Linear Programming as a Technique for Least Cost Furnish Analysis

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LINEAR PROGRAMMING AS A TECHNIQUE FOR LEAST COST FURNISH ANALYSIS

by

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ABSTRACT

This study was a limited laboratory scale investigation of whether or not linear programming was a viable technique for determining the least cost furnish blends. It is original in that it used actual laboratory developed data for input to determine the linear programming model, and the results were actually produced in the laboratory to see if constraints were met. The materials used were a bleached hardwood, a bleached softwood, tab cards, clay, and TiO_2. It was found that requirements of linearity and averaging inherent in the linear programming caused results which were not as accurate as needed. However, by using the technique several times in a successive approximation type procedure, readjusting between uses to compensate for the problems previously noted, results of sufficient accuracy to be realistically depended upon were obtained. It is felt this justifies considerable optimism for this technique as a means of constantly economizing furnish costs.

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INTRODUCTION

One of the largest costs involved in any papermaking operation is the cost of materials for the production of the final product. With today's scarce resources market, particularly with regard to the natural resources of fiber and mineral involved in the papermaking process, it is of the utmost necessity that maximum efficiency be applied by the papermaker in his utilization of these resources.

It was the objective of this study to evaluate linear programming as a means of determining least cost blends of various papermaking materials which are commonly found in the furnish. This seemed to be a particularly timely and appropriate concern since these material costs will probably be among the most rapidly rising costs to the industry in the next few years. Hopefully, it will be established that this means of evaluating the cost of each furnish against the final properties of the paper will yield a useful, everyday tool which the papermaker can apply on a routine basis to determine maximum utilization at minimum cost. In order to do this a general introduction to the meaning and assumptions of linear programming is necessary, as well as a review of what work has been done previously in this particular area, and work relating to the area in general.

HISTORICAL BACKGROUND

Linear programming may be regarded as a means of solving a system of equations with more variables than equations. Its result is a determination of some optimal course or method while being restricted to the various constraints of the equations which have been set up to describe the particular area of concern. A further general introduction to what linear programming is and the assumptions it makes will be attempted immediately as a necessary prerequisite to an understanding of this project.

As a generality, many problems can be thought of as a choice between a set of activities or activity levels. Each activity contributes to an objective but also interacts with other activities. Some problems which have been successfully analyzed with the aid of linear programming include production, scheduling, capital budgeting, portfolio selection, marketing, advertising, transportation, and personnel assignment.

Linear programming deals with activity levels (1,2). In its simplest form different activities represent different product output; more generally, activity levels represent different values of the controlled variables. Combinations of various activities are termed choice variables. A common problem is the choice of the best combination from the possible, feasible combinations. The limits of the various activities are represented by a system of constraint functions. Taken as a whole set, these indicate the possible, feasible programs. The rate at which each activity contributes to a given objective, such as profit, is represented by an objective function or equation. The optimal feasible programs are those which optimize the value of the objective function and also satisfy the constraint functions.

Whenever one is using a model, two things must be known: One is a knowledge of what is being described by the model; the other is a knowledge of the model itself. Without this information about how the model corresponds to reality and the abstractions and assumptions it makes, one cannot know how much confidence to place in the results it yields. This is true of linear programming, some of the assumptions of which will be discussed here.

Linear programming presumes that the objective function and every constraint function is of linear form. In relation to the objective function, this means that it is assumed that the contribution of each activity to the optimization of this function is directly proportional to the level of that activity. Economically speaking, marginal profit is constant within the interval for each individual production. In many real situations it is found that the objective function does not vary linearly against one or more of the activities. This problem can be dealt with by various methods but it will not be discussed in detail here. In terms of the constraint functions, linearity is assumed with regard to input and output, that is, doubling the input to any activity is expected to double its use of any factor of production and to double its output.

A further assumption of linear programming is additivity. Not only must the various activities contribute linearly to the objective function, but it is assumed that their total contribution to the objective function is the sum of their individual contributions. This should not represent much of a problem with blending, but as an illustration in terms of economics, two closely substitutable items might have interaction effects causing variations in contribution through an objective function when various levels are changed. In the same manner, direct interaction between various constraints is ruled out.

Another assumption which should be mentioned but which probably should not present a problem is divisibility. Linear programming assumes a continuous choice for the variable, that is, that any real value within the constraints may be used. In many problems this is not the case; for example, if one were deciding a capital cost problem, the solution to build 3.35 paper machines does not exist in reality and in certain cases rounding may remove one from the actual optimal solution. Special integer linear programming solutions do exist for whole-number problems such as these.

Constants representing certain values in the functions also can represent a problem, that of certainty. It may aid the solution of a problem to make assumptions of constants, but often these are predictions which may vary widely from actual values when the problem is encountered. If the condition of certainty exists this is not a problem. However, in cases of uncertainty, various techniques are available for recognition and dealing with the problem. Sensitivity analysis incorporated into the problem can study to a limited extent the ability of certain constants to vary before an optimal solution becomes non-optimal. A more systematic variation of the constants is used in a system referred to as parametric programming. A treatment of one or more constants as random variables together with the use of probability and statistical decision theory permits the most satisfactory resolution of the uncertainty problem in linear programming.

Finally, the assumption is made that only non-negative activity levels are feasible. This usually takes the form of part of the constraint functions in that the various activity levels are allowed only a positive value or zero. In terms of production this would mean that an activity cannot be reversed and thereby create a factor of production from a product.

The objective of this project was to evaluate linear programming as an effective tool for optimizing pulp furnishes in the real world; that is, actual properties of base pulps were determined and used to predict properties of sheets produced from them. Sheets were then made and analyzed for agreement with predicted values. This follows quite closely an article published by Foster (3).

Foster states that pulp blending is generally considered an art, supported partly by experience, partly by technical judgment, and partly by trial and error. He proceeds to discuss a generalized problem involving 116 pulps, the number arising by considering each as a separate combination of species, pulping method and degree of refining. His problem seeks to determine the least cost blend with controlled properties, including density, burst, tear, tensile, smoothness, and opacity. From this generalized problem he abstracts a specific problem limiting the input to six different pulps and setting constraint conditions uses linear programming to find an optimal solution. He does not, however, state whether this final solution was actually produced to check the results. He does state that input data was determined from actual laboratory tests and reminds the reader of the general rule of thumb that accuracy of results never exceeds the accuracy of the data.

Further study of this situation was introduced on a theoretical level by Lamer (4). Mr. Lamer introduces several different aspects to the problem. He states that, even though knowing the properties of each individual material entering the final pulp blend is the most accurate means of determining final properties, it is also possible to approach the problem by determining the properties of a large number of different mixtures, each mixture being significantly different from every other one. This is more applicable to mixed beating types of situations.

He also discusses the effect of filler additives, stating that these would probably have a negative effect on strength property, which would have to be determined, but that they, too, can easily be factored into the total problem. He concludes with a simple problem based on estimated values which illustrates the possibility of the technique, but again fails to have real input and output comparison to justify whether the technique actually proves out. Some of the assumptions, primarily those regarding linearity, have been dealt with before and are the prime points of interest for starting the basic problem. Therefore, these will be discussed next.

Linear programming depends on what might be called linear blending theory. At least this is the term chosen by D. R. Nordeman in his article on the subject published in the March 20, 1972, Paper Trade Journal (5). Nordeman uses the example of burst vs. freeness curves for a 100 percent softwood and 100 percent hardwood pulp. The theory states that by taking two points, one on either curve, a pulp resulting from a mixture of hardwood and softwood would lie the appropriate percentage distance along a line joining the two points that the mixture would call for. He also discusses application of the theory with regard to the liner board industry. Optimality in this situation would be a maximum hardwood utilization. Therefore, a target point lying somewhat between the hardwood and softwood curves would be chosen, and the possible combinations would result from all lines which pass through this point and intersected the two curves. Graphical optimalityis reached where the slopes of both curves are equal at the point of intersection with the line between these points and the target point. Beyond this point, increasing hardwood content would increase burst at a lesser rate than the removal of the softwood could justify.

The increased value of Nordeman's study was the fact that it was checked out using actual laboratory pulps. He states that some results did vary somewhat beyond normal 95 percent confidence limits, but that the actual variation was not beyond usual production capabilities in terms of correctability and that theory-wise some utility could be seen for on-line use.

Very little information could be found with regard to the linearity of substitution of one ingredient of a furnish for another. However, some further work was done on substitutability of softwood by hardwood in a study at the Department of Paper Science and Engineering at Western Michigan University on "Means and Techniques for Increasing the Utilization of Michigan Hardwoods" (6). This report found quite good linearity with regard to substitutions: "Pulp blends, ranging in hardwood content from zero to 100 percent (softwood content--100 to zero percent), showed near linearity with respect to all tests performed."

Much further information is contained in the report in the form of data on individual pulp species and many graphs relating various paper properties to percentage mixtures of hardwood and softwood pulps. This data, however, may not be particularly applicable beyond the general trends it shows. As has been stated previously, output data cannot be expected to be any more accurate than input data, and in an experiment in which one depends upon the other as in this one, input data must be determined quite closely within the actual experiment for true evaluation of the project as a whole. Therefore, some general preliminary work was necessary to establish the basic properties of certain papermaking substances and the degree to which substitutable linearity exists between them.

EXPERIMENTAL DESIGN

Since the object of this study was to determine the actual practicality of linear programming for determining least cost furnish blends, the essence of the experiment was to generate actual laboratory test data for input to the program and then actually generate sheets according to program specifications to determine if the model actually succeeded in determining a blend which provided the necessary properties. Previous work dealt only with linear programming theoretically, and did not use actual input data or check results.

Five basic papermaking materials were obtained: Espinola bleached softwood kraft, Burgess bleached hardwood kraft, tab cards, and a moderately bright filler clay, and a very bright TiO_2 from departmental stocks. It was then necessary to determine the interaction of each papermaking material towards certain properties of the final sheet to check whether the linearity assumption of the linear programming model was, in fact, accurate enough to use.

In order to do this the tab card, softwood, and hardwood stocks were refined using the Valley beater to 250, 330, and 380 ml. Canadian standard freeness respectively. Handsheets were then made using the standard procedure on the Noble and Wood sheet machine. These sheets were approximately $2.5 \pm .1$ g., which is equivalent to a 60 g. per meter square basis weight. Handsheets were made with 100% of each of the kinds of fibers. Then the interaction effects would be determined by decreasing one fiber percentage by 25 percentage point steps while increasing another fiber in the furnish by the same amount. In this way, the properties of the various mixtures of the three fiber elements in the furnish were determined. The properties of interest were tensile, tear, mullen, opacity, and brightness--all being determined by the usual TAPPI test methods. In order to determine the interaction effects of the various fillers, handsheets were made using five different loading levels of each of the two fillers with each of the fibers. These sheets were then tested for ash content to determine the actual loading level and the same properties as before were tested.

The data derived from these tests essentially constitutes the input to set up the linear programming model for the furnish interactions; it is also presented in tabular and graphical display subsequently.

The next step was to determine two sets of constraints and, after arranging the input material to fit the linear programming problem set-up, to use the Western Michigan University linear programming program (hereinafter referred to as LPR), to determine the least cost furnish and then to return to the laboratory and make this sheet according to the program generated furnish to determine whether the actual properties met the arbitrarily picked constraints.

This was done, and the handsheets produced from the program generated furnish by and large met most of the constraints. The program provides considerable other data with regard to predicted values of the constraint properties, such as marginal costs, limits on various properties, and data on which material would enter or leave the furnish if certain properties were extended beyond certain points. A more complete description of these results, that is the LPR output, and its meaning and significance will be given under results.

Arranging the data to fit the linear programming problem set-up corresponds to the statement of the problem included on the following separate sheet. What this amounts to is a function which the program seeks to maximize while maintaining itself within the constraints. The function consists of each component amount times its cost; therefore, the program minimizes total cost. The constraint equations represent the summing of the contribution of each furnish material to that particular property. Therefore, these are maintained according to the constraints plugged into the right hand side. The final four of these equations simply represent that each of the fillers must individually maintain itself below 20%, that the sum of the fillers must be below 30%, and that the sum of all the constituents must equal 100%. Some of the problems with this format will be brought up later under other headings.

STATEMENT OF PROBLEM IN THIS EXPERIMENT

Maximize: $F(X)=-295X1-285X2-321X3-90X4-530X5-5000X14$

Constraints:

$$
\begin{aligned}
-12.3X1-6.3X2-8.5X3+19.3X4+16X5+X6 &= -\text{Tensile Req.}\\
-75X1-42X2-53X3+40X4+103X5+X7 &= -\text{Tear Req.}\\
-42X1-20X2-21X3+75X4+76X5+X8 &= -\text{Mullen Req.}\\
-88.7X1-93.6X2-70X3-15X4-51X5+X9 &= -\text{Brtns Req.}\\
-67X1-79X2-76X3-66X4-85X5+X10 &= -\text{Op. Req.}\\
X4+X11 &= 20\\
X5+X12 &= 20\\
X4+X5+X13 &= 30\\
X1+X2+X3+X4+X5+X14 &= 100
\end{aligned}
$$

X1: softwood %
X2: hardwood %
X3: tab cards %
X4: clay %
X5: TiO_2 %

EXPERIMENTAL RESULTS

The preliminary results on the properties related to fiber composition and filler level are presented in Tables 1 through 4 and Figures 1 through 20. The tables are self-explanatory. The first set of graphs relates the various properties to percent composition. Which fiber is increasing in percentage and which is decreasing may be determined from the key. Since there was considerable data already generated on hardwood-softwood interactions, only the two 100 percent levels were determined and a straight-line relationship between them was presumed as is consistent with the previously determined data (6).

Many of these relationships turned out to be surprisingly linear; however, most have at least a slight curved character, and several have decidedly non-linear characteristics. It is possible, though not particularly valid scientifically, to hope that the problems this causes with the results of the linear programming and its presumption of linearity will tend to balance out against one another and have essentially a small effect. In fact, it was found that this is an invalid assumption and that further techniques either in an alteration of the linear programming model itself, or more realistically and easily accomplished, alterations on a short-term basis by the operator, were necessary to attain a sufficiently accurate result for furnish adjustment. This effect will be discussed more completely under "Discussion of Results."

Figures 6 through 20 show the relationship of the various properties to the percent ash by the type of filler; that is, one can see the relative gain in opacity or brightness as the percentage of clay or TiO_2 in the sheet increases. The relationship of the physical properties to the percent ash is also shown for each of the fillers and each of the sheets. Approximately the same remarks on non-linearity made above hold in this case.

The most interesting results and those most pertinent to the outcome of the project as a whole are in the output from LPR for each of the various furnishes and constraints. This is quite hard to interpret in the form which is printed out by the computer, so the most meaningful results have been tabulated in a more explanatory fashion in Tables 5 through 9.

The first two columns give the variable name and what it correlates with. The first five variables are assigned to these elements of the furnish so the correlation is obvious; X6 through X10 are the slack variables for the constraint equations on the properties with which they are associated, and this is where they derive their association with these properties. The following column simply gives the unit cost assigned each of the furnish constituents. These are actual prices supplied by a local mill. Following this are the number of units; that is, the percentage of each furnish constituent which according to the linear program can be combined to give all desired properties at a minimal cost.

The next three columns show the various values of the tested properties which are derived at various points in the project. First of these, the constraint value, is the arbitrarily chosen value of the property to be used as a constraint which the program must satisfy while attaining the least cost. The second column is the predicted value of the property. This is the number which according to LPR should be the value of this property in the sheet made according to the furnish it provided. The final column of this group is the actual value of this property in a sheet made according to the furnish provided by LPR. The various discrepancies and the meaning of these will be further discussed under "Discussion of Results."

After this column comes one labeled "Marginal Cost." In the case of the first five variables the number in this column is the amount by which the cost of these furnish constituents must decline before they become feasible for making this sheet. In the case of the second five variables, which reflect the various properties, the number in this column is the marginal cost of one more unit of the property. That is, it is the amount by which the furnish cost would increase if one more unit of the critical constraint were needed.

The last four columns display various limits on values in one of the previous columns. In the case of the first five rows, the lower limit number is the cost below which a furnish constituent cannot go without in some way altering the furnish and the variable affected by this is stated under lower limit variable. The same relationship holds for top limit variable and top limit on these rows. That is, if the top limit is exceeded, the indicated variable will enter the furnish. For rows six through ten these columns have similar but slightly different meaning. In these rows, if the columns are filled in, it is for a critical constraint, meaning one which is only barely met by the furnish and therefore has a marginal cost. The first of these columns shows the number of units by which this critical variable could be decreased, in the second column the variable which causes this limitation. The third and fourth columns reflect the same relationship for the upper limit.

DATA ON FIBER BLENDS

% Blend	Opacity	Brightness	Tensile	Tear	Mullen
100% S.W.	67.1	88.7	12.3	75	42
100% H.W.	79.2	93.6	6.3	42	20
100% T.C.	76.5	70.0	8.5	53	21.2
75% S.W 25% T.C.	71.1	80.9	12.6	80	38.9
50% S.W. 50% T.C.	73.4	75.7	10.4	59	28.5
25% S.W. 75% T.C.	74.5	74.2	11.2	60	30
75% H.W. 25% T.C.	78.0	82.9	7.7	48	22.6
50% H.W. 50% T.C.	78.1	80.4	8.0	46	27.7
25% H.W. 75% T.C.	77.8	75.4	8.8	50	24.3

Table 1

DATA ON % ASH FOR SOFTWOOD

% Ash	Opacity	Brightness	Tensile	Tear	Mullen
2.06 Clay	70.9	84.3	13	81.5	38
3.50 Clay	72.5	85.8	11.6	72.0	32
4.55 Clay	73.5	85.7	11.3	75.0	33
13.2 Clay	80.6	90.4	9.9	76	32
20.3 Clay	86.3	90.8	7.5	67	20.9
2.15 TiO_2	78.4	88.6	12	73	36.7
3.24 TiO_2	80.7	89	12	73	36.6
8.55 TiO_2	80.5	89.5	11.3	68	36.4
21.4 TiO_2	88.8	97.5	8.6	56	26.5
40.1 TiO_2	93.7	100.9	6.0	44	18.1

Table 2

DATA ON % ASH FOR HARDWOOD

% Ash	Opacity	Brightness	Tensile	Tear	Mullen
1.42 Clay	79.5	94.3	6.2	44	17.4
2.03 Clay	79.8	93.9	6.2	44	17
2.42 Clay	80.1	93.1	6.2	46	15.5
9.20 Clay	84.3	90.4	5.2	24	11.1
17.5 Clay	87.2	90.7	3.7	20	6.7
1.46 TiO_2	80.7	94.9	6.9	40	17
1.8 TiO_2	80.9	95.4	6.0	45	16.1
7.9 TiO_2	85.2	96.3	6.5	44	14.4
10.7 TiO_2	88.3	95.2	5.1	28	13.5
21.1 TiO_2	89.5	98.0	4.0	21	9.3

Table 3

DATA ON % ASH FOR TAB CARDS

% Ash	Opacity	Brightness	Tensile	Tear	Mullen
2.0 Clay	78.7	72.8	7.7	48	23
4.0 Clay	79.8	73.3	7.8	48.5	22
4.7 Clay	80.0	73.7	7.8	51	20.5
24 Clay	88.8	78.8	4.2	26	11.1
34 Clay	90.0	81.0	3.4	19	7.5
3.07 TiO_2	79.8	75.3	7.9 ~~50.6~~	50.6	24
5.75 TiO_2	84.1	76.0	7.6	48	22
4.85 TiO_2	84.8	76.1	7.8	48	20.5
12.7 TiO_2	92.5	82.0	5.4	28	15.2
20.5 TiO_2	94.0	87.2	4.4	20	10.5

Table 4

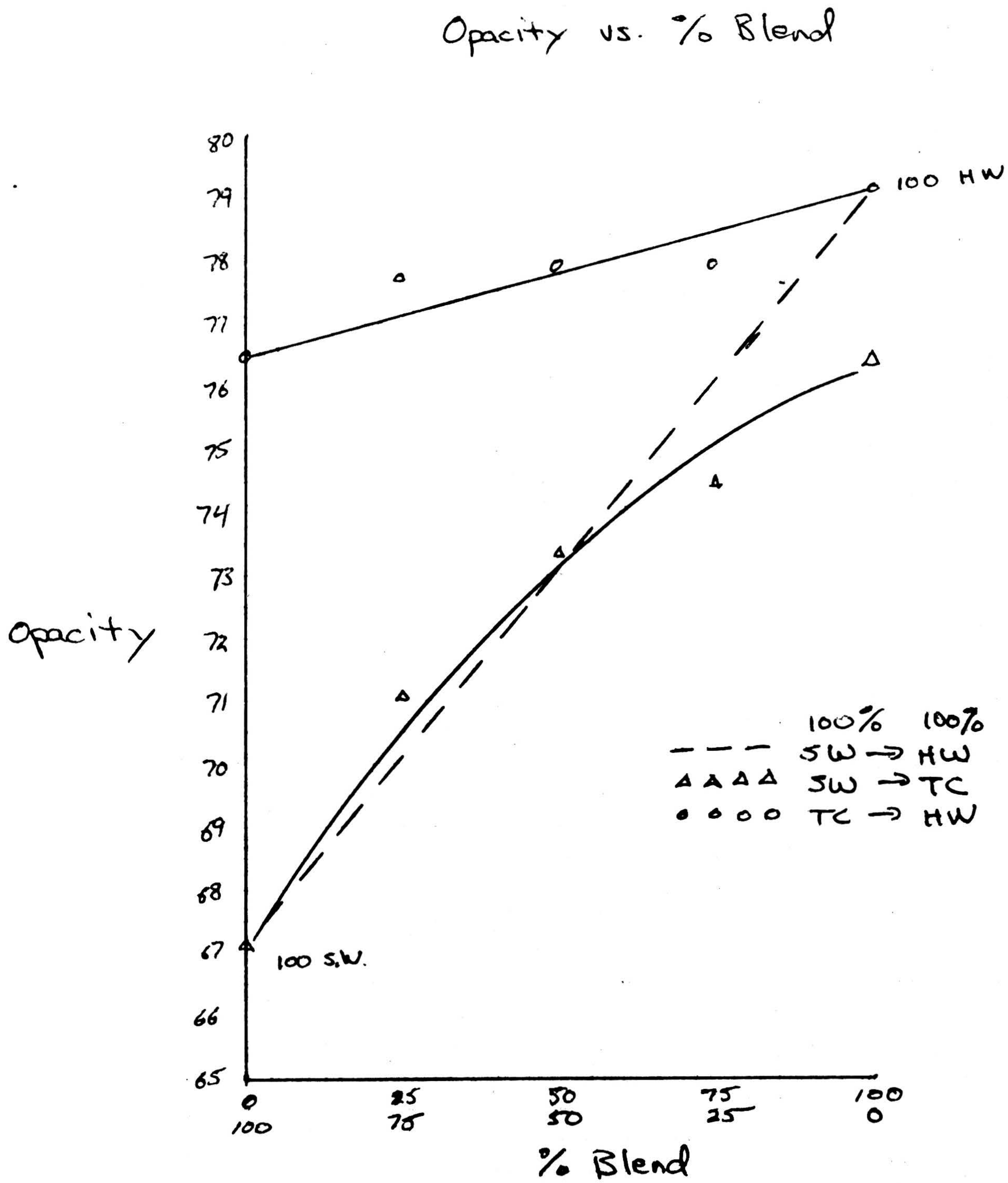

Opacity vs. % Blend
Opacity
80
79
78
77
76
75
74
73
72
71
70
69
68
67
66
65
100 HW
100 S.W.
100% 100%
SW → HW
SW → TC
TC → HW
0
100
25
75
50
50
75
25
100
0
% Blend

Figure 1

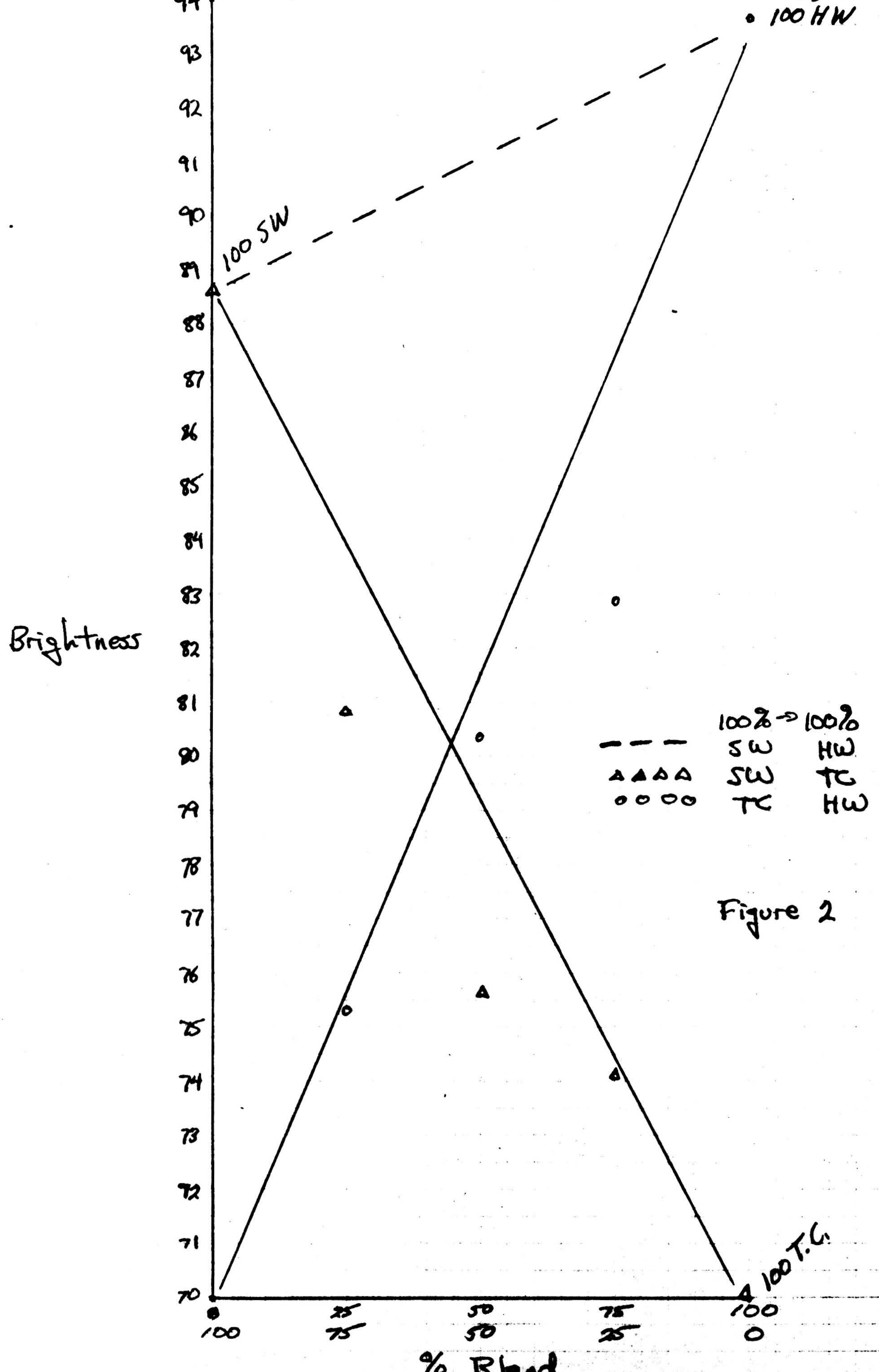

100 HW
100 SW
100 T.C.
Brightness
94
93
92
91
90
89
88
87
86
85
84
83
82
81
80
79
78
77
76
75
74
73
72
71
70
100% → 100%
SW HW
SW TC
TC HW
0 25 50 75 100
100 75 50 25 0
% Blend

Figure 2

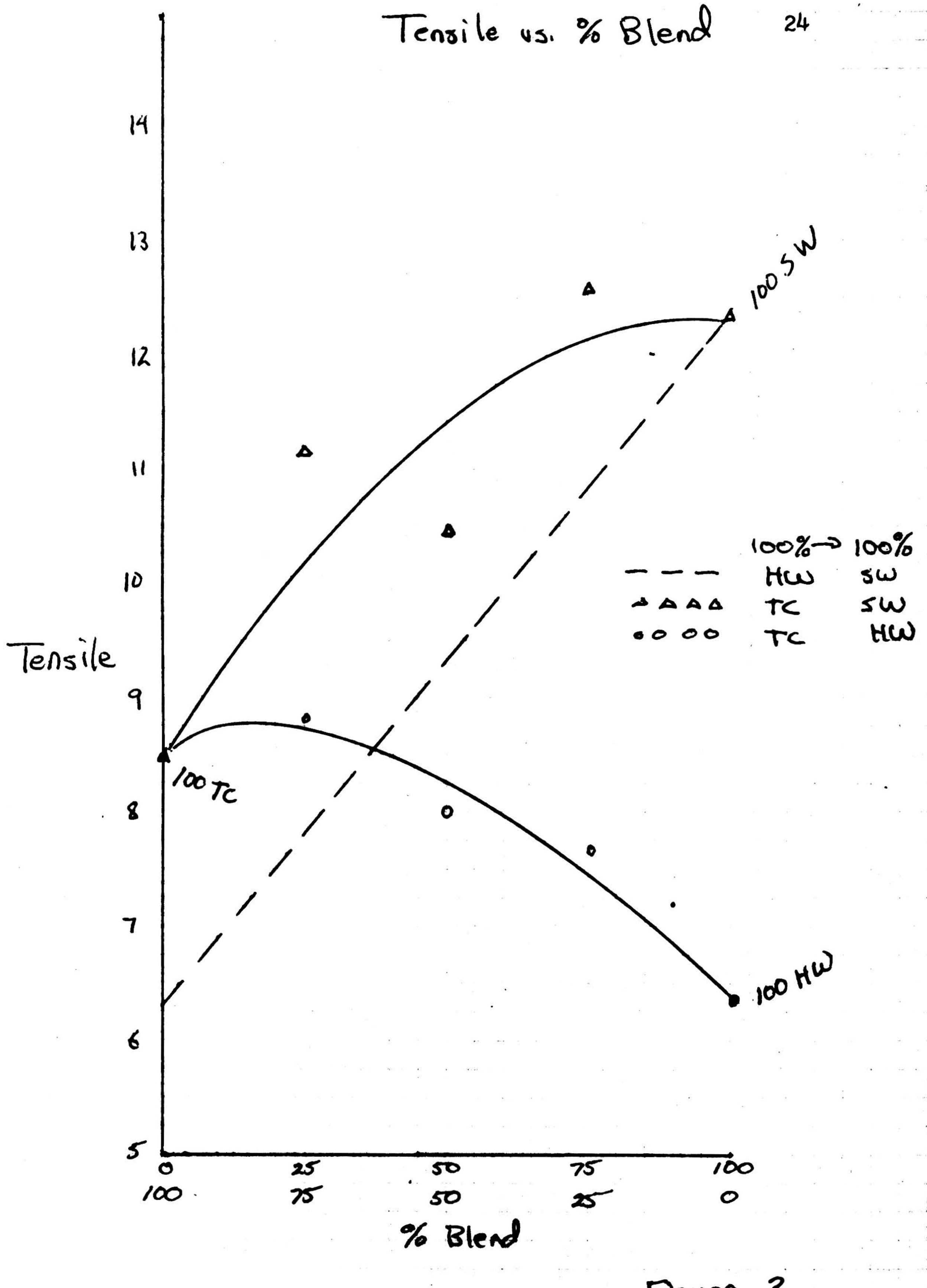

Tensile vs. % Blend
14
13
12
11
10
9
8
7
6
5
Tensile
100 SW
100 TC
100 HW
100% → 100%
HW SW
TC SW
TC HW
0 25 50 75 100
100 75 50 25 0
% Blend

Figure 3

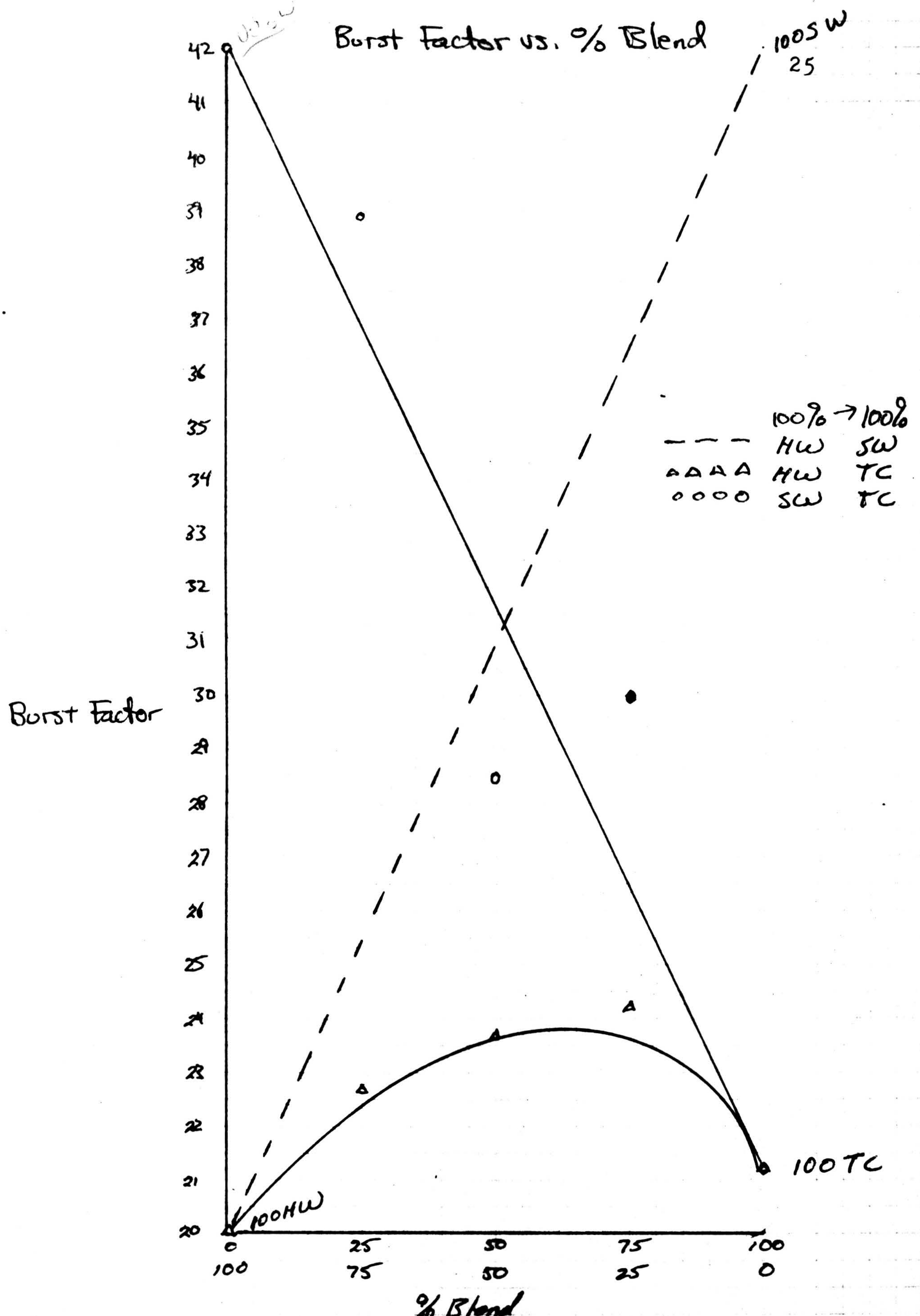
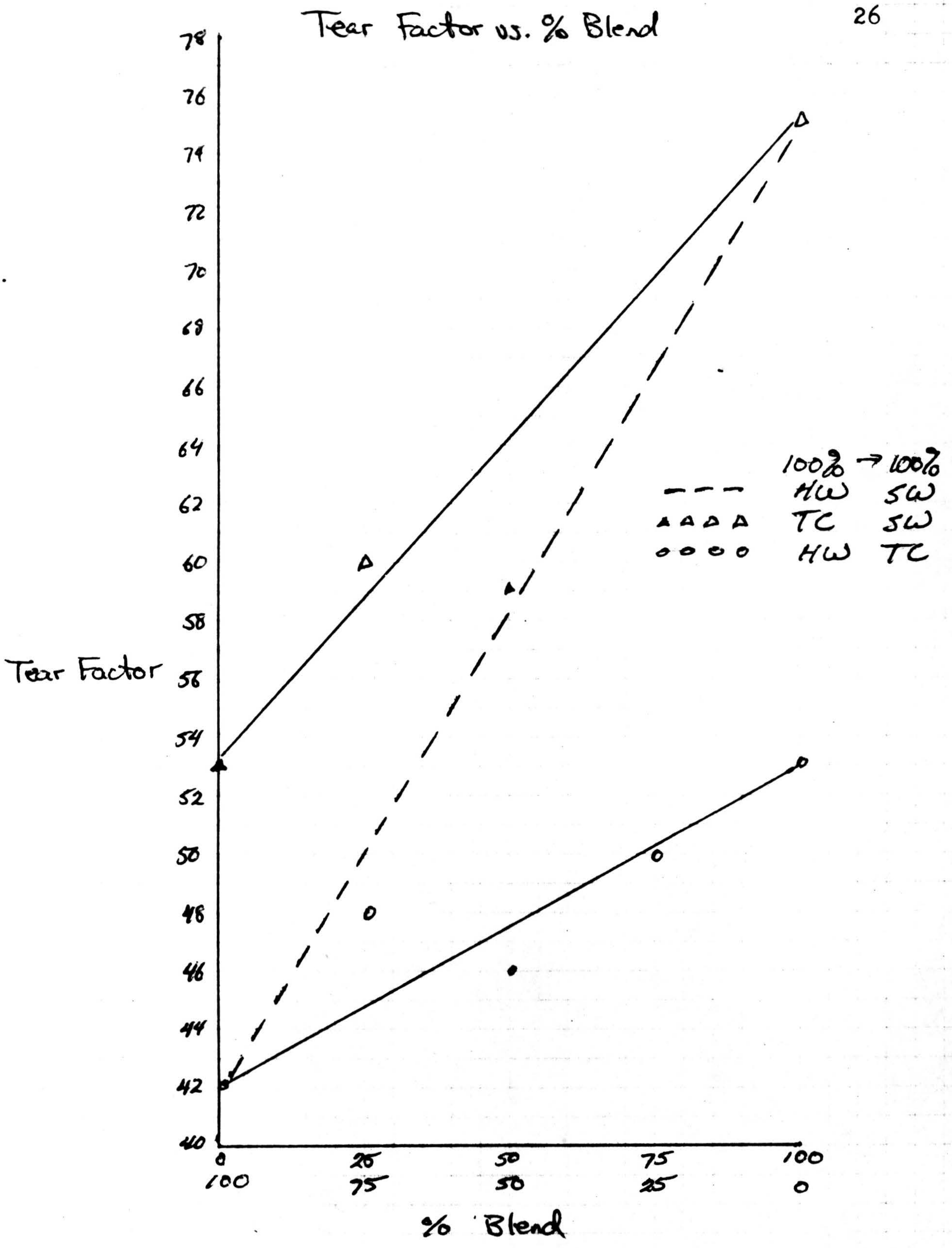

Burst Factor vs. % Blend
Burst Factor
% Blend
100SW
25
100%→100%
HW SW
HW TC
SW TC
100TC
100HW
42
41
40
39
38
37
36
35
34
33
32
31
30
29
28
27
26
25
24
23
22
21
20
0
100
25
75
50
50
75
25
100
0

Figure 4



Tear Factor vs. % Blend
78
76
74
72
70
68
66
64
62
60
58
56
54
52
50
48
46
44
42
40
Tear Factor
0
100
25
75
50
50
75
25
100
0
% Blend
100% → 100%
HW SW
TC SW
HW TC

Figure 5

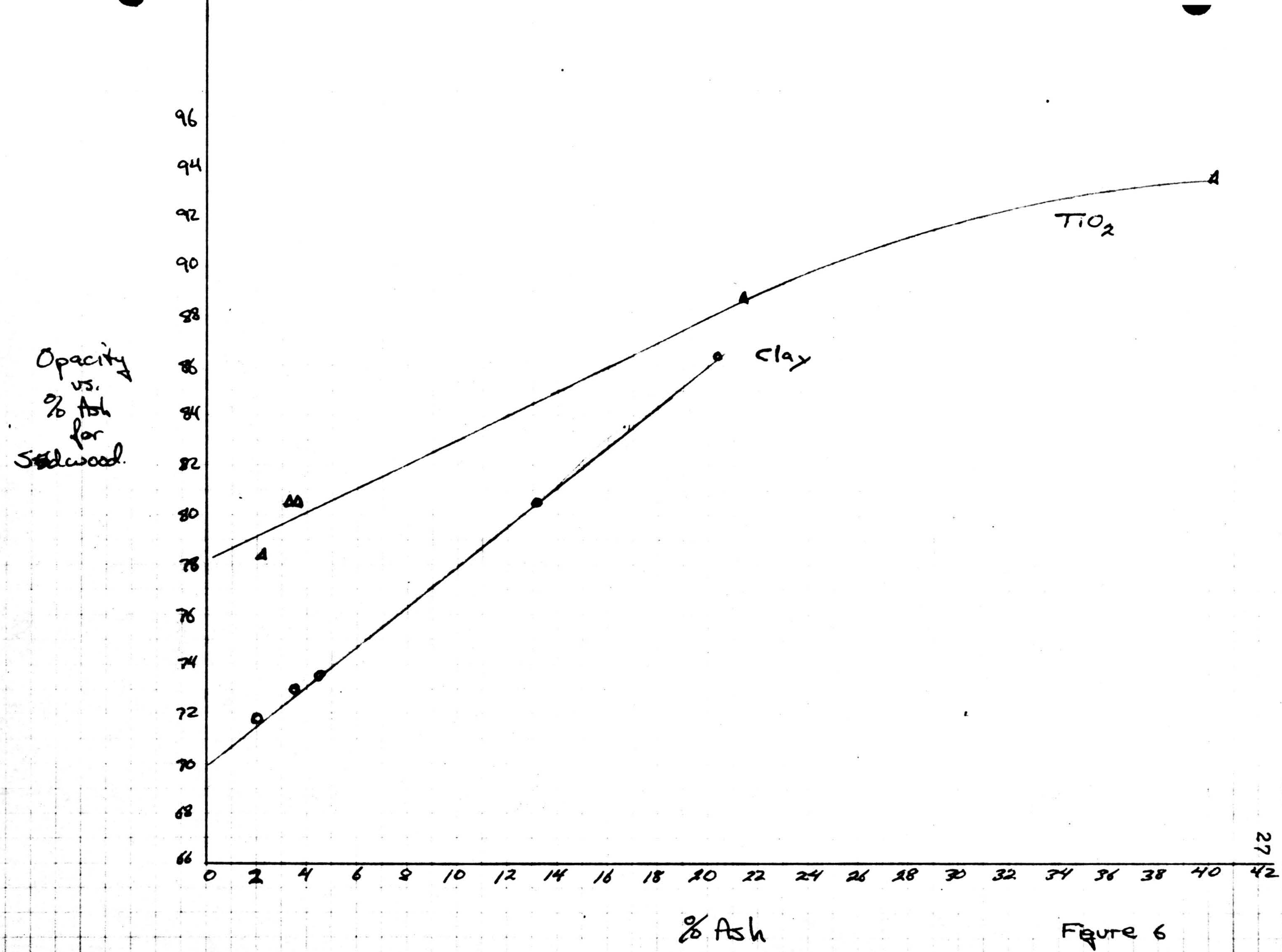
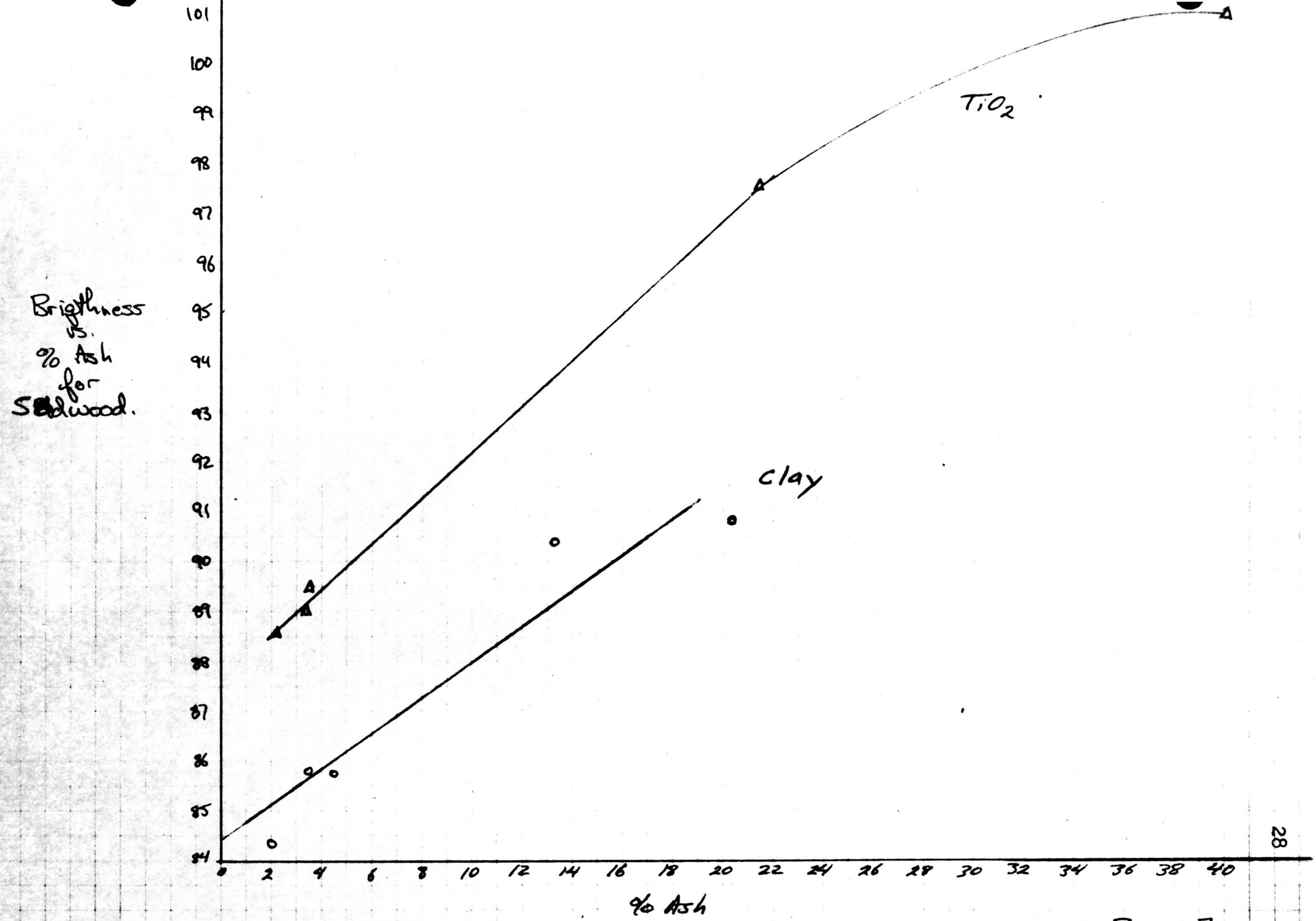

Opacity vs. % Ash for Softwood
96
94
92
90
88
86
84
82
80
78
76
74
72
70
68
66
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42
TiO_2
Clay
% Ash

Figure 6


Brigthness
vs.
% Ash
for
Softwood.
TiO2
Clay
% Ash
101
100
99
98
97
96
95
94
93
92
91
90
89
88
87
86
85
84
0
2
4
6
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20
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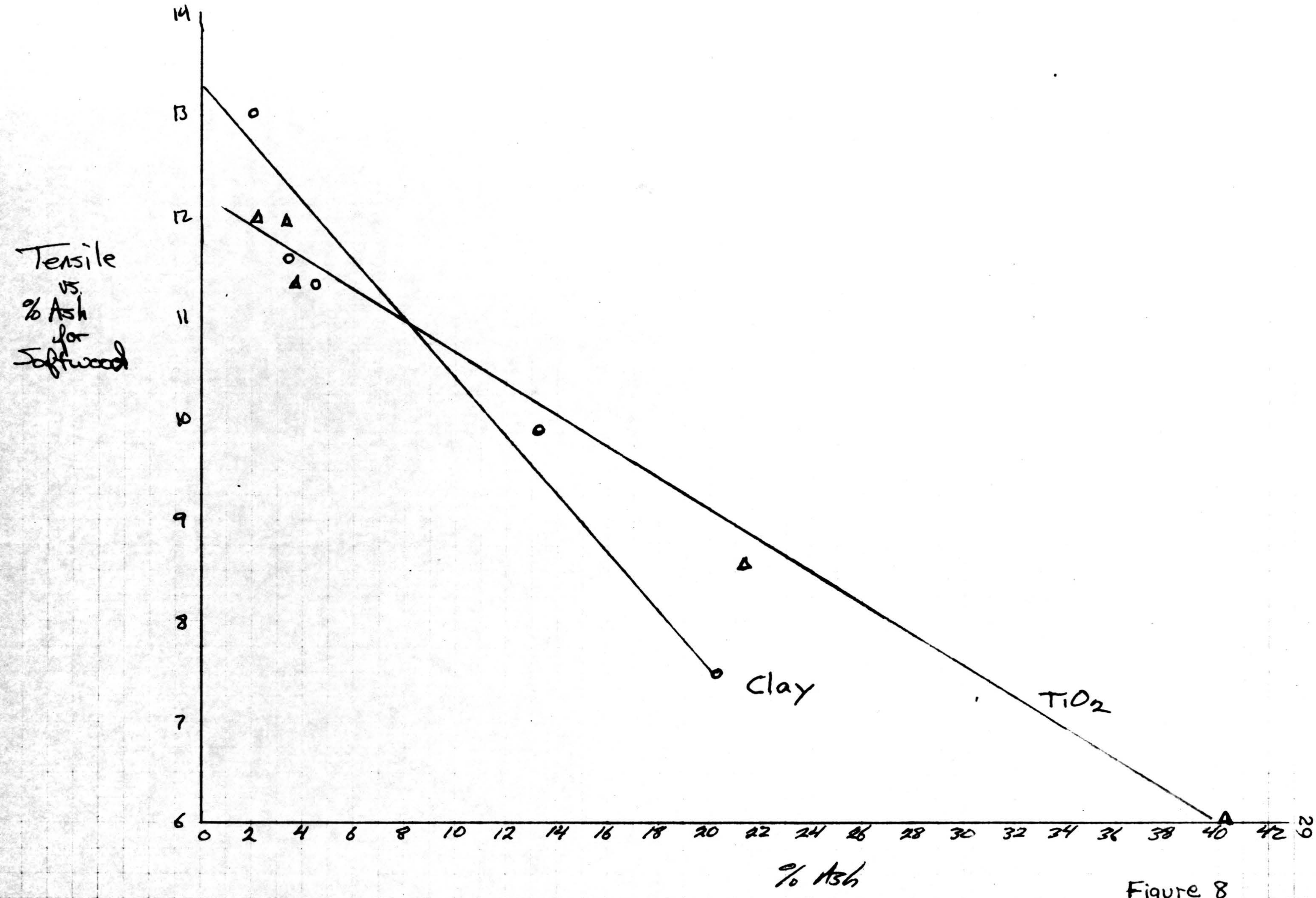
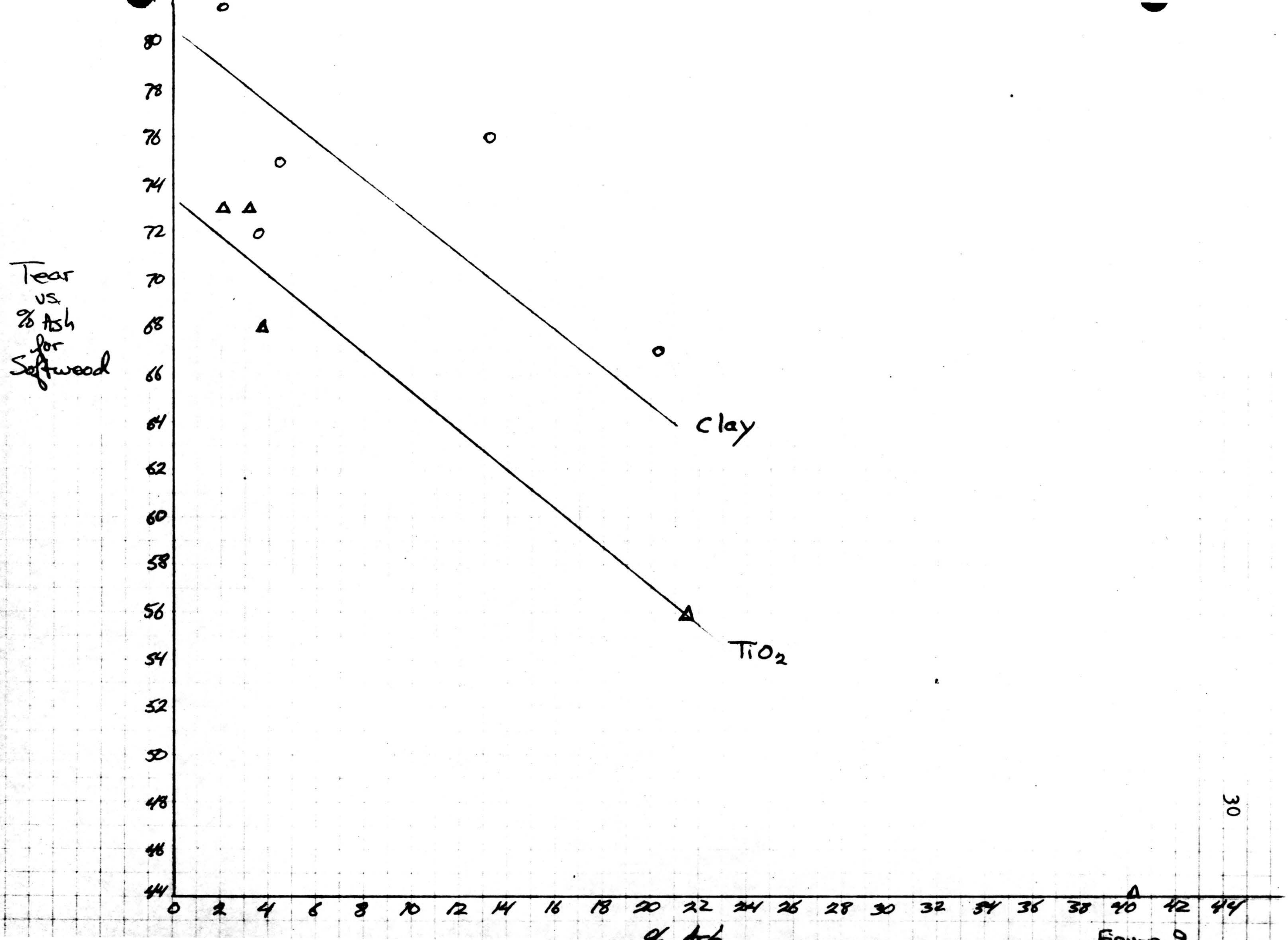
26
28
30
32
34
36
38
40

Figure 7



Tensile vs. % Ash for Softwood
14
13
12
11
10
9
8
7
6
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42
% Ash
Clay
TiO2

Figure 8


Tear
vs.
% Ash
for
Softwood
80
78
76
74
72
70
68
66
64
62
60
58
56
54
52
50
48
46
44
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
Clay
TiO2

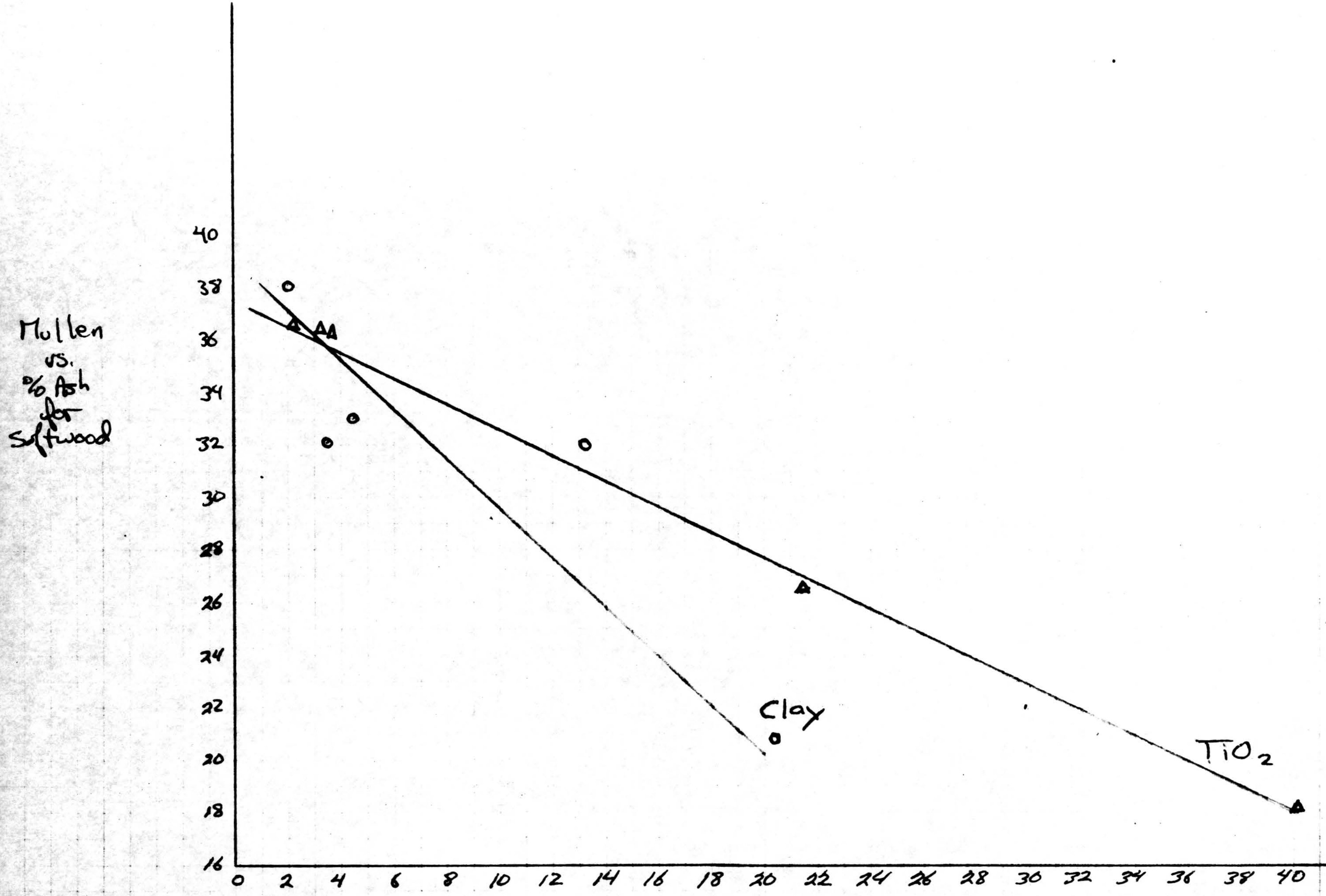

Mullen
vs.
% Ash
for
Softwood
40
38
36
34
32
30
28
26
24
22
20
18
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
Clay
TiO_2

Figure 10

Brightness vs. % Ash for Hardwood

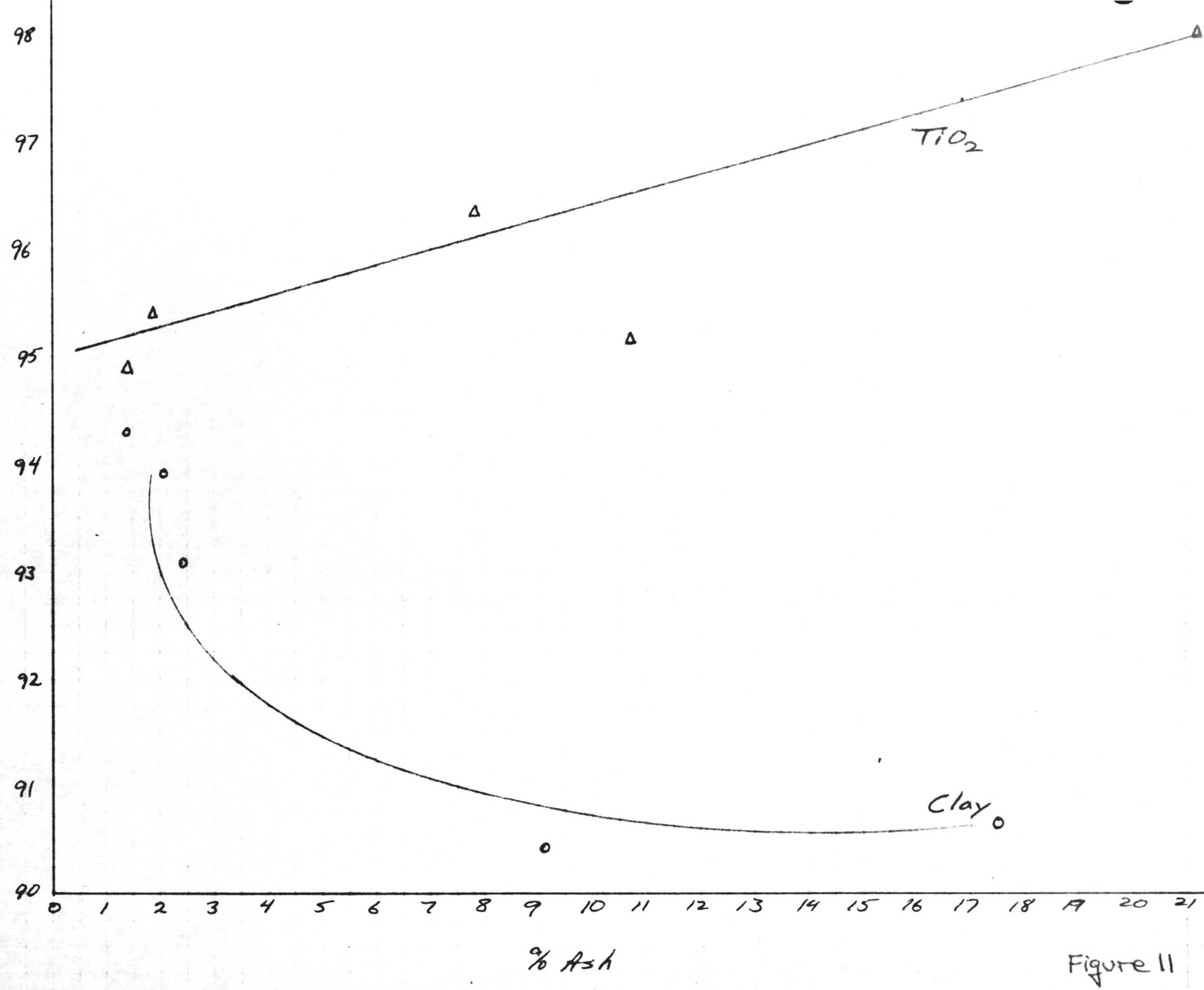
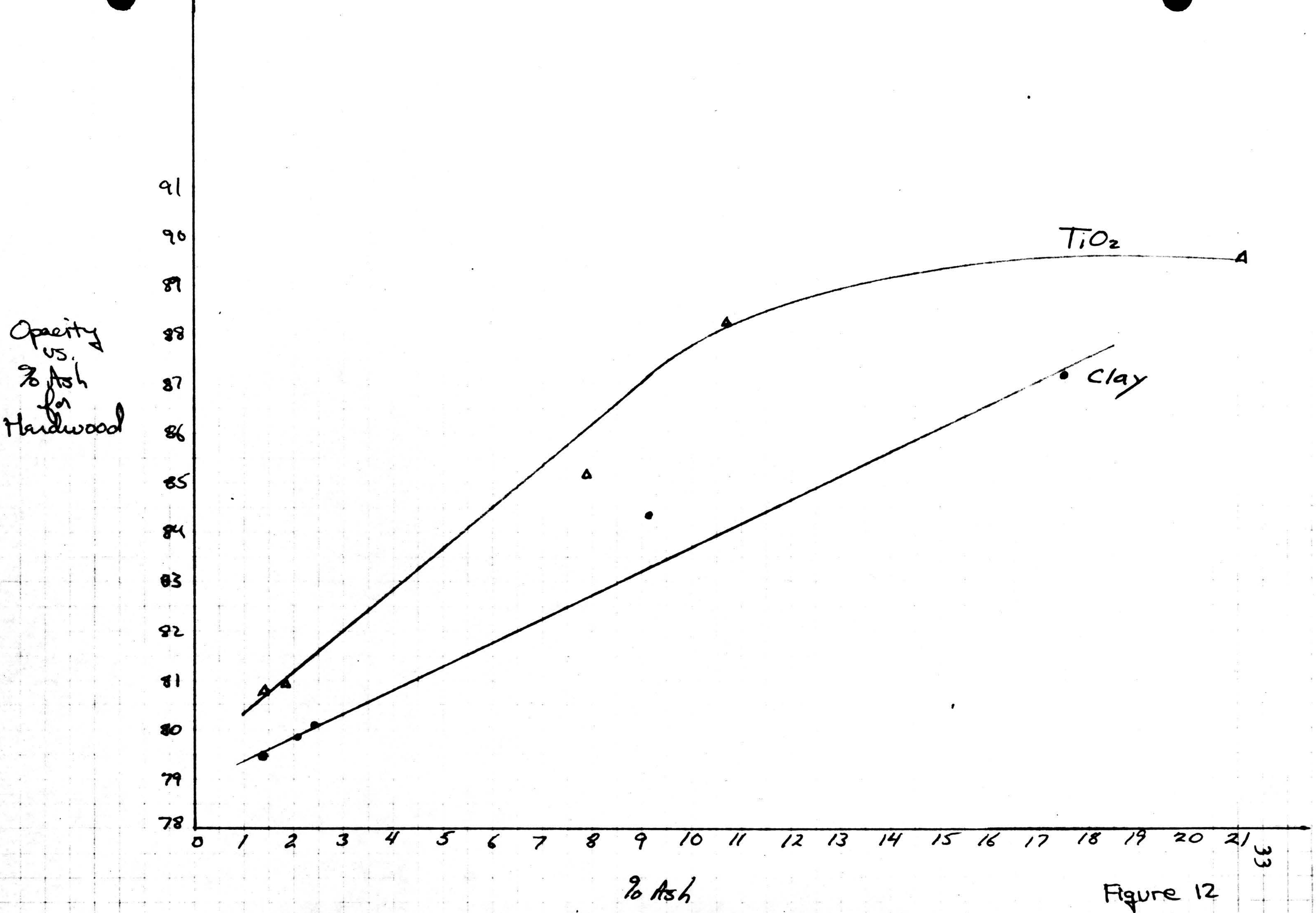


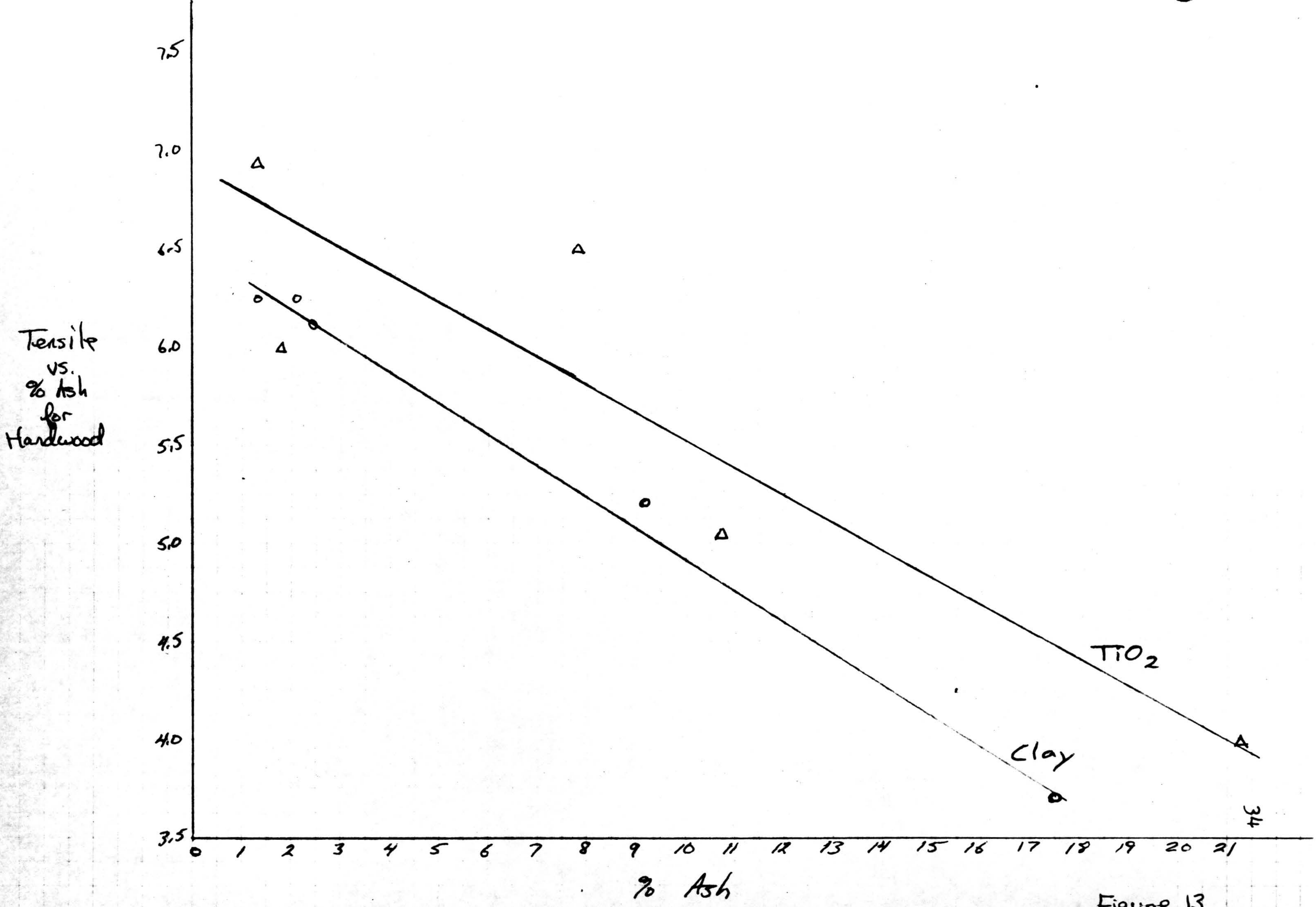
Figure 11



Opacity vs. % Ash for Hardwood
91
90
89
88
87
86
85
84
83
82
81
80
79
78
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
% Ash
TiO_2
Clay

33

Figure 12



Tensile
vs.
% Ash
for
Hardwood
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
% Ash
TiO_2
Clay

Figure 13

Tear vs. % Ash for Hardwood

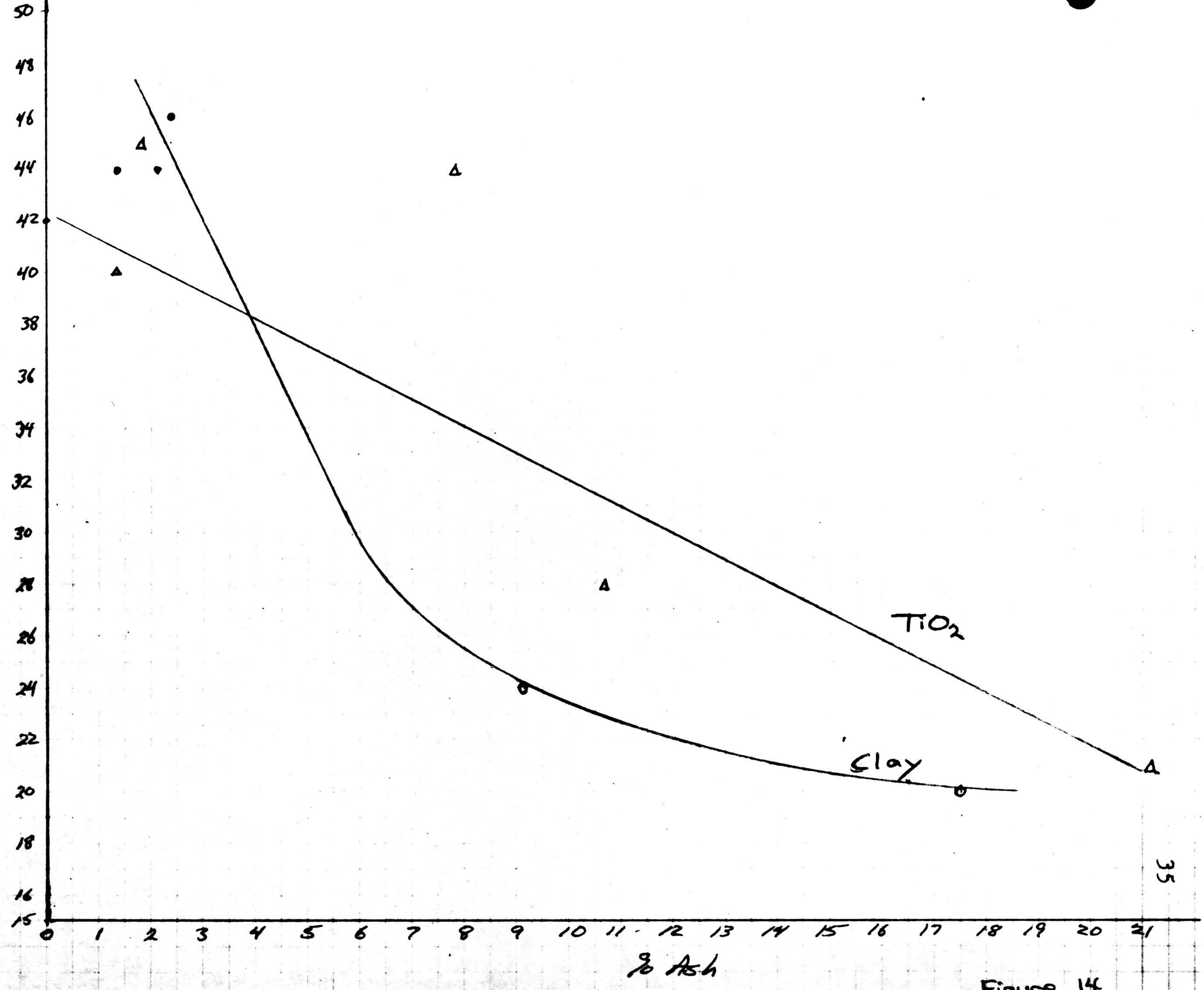


Figure 14

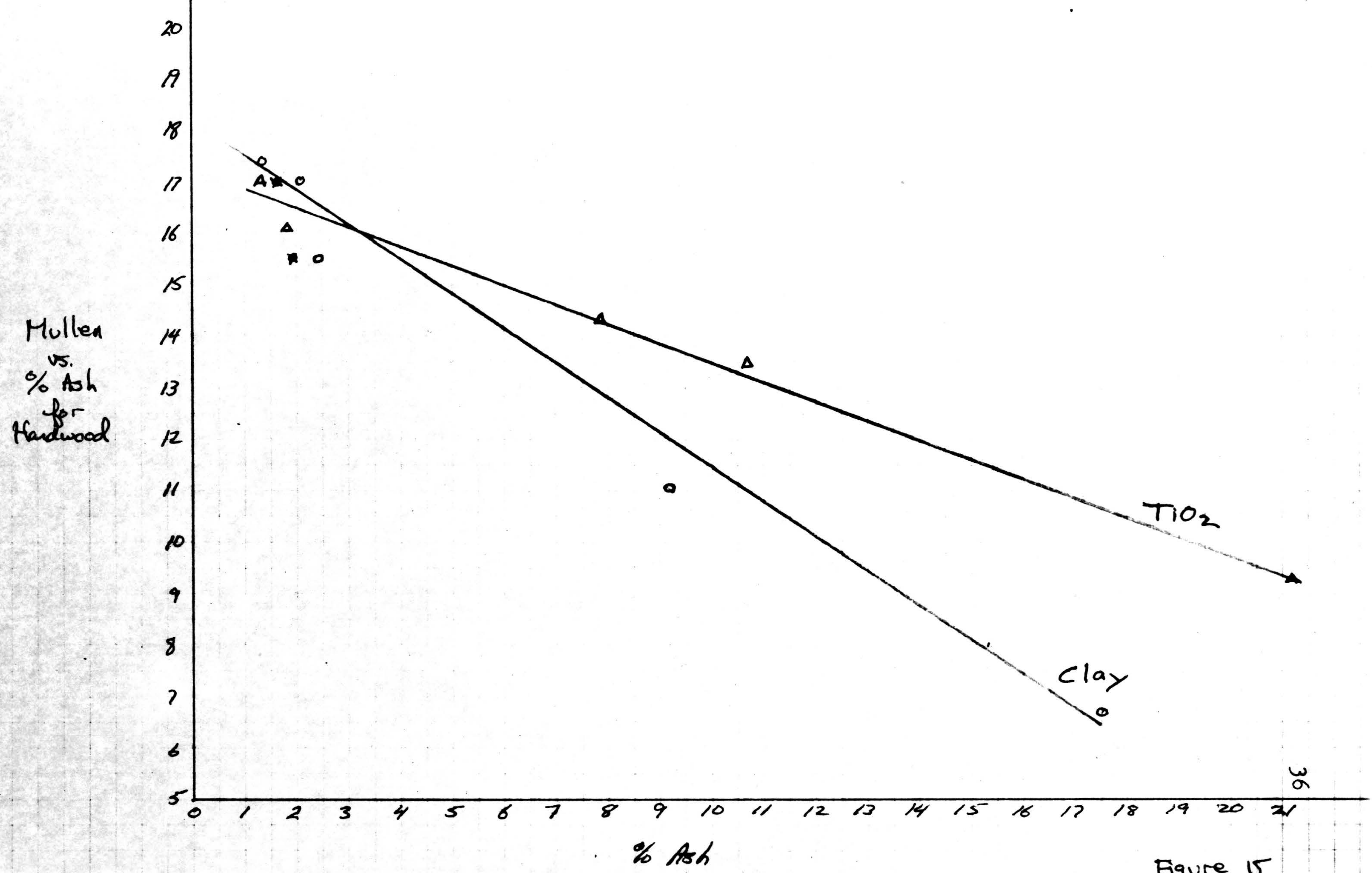

Mullen vs. % Ash for Hardwood
% Ash
TiO2
Clay
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

Figure 15

Brightness vs. % Ash for Tab Cards

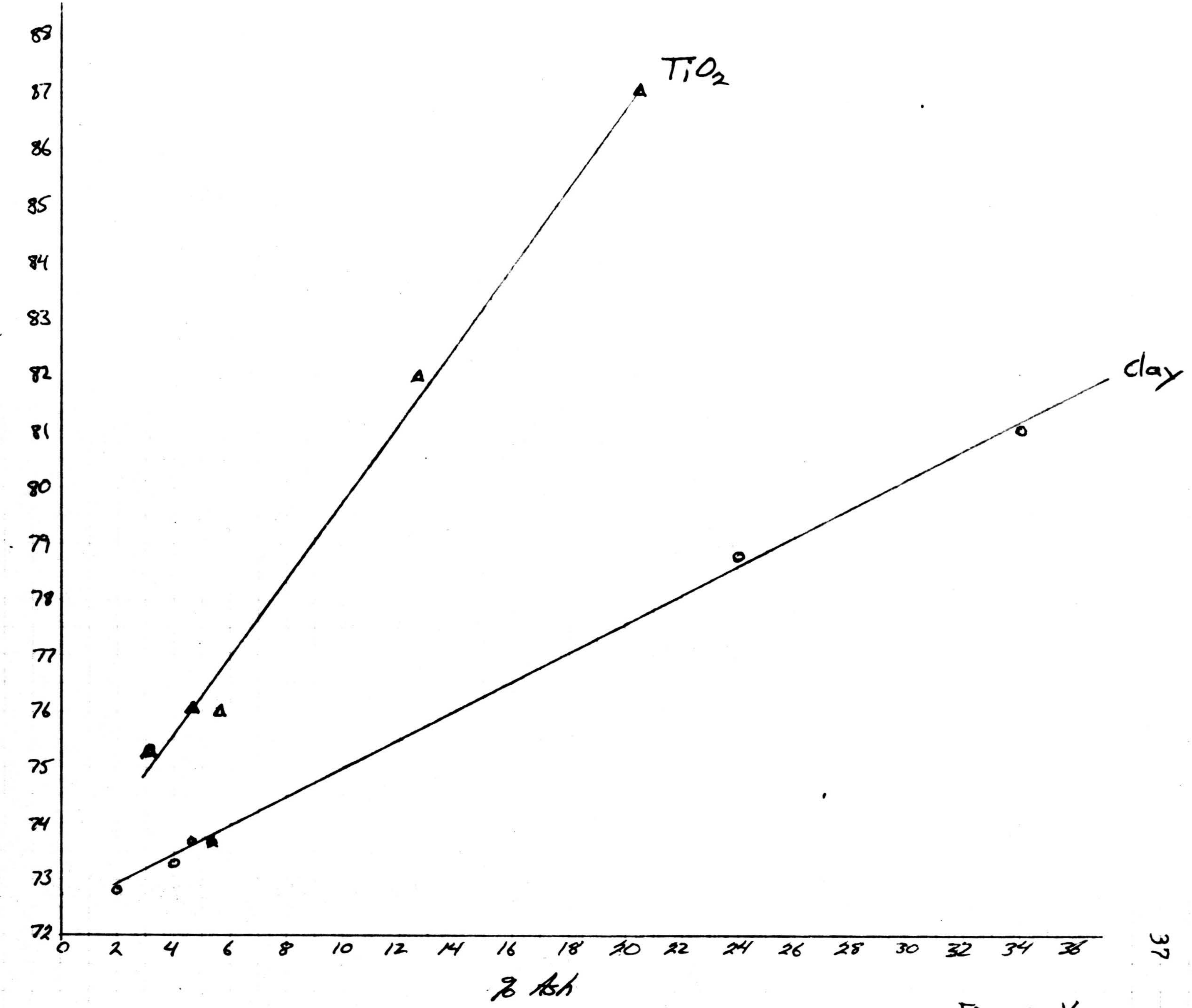


Figure 16

Opacity vs. % Ash for Tab Cards

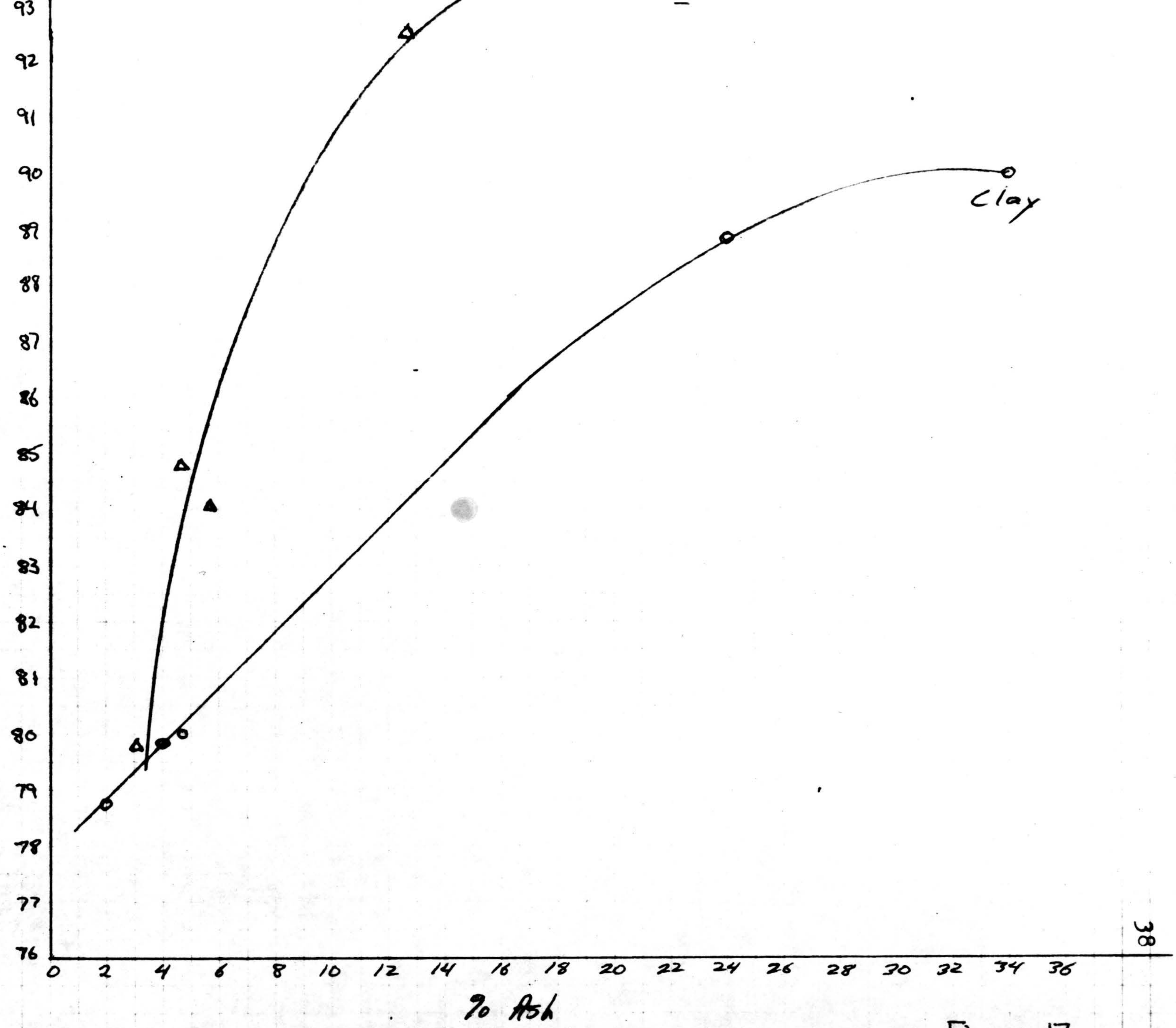


Figure 17

Tensile
vs.
% Ash
for
TAB CARDS

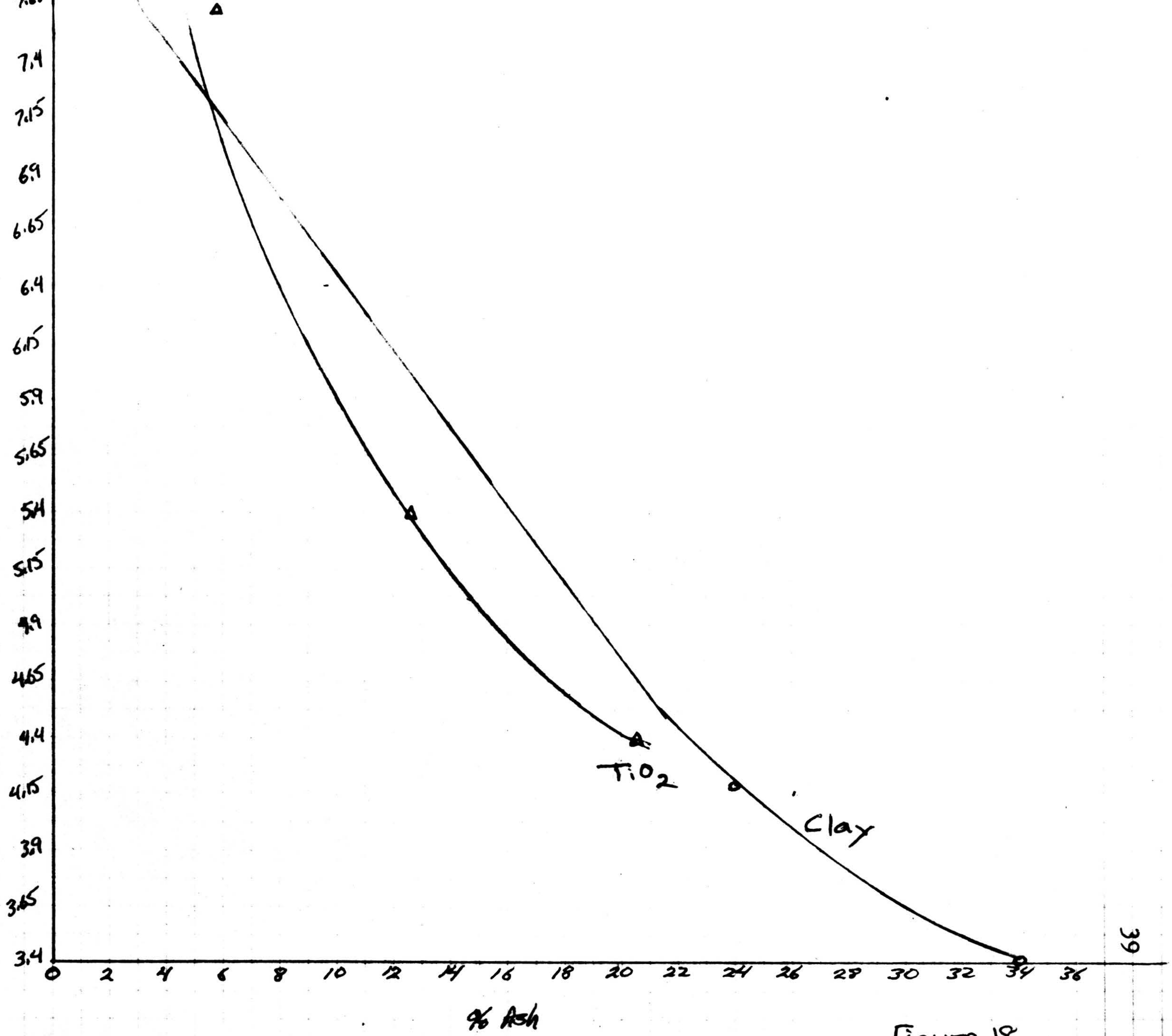


Figure 18

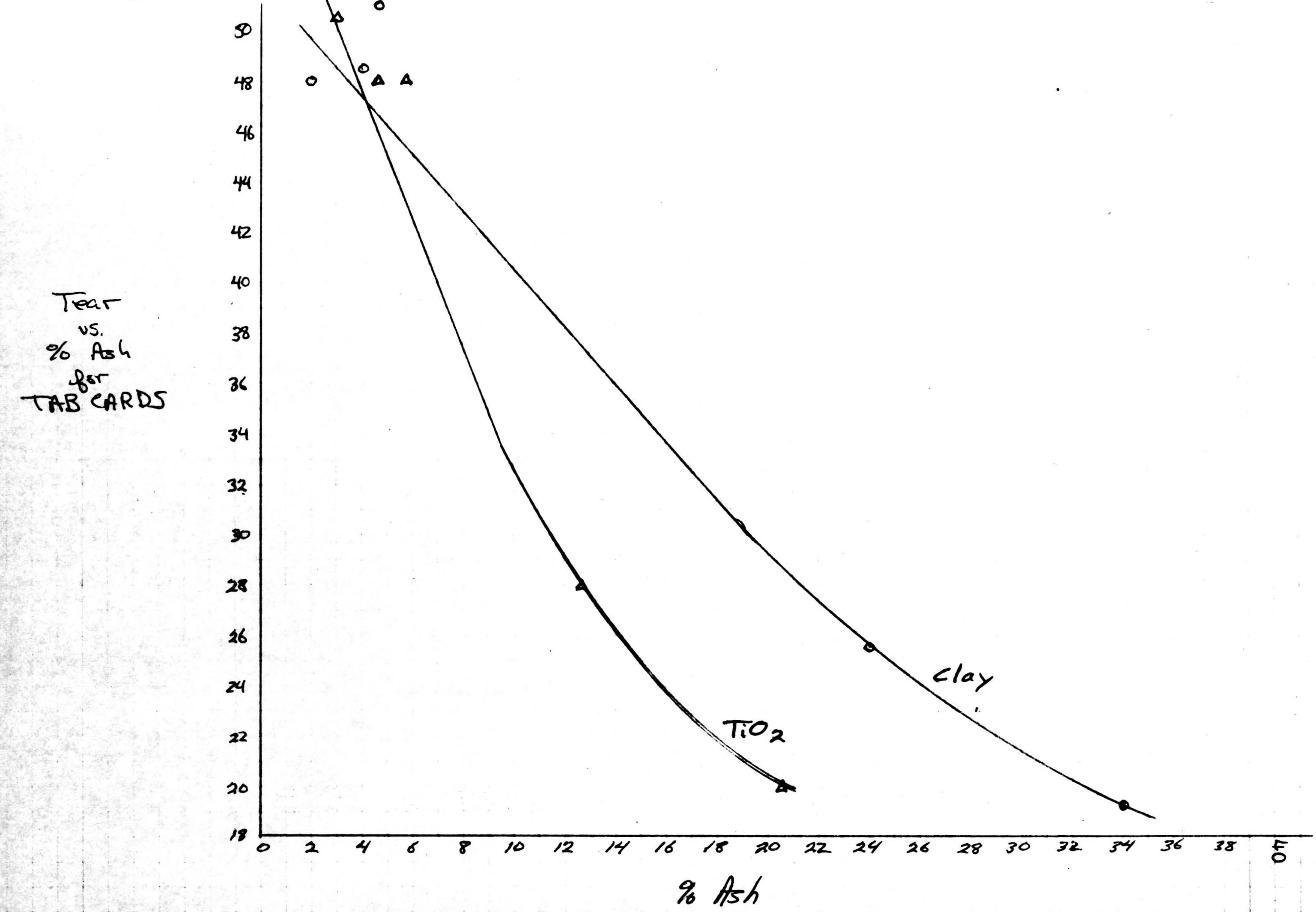

Tear
vs.
% Ash
for
TAB CARDS
50
48
46
44
42
40
38
36
34
32
30
28
26
24
22
20
18
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
% Ash
Clay
TiO_2

Figure 19

Mullen
vs.
% Ash
for
TAB CARDS

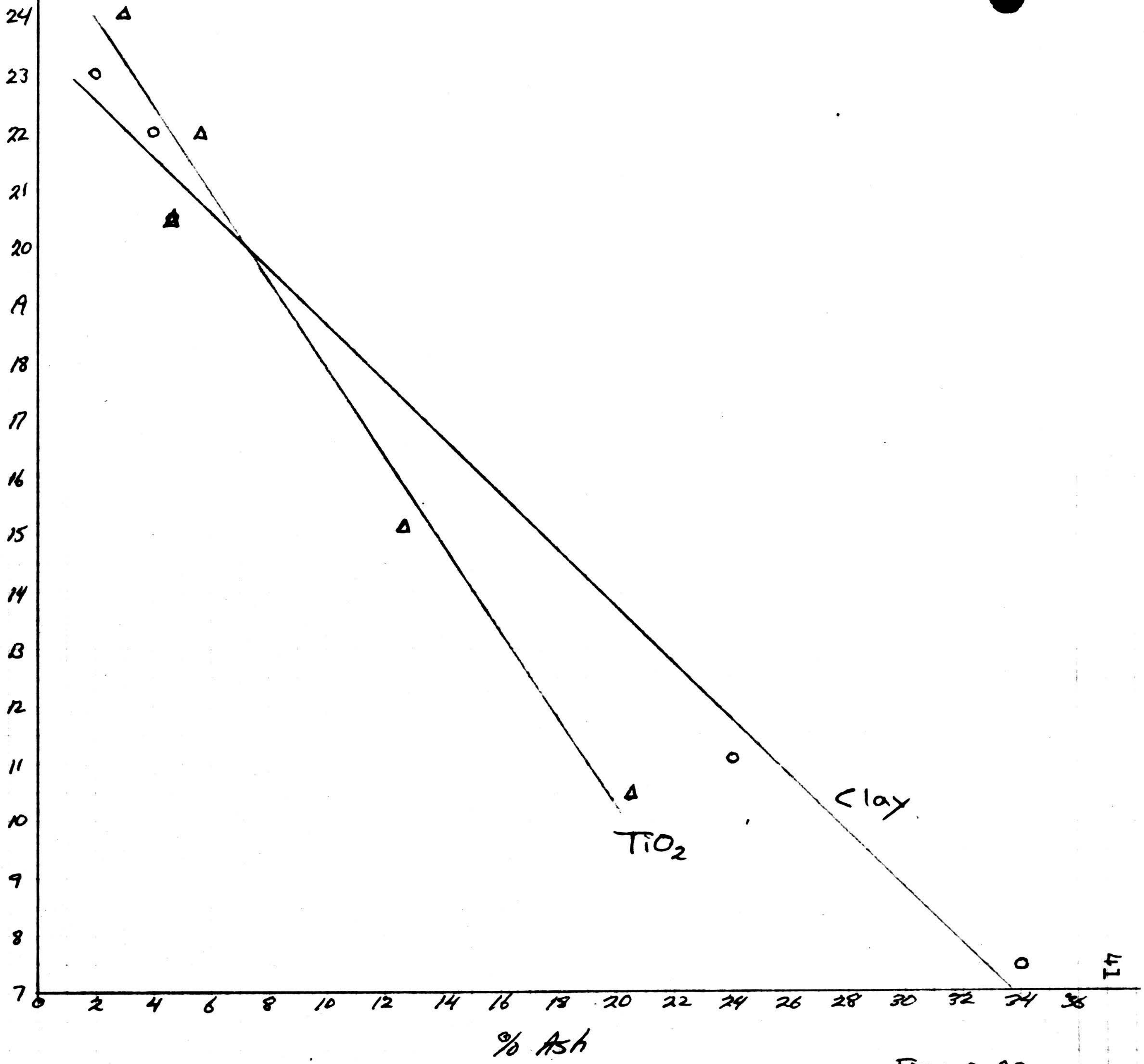


Figure 20

LPR OUTPUT FOR FURNISH 5

Minimal cost = 280.08

Correlation	Variable Name	Unit Cost	# of units	Constraint Value	Predicted Value	Actual Value	Marginal Cost	Low Limit	Low Limit Variable	Top Limit	Top Limit Variable
Softwood	X_1	295	92.7					90	X_6	331	X_2
Hardwood	X_2	285	0				28.9				
Tab Cards	X_3	321	0				50.7				
Clay	X_4	90	7.3					—	X_3	242	X_2
TiO_2	X_5	530	0				418.6				
Tensile	X_6			10	10	10.4	6.5	3.6 units	X_9	2.3 units	X_4
Tear	X_7			25	63	110					
Mullen	X_8			15	33	32					
Brightness	X_9			75	83	84.3					
Opacity	X_{10}			65	67	80					

Table 5

LPR OUTPUT FOR FURNISH 5A

minimal cost = 123.68

Correlation	Variable Name	Unit Cost	# of Units	Constraint Value	Predicted Value	Actual Value	Marginal Cost	Low Limit	Low Limit Variable	Top Limit	Top Limit Variable
Softwood	X_1	160	39.5					106.8	X_4	—	X_6
Hardwood	X_2	146	0				80.7			~~146.8~~	
Tab Cards	X_3	100	60.5					—	X_6	146.8	X_4
Clay	X_4	50	0				388.9				
TiO_2	X_5	460	0				746.8				
Tensile	X_6			10	10	9.6	15.8	.48 units	X_9	2.3 units	X_3
Tear	X_7			25	61	72					
Mullen	X_8			15	29	30.5					
Brightness	X_9			75	77	68.5					
Opacity	X_{10}			65	72	78					

Table 6

LPR OUTPUT FOR FURNISH 5A-E2

minimal cost 131.64

Correlation	Variable Name	Unit Cost	# of units	Constraint Value	Predicted Value	Actual Value	Marginal Cost	Low Limit	Low Limit Variable	Top Limit	Top Limit Variable
Softwood	X_1	160	45.2					139.9	X_4	—	X_4
Hardwood	X_2	146	9.8					65.3	X_9	170	X_6
Tab Cards	X_3	100	45.0					—	X_4	138.1	X_4
Clay	X_4	50	0				106.1				
TiO_2	X_5	460	0				459.4				
Tensile	X_6			10	10	9.6	3.6	1.24 units	X_7	.65 units	X_2
Tear	X_7			55	61.8	68					
Mullen	X_8			25	30.4	33					
Brightness	X_9			75	75	69.5	1.5	5.15 units	X_2	14.9 units	X_3
Opacity	X_{10}			65	72.2	77.5					

Table 7

LPR OUTPUT FOR FURNISH 6

minimal cost = 268.04

Correlation	Variable Name	Unit Cost	# of units	Constraint Value	Predicted Value	Actual Value	Marginal Cost	Low Limit	Low Limit Variable	Top Limit	Top Limit Variable
Softwood	X_1	295	19.8					273	X_6	331	X_9
Hardwood	X_2	285	70.5					256.1 [illegible]	2[illegible]1 X_9	308.6 ~~247~~	[illegible]
Tab Cards	X_3	321	0				65.7				
Clay	X_4	90	9.7					[illegible]	X_{14}	242	X_9
TiO_2	X_5	530	0				375.3				
Tensile	X_6			5	5	6.1	2.9	1.4 unit	X_8	.25 unit	X_{10}
Tear	X_7			20	35.7	62					
Mullen	X_8			10	15	16.5					
Brightness	X_9			85	85	84.2	1.5	.5 unit	X_{10}	4.6 units	X_1
Opacity	X_{10}			80	80.4	85.9					

Table 8

LPR OUTPUT FOR FURNISH 6A

minimal cost = 129.24

Correlation	Variable Name	Unit Cost	# of Units	Constraint Value	Predicted Value	Actual Value	Marginal Cost	Low Limit	Low Limit Variable	Top Limit	Top Limit Variable
Softwood	X_1	160	0				23.6				
Hardwood	X_2	146	63.6					121.5	X_4	175.7	X_1
Tab Cards	X_3	100	36.4					[redacted]	X[redacted]	117.2	X_1
Clay	X_4	50	0				57.2				
TiO_2	X_5	460	0				397				
Tensile	X_6			5	7.1	7.9					
Tear	X_7			20	46	60					
Mullen	X_8			10	20.4	21					
Brightness	X_9			85	85	73	1.9	15 units	X_2	8.6 units	X_3
Opacity	X_{10}			80	82.9	83.2					

Table 9

DISCUSSION OF RESULTS

As shown in Tables 5 through 9 the essence of the results is contained in the three columns reflecting the various values of the constraints for each of the outputs. While there are other significant aspects to be noted, this will be discussed first because it is by far of the most importance with regard to the long-term practicality of this scheme as a mill procedure.

In each of these furnishes the constraint value is merely the arbitrarily selected number which was used when picking the properties for the output sheet. However, if the model had operated perfectly the predicted value which is given by the output from LPR should have matched within experimental variability the actual value of the output sheet for each of the constraints. While these two values agreed closely enough of the time to encourage great optimism for the success of the procedure, they also disagreed significantly enough that steps had to be taken to determine the reason and what could be done about it.

The problem was that in presuming linearity the program was using only an approximation on a large scale of various constraint-constituent interactions. This means that the actual interrelationship on a critical variable would differ so much from this estimation that the predicted value would be significantly different from the actual value. This is not critical if there is a margin to work with above the actual limit on the constraint, but several times it can be seen that the critical variable was predicted to be nearly on the constraint and the value actually turned out considerably below. In a mill this would be an intolerable situation.

What could be done to remedy the problem? There are two possible courses of action. One would be to attempt to make enough alterations in the model by replacing curvilinear relationships with straight-line approximations of the same. This would be tedious, difficult to do accurately, and not entirely satisfactory in the end.

Therefore, for this project a second approach was adopted. This was to use the linear programming itself as a sort of estimation procedure in which each succeeding approximation was better than the one which preceded it. What this amounts to, in fact, is taking the first LPR output and seeing approximately where one stands as a furnish and then readjusting only those variables which need adjusting around this point and then rerunning LPR and getting a second estimation. If the papermaker's experience at this point tells him that in all likelihood this furnish would actually perform, then it could be tested out. However, if he feels it would not, the model could again be altered to a better description around the critical area described by the furnish and a new furnish generated which again should be more accurate than the old.

An example of this is the interaction of clay with hardwood and tab card fibers. The hardwood was quite bright and the addition of clay would actually decrease its brightness, whereas the tab cards were quite dull and clay would considerably help their brightness. Since in this means of programming a single coefficient must be used to describe the interaction of clay with the brightness constraint, there was no particularly satisfactory way this could be described. The means used was an average of the two correct values which was, in fact, a long way from describing either accurately. There is no way this could have been compensated for by the first procedure since this deals not with a non-linear relationship but simply with essentially two different interactions.

What the second procedure allowed was an iteration whereby, knowing the shortcomings, the operator could look at the first furnish and noting either a high amount of tab cards or of hardwood could either greatly increase the coefficient for the clay or greatly decrease it respectively. This would achieve the better description previously mentioned, enabling the second output to be more realistic than the first. Of course, this can be repeated many times in a small amount of time, and theoretically very accurate results could be obtained depending primarily on the skill and papermaking know-how of the individual adjusting the model.

The outcome of such a procedure is demonstrated by the LPR output labeled "5A-E2" (Table 7). This represents the second estimation by observing previous results and changing the model to a better description in the case of Furnish 5A. Although it is still a less-than-perfect match between predicted and actual results, enough improvement seems to be demonstrated to establish the validity of the procedure as a means of obtaining more realistic and accurate results.

Other significant results which demonstrate capabilities of this system are the minimum cost figures recorded in the upper right-hand corner of each output. Assuming proper entry of the problem, this is guaranteed to be the absolute lowest cost for which such properties could be obtained with these furnish elements. Therefore, any inaccuracy is only inherent in the model itself.

One cost-saving demonstrated by these outputs can be noted by looking at the contrast in furnish and cost between Furnishes 5 and 5A and 6 and 6A (Tables 5, 6, 8, and 9). Furnishes 5 and 6 are two different sets of constraints but the same set of costs, which are approximately the current prices for these materials. Furnishes 5A and 6A represent the same constraints but have costs representative of approximately two years ago. Although the costs have increased greatly, accounting for most of the increase in costs of the final output furnish, the item to be noted is that the actual percentage composition of the various materials in the output furnish in each case changes quite drastically. In other words the furnish used two years ago to attain certain properties in an output sheet is no longer what one could use most cheaply to achieve the same results today. In the rapidly changing price and availability structure of today, this procedure would be most useful in maintaining a constant least cost furnish, since actual constituents may change almost as rapidly as was seen in the two-year interval.

CONCLUSIONS

Linear programming by itself falls somewhat short of exactly producing least cost furnishes with certain constraints due to the inaccuracies of certain approximations within the model. However, by using it as a successive approximation or iteration procedure, and each time revising the model to better suit actual interactions around the point predicted by the last linear programming output, very accurate results can be obtained which would be greatly useful in maintaining constant maximum economy of furnish costs.

In addition, it was shown that under situations of fluctuating price and availability furnish blends which will provide certain properties at least cost will change rapidly over a short time period and such a technique is therefore a valuable tool in this sort of situation. Some work, of course, remains, probably the most important of which would be a basic feasibility study on a sort of project basis in a mill situation.

SUGGESTIONS FOR FURTHER STUDY

This study obviously has very limited scope with respect to the potential of the technique being investigated. Further lab work could be done by simply expanding the size of the model to many more variables, which should result in more realistic results in terms of actual furnish properties versus constraints. Another major area would be a study on basis weight correlation. This experiment was limited to a single basis weight. Probably the model could be extended to cover a whole range of basis weights with the same linear relationships over the short range, which make the rest of the technique viable. Other areas, such as coloring with dyes, could also be studied in the laboratory.

The other large segment of further work which would be necessary for implementation in a mill would be a study of the technique as regards certain variables introduced around a paper machine which are not encountered in a laboratory. This would mean a study of such paper machine effects as pressing, drying, drainage, sizing, and other effects introduced only in the paper machine situation. Probably it would also involve a large study of past production in terms of input versus output quality to determine a preliminary model for a particular mill situation. That is, a large statistical study, probably computerized, of this large amount of already generated data could yield a very good preliminary model from which to work in setting up actual equations for the linear programming problem itself.

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