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Privacy-preserving Energy Scheduling for Smart Grid with Renewables

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ABSTRACT We consider joint demand response and power procurement to optimize the average social welfare of a smart power grid system with renewable sources. The renewable sources such as wind and solar energy are intermittent and fluctuate rapidly. As a consequence, the demand response algorithm needs to be executed in real time to ensure the stability of a smart grid system with renewable sources. We develop a demand response algorithm that converges to the optimal solution with superlinear rates of convergence. In the simulation studies, the proposed algorithm converges roughly thirty time faster than the traditional subgradient algorithm. In addition, it is fully distributed and can be realized either synchronously or in asynchronous manner, which eases practical deployment.

INDEX TERMS Demand response, Smart Grid, Renewable sources, Distributed Algorithm, Convergence Analysis.

LOAD side management in the power grid is an active research area with a large body of existing work, ranging from traditional load control to real-time pricing [1], [9], [12], [14], [15], [17], [17], [18], [23], [25], [28]. However, as the penetration of renewable energy sources continues to grow, traditional load side management becomes inadequate. Demand response, on the other hand, can be employed to shift the load for economic benefits. Demand response is particular suitable for adapting elastic loads to renewable generations such as wind and power, which are intermittent and random [2], [7], [26], [27], [30]. Such demand response needs to act fast according to the dynamics of the renewable sources.

We consider a power system consists of multiple users. A single load-serving entity (LSE) serve all these users. It aggregates loads to ensure the efficiency of the whole sale market.

Our main contributions are fourfold:

- Uncertainty: Renewable sources such as wind and solar powers are considered , and thus the power supply is intermittent and uncertain.
- Supply and demand: LSE's supply decisions and the users' consumption decisions must be jointly optimized.
- Privacy-preserving: We assume the controller (e.g. a utility company) has no direct access to users' consumption requirements, which preserves the consumption data privacy.
- Two-timescale: We consider power procurement on both day-ahead wholesale market and user consumption adaption in real-time so that supply uncertainty of renewable energy can be mitigated.
- Fast convergence: Renewable energy supplies fluctuate rapidly. Thus demand response algorithm needs to adjust according to the dynamics in real time.

Energy management without renewable generation has been considered in [10]. However, the uncertainly in renewable generation requires real-time decision that can adjust according to the random renewable generations. The real-time consumption decision needs to be coordinated with day-ahead procurement decisions so that the overall expected welfare is maximized, which is the focus of this paper. Both the problem formulation and the solution are completely different from those in [10]. In addition, we believe the requirements of user consumptions are different and **private**. The algorithm presented in this paper is capable of achieving optimality without direct access to users' private information [27].

Demand response has been receiving significant research interest over the past decade. [4], [29] consider load control in smart buildings. [8], [22], [24] investigate cooperated scheduling among different appliances for residential load control. In addition, [6], [13], [21] consider the scheduling

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problem of electric vehicle charging, [11] on the optimal allocation of a supply deficit (rationing) among users using their supply functions. Similar models have been discussed in [7], [16]. These models consider both day-ahead and random energy generation. However, the objectives and operations considered in this paper are different from the aformenioned work.

A. NOTATIONS

Let $q_{ia}(t)$ denote the demands from appliance a of user i in t^{th} time period. In addition, we have $q_{ia} := (q_{ia}(t), t \in \mathcal{T})$ which represents the demand vector at different time instances. $q_i(t) := (q_{ia}(t), a \in \mathcal{A}_i)$ indicate the demand vector of i^{th} user for all appliances in the t^{th} time period. Likewise, we use $q_i := (q_{ia}, a \in \mathcal{A}_i)$ and $q := (q_i, \forall i)$ to denote the demand vector of i^{th} user and all users. For aggregate demands we have $Q_i(t) = \sum_{a \in \mathcal{A}_i} q_{ia}(t)$, $Q_{ia} := \sum_t q_{ia}(t)$, Q_i, Q, etc. Script letters represent sets. Small letters indicate individual quantities. Capital letters denote aggregate quantities. For clarity, we summarize the notations in the following table.

Symbol	Description
$\mathcal{N}$	a set of N users
$\mathcal{T}$	a set of T time points
$q_i(t)$,	energy assumption of appliance i
$q(t)$	energy consumption vector
$U_i(q_i(t); t)$	utility function for $q_i(t)$ at time t
$P_d(t)$, $c_d(P_d(t))$	day-ahead capacity and the cost
$P_o(t)$, $c_o(P_o(t))$	day-ahead energy and its cost
$P_r(t)$, $c_r(P_r(t))$	renewable energy and the cost
$P_b(t)$, $c_b(P_b(t))$	purchased balance energy and the cost
$Q(t)$, $\Delta Q(t)$	total demand and excess demand
$\mathbf{X}^{(t)}$, $\mathbf{Y}^{(t)}$	features from time $1, \ldots, t$

I. OPTIMIZATION MODEL AND FORMULATION

CONSIDER a set $\mathcal{N}$ of N users. We also assume these users are served by a single load-serving entity (LSE). A discrete-time model is considered, in which every day is split into T periods, which are indexed by $t \in \mathcal{T} = \{1, 2, \cdots, T\}$. The time duration for a period varis and can be 5, 15, or 60 mins in our study, representing the time resolution at which decisions including energy dispatch or demand response are made.

A. OPTIMIZATION MODEL

For notational simplicity, we assume that the i^{th} user $i \in \mathcal{N}$ only operates a single appliance so we only need one subscript for indexing appliances. In addition, $q_i(t)$ represents the energy consumption of i^{th} user in time period $t \in \mathcal{T}$. q_i denotes a vector $(q_i(t), \forall t)$ for the energy consumption over the whole day. The i^{th} appliance can be characterized as follows.

- a utility function given by $U_i(q_i(t); t)$ which quantifies the utility that i^{th} user obtains from using the i^{th} applicace and consuming $q_i(t)$ amount of energy in t^{th} time perdiod;
- constraintson energy consumptions:

$$\underline{q}_i(t) \leq q_i(t) \leq \overline{q}_i(t), \forall t \tag{1}$$

$$\underline{Q}_i \leq \sum_t q_i(t). \tag{2}$$

The first constraint ensures that the consumption in each time period is bounded in a certain range. The second constraint indicates that the total consumption is lower bounded by $\underline{Q}_i$. In addition, we define $\underline{q}_i(t) = \overline{q}_i(t) = 0$.if If the i^{th} appliance cannot use electricity in t^{th} time period.

For instance, if $(y_i(t), \forall t)$ is a desired consumption profile, then

$$\sum_t U_i(q_i(t); t) = -\sum_t (q_i(t) - y_i(t))^2$$

characterizes the utility of following $(y_i(t), \forall t)$.

The LSE power procurement process in t^{th} time period consists of two steps. In the first step, the LSE procures "day-ahead" power capacities $P_d(t)$ for the t^{th} time period and pays the capacity costs in the amount of $c_d(P_d(t); t)$. This means that LSE purchases up to $P_d(t)$ amount of energy in t^{th} time period of the following day at a pre-determined price. Let $P_o(t)$ denote the amount of the day-ahead energy that the LSE actually uses in t^{th} time period of the following day and $c_o(P_o(t); t)$ denote its cost. The renewable energy in the t^{th} time period is a nonnegative random variable $P_r(t)$ and the associated cost is $c_r(P_r(t); t)$. For notational simplicity, we assume $c_r(P_r; t) \equiv 0$ for all $P_r \geq 0$ and all t. At the time instance t^- (real time), the random variable $P_r(t)$ is realized to satisfy the demand. Then the LSE satisfies the excess demand via using $P_o(t)$ from the day-ahead capacity. If there is still excess demand, the LSE purchases the $P_b(t)$ from the real-time energy market at the cost in the amount of $c_b(P_b(t); t)$. Therefore we have $q_i(t) \geq 0$ and also the supplies $(P_d(t), P_r(t), P_o(t), P_b(t)) \geq 0$ should satisfy the following constraints:

$$\sum_i q_i(t) \leq P_r(t) + P_o(t) + P_b(t)$$

$$P_o(t) \leq P_d(t)$$

The following assumptions are made for the optimization model:

A1: For each time instance t, we assume the utility functions $U_i(q_i)$ are increasing, strictly concave, and continuously differentiable. In addition, the cost functions $c_d(\cdot; t)$, $c_o(\cdot; t)$ and $c_b(\cdot; t)$ are also assumed to be increasing, continuously differentiable, and convex with $c_d(0; t) = c_o(0; t) = c_b(0; t) = 0$.

A2: For each time instance t, $c_b'(0; t) > c_o'(P_o; t), \forall P_o \geq 0$. This indicates the marginal cost is strictly larger than the marginal cost of day-ahead energy.

In addition, we assume that $\underline{q}_i \geq 0$ for all i and $\underline{Q} \geq 0$.

The LST made the real-time decisions $(P_o(t), P_b(t))$ in order to minimize its total cost. Given the demand vector $q(t) := (q_i(t), \forall i)$, let $Q(t) := \sum_i q_i(t)$ indicate the total demand and $\Delta(Q(t)) := Q(t) - P_r(t)$ is the excess demand. Therefore, the LSE's decision in the t^{th} time period is given by:

$$\begin{aligned} P_o^*(t) &= [\Delta(Q(t))]_0^{P_d(t)} \\ P_b^*(t) &= [\Delta(Q(t)) - P_d(t)]_+ \end{aligned}$$

Please notice that for any real a, b, c, we have $[a]_+ := \max\{a, 0\}$ and $[a]_b^c := \max\{b, \min\{a, c\}\}$.

Therefore, we can calcuate the total supply cost as follows:

$$\begin{aligned} & c(Q(t), P_d(t); P_r(t), t) = \\ & \quad c_d(P_d(t); t) + c_o\left([\Delta(Q(t))]_0^{P_d(t)}; t\right) + \\ & \quad c_b\left([\Delta(Q(t)) - P_d(t)]_+; t\right) \end{aligned} \tag{3}$$

Please note that both the supply and user models can be extended, as shown in our technical report [19]. In addition, the algorithm we proposed can be employed to tackle more general models, with or without time correlation.

B. PROBLEM FORMULATION

The social welfare can be simply calcuated as different between standard user utility and supply cost, given by:

$$\begin{aligned} & W(q, P_d; P_r) := \\ & \sum_{t=1}^{T}\left(\sum_i U_i(q_i(t); t) - c(Q(t), P_d(t); P_r(t), t)\right) \end{aligned} \tag{4}$$

where $q := (q(t), \forall t)$, $Q(t) := \sum_i q_i(t)$, $P_d := (P_d(t), \forall t)$ and $P_r := (P_r(t), \forall t)$. Therefore, LSE aims to maximize the expected social welfare $E[W(q, P_d; P_r)]$. Please notice that this maximization problem needs to consider the random renewable generation P_r. Therefore, $q(t)$ should be decided after $P_r(t)$ have been realized at times t^- (i.e., real-time demand response). However, P_d should be decided a day ahead. Also, please notice that users' utility structure as well as the LSE's cost structure are private information. Therefore, ideally we should seek a solutions requiring no private information exchange.

II. ENERGY PROCUREMENT AND DEMAND RESPONSE

As mentioned above, we now focus on the case without the constraint (2) that couples the consumption decisions $q(t)$ across time. For brevity, t is dropped from the notation. In the t^{th} time period, the welfare maximization problem is give by:

$$\max_{P_d \geq 0}\left\{-c_d(P_d) + E \max_{q \in [\underline{q}, \overline{q}]} W_1(q; P_d, P_r)\right\} \tag{5}$$

where the real-time welfare is given by:

$$\begin{aligned} & W_1(q; P_d, P_r) := \\ & \sum_i U_i(q_i) - c_o\left([\Delta(Q)]_0^{P_d}\right) - c_b\left([\Delta(Q) - P_d]_+\right) \end{aligned} \tag{6}$$

E in (5) is the expectation that is taken for P_r. Therefore the optimization problem can be decomposed into two subproblems:

1) Real-time demand response: optimize real-time welfare W_1 over consumptions q given P_d, P_r:

$$\begin{aligned} \max_{q \in [\underline{q}, \overline{q}]} W_1(q; P_d, P_r) &= \sum_i U_i(q_i) - \\ & c_o\left([\Delta(Q)]_0^{P_d}\right) - c_b\left([\Delta(Q) - P_d]_+\right) \end{aligned} \tag{7}$$

Let $q(P_d, P_r)$ denote an optimizer.

2) Day-ahead capacity procurement: maximize expected welfare over P_d:

$$\max_{P_d \geq 0}\{-c_d(P_d) + EW_1(q(P_d, P_r); P_d, P_r)\} \tag{8}$$

We next solve each subproblem progressively.

A. REAL-TIME DEMAND RESPONSE

In this subsection, we present a distributed algorithm for real-time demand response based on "price" signal.

Note that problem (7) is equivalent to

$$\begin{aligned} \tilde{W}(P_d; P_r) \quad := \quad & \max_{q, y_o, y_b}\{\sum_i U_i(q_i) - c_o(y_o) - c_b(y_b)\} \\ s.t. \quad & \underline{q}_i \leq q_i \leq \bar{q}_i, \forall i \\ & 0 \leq y_o \leq P_d, y_b \geq 0 \\ & P_r + y_o + y_b \geq \sum_i q_i, \end{aligned} \tag{9}$$

where y_o and y_b indicate the amount of day-ahead capacity and real-time energy respectively.

In [20], we have proposed a distributed primal-dual algorithm for (9). However, the convergence of that algorithm relies on the proper selection of step sizes and it converges very slowly. Since two-way communication between the LSE and each user is required at every iteration, such slow converge speed may incur significant delay in demand response, especially for a large-scale power grid with many users.

Here we propose an algorithm based on combined secant [5] and bisection method that does not require any step-size selection and converges much faster than the primal-dual algorithm. For problem (9), we associate a Lagrangian multiplier $\lambda \geq 0$ (i.e., the "price" of electricity) to the last constraint, and obtain the dual function as

$$\begin{aligned} L(\lambda) \quad := \quad & \max_{q, y_o, y_b}\{\sum_i U_i(q_i) - c_o(y_o) - c_b(y_b) + \\ & \lambda(P_r + y_o + y_b - \sum_i q_i)\} \\ s.t. \quad & \underline{q}_i \leq q_i \leq \bar{q}_i, \forall i \\ & 0 \leq y_o \leq P_d, y_b \geq 0. \end{aligned} \tag{10}$$

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Problem (10) can be decomposed into a set of individual maximization problems for every user and two maximization problems for the LSE involving y_o and y_b respectively, given in the sequel,

$$L_i(\lambda) := \max_{q_i} \{U_i(q_i) - \lambda q_i\} \quad s.t. \quad \underline{q}_i \le q_i \le \bar{q}_i, \forall i, \tag{11}$$

$$L_{y_o}(\lambda) := \min_{y_o} \{c_o(y_o) - \lambda y_o\} \quad s.t. \quad 0 \le y_o \le P_d, \tag{12}$$

and

$$L_{y_b}(\lambda) := \min_{y_b} \{c_b(y_b) - \lambda y_b\} \quad s.t. \quad y_b \ge 0. \tag{13}$$

The master dual problem is,

$$\min_{\lambda} \; L(\lambda) \quad s.t. \; \lambda \ge 0, \tag{14}$$

where $L(\lambda) = \sum_i L_i(\lambda) - L_{y_o}(\lambda) - L_{y_b}(\lambda) + \lambda P_r$.

The following proposition follows from standard convex optimization theory.

Proposition 1. *For given λ, assume $q_i(\lambda)$, $y_o(\lambda)$, and $y_b(\lambda)$ are optimal solutions to (11), (12), and (13) respectively. Then $s(\lambda) = P_r + y_o(\lambda) + y_b(\lambda) - \sum_i q_i(\lambda)$ is a subgradient for the dual function $L(\lambda)$.*

Let $\partial(L(\lambda))$ denote the set of all subgradients of $L(\lambda)$ at λ, it follows from the definition of the subgradient that λ^* minimize $L(\lambda)$ if and only if $0 \in \partial(L(\lambda^*))$.

This fact motivates us to propose a combined secant and bisection algorithm that starts with an initial interval $[\lambda^0_{\min}\ \lambda^0_{\max}]$ and progressively reducing the searching range to find an optima solution λ^*. Specifically, let $[\lambda^k_{\min}\ \lambda^k_{\max}]$ denote the searching range at the k^{th} iteration. We use the secant method in (17) to compute a new price λ^{k+1} within this range, and use it as one end of the next searching range. The secant method typically converges rapidly, especially if the initial guess i.e., $\lambda^0_{\min}$ or $\lambda^0_{\max}$ is close to the optimal solution. However, for certain objective functions, this method may exhibit slow convergence, e.g., when $\frac{\lambda^k_{\max} - \lambda^k_{\min}}{\lambda^{k-1}_{\max} - \lambda^{k-1}_{\min}} > 0.5$. If this situation occurs for two consecutive iterations before the algorithm converges, we use the bisection updating rule in Eq. (15) to compute the next price.

We assume that the LSE knows an upper bound of the total demand, P_{max}, which satisfies $P_{max} > \sum_i \bar{q}_i$. The detailed algorithm is given in Algorithm 1.

The convergence property of the proposed algorithm is as follows.

Proposition 2. *Algorithm 1 converges to the set of optimum solutions of (14).*

Proof. If $s(\lambda^0_{\min}) = s(0) \ge 0$, then the algorithm returns

Algorithm 1: Combined secant and bisection method for real-time demand response

The LSE lets $k = 0$, $\lambda^{-1} = \lambda^0_{\min} = 0$, and $\lambda^0 = \lambda^0_{\max} > c_b^{'}(P_{max})$. (Note that since $\lambda^0 > c_b^{'}(P_{max})$, we have $y_b(\lambda^0) \ge P_{max} > \sum_i \bar{q}_i \ge \sum_i q_i(\lambda^0)$. Therefore $s(\lambda^0) > 0$.)
if $s(\lambda^{-1}) = s(0) \ge 0$ **then**
  The price is determined as $\lambda^* = 0$ **else**
  repeat
    if *the algorithm has been converging slowly in the last two iterations (specifically, if $k \ge 2$ and $\frac{\lambda^k_{\max} - \lambda^k_{\min}}{\lambda^{k-1}_{\max} - \lambda^{k-1}_{\min}} > 0.5$ and $\frac{\lambda^{k-1}_{\max} - \lambda^{k-1}_{\min}}{\lambda^{k-2}_{\max} - \lambda^{k-2}_{\min}} > 0.5$)* **then**
      The next price λ^{k+1} is calculated using the bisection updating rule as:

$$\lambda^{k+1} = \frac{\lambda^k_{min} + \lambda^k_{max}}{2} \tag{15}$$

    else
      λ^{k+1} is calculated using the secant method [5]:

$$\lambda^{k+1} \tag{16}$$

$$= \frac{\lambda^k_{min} s(\lambda^k_{max}) - \lambda^k_{max} s(\lambda^k_{min})}{s(\lambda^k_{max}) - s(\lambda^k_{min})}; \tag{17}$$

    end
    end
    if $s(\lambda^{k+1}) > 0$ **then**
      The end-points of the bracket are set as $\lambda^{k+1}_{min} = \lambda^k_{min}$ and $\lambda^{k+1}_{max} = \min(\lambda^{k+1}, \lambda^{k+1}_{max})$
    else
      if $s(\lambda^{k+1}) < 0$ **then**
        The end-points are set as $\lambda^{k+1}_{min} = \max(\lambda^{k+1}, \lambda^{k+1}_{min})$ and $\lambda^{k+1}_{max} = \lambda^k_{max}$.
      end
    end
    end
    k=k+1;
  until $\sum_i |q_i(\lambda^{k+1}) - q_i(\lambda^k)| < 1e-5$ *or* $s(\lambda^{k+1}) = 0$;
  $\lambda^* = \lambda^{k+1}$
end
The consumption decisions are $q_i(\lambda^*), \forall i$.
end

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0, which is an optimum solution. (This is because by the definition of subgradient, for any $\lambda > 0$, we have $L(\lambda) \geq L(0) + s(0)\lambda \geq L(0)$.)

So we only need to consider the case when the initial two end points are of different sign, i.e., $s(\lambda^0_{\min}) < 0$ and $s(\lambda^0_{\max}) > 0$. Due to the definition of the subgradient, we have that $L(\lambda) > L(\lambda^0_{\min})$ for any $\lambda < \lambda^0_{\min}$; and $L(\lambda) > L(\lambda^0_{\max})$ for any $\lambda > \lambda^0_{\max}$. Therefore, λ^* must belong to the interval $[\lambda^0_{\min},\ \lambda^0_{\max}]$. Also, at each iteration of the above algorithm, unless the algorithm terminates after finding $s(\lambda^{k+1}) = 0$ (in which case λ^{k+1} is clearly a solution), we always have $s(\lambda^k_{\min}) < 0$ and $s(\lambda^k_{\max}) > 0$. Therefore $\lambda^* \in [\lambda^k_{\min},\ \lambda^k_{\max}]$. Due to the bisection updating rule used in the proposed algorithm, the following inequality always hold before convergence, i.e.,

$$\frac{\lambda^{k+3}_{\max} - \lambda^{k+3}_{\min}}{\lambda^{k}_{\max} - \lambda^{k}_{\min}} \tag{18}$$

$$= \frac{\lambda^{k+3}_{\max} - \lambda^{k+3}_{\min}}{\lambda^{k+2}_{\max} - \lambda^{k+2}_{\min}} * \frac{\lambda^{k+2}_{\max} - \lambda^{k+2}_{\min}}{\lambda^{k+1}_{\max} - \lambda^{k+1}_{\min}} * \frac{\lambda^{k+1}_{\max} - \lambda^{k+1}_{\min}}{\lambda^{k}_{\max} - \lambda^{k}_{\min}} \tag{19}$$

$$\leq 1 * 1 * 0.5 = 0.5, \tag{20}$$

which means the length of the searching interval will converge to zero. Hence, the above algorithm must converge to optimal solution. □

Please note that if we replace (17) with (15), we obtain the traditional bisection method. Also, the secant method is of suplinear convergence rate while the bisection algorithm is of linear convergence rate.

Proposition 3. *Assume $s(*)$ is twice differentiable, $s^{''}(\lambda^*)$ is bounded and $s'(\lambda^*)$ is not equal to zero, Algorithm 1 converges superlinearly.*

Proof. The analysis of the convergence of Algorithm 1 is based on analyzing the error sequence $\{e^0, e^1, e^2, ...\}$, where $e^k = \lambda^* - \lambda^k$. According to Algorithm 1,

$$\lambda^{k+1} = \frac{\lambda^k_{min} s(\lambda^k_{max}) - \lambda^k_{max} s(\lambda^k_{min})}{s(\lambda^k_{max}) - s(\lambda^k_{min})}. \tag{21}$$

And the corresponding sequence of error is given by:

$$e^{k+1} = \frac{e^{k-1} s(\lambda^k) - e^k s(\lambda^{k+1})}{s(\lambda^k) - s(\lambda^{k-1})}, \tag{22}$$

We have proved that the proposed algorithm converges, thus we can use the following truncated Taylor expansions to calculate $s(\lambda^k)$, i.e.,

$$s(\lambda^k) = -s'(\lambda^*)e^k + \frac{1}{2}s^{''}(\lambda^*)(e^k)^2 + \mathcal{O}\left(|e^k|^3\right) \tag{23}$$

Substitute the Taylor expansion into (22) we have

$$e^{k+1} = \frac{\mathcal{O}\left(|e^k|^4\right) + \frac{1}{2}s^{''}(\lambda^*)e^k e^{k-1}}{s'(\lambda^* + \frac{1}{2}s^{''}(\lambda^*)(e^k + e^{k-1})) + \mathcal{O}\left(|e^k|^3\right)} \tag{24}$$

Since we have proved the convergence of Algorithm 1, the $\mathcal{O}\left(|e^k|^l\right)$ is dominated by $\mathcal{O}\left(|e^{k-1}|^l\right)$ for $l = 3$ and $l = 4$. Therefore, asymptotically we have

$$e^{k+1} \asymp \frac{s^{''}(\lambda^*)}{2s'(\lambda^*)} e^k e^{k-1} \tag{25}$$

Since $s^{''}(\lambda^*)$ is bounded and $s'(\lambda^*)$ does not equal to zero, $\frac{e^{k+1}}{e^k}$ is strictly smaller than 1/2 when k goes to infinity. This means after a finite number of iterations, the bisection updating rule will not be applied. Hence the convergence rate is equal to the scant method which is superlinear. □

Please note Algorithm 1 can be realized either in an asynchronous manner or synchronously, which is implementation friendly. Also, please notice that the message exchange required for Algorithm 1 to converge is much less than that of the primal-dual subgradient algorithm.

In the simulation studies, we will demonstrate that Algorithm 1 converges significantly faster than the primal-dual algorithm in [20]. In this particular case, it requires only fifteen iterations to converge to the optimal solution whereas it takes the subgradient method 500 iterations to converge. It is observed that the Algorithm 1 converges roughly thirty times faster than the traditional subgradient algorithm.

In addition, the proposed algorithm is **distributed** and **scalable** since each user only needs to solve a single optimization problem as given in (11) regardless of the total number of users in the system. Hence it is particularly suitable for large-scale system involves tens of thousands of users.

B. DAY-AHEAD CAPACITY PROCUREMENT

In this section, we present an algorithm for the LSE to compute the optimum day-ahead capacity that maximizes the expected social welfare. For this purpose, the LSE needs to solve (5), which is equivalent to

$$\min_{P_d \geq 0} \{-E[\tilde{W}(P_d; P_r)] + c_d(P_d)\}, \tag{26}$$

where $\tilde{W}(.)$ is defined in (9). Write the optimum dual variable λ^* as $\lambda^*(P_d; P_r)$ to reflect its dependency on P_d and P_r. The following result says that a subgradient of $-\tilde{W}(P_d; P_r)$ is $-[\lambda^*(P_d; P_r) - c_o^{'}(P_d)]_+$. Based on the definition of stochastic subgradient, we can obtain the following proposition.

Proposition 4. *(i) $\tilde{W}(P_d; P_r)$ is concave in P_d.*
(ii) For any $\tilde{P}_d, P_d \geq 0$, we have

$$\tilde{W}(\tilde{P}_d; P_r) \leq \tilde{W}(P_d; P_r) + [\lambda^*(P_d; P_r) - c_o^{'}(P_d)]_+ \cdot (\tilde{P}_d - P_d). \tag{27}$$

In particular, if $\tilde{W}(P_d; P_r)$ is differentiable at P_d, then

$$\partial \tilde{W}(P_d; P_r)/\partial P_d = [\lambda^*(P_d; P_r) - c_o^{'}(P_d)]_+.$$

Proof. Part (i): Since $\tilde{W}(P_d; P_r)$ is the optimal value of the convex optimization problem (9), it is concave in P_d [3].

Part (ii): To emphasize their dependency on P_d and/or P_r, we write $L(\lambda)$ in (10) as $L(\lambda; P_d, P_r)$, and $L_{y_o}(\lambda)$ in (12) as $L_{y_o}(\lambda; P_d)$. Then,

$$L(\lambda; P_d, P_r) = \sum_i L_i(\lambda) - L_{y_o}(\lambda; P_d) - L_{y_b}(\lambda) + \lambda P_r.$$

By strong duality, $\min_{\lambda \geq 0} L(\lambda; P_d, P_r) = \tilde{W}(P_d; P_r)$. It is not difficult to verify that for any $\tilde{P}_d, P_d \geq 0$

$$L_{y_o}(\lambda; \tilde{P}_d) \leq L_{y_o}(\lambda; P_d) + [\lambda - c_o^{'}(P_d)]_+ \cdot (\tilde{P}_d - P_d).$$

So

$$\begin{aligned} & L(\lambda^*(P_d; P_r); \tilde{P}_d, P_r) - L(\lambda^*(P_d; P_r); P_d, P_r) \\ & \leq [\lambda - c_o^{'}(P_d)]_+ \cdot (\tilde{P}_d - P_d). \end{aligned} \quad (28)$$

Note that $L(\lambda^*(P_d; P_r); P_d, P_r) = \tilde{W}(P_d; P_r)$. Also, since $\lambda^*(\tilde{P}_d; P_r)$ solves $\min_{\lambda \geq 0} L(\lambda; \tilde{P}_d, P_r)$, we have

$$L(\lambda^*(P_d; P_r); \tilde{P}_d, P_r) \geq L(\lambda^*(\tilde{P}_d; P_r); \tilde{P}_d, P_r) = \tilde{W}(\tilde{P}_d; P_r).$$

Plugging these into (28), we obtain (27). □

Using Prop. 4, a subgradient of the objective function in (26) is $-E([\lambda^*(P_d; P_r) - c_o^{'}(P_d)]_+) + c_d^{'}(P_d)$. A stochastic subgradient algorithm that simulates the system by drawing samples of P_r and converges to the set of optimal P_d is as follows.

Algorithm 2: Day-ahead capacity

1) Initially, let $P_d^0 = 0$.
2) In step $m = 0, 1, 2, \ldots$, independently generate a sample of P_r (denoted by P_r^m), run Algorithm 1 and obtain the optimum dual variable $\lambda^*(P_d^m; P_r^m)$. Then, compute

$$P_d^{m+1} = \left\{ P_d^m + \alpha^m \{ [\lambda^*(P_d^m; P_r^m) - c_o^{'}(P_d^m)]_+ - c_d^{'}(P_d^m) \} \right\}_0^{P_{max}}$$

where $\alpha^m = 1/(m+1)$ is the step size. The algorithm terminates when a certain convergence criterion is met (e.g., $|P_d^{m+1} - P_d^m| \leq 1e-4$).

C. COMPARISON WITH DAY-AHEAD PLANNING

In our proposed algorithms above, the demand is decided after the realization of the renewable energy P_r (i.e., real-time demand response). Another possible scheme for load-side participation is "day-ahead planning", in which the consumption q is planned one day ahead. The maximal expected social welfare with day-ahead planning is

$$W_d := \max_{P_d \geq 0, q \in [\underline{q}, \overline{q}]} \{-c_d(P_d) + E[W_1(q; P_d, P_r)]\}. \quad (29)$$

In this section, we compare the expected social welfare achieved by real-time demand response and day-ahead planning through an example, and show how their difference depends on the variance of P_r and the cost of real-time energy.

Proposition 5. *Assume that the utility function is $U_i(q_i) = -(q_i - z)^2, i = 1, 2, \ldots, N$ where $z > 0$ is the target demand of user i (for simplicity, we assume that the target is the same for all users). There is no upper-bound constraint on $q_i \geq 0$. The cost functions are assumed to be $c_d(P) = \beta \cdot P^2, c_o(P) = 0$ and $c_b(P) = \gamma \cdot P^2$. Denote the variance of P_r as $\sigma^2 = E[(P_r - \bar{P}_r)^2]$ where $\bar{P}_r = E(P_r)$. We also make the technical assumption that*

$$P_r^{max} - \bar{P}_r < \frac{\beta}{\beta\gamma N + \beta + \gamma}(Nz - \bar{P}_r) \quad (30)$$

where P_r^{max} is the upper bound of P_r.

Then, the expected social welfare with real-time demand response is higher than with day-ahead planning, and the difference is $\frac{N\gamma^2}{1+N\gamma}\sigma^2$.

Remark: Note that the difference increases with the variance of P_r. Also, the more expensive is the real-time balancing energy (i.e., the larger is γ), the larger is the difference.

Proof. First consider day-ahead planning. Since the constraint on q is $q \geq 0$, (29) becomes

$$W_d := \max_{P_d \geq 0, q \geq 0} \{-c_d(P_d) + E[W_1(q; P_d, P_r)]\}. \quad (31)$$

If the real-time energy $\sum_i q_i - P_d - P_r > 0$ for any realization of P_r, we have

$$\begin{aligned} & E[W_1(q, P_d; P_r)] \\ = & E[\sum_i U_i(q_i) - \gamma(\sum_i q_i - P_d - P_r)^2] \\ = & \sum_i U_i(q_i) - \gamma E\{[\sum_i q_i - P_d - \bar{P}_r + (\bar{P}_r - P_r)]^2\} \\ = & \sum_i U_i(q_i) - \gamma(\sum_i q_i - P_d - \bar{P}_r)^2 - \gamma\sigma^2 \\ = & W_1(q, P_d; \bar{P}_r) - \gamma\sigma^2. \end{aligned} \quad (32)$$

We first solve $\max_{P_d, q}\{-c_d(P_d) + W_1(q, P_d; \bar{P}_r) - \gamma \cdot \sigma^2\}$ without any constraint. The optimal solution is

$$\begin{aligned} \tilde{P}_d &= \frac{\gamma(Nz - \bar{P}_r)}{\beta\gamma N + \beta + \gamma}, \\ \tilde{q}_i &= \frac{z(\beta + \gamma) + \beta\gamma\bar{P}_r}{\beta\gamma N + \beta + \gamma}, \forall i. \end{aligned}$$

Clearly, $\tilde{q}_i > 0$. Since $P_r^{max} - \bar{P}_r \geq 0$, the assumption (30) implies that $Nz - \bar{P}_r > 0$, which implies that $\tilde{P}_d > 0$. Also,

$$\begin{aligned} & \sum_i \tilde{q}_i - \tilde{P}_d - P_r \\ = & \sum_i \tilde{q}_i - \tilde{P}_d - \bar{P}_r + (\bar{P}_r - P_r) \\ = & \frac{\beta}{\beta\gamma N + \beta + \gamma}(Nz - \bar{P}_r) + (\bar{P}_r - P_r) \\ > & 0 \end{aligned}$$

where the last step follows from (30).

Therefore, the constraints $P_d \geq 0, q \geq 0$ are not active, and the real-time energy $\sum_i \tilde{q}_i - \tilde{P}_d - P_r$ is indeed always positive. So, $\tilde{P}_d$ and $\tilde{q}$ are the optimal solution of (31). Consequently,

$$W_d = -\frac{\beta\gamma}{\beta\gamma N + \beta + \gamma}(N \cdot z - \bar{P}_r)^2 - \gamma\sigma^2.$$

Now we consider real-time demand response. Similarly, we solve the problem (5) without considering the constraints, and later show that the constraints are not active. We have

$$\max_q W_1(q; P_d, P_r) = -\frac{\gamma}{1 + N \cdot \gamma}(P_d + P_r - N \cdot z)^2,$$

and the maximizer is

$$q_i(P_d, P_r) = \frac{z + \gamma(P_d + P_r)}{1 + \gamma N}, \forall i.$$

Assuming that the real-time energy is positive (which will be justified later), we have (similar to (32))

$$E[\max_q W_1(q; P_d, P_r)] = -\frac{\gamma}{1 + N \cdot \gamma}(P_d + \bar{P}_r - N \cdot z)^2 - \frac{\gamma}{1 + N \cdot \gamma}\sigma^2.$$

To find the optimal day-ahead energy, we solve $\max_{P_d}\{-c_d(P_d) + E[\max_q W_1(q; P_d, P_r)]\}$, and find that the maximizer is

$$P_d^* = \frac{\gamma(Nz - \bar{P}_r)}{\beta\gamma N + \beta + \gamma}.$$

Clearly, $P_d^* > 0$ since $Nz - \bar{P}_r > 0$. Therefore, $q_i(P_d, P_r) > 0$. Finally, we have

$$\sum_i q_i(P_d^*, P_r) - P_d^* - P_r = \frac{\beta}{\beta\gamma N + \beta + \gamma}(Nz - \bar{P}_r) + \frac{\bar{P}_r - P_r}{1 + \gamma N} > 0,$$

using (30). Therefore the real-time energy under P_d^* and $q_i(P_d^*, P_r)$ is always positive.

So, P_d^* and $q_i(P_d^*, P_r)$ are the optimal solution of (5), and the optimal value is

$$W^* = -\frac{\beta\gamma}{\beta\gamma N + \beta + \gamma}(N \cdot z - \bar{P}_r)^2 - \frac{\gamma}{1 + N \cdot \gamma}\sigma^2.$$

Comparing W^* to W_d, we have

$$W^* - W_d = \frac{N\gamma^2}{1 + N\gamma}\sigma^2.$$

□

III. NUMERICAL EXAMPLES

NUMERICAL examples are provided in this section to illustrate the performance of our algorithms. We consider a time-slotted system with 24 hours, i.e., $T = 24$. The first, second, and the third time period is 8-9am, 9-10am, and so on. For the i^{th} user, the following utility function is considered, i.e., $U_i(q_i) = \sum_{t=1}^{T} U_i(q_i(t); t) = -\sum_{t=1}^{T}[q_i(t) - y_i(t)]^2$ where $y_i(t)$ is user i's target consumption in slot t. This quantify the deviation of the actual demand profile $\{q_i(t)\}$ from the target. Fig. 1 illustrates the target demand profiles of $N = 4$ users in our simulation. The unit of energy is kWh. We assume that $0 \le q_i(t) \le 20, \forall i, t$.

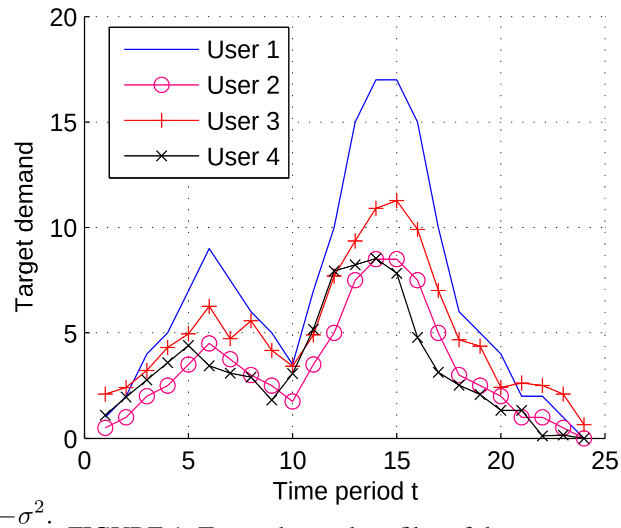

FIGURE 1: Target demand profiles of the users

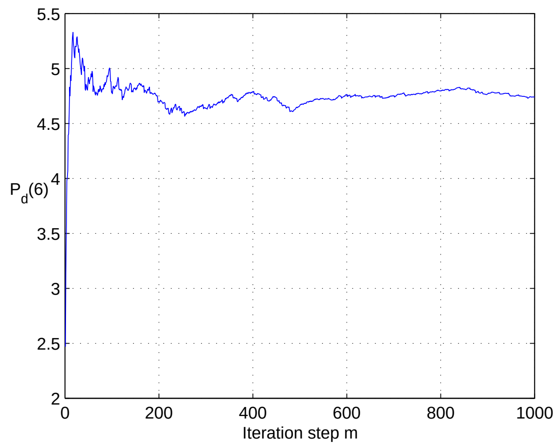

FIGURE 2: Convergence of $P_d(6)$

Assume that $P_r(t)$ is uniformly distributed between 0 and $2\bar{P}_r(t) > 0$, so that its mean is $E(P_r(t)) = \bar{P}_r(t)$. Also, $P_r(t)$'s are independent across t. The values of $(\bar{P}_r(t), \forall t)$ are (2, 3, 4, 5, 5, 6, 6, 7, 6, 5, 4, 3, 2, 2, 3, 4, 4, 4, 4, 3, 3, 2, 2, 2).

For each time period, assume that the cost functions are $c_d(P) = (P^2 + P)/2$, $c_o(P) = P/2$, and $c_b(P) = P^2/2 + 5P$.

Algorithm 2 is employed to determine the day-ahead capacity. Fig. 2 shows that the computed value of $P_d(6)$ converges. $P_d(t)$'s converges in a similar .

In Fig. 4 to Fig. 6, we show the convergence trajectories of a variety of algorithms for real-time demand response of four users, including the conventional primal-dual method (denoted by 'primal-dual'), the bisection method (denoted by 'bisection'), and the combined secant and bisection method (denoted by 'bisection + secant'). As we can see, the convergence speed of both bisection algorithm and the combined

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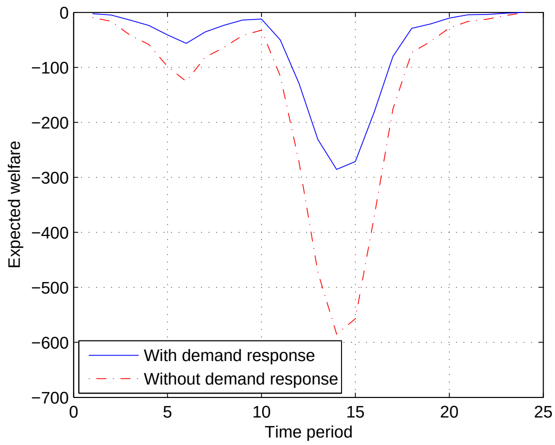

FIGURE 3: Expected welfare in each time period

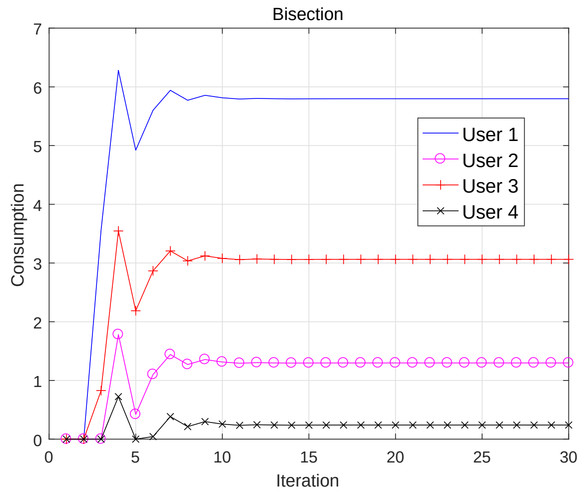

FIGURE 5: Convergence performance for real-time demand response using bisection method

secant and bisection algorithm significantly outperform the traditional primal-dual algorithm, since the formers at least have a linear convergence rate while the latter only has a sublinear convergence rate. Moreover, we observe that the combined secant and bisection method provides a better convergence performance than that of bisection method, since the convergence speed of the latter relies on the tightness of the initial guess interval that contains the optimal dual solution, while the former mitigate this issue by using the secant step. In Fig. 7, we present the average number of iterations required for convergence of Algorithm 1 at different time steps. As we can see, 'secant + bisection' always yields the fastest convergence speed.

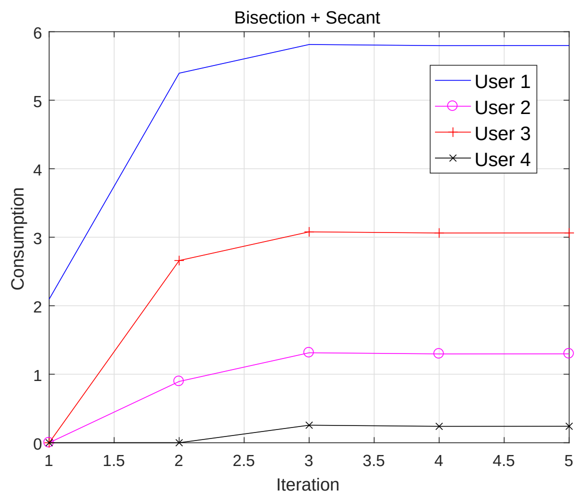

FIGURE 6: Convergence performance for real-time demand response using combined secant and bisection algorithm

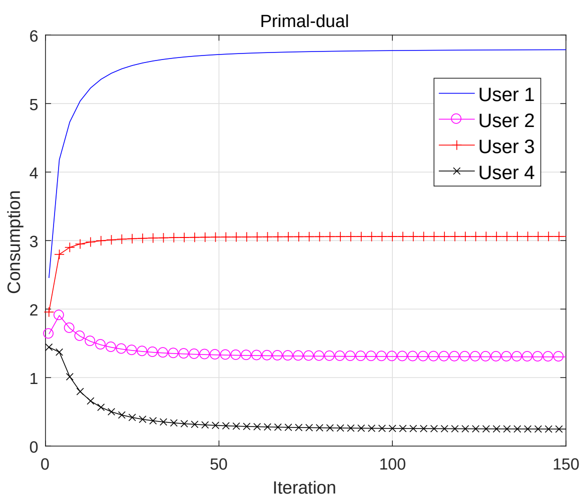

FIGURE 4: Convergence performance for real-time demand response using primal-dual method

In Fig. 8, we demonstrate the impact of various implementations of Algorithm 1 (based on 'primal-dual', 'bisection', and 'secant + bisection', respectively) on the convergence performance of Algorithm 2 while computing the day-ahead capacity at period 6, $P_d(6)$. As we can see, different implementations of Algorithm 1 has less effect on the accuracy of the solution of Algorithm 2. However, 'bisection' and 'secant + bisection' are the most efficient in computation. They also yield the same convergence trajectory of Algorithm 2. This is not surprising, since both of the methods converge fast and solve the master dual problem (14) at the same level of accuracy.

Finally, the expected social welfare under Algorithm 1 and 2, as given in Fig. 3, is significantly higher than the case without demand response (where the users consume the target demands and the LSE optimizes $P_d(t)$.)

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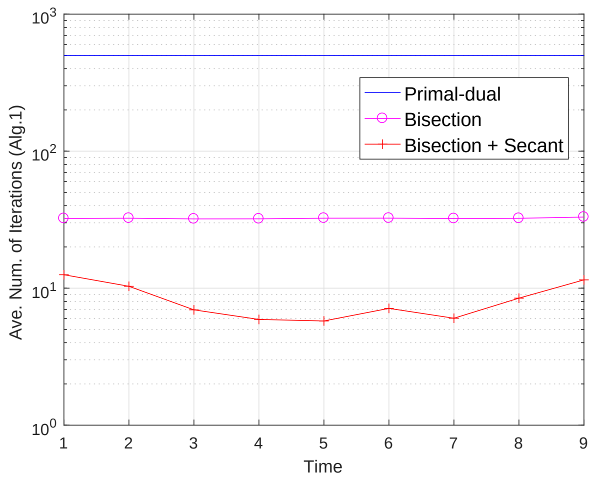

FIGURE 7: Average number of iterations required for convergence of Algorithm 1 for real-time demand response

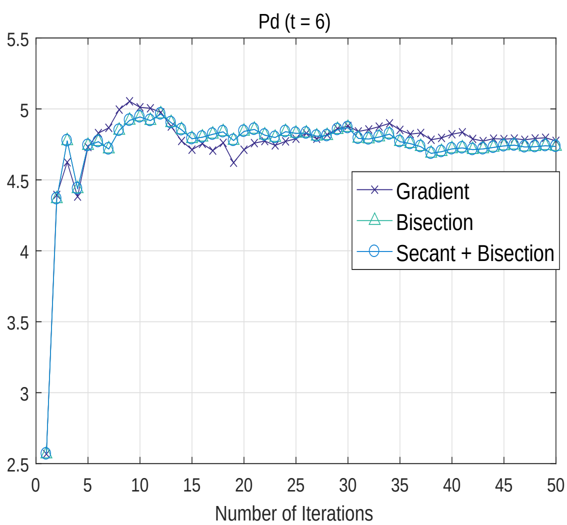

FIGURE 8: Convergence of day-ahead capacity $P_d(6)$ under different implementations of Algorithm 1.

IV. CONCLUSION

We have studied joint energy procurement and real-time demand responses for a smart power grid system integrated with renewable energy sources. In particular, a demand response algorithm with fast convergence has been presented and its performance has been both theoretically analyzed and studied through simulation studies. It exhibits much faster convergence speed than the traditional subgradient algorithm.

While we focused on one type of utility functions and consumption constraints in this paper, the proposed framework can be extended to other types of appliances as well. A possible future research topic is to consider the case with distributed renewable generation with users such as the case in a microgrid system.

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