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Efficiently finding the smallest k values in a large Cartesian product of lists

Patrick Kreitzberg

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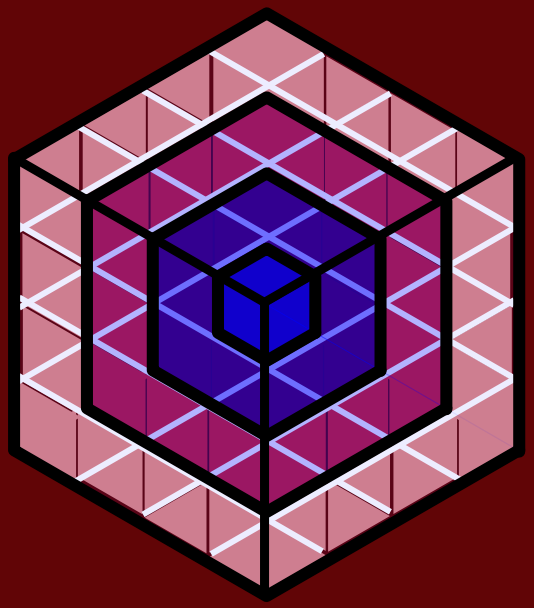


EFFICIENTLY FINDING THE SMALLEST *K* VALUES IN A LARGE CARTESIAN PRODUCT OF LISTS

PATRICK KREITZBERG
ADVISOR: OLIVER SERANG

UNIVERSITY OF MONTANA

SELECTING MEALS WITH SALAD, BEVERAGE, AND ENTRÉE BASED ON PRICE

Salads

Add Grilled Chicken, *Salmon or Shrimp for 5.49

GF *Sirloin Bleu Cheese
Fresh Greens, Tomato, Cucumber, Red Onion topped with charbroiled medium Angus Sirloin, grilled Portabellas and Bleu Cheese Crumbles. 14.99

GF Huckleberry Walnut
Fresh Greens, Walnuts, Dried Cranberries, Green Apples, Bleu Cheese Crumbles, Avocado, Shredded Carrots, and Montana Huckleberries. 12.99

Buffalo Chicken
Crispy Chicken Strips tossed in our zesty Buffalo Hot Sauce served with Tomato, Cucumber, Red Onion, Croutons, Crumbled Bleu Cheese and Celery. 11.99

California Cobb
Charbroiled Chicken Breast, Avocado, Olives, Cucumbers, Egg, Smoked Bacon, Tomatoes, Crumbled Bleu Cheese and Croutons. 14.99

Chinese Chicken
Shredded Cabbage, fresh Greens and Carrots tossed with Sesame Vinaigrette topped with toasted Almonds, Wonton Strips, and Mandarins. 12.99

GF Glacier
Fresh Greens sliced Strawberries, Cucumber, Dried Cranberries, Candied Pecans and Feta. 11.99

GF Seafood Louie
Our Crab and Shrimp Salad, Avocado, Tomato Slices, Cucumbers and Egg on fresh Greens. 14.99

Beverages

Tea
Fresh brewed Iced Tea 2.99
Pomegranate, Huckleberry, Blueberry, Strawberry Raspberry Iced Tea 2.99

Lemonade
Lemonade 2.99 • The Arnold Palmer 2.99
Pomegranate, Huckleberry, Blueberry, Raspberry or Strawberry Lemonade 2.99

Energy Drink
Red Bull: Sugar Free and Assorted Flavors 3.00

Juice
Orange Juice 2.99 • Grapefruit, Pineapple and Cranberry 2.59

Soda
Jackson Hole Huckleberry Soda 2.99
Weinhard's Bottled Rootbeer 2.99
Coke, Diet Coke, Sprite, Sprite Zero, Dr. Pepper, Cherry Coke, Powerade, Fanta 2.99
(Pepsi products available in Butte)

Hot Drinks
Fresh-brewed, French Roast Coffee 2.79
Hot Tea 2.29 Hot Chocolate 2.49

Miscellaneous
Dasani Bottled Water 1.99
Milk or Chocolate Milk 2.49

Specialty Steaks

GF Herb Crusted Filet
Our Club Angus Filet Mignon charbroiled with an Herb Seasoned crust and topped with Red Wine Rosemary butter reduction. 31.99

GF Blackberry Filet Mignon
Our tender 7 oz Club Angus Filet makes the perfect canvas for this incomparable Blackberry Brandy Steak Sauce. 31.99

GF House Marinated Ribeye
Our tender 10 oz Club Angus Ribeye, marinated overnight in our blend of Herbs and Spices. 28.99

Steak Oscar
Our tender Club Angus Sirloin topped with fresh Crab Meat, luscious housemade Béarnaise and Asparagus Spears. 8 oz 27.99

Garlic Button Sirloin
Button Mushrooms sautéed in our Creamy Garlic Butter Sauce then served with our tender Prime Grade Club Angus Sirloin and signature Parmesan Roll. 8 oz 26.99

GF The Royal New York
Our 12 oz Club Angus New York Strip charbroiled, topped with Caramelized Onions, sautéed Mushrooms, and melted Bleu Cheese Crumbles. 31.99

Lease expensive meal:

- Pick least expensive of each
  Glacier Salad, Water, and Garlic Button Sirloin $40.97

Second least expensive meal:

- Change salad, drink, or steak?

Buffalo Chicken, Water, and Garlic Button Sirloin
Glacier Salad, Hot Tea, and Garlic Button Sirloin
Glacier Salad, Water, and House Marinated Ribeye

Third least expensive meal:

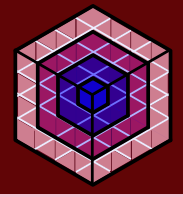
- Change salad, drink, or steak?

- Buffalo Chicken, Water, and Garlic Button Sirloin
Chinese Chicken, Water, and Garlic Button Sirloin
Buffalo Chicken, Hot Tea, and Garlic Button Sirloin
Buffalo Chicken, Water, and House Marinated Ribeye

- Glacier Salad, Hot Tea, and Garlic Button Sirloin
Buffalo Chicken, Hot Tea, and Garlic Button Sirloin
Glacier Salad, Hot Choc, and Garlic Button Sirloin
Glacier Salad, Hot Tea, and House Marinated Ribeye

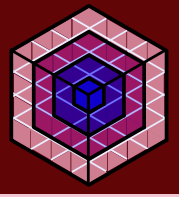
- Glacier Salad, Water, and House Marinated Ribeye
Buffalo Chicken, Water, and House Marinated Ribeye
Glacier Salad, Hot Tea, and House Marinated Ribeye
Glacier Salad, Water, and The Royal New York

NAÏVELY GENERATE ALL MEALS AND SORT



Rank	Price	Salad	Beverage	Steak
0	$40.97	Buffalo Chicken	Bottled water	Garlic button sirloin
1	$40.97	Glacier Salad	Bottled water	Garlic button sirloin
2	$41.27	Buffalo Chicken	Tea	Garlic button sirloin
3	$41.27	Glacier Salad	Tea	Garlic button sirloin
4	$41.47	Buffalo Chicken	Hot chocolate	Garlic button sirloin
5	$41.47	Buffalo Chicken	Milk	Garlic button sirloin
6	$41.47	Glacier Salad	Hot chocolate	Garlic button sirloin
7	$41.47	Glacier Salad	Milk	Garlic button sirloin
8	$41.57	Buffalo Chicken	Not orange juice	Garlic button sirloin
	⋮	⋮	⋮	⋮
668	$49.98	California Cobb	Red Bull	The Royal New York
669	$49.98	Seafood Louie	Red Bull	Berry filet mignon
670	$49.98	Seafood Louie	Red Bull	Herb crusted filet
671	$49.98	Seafood Louie	Red Bull	The Royal New York
672	$49.98	Sirloin blue cheese	Red Bull	Berry filet mignon

WHAT IS A CARTESIAN PRODUCT?

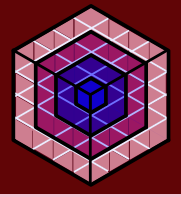


List A = {A1, A2,...}

List B = {B1, B2,...}

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
B1	A1*B1	A2*B1	A3*B1	A4*B1	●	●				
B2	A1*B2	A2*B2	A3*B2	●	●					
B3	A1*B3	A2*B3	●	●	●					
B4	●	●	●							
B5	●	●								
B6										
B7										
B8										
B9										
B10										

CARTESIAN PRODUCT WITH SORTED AXES

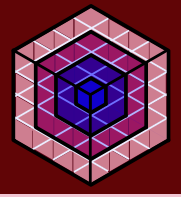


	Water $1.99	Tea $2.29	Milk $2.49	Hot Choc $2.8	Coffee $3.25	Not OJ $4.10
Garlic Butter Sirloin $26.99	1	2	3	4	6	11
Steak Oscar $27.99	5	7	8	9	12	16
House Marinated Ribeye $28.99	10	13	14	15	17	18
Berry Filet Mignon $31.99	19	22	25	28	31	34
Herb Crusted Filet $31.99	20	23	26	29	32	35
The Royal New York $31.99	21	24	27	30	33	36

SORTING BAD

IF YOU COULD JUST
GO AHEAD AND STOP SORTING
THAT WOULD BE GREAT
imgflip.com

CARTESIAN PRODUCT WITH SORTED AXES



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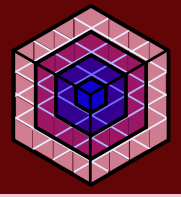
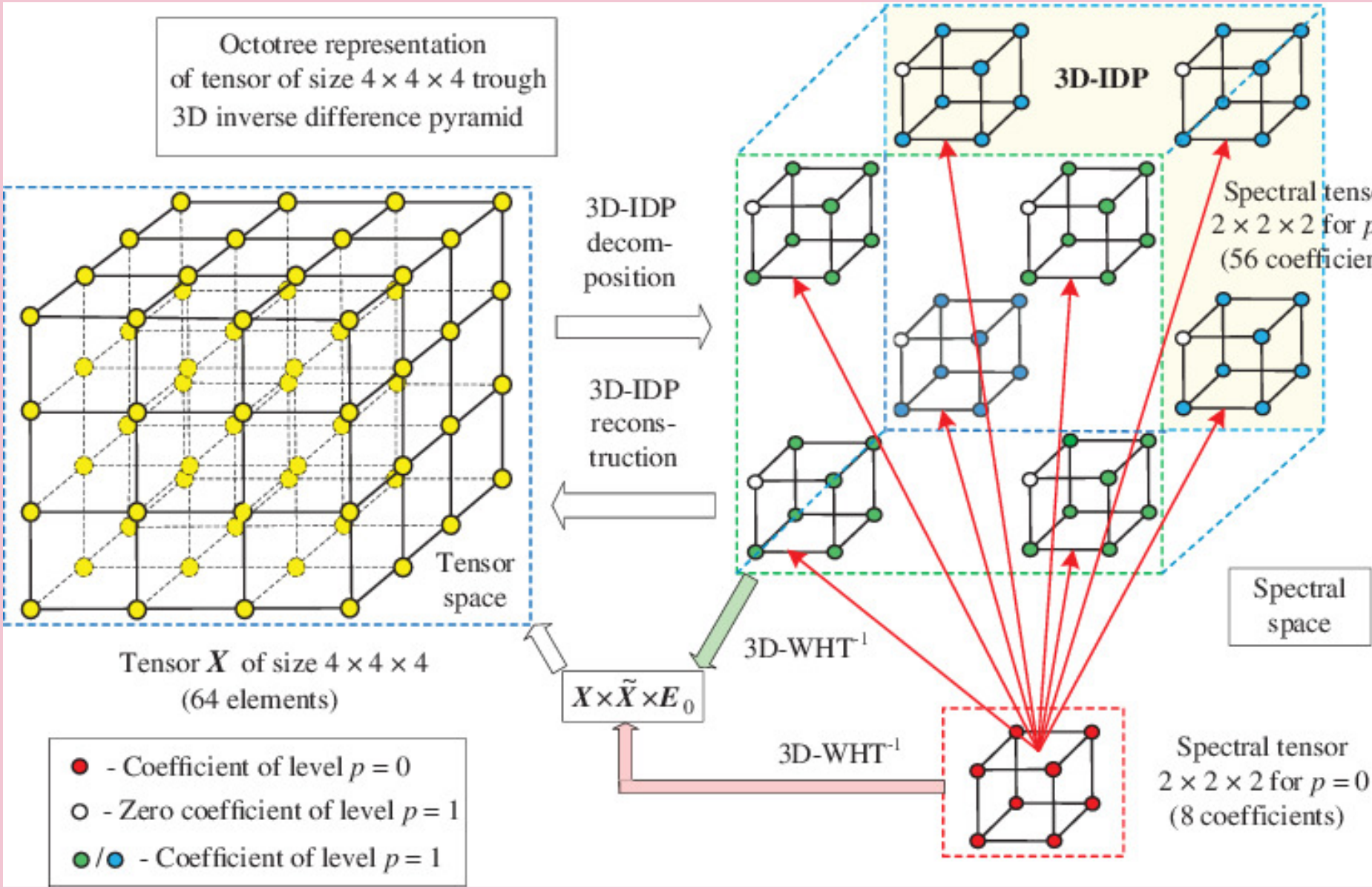
Herb Crusted Filet $31.99	20	23	26	29	32	35
The Royal New York $31.99	21	24	27	30	33	36

CARTESIAN PRODUCT WITH KIND-OF SORT-OF SORTED AXES



	Water $1.99	Milk $2.49	Tea $2.29	Not OJ $4.10	Coffee $3.25	Hot Choc $2.8
Garlic Butter Sirloin $26.99	1	3	2	11	6	4
Steak Oscar $27.99	5	8	7	16	12	9
House Marinated Ribeye $28.99	10	14	13	18	17	15
The Royal New York $31.99	21	27	24	36	33	30
Herb Crusted Filet $31.99	20	26	23	35	32	29
Berry Filet Mignon $31.99	19	25	22	34	31	28

M-dimensional version



M-dimensional version

Octotree representation
f tensor of size 4 × 4 × 4 trough
3D inverse difference pyramid

3D-IDP
decom-
position

3D-IDP
recons-
truction

Tensor
space

Tensor X of size 4 × 4 × 4
(64 elements)

$X \times \tilde{X} \times E_0$

3D-WHT^{-1}

3D-WHT^{-1}

3D-IDP

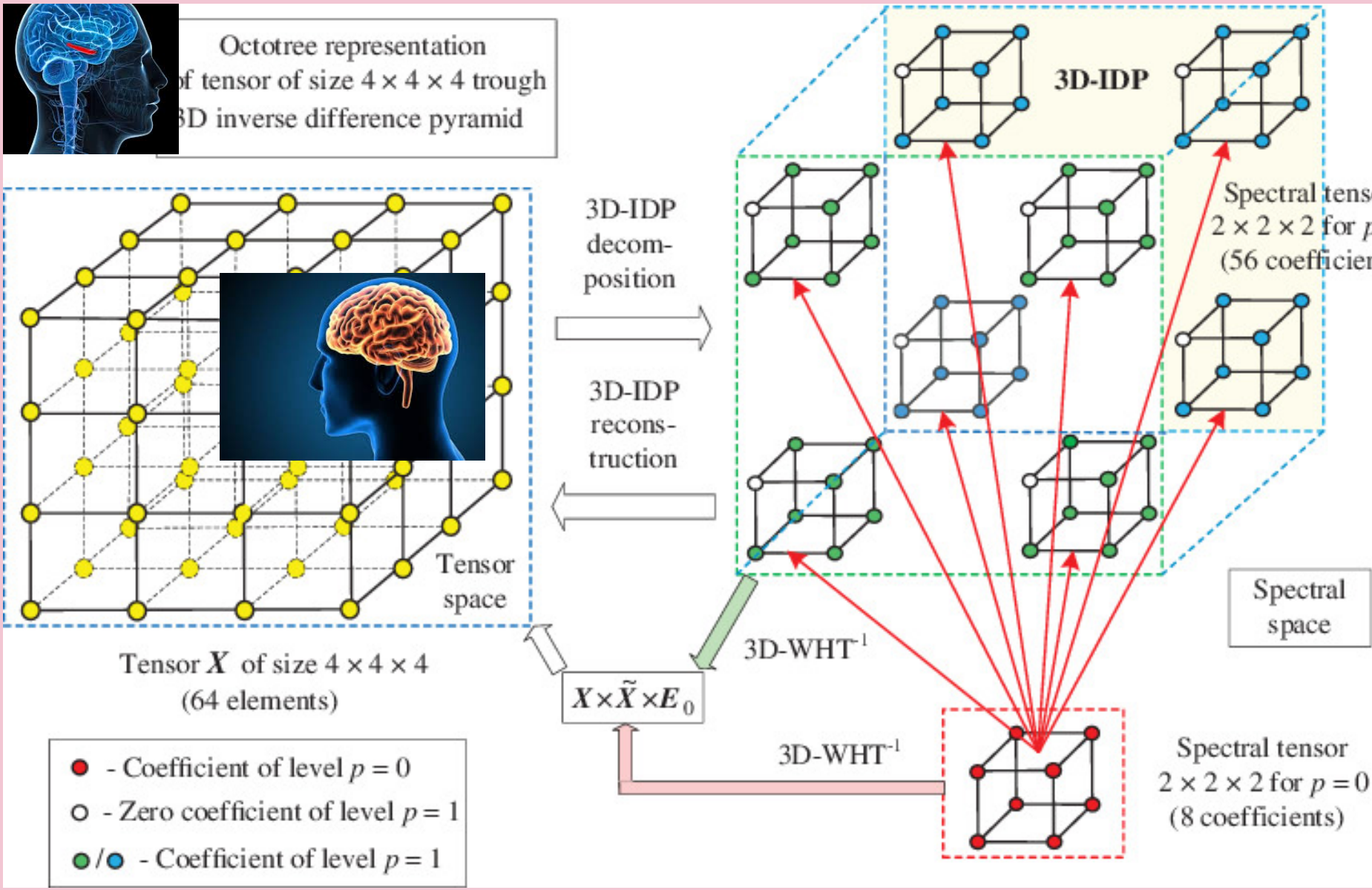
Spectral tens
2 × 2 × 2 for p
(56 coefficien

Spectral
space

Spectral tensor
2 × 2 × 2 for $p = 0$
(8 coefficients)

- Coefficient of level $p = 0$
- Zero coefficient of level $p = 1$
/ - Coefficient of level $p = 1$

M-dimensional version



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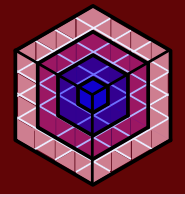
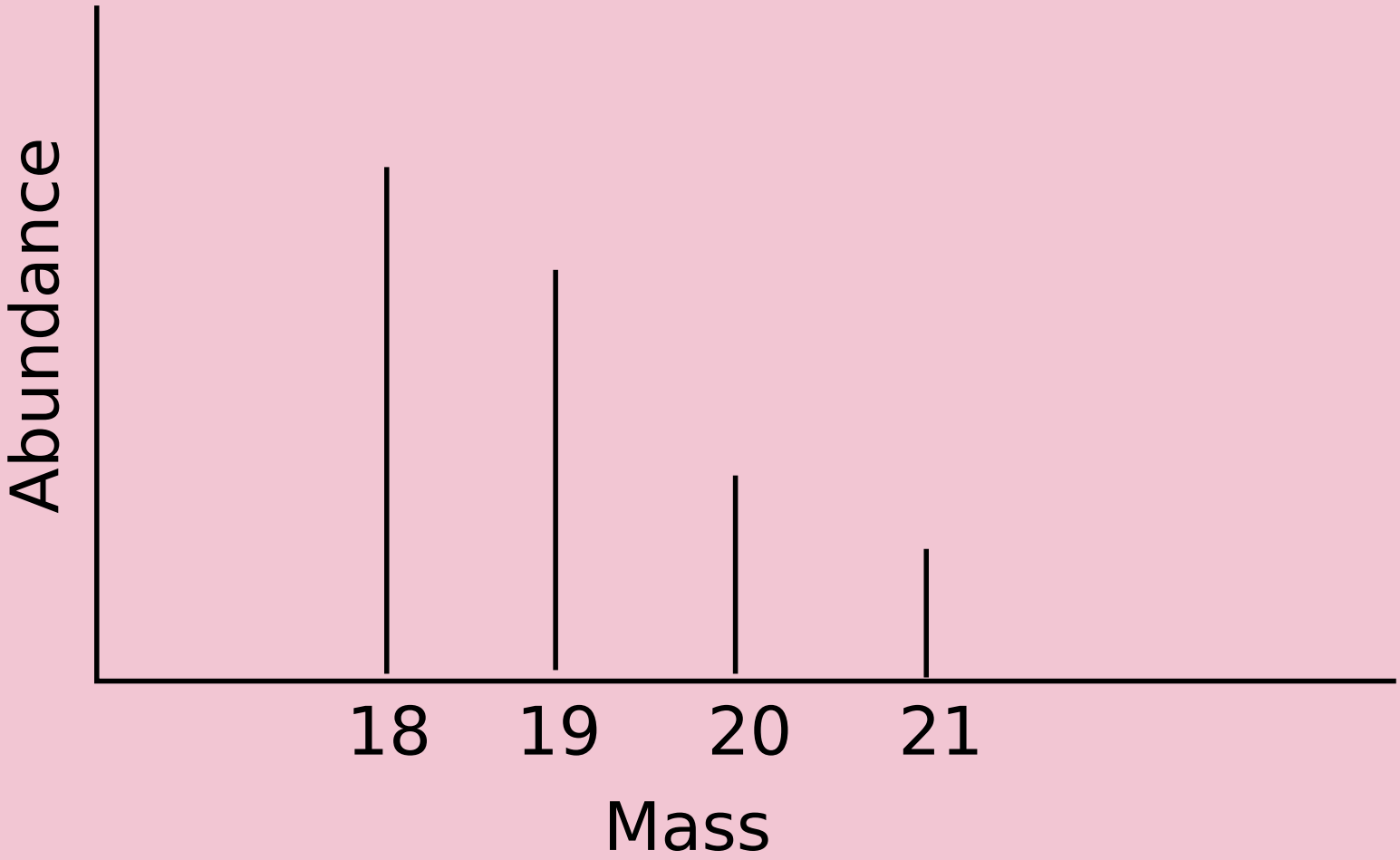
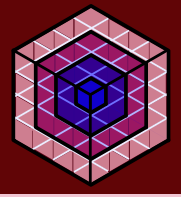
CALCULATING ISOTOPE MASS



Hydrogen masses 1, 2, 3
Oxygen masses 16, 17, 18



Math

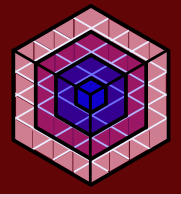


$$1 + 2 + 3 + 4$$

$$=$$

$$(1 + 2) + (3 + 4)$$

Math



$$1 + 2 + 3 + 4 + 5 + 6$$

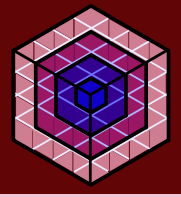
$$=$$

$$(1 + 2) + (3 + 4) + (5 + 6)$$

$$=$$

$$((1 + 2) + (3 + 4)) + (5 + 6)$$

Math



$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

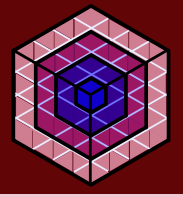
$$=$$

$$(1 + 2) + (3 + 4) + (5 + 6) + (7 + 8)$$

$$=$$

$$((1 + 2) + (3 + 4)) + ((5 + 6) + (7 + 8))$$

Math



$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

$$=$$

$$(1 + 2) + (3 + 4) + (5 + 6) + (7 + 8) + (9 + 10)$$

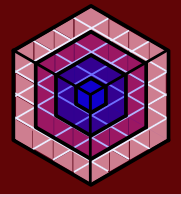
$$=$$

$$((1 + 2) + (3 + 4)) + ((5 + 6) + (7 + 8)) + (9 + 10)$$

$$=$$

$$(((1 + 2) + (3 + 4)) + ((5 + 6) + (7 + 8))) + (9 + 10)$$

Math



$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12$$

$$=$$

$$(1 + 2) + (3 + 4) + (5 + 6) + (7 + 8) + (9 + 10) + (11 + 12)$$

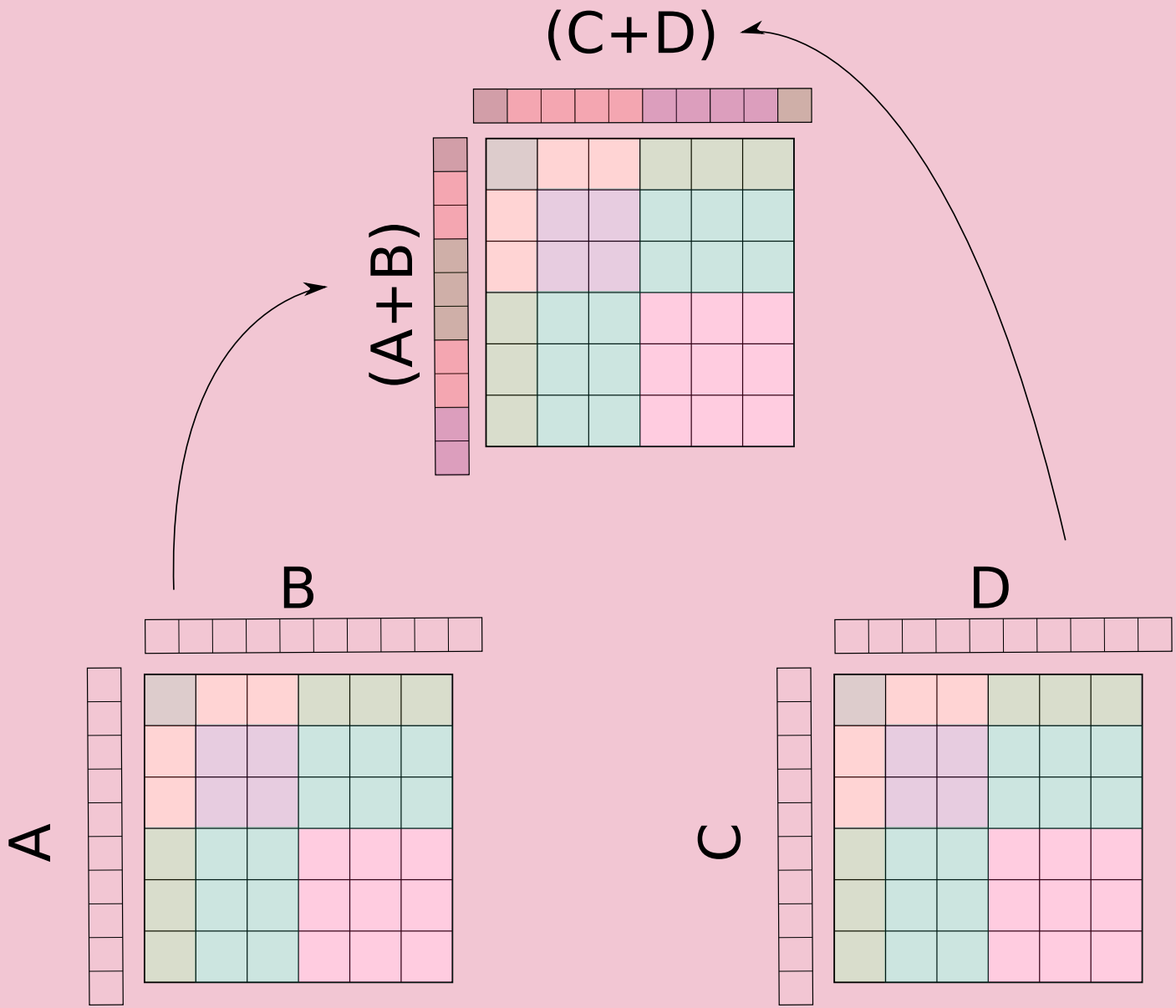
$$=$$

$$((1 + 2) + (3 + 4)) + ((5 + 6) + (7 + 8)) + ((9 + 10) + (11 + 12))$$

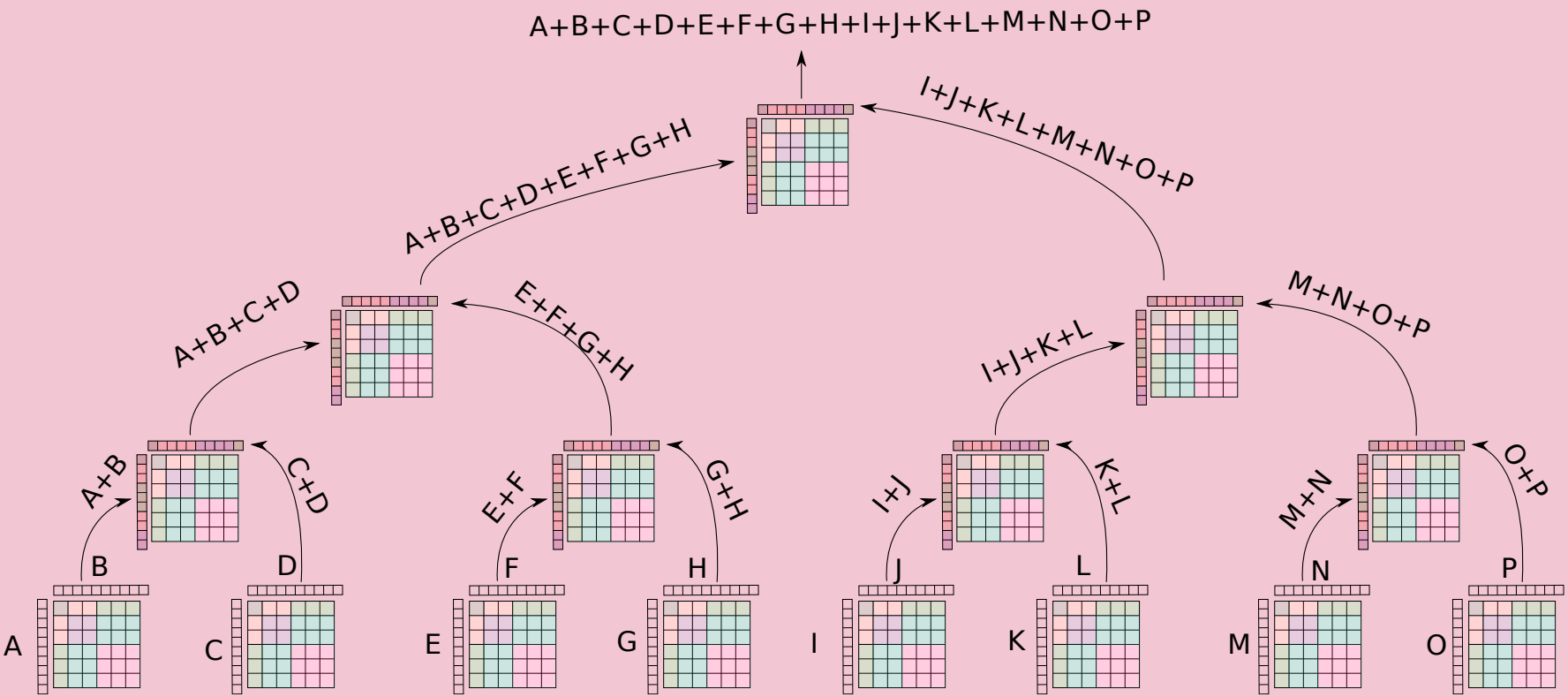
$$=$$

$$(((1 + 2) + (3 + 4)) + (5 + 6) + (7 + 8))) + ((9 + 10) + (11 + 12))$$

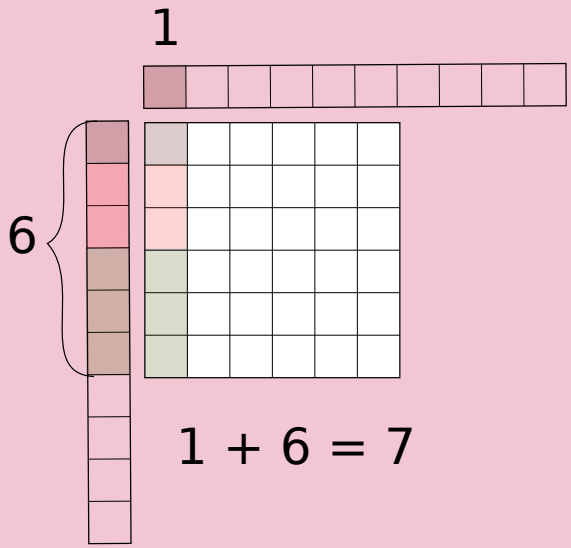
COMBINING PAIR-WISE CARTESIAN PRODUCTS

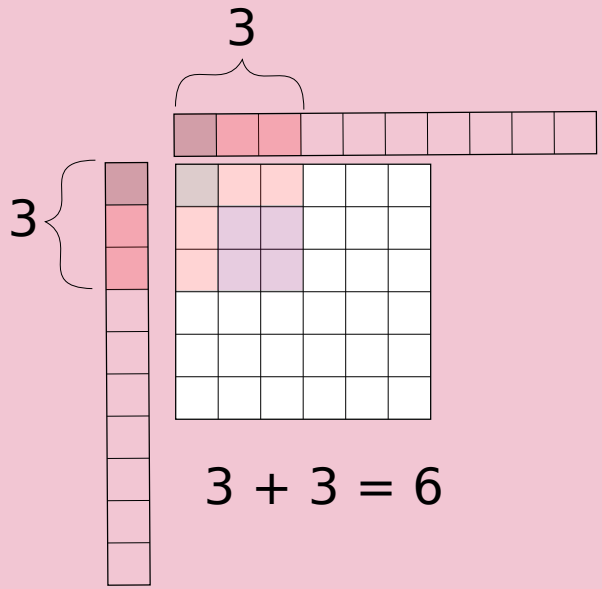


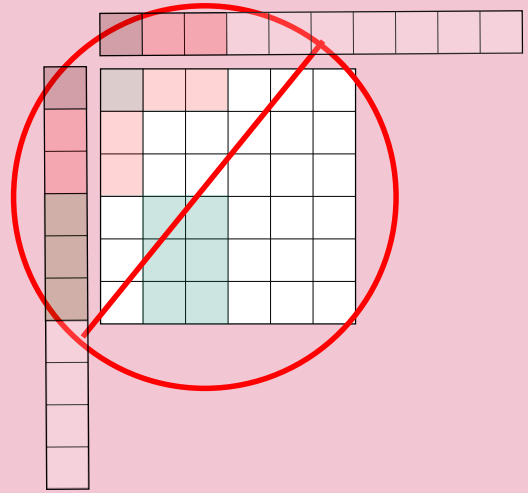
TREE OF PAIR-WISE CARTESIAN PRODUCTS

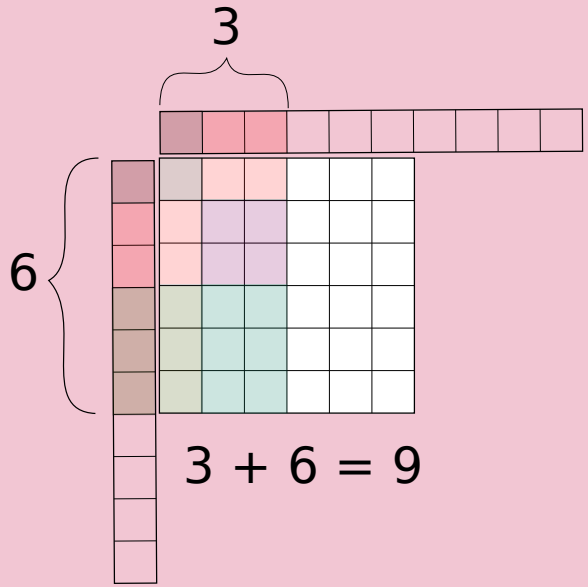


WHAT DOES A PARENT NEED OF THEIR CHILD?









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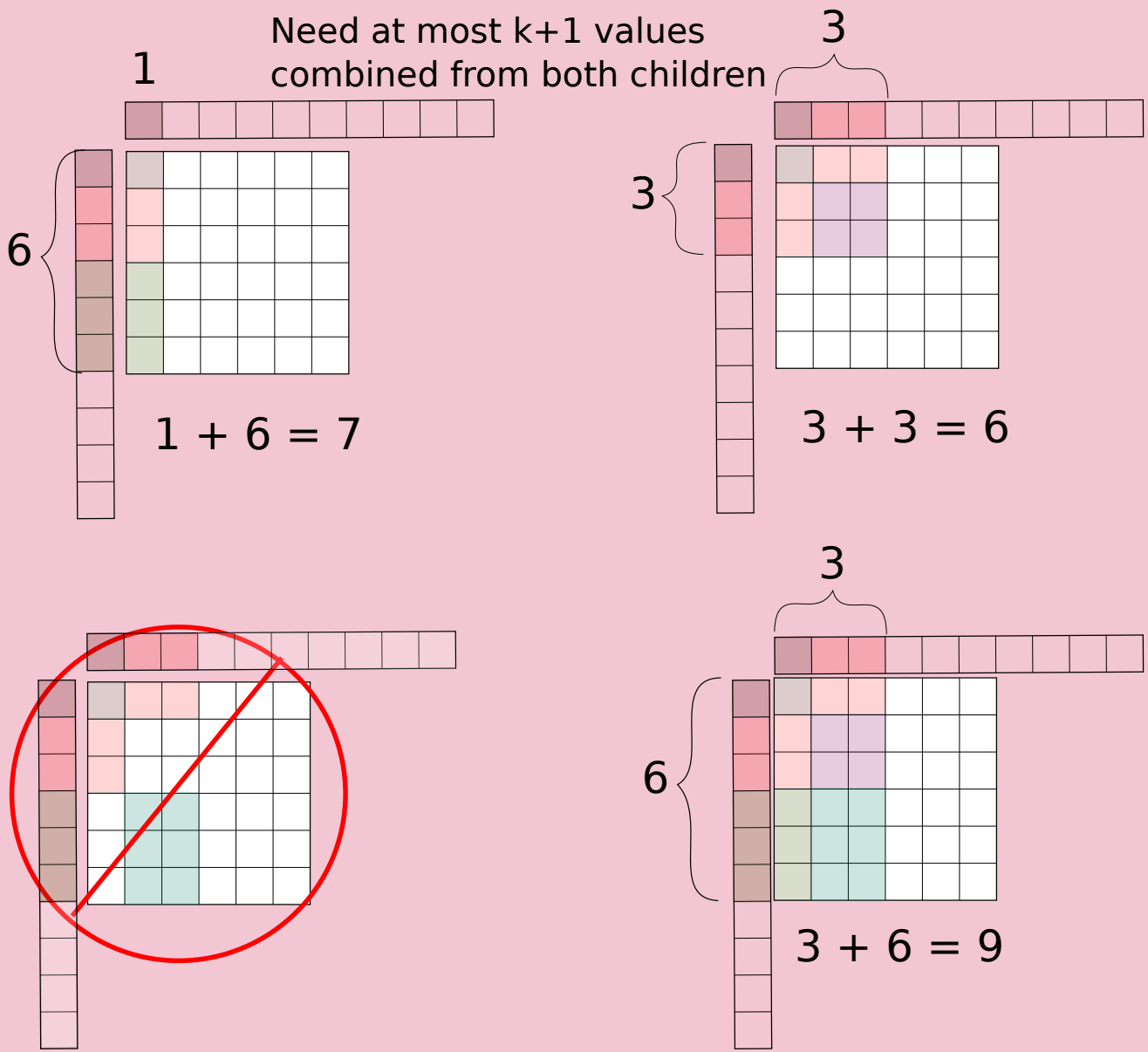
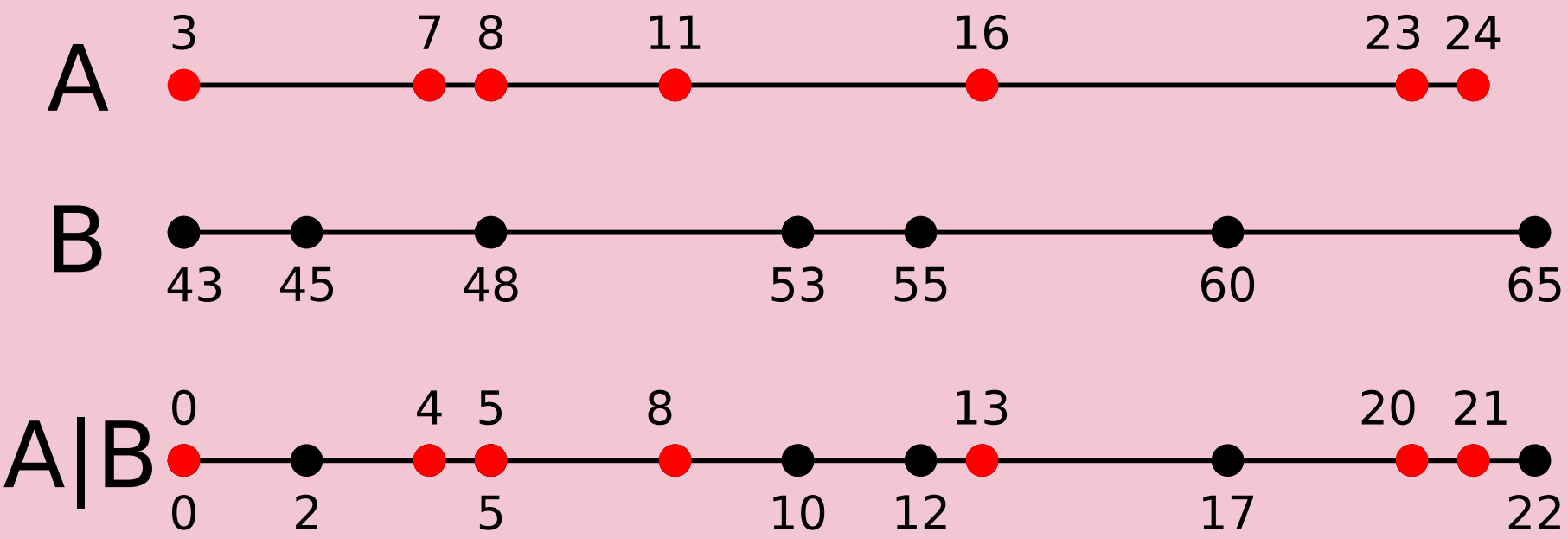
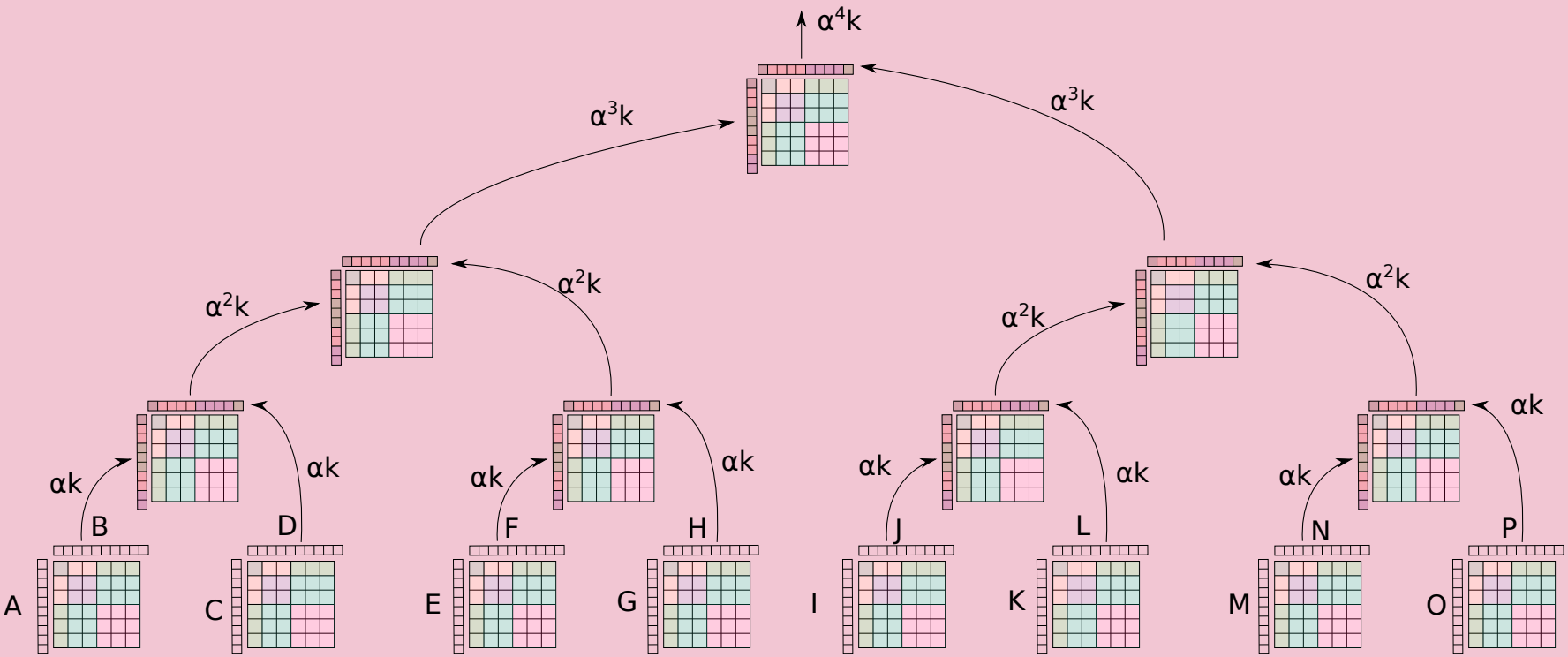


SELECTING WHICH VALUES FROM A AND B ARE NEEDED FOR $(A + B) + (C + B)$



SIZE OF CHILDREN'S OUTPUT

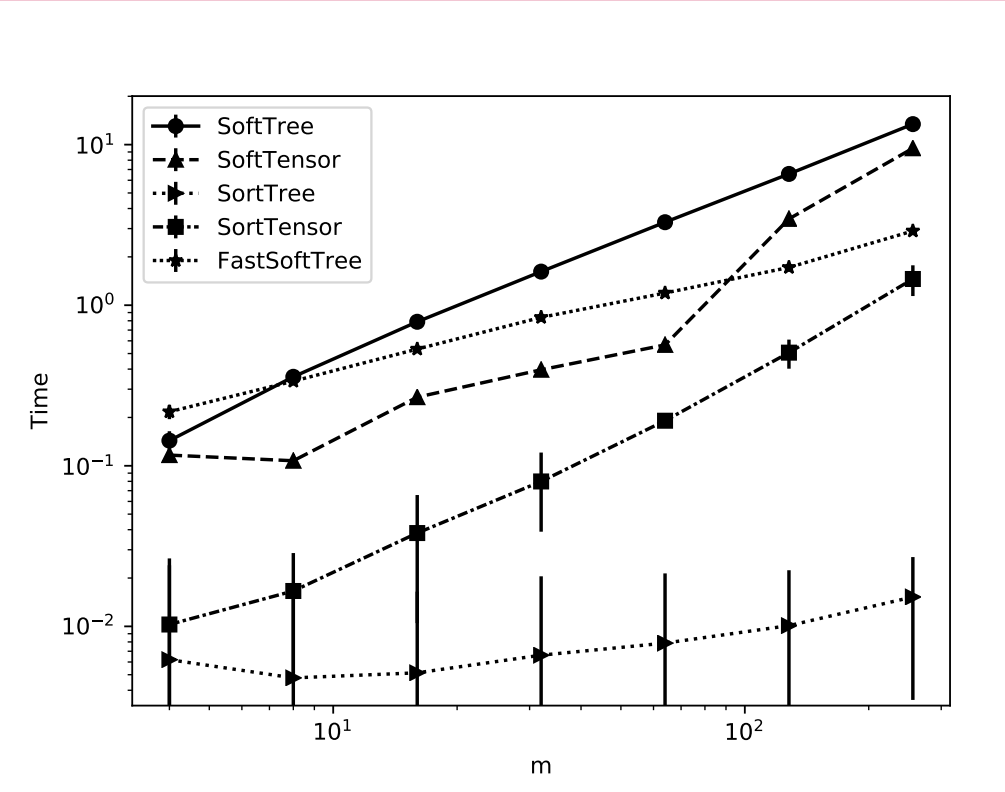


RUNTIME

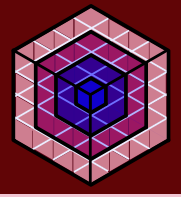
Runtimes are compared to several other methods:

1. FastSoftTree has runtime$\in O(n \cdot m + k \cdot m^{0.1})$
2. SoftTree has runtime $\in O(n \cdot m + k \cdot m)$
3. SortTree has runtime $\in O(n \cdot m + k \log(k) \cdot \log(m) + k \cdot m)$

The theoretical runtime is remarkable because the runtime scales sublinearly in m for more requested items, k.



ACKNOWLEDGEMENTS AND REFERENCES



Acknowledgements:

- This work was supported by grant number 1845465 from the National Science Foundation.
- Thanks to my advisor, Oliver Serang
- Members of Serang lab: Kyle Lucke, Jake Pennington

References:

- "Selection on $X_1 + X_2 + \cdots + X_m$ with layer-ordered heaps" By Kreitzberg, Lucke, and Serang
- "Optimal selection on $X + Y$ simplified with layer-ordered heaps" by Serang