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# Transportation Improvement and Hollowing-out of Urban Commercial Center: Do They Harm Consumer Welfare?

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## **Abstract:**

Concentration or dispersion of retail stores is the result of market interactions. If it involves market failures, then the spatial location equilibrium of retail stores is not optimal in terms of social welfare. We investigate two important market failures involving retail store location: “monopolistic competition among retail stores” and “shopping externality caused by multipurpose shopping”. Retail store locations in market equilibrium and those in a social optimum are derived. Next, we show that the degree of hollowing-out of urban centers is not always excessive from the perspective of the social optimum. It is believed that hollowing-out of urban commercial centers harms social welfare. But on the contrary, if the accessibility of suburban areas from residential areas is lower than that of the urban center, we confirm that hollowing-out of urban commercial centers is desirable. In this case, promotion of retail stores’ location in urban center, such as subsidies to locate in the city center or restrictions on location in suburbs, decreases social welfare. Instead, promotion of stores’ location in the suburbs is preferred.

## **Keywords:**

*hollowing-out, suburbanization, monopolistic competition, shopping externality*

## **JEL Classification:**

L11, L13, R12, R32

**Highlights:**

1. We show a divergence of the market boundary of retail stores between the market equilibrium and social optimum.
2. We find that the degree of hollowing-out of urban centers is not always excessive from the perspective of social welfare.
3. Hollowing-out of urban centers is excessive if the accessibility of suburban areas from residential areas is higher than that of the central commercial area.
4. Retail store location in suburban areas does not always harm social welfare. It depends on the difference in relative accessibility to the urban center and suburban areas.

# 1 Introduction

Hollowing-out of urban commercial center is a recent urban problem at issue because it is believed that it should worsen social welfare. In our paper, considering two important market failures involved in retail store location: “monopolistic competition among retail stores” and “shopping externality caused by multipurpose shopping”, we explore hollowing-out of urban commercial centers from the perspective of social welfare.

Spatial allocation of retail stores, including concentration and hollowing-out, is an outcome of market equilibrium. Therefore, the allocation is socially optimal if there are no market failures. Why do people believe that hollowing-out of urban commercial center harm social welfare? We guess that they believe concentration of commerce brings agglomeration economy. It is certainly common that the productivity of firms is influenced by proximity to other firms.<sup>1</sup>

Retail store location is footloose in the sense that stores’ locations are less constrained by inputs such as resources and labor supplies. Large markets are more attractive for retail stores. Their profit-seeking location brings concentration of many kinds of retail stores in urban areas. Therefore, a main cause of the recent hollowing-out is the suburbanization of commerce which results from motorization, improvement of transportation networks, and the expansion of suburban areas over the last few decades.

Again, Hollowing-out of urban commercial centers is not optimal if it involves technological externalities attributed to market failures. In fact, the relevant literature has described several factors of market failure in commerce: 1) spatial price competition of commercial location, 2) imperfect competition among retail stores, and 3) shopping externality (O’Sullivan (1993)) caused by multipurpose (one-stop) shopping. We will review some relevant studies.

First, 1) “spatial price competition of commercial locations” is a phenomenon among firms that supply a homogeneous good. Hotelling’s “ice cream vendor” problems (Hotelling (1929)) serve as an illustrative example. Each firm decides its price and the location under the prevailing competition with neighboring firms in the market area. Spatial price competition is examined in many studies using the framework of that model (see e.g., Cappaola and Order (1978), Anderson, de Palma and Thisse (1992), and Beckmann (1999)). However, although this spatial competition framework can represent

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<sup>1</sup> In the United States, the suburbanization of cities advanced explosively during the 1970s, but it was followed by the decay of city centers. In Germany and France in the 1970s, the necessity for revitalization of city centers was a common theme for policymakers and researchers. It remains an important policy issue in the United Kingdom. Following such trends in the US and Europe, since regulations of commercial development in suburban areas were loosened in Japan in the 1990s, the hollowing-out of urban commercial centers in most small cities has become an increasingly important social issue.

competition among suburban retail stores such as the same-sized shopping malls, gas stations, and convenience stores, it is not useful to consider goods of various kinds supplied in the urban center and suburban areas. Therefore, we investigate the two remaining factors of market failure.

Next, we review 2) “imperfect competition among stores”. Agglomeration of retail stores is advantageous for each because it creates an attractive area for consumption and more consumers visit the region. However, it can be disadvantageous for each; agglomeration of retail stores brings competition from other retail stores. In an urban center, a wide variety of retail stores locate and mutually compete for profits. Such a competitive framework is monopolistic competition. With price competition, each retail store differentiates its services and assortment to compete with other retail stores. To represent such locations of differentiated retail stores, the monopolistic competition model is suitable. Monopolistic competition is modeled by Dixit and Stiglitz (1977). The Dixit–Stiglitz model has already been used in many papers for analyses of various problems related to spatial issues<sup>2</sup>. The formulation of monopolistic competition in our model follows that of Fujita et al. (1999).

We review 3) “shopping externality (O’Sullivan (1993)) caused by multipurpose (one-stop) shopping”. If different categories of retail stores are agglomerated in one region, then consumers can purchase goods of several kinds with one trip, which is so-called “multipurpose shopping” or “one-stop shopping”. Consequently, agglomeration of retail stores provides a positive externality for consumers, which was designated as the “shopping externality” (but not fully demonstrated with a model) by O’Sullivan (1993).

Some papers have explained the mechanics generating commercial agglomeration with the existence of shopping externality. Wolinsky (1983) explains information asymmetries between stores and consumers cause a commercial agglomeration that sells imperfectly substitute goods. Information asymmetries mean that consumers do not know the price and the quality of goods perfectly until they visit each store. De Palma et al. (1985) show that the agglomerated configuration of three or more retail stores at the market center is stable in a Hotelling location model if goods are sufficiently differentiated and if the transport costs are sufficiently low. Ago (2008) presents the same result as that of de Palma et al. (1985) in the case of monopolistic competition. These papers present interesting implications for the formation of agglomeration. Ago (2008) demonstrates that monopolistic competition is a cause of agglomeration of firms in a Hotelling location model. However, he does not address shopping externality, which is one cause of agglomeration of retail stores. Additionally, neither paper addresses the change in utility level by transportation improvement, which the current paper

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<sup>2</sup> See, e.g., Fujita et al. (1999) and Baldwin et al. (2003).

targets. Kishi et al. (2015) explore a spatial price competition model with central and outlying commercial areas through a simple extension of Capozza and Van Order (1978). However this paper does not consider monopolistic competition with a variety of goods.

In our shopping behavior, we spend much time and money on obtaining some of our favorite goods. Therefore, consumers have the benefit of saving the cost for comparison of goods if there is a commercial agglomeration that sells imperfectly substitute goods. Accordingly, the shopping externality has a much greater impact on our choice of shopping place. Furthermore, if a commercial agglomeration sells complementary goods, then consumers have the benefit of purchasing goods of several kinds in one-stop shopping and can therefore save transport costs. These merits for consumers increase the demand accruing to neighboring stores.

Such shopping behavior of consumers is modeled in the previous literature (see e.g., Hanson (1980), Mulligan (1985, 1987), and Ingene and Ghosh (1990)). Our model, proposed in Section 2, specifically examines one-stop shopping behavior in the same manner as many previous studies<sup>3</sup>. Some analyze retail store location. For example, Eaton and Lipsey (1982) show the mechanism of retail store agglomeration caused by one-stop shopping behavior. Henkel et al. (2000) model one-stop shopping behavior to analyze coalition formation among retail service suppliers. Tabuchi (2009) models self-organization marketplaces under a one-stop shopping situation. Those previous papers focus on agglomeration mechanisms, while our research focuses on welfare analyses.

The existence of these two factors of market failure, monopolistic competition and shopping externality, brings socially “non-optimal” spatial allocation of retail stores in market equilibrium. As described above, these two factors are very common in an environment of retail store competition and consumer shopping activity. Uschev et al. (2015) introduce these two factors and study competition between urban retail stores and suburban retail stores. They focus on government regulation of retail store location. Their study is related to our objective. However, they assume a linear city which has a market place at each edge, and therefore transportation costs of both market places only depend on the distance from the residential area. Our study explores asymmetric transport improvements (e.g. construction of a public transportation system to the urban center or improvement of road facilities in the outskirts of the city).

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<sup>3</sup> Several papers model such consumer behaviour with random utility frameworks (see e.g. Popkowski et al. (2004) and Sinha (2000)) or the hazard model (see e.g. Popkowski et al. (2000)). In these models, all consumers buy goods at all shopping clusters scattered geographically. However, as Christaller (1933) modeled the hierarchical marketplace, a large marketplace usually has all kinds of products supplied in a smaller marketplace. In this case, modelling one-stop shopping behavior is sufficient for our paper’s purpose by appropriately adjusting the time interval.

In summary, our research objective is to analyze how these two factors affect consumer welfare through the change in retail store location caused by transportation improvement. From our analysis, we can theoretically clarify if hollowing-out or suburbanization of commerce caused by transportation improvement harm social welfare or not.

In the rest of our paper, we first outline the model in Section 2. Then, in Section 3 we derive retail stores' location in market equilibrium and in social optimum. We derive how transport improvement changes consumers' welfare depending on the retail store location by comparative statics with respect to transportation cost. Section 4 explains how social welfare changes with respect to transportation improvement through the change in retail store location. Section 5 concludes the paper.

## 2 Model

### 2.1 Image of the city

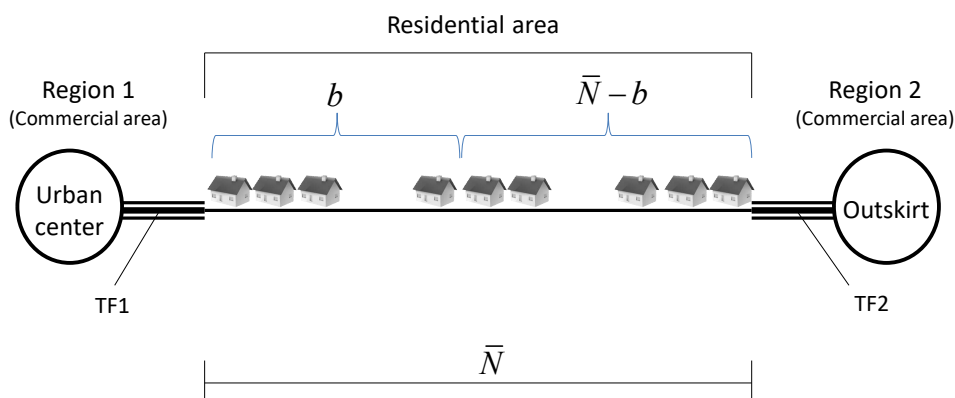
We consider a linear residential area, which is expressed as a line segment where homogeneous consumers are distributed uniformly and continuously. The total population of consumers is fixed as  $\bar{N}$ . Each consumer resides on a plot of land, the length of which is normalized to 1. Consequently, the length of the line segment is  $\bar{N}$ .

Each of the two ends of the interposed residential area has a transportation facility to a commercial area. Figure 1 shows that the transportation facilities are represented as "TF1" and "TF2". The two commercial areas are called, respectively, "region 1" and "region 2". Retail stores locate in the two commerce regions 1 and 2. However, they are not allowed to locate in the housing area. In real cities also, zoning regulations restrict large-size store locations in housing areas. Moreover, most cities have a central commercial area and suburban shopping centers. The two commercial regions in the model can be interpreted as them. In both regions, retail stores choose their locations in the existence of monopolistic competition<sup>4</sup>. Considering the transportation cost and the variety of the commodities that are supplied, each consumer purchases goods at either region. No congestion arises in relation to their shopping trips.

For convenience of our consideration, we merely assume that region 1 is an urban center, whereas region 2 is on the outskirts of a city. However, no technical difference exists among retail stores in regions 1 and 2 in the model. The exchange of 1 and 2 would not influence the following analysis.

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<sup>4</sup> For simplification, we do not model the land market of commercial areas. In our model, retail stores can locate anywhere with no land rent. Our conclusions are fundamentally identical to those in the case in which the land market is introduced into the model if no market failure exists in the land market.



**Figure 1** Shape of the City

## 2.2 Transportation cost

Transportation in the residential area and transportation facilities TF1 and TF2 requires some monetary expense, although it is costless in commercial areas<sup>5</sup>. It invariably compels consumers to purchase widely various goods during one shopping trip to save transportation costs between the residence and the commercial area. The greater the amount of goods obtained in one shopping trip, the lower the transportation cost per good. Put differently, scale economies relate to the consumers' shopping trips. To save their transportation cost, rational consumers purchase goods of many kinds in one shopping trip.

It is assumed that the transportation cost per unit distance in the residential area is one, and that the travel cost of TF1 is  $t_1$ , and that of TF2 is  $t_2$ . Therefore, for the  $n$ th consumer from region 1, the respective transportation costs  $L_1(n)$  for region 1 and  $L_2(n)$  for region 2 are given as  $L_1(n) = n + t_1$  and  $L_2(n) = (\bar{N} - n) + t_2$ .

## 2.3 Consumer behavior

<sup>5</sup> There is usually sufficient distance between the residential area and the commercial area; therefore, consumers spend a certain amount of transportation cost. However, in a commercial area, they will have lower transportation costs because retail stores are agglomerated. Our simplification is based on such circumstances.



In our model, we specifically examine one-shopping trip behaviour of consumers in a certain fixed time interval, as assumed by Eaton and Lipsey (1982). Each consumer shares a log linear utility function:

$$V_i = \mu \ln M_i + (1 - \mu) \ln A_i. \quad (1)$$

Therein, subscript  $i$  means that the consumer shops in region  $i$ ,  $M_i$  represents a composite index of the consumption of commercial goods<sup>6</sup>,  $A_i$  denotes the consumption of a numeraire good of which the price is one, and  $\mu$  signifies a constant representing the expenditure share of commercial goods. The quantity index,  $M_i$ , is a sub-utility function defined over a continuum of varieties of commercial goods. In addition,  $m_i(x)$  denotes the consumption of each available variety;  $f_i$  denotes the number of goods sold in region  $i$ , which is equivalent to the number of retail stores, and  $\sigma$  is the elasticity of substitution between any two varieties. We assume that  $M_i$  is defined by a constant elasticity of substitution (CES) function:

$$M_i = \left[ \int_0^{f_i} m_i(x)^{\sigma-1/\sigma} dx \right]^{\sigma/\sigma-1}.$$

Given income  $Y$  and price  $p_i(x)$  for each commercial good, and transportation costs for region  $i$ , the consumers' budget constraint is

$$A_i + \int_0^{f_i} p_i(x) m_i(x) dx + L_i(n) = Y.$$

The  $n$ th consumers' utility maximization is represented as

$$\max_{M_i, A_i} V_i \quad s.t. \quad A_i + G_i M_i + L_i(n) = Y. \quad (2)$$

Therein,  $G_i$  is the price index of commercial goods supplied in region  $i$ . Because it is assumed that no technical difference exists among retail stores, all commercial goods are sold at the same price  $p^*$ . Therefore,  $G_i$  is represented as

$$G_i = \left[ \int_0^{f_i} p(x)^{1-\sigma} dx \right]^{1/1-\sigma} = p^* f_i^{\frac{1}{1-\sigma}}. \quad (3)$$

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<sup>6</sup> Horizontally differentiated goods are goods that have elasticity of substitution with each other. In a real economy, a large number of retail stores supply quite a variety of goods: food, clothes, electronic equipment, furniture, and so on. Some have no elasticity of substitution with other goods. We specifically examine retail stores, which supply horizontally differentiated goods. Other goods are regarded as numeraire goods in the same manner as described in the literature (see e.g. Tabuchi (2009)).

Maximized utility by consumers' utility maximization (2), is expressed as a function of income, price of retail stores and number of goods, giving the indirect utility function

$$V_i = \ln\left(\frac{\mu}{p^*}\right)^\mu (1-\mu)^{1-\mu} + \ln(Y-L_i) + \frac{\mu}{\sigma-1} \ln f_i. \quad (4)$$

Equation (4) is derived from maximizing the utility of an  $n$ th consumer who goes shopping in region  $i$ . The first term in eq. (4) is a function of the good price; the second term is that of income. The third term is that of the number of kinds of goods. In eq. (4),  $L_i$  is a function of  $n$ , which represents the place where the consumer lives, whereas  $f_i$  is a function of market boundary  $b$  (see Appendix A). Therefore, we have

$$V_i \equiv V_i(n, b),$$

which is a utility function of an  $n$ th consumer to visit region  $i$  when the market boundary is  $b$ .

## 2.4 Retail store behavior

Each retail store supplies a horizontally differentiated good under conditions of free entry and exit. Under monopolistic competition, they do not supply the same kind of good as the others. Therefore, their number in a region is equivalent to the number of kinds of goods.

The technology is the same in both regions: it involves a fixed input cost  $F$  and marginal input cost requirement  $c$ . Consequently, the production of a quantity  $q$  of any good at any location requires the cost given as  $F + cq$ .

Considering a particular retail store supplying a specific good, its profit,  $\pi$ , is given as

$$\pi = pq - F - cq,$$

where  $p$  is the mill price. Its price index,  $G_i$ , is given for each retail store because it is determined according to all retail stores in region  $i$ . The perceived elasticity of demand is therefore  $\sigma$ . Consequently, the first order condition of profit maximization implies that equilibrium price  $p^*$  is

$$p^* = \frac{\sigma}{\sigma-1} c \quad (5)$$

for all retail stores. Given the pricing rule, the profit is

$$\pi = \frac{1}{\sigma-1} cq - F.$$

Therefore, the zero-profit condition implies that equilibrium output  $q^*$  is

$$q^* = \frac{F(\sigma-1)}{c}. \quad (6)$$

It is constant for every active retail store in the economy.

The number of kinds of goods in each region,  $f_i$ , depends on the demand in each region. Put differently,  $f_i$  is a function of the market area size, equivalent to the number of consumers who visit the region. Differentials of  $f_1$  and  $f_2$  with respect to their own market areas are

$$\frac{\partial f_1}{\partial b} = \frac{\mu}{F\sigma}(Y - b - t_1) > 0 \quad \text{and} \quad \frac{\partial f_2}{\partial (\bar{N} - b)} = \frac{\mu}{F\sigma}(Y - (\bar{N} - b) - t_2) > 0^7.$$

They show that the expansion of the market area increases the variety of goods in the region. It corresponds to such a situation that increasing consumers makes the commercial area more attractive for retail stores from the standpoint of profit and the start of new retail store operations.

### 3 Market equilibrium and social optimum

#### 3.1 Market boundary under market equilibrium

Let  $b^m$  be the market boundary under market equilibrium. If  $0 < b^m < \bar{N}$ , then consumers at  $n \in (0, b^m)$  go shopping in region 1, whereas consumers at  $n \in (b^m, \bar{N})$  go shopping to region 2. We call this market equilibrium ‘interior equilibrium’ (*IE*). If  $b^m = 0$  or  $b^m = \bar{N}$ , then all consumers go to one region for shopping, region 1 ( $b^m = \bar{N}$ ) or region 2 ( $b^m = 0$ ). We call this market equilibrium ‘corner equilibrium’ (*CE*).

Under *IE*, the consumer on a market boundary is indifferent to visiting either region. If the utility obtained from visiting one region is higher than that of another region for all consumers, then the *IE* does not exist and all consumers go to one region for shopping, which corresponds to *CE*. The condition of *IE* and *CE* is expressed as the following.

$$IE: b^m \in (0, \bar{N}) \quad \text{and} \quad V_1(n = b^m, b^m) = V_2(n = b^m, b^m)^8 \quad (7a)$$

<sup>7</sup> The derivation is explained in Appendix A.

<sup>8</sup>  $V_i(n = b^m, b^m)$  shows the utility of a consumer on market boundary when he or she visits region  $i$ .

$$CE: b^m = 0 \text{ and } V_1(n=0,0) < V_2(n=0,0),$$

$$\text{or } b^m = \bar{N} \text{ and } V_1(n=\bar{N},\bar{N}) > V_2(n=\bar{N},\bar{N}) \quad (7b)$$

A stability condition is necessary for the  $IE (b^m \in (0, \bar{N}))$ . It is shown as

$$\partial(V_1(n=b,b) - V_2(n=b,b)) / \partial b < 0, \quad (8)$$

which expresses that any consumer's change in a region for shopping decreases their own utility level. In this situation, no consumer has an incentive to change the shopping destination.

To capture the configuration of  $V_i(n=b,b)$  with respect to market boundary, differentiating  $V_1(n=b,b)$  and  $V_2(n=b,b)$  with respect to their own market areas  $b$  and  $\bar{N}-b$  yields

$$\frac{\partial V_1(n=b,b)}{\partial b} = -\frac{1}{Y-b-t_1} + \frac{\mu}{\sigma-1} \frac{\partial f_1}{\partial b} \frac{1}{f_1} \text{ and} \quad (9a)$$

$$\frac{\partial V_2(n=b,b)}{\partial(\bar{N}-b)} = -\frac{1}{Y-(\bar{N}-b)-t_2} + \frac{\mu}{\sigma-1} \frac{\partial f_2}{\partial(\bar{N}-b)} \frac{1}{f_2}. \quad (9b)$$

The first terms in eqs. (9a) and (9b) are negative, whereas the second terms are positive. We can derive the following properties:

$$\partial V_1(n=b,b) / \partial b \rightarrow +\infty \text{ when } b \rightarrow 0, \quad \partial V_2(n=b,b) / \partial(\bar{N}-b) \rightarrow +\infty \text{ when } \bar{N}-b \rightarrow 0,$$

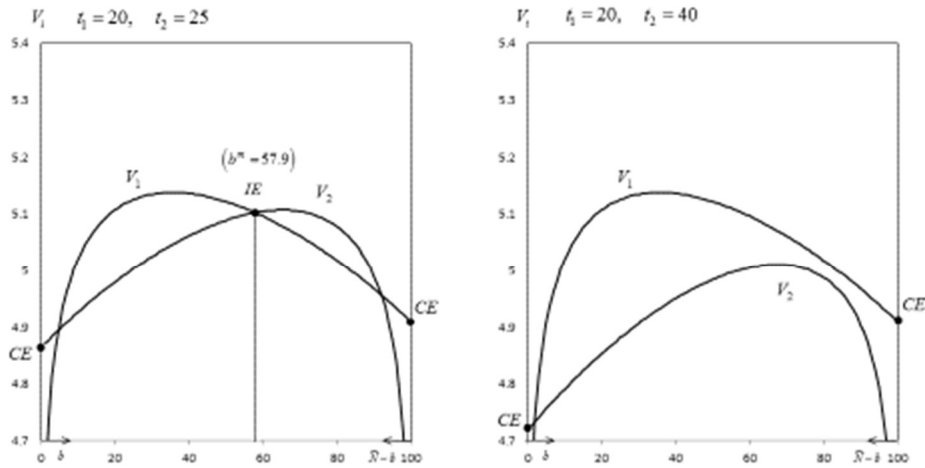
$$\partial^2 V_1(n=b,b) / \partial b^2 < 0, \text{ and } \partial^2 V_2(n=b,b) / \partial(\bar{N}-b)^2 < 0.$$

Therefore,  $V_i(n=b,b)$  is concave with respect to  $b$ .

Figure 2 shows examples of the configuration of  $V_i(n=b,b)$  derived by numerical simulation. In Fig. 2, the horizontal axis expresses the market area of each region:  $b$ , starting from the left, is the market area of region 1, equivalent to the number of consumers who visit region 1, and  $\bar{N}-b$ , starting from the right, is the market area of region 2, equivalent to the number of consumers who visit region 2. The difference in shape between  $V_1(n=b,b)$  and  $V_2(n=b,b)$  arises only from the difference of  $t_i$ ,

transportation cost depending on transportation facilities, which is included in the second and third terms in eq. (4)<sup>9</sup>.

The left side in Fig. 2 shows the case in which the difference between  $t_1$  and  $t_2$  is not so large ( $t_1 = 20, t_2 = 25$ ), whereas the right side is the case in which the difference between  $t_1$  and  $t_2$  is large ( $t_1 = 20, t_2 = 40$ ). The remaining exogenous variables and parameters are fixed throughout the numerical simulations in this paper ( $\mu = 0.2, \sigma = 2, Y = 250, F = 20, c = 3$ , and  $\bar{N} = 100$ ). The graph on the left side has three intersections of  $V_1(n = b, b)$  and  $V_2(n = b, b)$  whereas the graph on the right side has one intersection.



ie

**Figure 2** Configuration of  $V_i(n = b, b)$  and market boundary with market equilibrium

Among the three intersections in the left-hand graph in Fig. 2, only  $b^m = 57.9$  satisfies the stability condition (8). That means both  $CEs$  and  $IE$  exist if the difference between  $t_1$  and  $t_2$  is small. In the right-hand graph, the intersection does not satisfy the stability condition (8). That means only  $CEs$  exist if the difference between  $t_1$  and  $t_2$  is small.

### 3.2 Market boundary under social optimum

<sup>9</sup> The second and third terms in eq. (4) include  $f_i$ , which is a function of  $t_i$ , as shown in Appendix A.

We define social welfare  $SW$  as the sum of individuals' utility levels shown as

$$SW = \int_0^b V_1(n, b) dn + \int_{\bar{N}-b}^{\bar{N}} V_2(n, b) dn,$$

in which the first term is the sum of utility levels of consumers who visit region 1, whereas the second term is the sum of the utility levels of consumers who visit region 2.

Let  $b^s$  be the market boundary under social optimum.  $SW$  is not concave with respect to  $b$ . Moreover, it is not obvious whether  $SW$  is a monotonically increasing or decreasing function of  $b$ . Therefore, we cannot derive the condition which  $b^s$  satisfies. However, if we assume  $0 < b^s < \bar{N}$ , then  $b^s$  satisfies first-order condition  $\partial SW / \partial b = 0$ , shown as

$$V_1(n, b^s) + EX_1(n, b^s) = V_2(n, b^s) + EX_2(n, b^s), \text{ where} \quad (10)$$

$$EX_1(b) = \int_0^b \frac{\partial V_1(n, b)}{\partial b} dn = \frac{\mu}{\sigma-1} \frac{Y-t_1-b}{Y-t_1-\frac{b}{2}} \text{ and} \quad (11a)$$

$$EX_2(b) = \int_b^{\bar{N}} \frac{\partial V_2(n, b)}{\partial (\bar{N}-b)} dn = \frac{\mu}{\sigma-1} \frac{Y-t_2-(\bar{N}-b)}{Y-t_2-\frac{(\bar{N}-b)}{2}}. \quad (11b)$$

Comparing eq. (7a) to eq. (10), the difference of the condition between market equilibrium and social optimum is the second term in eq. (10), represented as  $EX_1$  and  $EX_2$ . They express that technical externality arises from multipurpose shopping and monopolistic competition. Equations (11a) and (11b) show that the change in utility caused by the infinitesimal change in market area  $\partial V_i(n, b) / \partial b$  applies to all consumers who visit region  $i$ . Put differently, if consumers switch their own personal destination from a marketplace to the other marketplace, then it changes not only their own utility but also that of all other consumers through the change in the number of goods in the region.

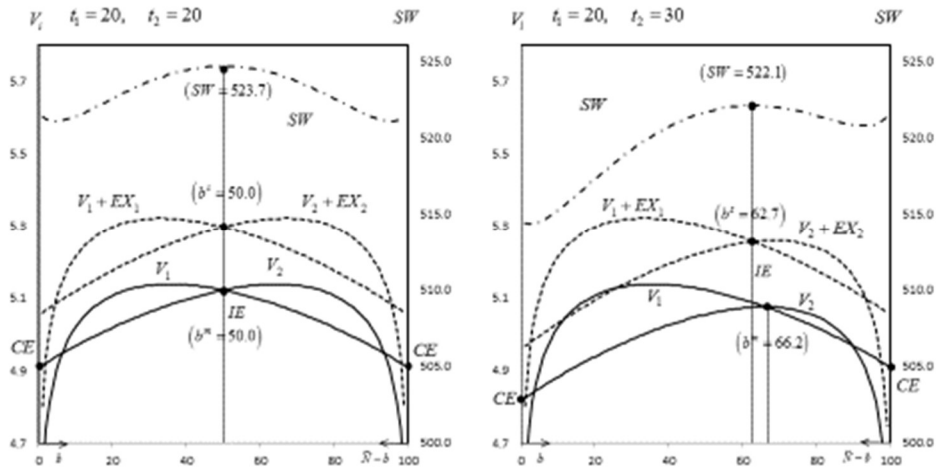
### 3.3 Comparing market equilibrium to social optimum

We quantitatively explore the relation between  $b^m$ , the market boundary with the market equilibrium, and  $b^s$ , that with social optimum.  $b^m$  can be  $IE$  and  $CE$ , therefore we determine them individually.

To capture the configuration of  $V_i(n=b,b)+EX_i(b)$  with respect to market boundary, we must capture the configuration of  $EX_i(b)$ . We can derive the following properties from eq. (11a) and (11b):  $EX_1(b) \rightarrow \mu/(\sigma-1)$  when  $b \rightarrow 0$ ,  $EX_2(b) \rightarrow \mu/(\sigma-1)$  when  $\bar{N}-b \rightarrow 0$ ,  $\partial EX_1/\partial b < 0$ , and  $\partial EX_2/\partial(\bar{N}-b) < 0$ . From these properties, we can draw the shape of  $V_i(n=b,b)$  and  $V_i(n=b,b)+EX_i(b)$  and corresponding  $SW$  as Fig. 3, which is derived through numerical simulation. In Fig. 3,  $V_i(n=b,b)$  is drawn as a solid line,  $V_i(n=b,b)+EX_i(b)$  is drawn as a dashed line, and the corresponding  $SW$  is drawn as a chain line.

In Fig. 3, three stable market boundaries with market equilibria are shown. One is  $b^m$  which corresponds to the intersection of  $V_1(n=b,b)$  and  $V_2(n=b,b)$  (interior equilibrium,  $IE$ ); the others are  $b^m = 0$  and  $b^m = \bar{N}$  (corner equilibrium,  $CE$ ). Socially optimum market boundary  $b^s$  is the intersection of  $V_1(n=b,b)+EX_1(b)$  and  $V_2(n=b,b)+EX_2(b)$ <sup>10</sup>.

The  $CEs$  ( $b^m = 0$  and  $b^m = \bar{N}$ ) can be stable market equilibria in Fig. 3. Either one of the two  $CEs$  ( $b^m = 0$  or  $b^m = \bar{N}$ ) implies that consumers are caught in a “utility trap”. At each  $CE$ , an individual’s change in shopping region decreases that individual’s utility level. Therefore no one changes their shopping region, and  $SW$  is far from the optimum level.



**Figure 3** Configuration of  $V_i(n=b,b)+EX_i(b)$  and corresponding  $SW$  ( $IE$  and  $CEs$ )

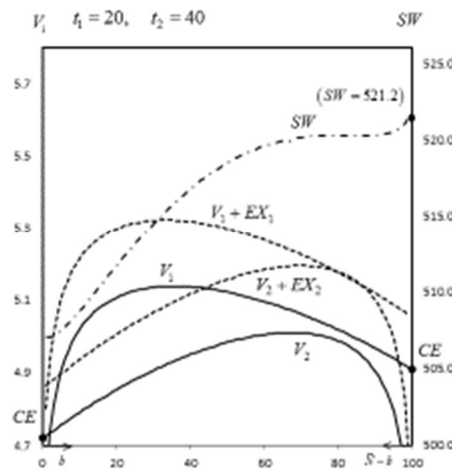
<sup>10</sup> Numerical simulation confirms that the social optimum is the interior solution.

However, if some large number of individuals change shopping regions simultaneously, then the utility level of each increases. They extricate themselves from a “utility trap” and the market boundary heads for *IE*.

The left side in Fig. 3 shows the case in which region 1 and region 2 are homogeneous;  $t_1 = t_2$  ( $t_1 = t_2 = 20$ ). In this case,  $V_1(n = b, b)$  and  $V_2(n = b, b)$  are symmetric. Therefore  $b^m$  and  $b^s$  are just middle:  $b^m = b^s = \bar{N}/2$ . The right side in Fig. 3 shows the case in which  $t_1 < t_2$  ( $t_1 = 20, t_2 = 30$ ). The increase in  $t_2$  affects the decrease in budget for goods in consumers who visit region 2. Therefore  $V_2(n = b, b)$  moves downward. In consequence,  $b^m$  and  $b^s$  are moved to the right.

In our model, the difference between  $b^m$  and  $b^s$  is determined by the difference between  $t_1$  and  $t_2$ . It is theoretically derived that  $b^m > b^s$  if  $t_1 < t_2$  and  $b^m < b^s$  if  $t_1 > t_2$ . The derivation of this property is shown in Appendix B.

Because of monopolistic competition and shopping externality, socially optimal  $b^s$  differs from  $b^m$  if the transportation cost to region 1 and region 2 differ ( $t_1 \neq t_2$ ). It is interpreted that a commercial area’s retail stores are agglomerated excessively if the accessibility from the residential area is better than that of another commercial area.



**Figure 4** Configuration of  $V_i(n = b, b) + EX_i(b)$  and corresponding  $SW$  ( $CEs$ )

If the difference between  $t_1$  and  $t_2$  is large, then stable *IE* does not exist and only *CE* ( $b^m = 0$  and  $b^s = \bar{N}$ ) exists. Figure 4 shows the case in which  $t_2$  is much larger than  $t_1$  ( $t_1 = 20, t_2 = 40$ ). The *CE*



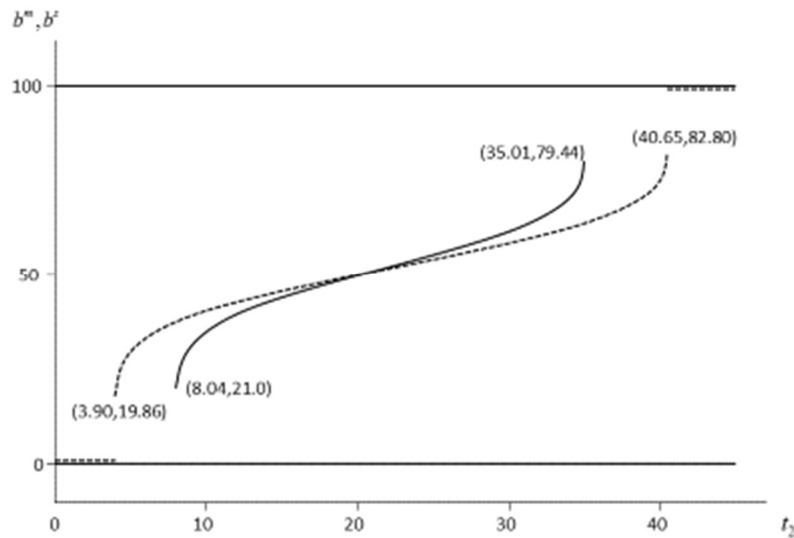
( $b^m = 0$ ) in Fig. 4 implies that consumers are caught in a “utility trap”. At the  $CE$  ( $b^m = 0$ ),  $SW$  is at its minimum. If consumers escape a “utility trap”, then by the same token, the market boundary heads for another  $CE$  ( $b^m = \bar{N}$ ). Numerical simulation clarifies that  $b^s = \bar{N}$ .

#### 4 Transportation improvement and hollowing-out of urban commercial centers

Regarding region 1 and region 2 as the urban center and the outskirts of a city in our model, we express the process of hollowing-out of urban commercial centers because of transportation improvement in suburban area, which corresponds to TF2 in our model.

##### 4.1 Transportation improvement and change in equilibrium

Fig. 5 shows a stable market boundary under market equilibrium and that under social optimum corresponding to the change in  $t_2$ ;  $t_1$  is fixed as  $t_1 = 20$ . Market boundary under market equilibrium is shown as a solid line whereas that under social optimum is shown as a dashed line.



**Figure 5** Bifurcation of market boundary and social optimum

As shown in Section 3,  $CEs$  ( $b^m = 0$  or  $b^m = \bar{N}$ ) exist in any  $t_2$ . Under  $8.04 \leq t_2 \leq 35.01$ ,  $IE$  ( $0 < b^m < \bar{N}$ ) also exists in addition to  $CEs$ .

## 4.2 Emergence of a new marketplace

From a historic perspective, formulation of cities were begun with an agglomeration of commerce in one marketplace. Therefore, almost every city has a city center, which has agglomeration of commerce. In our model,  $CE (b^m = \bar{N})$  corresponds to such formulation of cities that a city has only one agglomeration of retail store in city center (region 1) and all consumers go shopping in the city center (region 1). We assume this  $CE$  as the initial state<sup>11</sup>.

With transportation improvement to region 2 from the initial state,  $t_2$  becomes small and  $V_2(n=b,b)$  moves upward in Fig. 4. Decreasing  $t_2$  from the initial state eventually generates  $IE$ , intersection of  $V_1(n=b,b)$  and  $V_2(n=b,b)$ . Therefore, transportation improvement to region 2 creates the potential for retail stores to locate in region 2 with some other retail stores. If some retail stores locate simultaneously in region 2, then consumers living near region 2 can improve their utility by changing their shopping place from region 1 to region 2. Their utility losses occur by having fewer kinds of goods available in the outskirts, but they can obtain higher utility by saving transportation costs.

The introduction of many new retail stores into region 2 is the formation of a new marketplace in region 2. Formation of a new marketplace at the edge of a residential area is explored in Tabuchi (2009). The increasing number of consumers surrounding a central marketplace expands the residential area to the outskirts. Therefore, transportation costs from the outskirts to the central marketplace increase. Eventually, an increase in population causes the formation of an “edge city”, which is a new marketplace at the edge of a residential area. Our model fixes the population of consumers. However, expanding the residential area to the outskirts is equivalent to that of transportation improvement in region 2 in the sense that the transportation cost for the outskirts becomes low compared to that for the city center for consumers living near the outskirts.

In order to formulate a new marketplace in the outskirts (region 2), each retail store requires a certain number of other retail stores. That is, concerted conglomerate of retail stores can locate in the outskirts (region 2). This conglomerate corresponds to a shopping mall, which provides a wide variety of goods. This bifurcation from  $CE$  to  $IE$ , observed under  $t_2 \leq 35.01$  in Fig. 4, improves social welfare. It means that location of a conglomerate of retail stores in the outskirts should be promoted from the perspective of social welfare improvement.

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<sup>11</sup> Cities are historically formulated around one marketplace as a general rule. Therefore, if the transportation cost for the outskirts is sufficiently large, the corresponding  $CE$  is such that all retail stores are located in the city center.

When  $3.90 \leq t_2 \leq 8.04$ ,  $IE$  does not exist in market equilibrium. However, socially optimal boundary is  $0 < b^m < \bar{N}$ . Therefore, it means that location of a certain number of retail stores in the outskirts should be promoted even though there is no potential for collective retail stores to locate in the outskirts with some other retail stores from the perspective of social welfare maximization.

### 4.3 Controlling hollowing-out and suburbanization

Transportation improvement in the outskirts of a city is a main cause of the hollowing-out of shopping centers and suburbanization of retail stores. In fact, our model demonstrates that a decrease in  $t_2$  makes  $b^m$  smaller. Improving access to the outskirts of a city expands the market area of retail stores in the outskirts in a city.

Comparing the market boundary with the market equilibrium  $b^m$  and that with social optimum  $b^s$ ,  $b^m > b^s$  if  $t_1 < t_2$ , and  $b^m < b^s$  if  $t_1 > t_2$ . If the accessibility for the outskirts is worse ( $t_1 < t_2$ ), then the market area of the outskirts under market equilibrium is smaller than that under social optimum ( $b^m > b^s$ ). Therefore policies for regulation of suburbanization of retail stores or promotion and revitalization of retail stores in the city center decrease social welfare if the accessibility for the outskirts is worse.

Such regulation or promotion improves the utility of consumers who visit city centers through a decrease in transportation cost and increases the variety of retail stores in city centers. However, it simultaneously brings utility loss to consumers who visit the outskirts through a decrease in the variety of retail stores in the outskirts. Under the situation in which accessibility to the outskirts is worse, the utility loss is greater than the improvement of the utility.

Conversely, if the degree of suburbanization is large ( $t_1 > t_2$ ), then the market boundary with market equilibrium is smaller than that with social optimum ( $b^m < b^s$ ). Therefore policies for regulation of suburbanization of retail stores or promotion and revitalization of retail stores in city centers increase social welfare if accessibility to the outskirts is better.

Hollowing-out of an urban center's commerce has been promoted during the past few decades in developed countries as described early in this paper. In such countries, road construction and improvement in outlying areas is well underway. Therefore we can regard suburbanization of retail stores as excessive and the regulation of suburbanization of retail stores or promotion and revitalization of retail stores in the city center as effective from the perspective of social welfare improvement.

#### 4.4 Complete suburbanization and hollowing-out

Under circumstances in which transportation cost for region 1 and region 2 are similar, the *IE* comes into existence, retail stores locating in both regions. We assume that retail stores locate in both regions ( $0 < b^m < \bar{N}$ ) under  $8.04 < t_2 < 35.01$ , given the initial state.

Promoting transportation improvement for region 2 from the initial state,  $t_2$  becomes small and  $V_2(n=b,b)$  moves upward in Fig. 2. Decreasing  $t_2$  from the initial state eventually eliminates *IE* and generates a *CE*. If consumers cooperate to move in search of a better equilibrium, then the *CE* by which all retail stores locate in region 2 comes into existence. This change in equilibrium from the *IE* ( $0 < b^m < \bar{N}$ ) to the *CE* ( $b^m = 0$ ) by the decrease in  $t_2$  engenders complete suburbanization and hollowing-out of urban commercial centers.

Complete suburbanization and hollowing-out of urban commercial centers brings improvement of social welfare because welfare improvement by greater availability of goods in region 2 and saving of transportation cost to region 2 exceeds the welfare loss incurred by the increase in transportation cost for consumers living near region 1.

Under  $35.01 \leq t_2 \leq 40.65$ , *IE* does not exist in market equilibrium. However, the socially optimal boundary is  $0 < b^m < \bar{N}$ . Therefore, it means that location of a certain number of retail stores in the urban center should be promoted even though there is no potential for collective retail stores to locate in the urban center with some other retail stores from the perspective of social welfare maximization.

Retail store concentration in the outskirts is often subject to control<sup>12</sup> because suburbanization and the hollowing-out of urban commercial centers are believed to be inefficient from the conventional perspective of social welfare. However, our model shows that it is desirable given the existence of retail store monopolistic competition and shopping externality.

## 5. Conclusion

We have derived the mechanics generating a divergence of market boundary of retail stores between the market equilibrium and social optimum by constructing a model that introduces market failure. Although market failures of several kinds exist, we have specifically described two of them:

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<sup>12</sup> Some policies have been implemented to promote urban center revitalization: improving transportation accessibility of an urban center and compact city policy are typical policies. Compact city policy leads residents, retail stores, and public facilities such as schools and hospitals to locate in a city center against the unregulated suburbanization and hollowing-out of the commercial center.

shopping externality for consumers and monopolistic competition of retail stores, which have greater influence on retail store location.

Depending on the difference in  $t_1$  and  $t_2$ , stable multi-equilibria exist under the market equilibrium. If the difference is large, then only two *CEs* ( $b^m = 0$  and  $b^m = \bar{N}$ ) exist. One is equal to the social optimum. If the difference is small, then both *IE* and *CEs* exist. In this case, consumers can be caught in a “utility trap”<sup>13</sup>. If one of the two *CEs* ( $b^m = 0$  or  $b^m = \bar{N}$ ) comes into existence, then social welfare is less than *IE*.

Even if *IE* comes into existence under a market equilibrium, it is different from the social optimum except for the case in which both regions are indifferent ( $t_1 = t_2$ ). We showed that a commercial area’s market area is excessive if the accessibility from residential areas is better than that to another commercial area.

We make three conclusions relating our theoretical conclusion with some issues of retail store location and social welfare change caused by transportation improvement. The first conclusion shows that, if the transportation cost to the city center is sufficiently smaller than that to the suburbs, then all retail stores concentrate in the city center. Given such a central agglomeration case, a decrease in transportation cost to the suburb can spur the emergence of suburban retail stores. This emergence invariably increases social welfare, which implies that an improvement in transportation to suburbs is desirable if it generates an increase in stores in the suburbs. In that case, policy-makers should not restrict suburban locations.

The second conclusion shows that if the transportation cost to the city center is smaller than that to the suburb, the number of retail stores in the city center is greater than that in a social optimum, which implies that, in that case, promotion of retail stores’ location in the city center such as subsidies to locate in the city center or restrictions of location in the suburbs, worsens social welfare. Instead, promotion of stores’ location in the suburbs is preferred.

Third, transportation improvement for the suburbs eventually generates complete suburbanization and hollowing-out of urban commercial centers, which is optimal from the perspective of social welfare. Complete suburbanization and hollowing-out of urban commercial centers brings a greater variety of goods in suburbs and savings of transportation costs to the suburbs, with benefits

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<sup>13</sup> If there are many commercial areas, the probability of a utility trap is low because consumers can extricate themselves from a utility trap with a small quantity of individuals changing their shopping region at the same time. They can exit a utility trap by cooperating in choosing where to shop. However, few actual commercial areas exist for consumers’ usual shopping behaviour, and it seems unrealistic that consumers will cooperate, even if the number of consumers is fairly small. Therefore, consumers can be caught in a utility trap.

exceeding the shortcomings of hollowing-out of urban commercial centers.

Suburbanization and hollowing-out of urban commercial centers is believed to be inefficient from the conventional perspective of social welfare. Therefore, some policies have been implemented against suburbanization in many developed countries. Nevertheless, it is desirable in our model, because our model incorporates shopping externality and monopolistic competition among retail stores as factors exacerbating market failure; some other factors of market failure, such as agglomeration economies and congestion externality, are not addressed in the model. However, these factors, such as agglomeration economies and congestion externality, make little impact on location choices of individual retail stores.

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## APPENDICES

### A. Derivation of differentials of $f_1$ and $f_2$

$p(j)m(j)$ , the consumer expenditure on a good, is derived through formulation of consumer behavior as

$$p(j)m(j) = p(j) \left( \frac{p(j)}{G_i} \right)^{-\sigma} M_i, \quad (A1)$$

where  $M_i = \mu(Y - L_i)/G_i$  is the solution of consumers' utility maximization (2). Substituting  $M_i = \mu(Y - L_i)/G_i$  and eq. (3) into (A1) yields  $p(j)m(j) = \mu(Y - n - t_1)/f_1$  for a good supplied in region



1 and  $p(j)m(j) = \mu(Y - (\bar{N} - n) - t_2) / f_2$  for a good supplied in region 2. The sum of the expenditure with respect to all consumers visiting each region is equal to the sales turnover of each retail store  $p^*q^*$  when the market boundary is  $b$  ( $0 \leq b \leq \bar{N}$ ). This condition is shown as

$$p^*q^* = \int_0^b \frac{\mu}{f_1} (Y - n - t_1) dn \text{ in region 1 and } p^*q^* = \int_{\bar{N}-b}^{\bar{N}} \frac{\mu}{f_2} (Y - (\bar{N} - n) - t_2) dn \text{ in region 2.}$$

Substituting eq. (5) and (6) into this condition yields

$$f_1 = \frac{\mu}{F\sigma} \left( (Y - t_1)b - \frac{b^2}{2} \right) \text{ and } f_2 = \frac{\mu}{F\sigma} \left( (Y - t_2)(\bar{N} - b) - \frac{(\bar{N} - b)^2}{2} \right).$$

Therefore, the differentials of  $f_1$  and  $f_2$  with respect to their own market area are

$$\frac{\partial f_1}{\partial b} = \frac{\mu}{F\sigma} (Y - b - t_1) > 0 \text{ and } \frac{\partial f_2}{\partial (\bar{N} - b)} = \frac{\mu}{F\sigma} (Y - (\bar{N} - b) - t_2) > 0.$$

## B. Derivation of Property 2.

If  $EX_1(b)$  is larger/smaller than  $EX_2(b)$  at  $b^m$ , then  $V_1(n=b, b) + EX_1(b)$  is larger/smaller than  $V_2(n=b, b) + EX_2(b)$  at  $b^m$  because  $V_1(n=b, b) = V_2(n=b, b)$ . Therefore  $V_1(n=b, b) + EX_1(b)$  and  $V_2(n=b, b) + EX_2(b)$  intersect a point that is larger/smaller than  $b^m$ . Accordingly, whether  $b^m$  is larger than  $b^s$  or not depends on whether or not  $EX_1(b)$  is greater than  $EX_2(b)$  at  $b^m$ .

$\Delta EX(b^m) = EX_1(b^m) - EX_2(b^m)$  is derived from eqs. (11a) and (11b) as

$$\Delta EX(b^m) = \frac{\mu}{\sigma - 1} \frac{(\bar{N} - b^m)(Y - t_1) - b^m(Y - t_2)}{2 \left[ Y - t_1 - \frac{b^m}{2} \right] \left[ Y - t_2 - \frac{(\bar{N} - b^m)}{2} \right]}. \quad (\text{B1})$$

The denominator of eq. (A1) is positive. Therefore the sign of the numerator determines the sign of  $\Delta EX(b^m)$ .

To ascertain the sign of the numerator in eq. (B1), we use some related expressions. First, from  $V_1(n=b^m, b^m) = V_2(n=b^m, b^m)$  and eq. (4), we derive

$$\ln \left( \frac{\mu}{p^*} \right)^\mu (1 - \mu)^{1-\mu} + \ln(Y - L_1) + \frac{\mu}{\sigma - 1} \ln f_1 = \ln \left( \frac{\mu}{p^*} \right)^\mu (1 - \mu)^{1-\mu} + \ln(Y - L_2) + \frac{\mu}{\sigma - 1} \ln f_2$$

$$\Rightarrow \frac{Y - t_1 - b^m}{Y - t_2 - (\bar{N} - b^m)} = \left( \frac{f_2}{f_1} \right)^{\frac{\mu}{\sigma-1}}. \quad (\text{B2})$$

Second, as explained in the text, the decrease in  $t_1$  affects the increase in budget for goods of consumers who visit the region. Therefore  $V_1(n=b, b)$  moves upward. In consequence,  $b^m$  becomes larger and  $f_1$  becomes larger because  $\partial f_1 / \partial b > 0$ . Therefore, if  $t_1 < t_2$ , then  $f_1 > f_2$ .

If  $t_1 < t_2$ , then  $f_1 > f_2$ . Therefore  $Y - t_2 - (\bar{N} - b^m) > Y - t_1 - b^m$  from eq. (B2). This is altered as  $Y - t_1 > Y - t_2 - (\bar{N} - 2b^m)$ . Substituting this relation into the numerator in eq. (B1), we obtain the following relation:

$$(\bar{N} - b^m)(Y - t_1) - b^m(Y - t_2) < (\bar{N} - b^m)[Y - t_2 - (\bar{N} - 2b^m)] - b^m(Y - t_2).$$

Altering the right hand side in this relation yields

$$(\bar{N} - b^m)(Y - t_1) - b^m(Y - t_2) < (\bar{N} - 2b^m)[Y - t_2 - (\bar{N} - b^m)],$$

of which the first term on the right side is negative because  $b^m > \bar{N}/2$ . The second term on the right side is positive. Therefore, we can derive the following relation:

$$(\bar{N} - b^m)(Y - t_1) - b^m(Y - t_2) < (\bar{N} - 2b^m)[Y - t_2 - (\bar{N} - b^m)] < 0.$$

Accordingly,  $\Delta EX(b^m) < 0$  if  $t_1 < t_2$ . Therefore, we can obtain the following relation:

$$b^m > b^s \text{ if } t_1 < t_2.$$

In the same manner, the remaining property ( $b^m < b^s$  if  $t_1 > t_2$ ) is also derived.