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Comments

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Abstract

We derive a capital asset pricing model with market sentiment from a representative agent that exhibits two basic behavioral biases – ambiguity aversion and positive skewness preference. The asset pricing formula generalizes the classical CAPM by accounting for model uncertainty, positive skewness, disaster risk, and market sentiment, thereby linking four strands of the literature. We apply the Market Sentiment CAPM to provide a unified explanation for the beta anomaly and three other market anomalies, and to predict how they are affected by sentiment. The Market Sentiment CAPM provides a theoretical foundation for the pricing of sentiment in the cross-section of returns.

JEL classification: D81.

Keywords: Capital Asset Pricing Model, Beta Anomaly, Sentiment, Uncertainty, Skewness Preference, Disaster Risk.

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1 Introduction

The capital asset pricing model (CAPM) pioneered by Sharpe (1964) and Lintner (1965) was a major advance in financial theory. It has penetrated the finance industry where it is commonly used in capital budgeting decisions and to evaluate the performance of mutual fund managers. It has also reached the popular press where its relationship between an asset's expected return and systematic risk, beta, has become one of the best-known equations in economics.

Despite its widespread influence, research has uncovered asset pricing anomalies in which some assets earn systematically higher returns than other assets in a manner unexplained by beta. Early work by Black et al. (1972) documented the beta anomaly in which stocks with high (low) betas earn low (high) abnormal returns. The beta anomaly is now recognized to be "one of the most persistent anomalies in empirical asset pricing research." (Bali et al., 2017, p. 2369). More recent anomalies have been documented by Baker and Wurgler (2006), Bali et al. (2011), and Bégin et al. (2020), who find that market sentiment and idiosyncratic risk affect the cross-section of stock returns.

The CAPM can be derived from a representative investor who maximizes a linear tradeoff between the mean and standard deviation of portfolio returns. The model was traditionally derived under conditions of risk (under the assumption that the true distribution of portfolio returns is known). In practice, means and standard deviations are not known exactly and hence investors are exposed to model uncertainty – the risk of optimizing a portfolio with respect to the "wrong" probability distribution. Investors may therefore care about portfolio decisions that are robust to mis-specified probability models.

In this paper, we extend the CAPM to incorporate a role for model uncertainty. In particular, we consider an investor who cares about (i) expected returns with respect to some prior, (ii) risk or dispersion of returns with respect to that prior, and (iii) robustness of returns to a mis-specified prior. The resulting market sentiment capital asset pricing model incorporates a role for model uncertainty, positive skewness, disaster risk, and market sentiment. A literature pioneered by Hansen and Sargent (2001) considers investors who are concerned about holding mis-specified probability models of asset returns and who have preferences from robust control theory. Kraus and Litzenberger (1976), Harvey and Siddique (2000), Brunnermeier et al. (2007), and Barberis and Huang (2008), study investors who prefer positively skewed returns. Rietz (1988), Barro (2006), and Gabaix (2012) consider the impact of rare economic disasters on asset prices. De Long et al. (1990), Barberis et al. (1998), Hong and Stein (1999), and Barberis et al. (2015) model the effects of investor sentiment on asset prices. However, each of these papers on sentiment studies an economy with one safe asset and one risky asset. The market sentiment CAPM provides a simple approach to relate these literatures, and differs from the classical models of sentiment by focusing on the cross-section.

We make the following contributions:

1. **The Market Sentiment CAPM:** We derive the market sentiment CAPM from a representative investor who exhibits two basic behavioral biases: skewness preference and ambiguity aversion.

2. **A more comprehensive account of systematic risk:** The market sentiment CAPM incorporates a role for positive systematic skewness, systematic disaster risk, market uncertainty, and market sentiment as dimensions of systematic risk in addition to systematic volatility (beta).

3. **A unified explanation for four market anomalies:** We apply the market sentiment CAPM to provide a unified theoretical explanation for the beta anomaly (Black et al., 1972), the idiosyncratic volatility puzzle (Ang et al., 2006), the max premium (Bali et al., 2011), and the crash premium (Chabi-Yo et al., 2018).

4. **Predicting properties of the beta anomaly:** The market sentiment CAPM predicts the dependence of the beta anomaly on market uncertainty, its dependence on market sentiment, and its concentration among high beta stocks with high maximum returns, each of which has empirical support.

5. **Predicting the effects of market sentiment on the cross-section:** The market sentiment CAPM predicts how each anomaly we study is affected by market sentiment, consistent with the empirical evidence.

6. **Providing support for an approach to implement the Market Sentiment CAPM empirically:** We note how the market sentiment CAPM provides a theoretical foundation for the Level, Slope, and Curve factor model (Clarke, 2016), and we provide new evidence supporting this interpretation.

Our approach is related to two current papers. Ding et al. (2019) generalize the model of De Long et al. (1990) to include two risky assets, one of which is assumed to be more prone to sentiment. Our approach differs in that we generalize the CAPM, we consider a role for model uncertainty, positive skewness, and disaster risk in addition to market sentiment, and we apply our model to explain four prominent asset pricing anomalies.

In independent and concurrent work, Barberis et al. (2019) apply prospect theory to study anomalies in the cross-section of returns. Our work differs in several respects: Barberis et al. (2019) use the original version of cumulative prospect theory from Tversky and Kahneman (1992). Due to the complex rank-dependent probability weighting function in that model, that approach precludes the possibility of a closed form characterization of equilibrium returns, and instead the paper relies on simulations. Barberis et al. (2019) consider a larger number of anomalies than we do here. However, they do not consider the beta anomaly or the effects of market sentiment on market anomalies. In contrast, we obtain a closed form representation of asset risk premia resulting in a linear factor model for the cross-section and we investigate the comparative statics of our model with respect to changes in market sentiment and market uncertainty.

2 Skewness Preference and Ambiguity Aversion

Two robust behavioral biases for choice under risk and uncertainty are ambiguity aversion (Ellsberg, 1961) and skewness preference (Tversky and Kahneman, 1992). One form of ambiguity aversion is a preference for making decisions that are robust to mis-specified probability models (Wald, 1950; Hurwicz, 1951).

Ambiguity aversion and skewness preference are not only laboratory phenomena – they provide explanations for failures of economic theory in markets: ambiguity aversion provides an explanation for buying-selling price gaps in markets (Dow and da Costa Werlang, 1992) and for the equity premium puzzle (e.g., Ju and Miao, 2012). Skewness preference can explain much of the risk-seeking behavior observed in markets such as the simultaneous purchasing of lottery

tickets and insurance (Friedman and Savage, 1948), the overpricing of long-shots in betting markets (Weitzman, 1965), and the over-valuation of positively skewed financial assets (Barberis and Huang, 2008).

Given the strong behavioral support for ambiguity aversion and skewness preference and their relevance in applications, it seems desirable for a model of choice under risk and uncertainty to predict both behaviors. We observe that both behaviors are naturally accommodated by a model of robust decision making. Robust decision models typically focus on worst-case scenarios. A less conservative approach pioneered by Hurwicz (1951) maximizes the convex combination of the worst-case and best-case scenarios. The Hurwicz α-criterion provides a simple approach to incorporating a preference for robustness toward model uncertainty (by overweighting the worst outcome), and a preference for positively skewed returns (by overweighting the best outcome).

Formally, let there be a non-empty finite set of states S, a non-empty convex set of possible outcomes X in $\mathbb{R}$, and a set of ambiguous acts $\mathcal{F}$, where an act $f \in \mathcal{F}$ is a mapping $f : S \to X$. Denote by $f(s)$ the outcome that occurs if act f is chosen and state s occurs. Let $\Delta(S)$ denote the set of all possible probability distributions on S with generic element π.

We consider a representative investor who cares primarily about three features of asset returns: (i) the expected return on an asset with respect to the investor's prior π; (ii) risk or dispersion of returns with respect to π; and (iii) robustness of returns to different specifications of π. The investor is contemplating investment in n risky assets. Denote by $R_j \in \mathcal{F}$, for $j = 1, \ldots, n$, the act obtained by investing in the j-th asset, where, $r_{js} := R_j(s) \in X$ is the return of asset j in state s. Further, if the investor has subjective probability distribution $\pi \in \Delta(S)$ across states, denote by $r_j := E_\pi(R_j) = \sum_{s \in S} \pi_s r_{js}$ the expected return on asset j across states. Let $\bar{r}_j := \max_{s \in S} r_{js}$ and $\underline{r}_j := \min_{s \in S} r_{js}$. There is an additional safe asset with return $r_0 > 0$ in every state, corresponding to the constant act R_0 that maps every state to r_0. Given $w_j \in \mathbb{R}$, $j = 0, \ldots, n$, such that $\sum_{j=0}^{n} w_j = 1$, we call the mixture act $R := \sum_{j=0}^{n} w_j R_j$ a portfolio with holdings vector w.

The investor evaluates portfolios according to risk-return-robustness preferences given by

$$V(R) = \mu(R) - \rho\sigma(R) + \gamma\psi(R). \tag{1}$$

where $\mu(R) = \sum_{s \in S} \pi_s R(s) = w_0 r_0 + \sum_{j=1}^{n} w_j r_j$ is the expected return of the portfolio, $\sigma(R) = \left(\sum_{s \in S} \pi_s \left(R(s) - \mu(R)\right)^2\right)^{1/2} = \left(\sum_{s \in S} \pi_s \left(\sum_{j=1}^{n} w_j(r_{js} - r_j)\right)^2\right)^{1/2}$ is the standard deviation of the portfolio, and $\psi(R) = \alpha \max_{s \in S}(R(s) - w_0 r_0) + (1 - \alpha)\min_{s \in S}(R(s) - w_0 r_0) = \alpha \max_{s \in S}\left\{\sum_{j=1}^{n} w_j r_{js}\right\} + (1 - \alpha)\min_{s \in S}\left\{\sum_{j=1}^{n} w_j r_{js}\right\}$ is the robust (Hurwicz) value of the risky assets in the portfolio. The parameter $\rho \geq 0$ represents the agent's degree of risk aversion, $\alpha \in [0, 1]$ represents the agent's degree of optimism (the degree to which the agent overweights the best-case scenario), and $\gamma \geq 0$ represents the agent's perceived ambiguity (the degree of uncertainty in the agent's beliefs). Consequently, the agent places relatively greater weight on robustness (represented by the Hurwicz criterion) as the agent becomes more uncertain about her subjective prior distribution π, since μ and σ depend on π, but the Hurwicz criterion does not.

The preference function (1) spans three prominent decision models: risk-neutral subjective expected utility ($\rho = 0, \gamma = 0$), mean-variance analysis ($\rho > 0, \gamma = 0$), and a special case of prospect theory (Tversky and Kahneman, 1992) with a textbook prospect theory probability weighting function (Wakker, 2014) that overweights the tails of the distribution ($\rho = 0, \gamma > 0$).

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Further, it yields a separation of the investor's beliefs (represented by π), the investor's perceived ambiguity (represented by γ), and the investor's ambiguity attitudes (represented by α). When $\gamma = 0$, the agent perceives no ambiguity (the probabilities are known precisely) and (1) reduces to the classical mean-variance model (Markowitz, 1952). Our approach extends the CAPM to the more general domain of uncertainty where the probability model is not precisely known, in which case a concern for robustness becomes important.

3 The Market Sentiment CAPM

To set up the investor's portfolio problem, let r denote the vector $[r_1, \ldots, r_n]^T$, r^s denote the vector $[r_{1s}, \ldots, r_{ns}]^T$, w denote the vector $[w_1, \ldots, w_n]^T$, and C denote the matrix $\sum_{s \in S} \pi_s(r^s - r)(r^s - r)^T$. Matrix C is the covariance matrix of the risky assets and is assumed to be positive definite. Function V can be re-written as $V(R) = r_0 w_0 + r^T w - \rho(w^T C w)^{1/2} + \gamma \alpha \max_s w^T r^s + \gamma(1-\alpha) \min_s w^T r^s$. The investor wishes to maximize V subject to $w_0 + e^T w = 1$, where e denotes the n-dimensional all-ones vector. This is equivalent to maximizing the unrestricted function

$$g(w) := r_0 + (r - r_0 e)^T w - \rho(w^T C w)^{1/2} + \gamma \alpha \max_s w^T r^s + \gamma(1 - \alpha) \min_s w^T r^s. \qquad (2)$$

If g has an optimal solution w^*, then $r_m := w_0^* r_0 + \sum_{j=1}^n w_j^* r_j$ is the optimal expected value of the portfolio.

Proposition 1 *If function g from (2) has an optimal nonzero solution w^* and is differentiable at w^*, then*

$$r_j - r_0 = \beta_j(r_m - r_0) + \gamma \alpha(\beta_j \overline{r} - \overline{r}_j) + \gamma(1 - \alpha)(\beta_j \underline{r} - \underline{r}_j) \qquad (3)$$

for all $j = 1, \ldots, n$, where

$$\beta_j = \frac{Cov(R_j, R^*)}{\sigma^2(R^*)}, \qquad (4)$$

R^* is the optimal portfolio corresponding to holdings (w_0^*, w^*), $w_0^* = 1 - e^T w^*$, and $\overline{s}, \underline{s} \in S$ are such that $\overline{r} := w^{*T} r^{\overline{s}} \geq w^{*T} r^s$, and $\underline{r} := w^{*T} r^{\underline{s}} \leq w^{*T} r^s$, for all $s \in S$.

In addition to the CAPM risk premium, (3) includes a positive skewness premium which yields lower equilibrium expected returns for assets with high potential in the best market state, and an ambiguity premium which yields higher equilibrium expected returns for assets that are less robust in the worst market state. We refer to expression (3) as the *Market Sentiment CAPM* identity. Note that (3) can be written equivalently as a linear factor model:

$$r_j - r_0 = \beta_j(r_m - r_0) + \overline{\beta}_j(-\alpha\gamma) + \underline{\beta}_j(1 - \alpha)\gamma \qquad (5)$$

where $\overline{\beta}_j = \overline{r}_j - \beta_j \overline{r}$ and $\underline{\beta}_j = \beta_j \underline{r} - \underline{r}_j$. Our specification of $\overline{\beta}_j$ reflects the intuition that stocks with higher maximum returns (high $\overline{r}_j$) have *higher* exposure to bullish sentiment. The bullish sentiment factor is then given by $-\alpha\gamma$, which captures the intuition that investors pay a premium for assets that are exposed to investor optimism. Under the specification of $\underline{\beta}_j$, stocks with lower minimum returns (low $\underline{r}_j$) have higher exposure to bearish sentiment, where the

bearish sentiment factor is $\gamma(1 - \alpha) > 0$. Note that one could use an alternative two-parameter specification which does not require a dependence between bull and bear market factors. In particular, $\gamma\alpha$ and $\gamma(1 - \alpha)$ in the robust term in (1) can be replaced by non-negative weights $\overline{\alpha}$ and $\underline{\alpha}$, respectively. Identity (5) highlights a central implication of our approach – that investor sentiment is priced in equilibrium.

In practice, many retail investors face short-sale constraints and even institutional investors often have limited negative holdings. Under the assumption of no-short selling ($w_j \geq 0$ for all j), and the following construction of the market state space, function g from (2) does have an optimal solution that is differentiable at w^*. To construct the state space, let each asset j have possible returns given by a set S_j (e.g., returns the representative agent believes are possible). We define the market state space by the Cartesian product $S = \prod_{j=1}^{n} S_j$. Then the state space consists of all possible configurations of asset payoffs. It thus includes scenario $(\overline{s}_1, \ldots, \overline{s}_n)$, where $\overline{s}_j = \max_{s \in S} r_{js}$ for all $j = 1, \ldots, n$, and scenario $(\underline{s}_1, \ldots, \underline{s}_n)$, where $\underline{s}_j = \min_{s \in S} r_{js}$ for all $j = 1, \ldots, n$. Since we are assuming that $w^* \geq 0$, notice that $(\overline{s}_1, \ldots, \overline{s}_n) = \overline{s}$ and $(\underline{s}_1, \ldots, \underline{s}_n) = \underline{s}$, where $\overline{s}$ and $\underline{s}$ are as defined in Proposition 1, that is, they are the best and worst market scenarios, respectively. Under this state space, each asset attains its best (worst) returns in multiple states including states where other assets do not perform well (poorly). In this respect, the market sentiment CAPM predicts that idiosyncratic maximum and minimum returns $\overline{r}_j$ and $\underline{r}_j$ (tail risk) for an asset are priced in equilibrium but other forms of idiosyncratic risk are not priced. Consistent with this prediction, Bégin et al. (2020) find that "the normal component of idiosyncratic risk, which is easily diversifiable, is not priced after accounting for other sources of risk. Firm-specific jump risk, however, is priced" (p. 199). They conclude that "Tail risk thus plays a central role in the pricing of idiosyncratic risk" (p.155).

To highlight the pricing of idiosyncratic risk, note that (3) can be rewritten as

$$r_j - r_0 = \beta_j(r_m - r_0 + \gamma(\alpha\overline{r} + (1 - \alpha)\underline{r})) - \overline{r}_j\gamma\alpha - \gamma(1 - \alpha)\underline{r}_j. \tag{6}$$

In addition to systematic volatility β_j, the market sentiment CAPM accounts for market sentiment α, market uncertainty γ, positive systematic skewness $\overline{r}$, and systematic disaster risk $\underline{r}$. From (6), we see that the market sentiment CAPM also prices extreme idiosyncratic positive returns $\overline{r}_j$, idiosyncratic disaster risk $\underline{r}_j$, and consequently, idiosyncratic volatility (which depends on both $\overline{r}_j$ and $\underline{r}_j$).

4 Market Anomalies and Market Sentiment

We apply the market sentiment CAPM to provide a unified theoretical explanation for four asset pricing anomalies and their observed dependence on market sentiment.

4.1 The Crash Premium and Market Sentiment

Rietz (1988), Barro (2006), and Gabaix (2012) posit a role for disaster risk as a major determinant of asset prices. A similar implication emerges under the market sentiment CAPM since the representative investor overweights the worst market state. A prediction of these approaches is that assets that perform poorly in the worst states will command a large risk premium. Chabi-Yo et al. (2018) investigated this prediction empirically and identify a crash premium: stocks with

strong lower-tail dependence with the market return (LT) have higher average future returns than stocks with weak lower tail dependence (HT).

Corollary 1 (Crash premium): *Consider two stocks HT and LT such that $\beta_{HT} = \beta_{LT}$, $\bar{r}_{HT} = \bar{r}_{LT}$, and $\underline{r}_{HT} > \underline{r}_{LT}$. Then, for all $\alpha < 1$ and $\gamma > 0$, we have*

(i) (Crash premium) $r_{LT} - r_{HT} = \gamma(1 - \alpha)(\underline{r}_{HT} - \underline{r}_{LT}) > 0$.

(ii) (Crash premium and sentiment) $r_{LT} - r_{HT}$ is decreasing in α.

Prediction (ii) implies a negative relationship between sentiment and crash-sensitive stocks. This prediction is distinct from the rare disaster framework since it does not relate the risk of rare-disasters to market sentiment. Consistent with prediction (ii), Chabi-Yo et al. (2018, Table 5) find that an increase in sentiment is associated with a lower crash premium.

4.2 The Max Premium and Market Sentiment

Bali et al. (2011) identified a "max effect" in which stocks with high maximum returns (HR) earn lower average returns than stocks with low maximum returns (LR). This effect is predicted by the market sentiment CAPM since, under the Cartesian product structure of the state space, an asset's own maximum return is priced.

Corollary 2 (MAX effect): *Consider two stocks HR and LR such that $\beta_{HR} = \beta_{LR}$, $\bar{r}_{HR} > \bar{r}_{LR}$, and $\underline{r}_{HR} = \underline{r}_{LR}$. Then, for all $\alpha > 0$ and $\gamma > 0$, we have*

(i) (MAX effect) $r_{LR} - r_{HR} = \gamma\alpha(\bar{r}_{HR} - \bar{r}_{LR}) > 0$.

(ii) (MAX effect and sentiment) $r_{LR} - r_{HR}$ is increasing in α.

Corollary 2 (ii) is supported empirically by Fong and Toh (2014), who find that high maximum return stocks perform poorly relative to low maximum return stocks following periods of high sentiment. In addition, $r_{LR} - r_{HR}$ is increasing in market uncertainty γ. Consistent with this prediction, Cheon and Lee (2018) find the premium of low maximum return stocks over high maximum return stocks to be greater following periods of higher market uncertainty.

4.3 Idiosyncratic Volatility and Market Sentiment

Stocks with high idiosyncratic volatility earn low average returns (Ang et al., 2006). This idiosyncratic volatility puzzle is difficult to explain by traditional models in which idiosyncratic risk is not priced. But even if investors are under-diversified and are exposed to idiosyncratic risk, one might expect that stocks with high idiosyncratic volatility earn higher average returns. Under the market sentiment CAPM, stocks with high idiosyncratic volatility are predicted to earn low average returns if they also have high maximum returns. In contrast, stocks with high idiosyncratic volatility and low maximum returns are predicted to earn high average returns. These predictions are supported by Bali et al. (2011), who observed that the low returns on stocks with high idiosyncratic volatility (HV) are due to HV stocks with high maximum returns. For stocks with similar maximum daily returns, Bali et al. found that average returns are *higher* for stocks with higher idiosyncratic volatility. Baker and Wurgler (2006) found that HV stocks perform poorly relative to stocks with low idiosyncratic volatility periods following of high sentiment. These findings are predicted by the market sentiment CAPM:

Corollary 3 (Idiosyncratic volatility): *Consider two stocks HV and LV such that $\beta_{HV} = \beta_{LV}$ and $\sigma_{HV} > \sigma_{LV}$. Then, for all $0 < \alpha < 1$ and $\gamma > 0$, we have*

(i) (IVOL and high MAX stocks) $r_{HV} - r_{LV} < 0$ if $\bar{r}_{HV} > \bar{r}_{LV} + \frac{1-\alpha}{\alpha}(\underline{r}_{LV} - \underline{r}_{HV})$.

(ii) (IVOL and low MAX stocks) $r_{HV} - r_{LV} > 0$ if $\bar{r}_{HV} < \bar{r}_{LV} + \frac{1-\alpha}{\alpha}(\underline{r}_{LV} - \underline{r}_{HV})$.

(iii) (IVOL and sentiment) $r_{HV} - r_{LV}$ is decreasing in α if $\underline{r}_{HV} < \underline{r}_{LV}$ and $\bar{r}_{LV} < \bar{r}_{HV}$.

Corollary 3 implies that HV stocks with sufficiently high maximum returns earn low expected returns, while HV stocks with sufficiently low maximum returns earn high expected returns, consistent with Bali et al. (2011). The implication in Corollary 3 that high idiosyncratic volatility stocks have higher maximum and lower minimum returns than low idiosyncratic volatility stocks also has empirical support: Bali et al. (2011, Table 8) find that the average cross-sectional correlation between stocks with high idiosyncratic volatility and stocks with high maximum returns and the correlation between high idiosyncratic volatility and stocks with low minimum returns are each approximately 0.75.

4.4 The Beta Anomaly and Market Sentiment

Bali et al. (2017) remark "The positive (negative) abnormal returns of portfolios composed of low-beta (high-beta) stocks, which we refer to as the beta anomaly, is one of the most persistent and widely studied anomalies in empirical research of security returns" (p. 2370). The beta anomaly was first documented by Black et al. (1972) and challenges the central empirical prediction of the CAPM that investors demand higher expected returns for assets with higher systematic risk. The market sentiment CAPM predicts the comparative statics of the beta anomaly as presented in the following result.

Corollary 4 (Beta anomaly): *For two stocks HB and LB such that $\beta_{HB} > \beta_{LB}$, we have $r_{HB} < r_{LB}$ (the beta anomaly) if and only if*

$$\gamma\alpha\bar{r}_{HB} > (\beta_{HB} - \beta_{LB})(r_m - r_0 + \gamma(\alpha\bar{r} + (1-\alpha)\underline{r})) + \gamma\alpha\bar{r}_{LB} + \gamma(1-\alpha)(\underline{r}_{LB} - \underline{r}_{HB}). \quad (7)$$

In Corollary 4, the beta anomaly occurs if a high beta stock earns lower expected returns than a low beta stock. This definition is consistent with the empirical finding by Frazzini and Pedersen (2014) that "a betting against beta (BAB) factor, which is long leveraged low-beta assets and short high-beta assets, produces significant positive risk-adjusted returns" (p. 1). Under inequality (7), the beta anomaly occurs if and only if $\gamma\alpha\bar{r}_{HB}$ is sufficiently high. Hence, (7) implies: (i) the beta anomaly holds in times of high market uncertainty (high γ); (ii) the beta anomaly holds in times of high market sentiment (high α); (iii) The beta anomaly concentrates in high beta stocks with high maximum returns (high $\bar{r}_{HB}$). Each of these predictions has empirical support: Hong and Sraer (2016) find that the beta anomaly is greater in times of high market uncertainty. Antoniou et al. (2016) find that the beta anomaly is greater in times of high market sentiment (optimistic periods) than in times of low sentiment (pessimistic periods). Bali et al. (2017) find that the beta anomaly is due to high-beta stocks with high maximum returns.

Most studies of the beta anomaly have been empirical, identifying variables that mitigate the anomaly. A prominent model of the beta anomaly is Frazzini and Pedersen's (2014) model with leverage constraints. However, that model does not explain the empirical findings that the

beta anomaly depends on market sentiment (Antoniou et al., 2016) and that it is concentrated in high beta stocks with high maximum returns (Bali et al., 2017). Both of these predictions are novel implications of the market sentiment CAPM.

5 Market Sentiment CAPM Empirical Implementation

In bringing the model to data, a common practice is to use factor-mimicking portfolios that "represent" the risk factors in place of using the factors themselves. We propose that a plausible factor-mimicking bull sentiment portfolio is an optimistic-minus-pessimistic (OMP) portfolio that is long stocks with optimistic sentiment and short stocks with pessimistic sentiment. We propose that a plausible factor-mimicking bear sentiment portfolio is a weak-minus-strong (WMS) portfolio that is short stocks with strong sensitivity to sentiment-driven overpricing and long stocks with weak sensitivity to overpricing. An alternative specification for a bear sentiment portfolio is a safe-minus-risky (SMR) portfolio that is short safe (low volatility) stocks and long risky (high volatility) stocks.

We argue that a recently proposed factor model (Clarke, 2016) provides a natural specification of bull and bear sentiment portfolios. Clarke uses principal components analysis to extract the risk factors associated with expected returns. From the first three principal components he extracts a level factor, a slope factor and a curve factor which form a new three-factor model for the cross-section. The level factor is essentially a market factor as it has a 0.95 correlation with CRSP value-weighted market index. The slope factor is a portfolio that is long low expected return stocks and short high expected return stocks. The curve factor is short stocks with extreme positive or negative returns and long stocks with moderate returns.

5.1 The Slope Factor as a Bull Sentiment Factor

An OMP portfolio captures the intuition that bullish sentiment increases as stocks with optimistic sentiment have higher returns. It should: (i) earn negative expected returns (since in (3), $\alpha\gamma > 0$); (ii) be negatively related to the mispricing factors of Stambaugh and Yuan (2017), which are long underpriced (pessimistic) stocks and short overpriced (optimistic) stocks; (iii) be negatively related to the betting-against-beta (BAB) factor of Frazzini and Pedersen (2014), which is long low beta stocks and short high beta stocks (since high beta stocks are more sensitive to optimistic sentiment than low beta stocks). To evaluate the interpretation of the slope factor, we correlated its historical returns across the full sample of available data for the level, slope, and curve factors (July, 1964 through December, 2015) with the two mispricing factors from Stambaugh and Yuan (2017), and with the BAB factor. Consistent with these predictions: (i) the slope factor earns negative average returns; (ii) The two mispricing factors (MGMT and PERF) have, respectively, a -0.18 correlation and a -0.51 correlation with the slope factor; (iii) The BAB factor has a -0.25 correlation with the slope factor (in each case, $p < 0.001$). These observations indicate that the slope factor could be plausibly interpreted as an OMP portfolio that mimics the bull sentiment factor.

5.2 The Curve Factor as a Bear Sentiment Factor

A WMS portfolio captures the intuition that bearish sentiment increases as stocks with strong sensitivity to overpricing have lower returns. It should: (i) earn positive expected returns; (ii) be positively related to the Stambaugh and Yuan (2017) mispricing factors; (iii) be positively related to the BAB factor (since the MGMT, PERF, and BAB factors are each long (short) stocks with weak (strong) sensitivity to overpricing). Consistent with these predictions: (i) the curve factor earns positive average returns; (ii) The MGMT and the PERF mispricing factors have, respectively, a 0.31 correlation and a 0.14 correlation with the curve factor; The BAB factor has a 0.22 correlation with the curve factor (in each case, $p < 0.001$). These observations indicate that the curve factor could be plausibly interpreted as a WMS portfolio that mimics the bear sentiment factor.

An SMR portfolio also predicts a positive correlation between the curve factor and the BAB factor since the BAB factor is long safe (low-beta) stocks and short risky (high-beta) stocks. These observations indicate that the curve factor could be plausibly interpreted as a WMS or a SMR portfolio that mimics the bear sentiment factor. Alternatively, one might employ either mispricing factor from Stambaugh and Yuan (2017) as a factor-mimicking bear sentiment portfolio as these factors are long pessimistic stocks and short optimistic stocks.

6 Conclusion

We derived a generalization of the capital asset pricing model that accounts for model uncertainty, positive skewness, disaster risk, and market sentiment, thereby linking four primary strands of the asset pricing literature. The resulting market sentiment CAPM can be expressed as a three-factor asset pricing model for the cross-section of returns.

We applied the market sentiment CAPM to provide a unified explanation of four market anomalies. Notably, the market sentiment CAPM provides a novel explanation for the beta anomaly and predicts its observed dependence on market uncertainty, on market sentiment, and on stocks with extreme positive returns. We also discussed how one might construct factor-mimicking portfolios for the bull and bear sentiment factors. Our approach provides a theoretical foundation for the pricing of sentiment in the cross-section of stock returns.

Appendix

Proof of Proposition 1: Since g is differentiable at w^* and attains its maximum at that point, it satisfies the necessary condition $\nabla g(w^*) = 0$. Hence, w^* satisfies the following equation

$$r + \gamma \left(\alpha r^{\overline{s}} + (1 - \alpha) r^{\underline{s}} \right) = r_0 e + \rho \frac{Cw^*}{\left(w^{*T} C w^* \right)^{1/2}}. \tag{8}$$

Multiplying by w^* and then adding $r_0 w_0^*$ to both sides of (8), we obtain

$$r_m + \gamma \left(\alpha \overline{r} + (1 - \alpha) \underline{r} \right) - r_0 = \rho \left(w^{*T} C w^* \right)^{1/2}. \tag{9}$$

We multiply both sides of (8) by e_j, the j-th canonical vector in $\mathbb{R}^n$, to obtain

$$r_j + \gamma\left(\alpha r_{j\overline{s}} + (1-\alpha)r_{j\underline{s}}\right) - r_0 = \rho\frac{w^* C e_j}{\left(w^{*T}Cw^*\right)^{1/2}}. \tag{10}$$

Combining (9) and (10), we obtain

$$\frac{r_j + \gamma\left(\alpha r_{j\overline{s}} + (1-\alpha)r_{j\underline{s}}\right) - r_0}{r_m + \gamma\left(\alpha\overline{r} + (1-\alpha)\underline{r}\right) - r_0} = \frac{\rho\frac{w^* C e_j}{(w^{*T}Cw^*)^{1/2}}}{\rho\left(w^{*T}Cw^*\right)^{1/2}} = \frac{w^* C e_j}{w^{*T}Cw^*} = \beta_j,$$

from which

$$r_j + \gamma\left(\alpha r_{j\overline{s}} + (1-\alpha)r_{j\underline{s}}\right) - r_0 = \beta_j\left(r_m + \gamma\left(\alpha\overline{r} + (1-\alpha)\underline{r}\right) - r_0\right),$$

and (3) follows. ∎

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