Accumulation is late and brief in preferential choice.

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Preferential choices are often explained using models within the evidence accumulation framework: value drives the drift rate at which evidence is accumulated until a threshold is reached and an option is chosen. Although rarely stated explicitly, almost all such models assume that decision makers have knowledge at the onset of the choice of all available attributes and options. In reality however, choice information is viewed piece-by-piece, and is often not completely acquired until late in the choice, if at all. Across four eye-tracking experiments, we show that whether the information was acquired early or late is irrelevant in predicting choice: all that matters is whether or not it was acquired at all. Models with potential alternative assumptions were posited and tested, such as 1) accumulation of instantaneously available information or 2) running estimates as information is acquired. These provided poor fits to the data. We are forced to conclude that participants either are clairvoyant, accumulating using information before they have looked at it, or delay accumulating evidence until very late in the choice, so late that the majority of choice time is not time in which evidence is accumulated. Thus, although the evidence accumulation framework may still be useful in measurement models, it cannot account for the details of the processes involved in decision making.

In simple decision tasks, participants make a series of choices between two options, while researchers strive to explain their responses and reaction times[1,2]. The dominant account of these tasks is evidence accumulation, which characterises decision making as a process of updating a running total of evidence, either in favour of each option, or the relative evidence of an option compared to its competitor. A response is initiated once the evidence exceeds the decision maker's pre-defined threshold. Often, this process is simplified by using discrete time steps. Although the precise specifications of accumulation models vary, at a minimum they all include three key concepts. The *drift rate* is the mean rate at which the evidence of a given accumulator changes on each time step. The *boundary* is the threshold value at which evidence accumulation stops and a response is made. The *non-decision time* is the time not spent on accumulating but on other processes, such as perception and motor responses.

Evidence accumulation successfully explains many features of simple, fast, perceptual choice including the shape of the distribution of reaction times[3], speed-accuracy trade-offs[4], and fast errors[5]. It is also neurally plausible[6]. More recently, evidence accumulation models have been successfully applied to value-based choice, where participants choose their preferred option[7]. Extending the evidence accumulation framework from perceptual to preferential choices resulted in minimal changes to the model assumptions. Fundamentally the processes remain the same, but with money or subjective value ratings replacing perceptual properties such as luminance or motion coherence when defining the drift rate.

Whilst evidence accumulation processes have been successful in predicting preferential choice, some fundamental differences between value-based and perceptual choices are not always addressed. Particularly important are the assumptions made about the drift rate. In perceptual paradigms, evidence is often dynamic and stochastic, such as in the case of random dot kinematograms (RDK) where participants estimate the average direction of jittering dots[8]. In such tasks, longer deliberation times allow participants to build a more accurate representation of the options by integrating the stimulus signal for longer. Contrast this with a common preferential choice task where participants choose between two snack items[9,10]. In this case, the stimuli are static images. Once recognised, longer deliberation times do not increase the amount of information that a subject can collect, or improve the accuracy of their representation of the options.

Additionally, in applying these models to preferential choice, the time course of the acquisition of value information has been ignored. In an RDK task participants only need to attend to one input, a single patch of dots, to gain knowledge about all possible response options: left vs. right, or up vs. down. This means that evidence is being accumulated simultaneously and equally for all possible response options from the onset of the trial. However, in a snack choice task, the options must be presented in spatially distinct regions of the display. At the onset of the trial the subject knows nothing about the value of any of the options, then must gather information about each option sequentially. Further, stimulus displays are often designed so participants must shift their attention from item to item in spatially distinct locations. This means that in preferential choice evidence cannot be accumulated simultaneously and equally for the available options.

Therefore, these preferential choice experiments conflict with the assumptions of evidence accumulation models. These models incorrectly assume complete knowledge of item information from early in a choice—here referred to as Knowledge At Onset (KAO). The issues of KAO assumptions obviously apply to some perceptual tasks as well, such as luminance comparison choice or dot numerosity tasks, where there are multiple dot patches spatially separated. However, these issues are more pronounced in the vast majority of preferential choice tasks due to the static stimuli and the reduced role of peripheral vision in identifying the more visually complex properties of the options.

Furthermore, evidence accumulation models are being applied to vastly more complex choices in the preferential choice domain. Models such as decision field theory[11], leaking competing accumulators[12], the Poisson race model[13], the attentional drift diffusion model[9], associative accumulator model[14], and the multialternative linear ballistic accumulator model[15] have built upon the success of evidence accumulation in simple choice by applying similar frameworks to predict risky gamble and multi-attribute choices. The inclusion of many more pieces of information for the different attributes means that subjects must acquire much more information, and presumably process this more deeply, before a relative drift rate (or other parameters controlling attention switching likelihood etc.) can be known. However, all these models still assume full KAO.

As evidence accumulation models commonly assume some form of non-decision time, it would seem plausible that reading and information gathering could be incorporated into the choice process using this parameter. Essentially, if this were the case, evidence accumulation would only begin after all the information has been acquired. Indeed, similar multi-stage frameworks have been proposed outside of the evidence accumulator literature ([16,17] but see[18] for no changes in fixation pattern over time). These assume that in complex choice, the decision process begins with a reading phase prior to the choice process itself. Eye-tracking studies have provided evidence in support of this assumption. Commonly, these studies find that participants tend to begin by examining all attributes once, and then switch to a different pattern of refixations[16,19]. Specifically, attention early on tends to follow a systematic left to right, or top to bottom pattern, whilst during refixations there is no discernible pattern.

However, despite the potential compatibility of a reading phase with non-decision time, several pieces of evidence suggest this cannot be true. One issue is that of timing. Fitting evidence accumulation models tends to produce relatively short estimates for non-decision time periods. If they are estimated at all, they are typically between 100ms and 500ms e.g.[3,20–22]. Since a basic motor response requires 200ms or more to execute and the average length of a fixation is around 250ms, there is little time left for reading and information acquisition, even in very simple choices where all the information could be read quickly[18,23].

A further issue is that although many studies find a reading order effect from left to right and top to bottom at the beginning of a choice, this isn't consistent with a distinct reading phase that finishes prior to any choice processing[16,19]. This is because participants regularly begin re-fixating information before they have read all the available information, and often choose before having attended all the available information[18]. Therefore, information acquisition patterns appear incompatible with a strict interpretation of non-decision time as a reading phase.

Other families of evidence accumulation models have tried to address these issues by assuming serial information search and that decision makers can only attend to one piece of information at a time. Models such as piecewise linear ballistic accumulator[20] and the attentional drift diffusion model[9] assume that the drift rate is dependent upon the currently attended information. However, this dependency is incorporated as a bias towards the currently attended

Figure 1: Example trials from the poster and currency tasks.

information, and that the drift rate is still inherently relative (based on a summary difference between all values). That is, it is assumed at all times, that the value of all other information is known (KAO) so that relative drift rates can still be calculated from onset.

The question this paper asks is, how well do evidence accumulation models perform if we constrain their assumptions and do not allow information to be used before there is any way a decision maker could know it? That is, once we have ruled out clairvoyance, what can we conclude about evidence accumulation? We think that people are accumulating—in the sense of gathering the evidence needed to make a choice—for much of the decision time. But our stark conclusion is that people are not accumulating—in the evidence accumulation model sense of integrating incoming evidence over time, as instantiated, for example, in the drift diffusion model—for most of the time between stimulus onset and choice.

Models

Our approach to testing the accuracy of evidence accumulation models follows that introduced in Smith et al.[24] who demonstrated that a logistic regression framework simply and robustly recovers estimates of drift diffusion parameters whilst avoiding stochastic simulation. Specifically, this approach relies on predicting the choice proportions using a logistic regression. This allows us to test a range of plausible assumptions about the evidence accumulation mechanism using the same statistical model. For each assumption, we calculate the accumulated evidence as predicted by the attributes on each trial. The difference score is then used as a predictor in the logistic regression predicting choice. Specifically we model the choice on each trial in a logistic regression $Log\ Odds(ChooseA) = \beta_0 + \beta_\Delta \Delta$ where Δ is one of five estimates of the difference between the choice options A and B below.

Value Difference Model

The simplest and most common existing assumption is that the drift rate is defined by the difference in value (or subjective ratings) between the two options[1]. This is the assumption of accumulation of complete information from the beginning of the trial and of course implies KAO. We use this as our baseline. In multi-attribute choices, we assume that an object's value is represented as the mean value across its attributes.

To provide a concrete example across each of the following assumptions, we use the hypothetical trial described in Figure 2 and accompanying diagram. On this trial, participants had to choose between two options each with 3 attributes, which were 3 equally likely monetary outcomes (labelled a, b and c for Option 1 and x, y and z for Option 2). Formally, the value difference on this trial is given by

$$\Delta_{\text{VALUE}} = \frac{V_a + V_b + V_c}{3} - \frac{V_x + V_y + V_z}{3} \tag{1}$$

where V_i indicates the value of attribute i. Therefore, on this trial Δ_{VALUE} equals $(21+78+84)/3-(85+76+32)/3 = -3.33$.

Fixation number	Value fixated	$\hat{V}_a$	$\hat{V}_b$	$\hat{V}_c$	$\hat{V}_x$	$\hat{V}_y$	$\hat{V}_z$
pre-fixation	+	50	50	50	50	50	50
1	21	21	50	50	50	50	50
2	78	21	78	50	50	50	50
3	85	21	78	50	85	50	50
4	76	21	78	50	85	76	50
5	84	21	78	84	85	76	50
6	78	21	78	84	85	76	50

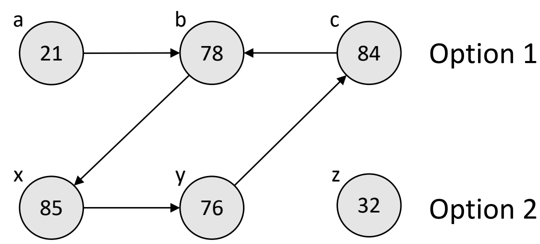

Figure 2: Illustrative fixation data for a single trial in the lottery experiment. In the table, the first row represents the estimated prior. Then, the subsequent rows show the six fixations to areas of interest representing the possible monetary rewards on this trial. The pattern of fixations is shown in the diagram on the right by the arrows between the nodes representing the attributes. Note the participant fixated twice on attribute b and never fixated on attribute z.

Fixation Weighted Value Difference Model

Another common assumption is that evidence is accumulated based upon the information currently attended at each point in time[9,25–27]. Here we take the stronger assumption, that the drift rate is solely determined by the currently attended information. Essentially, this removes any reliance on KAO as the decision maker is only accumulating currently attended information. We call this moment-by-moment shift in drift rate *fixation weighted value difference*. Here, we implement this by multiplying each value by the number of times it was fixated in a trial and divide by the total number of fixations to each item. We then take the difference between them.

For the example trial in Figure 2, this is would be formally represented as

$$\Delta_{\text{WEIGHTED VALUE}} = \frac{1}{N}\left(\sum_{i \in a,b,c} f_i V_i - \sum_{j \in x,y,z} f_j V_j\right) \tag{2}$$

where f_i is the frequency of fixations to attribute i across the trial and $N = \sum_{i\in a,b,c} f_i + \sum_{j\in x,y,z} f_j$ is the total number of fixations. Therefore, on the example trial $\Delta_{\text{WEIGHTED VALUE}} = 1/6\left((1 \times 21 + 2 \times 78 + 1 \times 84) - (1 \times 85 + 1 \times 76 + 0 \times 32)\right) = 16.67$ (because 78 is fixated twice and 32 is never fixated).

Updating Value Difference Model

Another assumption is that whilst accumulation starts at the beginning of the trial, it is initially based upon initial a priori expectations about the attribute values. Then, as information is learned about each option, the drift rate is updated to represent the currently known values[20]. Thus, the information acquired on each fixation in this model represents the change in drift rate, whereas in the Fixation Weighted Value Difference Model above it represents the drift rate itself.

The Updating Value Difference Model is equivalent to a Bayesian updating as information is received. The prior would be a uniform distribution over all attribute values. Then, when the attribute is fixated, this collapses to a point posterior distribution at the value of the attribute.

Here, we implement this using the prior v which is set to the mean of all values across the experiment (using the mean is sufficient instead of a uniform distribution because of the linear combination of values). In our Figure 2 example, this means the value of each lottery payout is assumed to be 50: the average payout value. (Other values for v account for less variance and thus this experiment-mean assumption is the best version of this model. Additionally, in Appendix A we considered a model where v was learned over previous trials—this made no difference.) As each attribute is fixated, its value is updated from the prior 50, to the true value and the drift rate is recalculated. Thus for each fixation we calculate the estimated value difference and then average this over fixations:

$$\Delta_{\text{UPDATING VALUE}} = \frac{1}{N}\left(\sum_{n=1}^{N}\left[\frac{\hat{V}_a + \hat{V}_b + \hat{V}_c}{3} - \frac{\hat{V}_x + \hat{V}_y + \hat{V}_z}{3}\right]_n\right) \tag{3}$$

where $[\hat{V}_i]_n$ is the participant's understanding of value for attribute i on fixation n and N is the total number of fixations. Therefore, on this example trial,

$$\begin{aligned}
\Delta_{\text{UPDATING VALUE}} = \frac{1}{6}\Bigg(& [\frac{1}{3}(21+50+50) - \frac{1}{3}(50+50+50)]_1 + \\
& [\frac{1}{3}(21+78+50) - \frac{1}{3}(50+50+50)]_2 + \\
& [\frac{1}{3}(21+78+50) - \frac{1}{3}(85+50+50)]_3 + \\
& [\frac{1}{3}(21+78+50) - \frac{1}{3}(85+76+50)]_4 + \\
& [\frac{1}{3}(21+78+84) - \frac{1}{3}(85+76+50)]_5 + \\
& [\frac{1}{3}(21+78+84) - \frac{1}{3}(85+76+50)]_6 \Bigg) \\
& = -10.22
\end{aligned}$$

Final Value Difference Model

Some have argued that there are multiple stages of decision making[19,28]. Most typically, this takes the form of a reading phase, followed by a decision phase that begins only once sufficient information has been collected or attended. This is implemented here by using only the information that a participant has acquired by the end of the trial. If a participant has fixated on all of the attributes by the end of the trial, this predictor is identical to the difference in expected value. However, if a participant has not fixated on all of the values, then the unfixated attributes are assumed to have a value v. Note that this model excludes information about when in the trial information was attended.

$$\Delta_{\text{FINAL VALUE}} = \left[\frac{\hat{V}_a + \hat{V}_b + \hat{V}_c}{3} - \frac{\hat{V}_x + \hat{V}_y + \hat{V}_z}{3}\right]_{n=N} \tag{4}$$

where N is the final fixation on that trial. Therefore, for the example trial $\Delta_{\text{FINAL VALUE}}$ would be equal to $1/3(21 + 78 + 84) - 1/3(85 + 76 + 50) = -9.33$.

Should this model prove as good a fit as the earlier models, then we will be forced to conclude that knowing when attributes are attended—either in the Fixation Weighted Value Difference Model by determining the instantaneous drift rate or in the Updating Value Difference Model by determining how the drift rate is updated—is not relevant for predicting choices.

Final Value Difference Plus History Model

Our last model is a more general test of whether it matters when an attribute value was acquired. Therefore, we also include time known terms for each attribute (a, b, c, x, y, z). Formally, this is defined as

$$\Delta_{\text{HISTORY}} = \Delta_{\text{FINAL VALUE}} + \sum_{i \in a,b,c,x,y,z} p_i(V_i - v) \tag{5}$$

where p_i equals the proportion of fixations for which the participant knew the attribute value.

The comparison of the final value different model with the final value difference plus history model tests whether, over and above the attribute information available by the final fixation, it matters how long the information about each attribute was known.

Attention Model

All the assumptions tested above share the property that the effect of attention depends upon the value of the information attended in some way. However, there is reason to believe that attention biases choice, independent of option values[10,29]— that more attention to an alternative increases the probability of choosing that alternative. Therefore we calculate the proportion of time where attention is directed to each option.

Here, attentional bias is defined as

$$A = \frac{T_a + T_b + T_c}{T_a + T_b + T_c + T_x + T_y + T_z} \tag{6}$$

where T_i is the amount of time in milliseconds the participant fixated within the area of interest around attribute i on each trial.

We estimate an additional version of each of the above models that includes the attentional bias A as a predictor.

Results

Figure 3 shows the results of fitting these models to four binary choice experiments. In two of the experiments, participants chose between two options, each represented by a single image. In the *food* experiment, participants selected their preferred snack food from two photographs and in the *posters* experiment, they chose which of two images from the International Affective Picture System[30] they preferred. In these experiments, participants rated each stimulus on a Likert scale, which enabled us to estimate the models using each participant's individual ratings. In the other two experiments, participants chose between two multi-attribute options: lotteries consisting of three possible tickets. In the *currency* experiment, these lotteries were presented in different currencies on each trial ($\pounds$, ¥ and Q). In the *lottery* experiment, all choices were presented as unitless "points". All experiments except the poster experiment were incentivised. In Figure 3 the bars indicate the variance in choices explained by each model (Nagelkerke pseudo R^2) with (light) and without (dark) a main effect of attention.

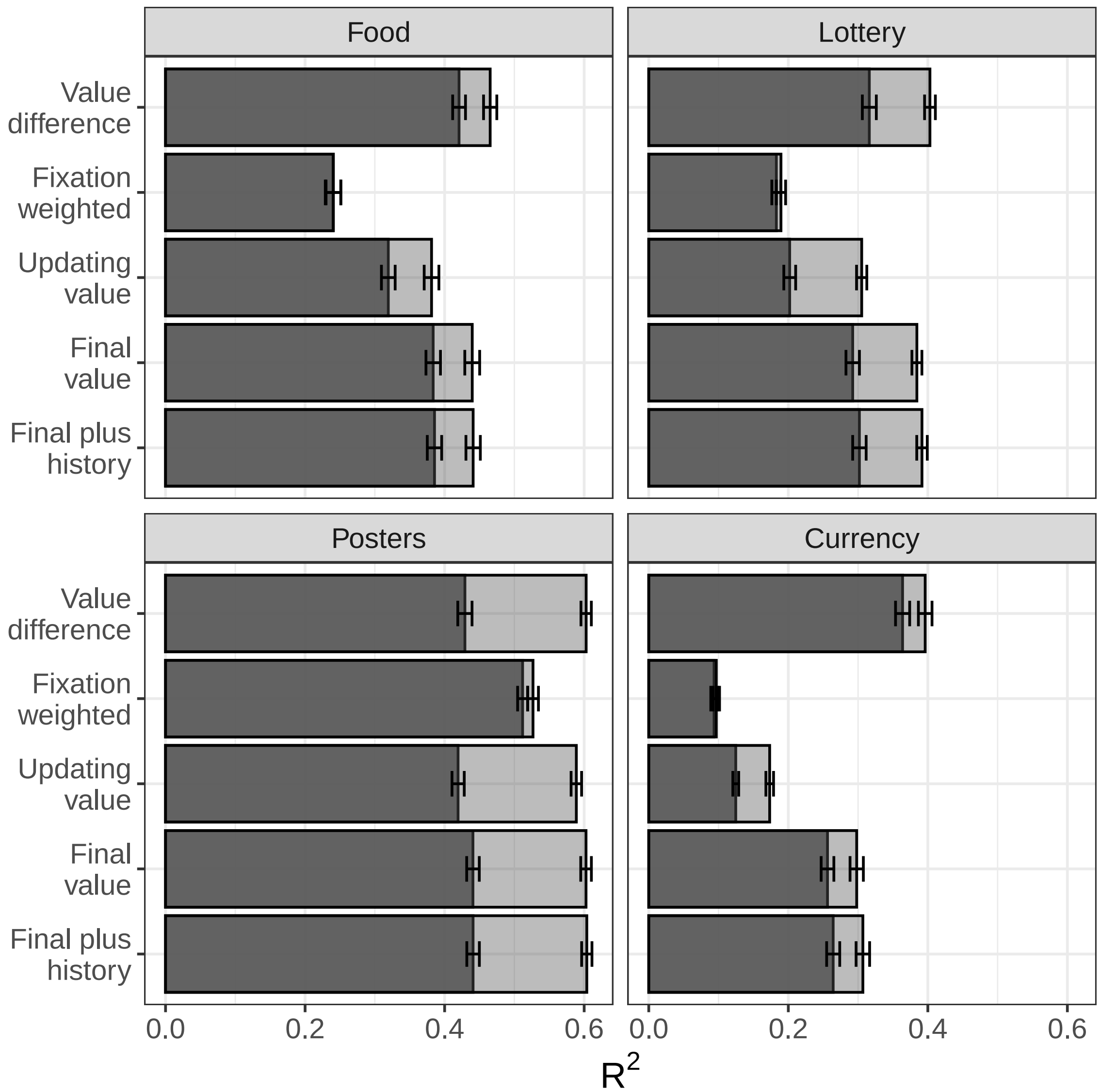

Figure 3: Model summaries for each experiment. Dark bars indicate the variance in choices captured by the model is measured by Nagelkerke pseudo R^2. Light bars include an additional main effect of attention.

The most striking result is that, for every experiment, the final value model explains more variance than the updating value model. That is, model fits worsen when they assume that the drift rate value is updated as new information is learned. This conclusion is supported by the fact that the final value plus history model accounts for no more variance than the final value (alone) model. In other words, adding in information about when an attribute value was acquired does not improve model performance. Furthermore, using this information to constrain accumulation models actually results in a substantially worse fit to the data.

In three of the four experiments, the final value model explains more variance than the fixation weighted model. That is, assuming that evidence is only accumulated based upon the currently attended information makes the model fit worse.

This is not true in the posters experiment, where the fixation weighted model explains more variance than the final value model. This is because there is a main effect of attention that is more than twice as large in the poster experiment than in the other experiments. The fixation weighted model is able to capture this large main effect of attention using the attention-by-value interaction [10], but the other models cannot capture this main effect. Thus the fixation weighted model is doing well not because people are weighting fixations, but because the model is able to capture a main effect of attention. (Our data do not allow us to say why there is a large main effect of attention in the posters experiment–we speculate this may be because of the inherent value of looking at pleasurable images–but this is not critical for the argument we make here.)

The value difference model accounts for similar amounts of variance as the final value model because, on the vast majority of trials in these tasks, participants view all of the attribute values and on these full-view trials the value difference model prediction *is* the final value model prediction. That is, when all values are known, the value difference model and the final value model are identical. And finding that the value difference model outperforms the fixation weighted model and the updating value model again shows that using information about when attribute information is gained is not useful for predicting choice.

Discussion

Here, we identify that a key assumption of evidence accumulation models is inherently impossible and wrong: that decision makers have perfect knowledge at the onset of a choice—KAO. We examine a number of alternative evidence accumulation models which remove this assumption. These alternatives retain the attractive properties of evidence accumulation models, including neural plausibility and the ability to predict choices and reaction times simultaneously. However, the results across four experiments show that the best performing models are ones which ignore the time when information is acquired.

This has serious implications for accumulation-based models of value-based choice [7,9,12–15,31]. We suggest that accumulation models, although they fit choice and reaction time data well, are fundamentally missing something. We are forced to a strong conclusion: If adding knowledge about when information becomes available to evidence accumulation models makes their fit worse, we must conclude that, if there is an accumulation process, it does not begin until about the time the final fixation is made. Effectively, our result confines the accumulation process to a small fraction of time at the end of a choice, because allowing it to start any earlier results in significantly poorer choice predictions. (Appendix B considers further how early, exactly, the accumulation process could start.) While the starting point for many avenues of research is an accumulation model, we should, perhaps, be looking more stringently at testing those underlying assumptions.

There are additional implications for research using process tracing methods [32]. A great deal of work has focused upon fitting models of choice to reaction time data, information search, and eye-tracking. However, these results suggest that this process tracing work may have been based upon a fundamental misunderstanding of the underlying process itself. For example, how are we to interpret findings testing the effect of choice difficulty upon drift rates and reaction times in light of the suggestion that evidence is not being accumulated over the majority of the decision time? This is not to say that process tracing is not valuable, but that research on properties of that process is very often structured around a particular model (or class of model). Such work is therefore inherently reliant upon the assumptions underlying that model, and when those assumptions prove to be faulty, the conclusions of process tracing efforts need to be revisited. A preferential approach might be to increase focus on model free tests, and independent characterisations of process data.

The conclusion from this paper is that it is perhaps futile to fit accumulator models like multialternative decision field theory [31], leaking competing accumulators [12], multialternative decision by sampling [33], the Poisson race model [13], the attention drift diffusion model[9,27], the associative accumulator model[14], and the multialternative linear ballistic accumulator model[15] to choice data from attribute by alternative matrices of multi-attribute choice options. For example, the model comparisons reviewed by Busemeyer et al.[7], which include Rieskamp[34], Scheibehenne et al.[35], Berkowitsch et al.[36], Hancock et al.[37], Bhatia[14], Trueblood et al.[15], Hotaling et al.[38], Turner et al.[39], Krajbich et al.[9,40] and Noguchi and Stewart[33] should be abandoned. This is because we show that much of the time taken to make a choice in multiattribute choice tasks is not time in which people are accumulating evidence—while in all of these models reaction time differences across choices are accounted for as differences in accumulation. Instead much of the choice time is for cognitive processes which precede the accumulation process. Thus the problem is that much modeling is targeting the accumulation process, but this is only a small fraction of the cognitive operations—at least as measured by time. This means that much of the cognitive processing in multiattribute choice remains to be explored and modelled.

Methods

Aside from the variations noted below, the experiments proceeded as follows. Participants were recruited from the University of Warwick participation pool and were paid for their participation. To limit movement, participants used a chin rest placed approximately 70cm away from the screen. Monocular eye movements were recorded at 500Hz with an EyeLink 1000 Plus (SR Research, Osgoode, ON, Canada) eye-tracker. Fixations were identified by the eye-tracker software using velocity algorithms. Participants initially underwent a 13 point calibration and validation cycle of the eye-tracker, which was repeated throughout the experiments. Stimulus presentation was controlled by MATLAB using Psychtoolbox extensions[41,42]. Trials were excluded if their reaction times were greater than 1.5 times the interquartile range above the mean reaction time across all trials within the experiment or less than 200ms.

Food[43]

Useable data was collected from 41 participants. Data was excluded for 4 additional subjects because of poor eye tracking data quality (as indicated by the proportion of time during trials where eye gaze was detected by the tracker being outside of the normal distribution across all participants). The stimuli were pictures of 50 different snack items. These were comprised of five types of snack: crisps, fruit candy, sweet carbonated drinks, health and sports drinks, and chocolate. This experiment was split into two parts. In the first, the participant rated the desirability of 50 snack items on a 1-9 scale. In the second, the 50 stimuli were paired to create 100 binary choice trials. Choice pairs were created such that the rating difference between the two items was 3 or less, and so that an individual snack item was not present in more than 5 trials. At the end of the experiment one trial was randomly selected and the subject was given the item they chose.

Posters[44]

Useable data was collected from 53 participants. The experiment was displayed on a widescreen monitor (1920 x 1080 resolution, 60Hz refresh). Additional data was collected from 13 participants but 12 were excluded due to a programming error and one because their gaze location could be measured for less than 70% of the time across all trials. The stimuli were chosen from the International Affective Picture System[30]. The pictures were all positive in affect (average ratings between 5=neutral and 7=mildly positive for both males and females) and had differences in value ratings of no more than 1.5 between male and female raters. After visual inspection, a further 7 images were removed for containing sexual images and 32 images were removed because they had a portrait aspect ratio. The 200 stimuli for each participant were randomly sampled without replacement from the 253 pictures that met these criteria.

All participants completed *binary choice* and *strength-of-preference* tasks in a counterbalanced order. Here, we only analyse the binary choice data. Two landscape pictures (each $514 \times 384px$) were displayed side by side following a fixation cross. The response scale was presented horizontally centered, below the stimuli. For the binary choice task, two labels ("Option A" and "Option B") were shown underneath the appropriate stimuli. The current choice was signified by a red, square marker ($30 \times 30px$) above the label. The marker was initially centered equidistant between the two images. To respond, the participants had to press the left mouse button. Reaction times were measured from the start of the trial to the mouse click onset. A blank, black screen was displayed for $500ms$ between each trial. Finally, participants had to rate their overall liking for each picture on a Likert scale, vertically displayed to the left of each image. The eye-tracker was recalibrated at the beginning of each condition and then after every 25 trials.

Lottery [45]

Useable data was collected from 54 participants. The data of an additional 5 subjects was excluded: 1 because they failed to complete the task within a reasonable time frame and withdrew, and 4 because of poor quality eye tracking data. The stimuli were gambles with three equally likely outcomes. Because the outcomes were equally likely, no probabilities or likelihoods were displayed during the trials themselves. For each trial, two gambles were presented, one on the top and one on the bottom of the screen. The three payouts of a single gamble were presented in horizontal alignment and were displayed as white text within a solid grey circle. This was done to reduce the discriminability of the numbers in peripheral vision, so that subjects had to directly fixate the number to determine its value. Each gamble consisted of three possible outcomes which were always one low value (10-30), one medium value (40-60), and one high value (70-90). The specific values were randomly drawn on each trial.

The main task consisted of 100 trials. Trials were presented in a random order, with trials from the different conditions intermixed. Each trial began with a fixation cross displayed in the centre of the screen, and the trial only began once the subject was looking at the fixation cross. Trials were split into 3 different conditions: no change, low to high, and high to low. There were 34 trials in the no change condition, and 33 in each of the other two. Here, we only analyse the no change condition. The eye-tracker was recalibrated at the beginning of the main block and then after every 20 trials.

Currency [44]

Usable data was collected from 46 participants from the California Institute of Technology participation pool. Additional data had been collected from 14 participants but four were excluded due to incomplete data (they ran over the experiment slot) and 10 because of poor eye tracking accuracy (their gaze location could be measured for less than 68% of the time averaged across all trials). The stimuli were two gambles with three equally likely outcomes, presented at the top and bottom of the screen. The three payouts of each option were presented side-by-side as white text within a solid grey circle. There were two within-subject conditions, counterbalanced for order. In the *commensurate* condition, there were 42 trials, in a third of trials all attributes were displayed in pounds, yen and "Q" respectively. In the *incommensurate* condition, on every trial, the three attributes of each option were presented in the three difference currencies. Here, we only analyse the commensurate condition. Participants were told the exchange rates for pounds, yen and Q to dollars. Here we analysed the equivalent dollar values. The eye-tracker was calibrated at the beginning and every 21 trials. Participants pressed the up key if they preferred the top lottery, and the down key if they preferred the bottom lottery. Areas of interested were defined horizontal distance of 320 pixels, and a vertical distance of 340 pixels apart on the screen.

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Appendices

Appendix A

Here, we compare updating value models that make different assumptions about the prior value for unattended attributes. In the updating value model, we assume that participants use the experiment-average attribute value as a static prior for all trials. However, this assumption obviously assumes some clairvoyance at the beginning of the experiment. Therefore, here we also include a model which assumes that participants use as a prior the updating mean of all the values they have seen on earlier trials in the experiment. Figure A1 shows that adopting this more realistic assumption makes almost no difference to model fit.

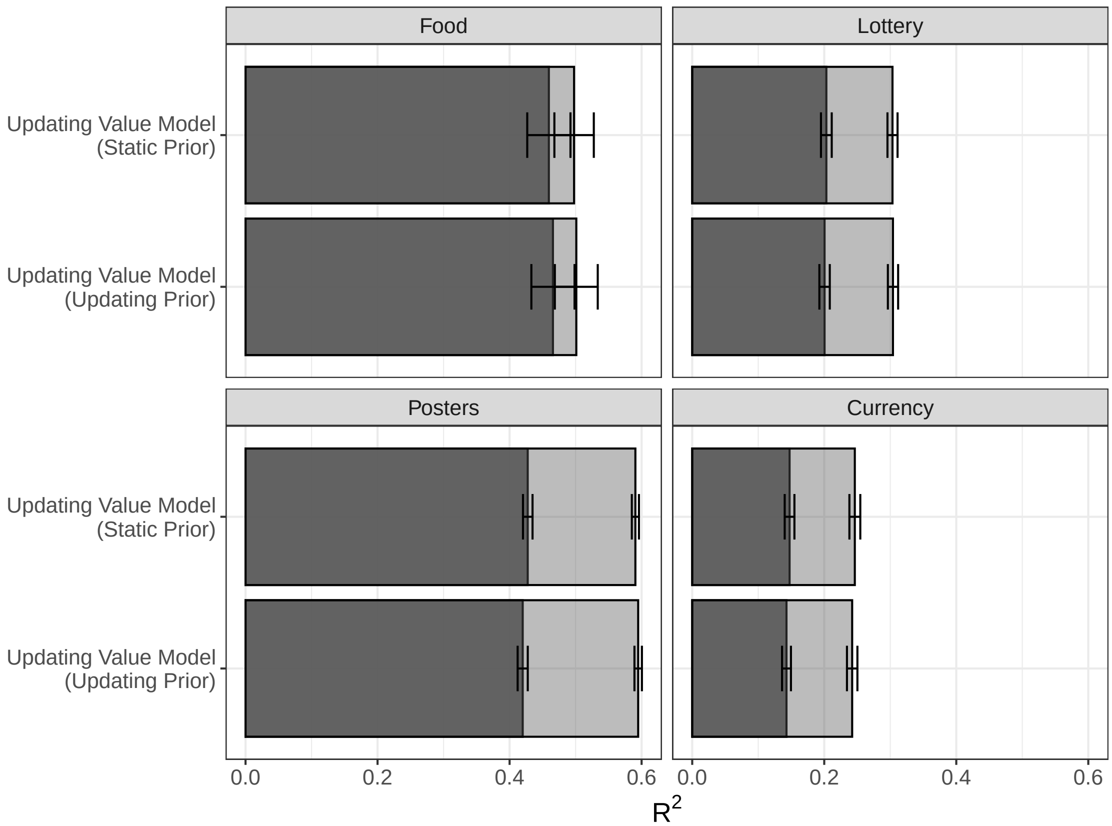

Figure A1: Comparison of updating models assuming two different types of prior across the four datasets.

Appendix B

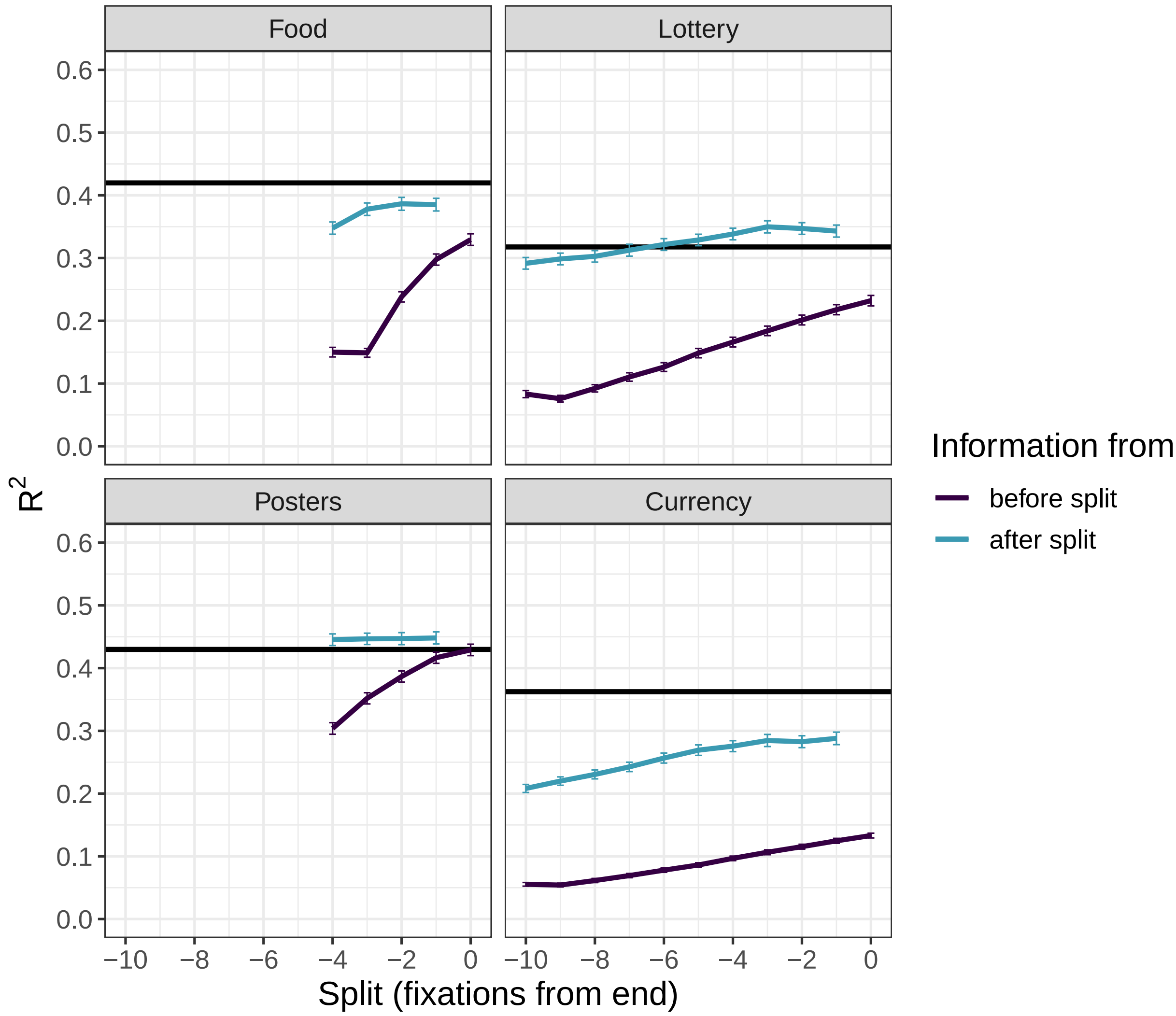

Figure B1: Updating value model split on the basis of each fixation averaged across trials.

The fixation weighted value difference and the updating value difference model both assume that people accumulate attribute value information as it is attended. The final value difference model assumes that accumulation is deferred until all of the information that will be acquired has been acquired. Finding that this final value difference model fits better allows us to reject the idea that accumulation begins as information is first acquired.

Here we explore the possibility that information acquisition begins later than the first fixations, but begins before the fixation upon which all of the information that will be acquired is acquired. To explore the issue of when accumulation starts directly, we split each trial into early and late fixations and model separately the early and late fixations with the updating value difference model.

Figure B1 splits each trial into early and late fixations. We took the late fixations and ran the updating value difference model as if accumulation only occurred during the late fixations. The "after-split" lines in Figure B1 show how model fit changes accumulation is assumed to start earlier and earlier (from right to left). Assuming that accumulation starts earlier than the last fixations makes the model fits slightly worse, or makes little difference. We have also included, for completeness, "before-split" lines which show what happens in accumulation updates only on

Table B1: Estimates of time spent accumulating. Columns from left to right: 1) mean reaction time, 2) average fixation length, 3) mean estimation of t_0 from the drift diffusion model, 4) mean time spent accumulating according to the drift diffusion model, 5) best fitting number of fixations from the end of the trial upon which accumulation starts, 6) the time we estimate participants start accumulating and 7) the percentage of trial in which the two approaches disagree about whether accumulation is taking place.

	Experiment means		DDM means		Our estimation		
	RT (ms)	Fixation length (ms)	t_0 (ms)	Accumulation time (ms)	Best-fitting split	Accumulation (ms)	Discr
Food	2210	242	532	1678	1	242	65
Posters	3514	264	897	2617	3	792	52
Lottery	6516	251	1885	4631	3	753	60
Currency	14574	385	2357	12217	1	385	81

the early fixations before the split. Assuming accumulation only during the early fixations provides a very poor fit.

We take the as the best-fitting split the point at which the R^2 value is highest. This point is, for example, 3 fixations back in the lottery experiment. Assuming accumulation starts earlier makes the model fit worse. Thus we have an estimate of how early accumulation starts according to the updating value-difference model.

We have also estimated the drift diffusion model (DDM) for each experiment for each participant. The t_0 parameter provides an estimate of the point at which accumulation starts according to the DDM. We contrast DDM and the updating value model estimates of the start time below.

We fitted the DDM to the response, reaction time and rating data (when applicable–i.e. food and posters) for each participant. We calculated model fits in R[46] using the `optim` function with the method "L-BFGS-B." The closed-form model was provided by the `rtdists` package[47]. Code for these model fits is available on OSF. The threshold separation a, drift rate v, non-decision time t_0, and inter-trial variability of non-decision time st_0 were estimated. The other parameters were set to the default provided by the package. We limited the search procedure to only consider positive values for a, t_0 and st_0. To get the best possible estimates, the model was fit to each participant's data 10 times with random starting parameters. The best parameters for each participant were those that maximised the log-likelihood of the responses and timing.

Table B1 shows that our estimates for the onset of accumulation are very different from those estimated by the drift diffusion model—we estimate that people begin accumulating much later than is estimated by t_0 in the DDM. Taking the food experiment as an example, we have a mean reaction time of $2210ms$ and an average fixation duration of $242ms$. Based on the drift diffusion models fits, we have an estimated t_0 of $532ms$, which is the estimated time at which accumulation begins. Subtracting this from the mean reaction time, we have an estimate of $1678ms$ as the average time in the trial during which accumulation is taking place according to the drift diffusion model. Using the updating value model, we selected the best-fitting number of fixations back from the end of the trial as the start of accumulation. For the food experiment, this is one fixation back. Thus we can also estimate the average time in the trial of $242ms$ during which accumulation is taking place according to the updating value evidence accumulation model (i.e., for the duration of the last fixation) The last column in Table B1 reports the fraction of the total reaction time where the drift diffusion model fit assumes participants are accumulating but our updating value model fitting assumes that they are not accumulating $(1678 - 242)/2210 = 65\%$. In every case, over half of the reaction time is time when the drift diffusion model assumes accumulation is taking place, but our modeling with the updating value model indicates that it is not.

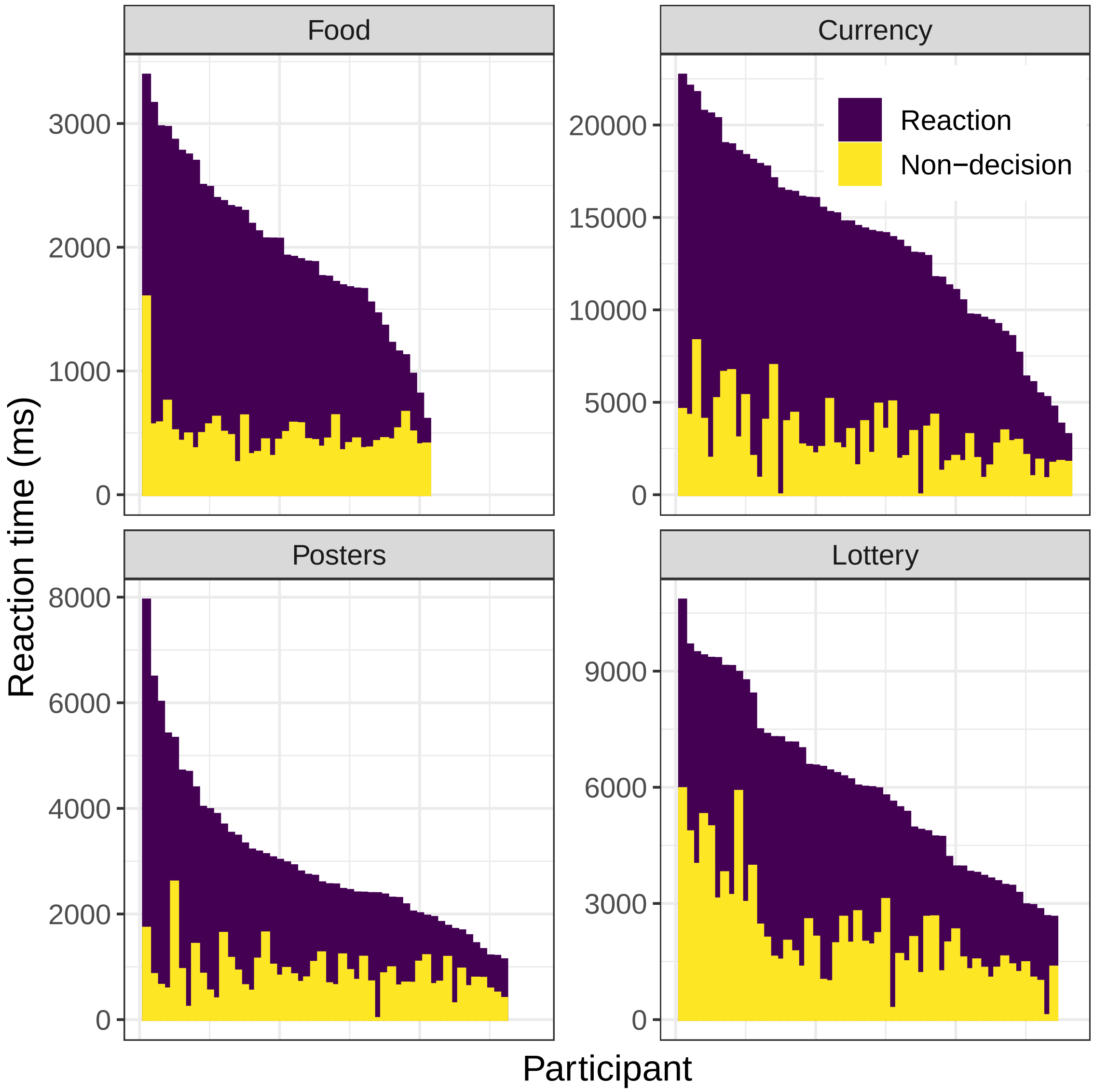

Figure B2: Reaction times and non-decision time t_0 for each participant.Participants are arbitrarily ordered from slowest to fastest.