LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

MASTER OF SCIENCE IN FINANCE

MASTER´S FINAL WORK

PROJECT

A MEAN-VARIANCE LOOK AT ROBO-ADVISING

ALESSANDRA ALVES RODRIGUES

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GLOSSARY

ARA - Absolute Risk Aversion

AUM - Assets Under Management

CAGR - Compound Annual Growth Rate

ETF - Exchange Traded Funds

EUT - Expected Utility Theory

FCA - Financial Conduct Authority

FINRA - Financial Industry Regulatory Authority

MFW - Masters Final Work.

MPT - Mean Portfolio Theory

TIPS - Treasury Inflation-Protected Securities

REITS - Real Estate

RRA - Relative Risk Averse

ABSTRACT

In the last few years the wealth management industry has experienced significant challenges and impactful trends, such as a decrease in customers' trust of traditional financial services, new regulatory burdens and increase of competition. In this context, the rise of automated investment managers, well known as 'robo-advisors' and the new combination of science and human capital has been challenging the wealth management industry to find new ways to create value benefiting the client. On this matter, this project contributes to a analysis of risk-return look and efficient frontiers of the recommended portfolio of five online platforms in United States in March 2017: Charles Schwab, SigFig, Wealthfront, ToleRisk and RiskAlyze. In this analysis, back-testing is conducted to assess performance, volatility, value at risk and sharpe ratios. This project is based on the Mean-Variance Theory and uses historical weekly closing prices of exchanged-traded-funds. Results indicates that the current practice of using questionnaires to determine investor risk profiles is of limited reliability. It also find that the robo-advisor model is seemingly benefiting conservative investors the most. Thus, this dissertation contribute to a view on Robo-advisors benefits and limitations, providing a parameter for better understanding its future potential.

Keywords: exchanged-traded-funds, efficient frontier, mean-variance theory, online investment platforms, robo-advisor and wealth management.

RESUMO

Nos últimos anos, a indústria de gestão de riquezas enfrentou desafios significativos e tendências impactantes, tais como a diminuição da confiança dos clientes nos serviços financeiros tradicionais, novos encargos regulatórios e aumento da concorrência. Neste contexto, a ascensão de gestores de investimento automatizados, conhecidos como "robo-advisors" e a nova combinação de ciência e capital humano tem desafiado a indústria de gestão de capital a encontrar novas formas de criar valor para beneficiar o cliente. Sobre esse assunto, esse projeto contribui para uma análise de risco-retorno e analise das fronteiras eficientes do portfólio recomendado de cinco plataformas online nos Estados Unidos em março de 2017: Charles Schwab, SigFig, Wealthfront, ToleRisk e RiskAlyze. Nessa análise, são realizados "backtesting" para avaliar o desempenho, a volatilidade, o valor em risco e os índices de Sharpe. Esse projeto é baseado na Teoria da Variação Média e é baseado em preços históricos de fechamento semanal de fundos de investimento abertos negociados em bolsa. Os resultados indicam que a prática atual de utilizar questionários para determinar o perfil de risco do investidor é de confiabilidade limitada. Os resultados também mostram que o modelo "robo-advisory" aparentemente beneficia investidores conservadores. Assim, esta dissertação contribui para uma visão sobre os benefícios e limitações das plataformas de investimento online, fornecendo um parâmetro para uma melhor compreensão do seu potencial futuro.

Palavras-chave: fundos de investimento abertos negociados em bolsa, fronteira eficiente, teoria da variação média, plataformas de investimento online, *robo-advisory* e gestão de riqueza.

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I dedicate this work to my parents and sisters who believed and supported me in the difficult decision I made when I was 18, I moved from São Paulo to Curitiba. Moving home so early opened my eyes to find out what I can do on my own. After 10 years, here I am, on the other side of the ocean thanking them again for supporting me unconditionally through my trajectory and for giving me the strength I need every day to be away from their embrace. I will never forget my first day in Lisbon, when I realized that my life would start over here, in a totally different environment. I thought about giving up several times for homesickness, sadness, neediness and so on...but they are always with me to support and encourage me to reach the end.

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1. Introduction

Over the last decade, the rise of automated investment managers, well known as ‘robo-advisors’ and the new combination of science and human capital has been challenging the wealth management industry to find new ways to create value benefiting the client (Delloitte, 2015). According to a Deustche Bank research (2016), automated investment managers have become one of the fastest growing areas within the fields of wealth management industry, pushing up the current business models, and expanding the wealth management client base, thanks to their user-friendly, automated process, and low-cost portfolio management.

The term robo-advisor consists in a combination of robotics “robo”, related to automated process without the influence of human beings, and “advisory”, related to the wealth management service which aim to create client portfolios. The combination of these terms results in online portfolio management platform that offers solutions for clients’ assets in an artificial intelligence advisory (Delloite, 2016).

The merger of investment theory and computer science came out in 1952, when Harry Markowitz introduced the era of modern portfolio theory with the mean-variance optimization. Markowitz introduced the mathematical formulation of risk and diversification arising from combinations of assets, concluding that the covariance across an given portfolio determines the additional risk, and diversification is key to reduce risk without sacrificing expected portfolio return. The work of Markowitz also was the kick-off for the use of sophisticated computer science in finance once his techniques for solving the portfolio selection required more advanced computational capacity, which led to the development of algorithms for solutions in Markowitz’s later work (Berk & DeMarzo, 2014; Markowitz, 1991). Relying on Markowitz’ efficient frontier of portfolios, Tobin (1958) also contribute finding an combination of an unique efficient portfolio of risky securities with a risk-free asset, allocated according to an given risk preference.

The contribution of Markowitz and Tobin are the basis for the construction of the main robo-advisor’s model nowadays, which are supported by the techniques of the modern portfolio theory way of constructing an optimal portfolio given the investor’s risk preference and the efficient market hypothesis rationale for passive investing. (Line Bjerknes, Ana Vukovic, 2017).

In order to get client's information and to manage investments, the Robo-advisors interact with clients digitally, by assessing investors risk preference and investment objectives through a questionnaire. The questions are developed in the form of a decision tree, designed to identify the client`s financial goals, risk preference, and investment horizon. The automated platform uses computer algorithms to offer investment selections, typically using Exchange Traded Funds (ETFs) as basic assets. Then, the automated system creates portfolio recommendations. They also manage their clients' portfolios on an ongoing basis, by automatically rebalancing portfolios to maintain the same asset allocation percentage targeted in advance and reinvesting dividends, redemptions, and interest payments. Some also provide tax-efficient solutions (Deutsche Bank, 2017).

Building up on the ongoing Digital Revolution, the numbers shows that robo-advisors have become an increasingly significant phenomenon. According to Statista, assets under management (AuM) in the robo-advisors industry amounts around US$980m in June 2019, and estimates for the future of this specific market are promising. Assets under management are expected to show an annual growth rate (CAGR 2019-2023) of 27.0% resulting in the total amount of US$2,6 billion by 2023. Furthermore, projections from Business Insider Intelligence expects automated financial advisor apps and services will manage approximately 10% of all global AuM by 2020. It means that more and more people are relying on the intelligence of algorithms to decide what to do with their assets.

The information about automated investment management currently available online is diverse in terms of the quality of the materials. However, little is known about the core portfolio management and asset allocation methods applied, as the robo-advisors do not fully disclose their methodology for strategy reasons. On this matter, before laying into the ground work of the mean-variance methodology, the first approach of this dissertation presents a brief overview of the main work steps currently used by online platforms to later presents an investment evaluation based on risk-return and efficient frontiers calculated by the portfolio allocation of five robo-advisors available in United States in 2017.

In this analysis, back-testing is conducted to access mean returns, standard deviations, covariances and Sharpe ratios, and an analysis in conducted to understand if the portfolio recommended by the Robo-advisors makes sense in relation to each type of risk aversion.

These metrics allow an investor to gauge an opinion about each of the portfolio's recommendations as the results of this work indicates that the current practice of using questionnaires to determine investor risk profiles is of limited reliability. It also find that the robo-advisor model is seemingly benefiting conservative investors the most.

This dissertation contribute to a view on robo-advisors asset allocation methodology, providing a parameter to understanding its future potential, also discussing some challenges and opportunities the industry may encounter.

2. ROBO-ADVISORS MAIN WORKING CARACTERISTICS

In a nutshell, the typical Robo-advisor employs three main building blocks of work:

1) Risk tolerance assessment: Determination of the investor's risk tolerance in order to select an efficient portfolio with the appropriate level of risk.

2) Asset class universe and investment vehicle selection: Identification of an ideal set of asset classes to invest in and an ideal investment vehicles to represent each asset class.

3) Asset allocation and portfolio management: Given the basic assets and market characteristics selected, the goal is to find an efficient frontier, and rebalance and tax-loss harvest when necessary.

Through the content of this section, the main characteristics of the robo-advisors are presented.

2.1. RISK TOLERANCE ASSESSMENT

In general, a wide range of tools is needed to assess risk tolerance. These tools must be able to combine the suggestions from classic economic literature, with behavioural finance and psychometrics, the science of measuring psychological magnitude. In relation to the classic portfolio theory based on assumption of rationality, choices under uncertainty are modelled within the framework of Expected utility theory (EUT) (Linciano, Soccorso, 2012).

The majority of the robo-advisors platforms is from the conception that understanding investor psychology and expectations is essential to create a safe financial strategy (Fish and Turner, 2017). Advisors should understand how investors make financial decisions and look at the difference between clients' decisions driven by their preferences and those that are driven by psychological biases is a strong argument for that.

The challenge is how advisors calibrate the theoretical framework with an efficient asset allocation based on investors needs and preferences. Although clients' needs are individual and very specific, portfolio managers tries to standardize to find the advisory process manageable. As said, Robo advisors evaluate customers risk profile using an online questionnaire following some metrics such as: age, income, liquid assets, investable assets and desired investing term. Each question is designed to help the robots determine the investor ability and willingness to take on risk. The result is a basic profile of risk and return that allows the robo-adviser to pick under the asset classes universe, their appropriate weightings (Line Bjerknes, Ana Vukovic, 2017).

The methodology behind the questionnaires basically rely on the EUT which is based on the assumption that investors maximize their final expected wealth when making investment decisions. According to Elton and Gruber (1995), the *nonsation* attribute, combined with the investor taste to risk make it possible to define the investor attitudes toward risk: risk aversion (conservative), risk neutral (moderate) and risk lover (aggressive).

As individuals do not care directly about the money from their outcomes but about utility that money provides instead, their ultimate goal is to maximize the expected utility. The portfolio problem is expressed basically as a choice between mean returns and standard deviation of return, resulting in an expected utility function by the maximization of the following function (Elton and Gruber, 1995):

$$f = R - \frac{\sigma^2}{T} \tag{1}$$

where T is referred to as risk tolerance and express the investor's trade-off between expected return and variance of return. The higher T, the "more tolerant" the investor is towards risk and the higher the risk of the portfolio selection (Elton and Gruber, 1995).

Knowing that the expected utility function measures the expected utility of a set of possible outcomes, it is mathematically represented by the sum of the products of the utility received from each outcome, multiplied by the respective probability of occurrence. Said so, the robo-advisor methodology basically applies the following academic formulation to define investor attitudes toward risk (Gill, 2017):

$$E[(U(X_i)] = p_1U(X_1) + p_2U(X_2) + \dots p_nU(X_n) = \sum_{i=1}^{n}[p_iU(X_i)] \qquad (2)$$

In general, an investor is considered risk averse when the second derivative of utility, with respect to wealth, is negative. That is, If U(W) is the utility function and U''(W) is the second derivative, then risk aversion is usually equated with an assumption that U''(W)<0. The assumption of risk aversion means an investor will reject a fair gamble because the disutility of the loss is greater than the utility of an equivalent gain. An individual is risk neutral if is indifferent to a fair gamble is undertaken, which implies a zero second derivative, U''(W)=0 (Elton and Gruber, 1995). Finally, an investor is considered risk loving if the expected utility from the outcome associated with a risky choice is greater than the utility from one outcome with certainty, that is U''(W)>0.

With the assessment to risk tolerance level, the goal is to answer the question: Given a certain risk attitude, what combination of different asset class do each risk profile investor tend to hold in their portfolio?

2.2 ASSET CLASS UNIVERSE AND INVESTMENT VEHICLE SELECTION

The next work step in the robo-advisors' investment methodology is to select the asset classes universe the desirable risk and return. According to MPT, it is recommended to choose asset classes with low correlation in order to increase the portfolios' diversification benefits. In general, asset classes are divided into main categories, such as equities, bonds and inflation assets, commodities and property, witch correspond to different functions in relation to the portfolio goal, such as growth, income, inflation protection, defensive assets and tax efficiency. The typical approach to asset allocation is a combination of index of stocks and bonds, and a cash position. Adjustments are made in order to include non-traditional asset classes, such as gold and other commodities depending on the strategy of the portfolio and to reflect the new market environment.

In order to choose an ideal set of asset classes for the current investment scenario, digital asset managers take the mean-variance approach for the inputs, and estimates for each asset class' expected return, standard deviation, and correlations with other asset classes. The platforms base on long-term historical values and short-term values to more accurately capture current conditions.

Regarding the investment vehicle universe, is a common approach to use ETFs in the portfolio selection. As ETFs are passive financial assets, they track the returns of the reference entities so the automated online platforms has the opportunity to lower costs, as they follow the track of a particular benchmarks and does not try to outplay it (CFA, 2018). More than that, ETFs are highly liquid because it is traded daily, so whenever the stock exchange is open it is possible to make changes to the portfolio quickly. They are an ideal investment vehicle for the robo-advisor approach, as it has the possibility to mitigate the idiosyncratic risk of individual securities through diversification and at the same time they allow investors to hold diversified portfolios without having thousands of individual securities.

2.3 ASSET ALLOCATION AND PORTFOLIO MANAGEMENT

Once the investor risk tolerance is stablished and the universe assets selection is done, the next step is to maximize the overall expected total return. As the asset allocation theory suggest that the higher risk tolerance level the higher the expected returns, assets class correlation are minimized in order to achieve greater diversification benefits. Also, the platforms optimize the diversification of the portfolio with the selection of numerous asset classes to satisfy their customer's situations. (Lam, 2016, Jorge da Silva, 2018).

Liquidity is also important for the robo-advisors asset allocation model. Customers using the online platforms have the ability to retract their assets at any time. Given this limitation, robo-advisors must select asset classes that are highly liquid and stay away from classes such as private equity as those funds are generally tied for certain time-frames (Jorge da Silva, 2018).

Overtime, there is a constant need to re-optimize by re-evaluating and re-balancing the portfolio resulting from price-level fluctuations and macroeconomic changes. Those are resulted in different risk-return profiles and therefore is necessary to manage investors overall risk tolerance (KeyPoint Financial, 2017).

Tax-loss harvesting is another value added feature that the main robo-advisors nowadays have been able to capitalize on. Tax-loss harvesting is achieved when a losing position is used to offset gains while still maintaining a portfolio’s variance/covariance mix. The robo-advisors algorithms are designed in a way to work in tandem with wash sale rules by selling something at a taxable loss and then repurchasing comparable assets which would yield a similar risk return profile (QPLUM, 2017).

3. LITERATURE OVERVIEW

Robo-advisors already passed through four generations, according to Moulliet, Stolzenbach, Majonek, and Völker (2016). The first two generations were simply online questionnaires and proposals, offering advice in an online access. In contrast, the last two generations are totally automated portfolio management, using quantitative methods and algorithms to construct and rebalance the portfolios. Therefore, only the third- and fourth-generation provide a truly automated investment portfolio advice service, starting from the selection of the assets and finishing with portfolio rebalancing and performance reporting. About 80% percent of European and American robo-advisors fall into the third generation category and the remainder are based on the first- or second-generation system.

Beketov, Lehmann, & Wittke (2018) show that only 73 out of 219 robo-advisors around the world disclose information about the asset allocations method used. The other systems either do not provide such information or do not use any specific asset allocation methods. Also, according to their survey modern portfolio theory is the prevalent model in the robo-advisory market. It seems that the almost 70-year-old theory is still attractive for the wealth management industry offering a passive portfolio allocation solution. However academic findings have evidenced the assumptions of modern portfolio theory to be severely flawed.

In the contribution of Lam (2016) shows mainly three limitations with MPT being the prevalent model in the robo-advisor market. First, they criticize the assumption of the

MPT in relation to the importance of investing in uncorrelated assets which is very unviable in in an increasingly globalized world. Second, the MPT approach might not provide the best passive investment model when correlations are unstable in situation of Market Stress. And third, MPT theory might underestimates risk as the assumption of normally distributed returns is flawed as don't have the ability to take into consideration times of markets distress. Thus, the proposed asset allocation by the robo-advisor using MPT might not provide an appropriate investment solution given the risk profile of the customer.

Still analyzing the contribution in Gill, (2017), the work presented the questionnaire used by the 5 robot-advising companies presented in this study and divided into on four behaviours: expectations, risk ability, risk preference and risk awareness. In relation to expectations, Charles Schwab, Riskalyze and Wealthfront ask the investors regarding the goal of the account, for example retirement, saving for major upcoming expenses, emergencies or wealth accumulation. Risk ability questions are more related to the understanding of financial investments, in the case of Charles Schwab, and the tolerance of the investor to take risk in certain period of time, which is the case of Tolerisk, Weatlhfront and Riskalyze. In relation to risk preference, all of them ask directly regarding how much risk the investor is willing to take and how they react to decline of their investments. Finally, when comes to the risk awareness, all platforms capture the behaviour of the investor by asking their recognition of the potential upside or downside of their investments.

Based on the research of Vishwarupe and Vu (2018), in order to evaluate how Robo-advisors work, they discussed their personally experience allocating money to validate the technology of the automated investment platform, Betterment Digital, over the course of one and a half months, starting from 1th of October to November 16. The authors discussed their experience and provided useful feedback by a customer evaluation scorecard created for five different areas: User friendliness, Financial Performance, Fees, Tools and Resources, and Investment Options. Results showed that robo-advisor services is more recommended to individuals who are new to the financial investing, like a college freshman who would be entering on a four years of college with student loans and thereby wanting to save their money. By transferring that money into a robo-advisor portfolio, these students would be able to find great return over their four years at college and, at the same time, learn more about the market and investing. Their experience with Betterment showed that the service is not for clients looking for a short-term gain due to the passive investment strategy been more beneficial for long-term returns.

As mentioned before, the idea of building efficient portfolios is based on MPT and this is one of the reasons that Vishwarupe and Vu (2018) are hesitant to recommend robo-advisor platforms. The use of the MPT although has been historically successful and keeps clients invested, the research indicated that robo-advisors fails to incorporate investors' varying degrees of risk aversion utilizing improper portfolio weights. An argument is that firms still tend to use the MPT because it is a common industry practice, so they overlook other theories despite their advantages. Full Scale Optimization, is recommended by the authors as an effective theory as it takes into account the reality of skewed returns and tweaks for loss aversion. Also, it is concluded that there is no clear answer whether robo-advisors in their current stage are better than conventional advisory since it all depended on a client's financial situation and needs.

4. METHODOLOGY

An investment portfolio analysis is provided by an assessment the portfolio of the 5 online platforms: Charles Schwab, Sigfig, Wealthfront, Tolerisk and Riskalyze. All robo-advisors discussed in this paper use mean-variance optimization to solve the efficient frontiers.

According to the first assumption of the Mean-Variance-Theory (Elton, Gruber and Brown), the investors only care about the mean and variance of future returns. That is, investors prefer higher means to lower means and lower variances to higher variances. As future expected returns and future covariance is not something that is observable in the market, historical data is used for the assessment of the past returns. Said so, this work analyses the historical means, variances and covariances to understand if the portfolio recommended by the Robo-advisors makes sense in relation to each type of risk aversion.

As robo-advisor's construct the best possible risky portfolio using the concept of efficient diversification, they supposed to choose a portfolio on the efficient-frontier as these are the portfolios with the highest expected returns and lowest volatility. An efficient frontier will normally differ according to the assumptions made about the short-sales, lending and borrowing. In this work, in particular is considered the following assumptions to the construction of the efficient frontiers: The Efficient Frontier with No Short Sales and The Efficient Frontier with Riskless Lending.

The following risk appetite are considered to build the different efficient frontiers presented in this dissertation: Conservative, Moderate and Aggressive.

- Conservative Allocation: for investors who seek current income and stability and are less concerned about growth;
- Moderate Allocation: recommended for an investor with long-term goals who do not demand current income and is looking for some growth potential. In this case, the investor also is likely to entail some fluctuations in value, but present less volatility than the overall equity market;
- Aggressive Allocation: suggested for long-term investor who want high growth potential and do not need current income. May entail substantial year-to-year volatility in value in exchange for potentially high long term returns.

After the selection of the profiles, the efficient frontiers and the investment opportunity set is calculated, based on the portfolio recommendations provided by each robo-advisory platform and are available in Appendix A.

The Mean-variance-analysis introduced by Markowitz (1952) is used for assembling the optimal portfolios. It is considered the problem of optimally investing capital in m risky assets $i = 1, 2, \ldots, m$ for a single period, with respective returns given by the following vector:

$$R = [R_1, R_2, \ldots, R_m] \tag{3}$$

The following vector of returns and covariance matrix represent the mean and covariance of these asset returns, respectively:

$$E[R] = \alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{pmatrix} \tag{4}$$

$$Cov[R] = \circ = \begin{bmatrix} \circ 1,1 & \cdots & \circ 1,m \\ \vdots & \ddots & \vdots \\ \circ m,1 & \cdots & \circ m,m \end{bmatrix} \tag{5}$$

The expected portfolio return is thus given by the linear combination of the underlying expectations:

$$\alpha_w = E[R_w] = w'\alpha \tag{6}$$

Similarly, the variance of the portfolio is given by the variance of the weighted average of the individual returns:

$$\sigma_w^2 = var[R_w] = w'\circ w \tag{7}$$

Given preferences for higher expected returns and lower variance, Markowitz posed the evaluation of different portfolios' is a quadratic programming problem, in which the objective is to maximize the expected return subject to a target return variance σ_0^2 :

$$\text{Maximize: } E[R_w] = w'\alpha \tag{8.0}$$

$$\text{Subject to:} \quad w'\circ w = \sigma_0^2\ 0 \tag{8.1}$$

$$w'1_m = 1 \tag{8.2}$$

Solving the maximization problem for every possible target variance, or the equivalent minimization problem for every possible target expected return α_0, yields the efficient frontier:

$$\{(\alpha_0,\ \sigma_0^2) = [E(\,R_{w0}), \text{var}\,(R_{w0})] \mid w_0\ optimal\ |\} \tag{9}$$

In order to identify the unique portfolio of risky investments to be optimally combined with borrowing or lending at the risk-free rate, the tangent portfolio is calculated by finding the tangency point on the efficient frontier of risky investments. The tangent portfolio is the portfolio with the highest Sharpe ratio. The Sharpe ratio is a measure of a portfolio's risk-adjusted return, presented in equation below. Here, r_p represents the portfolio risk; r_f the risk free rate; and σ_p the portfolio volatility:

$$S = \frac{r_p - r_f}{\sigma_p} \tag{10}$$

Given that the tangent portfolio has the highest Sharpe ratio, it provides the largest reward per unit of volatility of any portfolio available. The implication is that all investors should hold the tangent portfolio, weighted relative to the risk-free investment in accordance to the investor's ideal exposure to risk.

Following Tobin with the separation theorem, the optimal portfolio of risky assets is identified and then the appropriate ratio of investments in the tangent portfolio to risk-free assets is determined. Thus, all robo-advisor investors should have portfolios placed on the straight line representing the efficient frontier including risk-free investment.

The main goal of this dissertation is look into the portfolio allocation of this mentioned automated online platforms, and procced with a comparative analysis of their efficient frontiers resulted for each robo-advisor.

The methodology of the analysis available in this dissertation is divided into 2 exercises:

(1) Based on the 5-robo advisors asset allocation assessment, all historical closed prices data prior to March 31, 2017 is used to back test each portfolio. The following performance measures are used to evaluate portfolio recommendations: Risk profile of the returns, worst return in all period, 5% historical Value at Risk, 10% historical Value at Risk and 1 year Sharpe ratio.
The results are represented graphically per each type of risk profile. The inputs are based on the weekly closed prices of ETFs from the ETF's first closed price day to March 31, 2017.

(2) Based on the ETFs used by each of the platforms, the efficient frontier is determined for each platform for portfolios with a 5-year investment horizon, considering prohibited short selling scenario to respect the asset portfolio allocation at the given moment. The efficient frontier is represented graphically assuming that it is possible to deposit but not borrow money. The inputs are based on the weekly closed prices of ETFs from the ETF's first closed price day to March 31, 2017.

5. DATA

The portfolio investment universe included in this work is based on the investment universe assessment of Charles Schwab, SigFig, Wealthfront, ToleRisk and RiskAlyze, in March, 2017. As in Gill (2017), the selection of these specific online platforms follows characteristics such as: ease of opening accounts, reputation of robo-advisory platform, number of assets under management, number of clients using the platform and types of questions asked and their relevance to portfolio creation. The Table 1 gives a notion of general characteristics of each player discussed in this dissertation.

TABLE 1: ROBO-ADVISORS PLATFORM

Assessment of 5-online Robo-Advisors platform					
Robo-Advisor	charles SCHWAB	SIGFIG	wealthfront	TOLERI$K THE SCIENCE OF RISK TOLERANCE	riskalyze
Headquarters	EUA	EUA	EUA	EUA	EUA
Assets Under Management	$37 billions	$120 billions	$11 billions	-	-
Minimum Investment	$ 5.000	$ 2.000	$ 500	-	-
Fees	0.28% of assets	First $10,000 managed for free. 0.25% Annual fee	First $10,000 managed for free. 0.25% Annual fee	$70-$89/month	$165-$365/month per advisor
Automatic Rebalance	Yes	Yes	Yes	Yes	Yes
Advice	Hybrid	Automated	Automated	Automated	Automated

Source: Robo advisors official website

*Both Tolerisk and Riskalyze do not have assets under management since they correspond to advisors software available for costumers.

4.1 PORTFOLIO CONSTRUCTION OF EFFICIENT FRONTIERS

As the ultimate goal is to analyze the risk-returns of each robo-advisor platforms and build the correspondent efficient frontiers of the robo-advisors platforms based on the asset allocation of each risk tolerance profile, the questionaries' by the 5 robot-advising companies will not be disclosure. Going to the ground work, in total, 35 ETFs is taken into account for the evaluation of the 5 robo-platforms together and all of them is treated as risky assets.

The indexes are outlined in Table 2 and are mapped according to their correspondent asset class and the primary and secondary risk. The primary purpose and the assets underlying those ETFs and indexes are considered and mapped to those relevant risks mentioned above. The next step is assess each portfolio's asset classes and determine its primary and secondary risks relevant to each of the Index and ETF. Despite that some of the ETFs and indexes may have multiple risk exposures, only primary risk drivers are mapped (Gill, 2017).

TABLE 2: PORTFOLIO UNIVERSE SELECTION

Portfolio Universe Selection				
Index Code	**Index Type**	**Asset Class**	**Risk Exposure**	**Secondary Risk Exposure**
BND	Vanguard Total Bond Market ETF	Bonds	Interest Rate Risk	
DBC	PowerShares DB Commodity Tracking ETF	Bonds	Commodity Risk	
DBL	Doubleline Opportunistic Credit Fund	Bonds	Interest Rate Risk	
DGL	Gold and Other Precious metals	Inflation assets	Commodity Risk	
EEM	iShares MSCI Emerging Markets	Equity	Currency Risk	Equity Risk
EFA	iShares MSCI EAFE	Equity	Equity Risk	Currency Risk
EFR	Eaton Vance Senior Floating-Rate Fund	Bonds	Interest Rate Risk	
FLOT	iShares Floating Rate Bond	Bonds	Interest Rate Risk	
FPX	First Trust US IPO ETF	Equity	Equity Risk	
FXI	iShares China Large-Cap	Equity	Equity Risk	Currency Risk
HYG	iShares iBoxx $ High Yield Corporate Bd	Bonds	Interest Rate Risk	
IEMG	International Emerging Market Stocks	Equity	Currency Risk	Equity Risk
IGOV	International Developed Country Bonds	Bonds	Interest Rate Risk	Currency Risk
MBG	US Securitized Bonds	Bonds	Interest Rate Risk	
PDN	International Developed Small Company Stocks - Fundamental	Equity	Equity Risk	
PRF	US Large Company Stocks - Fundamental	Equity	Equity Risk	
PRFZ	US Small Company Stocks - Fundamental	Equity	Equity Risk	
PXF	International Developed Large Company Stocks - Fundamental	Equity	Equity Risk	
PXH	International Emerging Market Stocks - Fundamental	Equity	Currency Risk	Equity Risk
QQQ	QQQ · PowerShares QQQ ETF	Equity	Equity Risk	
SHY	SHY · iShares 1-3 Year Treasury Bond	Bonds	Interest Rate Risk	
SPY	US Equities	Equity	Equity Risk	
STIP	TIPS	Inflation assets	Inflation risk	Interest Rate Risk
TFI	Municipal Bonds	Bonds	Interest Rate Risk	
VB	US Small Company Stocks	Equity	Equity Risk	
VCIT	US Investment Grade Corporate Bonds	Bonds	Interest Rate Risk	
VEA	International Developed Large Company Stocks	Equity	Equity Risk	Currency Risk
VGIT	US Treasuries	Bonds	Interest Rate Risk	
VMMXX	Cash	Cash	Interest Rate Risk	
VNQ	US Exchange-Traded REITs	Property	Real Estate Risk	
VNRSQ	Natural Resources	Commodites	Commodity Risk	
VOO	US Large Company Stocks	Equity	Equity Risk	
VSS	International Developed Small Company Stocks	Equity	Equity Risk	Currency Risk
VWOB	International Emerging Market Bonds	Bonds	Interest Rate Risk	Currency Risk
VYM	US Corporate High Yield Bonds	Bonds	Equity Risk	
XLU	XLU · Utilities Select Sector SPDR® ETF	Equity	Equity Risk	

Source: Gill, Sinha, Azim, Jorge Da Silva & Bernal, 2017

In light of calculate the efficient frontier based on historical weekly close prices of the 35 ETFs above, the investment research firm Seeking Alpha is used for providing the inputs.

Excel is the tool used to calculate the asset class correlations, annual returns and standard deviations for the time period between Jan 01, 2000 (or first available close price date) and March 23, 2017 for each platform and risk profile. In some cases, when historical prices are not available for indices, the prices from the secondary indices are used to complement the price history. In this way it is possible to obtain more market trends as the secondary indices follow the same trend as they have the same asset classes allocation.

All the return calculations are done at the weekly basis. The weekly close prices is considered the best for this exercise because it captures the accurate standard deviations for the period since the launch of the asset and at the same without the necessity to work of daily data. The returns of each ETF are aggregated at the portfolio level to get the weekly return of the portfolio. The return aggregation of the portfolio is done based on the portfolio weight given by the robo-advisory platform. When the platform recommendation allocated a percentage in cash, it is assumed that it is invested in the risk-free rate.

The log returns are computed and the final aggregated returns are annualized by the multiplication with 52, with exception of the WVOB which the data is only available by Seeking Alpha in a monthly basis, resulting in aggregated return multiplication by 12. Additionally, final aggregated returns is done for also horizon of 5 years. The portfolio volatility is computed using the standard deviation of the log returns.

Mean-variance optimization and the efficient portfolio is performed in Excel to find the solution for the general efficient frontier. Once the vectors for portfolio returns are calculated, the formula for standard deviations is applied. The volatility is annualized to get the final value. The covariances matrices is done and also annualized. Also, it is assumed zero correlation between the various market risks mentioned for each index. The calculation are conducted for 1 year volatility for each portfolio selection. The efficient frontier is performed for short selling not allowed.

The risk-free U.S. 5 Year Treasury 1.93% at March 31, 2019 is used to calculate the efficient frontiers including risk-free investment, for each type of risk aversion.

6. RESULTS

In this section, it is presented the results obtained from back-testing the robo-advisor portfolios finding the risk returns and constructing the efficient frontier.

The overall performance of the portfolio are presented in the following tables. As show in Table 3, Tolerisk's portfolio returns is the highest over the investment period. As in this study of risk and return, the outputs obtained from the platforms for the same set of investors' characteristics are very different, the results also differ.

The results shows that although the higher the risk the higher return is consistent, is not proportional to compensate higher risk the investors are willing to take. For it, is not possible to figure out with this work if these mismatches are impacting the wealth growth of investors' portfolios and if it is resulted in piling up implicit losses when opting by theses automatic strategies.

In relation to the first assumption of MVT, that investors only care about the mean and variance of future returns, the overall performance of the portfolio might not be adjusted to client's risk profile. As in the real world most investor worry about bad outcomes, or the left tail of return distributions, the overall performance seems to not compensate the risk impose for each risk profile.

The three risk profiles used for all robo-platforms diverge from asset allocations. The three different approach on a risk-return selection has some pitfalls. It is notable that for some investors, the wiliness to take more risk is not been compensate by higher expected mean returns. Also, the best returns are related to the platform Tolerisk, which the diversification of assets is the lowest, comparing with other platforms, with a portfolio of just 2 ETFs. Those higher returns are consistent with higher risks. The possibility of introducing a framework for securing some level of protection is something that needs to be thoroughly analysed as the same stylized investor may end up with severe differences after being profiled in several platforms.

Table 3: Assessment of on-line platforms - Tolerisk

Tolerisk			
	Conservative	**Moderate**	**Aggressive**
Mean return annualized	6.4%	11.8%	13.0%
Mean Stardard Deviation annualized	4.3%	11.2%	13.4%
Worst Return	-3.0%	-5.6%	-6.8%
5% historical value at Risk	-0.6%	-6.7%	-9.1%
10% historical value at risk	0.9%	-2.6%	-4.2%
Sharpe Ratio	1.05	0.88	0.82

Table 3.1: Assessment of on-line platforms – Charles Schwab

Charles Schwab			
	Conservative	Moderate	Aggressive
Mean return annualized	5.3%	5.8%	7.3%
Mean Stardard Deviation annualized	5.7%	6.6%	9.3%
Worst Return	-2.4%	-2.9%	-4.3%
5% historical value at Risk	-4.1%	-5.0%	-7.9%
10% historical value at risk	-2.0%	-2.6%	-4.6%
Sharpe Ratio	0.59	0.59	0.58

Table 3.2: Assessment of on-line platforms – SigFig

SigFig			
	Conservative	Moderate	Aggressive
Mean return annualized	3.8%	6.3%	8.0%
Mean Stardard Deviation annualized	3.6%	7.9%	11.8%
Worst Return	-2.5%	-3.9%	-6.0%
5% historical value at Risk	-2.2%	-6.8%	-11.5%
10% historical value at risk	-0.9%	-3.9%	-7.2%
Sharpe Ratio	0.51	0.55	0.51

Table 3.3: Assessment of on-line platforms – Riskalze

Riskalze			
	Conservative	Moderate	Aggressive
Mean return annualized	3.9%	6.0%	7.1%
Mean Stardard Deviation annualized	2.7%	6.1%	8.6%
Worst Return	-1.4%	-3.0%	-4.5%
5% historical value at Risk	-0.5%	-4.0%	-7.1%
10% historical value at risk	0.4%	-1.8%	-4.0%
Sharpe Ratio	0.73	0.67	0.60

Table 3.4: Assessment of on-line platforms – Wealthfront

Wealtlfront			
	Conservative	Moderate	Aggressive
Mean return annualized	2.0%	4.1%	5.0%
Mean Stardard Deviation annualized	7.9%	10.0%	11.7%
Worst Return	-5.8%	-6.9%	-8.0%
5% historical value at Risk	-11.1%	-12.3%	-14.3%
10% historical value at risk	-8.2%	-8.7%	-10.1%
Sharpe Ratio	0.01	0.22	0.26

Figure 1: Results: Conservative portfolios

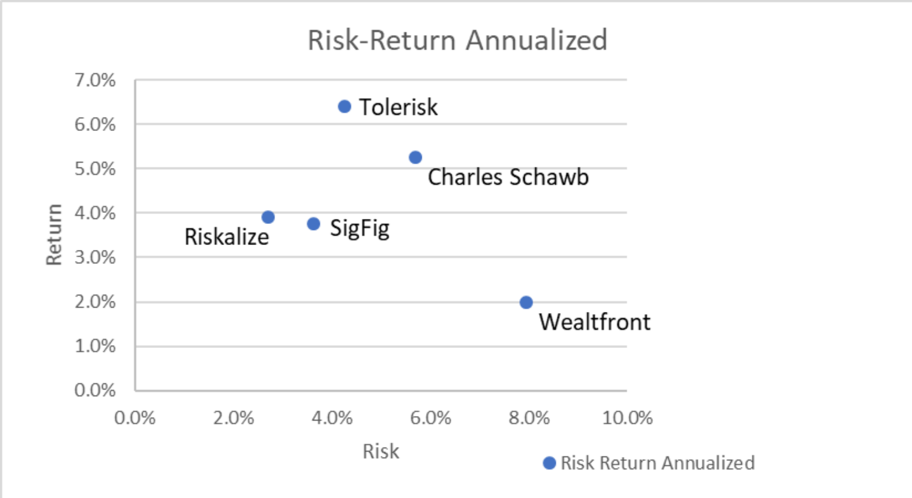


Figure 2: Results: Moderate portfolios

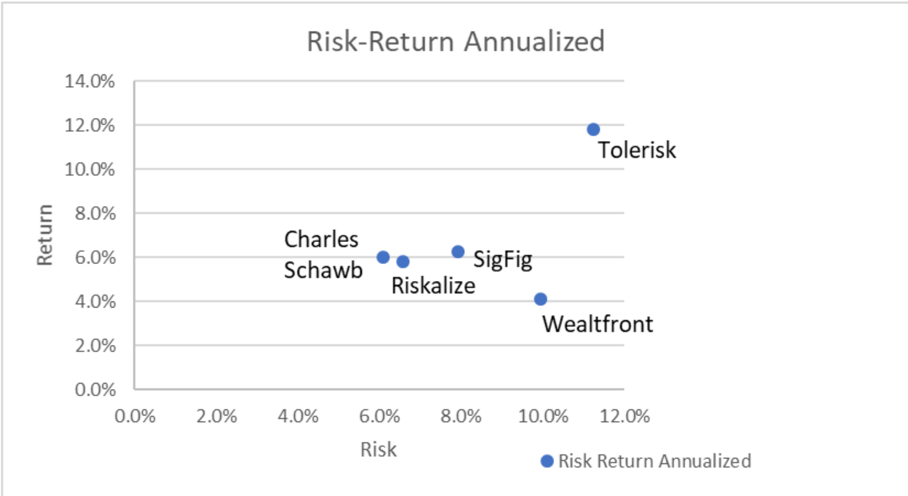
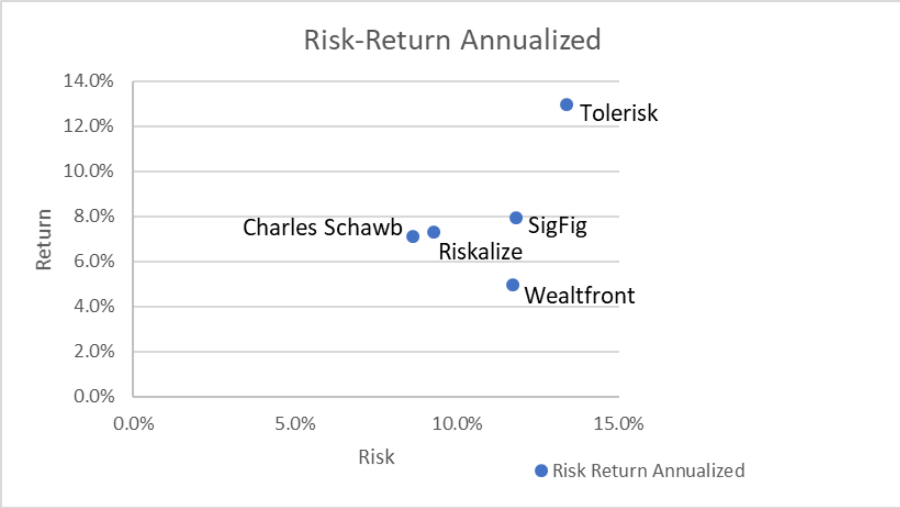


Figure 3: Results: Aggressive Portfolio



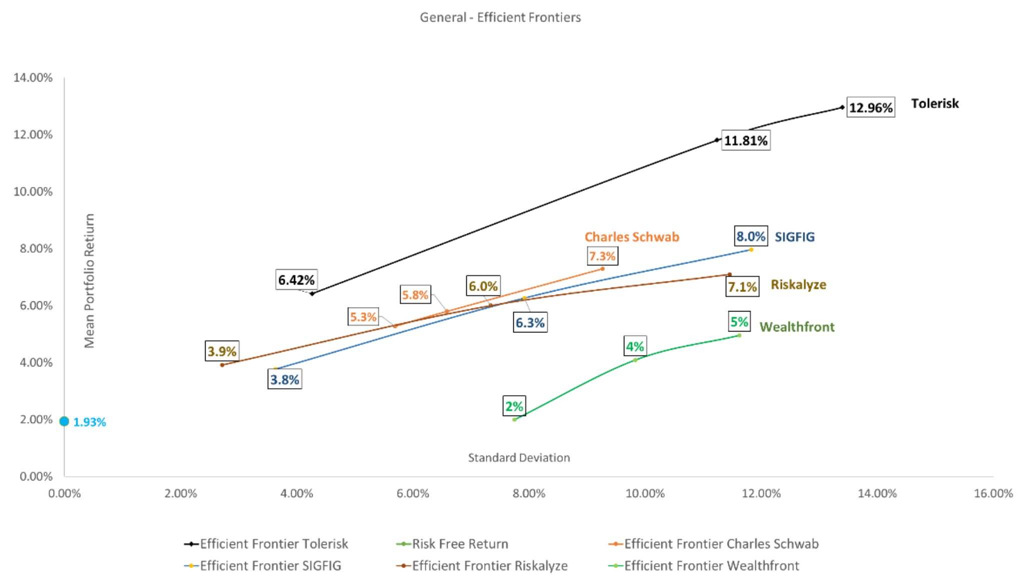
6.1 EFFICIENT FRONTIERS ASSESSMENT

The investment opportunity set is a hyperbola in standard deviation and mean return space. The efficient frontier is the upper part of the hyperbola.

Figures 4 shows the efficient frontiers, based on asset mean returns, volatilities and correlations for the time period between the first data the all the assets for each portfolio is available to March 2017. For each robo-advisor, the Figure 4 contains three data points representing the conservative, moderate and aggressive portfolios. As risk measured by volatility is increasing along thex-axis, the left most point represents the conservative portfolio and the right most represents the aggressive portfolio.

Comparing robo-advisor portfolios to one another, Figure 5 shows that Charles Schawb, SigFig and Riskalze have approximately the same level of return while the risk increase sparkly when the risk profile change. Also, it is observed that Tolerisk conservative portfolio has lower risk and higher return that most of the moderate and aggressive portfolio of the others robo-advisors, but when we check the portfolio allocation, it is represented by just 2 ETFs, which goes against the theory of diversification.

Figure 5: Results: Efficient Frontiers – General



Is well known in this work that just the assessment of the risk-returns of the Robo-advisors is not sufficient to measure the actual performance for the investments in conservative, moderate and aggressive robo-advisor portfolios. The actual performance should include substantial benefit of tax-loss harvesting outweighing costs and advisory fees which is not accounted in this project and do influence directly the management of the portfolios. Also, is difficult to measure how the volatilities of the portfolios are related to each investment since investors might have multiple goals and may have different investment horizons, where cross-sectional data may change the results.

6. CONCLUSION

Our estimations of the performances of notable robo-advisors – Tolerisk, Schwab Intelligent Portfolios, SigFig, Riskalze and Wealthfront– show that Tolerisk has the higher return of the portfolio for the lowest level of risk, going against the theory of diversification. It also find that the robo-advisor model is seemingly benefiting conservative investors the most.

Robo-advisors base their recommendations on the estimated frontier, which is always placed below the true frontier. Consequently, one cannot expect an investment manager basing asset allocations on mean-variance optimization to obtain an efficient portfolio.

From this work, evidence have shown that the analysis MPT methodology for the management of passive investment models, applied on online platforms suffers from flawed assumptions and model misspecification slowing down the potential of quantitative models used in robo-advisors. As the example of risk and return in this study, the allocation obtained from the platforms for the same set of investors' characteristics bring very different results.

This problem may arise in the beginning of the work robo-advisors standardize the investor risk-profiling process starting with definition and discussion of the investor’s situation and the goals that are to be achieved by the portfolio. The problem at this point may arise since investors might have multiple goals, and they are not necessarily able to quantify or set an investment time objective (CFA 2015-2016). Said so, the level of complexity necessary for defining frameworks should be developed.

The current standard process of risk profiling through questionnaires is found to be highly unreliable. The cause is primarily the design of the questionnaires, which focus on socioeconomic variables and hypothetical scenarios to elicit the investor’s behaviour (CFA 2015-2016). Risk profiling is still a very grey area when financial literacy is not robust enough for investors to comprehend the pitfalls of a wider class of protective strategies,

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APPENDICES

TABLE A – ROBO-ADVISORS ASSET ALLOCATION

Wealthfront Asset Allocation				
Index	**Index Type**	**Conservative**	**Moderate**	**Aggressive**
SPY	US Equities	27%	35%	35%
VEA	International Equities	12%	18%	25%
IEMG	Emerging market Equities	6%	15%	19%
VYM	Dividend Stocks	8%	6%	10%
VNRSQ	Natural Resources	6%	5%	5%
STIP	TIPS	6%	0%	0%
TFI	Municipal Bonds	35%	21%	6%

Schwab Asset Allocation				
Index	**Index Type**	**Conservative**	**Moderate**	**Aggressive**
PRF	US Large Company Stocks - Fundamental	7%	8%	11%
PXF	International Developed Large Company Stocks - Fundamental	5%	5%	8%
VOO	US Large Company Stocks	4%	5%	9%
PRFZ	US Small Company Stocks - Fundamental	4%	5%	8%
VEA	International Developed Large Company Stocks	3%	4%	5%
VNQ	US Exchange-Traded REITs	5%	5%	5%
VB	US Small Company Stocks	2%	3%	4%
PDN	International Developed Small Company Stocks - Fundamental	2%	3%	4%
IEMG	International Emerging Market Stocks	2%	5%	3%
PXH	International Emerging Market Stocks - Fundamental	2%	0%	5%
VSS	International Developed Small Company Stocks	1%	2%	3%
MBG	US Securitized Bonds	11%	9%	3%
VGIT	US Treasuries	8%	6%	0%
VYM	US Corporate High Yield Bonds	8%	8%	8%
VCIT	US Investment Grade Corporate Bonds	6%	6%	1%
STIP	TIPS	5%	1%	0%
IGOV	International Developed Country Bonds	5%	5%	3%
VWOB	International Emerging Market Bonds	4%	4%	7%
DGL	Gold and Other Precious metals	2%	4%	5%
VMMXX	Cash	14%	13%	9%

SigFig Asset Allocation				
Index	**Index Type**	**Conservative**	**Moderate**	**Aggressive**
SPY	US Equities	13%	35%	41%
VEA	International Equities	5%	13%	25%
IEMG	Emerging market Equities	4%	12%	24%
TFI	Municipal Bonds	14%	0%	0%
VCIT	US Investment Grade Bonds	22%	0%	0%
STIP	TIPS	22%	30%	3%
VGIT	Short Term Treasury	20%	0%	0%
VWOB	Emerging market Bonds	0%	10%	7%

Tolerisk Asset Allocation				
Index	**Index Type**	**Conservative**	**Moderate**	**Aggressive**
SPY	Equity	10%	80%	95%
VCIT	Bonds	90%	20%	5%

Riskalze Asset Allocation				
Index	**Index Type**	**Conservative**	**Moderate**	**Aggressive**
BND	BND · Vanguard Total Bond Market ETF	35%	25%	0%
SHY	SHY · iShares 1-3 Year Treasury Bond	30%	1%	0%
SPY	SPY · SPDR® S&P 500 ETF	13%	13%	26%
EFA	EFA · iShares MSCI EAFE	5%	15%	20%
HYG	HYG · iShares iBoxx $ High Yield Corporate Bd	5%	7%	0%
FLOT	FLOT · iShares Floating Rate Bond	5%	0%	0%
VMMXX	Cash	5%	0%	0%
VNQ	VNQ · Vanguard REIT ETF	2%	10%	12%
QQQ	QQQ · PowerShares QQQ ETF	0%	5%	17%
DBC	DBC · PowerShares DB Commodity Tracking ETF	0%	5%	7%
DBL	DBL · Doubleline Opportunistic Credit Fund	0%	7%	0%
EFR	EFR · Eaton Vance Senior Floating-Rate Fund	0%	7%	0%
XLU	XLU · Utilities Select Sector SPDR® ETF	0%	5%	0%
FXI	FXI · iShares China Large-Cap	0%	0%	5%
FPX	FPX · First Trust US IPO ETF	0%	0%	6%
EEM	EEM · iShares MSCI Emerging Markets	0%	0%	7%

Source: Gill, Sinha, Azim, Jorge Da Silva & Berna

FIGURE A1: RESULTS: EFFICIENT FRONTIER - TOLERISK

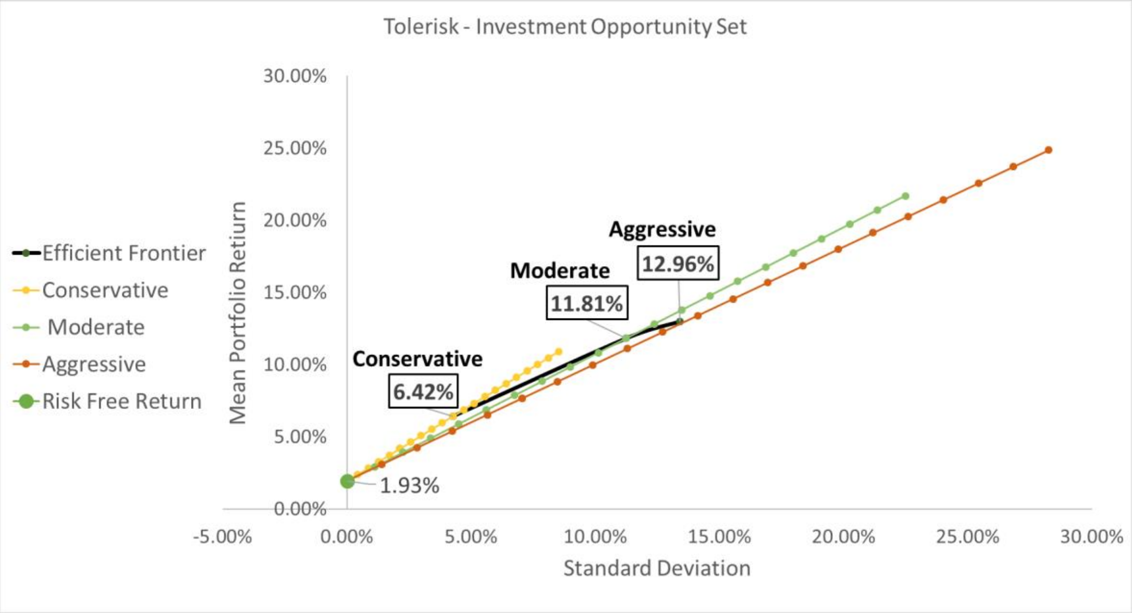

FIGURE A2: RESULTS: EFFICIENT FRONTIER – CHARLES SCHWAB

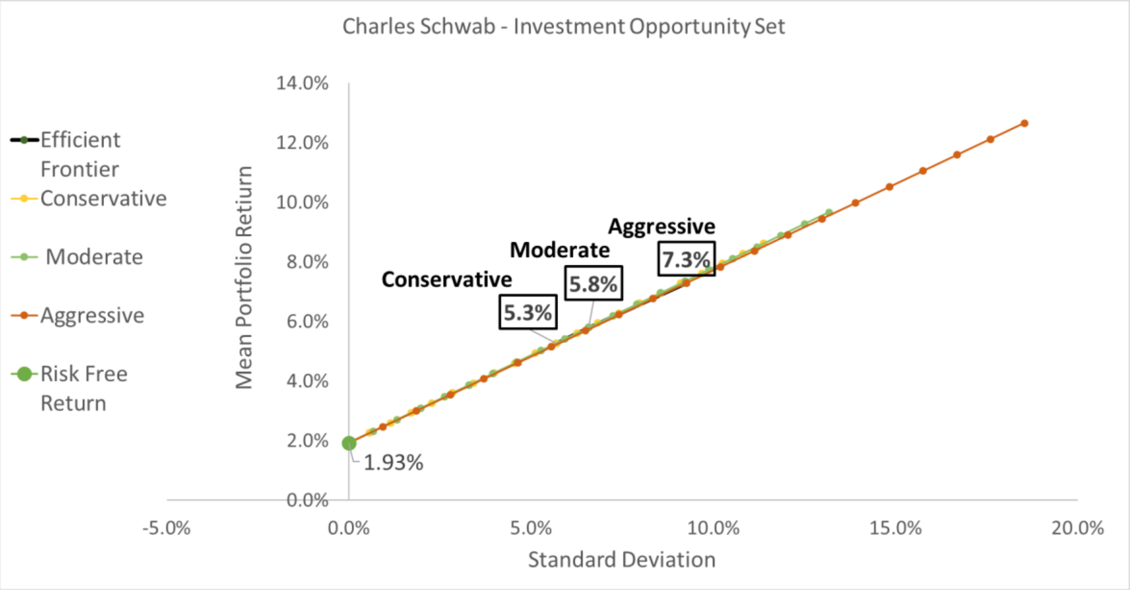

FIGURE A3: RESULTS: EFFICIENT FRONTIER – SIGFIG

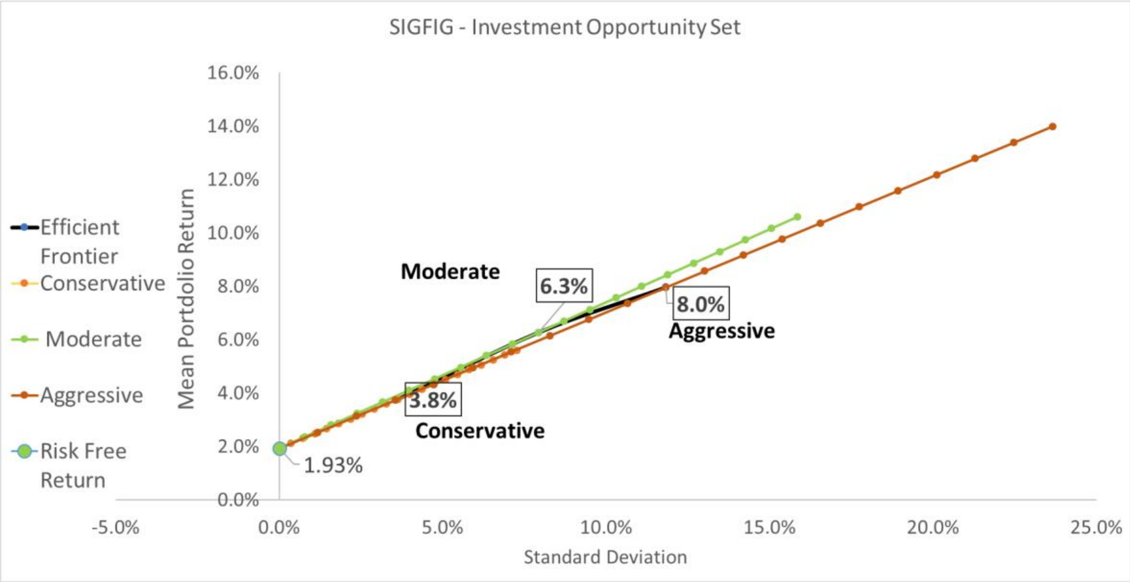

FIGURE A4: RESULTS: EFFICIENT FRONTIER – RISKALZE

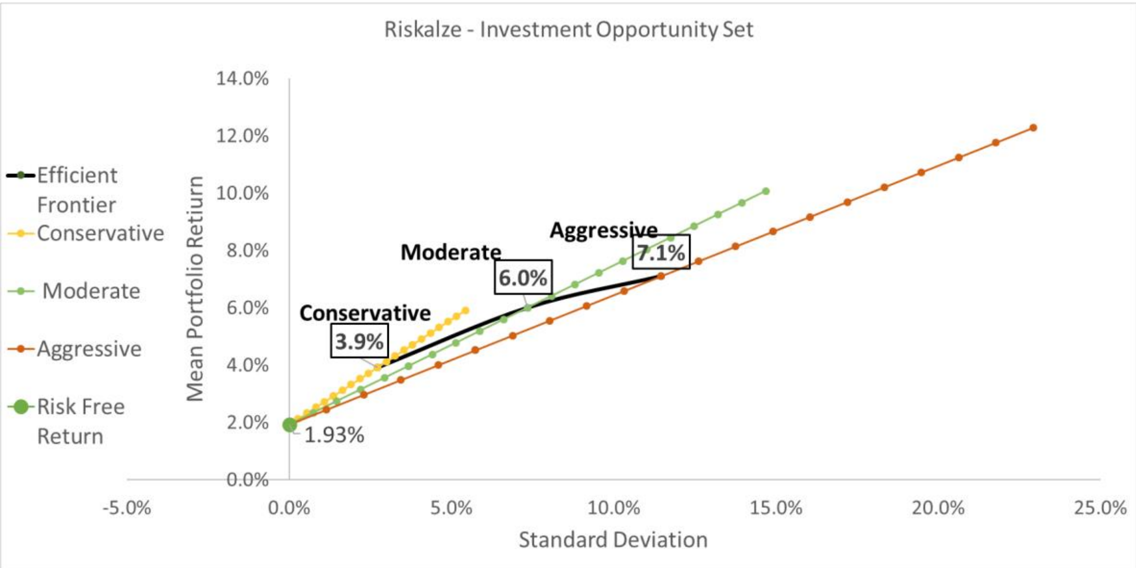

FIGURE A.5: RESULTS: EFFICIENT FRONTIER – WEALTHFRONT