Stock Market Volatility Analysis: A Case Study of TUNindex

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17 March 2020

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Abstract

Volatility is directly associated with risks and returns. This study aims to examine the volatility characteristics on Tunisian stock market index (5 days a weak TUNindex) that include clustering volatility, leptokurtosis, and leverage effect. The first objective is then to use the GARCH type models to estimate volatility of the daily returns series, consisting of 2191 observations from 01/02/2011 to 19/11/2019, with no significant weekdays effect. We use both symmetric and asymmetric models. The main findings suggest that the symmetric GARCHM and asymmetric TGARCH /APGARCH models can capture characteristics of TUNindex whereas EGARCH reveals no significant support for leverage effect existence. Looking at news impact curves, GJR model appears to be relatively better than other models. However, the volatility of stock returns is more affected by the past volatility than the related news from the previous period. The second objective is to use **GARCHM- X S** models to capture the effect of macro-economic instability via exchange rate growth and exchange rate volatility. For policy, GARCHM-XS2 turned to be the best model. The macroeconomic environment should be favourable to ensure growth in the stock market. Policies to reduce volatility in the the economy (more stable exchange rate) are a necessity for stock market.

Keywords : Tunisia, Stock Market, Tunindex, Volatility, Symmetric and Asymmetric GARCH Models, GARCH, TGARCH, GARCH-M, EGARCH, GARCHM-XS, Leverage Effect., Risk Premium, Stability.

JEL Codes: C22, D81, D82, E44, E47, O16.

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1. Introduction

The volatility analysis of stock markets is important for the investors in measuring and managing market risks more accurately which, in turn is useful in pricing capital assets, financial securities, and selecting portfolios. When volatility persists, securities firms are less able to use their available capital efficiently because of the need to reserve a larger percentage of cash-equivalent investments in order to reassure lenders and regulators; and greater volatility can reduce investor confidence in investing in stocks (Edwards, 2006). If volatility is changing at higher rate, it may results in high profits or huge losses (Hemanth & Basavaraj, 2016), and this should be boosted by providing empirical evidence from appropriate models.

Volatility is an important input to many investment decisions and portfolio selection.understanding the pattern of stock market volatility is important to investors as well as for investment policy. Volatility is directly associated with risks and returns. A large number of empirical studies have been accomplished to address the concept of volatility of stock markets using the family of ARCH/GARCH processes.

ARCH and GARCH models are used to capture both volatility and leptokurtosis. The so called “leverage effect” is also often observed in the financial time series (see (Black, (1976))). This usually occurs when **stock price changes** are negatively correlated with **changes in volatility**. Since ARCH and GARCH models are symmetric in nature, they fail to capture the leverage effect. In order to address this problem, many **nonlinear** extensions of the GARCH models have been proposed. These include **asymmetric class** of GARCH models such as exponential GARCH (EGARCH) model by (Nelson, 1991) the so-called GJR model by (Glosten, Jagannathan, & Runkle, 1993) and the power GARCH (APGARCH) model by (Ding, Engle, & Granger, 1993). In the light of these observations in the financial time series, a wide range of varying variance models have been used to estimate and predict volatility.

The aim of this paper is to use the General Autoregressive Conditional Heteroscedastic (GARCH) type models to estimate volatility of the daily returns of the Tunisian stock market: that is Tunindex. The volatility of the Tunisian stock market is modeled using daily return series consisting of 2191 observations from 01/02/2011 to 19/11/2019. ARCH effects test confirmed the use of GARCH family models. We use both symmetric and asymmetric models : GARCH(1, 1), GARCH-M(1, 1), EGARCH(1, 1), TGARCH(1, 1), PGARCH(1, 1) and APGARCH(1, 1) to capture the most common features of the stock market like leverage effect and volatility clustering. We consider also **GARCHM- X S-** models to capture the effect of macro-economic instability via exchange rate growth and exchange rate volatility. Post-estimation test for further ARCH effects were done for each model to confirm its adequacy. Also LR and LM test are used to select the more adequat model.

The rest of this paper is organized as follows. Following this introduction, Section 2 provides a brief empirical review of the methodology of modeling volatility using some well known symmetric and asymmetric GARCH models. A data description, summary statistics, and analysis is provided in Section 3. Methodology is given in section 4. The results of the estimated GARCH type models are discussed in Section 5. Lastly, section 6 concludes the paper.

2. Empirical review

The volatility analysis of stock markets is important for the investors in measuring and managing market risks more accurately which, in turn is useful in pricing capital assets, financial securities, and selecting portfolios. The main methodologies that are applied in modelling the stock market volatility are ARCH models introduced by (Engle R. F., 1982) and generalized as GARCH by (Bollerslev T. , 1986). The progress in such studies is generally provided for the purpose of estimation and prediction of the conditional variance of stock returns over the specified period.

Volatility is generally higher after the stock market falls than after it raises. Therefore, volatility of returns has an asymmetric predictable response to the changes in stock prices (it increases more when stock prices fall than when stock prices raise). So that there is a negative correlation between volatility and returns. This is so-called ***leverage effect*** and was reported by Black (1976).[1] However, (Black, (1976)) and (Schwert, 1989) found empirically that leverage alone can not explain all the asymmetry. Asymmetric ARCH (AARCH) by (Engle, Ito, & Lin, 1990), Exponential GARCH model (EGARCH) of (Nelson, 1991), Threshold ARCH model (TARCH) proposed by (Zakonian, 1990) and its modified version of (Glosten, Jagannathan, & Runkle, 1993) (GJR) are able to capture this predictable asymmetric effect.

For instance, the reader might get benefit from the research done by (Ahmed & Suliman, 2011), (Naimy, 2013), (Shamiri & Isa, 2009), (Kalu, 2010), and (Maqsood, Safda, & Shafi,, 2017). They used some models from GARCH family both symmetric and asymmetric to capture the stock market volatility. (Ahmed &

[1] When leverage of firms increases, uncertainty increases too.

Suliman, 2011) worked with the reference of Sudan stock market, while (Kalu, 2010) provides the volatility analysis of Nigerian stock exchange. Modeling volatility of Paris stock market using GARCH (1, 1) and compared with exponential weighted moving average (EWMA) was done by (Naimy, 2013). Similarly, (Shamiri & Isa, 2009) provide the comparison of usual GARCH model with the non linear asymmetric NAGARCH models based on Malaysian stock market. Table 6 give a sum up of more empirical review (sse Appendice).

3. Data Description and Basic Statistics

The time series data used for modeling volatility in this paper is the daily Tunisian Securities Exchange (TUNindex) index (5 days a weak) over the period from 2/01/2011 to 11/19/2019, resulting in total observations of 2191 observations from excluding public holidays. Figure 1 give daily TUNindex and exchange rate evolution for this period. We could see from the graph that there were larger fluctuations in the both series during 2017 until 2019 compared with the period between 2011 and 2016.

The daily returns (R_t) are calculated as the continuously compounded returns which are the first differences of log index of TUNindex of successive days: We used time series data sourced from Bourse de Tunis of Tunisia. We denote by

$$R_t = LSP_t - LSP_{t-1} = \Delta\, LSP_t,$$

where

$$\text{LSP} = \log(\text{TUNindex})$$

the return of stock market price TUNindex, where LSP_t and LSP_{t-1} are the t and $t-1$ th day Stock price in log.

Returns over the period is graphically shown below at Figure 2 (a).

The descriptive analysis of the underlying variables was carried out to check the characteristics of the series. Figure 2 (b) and (d) shows summary statistics of stock market return, R, and exchange rate, Exrate. Statistics consist of the daily sample mean return, standard deviation, minimum return and maximum return, skewness, kurtosis, and JB statistic.

The mean return is 0.000138 with the standard deviation of 0.005157. The mean exchange rate is 2.035 with the standard deviation of 0.499. For instance, the standard deviations indicate that Exchange Rate is more unstable/volatile compared with Stock Market return (R). There is also an excess in kurtosis as can be seen clearly for TUNindex returns. A high value of kurtosis 15.447 indicates a leptokurtic distribution that is an apparent departure from normality.

Another important test of normality is the Jarque-Bera (JB) statistic, which reject the null hypothesis of normality for the daily TUNindex returns at 5% level of significance. We can thus summarize that the TUNindex return series do not conform to normality but actually tend to have negative skewness (i.e. the distribution has not a thick tail).

Figure 2 (a) show us that there is evidence of volatility clustering, meaning that large or small asset price changes tend to be followed by other large or small price changes of either sign (positive or negative). This implies that stock return volatility changes over time.

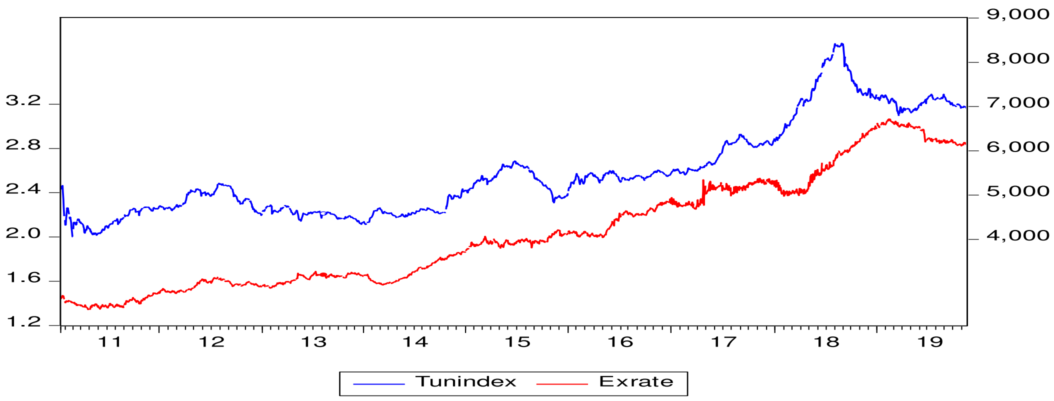


Figure 1: Daily mouvement of TUNindex and Exchange rate (Exrate)

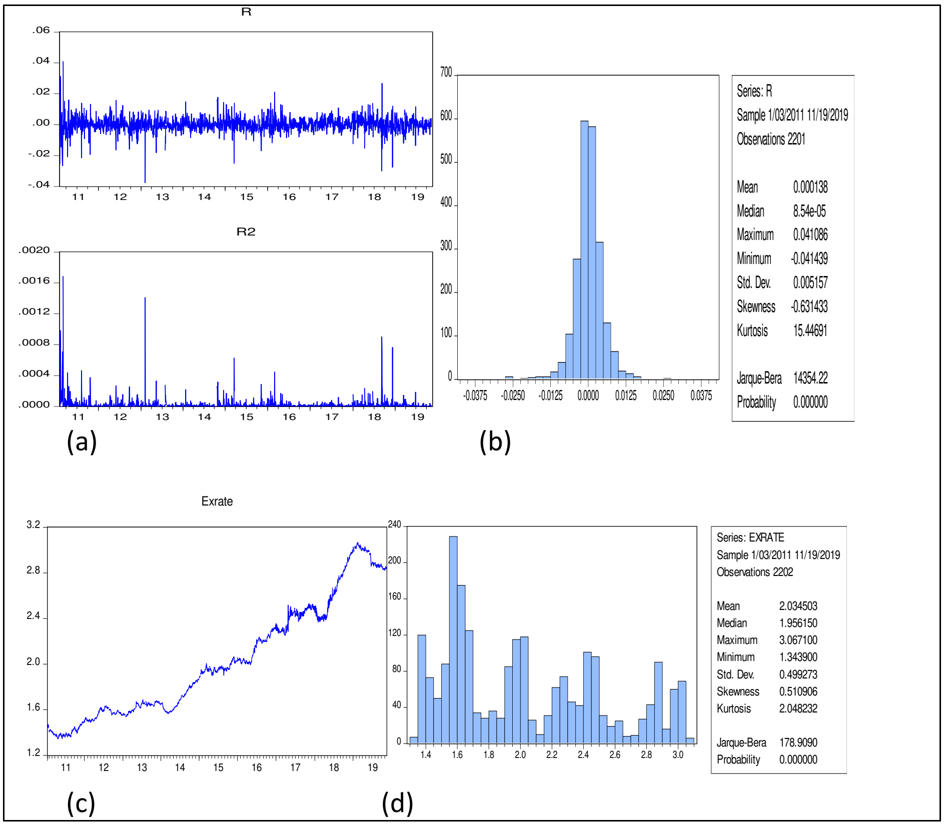


Figure 2: Return R_t and Exrate evolution and statistics

Figure 3 give evolution of exchange rate growth volatily (VEXG) and *Figure 4* illustrate evolution of VEX_t^- (and VEX_t^+) the partial sums of negative (and positive) changes in volatility of exchange rate (VEX) as defined by :

$$VEX_t^- = \sum_{j=1}^{t} \Delta VEX_j^- = \sum_{j=1}^{t} \min(\Delta VEX_j, 0),$$

and

$$VEX_t^+ = \sum_{j=1}^{t} \Delta VEX_j^+ = \sum_{j=1}^{t} \max(\Delta VEX_j, 0).$$

All these volatilities have different pattern **after 2016**.

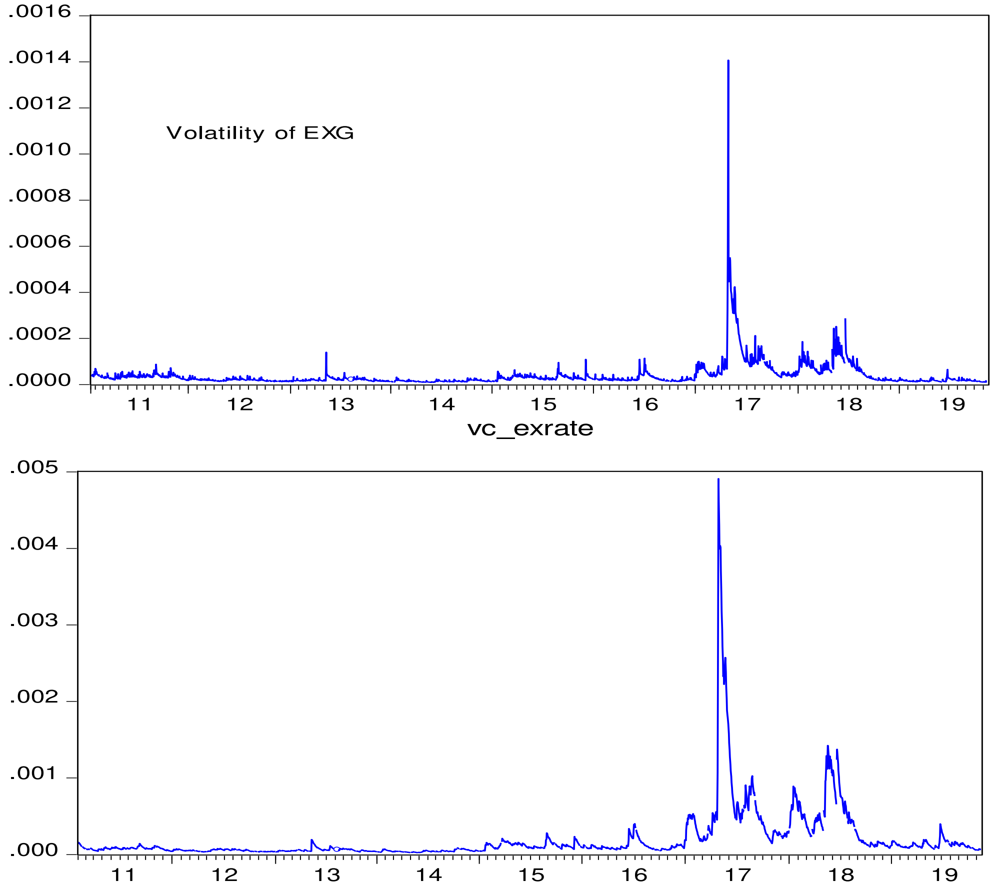
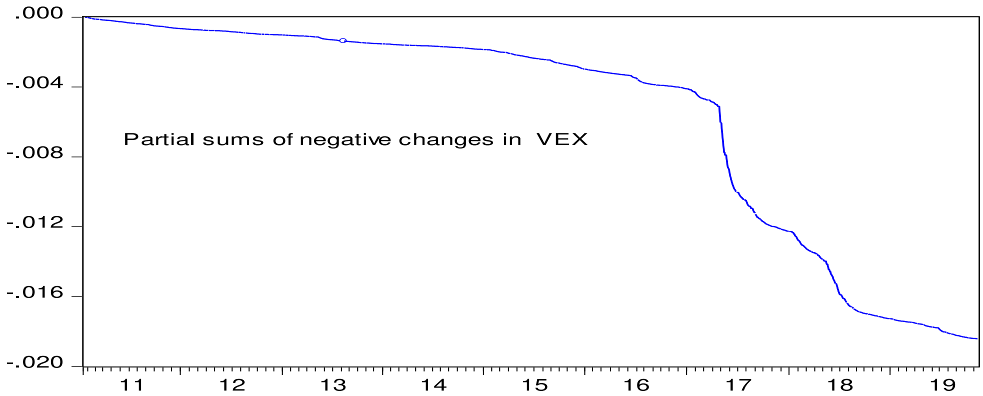
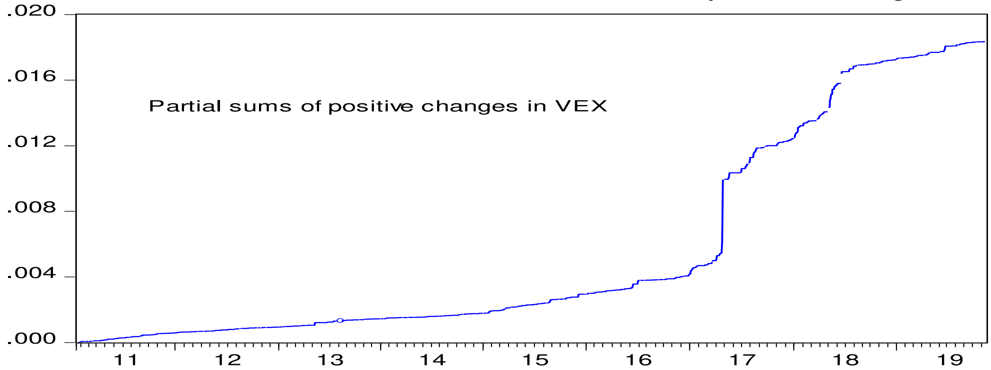


Figure 3: Volatilty of Exchange rate growth (VEXG) from AR(1)-GARCH(2,1) and volatility of Exrate.



VEX_t^- : Partial sums of negative changes in volatility of exchange rate



VEX_t^+ : Partial sums of positive changes in volatility of exchange rate

Figure 4: VEX_t^- and VEX_t^+ for exchange rate volatility, VEX, from

Before the application of AR(p)-GARCH technique, preliminary tests were conducted, such as the stationarity test of the variables (*Tunindex, R_t, and Exrate)* 'using the Augmented Dickey Fuller test (ADF), Philips Perron test (PP) and Kwiatkowski-Phillip-Schmidt-Shin (KPSS) test statistics. The test results are presented in Table 7 (see Appendice).

Table 7 revealed that *Tunindex* and Exrate series are not stationary, however the results for return R_t led towards the rejection of null hypothesis of unit root, and hence stationarity is present in return series.

Finally, it is important to examine the considered serie R_t to find the evidence of possible heteroscedasticity before applying the methodology of modeling conditional variance. In order to test the presence of heteroscedasticity in the TUNindex return series, the Lagrange Multiplier (LM) test will be applied to test the hypothesis that

$$\alpha_1 = \alpha_2 = \ldots = \alpha_q = 0,$$

where q is the order of ARCH effect.[2]

[2] The test procedure entails first obtaining the residuals u_t from the ordinary least square regression of TUNindex returns on a constant.

Results of LM test for various ARCH order q = 1, 2, 3 are presented in Table 1 which provide strong evidence of rejecting the null hypothesis of constant variance for all lags included. Rejecting H_0 indicates the presence of ARCH effect in the TUNindex returns series R_t and therefore we can conclude that the variance of the return of TUNindex is no-constant for all periods specified.[3]

Table 1 : Results of ARCH-LM test for different values of q.

ARCH order q	Test statistic TR^2	Probability
1	537.4039	0.0000
2	567.5435	0.0000
3	591.8380	0.0000

Notes: This table reports results from (Engle R. F., 1982)'s test of no ARCH, calculated on different lags.

Once the volatility is confirmed in data, we proceed our analysis further to estimate the parameters of both conditional mean and conditional variance equations.

4. Methodology : Conditional Mean and Variance specifications

To determine volatility, Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are widely used. Let R_t denote a real valued discrete time stochastic process and Ψ_{t-1} is the information set of all information through time t. We define the model that contains the features of both conditional mean and conditional variance as given below.

Conditional Mean Equation

$$R_t = E[R_t \;/\; \Psi_{t-1}\;] + u_t, \quad u_t \sim (0, {\sigma^2}_t) \quad (1)$$

Where $\Psi_{t-1}$is the information set, $E[R_t \;/\; \Psi_{t-1}\;] = \mu_t$ is the expression used to model the conditional mean of R_t given that the information through time t – 1, Ψ_{t-1}, which might be an autoregressive (AR) process, moving average (MA) process, or a combination of the two processes termed as ARMA process. The error u_t is assumed to be non constant quantity with respect to time and thus is given by

$$u_t = \sigma_t h_t$$

[3] We assume a constant mean model and the LM test is applied to compute the test statistic value TR^2, where T is the number of observations and R^2 is the coefficient of multiple correlation obtained from regressing the squared residuals on q own lagged values.

where $\sigma^2{}_t = V[R_t \ / \ \Psi_{t-1}]$ and $h_t \sim WN(0, 1)$. We briefly present a number of specifications of GARCH models to represent the situations for expressing the conditional variance.

Several Conditional Variance specifications :

Since the GARCH model was developed, a huge number of extensions and been suggested as a consequence of perceived problems with standard GARCH(p, q) models. First, the non-negativity conditions of parameters may be violated by the estimated model. Second, GARCH models cannot account for leverage effects, although they can account for volatility clustering and leptokurtosis in a series. Finally, the model does not allow for any direct feedback between the conditional variance and the conditional mean. The next few sections will discuss various models that are appropriate to capture the stylised features of volatility (most important extensions of GARCH model), that have been observed in the literature.[4] Pricisely, 3 symmetric and 3 asymmetric model will be presented. In Addition, since Macroeconomic instability can effect stock market price, 3 other models named GARCH-X type are considered when macro-economic variables are introduced in conditional mean and or in conditional variance. If these models are instable, they will be noted by GARCH-XS. And then, 3 instable GARCH-X models are considered.

A_ Symmetric GARCH Models :

1_ GARCH Model :

GARCH model is defined as the linear function of past squared residuals and the lagged conditional variances as given below :

$$\sigma^2{}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u^2{}_{t-i} + \sum_{i=1}^{q} \beta_i \sigma^2{}_{t-i} \quad (1)$$

in which all the coefficients must be positive (the non-negativity conditions), and the condition

$$\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1$$

is needed for covariance **stationarity**, where α_0 is the constant term, α_i , i = 1, …, 0, q are the parameters or coefficients of ARCH specifications, and β_i,i=1, …, p are the parameters or coefficients of GARCH specifications. The q and p are the

[4] Interested readers who wish to investigate further are directed to a comprehensive survey by (Bollerslev, Chou, & Kroner, ARCH Modelling in Finance: A Review of the Theory and Empirical Evidence, 1992).

respective orders of ARCH and GARCH processes. α parameter represents a magnitude effect or the symmetric effect of the model. β measures the **persistence** in conditional volatility irrespective of anything happening in the market. When **β is relatively large, then volatility takes a long time to die** out following a crisis in the market (Alexander, 2009).
The simplest specification of this model is GARCH (1, 1) model, that is,

$$\sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1}.$$

The non-stationarity in variance is the case where $\alpha_1 + \beta_1 \geq 1$ and the unconditional variance of u_t is not defined (negative). Moreover, $\alpha_1 + \beta_1 = 1$ is known as a unit root in variance, termed as IGARCH.

2_ GARCH-M Models :

Many theories in finance involve an explicit tradeoff between the risk and the expected return. The ARCH-in-Mean (ARCH-M) model introduced by (Engle, Lilien, & Robins, 1987) was developed to capture such relationship.

This variant of GARCH family allows the conditional mean of return series to depend on its conditional variance. A simple GARCH-M (1, 1) model is defined by the two equations, the one for conditional mean is given by

$$\mu_t = \mathrm{F}(R_t) + \beta X_t + GARCH\ effect, \quad (2)$$

where

$$\text{GARCH effect} \equiv \lambda\, \sigma^2{}_t, \lambda\, \sigma_t, or\ \lambda\ log(\sigma^2{}_t), \quad (3)$$

$\mathrm{F}(R_t)$ might be an autoregressive (AR) process, moving average (MA) process, or a combination of the two processes termed as ARMA process, and X is a vector of macro-economic variables. While the equation for conditional variance is same as provided by the GARCH (p, q) model. Depending on the sign of λ, an increase in the conditional variance will be associated with an increase or a decrease in the conditional mean. When dealing with market indices, $\boldsymbol{\lambda}$ is seen as a measure of **the risk aversion degree of agents**. If agents are **risk averse** they require a larger expected return from an **asset riskier** within a period when payoffs are riskier, leading to a **positive** sign of $\boldsymbol{\lambda}$. On the other hand, a larger expected return may not be required because investors may want to save more during riskier periods, leading to a **negative** sign of λ, see (Glosten, Jagannathan, & Runkle, 1993).[5]

[5] These authors argued that **positive** or zero relations between returns and volatility come from studies that use the standard GARCH-M model as (French, Schw ert, & Stamb augh, 1987) did. In their work, (Glosten, Jagannathan, & Runkle, 1993) used standard GARCH-M and got a **positive** correlation. However, they used Threshold GARCH model of (Zakonian, 1990) to allow positive and negative innovations to returns to have different impacts on volatility and they got a **negative** correlation. In contrast, (Campbell & Hentschel, 1992) using QGARCH (2,

3_ PGARCH (Power GARCH) :

Unlike the GARCH family, these models capture more regularities like **long memory** effect in just one model. The PGARCH (p, q) specification is as under;

$$\sigma^{\delta}{}_{t} = \alpha_0 + \sum_{i=1}^{q} \alpha_i(|u_{t-i}|)^{\delta} + \sum_{i=1}^{p} \beta_i \sigma^{\delta}{}_{t-i} \quad (4)$$

where δ is the parameter for power term such that δ > 0.

B_ Asymmetric GARCH Models

4_ EGARCH Models :

(Nelson, 1991) proposed the **exponential** GARCH (EGARCH) models particularly designed to allow **positive and negative** shocks to have a different impact on volatility. EGARCH model allows **big shocks** to have a greater impact on volatility than the standard GARCH model" (Engle & Ng, 1993). The EGARCH (p, q) specification is given by

$$log(\sigma^{2}{}_{t}) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left|\frac{u_{t-i}}{\sigma_{t-i}}\right| + \sum_{i=1}^{p} \beta_i log(\sigma^{2}{}_{t-i}) + \sum_{i=1}^{q} \gamma_i \frac{u_{t-i}}{\sigma_{t-i}} \quad (5)$$

where γ_i is the **asymmetric or leverage** effect parameter. This make the leverage effect exponential instead of quadratic, and therefore the estimates of the conditional **variance are garanteed to be non-negative**. The EGARCH model allaws for testing asymmetries and TARCH. Conditional variance is modeled to capture the leverage effect of volatility. If $\gamma_i = 0$, then the model is **symmetric**. When $\gamma_i < 0$, then positive shocks (good news) generate less volatility than negative shocks (bad news) and it implies that the relationship between volatility and returns is negative. When $\gamma_i > 0$, it implies that positive innovations are more destabilizing than negative innovations.

If the relationship between the current return and future volatility is **negative** then γ will be **negative** and hence the leverage effect is confined.

5_ TGARCH Models :

Another important volatility model commonly used to handle the **leverage effect** is the threshold GARCH (TGARCH) model developed by Glosten, Jagannathan, and Runkle in 1993 (noted also by **GJR** model). The TGARCH (p, q) framework of conditional variance is given by

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1)-M, which captures predictable asymetries, found **a positive** correlation for daily excess stock returns.

$$\sigma^2_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u^2_{t-i} + \sum_{i=1}^{p} \beta_i \sigma^2_{t-i} + \sum_{i=1}^{q} \gamma_i u^2_{t-i} I_{t-i} \quad (6)$$

Where $I_{t-i} = 1$ if $u_{t-i} < 0$ and 0 otherwise, γ_i is the parameter of **leverage** effect. If γ_i **= 0** the model collapses to the classical GARCH (p, q) process. Otherwise, when the shock **is positive**, the effect on volatility is α_i (i.e, $I_{t-i} = 0$), *and* when the shock **is negative**, the effect on volatility is $\alpha_i + \gamma_i$ (i.e, $I_{t-i} = 1$). Hence, we can say that for $\gamma_i > 0$, the effect of **bad** news (negative shock) have **larger impact** on conditional variance than does **good** news (it assumes that negative shocks have a higher impact than positive ones).

6_ APGARCH Model

Asymmetric Power GARCH is proposed by (Ding, Engle, & Granger, 1993)**.** Unlike the PGARCH, this model capture **leverage** effects. The APGARCH (p, q) specification is as under;

$$\sigma^\delta_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i (|u_{t-i}| - \gamma_i u_{t-i})^\delta + \sum_{i=1}^{p} \beta_i \sigma^\delta_{t-i} \quad (7)$$

where γ_i is the **leverage** effect parameter (it allows positive and negative **innovations** to have a different effect in the expected volatility), and δ is the parameter for power term such that δ > 0 .[6]

Comparisons or selection of more accurate model can be based on Likelihood ratio (LR) test. Test of GARCH(1, 1) against EGARCH(1, 1) or TGARCH(1, 1) is equivalent to test

$$H_{01} : \gamma_1 = 0,$$

while test against PGARCH(1, 1) model is equivalent to test

$$H_{02} : \delta_1 = 0,$$

and against GARCH-M(1, 1) model is equivalent to test

$$H_{03} : \lambda = 0,$$

while test against APGARCH(1, 1) model is equivalent to test

$$H_{04} : \delta_1 = \gamma_1 = 0.$$

[6] Note that, the APGARCH model includes several other ARCH extensions as special cases :

- The ARCH of (Engle R. F., 1982), when , δ =2, γ_i= 0 (i = 1, ..., p) and β_i= 0 (j = 1,…, p).
- The GARCH of (Bollerslev T. , 1986), when , δ = 2 γ_i = 0 = 0 (i = 1, ..., p).
- The GJR of (Glosten, Jagannathan, & Runkle, 1993), when δ = 2.
- The TARCH of (Zakonian, 1990) when δ = 1.

The LR test statistic is given by

$$LR = -2(LL_R - LL_U) \sim \chi^2_{(m)};$$[7]

where LL_R and LL_U are respectively restricted and unrestricted log-likelihood, and m is the number of restrictions (m = 1 for $H_{0i}, i = 1, 2, 3$ and m = 2 for H_{04}).[8]

C_ GARCHM-XS models

To examine the behavior of the GARCH models, as suggested by (Engle & Ng, 1993), different diagnostics tests for volatility models are used. These tests examine whether we can predict volatility by some variables observed in the past which are not included in the volatility model being considered.[9]

The diagnostics tests are derived by writing the volatility model in a more general form, of which the volatility model under the null hypothesis is a special case :

$$\sigma^2_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u^2_{t-i} + \sum_{i=1}^{q} \beta_i \sigma^2_{t-i} + \gamma_1{}' Z_t$$

where γ_1 is the (m×1) vector of additional parameters and Z_t is the vector of m corresponding additional explanatory variables, which are missing in the original volatility model. For example, these may be the variables which incorporate the instability and or asymmetry in the volatility model.

This type of model can be used to capture the effect of macro-economic instability. In order to approximate and quantify this instability, we use daily exchange rate growth volatility**,** $VEXG_t$**, and** the partial sums of negative and the partial sums of positive changes in volatility of exchange rate, VEX_t^- and VEX_t^+. Three model are then considered. Since Volatilty of Exchange rate growth (VEXG), volatility of Exrate (VEX*),* partial sums of negative and the partial sums of positive changes in volatility of exchange rate (VEX_t^-, and VEX_t^+) evolutions take different patturns after 2016 (see Figure 3 and Figure 4), they may have different effects on TUNindex return volatility. Then three other models are considered to take account of possible structure change in original three specifications.

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[7] Chi-square critical points are $\chi^2_{(1)} = 3.84$ and $\chi^2_{(2)} = 5.99$ at 5% and $\chi^2_{(1)} = 2.71$ and $\chi^2_{(2)} = 4.61$ at 10%.

[8] LR statistic follows asymptotically a Chi-squared distribution.

[9] If these variables can predict the squared normalized residual, then the variance model is misspecified. That is, if the test of significance of the other explanatory variables shows significant results, then we may conclude that the volatility model is not performing well.

7_GARCHM-XS1 model :

For this model, macro-economic **instability** for conditional mean and conditional variance is measered by explanatory variables : exchange rate growth volatility, VEXG*D2017, and exchange rate growth, EXG_t, as follow :

$$\mu_t = c + \phi R_{t-1} + \lambda \sigma_t + \beta \ VEXG_t \times D2017 + \beta' \ EXG_t \ (8)$$

$$\sigma^2{}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u^2{}_{t-i} + \sum_{i=1}^{q} \beta_i \sigma^2{}_{t-i} + \gamma_1 \ \text{VEXG} \times \text{D2017} \ (9)$$

where D2017 = 1 if year ≥ 2017 and 0 if not, where VEXG is the volatility of EXG ;

$$\text{EXG} = \Delta\text{Log(EXRate)}$$

with EXRate denote exchange rate. Both $VEXG_t$ and EXG_t and R_t are stationary processes (see Table 7 and Table 8 in Appendice).

The following two models take account of macro-economic instability only in conditional variance equations.

8_ GARCHM-XS2 model :

$$\mu_t = c + \phi R_{t-1} + \lambda \sigma_t \ (10)$$

and

$$\sigma^2{}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u^2{}_{t-i} + \sum_{i=1}^{q} \beta_i \sigma^2{}_{t-i} + \gamma_1 \ VEX_t^- + \boldsymbol{\gamma_2} \ \boldsymbol{VEX_t^-} \times \mathbf{D2017} \ (10)$$

where

$$VEX_t^- = \sum_{j=1}^{t} \Delta VEX_j^- = \sum_{j=1}^{t} min\,(\Delta VEX_j, 0), \quad (11)$$

is the partial sums of negative changes in volatility of exchange rate, and VEX is the volatility of exchange rate.

9_ GARCHM-XS3 model :

$$\sigma^2{}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u^2{}_{t-i} + \sum_{i=1}^{q} \beta_i \sigma^2{}_{t-i} + \gamma_1 \ VEX_t^+ + \boldsymbol{\gamma_2} \ \boldsymbol{VEX_t^+} \times \mathbf{D2017} \ (12)$$

where

$$VEX_t^+ = \sum_{j=1}^{t} \Delta VEX_j^+ = \sum_{j=1}^{t} max\,(\Delta VEX_j, 0), \quad (13)$$

is the partial sums of positive changes in volatility of exchange rate. Both VEX_t^- and VEX_t^+ are stationary series (see Table 8 in Appendice).

Note that if $\gamma_2 = 0$, GARCHM-XS2 and GARCHM-XS3 will be stable and will be denoted respectively by GARCHM-X2 and GARCHM-X3.

Also, if $\beta = 0$ and $\gamma_1 = 0$, GARCHM-XS1 will be stable and will be denoted by GARCHM-X1.

We test the null hypothesis that these additional missing variables are not significant vs the alternative that they are significant, that is, $H_0 : \gamma_1 = 0$. The test statistic is computed as

$$LM = T.R^2$$

where R^2 is the squared multiple correlation of above regression, and T is the number of observations in the sample.[10]

5. Empirical results

D_ GARCH type models

The time series data used for modeling volatility in this paper is the daily Tunisian stock index, that is TUN-index return, (5 days a weak) over the period from 03/01/2011 to 19/11/2019. To remove the autocorrelation effect and to get a white noise sequence, we fitted the Box-Jenkins models to the data. It was found that the AR (1) model was fitted well to the returns series, the results of which are given in Table 2.[11]

Table 2 : Results of AR(1) fit for TUNindex return

Variable R_t	Coefficient	Std. Error	t-Statistic	Prob.
R_{t-1}	0.242874	0.020600	11.78975	0.0000
C	0.000167	0.000101	1.653485	0.0984
F-statistic	138.9981			
Prob(F-statistic)	0.000000			
Sum squared resid	0.048806			
Akaike info criterion	-7.871855			
Durbin-Watson stat	1.986866			

The plot of the autocorrelation (**Figure 5**) of squared residuals, indicate the high volatility present in the data set, as there are many spikes showing the significant

[10] The LM test statistic is distributed asymptotically as chi-square with m degrees of freedom, where m is the number of additional parameters in the model. We refer to (Engle, R. F., 1984) for more details on the asymptotic theory of the LM test.

[11] Returns serie present significant autocorrelation, then $E[R_t / \Psi_{t-1}] = \mu_t$ is an autoregressive (AR(1)) process ;

$$\mu_t = F(R_t) = C + \emptyset_1 R_{t-1}.$$

autocorrelations of different orders. This suggests that we should fit a volatility model to this data.

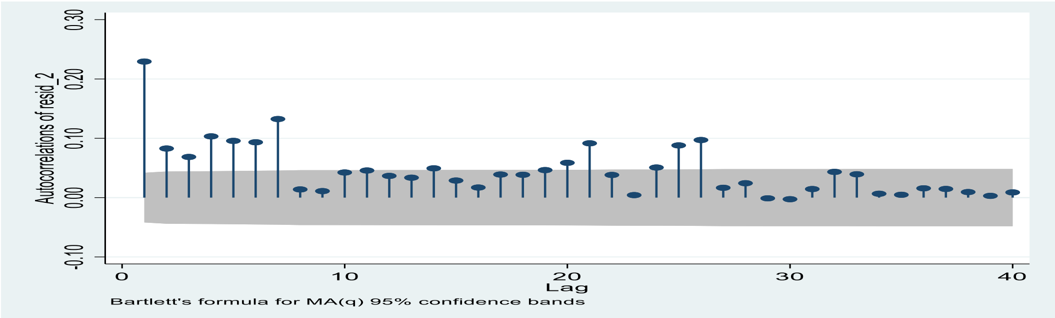

Figure 5 : Autocorrelation of squared residuals.

Before fitting the volatility models, it is important to check whether a day of the week effect is present in the series. Usually it is seen that the average return on Monday is significantly less than the average return over the other days of the week. It is important to know whether there are variations in volatility of stock returns by day of the week patterns and whether a high (low) return is associated with a corresponding high (low) return for a given day.[12] To get the data free of the effect of the week days, the day of the week effect is estimated in return equation by using Ordinary Least Square method (OLS) (see (Pagan & Schwert, 1990) and (Engle & Ng, 1993)). We fit the simple linear regression using OLS with return series on dummy variables for Monday, Tuesday, Wednesday, Thursday, and Friday, which are noted respectively by D_M, D_T, D_W, D_{Th}, D_F as regressors. Each of these day-of-the-week dummies takes a value of 1 on the corresponding weekday and a value of 0 otherwise. To avoid problem of multicollinearity, the dummy variable for Friday, D_F, is dropped from the regression equation. The regression equation results are presented in Table 3.

Table 3 : Day of the week effect for TUNindex return.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.20E-05	0.000108	-0.481232	0.6304
D_M	0.000780	0.000630	1.238882	0.2155
D_T	0.000418	0.000560	0.747763	0.4547
D_W	0.000362	0.000567	0.638554	0.5232
D_{Th}	0.000156	0.000560	0.279429	0.7799
F-statistic	0.600283			
Prob(F-statistic)	0.662463			
Sum squared resid	0.048752			

[12] Such a knowledge may allow investors to adjust their portfolios by taking into account day of the week variations in volatility. Finding certain patterns in volatility may be useful in several ways, including the use of predicted volatility patterns in hedging and speculative purposes and use of predicted volatility in valuation of certain assets.

Akaike info criterion	-7.870214
Durbin-Watson stat	1.989127

From Table 3, we conclude that there is no significant effect of the weekdays on Tunindex return. Then, from white noise sequence : residuals of the regression $R_t = C + \emptyset_1 R_{t-1}$, we can estimate the different volatility models using the maximum likelihood estimation procedure.

We consider the symmetric and asymmetric GARCH models including GARCH (1, 1), GARCH-M (1, 1), EGARCH (1, 1), TGARCH (1, 1), PGARCH (1, 1), and APGARCH (1, 1).[13]

We fit the above models to our Return serie looking for some relationship between return and its own estimated conditional variance. The estimation results are reported at Table 4 Panel A using Gaussian and Student-t distribution assumptions.[14] Table 4We conducted at first stage comparative tests of the models against conventional models presented in previous section. The criteria used to determine the performance include the log likelihood (LL) value comparison and likelihood ratio (LR) test as given by (Brooks, 2008) and according to (Alexander, 2009). Table 9 (see Appendice) reports the log-likelihood (LL) values for each estimated model. Models based on the Student-t distribution generally produce the largest LL value.[15] Only models based on Student t distribution will be then discussed.

The constant mean parameter in mean equation is insignificant except in GARCH-M (1, 1) model of these underlying models.[16] However, we observe the significant constant (α_0) in all models except in PGARCH and APGARCH, and significant ARCH effect (α_1) and significant GARCH effect (β_1) in all conditional variance equations.

[13] The estimation procedure uses the Broyden-Fletcher-Goldfarb Shanno (BFGS) optimization method useful for solving unconstrained non-linear problems.

[14] In the context of nonnormality, the usual standard error estimates will be inappropriate, and a different variance-covariance matrix estimator that is robust to nonnormality, due to (Bollerslev & Wooldridge, Quasi Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances, 1992), are used. This procedure (i.e. maximum likelihood with Bollerslev-Wooldridge standard errors) is known as *Quasi-Maximum Likelihood*, QML.

[15] Whereas the LL value for models that assume the Gaussian distribution are consistently much worse than those associated with Student-t distributions. This informs that the more leptokurtic Student-t assumption is generally better than the competing Gaussian distribution.

[16] These results are not reported in Table 4.

There is a significant contribution of GARCH effect in conditional mean of GARCH-M (1, 1) model. GARCH-M (1, 1) model reports a significant **positive risk-premium** (the $\boldsymbol{\lambda}$ estimated parameter : 0.224896) indicating that data series is positively related to its volatility. So there is feedback from the conditional variance to the conditional mean.[17] This is not surprising because if agents are risk averse they require a larger expected return from an asset riskier within a period. Our results are in agreement with (Campbell & Hentschel, 1992)'s results and against (Glosten, Jagannathan, & Runkle, 1993)' one. This result underscore that high and low of TUNindex are associated with the rise and fall of the returns volatility, that is, an increase in the risk leads to an increase in the amount of the risk premium demanded by investors to compensate for the additional amount of risk to which they are exposed.

Using the AIC criteria, GARCH-M is more adequate than the GARCH model. The AR terms are all significant for both models for the mean equation.[18] The parameter's estimates of the both GARCH(1, 1) and GARCH-M (1, 1) models are statistically significant.[19] The significance of the parameters shows that there exists volatility clustering. The results also indicate that the persistence in volatility, as measured by the sum $\alpha_1 + \beta_1$ in both models, is closer to one [0.817651& 0.817116], suggesting an important presence of ARCH and GARCH effects. This implies that current volatility of daily return can be explained by past volatility that tends to not persist over time and one can conclude that the volatilities associated with each of the significant variables do not last for long before it fades away. The conclusion of persistence volatility is not a strong conclusion for these specifications because sum of α_1 and β_1 is lower than one, indicating that the conditional variance process is stationary.[20] Also, the GARCH is greater than ARCH estimates ($\beta_1 > \alpha_1$) in the two models, it implies that the volatility of stock prices is more affected by the past volatility than the related news from the previous period.

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[17] The existence of risk premium is, therefore, another reason that some historical stock returns have serial correlations.

[18] The AR terms are positive for all considered models, implying that past returns have positive impact. $\emptyset_1 \approx 0.2$ for all models.

[19] Both constants for the variance equation are approximately equal to zero. This shows that current volatility is heavily premised on squared lagged residuals and previous stock return volatility.

[20] **The sum** of α_1 and $\beta_1$in GARCH (1, 1), GARCH-M (1, 1), TGARCH(1, 1), EGARCH(1, 1), are respectively equal to 0.817651, 0.817116, 0.761878, and 1.278964. Estimated parameters (α_1 and β_1) of almost all the models that when added were not very close to unity (with one exception of EGARCH model). The sum of ARCH and GARCH coefficients is not very close to one, indicating that volatility shocks are not persistent, indicating that large changes in returns tend not to be followed by larger changes and small changes tend not to be followed by smaller changes.

The leverage effect (γ) is estimated for three asymmetric GARCH models taking the values -0.028328, 0.115199 and 0.10416 respectively for **EGARCH**, **TGARCH,** and **APGARCH** models. We found significance for two from these three process (**TGARCH,** and **APGARCH** models) that confirm **the leverage** effect. The positive value of asymmetry coefficient $\boldsymbol{\gamma}$ indicates that «good news» increase the future volatility more than the «bad news». For significant asymmetry coefficient, we say that Bad and good news will increase volatility of stock market returns in different magnitude, and that investors on the Tunisian stock exchange react differently to information depending be it good or bad in making investment decisions.

For **EGARCH** model, the persistence parameter, $\beta_1 = 0.808820$ is not very large, implying that the variance moves slowly through time. The coefficient $\boldsymbol{\gamma_1} = -0.028328$ measures the presence of asymmetry. It is statistically **not significant** implying the **absence of asymmetry** and hence the **TGARCH** and **APGARCH** models are more adequate than **EGARCH** model.[21]

In the models with a significant power parameter we found $\boldsymbol{\delta}$ smaller than 2, in concordance with (Ding, Engle, & Granger, 1993) results for **APGARCH** model, while for PGARCH model, estimated $\boldsymbol{\delta}$ is more than 2. The estimated power parameter ($\boldsymbol{\delta}$) in Power GARCH model is found to be **2.32379** and **1.76977** repectively for symmetric and for asymmetric case (which are significant at 1% level). In addition, PGARCH and APGARCH models provide significant GARCH and ARCH effects.

The performance of these estimated models are determined on the basis of some accuracy measures and some tests statistics. In our study, we compute the Akaike information criteria (AIC), ARCH-LM test, DW (Durbin-Watson) statistic, and log of likely-houd function (LL). The results are displayed in Table 9 (see Annexe). A look on the table (Panel A) reveals that there are not big differences seen among the values of accuracy measures (AIC) obtained for all of estimated models. For symmetric models, LR tests , LL, and AIC criteria reveal that GARCH-M model is the best. Likelihood ratio test (LR) results for GARCH(1, 1) against one model of the considered models are also reported in Table 9 (see Appendice). Only H_{03} which is rejected at 5% level. We can conclude that,

[21] When, the negative asymmetry coefficient, γ_1, is significant, we conclude that the variance may goes up more after negative residuals than after positive residuals. Positive and negative shocks have different effects on the stock market returns series.

GARCH-M(1, 1), as **symmetric** model, is superior to GARCH model and then may be the adequate specification for TUNindex return.[22]

For asymmetric models, based on AIC criteria, we may suggest that **TGARCH (1, 1)** is more suitable process to capture the main features of TUNindex return like the **volatility and the leverage** effect. While, based on LL values, we suggest rather that APGARCH is the more suitable model. Likelihood ratio test (LR) results for GARCH(1, 1) against one of the asymmetric models reveal at 10% level, H_{01} is rejected and then only TGARCH(1, 1) and EGARCH(1, 1) are adequate model for TUNindex return.[23]

The **forecasting accuracy** of each model is measured with the root mean square error (RMSE), the mean absolute error (MAE), the Symmetric mean absolute percentage error (SMAPE), and Theil inequality coefficient (TIC). Model(s) with the lowest value of the error measure would be argued to be the most accurate. Table 10 (see Appendice) outlines the values of these forecasting accuracy criteria for the out-of-sample TUNindex forecast. We may conclude that a model which will give a low RMSE (MAE and SMAPE) error must probably will produce suitable prediction. A look on the table say that these results are not conclusive. And, no model over perform the others.

We plot then the observed and estimated TUNindex prices for the period from 03/01/2011 to 19/11/2019 in Figure 8 (a)-(f) ; see Appendice. These graphs show a close match to the data exhibiting that all these estimated econometric models provide a good fit to the observed TUNindex time series. No model seems to be the better.

But it is difficult to choose any one of these models on just the basis of the likelihood, AIC, etc. So we go for **the news impact curve** introduced by (Pagan & Schwert, 1990).[24] A representation of the degree of asymmetry of volatility to positive and negative shocks is given by the news impact curve. Generally, there are two ways of bringing the asymmetry in volatility models, either by bringing a shift or by allowing a rotation of the news impact curve. EGARCH and GJR bring

[22] With Models based on Gaussian distribution, LR test reject all null hypothesis (LR statistic is equal to 10.606, 11.946, 9.144, 16.116, and 8.668 respectively against EGARCH, TGARCH, PARCH, APGARCH, and GRCH-M).

[23] H_{04} is not rejected even at 10% level.

[24] The news impact curve plots the next-period volatility ($\sigma^2{}_t$) that would arise from various positive and negative values of u_{t-1}, given an estimated model. The curves are drawn by using the estimated conditional variance equation for the model under consideration, with its given coefficient estimates, and with the lagged conditional variance set to the unconditional variance. The news impact curve is capable of depicting the symmetric and asymmetric behavior of the different volatility models with respect to news.

the asymmetry by allowing the rotation in the news impact curve.[25] The resulting news impact curves for the GARCH vs GJR, vs EGARCH and vs APGARCH models are given at Figure 6.[26]

As can be seen from Figure 6, the GARCH news impact curve (the Black line) is of course symmetrical about zero (centered around u_{t-1} = 0), so that a shock of given magnitude will have the same impact on the future conditional variance (CV) whatever its sign. That is, positive and negative return shocks of the same magnitude produce the same amount of volatility. Then, since behavior of stock market is symmetric with respect to news, the GARCH model under predicts the amount of volatility following bad news (negative shock) and over predicts the amount of volatility following good news (positive shock).

On the other hand, the GJR news impact curve (the Green line) is asymmetric, with negative shocks having more impact on future volatility than positive shocks of the same magnitude. It can also be seen that a negative shock of given magnitude will have a bigger impact under **EGARCH** (Blue line) than would be implied by a GARCH, GJR, and APGARCH models, while a positive shock of given magnitude will have more impact under GARCH than GJR, APGARCH, and EGARCH models.[27] Thus, EGARCH highly over predict the volatility, which is absolutely wrong (γ_1 is not significant). Therefore, we can not consider this model too in this situation. Overall, GJR model appears to be relatively better than other models for given data set. With GRJ model , when the shock **is negative**, the effect on volatility is $\alpha_1 + \gamma_1 = 0.240710 + 0.115199 = 0.355909$. Also, the GARCH estimates is greater than ARCH effect ($\beta_1 = .521168 > \alpha_1 + \gamma_1$) in this model, it implies that the volatility of stock returns is more affected by the past volatility than the related news from the previous period.

[25] The nonlinear-asymmetric ARCH model of (Engle & Ng, 1993) employ a shifted news impact curve to achieve asymmetry, while (Glosten, Jagannathan, & Runkle, 1993) and EGARCH of (Nelson, 1991) allow the asymmetry by rotation.

[26] These models allow several types of asymmetry in the impact of news on volatility.

[27] The latter result arises as a result of the reduction in the value of α_1, the coefficient on the lagged squared error, when the asymmetry term is included in the model (see Table 44 Panel A).

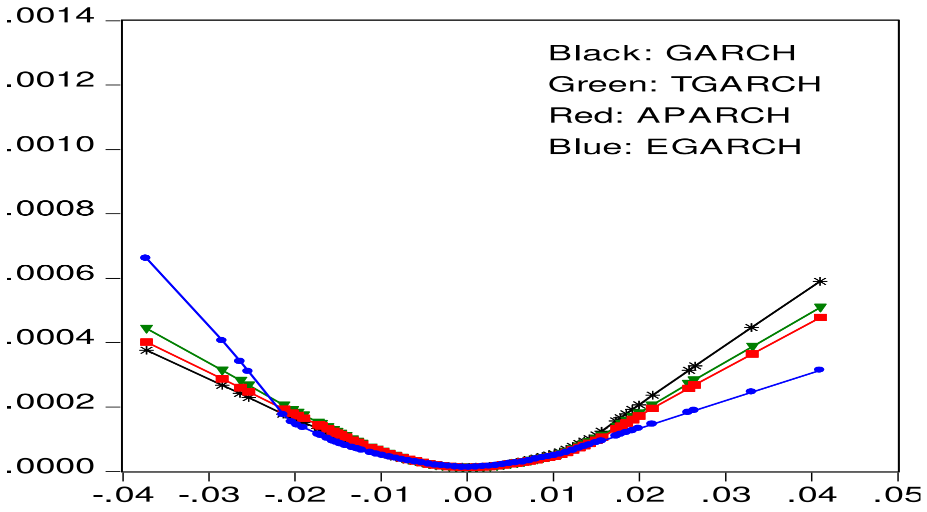

Figure 6 : News impact curves for TUNindex returns from various model (GARCH, TGARCH, EGARCH, and APGARCH) estimates .

Table 4 : Estimation results of GARCH (1,1), GARCH-M (1,1) , EGARCH (1, 1), TGARCH (1,1), PARCH (1,1), APARCH (1,1) and GARCH-M(1, 1) models. Panel A :.[28]

GARCH : $\sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1}$				
	α_0	α_1	β_1	
Normal	4.11E-06*	0.268236*	0.546047*	
Student	4.03E-06*	0.300366*	0.517285*	
EGARCH : $\text{Log}(\sigma^2{}_t) = \alpha_0 + \alpha_1 \left\lvert \frac{u_{t-1}}{\sigma_{t-1}} \right\rvert + \gamma_1 \frac{u_{t-1}}{\sigma_{t-1}} + \beta_1 \log(\sigma^2{}_{t-1})$				
	α_0	α_1	γ_1	β_1
Normal	−2.39039*	0.44417*	−0.039828**	0.811566*
Student	−2.449064*	0.470144*	**−0.028328**	0.808820*
TGARCH : $\sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \gamma_1 u^2{}_{t-1} I_{t-1} + \beta_1 \sigma^2{}_{t-1}$				
	α_0	α_1	γ_1	β_1
Normal	4.10E-06*	0.176280*	0.158233*	0.558036*
Student	4.01E-06*	0.240710*	0.115199***	0.521168*
PGARCH : $\sigma^\delta{}_t = \alpha_0 + \alpha_1(\lvert u_{t-1} \rvert)^\delta + \beta_1 \sigma^\delta{}_{t-1}$				
	α_0	α_1	β_1	δ
Normal	4.51E-07	0.403249*	0.040795	2.611111*
Student	2.10E-06	0.436370*	0.117313***	2.323790*

[28] Data analysis are done using Eviews and Stata statistical software.

APGARCH : $\sigma^{\delta}_t = \alpha_0 + \alpha_1(\lvert u_{t-1}\rvert - \gamma_1 u_{t-1})^{\delta} + \beta_1 \sigma^{\delta}_{t-1}$					
	α_0	α_1	γ_1	β_1	δ
Normal	0.000133	0.257792*	0.144572*	0.594149*	1.352831*
Student	1.37E-05	0.293028*	0.10416***	0.543048*	1.769747*
GARCH − M : $\mu_t = C + \emptyset_1 R_{t-1} + \lambda\, \sigma_t$ and $\sigma^2_t = \alpha_0 + \alpha_1 u^2_{t-1} + \beta_1 \sigma^2_{t-1}$					
	α_0	α_1	β_1	λ	
Normal	4.20E-06*	0.267014*	0.540360*	0.278879*	
Student	4.02E-06*	0.302543*	0.514573*	0.224896*	

Legend : * p<.1 ; ** p<.05 ; *** p<.01. Note : $I_{t-i} = 1$ if $u_{t-i} < 0$ and 0 otherwise.

F_ GARCHM model with Macroeconomic Effects (GARCHM-X and GARCHM-XS)

In previous section, the news impact curve [introduced by (Pagan & Schwert, 1990) and discussed in detail by (Engle & Ng, 1993)], was used for measuring the effect of news on volatility. All considered models till now suppose stable specifications. However, since 2011 (YESAMIN revolution), economic and political situation is instable in Tunisia. Then in the following, we investigate macroeconomic effects via exchange rate on TUNindex return volatility. This section employed rather the AR(1)-GARCHM-X (or GARCHM-XS : GARCHM-X with structural shift) model to investigate the effect of exchange rate growth and exchange rate volatility on stock Return volatility in Tunisia economy. All investigations are given at Table 11 Panel B for GARCHM-X models (see Appendice) and Table 5 Panel C for GARCHM-XS models as explained in section 4 (sub-section C).

After the preliminary tests, the exchange rate growth, EXG, and its volatility, VEXG, using an AR(1)-GARCH(2, 1) model, are calculated (see Figure 3). Given, the predicted volatility for exchange rate growth, the relationship between the conditional volatility in exchange rate and stock return is examined by estimating the conditional mean and conditional variance equations.

Again, **the performance** of these estimated models are determined on the basis of some accuracy measures. We compute the Akaike information criteria (AIC), ARCH-LM test, Durbin-Watson (DW) statistic, and log of likely-houd function (LL). The results are displayed at Table 12. A look on this table (Panel B and C) reveals that GARCHM-XS2 is more suitable process to capture the main features of TUNindex return. Comparisons or selection of more accurate model based on

Likelihood ratio (LR) tests and LM tests is also done. LM test results for GARCH(1, 1) against one model of the considered models (GARCHM- X1 or GARCHM-X2 or GRCHM-X3) are also reported in Table 12.[29] Only GARCHM-X2 and GARCHM-X1 which are significant. Then, LR test conclude that GARCHM-X2 is instable. That is, in all, only GARCHM-XS2 model will be then discussed.

Again, ARCH and GARCH coefficients in this model are found to be significant. The significance of the parameters shows that there exists volatility clustering. Also, the results indicate coefficients α_1 (0.266561) and β_1 (0.533351) are less than ones in GARCHM-XS2 model. With low values of $\beta_1,$ one can conclude that the volatilities do not last for long before it fades away. Also, the GARCH is greater than ARCH estimates in the model implying that the volatility of stock return is more affected by the **past volatility** than the **related news** from the previous period.

GARCHM-XS2 model reports a significant **positive risk-premium** (the $\boldsymbol{\lambda}$ estimated parameter : 0.286812) indicating that data series is positively related to its volatility. This mean that since agents are risk averse they require a larger expected return from riskier asset within a period.

Now, with respect to exchange rate volatility, this result is predicated on the fact bad news about the volatilities of exchange rate (referred to as exchange rate depreciation) correspond to positive volatility of stock return as it increases the conditional volatility ; γ_1 =0.000743 (see results from model GARCHM-XS2).[30]

[29] There is no big difference between estimates results of GARCHM-X2 and GARCHM-X3 model.

[30] While that good news about the volatilities of exchange rate (referred to exchange rate appreciation) correspond to negative volatility of stock return, since it reduces the conditional volatility ; $\gamma_1 = -0.000718$ (see results from model GARCHM-XS3). This result is inconsistent with (Zakaria & Shamsuddin, 2012).

Table 5 : Results of three GARCHM-XS (1, 1) models with structural shift- Panel C.

Conditional variance

Conditional mean

$\mu_t = c + \phi R_{t-1} + \lambda\,\sigma_t + \boldsymbol{\beta EXG} + \boldsymbol{\beta' VEXG \times D2017}$

C	$\emptyset_1$	λ	β	β'
−0.001091	00.227084*	0.284238*	**0.009069**	3.194222*

$\textbf{GARCHM} - \textbf{XS1}: \sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1} + \boldsymbol{\gamma_1\ VEXG \times D2017}$

α_0	α_1	β_1	γ_1
4.22E-06*	0.268286*	0.537486*	**−0.000194**

$\textbf{GARCHM} - \textbf{XS2}: \sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1} + \boldsymbol{\gamma_2\ VEX^-_t \times D2017} + \boldsymbol{\gamma_1\ VEX^-_t}$

$\mu_t = C + \lambda\,\sigma_t + \emptyset_1 R_{t-1}$

C	λ	$\emptyset_1$	α_0	α_1	β_1	γ_2	γ_1
−0.001030**	0.286812*	0.224690*	5.75E-06*	0.266561*	0.533351*	**−0.000624***	0.000743*

$\textbf{GARCHM} - \textbf{XS3}: \sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1} + \boldsymbol{\gamma_2\ VEX^+_t \times D2017} + \boldsymbol{\gamma_1\ VEX^+_t}$

C	λ	$\emptyset_1$	α_0	α_1	β_1	γ_2	γ_1
−0.001024*	0.285318*	0.224959*	5.67E-06*	0.265448*	0.534769*	0.000606*	−0.000718*

6. Conclusion

This paper has presented results from modeling volatility in an empirical investigation of equity return series from the Tunisian Stock Exchange. The time series data 27se dis the daily Tunisian Securities Exchange (TUNindex) index (5 days a weak) over the period from 03/01/2011 to 19/11/2019. The study compared varying GARCH-type models. Among many symmetric and asymmetric type heteroscedastic processes, we estimated 12 models : GARCH (1, 1), GARCH-M (1, 1), PGARCH (1, 1), EGARCH (1, 1), TGRACH (1, 1), APGARCH (1, 1), three GARCHM-X models, and three GARCHM-XS models.

At the begenning, to remove the autocorrelation effect and to get a white noise sequence, we fitted the Box-Jenkins models to the data. It was found that an AR (1) model was fitted well to the conditional mean of returns series. In addition, there is no significant effect of the weekdays on Tunindex return. And Applied ARCH LM test confirm presence of volatility clustering in Tunindex return.

Then, the presence of volatility **clustering** is strongly confined from all these estimated models as we obtained the significant estimates corresponding to ARCH effect and GARCH effect parameters.

The results show also that the volatility process is not highly persistent, thus, giving evidence of the existence of **risk premium** for the TUNindex return series. This in turn supports the positive correlation hypothesis : that is between volatility and expected stock returns.

Another fact revealed by the results is that the asymmetric TGARCH models provide better fit for Tunindex than the symmetric models. The asymmetric TGARCH (1, 1) and APGARCH models have significant estimates of the leverage effect. This proves the presence of leverage effect in the Tunindex return series. On the other hand, with asymmetric coefficient being not significant, the Exponential GARCH (1, 1) model proved to be not efficient model for modelling volatility. Analysis based on AIC criteria and LR test say that the **TGARCH** (1, 1) model is more appropriate in term of capturing the volatility clustering and leverage effect of the TUNindex stock market within the six first considered models.

For policy, GARCHM-X2 (1, 1) turned to be the best model using both the AIC and LL criterions, with the presence of **instability** found to be significant using LR test results.

The study concludes that positive and negative shocks impact differently on the stock market returns. Bad (and good) news will increase volatility of stock market returns in different magnitudes. The study results implies that the investment climate including the stability in the macroeconomic environment should be favourable to ensure growth in the stock market. Investors require the predictability of the future to make sound investment decisions. Policies to reduce volatility in the the economy (more stable exchange rate) are a necessity for stock market.

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Appendice

Figures

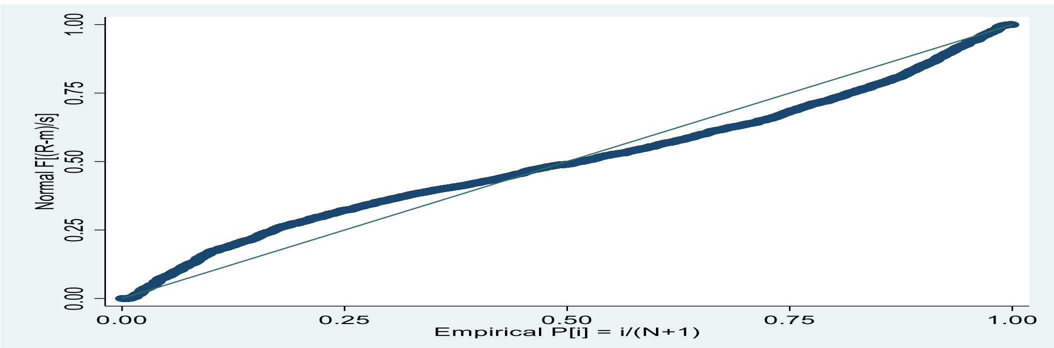
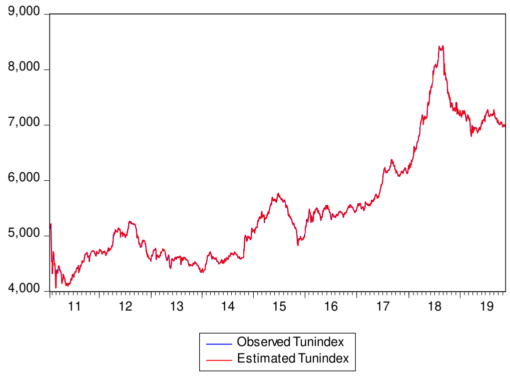
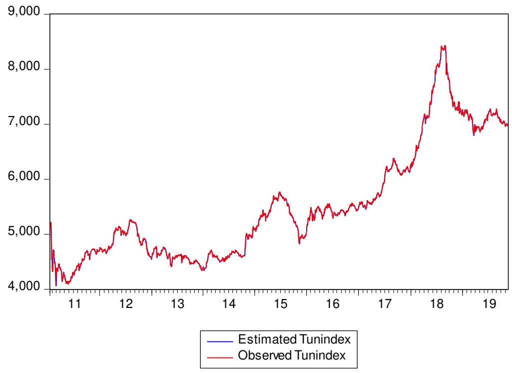


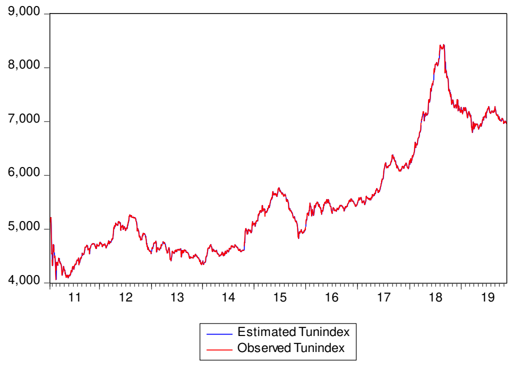
Figure 7 : Q-Q plot for daily Tunindex return



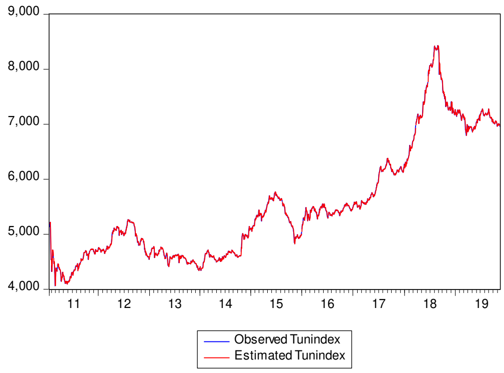
(a) **AR(1)-GARCH(1, 1)**



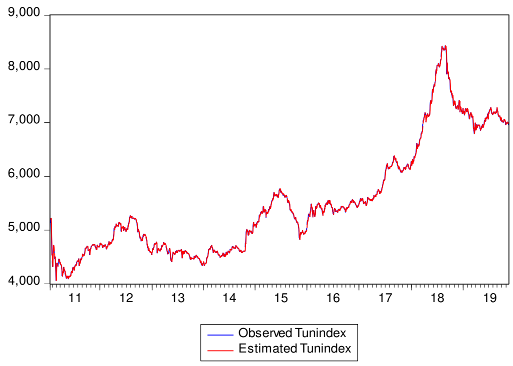
(b) **AR(1)-GARCHM**



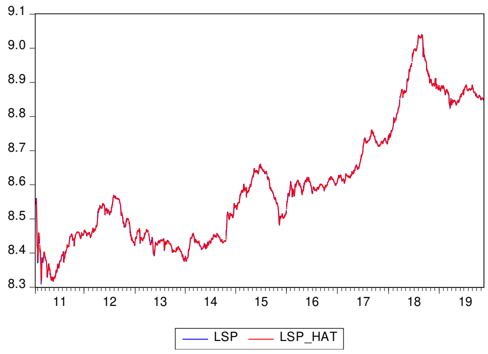
(c) **AR(1)-EGARCH**



(d) AR(1)-TGARCH



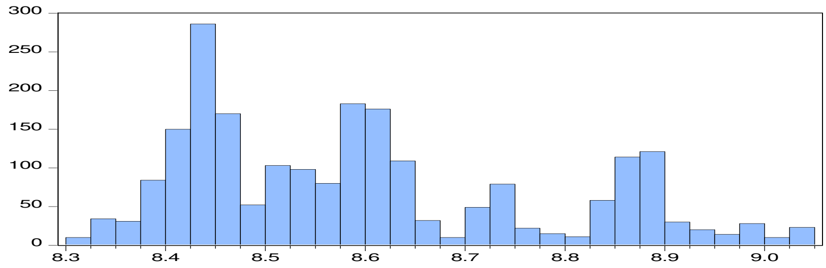
(e) AR(1)-PGARCH



(f) **AR(1)-GARCH-X1(1, 1)**

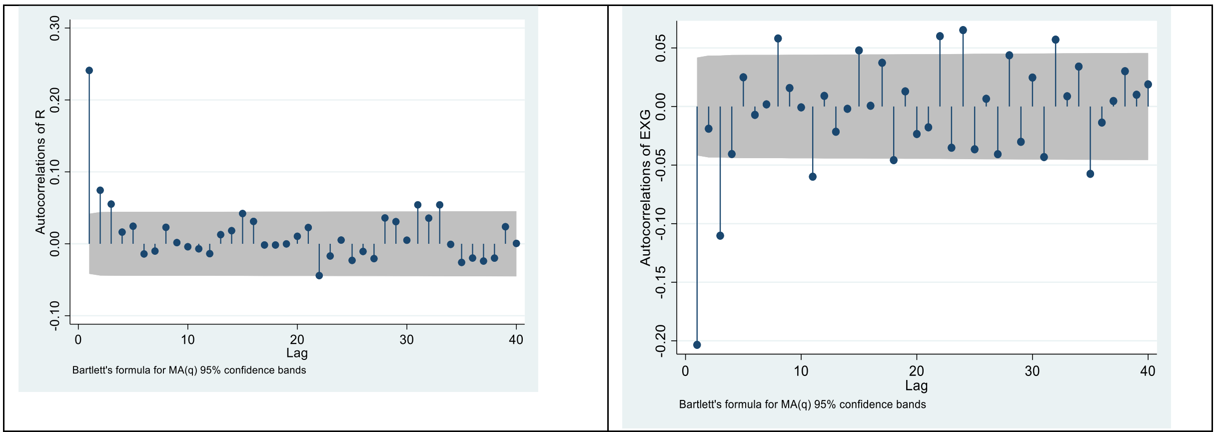
Figure 8 : (a) Plot of observed and estimated Tunindex from GARCH (1,1) Model. (b) Plot of observed and estimated Tunindex from GARCH-M (1,1) Model. (c) Plot of observed and estimated Tunindex from EGARCH (1,1) Model. (d) Plot of observed and estimated Tunindex from TGARCH (1,1) Model. (e) Plot of observed and estimated Tunindex from PGARCH (1,1) Model (f) Plot of observed and estimated Tunindex from. GARCH-X1(1,1).

All for daily data



Series: LSP	
Sample 1/03/2011 11/19/2019	
Observations 2202	
Mean	8.598540
Median	8.575272
Maximum	9.039747
Minimum	8.308576
Std. Dev.	0.177522
Skewness	0.662891
Kurtosis	2.367397
Jarque-Bera	197.9859
Probability	0.000000

Figure 9 : Descriptive statistics for daily data TUNindex



Stock Retun	Exchange rate

Figure 10 : correlogram of autocorrelation function for Daily data

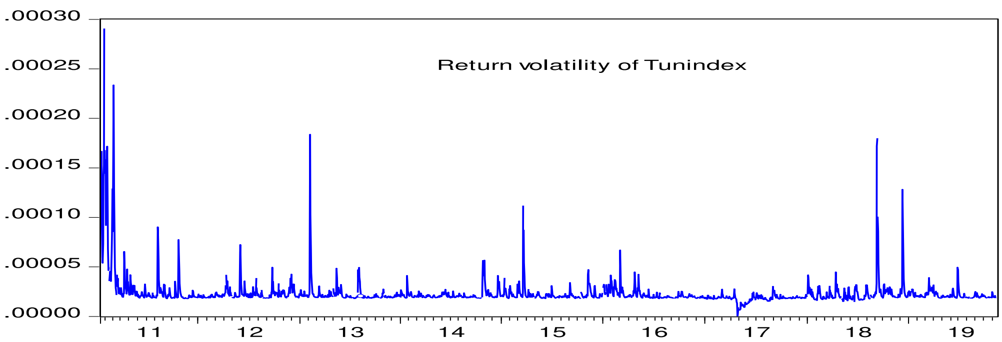


Figure 11 : Tunindex return volatility

Table Empirical Review

Table 6 : Empirical Review

Authors	Variables	Model	Sample	Results
Al Khazali (2003)	Share prices -CPI -Industrial production index	-Johansen cointegration test -GARCH	**-Countries :** 21 emerging countries **-Period :** 1980-2001 -Monthly data.	-Negative short-term relationship between stock market returns and inflation. -Positive long-term relationship between stock market returns and inflation.
(Hammoudeh & Li, 2008)		GARCH model	Arab Gulf stock markets	Volatility was very high
(Surya, 2008)		GARCH (1,1)	Nepalese stock market 1297 observations from 2003 to 2009	No significant asymmetry in the conditional volatility of returns high persistence and predictability of volatility
(Ahmed & Suliman, 2011)	Khartoum Stock Exchange – KSE)		Sudan January 2006 to November 2010	Conditional variance process was highly persistent Existence of risk premium for the KSE index return series Presence of leverage effect.
(Goyal, 2012)		GARCH and PGARCH	Indian stock price daily returns from 2000 to 2010	Symmetric and asymmetric effect
(Ndako, 2012)			South Africa	Financial liberalisation is statistically important and not positive
(Sharaf & Abdalla, 2013)		GARCH(1,1), GARCH-M(1,1), EGARCH(1,1) and GJR-GARCH(1,1) models.	Khartoum Stock Exchange (KSE) daily closing prices over the period from 2nd January 2006 to 31st August 2010	High volatility process is present in KSE Index Existence of risk premium and indicates the presence of the leverage effect in the KSE index returns series
(Ananzeh, Jdaitawi, & Al-Jayousi, 2013)	Amman stock Exchange		Amman Stock Exchange for 27 individual stocks daily data for the period 2002-2012.	Trading volume has no significant effect on the reduction of the volatility persistence for majority of stocks Trading volume significantly contributes to the return volatility process
(Khositkulporn, 2013),		Multiple regression and GARCH	Thailand	S&P 500 had a major influence on Thailand's stock market, followed by the BSI and oil price
(Koima, Mwita, & and Nassiuma, 2015))		GARCH (1, 1)	Kenyan stock market	Iin a financial crisis ; the negative returns shocks have higher volatility than positive returns shocks
(Banumathy & Azhagaiah, 2015)	daily closing prices of S&P CNX Nifty	GARCH (1,1) and TARCH (1,1), EGARCH (1,1) and TGARCH (1,1)	Indian stock Market Period : from 2003 to 2012.	Negative shocks have significant effect

	Index for 10 years			
(Cheteni, 2016)	Johannesburg Stock Exchange FTSE/JSE Albi index and the Shanghai Stock Exchange Composite Index	GARCH model	**Countries** South Africa and China stock markets **Period :** January 1998 to October 2014	Volatility was persistent in both exchange markets
(Emeka & Aham, 2016)	-Share price index -Inflation rates -Exchange rates	-Johansen's integration -AR (1) GARCH-S (1.1) - GARCH-X	**-Country :** Nigeria **-Period :** 1986-2012 -Quarterly data	-Negative relationship between stock price volatility and inflation rate. -Negative relationship between equity price volatility and the exchange rate.
(Murekachiro, 2016)	ZSE industrial index returns	GARCH (1,1) and EGARCH (1,1).	**Countries** Zimbabwe stock market **Period :** 19 February 2009 to 31 December 2014	Asymmetric EGARCH (1 ;1) model outperformed the symmetric GARCH (1 ;1)

Tables

Table 7 : Results of unit root test for original Tuindex series, and return series (Tunindex at first difference in log)[31] for daily data

				(PP)			
			Level			**1st Diff**	
		TUNINDEX	**R**	**EXRATE**	**d(TUNINDEX)**	**d®**	**d(EXRATE)**
With Constant	**t-Statistic**	-0.2139	-35.1895	0.2683	-37.0465	-326.1843	-74.7960
	Prob.	0.9343	0.0000	0.9766	0.0000	0.0001	0.0001
		n0	***	n0	***	***	***
With Constant & Trend	**t-Statistic**	-2.4578	-35.2841	-2.3926	-37.0514	-325.6233	-74.8643
	Prob.	0.3494	0.0000	0.3833	0.0000	0.0001	0.0001
		n0	***	n0	***	***	***
Without Constant & Trend	**t-Statistic**	1.0627	-35.2227	3.2466	-37.0503	-326.5386	-73.6475
	Prob.	0.9252	0.0000	0.9998	0.0000	0.0001	0.0001
		n0	***	n0	***	***	***
				(ADF)			
			Level			**1st Diff**	
		TUNINDEX	**R**	**EXRATE**	**d(TUNINDEX)**	**d®**	**d(EXRATE)**
With Constant	**t-Statistic**	-0.2505	-34.8711	0.4788	-36.6975	-22.2497	-35.7930
	Prob.	0.9295	0.0000	0.9860	0.0000	0.0000	0.0000
		n0	***	n0	***	***	***
With Constant & Trend	**t-Statistic**	-2.5670	-34.8891	-2.2895	-36.7177	-22.2442	-30.1716
	Prob.	0.2957	0.0000	0.4390	0.0000	0.0000	0.0000
		n0	***	n0	***	***	***
Without Constant & Trend	**t-Statistic**	0.9849	-34.8611	3.6985	-36.6817	-22.2557	-35.5003
	Prob.	0.9147	0.0000	1.0000	0.0000	0.0000	0.0000
		n0	***	n0	***	***	***
				(KPSS)[32]			
			Level			**1st Diff**	
		TUNINDEX	**R**	**EXRATE**	**d(TUNINDEX)**	**d®**	**d(EXRATE)**
With Constant	**t-Statistic**	4.9611	0.2164	5.9091	0.2550	0.0350	0.1759
	Prob.	***	n0	***	n0	n0	n0

[31] **This Result is The Out-Put of Program Has Developed By Dr. Imadeddin AlMosabbeh , College of Business and Economics, Qassim University-KSA**

[32] Null Hypothesis: the variable is stationary.

With Constant & Trend	t-Statistic	0.8320	0.0972	0.9962	0.1155	0.0308	0.0788
	Prob.	***	n0	***	n0	n0	n0

Table 8 : Descriptive analysis for daily data : Exchange rate growth (EXG), its volatility (VEXG), and partial sums of positive and negative changes in volatility of Exchange rate (VEX).

	EXG	VEXG	VEX_t^-	VEX_t^+
Mean	0.000309	3.95E-05	-0.005632	0.005712
Median	0.000278	2.34E-05	-0.002334	0.002308
Maximum	0.044029	0.000898	0.000000	0.018373
Minimum	-0.095449	9.91E-06	-0.018422	2.77E-06
Std. Dev.	0.006685	6.01E-05	0.006254	0.006365
Skewness	-1.055239	7.630818	-1.040332	0.994686
Kurtosis	26.81967	80.94890	2.406274	2.290192
Jarque-Bera	52441.65	578320.3	428.7633	408.5924
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2201	2200	2198	2198

Unit root test results

Null Hypothesis : considered series has a unit root

	EXG	VEXG		VEX_t^-	VEX_t^+
PP test statistic[33]	-69. 75309	-6.603327	Min-t[34]	-5.72584	-5.61725
Prob.*	0.0001	0.0000	Prob	< 0.01	< 0.01
ADF test statistic	-34.22572	-6.881947	Max-t[35]	3.807805	-5.597716
Prob.*	0.0000	0.0000		> 0.99	< 0.01
Conclusion	SL2	SL2		SL2	SL2

Heteroskedasticity Test : ARCH(1)

F-statistic	0.390477	Prob. F(1,2197)	0.5321
Obs*R-squared	0.390763	Prob. Chi-Square(1)	0.5319

Note : Min-t : Minimize Dickey-Fuller t-statistic is applied. Break Date : 4/25/2017 for VEX_t^- and 4/20/2017 for VEX_t^+. Max-t : Maximize intercept break t-statistic. Break Date : 5/12/2017 for VEX_t^- and 4/03/2017 for VEX_t^+.

[33] Test critical values :
-3.433127, -2.862653, -2.567408 For **1% level** 5% level 10% level.

[34] Test critical values :
-4.949133, -4.443649, -4.193627 For 1% level 5% level 10% level

[35] Test critical values :
-4.734858, -4.193627, -3.863839 For 1% level 5% level 10% level

Table 9 : Post estimation statistics for GARCH type models - **Panel A** .

Model	LL_N and LL_S		LR	Durbin-Watson		AIC		ARCH LM test (1)	
	Normal	Student		Normal	Student	Normal	Student	Normal	Student
GARCH	8888.347	9019.677		1.97339	1.92224	-8.1126	-8.2316	0.8526	0.8693
GRCH-M	8892.681	9023.337	7.32>3.84	1.952344	1.905654	-8.11569	-8.23409	0.9440	0.7040
PGARCH	8892.919	9019.861	0.368	1.919251	1.920343	-8.11590	-8.23092	0.3689	0.9872
EGARCH	8893.650	9017.814	3.726	1.959066	1.925533	-8.11657	-8.22905	0.7872	0.9874
TGARCH	8894.320	**9021.559**	3.764>2.71	1.996806	1.928954	-8.11718	-8.23247	0.9167	**0.7799**
APGARCH	8896.405	**9021.822**	4.29	1.955184	1.927031	-8.11817	-8.23180	0.5979	0.8804

Note : LL_N : log-likelihood with Normal distribution. LL_S : log-likelihood with Student distribution. Chi-square critical points for LR test statistic are $\chi^2_{(1)} = 3.84$ and $\chi^2_{(2)} = 5.99$ at 5% and $\chi^2_{(1)} = 2.71$ and $\chi^2_{(2)} = 4.61$ at 10%. For ARCH LM test, p-value is reported.

Table 10 : Evaluation of out-of-sample volatility forecasts for GARCH type models.

Panel A Conditional Volatility Model	RMSE	MAE	SMAPE	TIC
GARCH	0.004875	0.003270	181.4921	0.966455
GARCH-M	0.004865	0.003279	165.0669	0.922021
PGARCH	0.004875	0.003270	181.8473	0.967090
EGARCH	0.004876	0.003269	185.8639	0.972218
TGARCH	0.004876	0.003269	184.5602	0.970621
APGARCH	0.004876	0.003269	185.0673	0.971294

Note : RMSE :mean square error, MAE :mean absolute error, SMAPE :Symmetric mean absolute percentage error, and TIC :Theil inequality coefficient.

Table 11 : Results of three GARCHM –X (1, 1) models –Panel B :[36]

Conditional mean | **Conditional variance**

$\mu_t = c + \phi R_{t-1} + \beta \text{ VEXG} + \lambda\, \sigma_t$

C	$\emptyset_1$	β	λ
−0.001114*	0.226460*	3.495209*	0.276129*

GARCHM − X1 : $\sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1}$

α_0	α_1	β_1
4.26E-06*	0.271433*	0.532988*

GARCHM − X2 : $\sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1} + \gamma_1\, VEX_t^-$

$\mu_t = C + \lambda\, \sigma_t + \emptyset_1 R_{t-1}$

C	λ	$\emptyset_1$	α_0	α_1	β_1	γ_1
−0.000969*	0.271770*	0.227173*	4.44E-06*	0.267799*	0.542991*	5.19E-05*

GARCHM − X3 : $\sigma^2{}_t = \alpha_0 + \alpha_1 u^2{}_{t-1} + \beta_1 \sigma^2{}_{t-1} + \gamma_1\, VEX_t^+$

C	λ	$\emptyset_1$	α_0	α_1	β_1	γ_1
−0.000963*	0.270424*	0.227340*	4.43E-06*	0.267479*	0.543287*	−4.90E-05*

Table 12 : Post Estimation Statistics for GARCHM-X and GARCH-XS models

Model	LL_N	LM	Durbin-Watson	AIC	ARCH LM test (1)
Panel B					
GRCHM-X1	8896.347	7.332 > 3.84	1.952708	-8.11812	0.8646
GRCHM-X2	8894.064	2.766 > 2.71	1.949301	-8.11600	0.8425
GRCHM-X3	8893.964	2.566	1.949751	-8.11594	0.8414
Panel C		**LR**			
GRCHM-XS1	**8896.720**	0.746	1.953829	-8.1166	0.9329
GRCHM-XS2	**8898.488**	8.848 > 5.99	1.943178	**-8.1191**	0.9725
GRCHM-XS3	**8898.218**	8.508	1.943841	**-8.1189**	0.9364

Note : LL_N : log-likelihood with Normal distribution. Chi-square critical points for LR test statistic are $\chi^2_{(1)} = 3.84$ and $\chi^2_{(2)} = 5.99$ at 5% and $\chi^2_{(1)} = 2.71$ and $\chi^2_{(2)} = 4.61$ at 10%. For ARCH LM test, p-value is reported. LR = $-2(LL_R - LL_U)$ is test statistic to test GARCHM-X vs GARCHM-XS model. LM = $T.R^2$ is test statistic to test GARCHM vs GARCHM-X model.

[36] The Heteroskedasticity Consistent Covariance option is used to compute the quasi-maximum likelihood (QML) covariances and standard errors using the methods described by (Bollerslev & Wooldridge, Quasi Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances, 1992).