}$$

(iii) $\gamma_{M_1} \leq \gamma_{N_2}$ *and* $\gamma_{M_2} \geq \gamma_{N_1}$ then $\forall (m_1, m_2) \in \ V_1 \times V_2$

$$(td_\gamma)_{G_1 \oplus G_2}(m_1, m_2) = q(td_\gamma)_{G_1}(m_1) + s(td_\gamma)_{G_2}(m_2) \\ - (q-1)F_{G_1}(m_1) - \min\{F_{G_1}(m_1), F_{G_1}(m_1)\}$$

and

$\theta_{M_1} \geq \theta_{N_2}$ *and* $\theta_{M_2} \leq \theta_{N_1}$ then $\forall (m_1, m_2) \in \ V_1 \times V_2$

$$(td_\theta)_{G_1 \oplus G_2}(m_1, m_2) = q(td_\theta)_{G_1}(m_1) + s(td_\theta)_{G_2}(m_2) \\ - (q-1)F_{G_1}(m_1) - \max\{F_{G_1}(m_1), F_{G_1}(m_1)\}$$

$\forall (m_1, m_2) \in V_1 \times V_2$,s=$\mid V_1 \mid$ -$(d)_{G_1}(m_1)$ and q=$\mid V_2 \mid$ -$(d)_{G_2}(m_2)$.

Proof. $\forall (m_1, m_2) \in V_1 \times V_2$

$$
\begin{aligned}
(td_\alpha)_{\mathtt{G}_1 \oplus \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\alpha_{N_1} \oplus \alpha_{N_2})((m_1, m_2)(n_1, n_2)) + (\alpha_{M_1} \oplus \alpha_{M_2})(m_1, m_2) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \min\{\alpha_{M_1}(m_1), \alpha_{N_2}(m_2 n_2)\} \\
&+ \sum_{m_1 n_1 \in E_1, m_2 = n_2} \min\{\alpha_{N_1}(m_1 n_1), \alpha_{M_2}(m_2)\} \\
&+ \sum_{m_1 n_1 \notin E_1 and\ m_2 n_2 \in E_2} \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{N_2}(m_2 n_2)\} \\
&+ \sum_{m_1 n_1 \in E_1 and\ m_2 n_2 \notin E_2} \min\{\alpha_{N_1}(m_1 n_1), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\} \\
&+ \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\} \\
&= \sum_{m_2 n_2 \in E_2} \alpha_{N_2}(m_2 n_2) + \sum_{m_1 n_1 \in E_1} \alpha_{N_1}(m_1 n_1) \\
&+ \sum_{m_1 n_1 \notin E_1 and\ m_2 n_2 \in E_2} \alpha_{N_2}(m_2 n_2)\} + \sum_{m_1 n_1 \in E_1 and\ m_2 n_2 \notin E_2} \alpha_{N_1}(m_1 n_1) \\
&+ \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\} \\
&= \sum_{m_2 n_2 \in E_2} \alpha_{N_2}(m_2 n_2) + \sum_{m_1 n_1 \in E_1} \alpha_{N_1}(m_1 n_1) + \sum_{m_1 n_1 \notin E_1 and\ m_2 n_2 \in E_2} \alpha_{N_2}(m_2 n_2)\} \\
&+ \sum_{m_1 n_1 \in E_1 and\ m_2 n_2 \notin E_2} \alpha_{N_1}(m_1 n_1) + \alpha_{M_1}(m_1) + \alpha_{M_2}(m_2) - \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\} \\
&= q(td_\alpha)_{\mathtt{G}_1}(m_1) + s(td_\alpha)_{\mathtt{G}_2}(m_2) \\
&- (q-1)T_{\mathtt{G}_1}(m_1) - \max\{T_{\mathtt{G}_1}(m_1), T_{\mathtt{G}_1}(m_1)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\delta)_{\mathtt{G}_1 \oplus \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\delta_{N_1} \oplus \delta_{N_2})((m_1, m_2)(n_1, n_2)) + (\delta_{M_1} \oplus \delta_{M_2})(m_1, m_2) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \max\{\delta_{M_1}(m_1), \delta_{N_2}(m_2 n_2)\} \\
&+ \sum_{m_1 n_1 \in E_1, m_2 = n_2} \max\{\delta_{N_1}(m_1 n_1), \delta_{M_2}(m_2)\} \\
&+ \sum_{m_1 n_1 \notin E_1 and\ m_2 n_2 \in E_2} \max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1), \delta_{N_2}(m_2 n_2)\} \\
&+ \sum_{m_1 n_1 \in E_1 and\ m_2 n_2 \notin E_2} \max\{\delta_{N_1}(m_1 n_1), \delta_{M_2}(m_2), \delta_{M_2}(n_2)\} \\
&+ \min\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\} \\
&= \sum_{m_2 n_2 \in E_2} \delta_{N_2}(m_2 n_2) + \sum_{m_1 n_1 \in E_1} \delta_{N_1}(m_1 n_1)
\end{aligned}
$$

$$+\sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2}\delta_{N_2}(m_2n_2)\}+\sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2}\delta_{N_1}(m_1n_1)$$
$$+\min\{\delta_{M_1}(m_1),\delta_{M_2}(m_2)\}$$
$$=\sum_{m_2n_2\in E_2}\delta_{N_2}(m_2n_2)+\sum_{m_1n_1\in E_1}\delta_{N_1}(m_1n_1)+\sum_{m_1n_1\notin E_1 and\ m_2n_2\in E_2}\delta_{N_2}(m_2n_2)\}$$
$$+\sum_{m_1n_1\in E_1 and\ m_2n_2\notin E_2}\delta_{N_1}(m_1n_1)+\delta_{M_1}(m_1)+\delta_{M_2}(m_2)-\min\{\delta_{M_1}(m_1),\delta_{M_2}(m_2)\}$$
$$=q(td_\delta)_{\mathtt{G}_1}(m_1)+s(td_\delta)_{\mathtt{G}_2}(m_2)$$
$$-(q-1)T_{\mathtt{G}_1}(m_1)-\min\{T_{\mathtt{G}_1}(m_1),T_{\mathtt{G}_1}(m_1)\}$$

In a similar way others four will proved obviously.
where s=| V_1 | -$(d)_{G_1}(m_1)$ and q=| V_2 | -$(d)_{G_2}(m_2)$ □

Example 2.12. In Example 2.6 we have to find the degree and total degree of vertices of $\mathtt{G}_1 \oplus \mathtt{G}_2$ by using Figure 2, Figure 3, and Figure 4.

$$(d_\alpha)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c)=q(d_\alpha)_{\mathtt{G}_1}(a)+s(d_\alpha)_{\mathtt{G}_2}(c)$$

where s=| V_1 | -$(d)_{G_1}(a)$ and q=| V_2 | -$(d)_{G_2}(e)$

$$s=\mid V_1\mid -(d)_{G_1}(a)=2-1=1,\ q=\mid V_2\mid -(d)_{G_2}(e)=2-1=1$$

$$(d_\alpha)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c)=q(d_\alpha)_{\mathtt{G}_1}(a)+s(d_\alpha)_{\mathtt{G}_2}(c)=1(0.4)+1(0.5)=0.4+0.5=0.9$$

$$(d_\beta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c)=q(d_\beta)_{\mathtt{G}_1}(a)+s(d_\beta)_{\mathtt{G}_2}(c)=1(0.2)+1(0.4)=0.2+0.4=0.6$$

$$(d_\gamma)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c)=0.7,\ (d_\delta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c)=-1.1$$

$$(d_\eta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c)=-0.5,\ (d_\theta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c)=-0.7$$

$$So\ (d)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,e)=(0.9,0.6,-1.1,-0.5,-0.7)$$

By applying this technique we can find degree of all vertices in a similar way. Now we will find total degree of vertices. For this select vertex (a,e)

$$(td_\alpha)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c)=q(td_\alpha)_{\mathtt{G}_1}(a)+s(td_\alpha)_{\mathtt{G}_2}(c)$$
$$-(s-1)\alpha_{\mathtt{G}_2}(c)-(q-1)\alpha_{\mathtt{G}_1}(a)-\max\{\alpha_{\mathtt{G}_1}(a),\alpha_{\mathtt{G}_2}(c)\}$$
$$=1(0.7+0.4)+1(0.6+0.5)-(1-1)(0.6)-(1-1)(0.7)$$
$$-\max\{0.6,0.7\}=1(1.1)+1.1-0.7=1.5$$

$$
\begin{aligned}
(td_\delta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c) &= q(td_\delta)_{\mathtt{G}_1}(a) + s(td_\delta)_{\mathtt{G}_2}(c) \\
&- (s-1)\delta_{\mathtt{G}_2}(c) - (q-1)\delta_{\mathtt{G}_1}(a) - \min\{\delta_{\mathtt{G}_1}(a), \delta_{\mathtt{G}_2}(c)\} \\
&= 1(-0.6-0.2) + 1(-0.5-0.3) - (1-1)(-0.5) - (1-1)(-0.6) \\
&- \min\{-0.5,-0.6\} = (-0.8-0.8+0.6 = -1.0
\end{aligned}
$$

$$(td_\beta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c) = 1.0,\ (td_\gamma)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c) = 1.3$$

$$(td_\eta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c) = -1.1,\ (td_\theta)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c) = -1.7$$

$$(td)_{\mathtt{G}_1\oplus\mathtt{G}_2}(a,c) = (1.5, 1.0, 1.3, -1.0, -1.1, -1.7)$$

By applying this technique we can find total degree of all vertices in a similar way.

Definition 2.13. let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ are two bipolar single valued neutrosophic fuzzy graphs defined on $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ respectively. The Residue product of $\mathtt{G}_1$ and $\mathtt{G}_2$ is represented by $\mathtt{G}_1 \bullet \mathtt{G}_2 = (M_1 \bullet M_2, N_1 \bullet N_2)$. Residue product of $\mathtt{G}_1$ *and* $\mathtt{G}_2$ is defined as the following conditions: (i)

$$
\begin{aligned}
(\alpha_{M_1} \bullet \alpha_{M_2})((m_1,m_2)) &= \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\},\ (\beta_{M_1} \bullet \beta_{M_2})((m_1,m_2)) \\
&= \min\{\beta_{M_1}(m_1), \beta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(\gamma_{M_1} \bullet \gamma_{M_2})((m_1,m_2)) &= \min\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\},\ (\delta_{M_1} \bullet \delta_{M_2})((m_1,m_2)) \\
&= \min\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(\eta_{M_1} \bullet \eta_{M_2})((m_1,m_2)) &= \max\{\eta_{M_1}(m_1), \eta_{M_2}(m_2)\},\ (\theta_{M_1} \bullet \theta_{M_2})((m_1,m_2)) \\
&= \max\{\theta_{M_1}(m_1), \theta_{M_2}(m_2)\}
\end{aligned}
$$

$\forall (m_1, m_2) \in (V_1 \times V_2)$

(ii)

$$(\alpha_{N_1} \bullet \alpha_{N_2})((m_1,m_2)(n_1,n_2)) = \alpha_{N_1}(m_1n_1),\ (\beta_{N_1} \bullet \beta_{N_2})((m_1,m_2)(n_1,n_2)) = \beta_{N_1}(m_1n_1)$$

$$(\gamma_{N_1} \bullet \gamma_{N_2})((m_1,m_2)(n_1,n_2)) = \gamma_{N_1}(m_1n_1),\ (\delta_{N_1} \bullet \delta_{N_2})((m_1,m_2)(n_1,n_2)) = \delta_{N_1}(m_1n_1)$$

$$(\eta_{N_1} \bullet \eta_{N_2})((m_1,m_2)(n_1,n_2)) = \eta_{N_1}(m_1n_1),\ (\theta_{N_1} \bullet \theta_{N_2})((m_1,m_2)(n_1,n_2)) = \theta_{N_1}(m_1n_1)$$

$\forall m_1n_1 \in E_1, m_2 \neq n_2.$

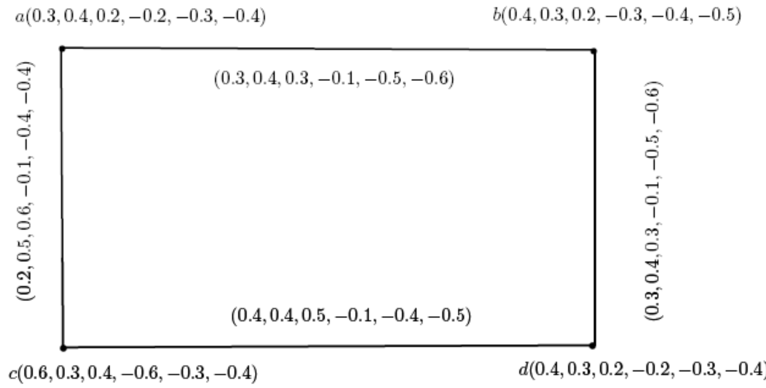

FIGURE 5. G_1

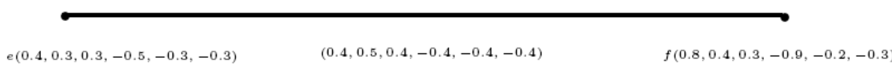

FIGURE 6. G_2

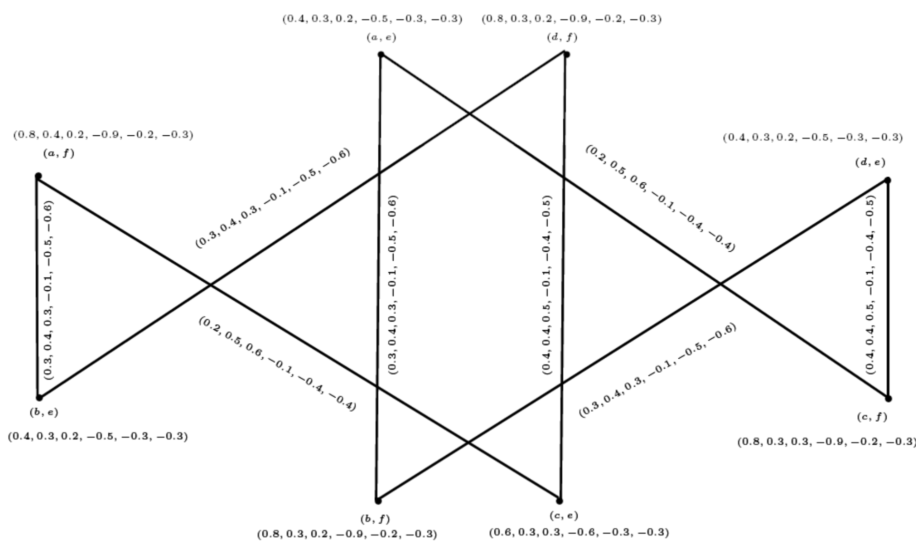

FIGURE 7. $G_1 \bullet G_2$

Example 2.14. Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be two BSVNGs on $V_1 = \{a, b, c, d\}$ *and* $V_2 = \{e, f\}$ respectively which shown in Figure 5 and Figure 6. Also Residue product is shown in Figure 7.

Proposition 2.15. Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be two BSVNGs of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively. Then the Residue product $G_1 \bullet G_2$ of $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a BSVNG.

Proof. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively. Let $(m_1, m_2)(n_1, n_2) \in E_1 \times E_2$ If $m_1 n_1 \in E_1$ *and* $m_2 \neq n_2$ then

$$\begin{aligned}
(\alpha_{N_1} \bullet \alpha_{N_2})((m_1, m_2)(n_1, n_2)) &= \alpha_{N_1}(m_1 n_1) \\
&\leq \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1)\} \\
&\leq \max\{\min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1)\}, \min\{\alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\}\} \\
&= \min\{\max\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1)\}, \max\{\alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\}\} \\
&= \min\{(\alpha_{M_1} \bullet \alpha_{M_2})(m_1, m_2), (\alpha_{M_1} \bullet \alpha_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}
(\beta_{N_1} \bullet \beta_{N_2})((m_1, m_2)(n_1, n_2)) &= \beta_{N_1}(m_1 n_1) \\
&\geq \max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1)\} \\
&\geq \min\{\max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1)\}, \max\{\beta_{M_2}(m_2), \beta_{M_2}(n_2)\}\} \\
&= \max\{\min\{\beta_{M_1}(m_1), \beta_{M_1}(n_1)\}, \min\{\beta_{M_2}(m_2), \beta_{M_2}(n_2)\}\} \\
&= \max\{(\beta_{M_1} \bullet \beta_{M_2})(m_1, m_2), (\beta_{M_1} \bullet \beta_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}
(\gamma_{N_1} \bullet \gamma_{N_2})((m_1, m_2)(n_1, n_2)) &= \gamma_{N_1}(m_1 n_1) \\
&\geq \max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1)\} \\
&\geq \min\{\max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1)\}, \max\{\gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\}\} \\
&= \max\{\min\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1)\}, \min\{\gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\}\} \\
&= \max\{(\gamma_{M_1} \bullet \gamma_{M_2})(m_1, m_2), (\gamma_{M_1} \bullet \gamma_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}
(\delta_{N_1} \bullet \delta_{N_2})((m_1, m_2)(n_1, n_2)) &= \delta_{N_1}(m_1 n_1) \\
&\geq \max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1)\} \\
&\geq \min\{\max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1)\}, \max\{\delta_{M_2}(m_2), \delta_{M_2}(n_2)\}\} \\
&= \max\{\min\{\delta_{M_1}(m_1), \delta_{M_1}(n_1)\}, \min\{\delta_{M_2}(m_2), \delta_{M_2}(n_2)\}\} \\
&= \max\{(\delta_{M_1} \bullet \delta_{M_2})(m_1, m_2), (\delta_{M_1} \bullet \delta_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}
(\eta_{N_1} \bullet \eta_{N_2})((m_1, m_2)(n_1, n_2)) &= \eta_{N_1}(m_1 n_1) \\
&\leq \min\{\eta_{M_1}(m_1), \eta_{M_1}(n_1)\} \\
&\leq \max\{\min\{\eta_{M_1}(m_1), \eta_{M_1}(n_1)\}, \min\{\eta_{M_2}(m_2), \eta_{M_2}(n_2)\}\} \\
&= \min\{\max\{\eta_{M_1}(m_1), \eta_{M_1}(n_1)\}, \max\{\eta_{M_2}(m_2), \eta_{M_2}(n_2)\}\} \\
&= \min\{(\eta_{M_1} \bullet \eta_{M_2})(m_1, m_2), (\eta_{M_1} \bullet \eta_{M_2})(n_1, n_2)\}
\end{aligned}$$

$$\begin{aligned}(\theta_{N_1} \bullet \theta_{N_2})((m_1, m_2)(n_1, n_2)) &= \theta_{N_1}(m_1 n_1)\\ &\leq \min\{\theta_{M_1}(m_1), \theta_{M_1}(n_1)\}\\ &\leq \max\{\min\{\theta_{M_1}(m_1), \theta_{M_1}(n_1)\}, \min\{\theta_{M_2}(m_2), \theta_{M_2}(n_2)\}\}\\ &= \min\{\max\{\theta_{M_1}(m_1), \theta_{M_1}(n_1)\}, \max\{\theta_{M_2}(m_2), \theta_{M_2}(n_2)\}\}\\ &= \min\{(\theta_{M_1} \bullet \theta_{M_2})(m_1, m_2), (\theta_{M_1} \bullet \theta_{M_2})(n_1, n_2)\}\end{aligned}$$

□

Definition 2.16. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs.For any vertex$(m_1, m_2) \in V_1 \times V_2$

$$\begin{aligned}(d_\alpha)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\alpha_{N_1} \bullet \alpha_{N_2})((m_1, m_2)(n_1, n_2))\\ &= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \alpha_{N_1}(m_1 n_1) = (d_\alpha)_{\mathtt{G}_1}(m_1)\end{aligned}$$

$$\begin{aligned}(d_\beta)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\beta_{N_1} \bullet \beta_{N_2})((m_1, m_2)(n_1, n_2))\\ &= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \beta_{N_1}(m_1 n_1) = (d_\beta)_{\mathtt{G}_1}(m_1)\end{aligned}$$

$$\begin{aligned}(d_\gamma)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\gamma_{N_1} \bullet \gamma_{N_2})((m_1, m_2)(n_1, n_2))\\ &= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \gamma_{N_1}(m_1 n_1) = (d_\gamma)_{\mathtt{G}_1}(m_1)\end{aligned}$$

$$\begin{aligned}(d_\delta)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\delta_{N_1} \bullet \delta_{N_2})((m_1, m_2)(n_1, n_2))\\ &= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \delta_{N_1}(m_1 n_1) = (d_\delta)_{\mathtt{G}_1}(m_1)\end{aligned}$$

$$\begin{aligned}(d_\eta)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\eta_{N_1} \bullet \eta_{N_2})((m_1, m_2)(n_1, n_2))\\ &= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \eta_{N_1}(m_1 n_1) = (d_\eta)_{\mathtt{G}_1}(m_1)\end{aligned}$$

$$\begin{aligned}(d_\theta)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\theta_{N_1} \bullet \theta_{N_2})((m_1, m_2)(n_1, n_2))\\ &= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \theta_{N_1}(m_1 n_1) = (d_\theta)_{\mathtt{G}_1}(m_1)\end{aligned}$$

Definition 2.17. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs. For any vertex$(m_1, m_2) \in V_1 \times V_2$

$$
\begin{aligned}
(td_\alpha)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\alpha_{N_1} \bullet \alpha_{N_2})((m_1, m_2)(n_1, n_2)) + (\alpha_{M_1} \bullet \alpha_{M_2})(m_1, m_2)) \\
&= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \alpha_{N_1}(m_1 n_1) + \min\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\} \\
&= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \alpha_{N_1}(m_1 n_1) + \alpha_{M_1}(m_1) + \alpha_{M_2}(m_2) - \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\} \\
&= (td_\alpha)_{\mathtt{G}_1}(m_1) + \alpha_{M_2}(m_2) - \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\beta)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\beta_{N_1} \bullet \beta_{N_2})((m_1, m_2)(n_1, n_2)) + (\beta_{M_1} \bullet \beta_{M_2}(m_1, m_2)) \\
&= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \beta_{N_1}(m_1 n_1) + \max\{\beta_{M_1}(m_1), \beta_{M_2}(m_2)\} \\
&= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \beta_{N_1}(m_1 n_1) + \beta_{M_1}(m_1) + \beta_{M_2}(m_2) - \min\{\beta_{M_1}(M_1), \beta_{M_2}(m_2)\} \\
&= (td_\beta)_{\mathtt{G}_1}(m_1) + \beta_{M_2}(m_2) - \min\{\beta_{M_1}(m_1), \beta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\gamma)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\gamma_{N_1} \bullet \gamma_{N_2})((m_1, m_2)(n_1, n_2)) + (\gamma_{M_1} \bullet \gamma_{M_2}(m_1, m_2)) \\
&= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \gamma_{N_1}(m_1 n_1) + \max\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\} \\
&= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \gamma_{N_1}(m_1 n_1) + \gamma_{M_1}(m_1) + \gamma_{M_2}(m_2) - \min\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\} \\
&= (td_\gamma)_{\mathtt{G}_1}(m_1) + \gamma_{M_2}(m_2) - \min\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\delta)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\delta_{N_1} \bullet \delta_{N_2})((m_1, m_2)(n_1, n_2)) + (\delta_{M_1} \bullet \delta_{M_2})(m_1, m_2)) \\
&= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \delta_{N_1}(m_1 n_1) + \max\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\} \\
&= \sum_{m_1 n_1 \in E_1, m_2 \neq n_2} \delta_{N_1}(m_1 n_1) + \delta_{M_1}(m_1) + \delta_{M_2}(m_2) - \min\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\} \\
&= (td_\delta)_{\mathtt{G}_1}(m_1) + \delta_{M_2}(m_2) - \min\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\eta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\eta_{N_1}\bullet\eta_{N_2})((m_1,m_2)(n_1,n_2)) + (\eta_{M_1}\bullet\eta_{M_2}(m_1,m_2)) \\
&= \sum_{m_1n_1\in E_1, m_2\neq n_2} \eta_{N_1}(m_1n_1) + \min\{\eta_{M_1}(m_1), \eta_{M_2}(m_2)\} \\
&= \sum_{m_1n_1\in E_1, m_2\neq n_2} \eta_{N_1}(m_1n_1) + \eta_{M_1}(m_1) + \eta_{M_2}(m_2) - \max\{I^{-}_{M_1}(m_1), \eta_{M_2}(m_2)\} \\
&= (td_\eta)_{\mathtt{G}_1}(m_1) + \eta_{M_2}(m_2) - \max\{\eta_{M_1}(m_1), \eta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\theta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\theta_{N_1}\bullet\theta_{N_2})((m_1,m_2)(n_1,n_2)) + (\theta_{M_1}\bullet\theta_{M_2}(m_1,m_2)) \\
&= \sum_{m_1n_1\in E_1, m_2\neq n_2} \theta_{N_1}(m_1n_1) + \min\{\theta_{M_1}(m_1), \theta_{M_2}(m_2)\} \\
&= \sum_{m_1n_1\in E_1, m_2\neq n_2} \theta_{N_1}(m_1n_1) + \theta_{M_1}(m_1) + \theta_{M_2}(m_2) - \max\{\theta_{M_1}(m_1), \theta_{M_2}(m_2)\} \\
&= (td_\theta)_{\mathtt{G}_1}(m_1) + \theta_{M_2}(m_2) - \max\{\theta_{M_1}(m_1), \theta_{M_2}(m_2)\}
\end{aligned}
$$

Example 2.18. In Example 2.14 we have to find the degree and total degree of vertices of $\mathtt{G}_1 \bullet \mathtt{G}_2$ by using Figure 5, Figure 6, and Figure 7.

$$(d_\beta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = (d_\beta)_{\mathtt{G}_1}(a) = 0.5 + 0.4 = 0.9$$

$$(d_\eta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = (d_\eta)_{\mathtt{G}_1}(a) = -0.4 - 0.5 = -0.9$$

$$(d_\alpha)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = 0.5,\ (d_\gamma)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = 0.9$$

$$(d_\delta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = -0.2,\ (d_\theta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = -1.0$$

$$(d)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = (0.5, 0.9, 0.9, -0.2, -0.9, -1.0)$$

By applying same method we can find degree of all vertices. Now we are to find total degree of vertices. For this select vertices (a,f)

$$
\begin{aligned}
(td_\beta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) &= (td_\beta)_{\mathtt{G}_1}(a) + \beta_{M_2}(f) - \min\{\beta_{M_1}(a), \beta_{M_2}(f)\} \\
&= (0.5 + 0.4 + 0.4) + 0.8 - \min(0.3, 0.8) \\
&= 1.3 + 0.8 - 0.3 = 1.8
\end{aligned}
$$

$$
\begin{aligned}
(td_\eta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) &= (td_\eta)_{\mathtt{G}_1}(a) + \eta_{M_2}(f) - \max\{\eta_{M_1}(a), \eta_{M_2}(f)\} \\
&= (-0.4 - 0.3 - 0.5) + (-0.2) - \max(-0.3, -0.2) \\
&= -1.2 - 0.2 + 0.2 = -1.2
\end{aligned}
$$

$$(td_\gamma)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = 1.1,\ (td_\delta)_{\mathtt{G}_1\bullet\mathtt{G}_2}(a,f) = -0.4$$

$$(td_{\theta})_{\mathtt{G}_1 \bullet \mathtt{G}_2}(a,f) = -1.4,\ (td_{\alpha})_{\mathtt{G}_1 \bullet \mathtt{G}_2}(a,f) = 0.8$$

$$So\ (td)_{\mathtt{G}_1 \bullet \mathtt{G}_2}(a,f) = (0.8, 1.8, 1.1 - 0.4, -1.2, -1.4)$$

by applying similar method we can find total degree of all others vertices in a similar way.

Definition 2.19. let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ are bipolar single valued neutrosophic fuzzy graphs defined on $G_1 = (V_1, E_1)$ and $G_2 = (V_1, E_2)$ respectively. The maximal product of $\mathtt{G}_1$ and $\mathtt{G}_2$ is represented by $\mathtt{G}_1 * \mathtt{G}_2 = (M_1 * M_2, N_1 \oplus N_2)$. The Maximal product of $\mathtt{G}_1$ *and* $\mathtt{G}_2$ is defined as the following conditions (i)

$$(\alpha_{M_1} * \alpha_{M_2})((m_1, m_2)) = \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\},\ (\beta_{M_1} * \beta_{M_2})((m_1, m_2)) = \min\{\beta_{M_1}(m_1), \beta_{M_2}(m_2)\}$$

$$(\gamma_{M_1} * \gamma_{M_2})((m_1, m_2)) = \min\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\},\ (\delta_{M_1} * \delta_{M_2})((m_1, m_2)) = \min\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\}$$

$$(\eta_{M_1} * \eta_{M_2})((m_1, m_2)) = \max\{\eta_{M_1}(m_1), \eta_{M_2}(m_2)\},\ (\theta_{M_1} * \theta_{M_2})((m_1, m_2)) = \max\{\theta_{M_1}(m_1), \theta_{M_2}(m_2)\}$$

$\forall\ (m_1, m_2) \in (V_1 \times V_2)$

(ii)

$$(\alpha_{M_1} * \alpha_{M_2})((m, m_2)(m, n_2)) = \max\{\alpha_{M_1}(m), \alpha_{N_2}(m_2 n_2)\},\ (\beta_{M_1} * \beta_{M_2})((m, m_2)(m, n_2)) = \min\{\beta_{M_1}(m), \beta_{N_2}(m_2 n_2)\}$$

$$(\gamma_{M_1} * \gamma_{M_2})((m, m_2)(m, n_2)) = \min\{\gamma_{M_{m_1}}(m), \gamma_{N_2}(m_2 n_2)\},\ (\delta_{M_1} * \delta_{M_2})((m, m_2)(m, n_2)) = \min\{\delta_{M_1}(m), \delta_{N_2}(m_2 n_2)\}$$

$$(\eta_{M_1} * \eta_{M_2})((m, m_2)(m, n_2)) = \max\{\eta_{M_1}(m), \eta_{N_2}(m_2 n_2)\},\ (\theta_{M_1} * \theta_{M_2})((m, m_2)(m, n_2)) = \max\{\theta_{M_{m_1}}(m), \theta_{N_2}(m_2 n_2)\}$$

$\forall\ m\ \in\ V_1\ and\ m_2 n_2\ \in\ E_2$

(iii)

$$(\alpha_{M_1} * \alpha_{M_2})((m_1, m)(n_1, m)) = \max\{\alpha_{N_1}(m_1 n_1), \alpha_{M_2}(m)\},\ (\beta_{M_1} * \beta_{M_2})((m_1, m)(n_1, m)) = \min\{\beta_{N_1}(m_1 n_1), \beta_{M_2}(m)\}$$

$$(\gamma_{M_1} * \gamma_{M_2})((m_1, m)(n_1, m)) = \min\{\gamma_{N_1}(m_1 n_1), \gamma_{M_2}(m)\},\ (\delta_{M_1} * \delta_{M_2})((m_1, m)(n_1, m)) = \min\{\delta_{N_1}(m_1 n_1), \delta_{M_2}(m)\}$$

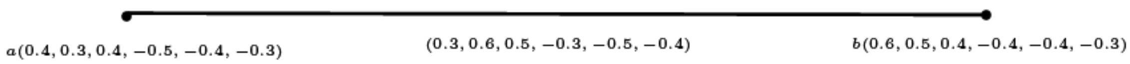

FIGURE 8. G_1

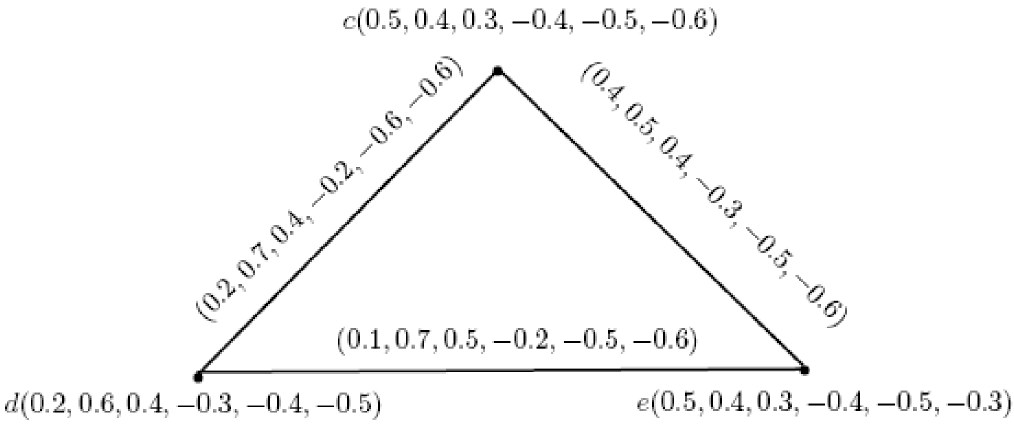

FIGURE 9. G_2

$$(\eta_{M_1} * \eta_{M_2})((m_1,m)(n_1,m)) = \max\{\eta_{N_1}(m_1n_1), \eta_{M_2}(m)\},\ (\theta_{M_1} * \theta_{M_2})((m_1,m)(n_1,m)) = \max\{\theta_{N_1}(m_1n_1), \theta_{M_2}(m)\}$$

$\forall\, m \in V_2$ *and* $m_1n_1 \in E_1$

Example 2.20. Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be two BSVNGs on $V_1 = \{a, b\}$ *and* $V_2 = \{c, d, e\}$ respectively which shown in Figure 8 and Figure 9. Also maximal product is shown in Figure 10.

Proposition 2.21. Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be two BSVNGs of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively. Then then maximal product $G_1 * G_2$ of $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a BSVNG.

Proof. Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be two BSVNGs of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively. Then the Maximal product $G_1 * G_2$ of $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ can be proved. Let $(m_1, m_2)(n_1, n_2) \in E_1 \times E_2$

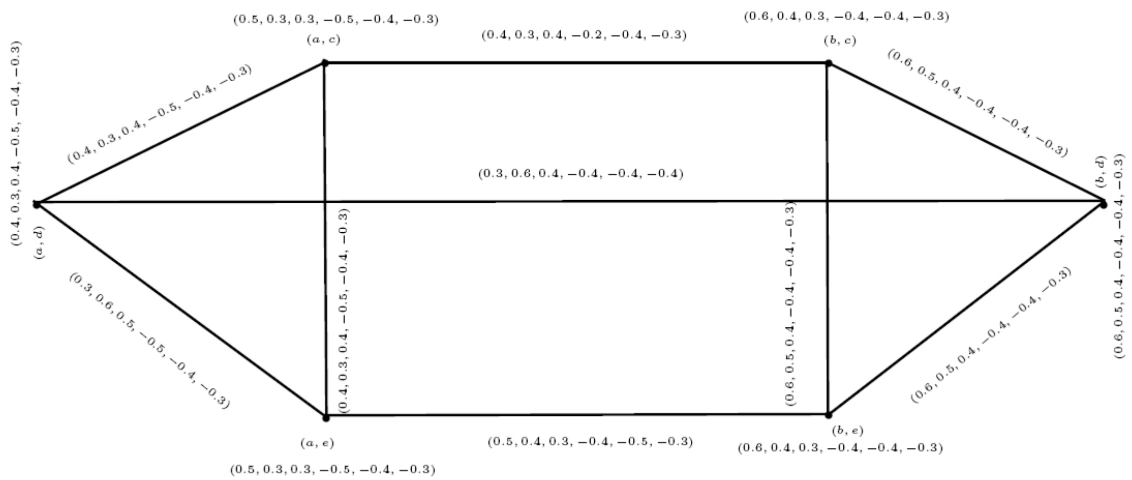

FIGURE 10. $G_1 * G_2$

(i) If $m_1 = n_1 = m$

$$\begin{aligned}(\alpha_{N_1} * \alpha_{N_2})((m, m_2)(m, n_2)) &= \max\{\alpha_{M_1}(m), \alpha_{N_2}(m_2n_2)\}\\ &\leq \max\{\alpha_{M_1}(m), \min\{\alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\}\}\\ &= \min\{\max\{\{\alpha_{M_1}(m), \alpha_{M_2}(m_2)\}, \max\{\{\alpha_{M_1}(m), \alpha_{M_2}(n_2)\}\}\\ &= \min\{(\alpha_{M_1} * \alpha_{M_2})(m, m_2), (\alpha_{M_1} * \alpha_{M_2})(m, n_2)\}\end{aligned}$$

$$\begin{aligned}(\beta_{N_1} * \beta_{N_2})((m, m_2)(m, n_2)) &= \min\{\beta_{M_1}(m), \beta_{N_2}(m_2n_2)\}\\ &\geq \min\{\beta_{M_1}(m), \max\{\beta_{M_2}(m_2), \beta_{M_2}(n_2)\}\}\\ &= \max\{\min\{\{\beta_{M_1}(m), \beta_{M_2}(m_2)\}, \min\{\{\beta_{M_1}(m), \beta_{M_2}(n_2)\}\}\\ &= \max\{(\beta_{M_1} * \beta_{M_2})(m, m_2), (\beta_{M_1} * \beta_{M_2})(m, n_2)\}\end{aligned}$$

$$\begin{aligned}(\gamma_{N_1} * \gamma_{N_2})((m, m_2)(m, n_2)) &= \min\{\gamma_{M_1}(m), \gamma_{N_2}(m_2n_2)\}\\ &\geq \min\{\gamma_{M_1}(m), \max\{\gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\}\}\\ &= \max\{\min\{\{\gamma_{M_1}(m), \gamma_{M_2}(m_2)\}, \min\{\{\gamma_{M_1}(m), \gamma_{M_2}(n_2)\}\}\\ &= \max\{(\gamma_{M_1} * \gamma_{M_2})(m, m_2), (\gamma_{M_1} * \gamma_{M_2})(m, n_2)\}\end{aligned}$$

$$\begin{aligned}(\delta_{N_1} * \delta_{N_2})((m,m_2)(m,n_2)) &= \min\{\delta_{M_1}(m), \delta_{N_2}(m_2n_2)\}\\ &\geq \min\{\delta_{M_1}(m), \max\{\delta_{M_2}(m_2), \delta_{M_2}(n_2)\}\}\\ &= \max\{\min\{\{\delta_{M_1}(m), \delta_{M_2}(m_2)\}, \min\{\{\delta_{M_1}(m), \delta_{M_2}(n_2)\}\}\\ &= \max\{(\delta_{M_1} * \delta_{M_2})(m,m_2), (\delta_{M_1} * \delta_{M_2})(m,n_2)\}\end{aligned}$$

$$\begin{aligned}(\eta_{N_1} * \eta_{N_2})((m,m_2)(m,n_2)) &= \max\{\eta_{M_1}(m), \eta_{N_2}(m_2n_2)\}\\ &\leq \max\{\eta_{M_1}(m), \min\{\eta_{M_2}(m_2), \eta_{M_2}(n_2)\}\}\\ &= \min\{\max\{\{\eta_{M_1}(m), \eta_{M_2}(m_2)\}, \max\{\{\eta_{M_1}(m), \eta_{M_2}(n_2)\}\}\\ &= \min\{(\eta_{M_1} * \eta_{M_2})(m,m_2), (\eta_{M_1} * \eta_{M_2})(m,n_2)\}\end{aligned}$$

$$\begin{aligned}(\theta_{N_1} * \theta_{N_2})((m,m_2)(m,n_2)) &= \max\{\theta_{M_1}(m), \theta_{N_2}(m_2n_2)\}\\ &\leq \max\{\theta_{M_1}(m), \min\{\theta_{M_2}(m_2), \theta_{M_2}(n_2)\}\}\\ &= \min\{\max\{\{\theta_{M_1}(m), \theta_{M_2}(m_2)\}, \max\{\{\theta_{M_1}(m), \theta_{M_2}(n_2)\}\}\\ &= \min\{(\theta_{M_1} * \theta_{M_2})(m,m_2), (\theta_{M_1} * \theta_{M_2})(m,n_2)\}\end{aligned}$$

(ii) If $m_2 = n_2 = m$

$$\begin{aligned}(\alpha_{N_1} * \alpha_{N_2})((m_1,m)(n_1,m)) &= \max\{\alpha_{N_1}(m_1n_1), \alpha_{M_2}(m)\}\\ &\leq \max\{\min\{\alpha_{N_1}(m_1n_1), \alpha_{M_2}(m)\}\\ &= \min\{\max\{\{\alpha_{N_1}(m_1), \alpha_{M_2}(m)\}, \max\{\{\alpha_{M_1}(n_1), \alpha_{M_2}(m)\}\}\\ &= \min\{(\alpha_{M_1} * \alpha_{M_2})(m_1,m), (\alpha_{M_1} * \alpha_{M_2})(n_1,m)\}\end{aligned}$$

$$\begin{aligned}(\beta_{N_1} * \beta_{N_2})((m_1,m)(n_1,m)) &= \min\{\beta_{N_1}(m_1n_1), \beta_{M_2}(m)\}\\ &\geq \min\{\max\{\beta_{N_1}(m_1n_1), \beta_{M_2}(m)\}\\ &= \max\{\min\{\{\beta_{N_1}(m_1), \beta_{M_2}(m)\}, \min\{\{\beta_{M_1}(n_1), \beta_{M_2}(m)\}\}\\ &= \max\{(\beta_{M_1} * \beta_{M_2})(m_1,m), (\beta_{M_1} * \beta_{M_2})(n_1,m)\}\end{aligned}$$

$$\begin{aligned}(\gamma_{N_1} * \gamma_{N_2})((m_1, m)(n_1, m)) &= \min\{\gamma_{N_1}(m_1n_1), \gamma_{M_2}(m)\}\\ &\geq \min\{\max\{\gamma_{N_1}(m_1n_1), \gamma_{M_2}(m)\}\\ &= \max\{\min\{\{\gamma_{N_1}(m_1), \gamma_{M_2}(m)\}, \min\{\{\gamma_{M_1}(n_1), \gamma_{M_2}(m)\}\}\\ &= \max\{(\gamma_{M_1} * \gamma_{M_2})(m_1, m), (\gamma_{M_1} * \gamma_{M_2})(n_1, m)\}\end{aligned}$$

$$\begin{aligned}(\delta_{N_1} * \delta_{N_2})((m_1, m)(n_1, m)) &= \min\{\delta_{N_1}(m_1n_1), \delta_{M_2}(m)\}\\ &\geq \min\{\max\{\delta_{N_1}(m_1n_1), \delta_{M_2}(m)\}\\ &= \max\{\min\{\{\delta_{N_1}(m_1), \delta_{M_2}(m)\}, \min\{\{\delta_{M_1}(n_1), \delta_{M_2}(m)\}\}\\ &= \max\{(\delta_{M_1} * \delta_{M_2})(m_1, m), (\delta_{M_1} * \delta_{M_2})(n_1, m)\}\end{aligned}$$

$$\begin{aligned}(\eta_{N_1} * \eta_{N_2})((m_1, m)(n_1, m)) &= \max\{\eta_{N_1}(m_1n_1), \eta_{M_2}(m)\}\\ &\leq \max\{\min\{\eta_{N_1}(m_1n_1), \eta_{M_2}(m)\}\\ &= \min\{\max\{\{\eta_{N_1}(m_1), \eta_{M_2}(m)\}, \max\{\{\eta_{M_1}(n_1), \eta_{M_2}(m)\}\}\\ &= \min\{(\eta_{M_1} * \eta_{M_2})(m_1, m), (\eta_{M_1} * \eta_{M_2})(n_1, m)\}\end{aligned}$$

$$\begin{aligned}(\theta_{N_1} * \theta_{N_2})((m_1, m)(n_1, m)) &= \max\{\theta_{N_1}(m_1n_1), \theta_{M_2}(m)\}\\ &\leq \max\{\min\{\theta_{N_1}(m_1n_1), \theta_{M_2}(m)\}\\ &= \min\{\max\{\{\theta_{N_1}(m_1), \theta_{M_2}(m)\}, \max\{\{\theta_{M_1}(n_1), \theta_{M_2}(m)\}\}\\ &= \min\{(\theta_{M_1} * \theta_{M_2})(m_1, m), (\theta_{M_1} * \theta_{M_2})(n_1, m)\}\end{aligned}$$

□

Definition 2.22. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs. $\forall (m_1, m_2) \in V_1 \times V_2$

$$\begin{aligned}(d_\alpha)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\alpha_{N_1} * \alpha_{N_2})((m_1, m_2)(n_1, n_2))\\ &= \sum_{m_1=n_1, m_2n_2 \in E_2} \max\{\alpha_{M_1}(m_1), \alpha_{N_2}(m_2n_2)\}\\ &+ \sum_{m_1n_1 \in E_1, m_2=n_2} \max\{\alpha_{N_1}(m_1n_1), \alpha_{M_2}(m_2)\}\end{aligned}$$

$$
\begin{aligned}
(d_\beta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\beta_{N_1}*\beta_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\beta_{M_1}(m_1), \beta_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\beta_{N_1}(m_1n_1), \beta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(d_\gamma)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\gamma_{N_1}*\gamma_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\gamma_{M_1}(m_1), \gamma_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\gamma_{N_1}(m_1n_1), \gamma_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(d_\delta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\delta_{N_1}*\delta_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \min\{\delta_{M_1}(m_1), \delta_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \min\{\delta_{N_1}(m_1n_1), \delta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(d_\eta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\eta_{N_1}*\eta_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \max\{\eta_{M_1}(m_1), \eta_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \max\{\eta_{N_1}(m_1n_1), \eta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(d_\theta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\theta_{N_1}*\theta_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1, m_2n_2\in E_2} \max\{\theta_{M_1}(m_1), \theta_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1, m_2=n_2} \max\{\theta_{N_1}(m_1n_1), \theta_{M_2}(m_2)\}
\end{aligned}
$$

Theorem 2.23. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ are two BSVNGs. If $\alpha_{M_1} \geq \alpha_{N_2}, \beta_{M_1} \leq \beta_{N_2}, \gamma_{M_1} \leq \gamma_{N_2}$ and $\alpha_{M_2} \geq \alpha_{N_1}, \beta_{M_2} \leq \beta_{N_1}, \gamma_{M_2} \leq \gamma_{N_1}$. Also If $\delta_{M_1} \leq \delta_{N_2}, \eta_{M_1} \geq \eta_{N_2}, \theta_{M_1} \geq \theta_{N_2}$ and $\delta_{M_2} \leq \delta_{N_1}, \eta_{M_2} \geq \eta_{N_1}, \theta_{M_2} \geq \theta_{N_1}$ Then for every $\forall (m_1, m_2) \in V_1 \times V_2$
$(d_\alpha)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) = (d)_{G_2}(m_2)\alpha_{M_1}(m_1) + (d)_{G_1}(m_1)\alpha_{M_2}(m_2)$
$(d_\beta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) = (d)_{\mathtt{G}_2}(m_2)\beta_{M_1}(m_1) + (d)_{G_1}(m_1)\beta_{M_2}(m_2)$
$(d_\gamma)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) = (d)_{G_2}(m_2)\gamma_{M_1}(m_1) + (d)_{G_1}(m_1)\gamma_{M_2}(m_2)$
$(d_\delta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) = (d)_{G_2}(m_2)\delta_{M_1}(m_1) + (d)_{G_1}(m_1)\delta_{M_2}(m_2)$

$(d_\eta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2)=(d)_{\mathtt{G}_2}(m_2)\eta_{M_1}(m_1)+(d)_{G_1}(m_1)\eta_{M_2}(m_2)$
$(d_\theta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2)=(d)_{G_2}(m_2)\theta_{M_1}(m_1)+(d)_{G_1}(m_1)\theta_{M_2}(m_2)$

Proof.

$$\begin{aligned}
(d_\alpha)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\alpha_{N_1}*\alpha_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{\alpha_{M_1}(m_1),\alpha_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \max\{\alpha_{N_1}(m_1n_1),\alpha_{M_2}(m_2)\}\\
&= \sum_{m_2n_2\in E_2,m_1=n_1} \alpha_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} \alpha_{N_1}(m_1n_1)\\
&= (d)_{G_2}(m_2)\alpha_{M_1}(m_1)+(d)_{G_1}(m_1)\alpha_{M_2}(m_2)
\end{aligned}$$

$$\begin{aligned}
(d_\delta)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\delta_{N_1}*\delta_{N_2})((m_1,m_2)(n_1,n_2))\\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \min\{\delta_{M_1}(m_1),\delta_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \min\{\delta_{N_1}(m_1n_1),\delta_{M_2}(m_2)\}\\
&= \sum_{m_2n_2\in E_2,m_1=n_1} \delta_{N_2}(m_2n_2) + \sum_{m_1n_1\in E_1,m_2=n_2} \delta_{N_1}(m_1n_1)\\
&= (d)_{G_2}(m_2)\delta_{M_1}(m_1)+(d)_{G_1}(m_1)\delta_{M_2}(m_2)
\end{aligned}$$

In a similar way others four will proved obviously. □

Definition 2.24. Let $\mathtt{G}_1=(M_1,N_1)$ and $\mathtt{G}_2=(M_2,N_2)$ be two BSVNGs. $\forall(m_1,m_2)\in\ V_1\times V_2$

$$\begin{aligned}
(td_\alpha)_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\alpha_{N_1}*\alpha_{N_2})((m_1,m_2)(n_1,n_2)) + (\alpha_{M_1}*\alpha_{M_2}(m_1,m_2)\\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{\alpha_{M_1}(m_1),\alpha_{N_2}(m_2n_2)\}\\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \max\{\alpha_{N_1}(m_1n_1),\alpha_{M_2}(m_2)\}\\
&+ \max\{\alpha_{M_1}(m_1),\alpha_{M_2}(m_2)\}
\end{aligned}$$

$$
\begin{aligned}
(td_{\beta})_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\beta_{N_1}*\beta_{N_2})((m_1,m_2)(n_1,n_2)) + (\beta_{M_1}*\beta_{M_2}(m_1,m_2) \\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \min\{\beta_{M_1}(m_1),\beta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \min\{\beta_{N_1}(m_1n_1),\beta_{M_2}(m_2)\} \\
&+ \min\{\beta_{M_1}(m_1),\beta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_{\gamma})_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\gamma_{N_1}*\gamma_{N_2})((m_1,m_2)(n_1,n_2)) + (\gamma_{M_1}*\gamma_{M_2}(m_1,m_2) \\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \min\{\gamma_{M_1}(m_1),\gamma_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \min\{\gamma_{N_1}(m_1n_1,\gamma_{M_2}(m_2)\} \\
&+ \max\{\gamma_{M_1}(m_1),\gamma_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_{\delta})_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\delta_{N_1}*\delta_{N_2})((m_1,m_2)(n_1,n_2)) + (\delta_{M_1}*\delta_{M_2}(m_1,m_2) \\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \min\{\delta_{M_1}(m_1),\delta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \min\{\delta_{N_1}(m_1n_1),\delta_{M_2}(m_2)\} \\
&+ \min\{\delta_{M_1}(m_1),\delta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_{\eta})_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\eta_{N_1}*\eta_{N_2})((m_1,m_2)(n_1,n_2)) + (\eta_{M_1}*\eta_{M_2}(m_1,m_2) \\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{\eta_{M_1}(m_1),\eta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \max\{\eta_{N_1}(m_1n_1),\eta_{M_2}(m_2)\} \\
&+ \max\{\eta_{M_1}(m_1),\eta_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_{\theta})_{\mathtt{G}_1*\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\theta_{N_1}*\theta_{N_2})((m_1,m_2)(n_1,n_2)) + (\theta_{M_1}*\theta_{M_2}(m_1,m_2) \\
&= \sum_{m_1=n_1,m_2n_2\in E_2} \max\{\theta_{M_1}(m_1),\theta_{N_2}(m_2n_2)\} \\
&+ \sum_{m_1n_1\in E_1,m_2=n_2} \max\{\theta_{N_1}(m_1n_1,\theta_{M_2}(m_2)\} \\
&+ \max\{\theta_{M_1}(m_1),\theta_{M_2}(m_2)\}
\end{aligned}
$$

Theorem 2.25. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs. If $\alpha_{M_1} \geq \alpha_{N_2}, \beta_{M_1} \leq \beta_{N_2}, \gamma_{M_1} \leq \gamma_{N_2}$ and $\alpha_{M_2} \geq \alpha_{N_1}, \beta_{M_2} \leq \beta_{N_1}, \gamma_{M_2} \leq \gamma_{N_1}$. Also If $\delta_{M_1} \leq \delta_{N_2}, \eta_{M_1} \geq \eta_{N_2}, \theta_{M_1} \geq \theta_{N_2}$ and $\delta_{M_2} \leq \delta_{N_1}, \eta_{M_2} \geq \eta_{N_1}, \theta_{M_2} \geq \theta_{N_1}$ Then for every $\forall(m_1, m_2) \in \ V_1 \times V_2$

$(d_\alpha)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) = (d)_{G_2}(m_2)\alpha_{M_1}(m_1) + (d)_{G_1}(m_1)\alpha_{M_2}(m_2)$

$(d_\beta)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) = (d)_{\mathtt{G}_2}(m_2)\beta_{M_1}(m_1) + (d)_{G_1}(m_1)\beta_{M_2}(m_2)$

$(d_\gamma)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) = (d)_{G_2}(m_2)\gamma_{M_1}(m_1) + (d)_{G_1}(m_1)\gamma_{M_2}(m_2)$

$(d_\delta)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) = (d)_{G_2}(m_2)\delta_{M_1}(m_1) + (d)_{G_1}(m_1)\delta_{M_2}(m_2)$

$(d_\eta)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) = (d)_{\mathtt{G}_2}(m_2)\eta_{M_1}(m_1) + (d)_{G_1}(m_1)\eta_{M_2}(m_2)$

$(d_\theta)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) = (d)_{G_2}(m_2)\theta_{M_1}(m_1) + (d)_{G_1}(m_1)\theta_{M_2}(m_2)$

Proof.

$$
\begin{aligned}
(td_\alpha)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\alpha_{N_1} * \alpha_{N_2})((m_1, m_2)(n_1, n_2)) + (\alpha_{M_1} * \alpha_{M_2})(m_1, m_2) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \max\{\alpha_{M_1}(m_1), \alpha_{N_2}(m_2 n_2)\} \\
&+ \sum_{m_1 n_1 \in E_1, m_2 = n_2} \max\{\alpha_{N_1}(m_1 n_1), \alpha_{M_2}(m_2)\} \\
&+ \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\} \\
&= \sum_{m_2 n_2 \in E_2, m_1 = n_1} \alpha_{N_2}(m_2 n_2) + \sum_{m_1 n_1 \in E_1, m_2 = n_2} \alpha_{N_1}(m_1 n_1) \\
&+ \max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\} \\
&= (d)_{G_2}(m_2)\alpha_{M_1}(m_1) + (d)_{G_1}(m_1)\alpha_{M_2}(m_2) + max\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\delta)_{\mathtt{G}_1 * \mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2) \in E_1 \times E_2.} (\delta_{N_1} * \delta_{N_2})((m_1, m_2)(n_1, n_2)) + (\delta_{M_1} * \delta_{M_2})(m_1, m_2) \\
&= \sum_{m_1 = n_1, m_2 n_2 \in E_2} \min\{\delta_{M_1}(m_1), \delta_{N_2}(m_2 n_2)\} \\
&+ \sum_{m_1 n_1 \in E_1, m_2 = n_2} \min\{\delta_{N_1}(m_1 n_1), \delta_{M_2}(m_2)\} \\
&+ \min\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\} \\
&= \sum_{m_2 n_2 \in E_2, m_1 = n_1} \delta_{N_2}(m_2 n_2) + \sum_{m_1 n_1 \in E_1, m_2 = n_2} \delta_{N_1}(m_1 n_1) \\
&+ min\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\} \\
&= (d)_{G_2}(m_2)\delta_{M_1}(m_1) + (d)_{G_1}(m_1)\delta_{M_2}(m_2) + min\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\}
\end{aligned}
$$

In a similar way others four will proved obviously. □

Example 2.26. In Example 2.20 we have to find the degree and total degree of vertices of $\mathtt{G}_1 * \mathtt{G}_2$ by using Figure 8, Figure 9, and Figure 10. Select the vertex (e,a).

$$(d_\alpha)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = (d)_{G_2}(c)\alpha_{M_1}(a) + (d)_{G_1}(a)\alpha_{M_2}(c)$$
$$= 2(0.4) + 1(0.5) = 0.8 + 0.5 = 1.3$$

$$(td_\delta)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = (d)_{G_2}(c)\delta_{M_1}(a) + (d)_{G_1}(a)\delta_{M_2}(c)$$
$$= 2(-0.5) + 1(-0.4) = -1.0 - 0.4 = -1.4$$

, $(d_\beta)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = 1.0$, $(d_\gamma)_{\mathtt{G}_1*\mathtt{G}_2}(e,a) = 1.1$, $(td_\eta)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = -1.3$, $(td_\theta)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = -1.2$. By applying the same method we can find the degree of all vertices.now we are find the total degree of vertices in maximal product. For this select the same vertex (e,a).

$$(td_\alpha)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = (d)_{G_2}(c)\alpha_{M_1}(a) + (d)_{G_1}(a)\alpha_{M_2}(c) + \max\{\alpha_{M_1}(a), \alpha_{M_2}(c)\}$$
$$= 2(0.4) + 1(0.5) + \max(0.4, 0.5) = 0.8 + 0.5 + 0.5 = 1.8$$

$$(td_\theta)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = (d)_{G_2}(c)\theta_{M_1}(a) + (d)_{G_1}(a)\theta_{M_2}(c) + \min\{\theta_{M_1}(a), \theta_{M_2}(c)\}$$
$$= 2(-0.3) + 1(-0.6) + \min(-0.3, -0.6) = -0.6 - 0.6 - 0.6 = -1.8$$

$(td_\beta)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = 1.3$,$(td_\gamma)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = 1.4$, $(td_\delta)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = -1.8$,$(td_\eta)_{\mathtt{G}_1*\mathtt{G}_2}(a,c) = -1.8$. By applying same method or technique we can find all other vertices total degree.

Definition 2.27. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ are two bipolar single valued neutrosophic fuzzy graphs defined on $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ respectively. The rejection of $\mathtt{G}_1$ and $\mathtt{G}_2$ is represented by $\mathtt{G}_1|\mathtt{G}_2 = (M_1|M_2, N_1|N_2)$. Rejection of $\mathtt{G}_1 and\ \mathtt{G}_2$ is defined as the following conditions:

(i)

$$(\alpha_{M_1}|\alpha_{M_2})((m_1, m_2)) = \min\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2)\},\ (\beta_{M_1}|\beta_{M_2})((m_1, m_2)) = \max\{\beta_{M_1}(m_1), \beta_{M_2}(m_2)\}$$

$$(\gamma_{M_1}|\gamma_{M_2})((m_1, m_2)) = \max\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2)\},\ (\delta_{M_1}|\delta_{M_2})((m_1, m_2)) = \max\{\delta_{M_1}(m_1), \delta_{M_2}(m_2)\}$$

$$(\eta_{M_1}|\eta_{M_2})((m_1, m_2)) = \min\{\eta_{M_1}(m_1), \eta_{M_2}(m_2)\},\ (\theta_{M_1}|\theta_{M_2})((m_1, m_2)) = \min\{\theta_{M_1}(m_1), \theta_{M_2}(m_2)\}$$

$\forall\ (m_1, m_2) \in (V_1 \times V_2)$.

(ii)

$$(\alpha_{N_1}|\alpha_{N_2})((m, m_2)(m, n_2)) = \min\{\alpha_{M_1}(m), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\},\ (\beta_{N_1}|\beta_{N_2})((m, m_2)(m, n_2))$$
$$= \max\{\beta_{M_1}(m), \beta_{M_2}(m_2), \beta_{M_2}(n_2)\}$$

$(\gamma_{N_1}|\gamma_{N_2})((m,m_2)(m,n_2)) = \max\{\gamma_{M_1}(m), \gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\},\ (\delta_{N_1}|\delta_{N_2})((m,m_2)(m,n_2))$
$= \max\{\delta_{M_1}(m), \delta_{M_2}(m_2), \delta_{M_2}(n_2)\}$

$(\eta_{N_1}|\eta_{N_2})((m,m_2)(m,n_2)) = \min\{\eta_{M_1}(m), \eta_{M_2}(m_2), \eta_{M_2}(n_2)\},\ (\theta_{N_1}|\theta_{N_2})((m,m_2)(m,n_2))$
$= \min\{\theta_{M_1}(m), \theta_{M_2}(m_2), \theta_{M_2}(n_2)\}$

$\forall\ m \in V_2$ *and* $m_2n_2 \notin E_2$.

(iii)

$(\alpha_{N_1}|\alpha_{N_2})((m,m_2)(m,n_2)) = \min\{\alpha_{M_1}(m), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\},\ (\beta_{N_1}|\beta_{N_2})((m,m_2)(m,n_2))$
$= \max\{\beta_{M_1}(m), \beta_{M_2}(m_2), \beta_{M_2}(n_2)\}$

$(\gamma_{N_1}|\gamma_{N_2})((m,m_2)(m,n_2)) = \max\{\gamma_{M_1}(m), \gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\},\ (\delta_{N_1}|\delta_{N_2})((m,m_2)(m,n_2))$
$= \max\{\delta_{M_1}(m), \delta_{M_2}(m_2), \delta_{M_2}(n_2)\}$

$(\eta_{N_1}|\eta_{N_2})((m,m_2)(m,n_2)) = \min\{\eta_{M_1}(m), \eta_{M_2}(m_2), \eta_{M_2}(n_2)\},\ (\theta_{N_1}|\theta_{N_2})((m,m_2)(m,n_2))$
$= \min\{\theta_{M_1}(m), \theta_{M_2}(m_2), \theta_{M_2}(n_2)\}$

$\forall\ z \in V_2$ *and* $m_1n_1 \notin E_1$.

(i∨) $(\alpha_{N_1}|\alpha_{N_2})((m_1,m_2)(n_1,n_2)) = \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\}$,
$(\beta_{N_1}|\beta_{N_2})((m_1,m_2)(n_1,n_2)) = \max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1), \beta_{M_2}(m_2), \alpha_{N_2}(n_2)\}$, $(\gamma_{N_1}|\gamma_{N_2})((m_1,m_2)(n_1,n_2)) = \max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1), \gamma_{M_2}(m_2), \alpha_{M_2}(n_2)\}$,
$(\delta_{N_1}|\delta_{N_2})((m_1,m_2)(n_1,n_2)) = \max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1), \delta_{M_2}(m_2), \delta_{M_2}(n_2)\}$
,
$(\eta_{N_1}|\eta_{N_2})((m_1,m_2)(n_1,n_2)) = \min\{\eta_{M_1}(m_1), \eta_{M_1}(n_1), \eta_{M_2}(m_2), \delta_{N_2}(n_2)\}$
,
$(\theta_{N_1}|\theta_{N_2})((m_1,m_2)(n_1,n_2)) = \min\{\theta_{M_1}(m_1), \theta_{M_1}(n_1), \theta_{M_2}(m_2), \delta_{M_2}(n_2)\}$

$\forall\ m_1n_1 \notin E_1$ and $m_2n_2 \notin E_2$.

Example 2.28. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs on $V_1 = \{a, b, c, d\}$ *and* $V_2 = \{e, f\}$, respectively which shown in Figure 11 and Figure 12. Also rejection shown in Figure 13.

Proposition 2.29. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, respectively. Then the rejection $\mathtt{G}_1|\mathtt{G}_2$ of $\mathtt{G}_1 = (V_1, E_1)$ and $\mathtt{G}_2 = (V_2, E_2)$ is a BSVNG.

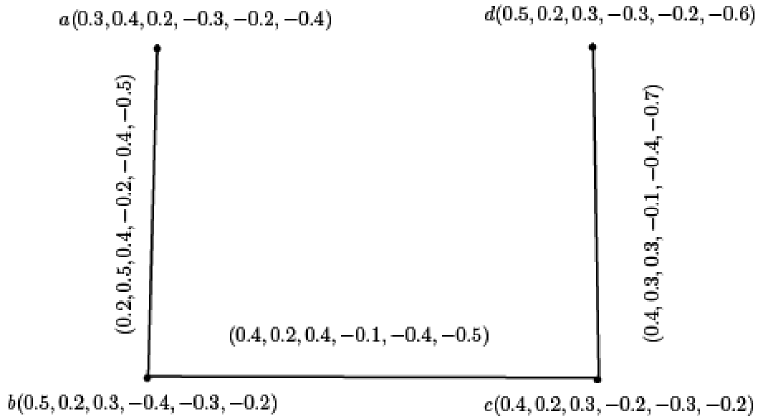

FIGURE 11. $\mathtt{G}_1$

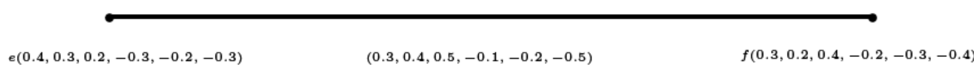

FIGURE 12. $\mathtt{G}_2$

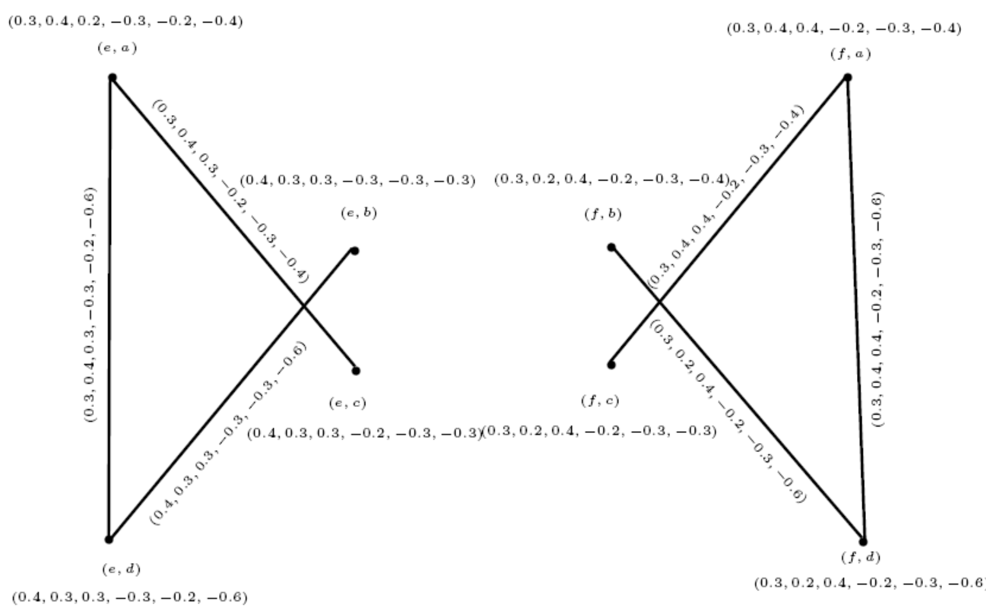

FIGURE 13. $\mathtt{G}_1 \mid \mathtt{G}_2$

Proof. Suppose that $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ respectively. Then for $(m_1, m_2)(n_1, n_2) \in E_1 \times E_2$.

(i) If $m_1 = n_1, m_2n_2 \notin E_2$

$$\begin{aligned}(\beta_{N_1}|\beta_{N_2})((m_1,m_2)(n_1,n_2)) &= \max\{\beta_{M_1}(m_1),\beta_{M_2}(m_2),\beta_{M_2}(n_2)\}\\ &= \max\{\max\{\beta_{M_1}(m_1),\beta_{M_2}(m_2)\},\max\{\beta_{M_1}(n_1),\beta_{M_2}(n_2)\}\}\\ &= \max\{(\beta_{M_1}|\beta_{M_2})(m_1,m_2),(\beta_{M_1}|\beta_{M_2})(n_1,n_2)\}\end{aligned}$$

$$\begin{aligned}(\eta_{N_1}|\eta_{N_2})((m_1,m_2)(n_1,n_2)) &= \min\{\eta_{M_1}(m_1),\eta_{M_2}(m_2),\eta_{M_2}(n_2)\}\\ &= \min\{\min\{\eta_{M_1}(m_1),\eta_{M_2}(m_2)\},\min\{\eta_{M_1}(n_1),\eta_{M_2}(n_2)\}\}\\ &= \min\{(\eta_{M_1}|\eta_{M_2})(m_1,m_2),(\eta_{M_1}|\eta_{M_2})(n_1,n_2)\}\end{aligned}$$

In a similar way others four will proved obviously.

(ii) If $m_2 = n_2, m_1n_1 \notin E_1$

$$\begin{aligned}(\alpha_{N_1}|\alpha_{N_2})((m_1,m_2)(n_1,n_2)) &= \min\{\alpha_{M_1}(m_1),\alpha_{M_1}(n_1),\alpha_{M_2}(m_2)\}\\ &= \min\{\min\{\alpha_{M_1}(m_1),\alpha_{M_2}(m_2)\},\min\{\alpha_{M_1}(n_1),\alpha_{M_2}(n_2)\}\}\\ &= \min\{(\alpha_{M_1}|\alpha_{M_2})(m_1,m_2),(\alpha_{M_1}|\alpha_{M_2})(n_1,n_2)\}\end{aligned}$$

$$\begin{aligned}(\delta_{N_1}|\delta_{N_2})((m_1,m_2)(n_1,n_2)) &= \max\{\delta_{M_1}(m_1),\delta_{M_1}(n_1),\delta_{M_2}(m_2)\}\\ &= \max\{\max\{\delta_{M_1}(m_1),\delta_{M_2}(m_2)\},\max\{\delta_{M_1}(n_1),\delta_{M_2}(n_2)\}\}\\ &= \max\{(\delta_{M_1}|\delta_{M_2})(m_1,m_2),(\delta_{M_1}|\delta_{M_2})(n_1,n_2)\}\end{aligned}$$

In a similar way others four will proved obviously.

(iii) If $m_1n_1 \notin E_1 and\ m_2n_2 \notin E_2$

$$\begin{aligned}(\gamma_{N_1}|\gamma_{N_2})((m_1,m_2)(n_1,n_2)) &= \max\{\gamma_{M_1}(m_1),\gamma_{M_1}(n_1),\gamma_{M_2}(m_2),\gamma_{M_2}(n_2)\}\\ &= \max\{\max\{\gamma_{M_1}(m_1),\gamma_{M_2}(m_2)\},\max\{\gamma_{M_1}(n_1),\gamma_{M_2}(n_2)\}\}\\ &= \max\{(\gamma_{M_1}|\gamma_{M_2})(m_1,m_2),(\gamma_{M_1}|\gamma_{M_2})(n_1,n_2)\}.\end{aligned}$$

$$\begin{aligned}(\theta_{N_1}|\theta_{N_2})((m_1,m_2)(n_1,n_2)) &= \min\{\theta_{M_1}(m_1),\theta_{M_1}(n_1),\theta_{M_2}(m_2),\theta_{M_2}(n_2)\}\\ &= \min\{\min\{\theta_{M_1}(m_1),\theta_{M_2}(m_2)\},\min\{\theta_{M_1}(n_1),\theta_{M_2}(n_2)\}\}\\ &= \min\{(\theta_{M_1}|\theta_{M_2})(m_1,m_2),(\theta_{M_1}|\theta_{M_2})(n_1,n_2)\}.\end{aligned}$$

In a similar way others four will proved obviously.

Hence all properties are satisfied truly, so in all cases $N_1|N_2$ is a BSVNG on $M_1|M_2$. Therefore we can say $\mathtt{G}_1|\mathtt{G}_2 = (M_1|M_2, N_1|N_2)$ is a BSVNG. □

Definition 2.30. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, Y_2)$ be two BSVNGs. $\forall (m_1, m_2) \in V_1 \times V_2$

$$\begin{aligned}(d_\alpha)_{\mathtt{G}_1|\mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\alpha_{N_1}|\alpha_{N_2})((m_1, m_2)(n_1, n_2)) \\ &= \sum_{m_1=n_1, m_2n_2\notin E_2} \min\{\alpha_{M_1}(m_1), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\} \\ &+ \sum_{m_2=n_2, m_1n_1\notin E_1} \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{M_2}(m_2)\} \\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \min\{\alpha_{M_1}(m_1), \alpha_{M_1}(n_1), \alpha_{M_2}(m_2), \alpha_{M_2}(n_2)\}\end{aligned}$$

$$\begin{aligned}(d_\beta)_{\mathtt{G}_1|\mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\beta_{N_1}|\beta_{N_2})((m_1, m_2)(n_1, n_2)) \\ &= \sum_{m_1=n_1, m_2n_2\notin E_2} \max\{\beta_{M_1}(m_1), \beta_{M_2}(m_2), \beta_{M_2}(n_2)\} \\ &+ \sum_{m_2=n_2, m_1n_1\notin E_1} \max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1), \beta_{M_2}(m_2)\} \\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \max\{\beta_{M_1}(m_1), \beta_{M_1}(n_1), \beta_{M_2}(m_2), \beta_{M_2}(n_2)\}\end{aligned}$$

$$\begin{aligned}(d_\gamma)_{\mathtt{G}_1|\mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\gamma_{N_1}|\gamma_{N_2})((m_1, m_2)(n_1, n_2)) \\ &= \sum_{m_1=n_1, m_2n_2\notin E_2} \max\{\gamma_{M_1}(m_1), \gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\} \\ &+ \sum_{m_2=n_2, m_1n_1\notin E_1} \max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1), \gamma_{M_2}(m_2)\} \\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \max\{\gamma_{M_1}(m_1), \gamma_{M_1}(n_1), \gamma_{M_2}(m_2), \gamma_{M_2}(n_2)\}\end{aligned}$$

$$\begin{aligned}(d_\delta)_{\mathtt{G}_1|\mathtt{G}_2}(m_1, m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\delta_{N_1}|\delta_{N_2})((m_1, m_2)(n_1, n_2)) \\ &= \sum_{m_1=n_1, m_2n_2\notin E_2} \max\{\delta_{M_1}(m_1), \delta_{M_2}(m_2), \delta_{M_2}(n_2)\} \\ &+ \sum_{m_2=n_2, m_1n_1\notin E_1} \max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1), \delta_{M_2}(m_2)\} \\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \max\{\delta_{M_1}(m_1), \delta_{M_1}(n_1), \delta_{M_2}(m_2), \delta_{M_2}(n_2)\}\end{aligned}$$

$$\begin{aligned}(d_\eta)_{\mathtt{G}_1|\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\eta_{N_1}|\eta_{N_2})((m_1,m_2)(n_1,n_2))\\ &= \sum_{m_1=n_1,m_2n_2\notin E_2} \min\{\eta_{M_1}(m_1),\eta_{M_2}(m_2),\eta_{M_2}(n_2)\}\\ &+ \sum_{m_2=n_2,m_1n_1\notin E_1} \min\{\eta_{M_1}(m_1),\eta_{M_1}(n_1),\eta_{M_2}(m_2)\}\\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \min\{\eta_{M_1}(m_1),\eta_{M_1}(n_1),\eta_{M_2}(m_2),\eta_{M_2}(n_2)\}\end{aligned}$$

$$\begin{aligned}(d_\theta)_{\mathtt{G}_1|\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\theta_{N_1}|\theta_{N_2})((m_1,m_2)(n_1,n_2))\\ &= \sum_{m_1=n_1,m_2n_2\notin E_2} \min\{\theta_{M_1}(m_1),\theta_{M_2}(m_2),\theta_{M_2}(n_2)\}\\ &+ \sum_{m_2=n_2,m_1n_1\notin E_1} \min\{\theta_{M_1}(m_1),\theta_{M_1}(n_1),\theta_{M_2}(m_2)\}\\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \min\{\theta_{M_1}(m_1),\theta_{M_1}(n_1),\theta_{M_2}(m_2),\theta_{M_2}(n_2)\}\end{aligned}$$

Definition 2.31. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, Y_2)$ be two BSVNGs. $\forall (m_1, m_2) \in\ V_1 \times V_2$

$$\begin{aligned}(td_\alpha)_{\mathtt{G}_1|\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\alpha_{N_1}|\alpha_{N_2})((m_1,m_2)(n_1,n_2)) + (\alpha_{M_1}|\alpha_{M_2})(m_1,m_2)\\ &= \sum_{m_1=n_1,m_2n_2\notin E_2} \min\{\alpha_{M_1}(m_1),\alpha_{M_2}(m_2),\alpha_{M_2}(n_2)\}\\ &+ \sum_{m_2=n_2,m_1n_1\notin E_1} \min\{\alpha_{M_1}(m_1),\alpha_{M_1}(n_1),\alpha_{M_2}(m_2)\}\\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \min\{\alpha_{M_1}(m_1),\alpha_{M_1}(n_1),\alpha_{M_2}(m_2),\alpha_{M_2}(n_2)\}\end{aligned}$$

$$\begin{aligned}(td_\beta)_{\mathtt{G}_1|\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\beta_{N_1}|\beta_{N_2})((m_1,m_2)(n_1,n_2)) + (\beta_{M_1}|\beta_{M_2})(m_1,m_2)\\ &= \sum_{m_1=n_1,m_2n_2\notin E_2} \max\{\beta_{M_1}(m_1),\beta_{M_2}(m_2),\beta_{M_2}(n_2)\}\\ &+ \sum_{m_2=n_2,m_1n_1\notin E_1} \max\{\beta_{M_1}(m_1),\beta_{M_1}(n_1),\beta_{M_2}(m_2)\}\\ &+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \max\{\beta_{M_1}(m_1),\beta_{M_1}(n_1),\beta_{M_2}(m_2),\beta_{M_2}(n_2)\}\end{aligned}$$

$$
\begin{aligned}
(td_\gamma)_{\mathtt{G}_1|\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\gamma_{N_1}|\gamma_{N_2})((m_1,m_2)(n_1,n_2)) + (\gamma_{M_1}|\gamma_{M_2})(m_1,m_2)\\
&= \sum_{m_1=n_1, m_2n_2\notin E_2} \max\{\gamma_{M_1}(m_1),\gamma_{M_2}(m_2),\gamma_{M_2}(n_2)\}\\
&+ \sum_{m_2=n_2, m_1n_1\notin E_1} \max\{\gamma_{M_1}(m_1),\gamma_{M_1}(n_1),\gamma_{M_2}(m_2)\}\\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \max\{\gamma_{M_1}(m_1),\gamma_{M_1}(n_1),\gamma_{M_2}(m_2),\gamma_{M_2}(n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\delta)_{\mathtt{G}_1|\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\delta_{N_1}|\delta_{N_2})((m_1,m_2)(n_1,n_2)) + (\delta_{M_1}|\delta_{M_2})(m_1,m_2)\\
&= \sum_{m_1=n_1, m_2n_2\notin E_2} \max\{\delta_{M_1}(m_1),\delta_{M_2}(m_2),\delta_{M_2}(n_2)\}\\
&+ \sum_{m_2=n_2, m_1n_1\notin E_1} \max\{\delta_{M_1}(m_1),\delta_{M_1}(n_1),\delta_{M_2}(m_2)\}\\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \max\{\delta_{M_1}(m_1),\delta_{M_1}(n_1),\delta_{M_2}(m_2),\delta_{M_2}(n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\eta)_{\mathtt{G}_1|\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\eta_{N_1}|\eta_{N_2})((m_1,m_2)(n_1,n_2)) + (\eta_{M_1}|\eta_{M_2})(m_1,m_2)\\
&= \sum_{m_1=n_1, m_2n_2\notin E_2} \min\{\eta_{M_1}(m_1),\eta_{M_2}(m_2),\eta_{M_2}(n_2)\}\\
&+ \sum_{m_2=n_2, m_1n_1\notin E_1} \min\{\eta_{M_1}(m_1),\eta_{M_1}(n_1),\eta_{M_2}(m_2)\}\\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \min\{\eta_{M_1}(m_1),\eta_{M_1}(n_1),\eta_{M_2}(m_2),\eta_{M_2}(n_2)\}
\end{aligned}
$$

$$
\begin{aligned}
(td_\theta)_{\mathtt{G}_1|\mathtt{G}_2}(m_1,m_2) &= \sum_{(m_1,m_2)(n_1,n_2)\in E_1\times E_2.} (\theta_{N_1}|\theta_{N_2})((m_1,m_2)(n_1,n_2)) + (\theta_{M_1}|\theta_{M_2})(m_1,m_2)\\
&= \sum_{m_1=n_1, m_2n_2\notin E_2} \min\{\theta_{M_1}(m_1),\theta_{M_2}(m_2),\theta_{M_2}(n_2)\}\\
&+ \sum_{m_2=n_2, m_1n_1\notin E_1} \min\{\theta_{M_1}(m_1),\theta_{M_1}(n_1),\theta_{M_2}(m_2)\}\\
&+ \sum_{m_1n_1\notin E_1 and\ m_2n_2\notin E_2} \min\{\theta_{M_1}(m_1),\theta_{M_1}(n_1),\theta_{M_2}(m_2),\theta_{M_2}(n_2)\}
\end{aligned}
$$

Example 2.32. Let $\mathtt{G}_1 = (M_1, N_1)$ and $\mathtt{G}_2 = (M_2, N_2)$ be two BSVNGs as in Example 2.28. Their rejection is also shown in Figure 13. We will find the vertex degree in rejection. Consider the vertex (d,a) here:

$$\begin{aligned}(d_\gamma)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) &= \max\{\gamma_{M_2}(e),\gamma_{M_1}(a),\gamma_{M_1}(d)\} + \max\{\gamma_{M_2}(a),\gamma_{M_1}(a),\gamma_{M_1}(c)\}\\ &= \max\{0.2,0.2,0.3\} + \max\{0.2,0.2,0.3\}\\ &= 0.3+0.3\\ &= 0.6\end{aligned}$$

$$\begin{aligned}(d_\theta)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) &= \min\{\theta_{M_2}(e),\theta_{M_1}(a),\theta_{M_1}(d)\} + \min\{\theta_{M_2}(a),\theta_{M_1}(a),\theta_{M_1}(c)\}\\ &= \min\{-0.3,-0.4,-0.6\} + \min\{-0.3,-0.4,-0.2\}\\ &= -0.6-0.4\\ &= -1.0\end{aligned}$$

$$(d_\alpha)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) = 0.6,\ \ (d_\beta)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) = 0.8$$

$$(d_\delta)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) = -0.5,\ \ (d_\eta)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) = -0.5$$

In a similar way, we can find degree of all vertices of a graph in rejection. Now we will find out the total vertex degree of graph in rejection. Consider the same vertex (d,a) here:

$$\begin{aligned}(td_\gamma)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) &= \max\{\gamma_{M_2}(e),\gamma_{M_1}(a),\gamma_{M_1}(d)\} + \max\{\gamma_{M_2}(a),\gamma_{M_1}(a),\gamma_{M_1}(c)\} + \min\{\gamma_{M_2}(e),\gamma_{M_1}(a)\}\\ &= \max\{0.2,0.2,0.3\} + \max\{0.2,0.2,0.3\} + \min\{0.2,0.2\}\\ &= 0.3+0.3+0.2\\ &= 0.8\end{aligned}$$

$$\begin{aligned}(td_\theta)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) &= \min\{\theta_{M_2}(e),\theta_{M_1}(a),\theta_{M_1}(d)\} + \min\{\theta_{M_2}(a),\theta_{M_1}(a),\theta_{M_1}(c) + \min\{\theta_{M_2}(e),\theta_{M_1}(a)\}\}\\ &= \min\{-0.3,-0.4,-0.6\} + \min\{-0.3,-0.4,-0.2\} + \min\{-0.3,-0.4\}\\ &= -0.6-0.4-0.4\\ &= -1.4\end{aligned}$$

$$(td_\alpha)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) = 0.9,\ \ (td_\beta)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) = 1.1$$

$$(td_\delta)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) = -0.8,\ \ (td_\eta)_{\mathtt{G}_1|\mathtt{G}_2}(e,a) = -0.7$$

In a similar way we can find total vertex degree in rejection.

3. **Application of bipolar single valued neutrosophic graph (BSVNG)**

3.1. *Educational Designation participation*

Let {Bilal, Asif, Shoaib, Ijaz } be the set of four applicants for designations {Head of department(HOD),Director of Department(DOD),Assistant director of department(ADOD)}. For this purpose p=4 (say) be number of applicants and d=3 be number of designations. Consider bipolar single valued-neutrosophic diagraph which is shown in figure **??** representing the competition between applicants for designation in organization. $\alpha(y)$ is the positive degree of membership for every applicants denote the percentage of ability toward the purpose of organization , $\beta(y)$ and $\gamma(y)$ are indeterminacy and false in percentage. $\delta(y)$ is the is the negative degree of membership for every applicants denote the percentage of non ability toward the purpose of organization, $\eta(y)$ and $\theta(y)$ are represents the indeterminacy and false in percentage. $\alpha(y)$ of every directed edge between both designations and applicants denote the eligibility or positive response from designation in organization , $\beta(y)$ and $\gamma(y)$ are indeterminacy and false in this percentage. $\delta(y)$ of every directed edge between both designations and applicants denote the non-eligibility or negative response from designation in organization , $\eta(y)$ and $\theta(y)$ are indeterminacy and false in this percentage. Edge membership degree of

Table 1

y∈ Y	N(y)
Bilal	$\{(ADOD,0.5,0.3,0.4,-0.4,-0.5,-0.8),(HOD,0.6,0.4,0.2,-0.4,-0.6,-0.5)\}$
Asif	$\{(ADOD,0.8,0.6,0.5,-0.1,-0.4,-0.5),(HOD,0.5,0.6,0.6,-0.3,-0.4,-0.7),(DOD,0.4,0.6,0.4,-0.2,-0.3,-0.5)\}$
Shoaib	$\{(DOD,0.5,0.4,0.5,-0.5,-0.4,-0.4)\})$
Ijaz	$\{(HOD,0.7,0.5,0.6,-0.3,-0.5,-0.4),(DOD,0.7,0.4,0.5,-0.4,-0.3,-0.2)\})$

graph is also determined by the following

$$N(Bilal)\cap N(Asif) = \{(ADOD,0.5,0.6,0.5,-0.1,-0.5,-0.8),(HOD,0.5,0.6,0.6,$$
$$-0.3,-0.6,-0.7)\}$$
$$N(Bilal)\cap N(Shoaib) = \emptyset$$
$$N(Bilal)\cap N(Ijaz) = \{(HOD,0.6,0.5,0.6,-0.3,-0.6,-0.5)\}$$
$$N(Asif)\cap N(Shoaib) = \{(DOD,0.4,0.6,0.5,-0.2,-0.4,-0.5)\}$$
$$N(Asif)\cap N(Ijaz) = \{(HOD,0.5,0.6,0.6,-0.3,-0.5,-0.7),(DOD,0.4,0.6,0.5,-0.2,$$
$$-0.3,-0.5)\}$$
$$N(Shoaib)\cap N(Ijaz) = \{(DOD,0.5,0.4,0.5,-0.4,-0.4,-0.4)\}$$

There is no edge between Shoaib and Bilal because there is no common designation.

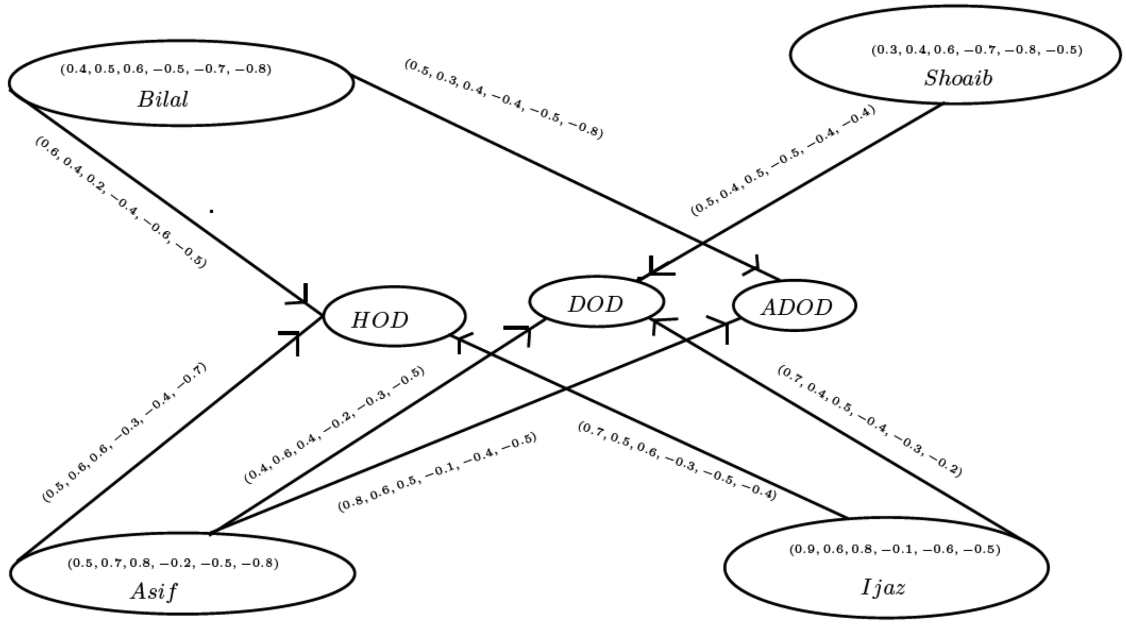

FIGURE 14. Bipolar single valued neutrosophic digraph

$$\begin{aligned}
(Bilal, Asif) &= (0.4, 0.7, 0.8, -0.2, -0.7, -0.8)(0.5, 0.6, 0.5, 0.3, 0.6, 0.7)\\
&= (0.20, 0.42, 0.40, -0.06, -0.42, -0.56)\\
(Bilal, Shoaib) &= \emptyset\\
(Bilal, Ijaz) &= (0.4, 0.6, 0.8, -0.1, -0.7, -0.8)(0.6, 0.5, 0.6, 0.3, 0.6, 0.5)\\
&= (0.24, 0.30, 0.48, -0.03, -0.42, -0.40)\\
(Asif, Shoaib) &= (0.3, 0.7, 0.8, -0.2, -0.8, -0.8)(0.4, 0.6, 0.5, 0.2, 0.4, 0.5)\\
&= (0.12, 0.42, 0.40, -0.04, -0.32, -0.40)\\
(Asif, Ijaz) &= (0.5, 0.7, 0.8, -0.1, -0.6, -0.8)(0.5, 0.6, 0.5, 0.3, 0.3, 0.5)\\
&= (0.25, 0.42, 0.40, -0.03, -0.18, -0.40)\\
(Shoaib, Ijaz) &= (0.3, 0.6, 0.8, -0.1, -0.8, -0.5)(0.5, 0.4, 0.5, 0.4, 0.4, 0.4)\\
&= (0.15, 0.24, 0.40, -0.04, -0.32, -0.20)
\end{aligned}$$

Bipolar single-valued neurotrophic graph for competition of all participant is shown in figure 15. Competition between two individually applicants and when applicant competing for designation is also given in graph 15.

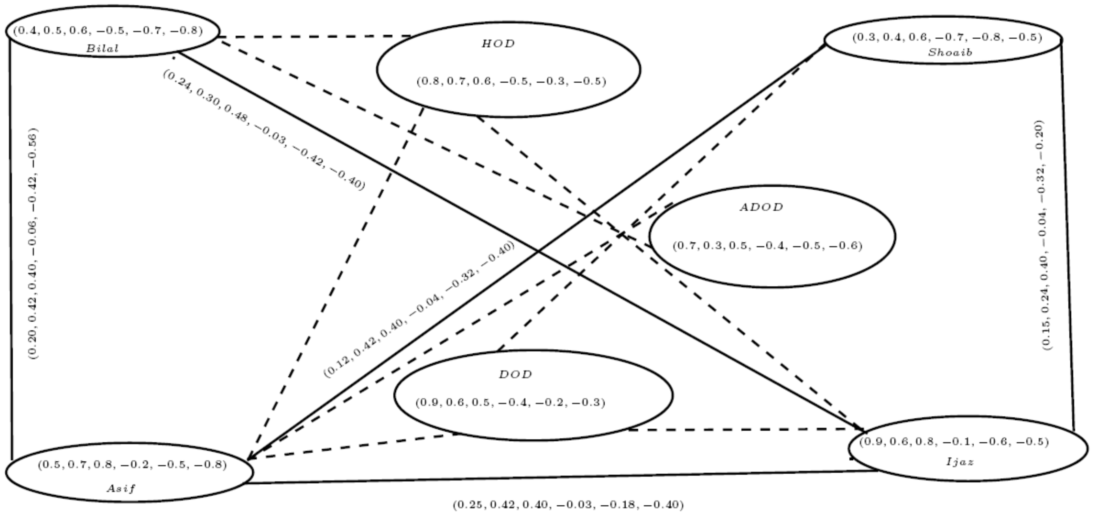

FIGURE 15. Bipolar single valued neutrosophic competition graph

$$R(Bilal, HOD) = (\frac{0.20+0.24}{2}, \frac{0.20+0.24}{2}, \frac{0.42+0.30}{2}, \frac{0.40+0.48}{2}, \frac{-0.06-0.03}{2}, \frac{-0.42-0.42}{2}, \frac{-0.56-0.40}{2}) = (0.22, 0.36, 0.44, -0.045, -0.42, -0.48)$$

Similarly we will find others R(applicant,Designation).

$$S(Bilal, HOD) = 1 + 0.22 - 0.045 - (0.36 + 0.44 - 0.42 - 0.48) = 1.275$$

$$S(Asif, HOD) = 1 + 0.225 - 0.045 - (0.42 + 0.40 - 0.30 - 0.48) = 1.14$$

$$S(Ijaz, HOD) = 1 + 0.245 - 0.03 - (0.36 + 0.44 - 0.225 - 0.29) = 0.93$$

$$S(Bilal, ADOD) = 1 + 0.20 - 0.06 - (0.42 + 0.40 - 0.42 - 0.56) = 1.30$$

$$S(Asif, ADOD) = 1 + 0.20 - 0.06 - (0.42 + 0.40 - 0.42 - 0.56) = 1.30$$

$$S(Asif, DOD) = 1 + 0.185 - 0.035 - (0.42 + 0.40 - 0.25 - 0.40) = 0.98$$

$$S(Shoaib, DOD) = 1 + 0.135 - 0.04 - (0.33 + 0.40 - 0.32 - 0.30) = 0.985$$

$$S(Ijaz, DOD) = 1 + 0.20 - 0.035 - (0.33 + 0.40 - 0.25 - 0.30) = 0.985$$

Black solid lines show comparison between two applicants and dot line means applicant compete for designation. From above table, applicants compete other if it has a more strength. For example, in HOD designation Bilal has more strength from all. Its eligibility is strong than other. In ADOD designation Asif and Bilal are in equal position. In DOD designation Shoaib and Ijaz compete the others but equally compete to each other. [H]In this algorithm these are the steps

Step 1: Start. **Step 2:** Input $\alpha(y), \beta(y)$ *and* $\gamma(y)$ membership values for set p applicants.

TABLE 2

(Applicant,designation)	in competition	R(applicant,Designation)	S(applicant,Designation)
(Bilal,HOD)	Asif, Ijaz	(0.22,0.36,0.44,-0.045,-0.42,-0.48)	1.275
(Asif,HOD)	Bilal,Ijaz	(0.225,0.42,0.40,-0.045,-0.30,-0.48)	1.14
(Ijaz,HOD)	Bilal,Asif	(0.245,0.36,0.44,-0.03,-0.225,-0.29)	0.93
(Bilal,ADOD)	Asif	(0.20,0.42,0.40,-0.06,-0.42,-0.56)	1.30
(Asif,ADOD)	Bilal	(0.20,0.42,0.40,-0.06,-0.42,-0.56)	1.30
(Asif,DOD)	Shoaib,Ijaz	(0.185,0.42,0.40,-0.035,-0.25,-0.40)	0.98
(Shoaib,DOD)	Asif,Ijaz	(0.135,0.33,0.40,-0.04,-0.32,-0.30)	0.985
(Ijaz,DOD)	Asif,Shoaib	(0.20,0.33,0.40,-0.035,-0.25,-0.30)	0.985

Step3: For any two vertices x_i *and* xj *taking* $\alpha(x_ix_j), \beta(x_ix_j)$ *and* $\gamma(x_ix_j)$ are positive but $\delta(x_ix_j), \eta(x_ix_j)$ *and* $\theta(x_ix_j)$ are negative. Then $(x_i, \alpha(x_ix_j), \beta(x_ix_j), \gamma(x_ix_j), \delta(x_ix_j), \eta(x_ix_j), \theta(x_ix_j))$
Step4: To obtain bipolar single valued neutrosohic out-neighbourhoods $N(x_i)$ Repeat step 3 for all vertices x_i *and* x_j.

Step5: Find out $N(x_i) \cap N(x_j)$. **Step6:** Calculate height $h(N(x_i) \cap N(x_j))$. **Step7:** Draw all edge where $N(x_i) \cap N(x_j)$ is non empty. **Step8:** Give a membership value to every edge x_ix_j by using the following conditions
$\alpha(x_ix_j = (\min\{x_i \cap x_j\})[N(x_i \cap N(x_j)],\ \beta(x_ix_j = (\max\{x_i \cap x_j\})[N(x_i \cap N(x_j)]$
$\gamma(x_ix_j = (\max\{x_i \cap x_j\})[N(x_i \cap N(x_j)],\ \delta(x_ix_j = (\max\{x_i \cap x_j\})[N(x_i \cap N(x_j)]$
$\eta(x_ix_j = (\min\{x_i \cap x_j\})[N(x_i \cap N(x_j)],\ \theta(x_ix_j = (\min\{x_i \cap x_j\})[N(x_i \cap N(x_j)]$
Step9: If x, $z_1, z_2, z_3, ..., z_p$ are applicants for designations d, then strength of applicants competition is R(x,d)=$(\alpha(x,d), \beta(x,d), \gamma(x,d), \delta(x,d), \eta(x,d), \theta(x,d))$ of every applicants x and designation d is given by the following R(x,d)=$(\frac{\alpha(xz_1)+...\alpha(xz_p)}{p}, \frac{\beta(xz_1)+...\beta(xz_p)}{p}, \frac{\gamma(xz_1)+...\gamma(xz_p)}{p}, \frac{\delta(xz_1)+...\delta(xz_p)}{p}, \frac{\eta(xz_1)+...\eta(xz_p)}{p}, \frac{\theta(xz_1)+...\theta(xz_p)}{p})$
Step10: Find out $S(x,d) = 1+\alpha(x,d)+\delta(x,d)-(\beta(x,d)+\gamma(x,d)+\eta(x,d)+\theta(x,d))$. **Step11:** End

4. **Conclusion**

There are more advantages of a bipolar fuzzy set than fuzzy set in real life phenomenon. A BSVNG has many applications in the field of economics, medical science as well as in scientific engineering. The flexibility and compatibility of BSVNG are higher than SVNG. We presented the new properties on a bipolar single-valued neutrosophic graph known as Residue product, maximal product, Symmetric difference and Rejection of a graph. These all graph products are suggestive of some aspects of network design. They can be applicable for the configuration processing of space structures. The repeated application of these operations in constructing a network generates graphs that display fractal properties. We also discussed the idea with examples to find the degree and total degree of vertices of some graphs. We have established some related theorems of these graphs. We have also proved the theorems which are related to these properties. In the future, our goal is to extend this work on the (1) complex neutrosophic graphs and some (2) bipolar complex neutrosophic graph.

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