IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received November 25, 2019, accepted January 8, 2020, date of publication January 28, 2020, date of current version February 10, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2970083

GNSS-Based Attitude Determination Techniques–A Comprehensive Literature Survey

ALMAT RASKALIYEV[1], **SAROSH HOSI PATEL**[1], **TAREK M. SOBH**[1], **AND AIDOS IBRAYEV**[2]
[1]Robotics, Intelligent Sensing and Control (RISC) Laboratory, School of Engineering, University of Bridgeport, Bridgeport, CT 06604, USA
[2]Department of Mechanical Engineering, Faculty of Mathematical Engineering, Al-Farabi Kazakh National University, Almaty 050061, Kazakhstan

Corresponding author: Aidos Ibrayev (ibraev_aidos@mail.ru)

This work was supported by the University of Bridgeport.

ABSTRACT GNSS-based Attitude Determination (AD) of a mobile object using the readings of the Global Navigation Satellite Systems (GNSS) is an active area of research. Numerous attitude determination methods have been developed lately by making use of various sensors. However, the last two decades have witnessed an accelerated growth in research related to GNSS-based navigational equipment as a reliable and competitive device for determining the attitude of any outdoor moving object using data demodulated from GNSS signals. Because of constantly increasing number of GNSS-based AD methods, algorithms, and techniques, introduced in scientific papers worldwide, the problem of choosing an appropriate approach, that is optimal for the given application, operational environment, and limited financial funding becomes quite a challenging task. The work presents an extensive literature survey of the methods mentioned above which are classified in many different categories. The main aim of this survey is to help researchers and developers in the field of GNSS applications to understand pros and cons of the current state of the art methods and their computational efficiency, the scope of use and accuracy of the angular determination.

INDEX TERMS Attitude determination, GNSS, angular resolution, ambiguity resolution.

I. INTRODUCTION

The attitude of any object is its spatial orientation with respect to the object's mass center. This parameter is usually represented by Euler angles, Rodriguez parameters, and quaternion or direction cosine matrix. Attitude Determination is an operation of attitude computation of the object relative to some inertial reference frame or Earth. AD is usually provided by sensors installed on the object and mathematical computations made on the microcontroller. Algorithms and techniques applied and computational power of the processing unit usually define the accuracy constraints. Attitude determination subsystems are widely implemented in satellites, vehicles, boats, aircrafts and other mobile objects. Inertial Measurement Units (IMU), GNSS receivers, magnetometers, digital compasses, gyroscopes, and accelerometers might be used as sensors in AD subsystems. Then, point to point or recursive attitude determination algorithms process readings from the sensors in order to estimate the attitude by means of kinematic and dynamic models.

Recent studies have proved that the GNSS can take an important place in numerous applications, including attitude determination, because of its stable operation, cost effectiveness and low power consumption. Computational methods and algorithms developed for solving problems in the area of GNSS-based AD are described in a great amount of scientific reports and journals that is why strong demand for providing well-structured reviews of these methods arises.

In [1], the authors provide a short overview of GNSS-based methods and models for AD of a spacecraft by means of phase measurements. The authors of [2] compare distinct methods, constrained Least-squares Ambiguity Decorrelation Adjustment (LAMBDA) and multivariate constrained LAMBDA, which represent two different approaches in the area of GNSS-based AD. The researchers in [3] make a comparison between methods based on single and double differenced carrier phase measurements. Baroni and Kuga in [4] made theoretical and experimental analysis that compared Least-Squares Ambiguity Search Technique (LSAST) and LAMBDA algorithms using quaternion formulation. The same researchers in [5] made a comparison between LSAST and LAMBDA methods providing their findings in analysis of the computational process of ambiguity resolution.

The associate editor coordinating the review of this manuscript and approving it for publication was Masood Ur-Rehman.

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see http://creativecommons.org/licenses/by/4.0/

However, the works mentioned above do not encompass all contemporary methods of GNSS-based AD in a single entity written in some structured way.

With the background presented in Section 1, the second section provides an overview and classification of current mathematical algorithms and techniques utilized in the process of GNSS-based AD. It outlines roughly three algorithmic steps of attitude determination and ambiguity resolution during computational operations, as well as baseline and attitude approaches in constructing GNSS-based AD methods. The third section elaborates on the interrelation between the sensors used for AD and corresponding computational techniques. The forth section discusses optimal ways of processing GNSS measurements depending on the mathematical model chosen for solving an attitude determination problem. This section also explains positioning and geometry free GNSS models that are used while employing various types of GNSS readings. In the fifth section, we describe the methods applied based on various dynamic and kinematic models of the moving object given in the problem. Finally, we make some conclusions and illustrate a representative list of research articles to emphasize the performance of GNSS-based AD methods.

II. GNSS-BASED ATTITUDE DETERMINATION METHODS

Current GNSS-based methods employed in AD algorithms can be generally divided into three operational groups, which are aimed for line bias/baseline computation, integer ambiguity resolution (IAR) and estimation of attitude angles [6]. ‘‘Line biases commonly occur due to the differences in cable lengths between antennas and the receiver’’ [7] or differences in radio frequency (RF) front ends in the receiver [8] or a combination of both. These parameters are generally given as constant variables and calibrated using some technique before launching a custom AD algorithm in GNSS attitude determination receivers, for example, Trimble’s TANS (Trimble Advanced Navigation Sensor) Vector [6] and Space Systems/Loral’s GPS Tensor. Another method is that the line biases are treated as components of the state vector of the system, and therefore, estimated along with other state components (for example [8]).

A GNSS receiver is able to measure only the fractional part of the carrier phase. The integer number of wavelengths between an antenna and a satellite cannot be measured directly. This is a well-known problem of integer ambiguity. Two main approaches have evolved to resolve the problem of integer ambiguity in the area of GNSS-based attitude determination. The approaches are either motion-based (for instance, [7]) or search-based (for example, ([9]). Motion-based methods require acquiring measurements for a period of time during which considerable changes of a visible navigational satellites constellation or the antenna platform motion have happened. The search-based algorithms utilize only single epoch readings to retrieve the most likely solution that is why they are sometimes subject to incorrect solutions because of measurement noise. Two search-based techniques have been developed. In the first technique, the search is conducted in a real number domain. The search domain is composed of all potential grid points in the coordinate system of solution parameters. These parameters might be the actual length and azimuth angle of a baseline (for instance, [9]) or Euler angles of the antenna platform [10]. In the second search-based technique, the search environment is chosen in the integer number domain. ‘‘The search domain is composed of all potential combinations of integer ambiguity candidates’’ [9].

In scientific publications, various integer ambiguity resolution methods have been elaborated [1], like Least-Squares Ambiguity Search Technique (LSAST) [11], Fast Ambiguity Resolution Approach (FARA) [12], Modified Cholesky decomposition [13], most widely applied Least-squares Ambiguity Decorrelation Adjustment (LAMBDA) [14], Null method [15], Fast Ambiguity Search Filter (FASF) [16], Three Carrier Ambiguity Resolution (TCAR) [17], Integrated TCAR [18], Optimal Method for Estimating GPS Ambiguities (OMEGA) [19] and Cascade Integer Resolution (CIR) [20]. A comparison of LAMBDA with CIR, TCAR, ITCAR and the Null-method is provided in [21] and [22].

Nowadays the LAMBDA method is a common method for solving GNSS integer ambiguity resolution problems with unconstrained baselines. For nonlinearly constrained ambiguity resolution problems, the single baseline constrained LAMBDA method [23] was introduced and the newly suggested The Multivariate Constrained LAMBDA method ([24] and [25]) determines the integer ambiguities and Euler’s angles in an integral manner.

The third operational group of AD algorithms, implemented in the procedure of attitude angles estimation, is composed of three consecutive steps [26]. Firstly, a typical least-squares adjustment is employed in order to achieve the so-called float solution. All unknown parameters are evaluated to be real-valued. In the second step, the integer constraint on the ambiguities is taken into account. This implies that the float ambiguities are mapped to integer parameters. Various options of the mapping function are viable. The float ambiguities can merely be rounded to the closest integer values or rounded to some extent so that the correlation between the ambiguities reaches its minimum. Application of the integer least-squares estimator becomes optimal, that increases the probability of valid integer estimation. In the third step, after fitting the ambiguities to their integer counterparts, the remaining unknown parameters are resolved based on their correlation with the fixed ambiguities [27].

Among IAR methods, the integer estimators widely utilized in GNSS applications [28] are Integer least squares (ILS) [29] Integer Bootstrapping (IB) [30], and Integer Rounding (IR) [31]. They present various options of mapping to integer parameters.

From the point of view of measurement processing, the techniques for attitude angles estimation can be approximately divided into the following two types: (a) point estimation techniques (for example [7]) and (b) stochastic filtering techniques (for instance [32]). There are two categories of point estimation techniques. The first category of point estimation technique employs vectorized measurements [33] and can be regarded as a two-level optimal estimation problem, the least squares problem and Wahba's problem [34]. The second category of point estimation techniques is related to the differenced carrier phase measurements directly. It either utilizes a non-linear, least-square fit (NLLSFit) method [35] or transforms the problem interchangeably into Wahba's problem [7].

III. SENSORS USED IN GNSS-BASED AD

To solve the problem of attitude determination of the moving object, one can use only GPS and other satellite navigation systems (standalone GNSS AD), make integration of GNSS receivers with INS (for example accelerometer and gyroscope), as well as integrate with other navigational sensors (for instance magnetic antenna, digital compass and magnetometer).

Standalone GNSS AD may require integration of GPS receivers with other satellite navigation sensors such as GLONASS, Russian Global Navigation Satellite System, Galileo, Europe's own global navigation satellite system, and Compass (Chinese second-generation satellite navigation system also known as Beidou-2). GNSS-based AD may be categorized as dedicated or non-dedicated. In the dedicated AD system, a single exclusively focused GNSS receiver is used while in the non-dedicated AD system, a set of independent, general-purpose GNSS receivers is used for the attitude determination of the object. Many companies that manufacture GNSS receivers like Trimble [36], [37]; Texas Instruments [38], [39]; Ashtech [40]–[42]; Adroit Systems [43], and others have been designing dedicated GNSS receivers with multiple antennas for attitude determination.

If we consider measurement types used for standalone GNSS-based AD, then they are divided as L1 frequency and L1/L2 frequency GNSS receivers. Double frequency receivers have much higher cost, but they lead to improvement of ambiguity resolution because of tackling the dispersive ionosphere delays.

The number of GNSS antennas, used for AD, might also vary. There are computational methods that use only one GNSS antenna (for instance [44], [45]), two GNSS antennas [46]–[48] that lead to a single computational baseline and three or more GNSS antennas. Employing one or two antennas usually provides an opportunity to calculate with good precision only two of three attitude angles.

GNSS receivers can also be divided as sensors able to register code phase (code) and carrier phase measurements or only code measurements. Furthermore, the majority of GNSS receivers provide code and carrier phase measurements. Because of the nature of these readings, AD based on carrier phase measurements can result in much more accuracy in comparison with code (pseudo range) measurements. That is why a centimeter level GNSS-based AD requires GNSS receivers that can measure carrier phase with a good precision. However, code measurements are often utilized in code-phase smoothing and cycle slip detection and repair algorithms, while utilization of carrier phase measurements requires implementation of IAR algorithm.

IV. GNSS-BASED ATTITUDE DETERMINATION MODELS

An overview of GNSS models and their applications in different areas [1] are presented in textbooks such as [49]–[54]) GNSS models have two main types: non-positioning or geometry-free models and the positioning or geometry-based models. Furthermore, various GNSS models might be distinguished by virtue of the differencing utilized. By differencing, we mean to take the differences between measurements from different receivers and/or different satellites. It is usually used in order to get rid of several error types from the observation equations [23]. The single difference and double-difference methods are discussed in [3]. Unconstrained baselines are baselines for which preliminary information about the length is not known and constrained baselines are baselines for which the length is a-priori known and constant [55].

In general, all GNSS baseline models can be put in the following formula of linearized observation equations according to a Gauss-Markov model [56]:

$$E(y) = Aa + Bb; \quad D(y) = Q_{yy} \tag{1}$$

where y is the known vector of GNSS observables, a and b are the unknown parameter vectors of integer ambiguities and real-valued baselines correspondingly. $E(:)$ and $D(:)$ denote the expectation and dispersion operators, and A and B are the given design matrices which bound the data vector to the unknown parameters. Matrix A includes the carrier wavelengths and the geometry matrix B includes the receiver-satellite unit line-of-sight vectors. The variance matrix of y is set to the positive definite matrix Q_{yy}.

The model defined by (1) is referred as the unconstrained model and its Integer Least Squares (ILS) solution is found in [56]. For GNSS-based attitude determination applications, one often may take advantage of the knowledge of the additional constraint on the baseline vector length, so that the Integer Least-Squares minimization problem can be reorganized as a Quadratically Constrained Integer Least-Squares (QC-ILS) problem [56].

After applying double differencing, the model (1) can be converted into a single-epoch, multi-frequency GNSS array model in a multivariate form as:

$$E(Y) = MX + NZ, \quad X \in R^{3\times r}, \quad Z \in Z^{fs\times r}, \tag{2}$$

with r number of baselines, f number of GNSS frequencies, $s+1$ number of GNSS satellites tracked, Y the $2fs \times r$ data matrix of double-differenced observables, (M, N) the $2fs \times (3 + fs)$ design matrix, $X \in R^{3\times r}$ the unknown real-valued baseline matrix in the reference frame and $Z \in Z^{fs\times r}$ the unknown integer ambiguity matrix.

Depending from the types of constraints employed, the model (1) can be developed into a general GNSS attitude model, an affine model, or a quadratically-constrained model. The general GNSS attitude model contains the orthonormality constraint of the rotation matrix between body frame and the reference frame [25], [57] and it is formed by the (2) and (3):

$$D(vec(Y)) = P \otimes Q \tag{3}$$

where

$$P = D_r^T Q_r D_r, \quad Q = blockdiag(Q_\Phi, Q_P) \tag{4}$$

$$Q_\Phi = Q_f \otimes D_s^T Q_\Phi D_S, \quad Q_P = Q_f \otimes D_S^T Q_P D_S \tag{5}$$

The matrices Q_r, Q_f, Q_Φ and Q_P are co-factor matrices and the matrices D_r^T and D_s^T are differencing matrices.

For some special cases when the number of baselines equals the dimension of the space spanned by those baselines, the GNSS attitude model reduces to the GNSS quadratically-constrained model [25], [24]:

$$E(Y) = MX + NZ, \quad X^T X = C, \quad D(vec(Y)) = P \otimes Q, \tag{6}$$

with $Y \in R^{2fs \times r}, X \in R^{3 \times r}$, and $Z \in Z^{fs \times r}$. This model formulation is equivalent to that of the general GNSS attitude model for $rank(X) = r \leq 3$, that is, when the baseline matrix in a body frame is invertible.

When the quadratic constraints are neglected, and only linear constraints are taken into account, the GNSS attitude model takes the linear form of the affine constrained GNSS attitude model [58]:

$$E(Y) = MX + NZ, \quad XS = 0, \quad D(vec(Y)) = P \otimes Q, \tag{7}$$

With $Y \in R^{2fs \times r}, M \in R^{2fs \times 3}, N \in R^{2fs \times fs}, S \in R^{r \times (r-q)}, P \in R^{r \times r}, Q \in R^{2fs \times 2fs}$ and the unknown parameter matrices $X \in R^{3 \times r}$ and $Z \in Z^{fs \times r}$. This model is referred to as the affine constrained GNSS attitude model, since the linear matrix constraint $XS = 0$ implies the formation of an affine transformation between reference- and body frames.

Models employed for integer ambiguity resolution and attitude angles estimation are chosen depending on the application requirements which are related to the expected accuracy of output angular estimates, success rate and the initialization time of the accurate estimation as well as a predicted dynamic model of the object's motion. IAR is referred to be an initialization procedure because integer ambiguities are constant over time (while currently visible GNSS satellites are tracked continuously) and they do not require being resolved again for a short time span once they have been fixed. "The IAR and attitude angles estimation are therefore often considered as two stand-alone processes, so that they have been discussed separately in the research papers" [7], [9], [33], [35]. The line bias computation is produced together with the single-differenced IAR problem when the line biases correspond to the single-differenced measurements. This means that it is suggested to conduct data pre-processing for the line bias computation when IAR is produced in the single difference domain (for example [6]). The choice of employing models of undifferenced, single-differenced and double-differenced domains is often related with an aim of acquiring minimal correlation among output linearized observation equations.

V. APPLICATION ENVIRONMENTS

The range of GNSS-based AD methods and techniques vary depending on the environment of proposed application due to the opportunity of using dynamic and kinematic models simulating the motion of the given object. These methods can be roughly categorized as suitable for a satellite, a rocket, an aircraft / UAV (unmanned aerial vehicle) and an object moving on the surface of Earth.

In literature, various GNSS-based AD satellite missions have been discussed namely GADACS (the GPS Attitude Determination and Control System) [59], SPARTAN (Station Power, Articulation, Thermal, and Analysis) [60], RadCal (Radar Calibration) [61], REX (Radiation Experiment) II [62], Gravity Probe B [63], UniSat (University Satellite) [64], Gyrostat [65], UoSat-12 (University of Surrey) [66] and AlSat-1 (Algeria Satellite) [67]. The equipment installed on satellites usually has tight constraints on its mass, volume and energy consumption. Therefore, the AD subsystem of the satellite is often composed of a dedicated GNSS receiver with one or two antennas.

An attitude determination problem of the spinning rocket was solved in [68], [69]. To provide reliable and continuous angular estimates, the researchers have to make use of on-the-fly cycle slip detection and repair technique as well as a specific dynamic model that takes into account fast angular spinning rotation of the rocket.

A solution of GNSS-based AD problem for an aircraft and UAV was provided in a number of research articles [44], [70]–[74]. It is featured by integration with INS unit for more reliable and undisturbed angular estimation. Kalman filter is usually implemented in tightly or loosely integrated GNSS and INS based AD systems. In the case of tight integration, INS measurements are utilized for assisting in integer ambiguity resolution, as well as in an inertial integrated cycle slip detection algorithm.

On-the-fly GNSS-based attitude determination for land and sea vehicles was considered in [75]–[78]. That kind of applications generally do not require high precision in angular estimates, but they are more demanding for the time of initialization and computation, as well as the cost of hardware and equipment.

VI. CONCLUSION

It was discussed above that the problem of GNSS-based attitude determination is solved through implementing a wide range of methods within three consecutive steps that can be repeated during the computational process in order to obtain more accurate AD results. The same methods or their variations might be also employed for integration of

TABLE 1. Comparison of GNSS-based attitude determination methods.

Computational methods and techniques	Name of the article	Hardware and infrastructure	Accuracy	Best environment	Average baseline
Carrier phase double difference technique; integer ambiguity searches through applying Kalman filter algorithm, Gaussian transformation, and Cholesky factorization	"Ambiguity Resolution in GPS-based, Low-cost Attitude Determination" [47]	Two low-cost Allystar GPS OEM boards, two single frequency antennas	0.045 deg (RMS) for heading angle	Kinematic attitude determination	3 meters
Carrier phase double difference technique; integer ambiguity resolution by mean field annealing neural network (MFANN) algorithm, which is a combination of Hopfield neural network and the stochastic simulated annealing technique	"Application of Mean Field Annealing Algorithms to GPS-based Attitude Determination" [48]	Two GG-24, two single frequency antennas	Not described in the article	Real-time attitude determination is possible if parallel computing technology is used	1 meter
A BPNN algorithm is used in the adaptive Kalman filtering for GPS/INS integration with attitude determination	"Neural Network-based GPS/INS Integrated System for Spacecraft Attitude Determination" [79]	Simulation software of GPS/INS integrated the system	Not described in the article	Real-time spacecraft attitude determination	not described in the article
Carrier phase doubles difference technique; constrained LAMBDA method; the nonlinear least-square fit (NILSFit) method	"Comparison of attitude determination approaches using multiple Global Positioning System (GPS) antennas" [2]	Four single frequency antennas simulated by means of VISUAL software	Standard deviation (degree): Yaw (0.067), Roll (0.228), Pitch (0.474)	Real-time attitude determination with noise levels for code PR - 15 cm and phase PR - 3 mm	about 1 meter
A multivariate Constrained LAMBDA method (MC-LAMBDA) which solves for the GPS integer ambiguities and the attitude matrix in an integral manner	"Comparison of attitude determination approaches using multiple Global Positioning System (GPS) antennas" [2]	Four single frequency antennas simulated by means of VISUAL software	Standard deviation (degree): Yaw (0.034), Roll (0.059), Pitch (0.045)	Real-time attitude determination with noise levels for code PR - 15 cm and phase PR - 3 mm	about 1 meter
Double difference carrier phase technique; integer ambiguity search by means of modified Least-SquaresAmbiguity Search Technique (LSAST); least squares adjustment method	"Development of a Low-cost GPS-based Attitude Determination System" [80]	Two CMC Allstar single frequency GPS receivers with dual antenna	RMS of Heading = 3.2 arc minutes, RMS of Pitch =7.6 arc minutes	Examined by Spirent STR-4760 (Global Simulation Systems 2000) hardware simulator	1 meter

TABLE 1. ***(Continued.)*** **Comparison of GNSS-based attitude determination methods.**

Double difference carrier phase technique; integer ambiguity search by means of modified Least-Squares Ambiguity Search Technique (LSAST); least squares adjustment method; the centralized Kalman filter for gyro measurement integration	"Development of a Low-cost GPS-based Attitude Determination System" [80]	Three Murata ENV-05D-52 gyroscopes, Three CMC Allstar single frequency GPS receivers, and corresponding AT575-70 antennas	STD of Heading = 0.17 degrees, STD of Pitch = 0.43 degrees, STD of Roll = 1.32 degrees	Examined by Spirent STR-4760 (Global Simulation Systems 2000) hardware simulator	0.85 meter
Double difference carrier phase technique; Kalman filter for loosely coupled GPS receiver and Gyro integration	"Autonomous fault detection on a low-cost GPS-aided attitude determination system" [81]	Three-axis silicon Vibrating Ring Gyros and single frequency GPS receivers	Not described in the article	Autonomous fault detection in flight conditions	1 meter
Constrained unscented Kalman Filter, quaternion method, and complementary filter	"Unscented Kalman Filtering for Attitude Determination Using MEMS Sensors" [82]	GPS receiver, MEMS three-axis accelerometer, MEMS three-axis gyroscope, electronic compass	Quite low	Flight kinematic conditions	Not found
Antenna design of low-multipath radiation pattern	"Shorted annular patch antennas for multipath rejection in GPS-based attitude determination systems"[46]	Two double-frequency GPS receivers with a shorted annular patch antenna	Standard deviation (deg) = 10.0; RMS (deg) = 13.1	Medium-accuracy attitude sensor for low-earth-orbit missions	Not described in the article
SNR weighed model approach	"Single Antenna Attitude Determination for FedSat" [45]	One Blackjack double frequency GPS receiver	Uncertainty ranges from +- 10 degree to +- 30 degree	When used in a wide-open area with as many satellites in view as possible	Not found
Improved carrier phase measurement model with open-circuit antenna voltage	"Attitude Determination from Single-Antenna Carrier-Phase Measurements" [44]	One single-frequency GPS receiver	Not described in the article	Preferably in micro air vehicles (MAV), nanosatellites and in general aviation aircraft	Not found
Carrier phase doubles difference technique; direct attitude determination method based on the non-linear least-square solution	"A Direct Attitude Determination Approach Based on GPS Double-Difference Carrier Phase Measurements" [83]	Three single frequency GPS receivers	Precision estimation is 0.1 degree	Real-time attitude determination in static mode	3.3 meter
Carrier phase double difference technique; attitude parameterization using quaternions; least-squares ambiguity decorrelation adjustment (LAMBDA) method for integer	"Analysis of Attitude Determination Methods Using GPS Carrier Phase Measurements" [4]	3 AllStar CMC GPS receivers of Canadian Marconi Space Company and	The accuracy of the method is 0.15 degree	Real-time attitude determination in static mode	1 meter

TABLE 1. ***(Continued.)*** **Comparison of GNSS-based attitude determination methods.**

ambiguity resolution		3 AllStar CMC model AT 575-70 GPS antennas			
Carrier phase double difference technique; attitude parameterization using quaternions; Least-Squares Ambiguity Solution Technique (LSAST) method for integer ambiguity resolution	"Analysis of Attitude Determination Methods Using GPS Carrier Phase Measurements" [4]	3 AllStar CMC GPS receivers Canadian of Marconi Space Company and 3 AllStar CMC model AT 575-70 GPS antennas	The accuracy of the method is 0.15 degree	Real-time attitude determination in static mode	1 meter
Attitude parameterization using quaternions; Lagrange multiplier techniques; Constrained filtering method; Newton-Raphson method	"Constrained filtering method for attitude determination using GPS and gyro" [84]	A simulation model of three gyros of Litton (SIRU) and three GPS L1 carrier phase receivers is used	Steady-state errors of Monte-Carlo simulation are roll = 0.3733 degree, pitch = 0.3609 degree, yaw = 0.0727 degree	If it is desired to use the proposed method in real time, one may need to employ a powerful signal processor	1.4 meter
The geometric technique of carrier phase first differences; extended Kalman filter method for integer ambiguity resolution	"Investigation of Models and Estimation Techniques for GPS Attitude Determination" [85]	Simulation of three double frequency GPS receivers	0.2 degrees and 0.5 degrees accuracy depending on the measurement noise and how accurately the noise is compensated for in the filter	Spacecraft attitude determination	1 meter
The geometric technique of carrier phase first differences; Single Frame Estimation (OUEST) method for integer ambiguity resolution	"Investigation of Models and Estimation Techniques for GPS Attitude Determination" [85]	Simulation of three double frequency GPS receivers	An accuracy between 0.5 and 1.0 degrees	Spacecraft attitude determination	1 meter
Carrier phase doubles difference technique; attitude parameterization using quaternion; combined Kalman filter for combined attitude, line bias, and baseline estimation which is an improvement of a bootstrapping method	"GPS-based attitude determination final technical report" [86]	Simulation of four double frequency GPS receivers	RMS of yaw = 0.274 degrees; RMS of roll = 0.450 degrees; RMS of pitch = 0.418 degrees	Spacecraft attitude determination	0.3 meter
Pseudo-range and carrier phase single difference technique; pseudo range smoothing method; early exit strategy and the active method for integer ambiguity	"An integer ambiguity resolution algorithm for real-time GPS attitude determination" [87]	Simulation of three single frequency GPS receivers	Not described in the article	Real-time attitude determination in static mode	1 meter

TABLE 1. ***(Continued.)*** **Comparison of GNSS-based attitude determination methods.**

resolution					
Carrier phase double difference technique; multipath frequency compensation based on Kalman filtering of the first order white noise; this KF estimator is also used for ultimate attitude determination	“Experimental Results on Three Multipath Compensation Techniques for GPS-based Attitude Determination” [88]	Single frequency GEC-Plessey (UK) GPS Builder-2 system receiver with two antennas	It can yield an accuracy of 0.5 to 1 deg	Satellite real-time attitude determination in static mode	1 meter
Triple difference carrier phase technique; tight integration of GPS and INS by Kalman filter for integer ambiguity resolution; an inertial integrated cycle slip detection algorithm	“Attitude Determination GPS/INS Integration System Design Using Triple Difference Technique” [74]	Simulation of an automotive grade IMU and three L1 C/A code GPS receivers	Maximum roll, pitch, and heading errors are 1.05°, 1.06° and 5.41° root-mean-square (RMS), respectively	Aircraft attitude determination in the real-time dynamic mode	0.7 meter
Single point positioning for master antenna by using Extended Kalman Filter (EKF); pseudo-range and carrier phase double difference technique; the system uses Kalman and Bayes filters to track the three baselines and their float ambiguities; Goad's wide lane technique is used to fix the float ambiguities prior to the baseline estimation	“Attitude Determination Using GPS” [89]	A Javad AT4 (four integrated dual frequency receivers with 20 channels on both carrier phases) channel receiver with four antennas	STD of pitch = 0.89 degrees, STD of Roll = 0.97 degrees, STD of yaw = 0.82 degrees	Automobile attitude determination in real-time on-the-fly mode	0.8 meter
Phase-based attitude determination in on-the-fly kinematic mode	“Evaluation of GPS-Based Attitude Parameters Applied to Bathymetric Measurements” [90]	Four antennas with some GPS receiver	The precision of the estimates was around 1.6′ for heading, 2.3′ for pitch, 9.9′ for roll	Sea vessel attitude determination in static mode	Not described in the article
Carrier phase double difference technique; baseline vector resolution using integer least square estimation; discrete particle swarm optimization (DPSO) to search for the optimal integer ambiguity	“Study on attitude determination based on discrete particle swarm optimization “ [90]	Two GPS-701-GG antennas and OEMV-1G, single frequency GPS receiver	Heading error ≤0.2°, pitch error ≤0.4°	Suitable for dynamic attitude determination	1 meter
Carrier phase differential GPS (CPDGPS) technique; Kalman Filter to calculate the float ambiguity estimate; the attitude space search method for integer ambiguity resolution	“Analysis of GPS-based Real-Time Attitude Determination System for ITS Application” [77]	Three sets of [Hemisphere Crescent OEM board 280, Titan III antenna 70, Serial data logger 25, Voltage converter and power regulator 18, Micro SD Card]	Phase center variation error becomes a significant error component that needs to be calibrated	Automobile attitude determination in post-processing dynamic mode with short baseline (< 3 * wavelength)	36 cm
Carrier phase and code double differences technique; code smoothing by means of	“Attitude Determination Using Multiple L1 GPS	Three Magellan AC12 GPS	STD of pitch = 1.001 degrees,	Real-time attitude	1 meter

TABLE 1. ***(Continued.)*** **Comparison of GNSS-based attitude determination methods.**

complementary Kalman Filer; integer ambiguity resolution by means of LAMBDA method and the Ambiguity filter; attitude determination technique based on Least Square Estimation algorithm and rotation matrix	Receivers" [91]	Receivers with three NAIS magnetic antennas, one U-Blox 6 GPS receiver with one ANN-MS-0-005 magnetic antenna	STD of Roll = 0.709 degrees, STD of heading = 0.261 degrees	determination in static mode	
Carrier phase and code double differences technique; code smoothing by means of complementary Kalman Filer; integer ambiguity resolution by means of LAMBDA method and the Ambiguity filter; attitude determination technique based on rotation quaternion found out resorting to an Extended Kalman Filter	"Attitude Determination Using Multiple L1 GPS Receivers" [91]	Three Magellan AC12 GPS receivers with three NAIS magnetic antennas, one U-Blox 6 GPS receiver with one ANN-MS-0-005 magnetic antenna	STD of pitch = 0.708 degrees, STD of Roll = 0.751 degrees, STD of heading = 0.212 degrees	Real-time attitude determination in static mode	1 meter
GPS solutions are integrated into a Kalman filter by providing external velocity and position observations. A Kalman dynamic model is designed to appropriate for MEMS sensor noise characteristics. The bias and drift are estimated by the integrated Kalman filter, which enables the online calibrations of MEMS sensor	"Low-cost MEMS Sensor-based Attitude Determination System by Integration of Magnetometers and GPS: A Real-Data Test and Performance Evaluation" [92]	MEMSenseTM's nano Inertial Measurement Unit integrated by magnetometer and one GPS receiver	Errors are 2.9º, 2.7º and 11º respectively in roll, pitch and heading angle	Dynamic movement when no direct attitude observation available in the navigation filter	Not found
Measurement equations have attitude analytical resolutions by using simultaneous single difference carrier phase equations for two in-view satellites. In addition, the algorithm is capable of reducing the search integer space into countable 2D discrete points and the ambiguity function method (AFM) resolves the ambiguity function within the analytical solutions space	"Hybrid analytical resolution approach based on ambiguity function for attitude determination" [93]	A multi-antenna single-frequency GPS attitude determination system with two antennas	The standard deviations of yaw and pitch angles are 0.17° and 0.29°, respectively	Real-time epoch-by-epoch attitude determination in static mode	1 meter
Optimization method whose updated at each time step requires the integration of the dynamic equations and employs a forgetting factor to weight the history of accumulated GPS measurements; a cubic spline interpolation and the geometric version of Newton's algorithms on the rotation group for dynamic model	"Dynamics-Based Attitude Determination Using the Global Positioning System" [94]	Simulation of four single-frequency GPS receivers; noise variance is equal to 0.01	Roll, pitch, and yaw angles are equal to 0.423, 0.501 and 0.470 respectively	Satellite real-time attitude determination in dynamic mode	0.6 meter
Optimization method whose updated at each time step requires the integration of the dynamic equations and employs a forgetting factor to weight the history of accumulated GPS measurements; a cubic spline	"Dynamics-Based Attitude Determination Using the Global Positioning System" [94]	Simulation of four single-frequency GPS receivers; noise variance is equal to 0.01	Roll, pitch, and yaw angles are equal to 3.090, 3.505 and 3.309 respectively	Satellite real-time attitude determination in static mode	0.6 meter

TABLE 1. *(Continued.)* **Comparison of GNSS-based attitude determination methods.**

interpolation and the geometric version of Newton's algorithms on the rotation group for dynamic model					
Carrier phase double difference technique; integer ambiguity search through applying LAMBDA method; determining baseline by means of Receding-horizon State Estimation (RHSE) method; the direct computation method for attitude determination	"A Study of GPS Based Attitude Determination Technique" [95]	Three double frequency GPS receivers: Trimble 4800, Trimble 4700 and Leica AT502; three antennas: Trimble 4800 Internal, MICRO-CTR L1/L2 GND, and MICRO-CTR L1/L2 GND	The attitude determination accuracy was about 0.5° in yaw, 0.75° in pitch and 0.5° in roll	Real-time attitude determination in static mode	1 meter
A very broad range of methods	"Exploring the Ambiguity Resolution in Spacecraft Attitude Determination Using GNSS Phase Measurement" [1]	Single-frequency, single-epoch GNSS-based attitude determination	Millimeter level accuracy	Spacecraft Attitude Determination in real-time mode	Not described in the article
Differential GPS carrier phase solution; the least squares adjustment model for attitude determination; Wing Flexure Modeling technique	"Assessment of a non-dedicated GPS receiver system for precise airborne attitude determination" [96]	Four independent NovAtel GPS Card receivers	RAMS for roll = 4.2 arcmin, for pitch = 5.8 arcmin, for heading = 3.5 arcmin	Airborne attitude determination in dynamic mode	8.5 meter
Carrier phase single difference technique; cycle slip detection based on Doppler frequency; guessed baseline search method (modification of LAMBDA) for integer ambiguity determination	"GPS-based attitude determination" [97]	Javad Navigation Systems JNSGyro-2 which consists of two high-quality dual frequency capable GPS OEM boards that are connected to each other through a serial data interface, and Javad Marant geodetic quality dual frequency antennas	STD of pitch (deg) = 0.4 for 7 visible satellites	Automobile attitude determination in the urban area	0.74 meter
Carrier phase single difference technique; attitude determination from vectorized measurements; a general method which transforms the general GPS cost function into a Wahba cost function was presented	"Attitude determination using Global Positioning System signals" [98]	Simulation of four double frequency GPS receivers	Accuracies between 0.5 to 1.0 degrees (root-mean-square) were achieved	Vehicle attitude determination in the sight of at least two GPS satellites	1.1 meter

TABLE 1. ***(Continued.)*** **Comparison of GNSS-based attitude determination methods.**

An attitude point solution algorithm which performs iterative least squaresfit a set of simultaneous observations	"GPS Based Attitude Determination The REX II Flight Experience" [99]	Trimble Advanced Navigation Sensor (TANS) Vector receiver with four antennas	Sigma = 1.65° for Roll, 0.94° for Pitch and 1.96°for Yaw	Spacecraft attitude determination in the real-time dynamic mode	48 cm
A Levenberg-Marquardt algorithm (LMA) batch filter performs a least-squares fit on a bank of correlators to generate GPS observables for multiple satellites. These observables are then used as measurements in a Rauch-Tung-Striebel (RTS) smoother that optimizer estimates of carrier phase, Doppler shift, and code phase. Attitude determination is carried out by another batch filter that uses the single-differenced optimized carrier phase estimates between two antennas. The batch filter itself is a combination of a substantially modified form of the LMA and the Least-Squares Ambiguity Decorrelation (LAMBDA) method	"GPS-Based Attitude Determination for a Spinning Rocket" [69]	Two GPS antennas linked to two RF front-ends with a common clock	The accuracy results were reasonable when compared with those from a magnetometer and horizon crossing indicator	An algorithm is developed for determining the attitude of a spinning sounding rocket. This algorithm is able to track signals with low availability duty cycles, but with enough accuracy to yield phase observables for the precise, 3-axis attitude determination of a nutating rocket.	Not described in the article
Carrier phase doubles difference technique	"High Precision Attitude Using Low-Cost GPS Receivers" [100]	Sentek Systems GPS interferometric attitude system based on three Allstar CMT-1200 single-frequency GPS receivers	SD of yaw is 0.245°, SD of pitch and roll is 0.481° and 0.473° respectively	Real-time attitude determination in static mode	80 cm

GNSS equipment with other navigational sensors, such as an accelerometer, gyroscope, electronic compass etc. That kind of integration results in more reliable, accurate and robust attitude determination. The set of methods suitable and optimal for usage in an abstract situation may change depending on application environment, applied sensors, mathematical and dynamic models. This also causes large differences in the accuracy obtained when utilizing different set of methods in a particular application.

The article summarizes some part of the findings based on the comprehensive literature survey provided in Table 1. The information given in the table has focused on the performance metrics (accuracy, precision, success rate of getting a correct solution, average baseline) that can be achieved while using various sets of sensors for GNSS-based attitude determination.

The literature survey has suggested that the performance metrics should be investigated in two different modes. First, according to the table, the Multivariate Constrained LAMBDA method has shown the best performance in static mode. Second, taking into account the literature survey, it has been assumed that combined implementation of extended Kalman filter based on the dynamic model and a baseline constrained LAMBDA method should be the most optimal in dynamic mode. This assumption will be tested and verified in future works.

REFERENCES

[1] L. Meibo, N. N. Abbas, and L. YanJun, ''Exploring the ambiguity resolution in spacecraft attitude determination using GNSS phase measurement,'' *Int. J. Social, Behav., Educ., Econ., Bus. Ind. Eng.*, vol. 7, no. 7, pp. 1976–1980, Jul. 2013.

[2] W. Bing, S. Lifen, X. Guorui, D. Yu, and Q. Guobin, ''Comparison of attitude determination approaches using multiple global positioning system (GPS) antennas,'' *Geodesy Geodyn.*, vol. 4, no. 1, pp. 16–22, Feb. 2013.

[3] Y. Li, K. Zhang, C. Roberts, and M. Murata, ''On-the-fly GPS-based attitude determination using single-and double-differenced carrier phase measurements,'' *GPS Solutions*, vol. 8, no. 2, pp. 93–102, Jul. 2004.

[4] L. Baroni and H. K. Kuga, ''Analysis of attitude determination methods using GPS carrier phase measurements,'' *Math. Problems Eng.*, vol. 2012, pp. 1–10, Feb. 2012.

[5] L. Baroni and H. K. Kuga, ''Analysis of ambiguity resolution methods for attitude determination using GPS carrier phase measurements,'' *Adv. Astron. Sci.*, vol. 145, pp. 301–309, 2012.

[6] J. Um and E. G. Lightsey, ''GPS attitude determination for the SOAR experiment,'' *Navigation*, vol. 48, no. 3, pp. 180–194, Sep. 2001.

[7] C. E. Cohen and B. W. Parkinson, ''Integer ambiguity resolution of the GPS carrier for spacecraft attitude determinationn,'' in *Proc. Annu. AAS Rocky Mountain Conf.*, Keystone, CO, USA, 1992, pp. 107–118.

[8] S. Purivigraipong, Y. Hashida, and M. J. Unwin, ''GPS attitude determination for microsatellites,'' in *Proc. ION GPS*, Nashville, TN, USA, 1999, pp. 2017–2026.

[9] D. Knight, ''A new method of instantaneous ambiguity resolution,'' in *Proc. ION GPS*, Salt Lake City, UT, USA, 1994, pp. 707–716.

[10] M. Ziebart and P. Cross, ''LEO GPS attitude determination algorithm for a micro-satellite using boom-arm deployed antennas,'' *GPS Solutions*, vol. 6, no. 4, pp. 242–256, Mar. 2003.

[11] S. Verhagen, ''The GNSS integer ambiguities: Estimation and validation,'' Ph.D. dissertation, Delft Inst. Earth Observ. Space Syst., Delft Univ. Technol., Delft, Netherlands, 2005.

[12] R. Hatch, ''Instantaneous ambiguity resolution,'' in *Proc. Kinematic Syst. Geodesy, Surveying, Remote Sens., Symp.*, Banff, AB, Canada, no. 107, 1991, pp. 299–308.

[13] E. Frei and G. Beutler, ''Rapid static positioning based on the fast ambiguity resolution approach FARA: Theory and first results,'' *Manuscripta Geodaetica*, vol. 15, no. 6, pp. 325–356, 1990.

[14] H.-J. Euler and H. Landau, ''Fast GPS ambiguity resolution on-the-fly for real-time applications,'' in *Proc. 6th Int. Geodesy Symp. Satell. Positioning*, Columbus, OH, USA, 1992, pp. 650–659.

[15] P. J. G. Teunissen, ''Least-squares estimation of the integer GPS ambiguities,'' in *Proc. IAG Gen. Meeting, IV Theory Methodol.*, Beijing, China, 1993, pp. 1–16.

[16] M. Martin-Neira, M. Toledo, and A. Pelaez, ''The null space method for GPS integer ambiguity resolution,'' in *Proc. DSNS*, Bergen, Norway, 1995, pp. 24–28.

[17] D. Chen and G. Lachapelle, ''A comparison of the FASF and least squares search algorithms for on the fly ambiguity resolution,'' *Navigation*, vol. 42, no. 2, pp. 371–390, 1995.

[18] R. A. Harris, ''Direct resolution of carrier-phase ambiguity by 'bridging the wavelength gap,''' ESA Publication, Auckland, New Zealand, Tech. Rep. TST/60107/RAH/Word, 1997.

[19] U. Vollath, S. Bimbach, H. Landau, J. M. Fraile-Ordofnez, and M. Martfin-Neira, ''Analysis of three-carrier ambiguity resolution (TCAR) technique for precise relative positioning in GNSS-2,'' in *Proc. ION GPS*, Nashville, TN, USA, 1998, pp. 417–426.

[20] D. Kim and R. B. Langley, ''An optimized least-squares technique for improving ambiguity resolution and computational efficiency,'' in *Proc. ION GPS*, Nashville, TN, USA, 1999, pp. 1579–1588.

[21] J. Jung, P. Enge, and B. Pervan, ''Optimization of cascade integer resolution with three civil GPS frequencies,'' in *Proc. ION GPS*, Salt Lake City, UT, USA, 2000, pp. 2191–2200.

[22] P. Joosten and S. Verhagen, ''Analysis of ambiguity resolution algorithms and quality control parameters for global navigations satellite systems,'' ESA Publication, Auckland, New Zealand, Tech. Rep. 16793/02/NL/LvH, 2003.

[23] P. Buist, ''The baseline constrained LAMBDA method for single epoch, single frequency attitude determination applications,'' in *Proc. ION-GNSS*, Fort Worth, TX, USA, 2007, pp. 2962–2973.

[24] G. Giorgi and P. J. G. Teunissen, ''Carrier phase GNSS attitude determination with the multivariate constrained LAMBDA method,'' in *Proc. IEEE Aerosp. Conf.*, Big Sky, MT, USA, 2010, pp. 1–12.

[25] P. J. G. Teunissen, ''A general multivariate formulation of the multi-antenna GNSS attitude determination problem,'' *Artif. Satell.*, vol. 42, no. 2, pp. 97–111, Feb. 2007.

[26] P. J. G. Teunissen, ''The least-squares ambiguity decorrelation adjustment: A method for fast GPS integer ambiguity estimation,'' *J. Geodesy*, vol. 70, nos. 1–2, pp. 65–82, Nov. 1995.

[27] G. Giorgi and P. J. Buist, ''Single-epoch, single-frequency, standalone full attitude determination: Experimental results,'' in *Proc. NAVITEC*, Noordwijk, The Netherlands, 2008, pp. 1–8.

[28] S. Verhagen and P. Joosten, ''Analysis of integer ambiguity resolution algorithms,'' *Eur. J. Navigat.*, vol. 2, no. 4, pp. 38–50, Jan. 2004.

[29] P. J. G. Teunissen and S. Verhagen, ''GNSS carrier phase ambiguity resolution: Challenges and open problems,'' in *Proc. Sci. Meetings IAG Gen. Assem.*, Berlin, Germany, 2007, pp. 785–792.

[30] P. J. G. Teunissen, ''An optimality property of the integer least-squares estimator,'' *J. Geodesy*, vol. 73, no. 11, pp. 587–593, Dec. 1999.

[31] P. J. G. Teunissen, ''A class of unbiased integer GPS ambiguity estimators,'' *Artif. Satell.*, vol. 33, no. 1, pp. 3–10, 1998.

[32] D. Choukroun, ''A novel quaternion Kalman filter using GPS measurements,'' in *Proc. ION GPS*, Portland, OR, USA, 2002, pp. 1117–1128.

[33] I. Y. Bar-Itzhack, P. Y. Montgomery, and J. C. Garrick, ''Algorithms for attitude determination using the global positioning system,'' *J. Guid., Control, Dyn.*, vol. 21, no. 6, pp. 846–852, Nov./Dec. 1998.

[34] G. Wahba, ''A least squares estimate of satellite attitude,'' *SIAM Rev.*, vol. 7, no. 3, p. 409 Jul. 1965.

[35] C. E. Cohen, ''Attitude determination,'' in *Global Positioning System: Theory and Applications*, vol. 2. Reston, VA, USA, 1996, ch. 19, pp. 519–538.

[36] C. E. Cohen, ''Attitude determination using GPS,'' Ph.D. dissertation, Dept. Aeronaut. Astronaut., Stanford Univ., Stanford, CA, USA, 1993.

[37] L. R. Kruczynski, P. C. Li, A. G. Evans, and B. R. Hermann, ''Using GPS to determine vehicle attitude: USS yorktown test results,'' in *Proc. ION GPS*, Colorado Spring, CO, USA, 1989, pp. 163–171.

[38] J. Rath and P. Ward, ''Attitude estimation using GPS,'' in *Proc. 2nd Nat. Tech. Meeting Inst. Navigat.*, San Mateo, CA, USA, 1989, pp. 169–178.

[39] R. A. Brown and A. G. Evans, ''GPS pointing system performance,'' in *Proc. ION GPS*, Broadmoor, CO, USA, 1990, pp. 645–654.

[40] K. Ferguson, J. Kosmalska, M. Kuhl, J.-M. Eichner, and K. Kepski, ''Three-dimensional attitude determination with the Ashtech 3DF 24-channel GPS measurement system,'' in *Proc. ION GPS*, Phoenix, AZ, USA, 1991, pp. 35–41.

[41] F. Graas and M. Braasch, ''GPS interferometric attitude and heading determination: Initial flight test results,'' *Navigation*, vol. 38, no. 4, pp. 297–316, Dec. 1991.

[42] G. Lu, M. E. Cannon, G. Lachapelle, and P. Kielland, ''Attitude determination in a survey launch using multi-antenna GPS technologies,'' in *Proc. Nat. Tech. Meeting*, San Francisco, CA, USA, 1993, pp. 251–259.

[43] R. D. Jurgens and C. E. Rodgers, ''Advances in GPS attitude determining technology as developed for the strategic defense command,'' in *Proc. ION GPS*, Albuquerque, NM, USA, 1991, pp. 991–999.

[44] T. B. Bahder, ''Attitude determination from single-antenna carrier-phase measurements,'' *J. Appl. Phys.*, vol. 91, no. 7, pp. 4677–4684, Apr. 2002, doi: 10.1063/1.1448871.

[45] C. Wang, R. A. Walker, and W. Enderle, ''Single antenna attitude determination for FedSat,'' in *Proc. ION GPS*, Portland, OR, USA, 2002, pp. 134–144.

[46] L. Boccia, G. Amendola, G. Di Massa, and L. Giulicchi, ''Shorted annular patch antennas for multipath rejection in GPS-based attitude determination systems,'' *Microw. Opt. Technol. Lett.*, vol. 28, no. 1, pp. 47–51, Jan. 2001.

[47] S. Fan, K. Zhang, and F. Wu, ''Ambiguity resolution in GPS-based, low-cost attitude determination,'' *J. Global Positioning Syst.*, vol. 4, nos. 1–2, pp. 207–214, Jul. 2010.

[48] H.-Y. Liu and H.-N. Wang, ''Application of mean field annealing algorithms to GPS-based attitude determination,'' *Chin. J. Aeronaut.*, vol. 17, no. 3, pp. 165–169, Aug. 2004.

[49] B. W. Parkinson, "Attitude determination," in *Progress in Astronautics and Aeronautics: Global Positioning System: Theory and Applications*, vol. 2, Washington, SW, USA: AIAA, 1996, ch. 19, pp. 519–538.

[50] G. Strang and K. Borre, *Linear Algebra, Geodesy, and GPS*, Wellesley, MA, USA: Wellesley Cambridge Press, 1997, pp. 481–505.

[51] P. J. G. Teunissen and A. Kleusberg, *GPS for Geodesy*. Berlin, Germany: Springer, 1998.

[52] A. Leick, L. Rapoport, and D. Tatarnikov, *GPS Satellite Surveying*, 3rd ed. New York, NY, USA: Wiley, 2015.

[53] P. Misra and P. Enge, *Global Positioning System: Signals, Measurements and Performance*, 2nd ed. Lincoln, MA, USA: Ganga-Jamuna Press, 2006.

[54] B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins, "Applications of GPS," in *Global Positioning System: Theory and Practice*, 5th ed. New Delhi, India: Springer, 2007, ch. 12, sec. 2, pp. 327–329.

[55] P. J. Buist, P. J. G. Teunissen, G. Giorgi, and S. Verhagen, "Instantaneous GNSS-based kinematic relative positioning and attitude determination using multi-antenna configurations," in *Proc. Int. Symp. GPS/GNSS*, Jeju, South Korea, 2009, pp. 1–11.

[56] P. J. G. Teunissen, "GNSS ambiguity resolution for attitude determination: Theory and method," in *Proc. Int. Symp. GPS/GNSS*, Tokyo, Japan, 2008, pp. 1–10.

[57] P. J. G. Teunissen, "A-PPP: Array-aided precise point positioning with global navigation satellite systems," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 2870–2881, Jun. 2012.

[58] P. J. G. Teunissen, "The affine constrained GNSS attitude model and its multivariate integer least-squares solution," *J. Geodesy*, vol. 86, no. 7, pp. 547–563, Jul. 2012.

[59] F. Bauer, E. Lightsey, J. Mccullough, J. O'Donnell, and R. Schnurr, "GADACS: A GPS attitude determination and control experiment on a spartan spacecraft," *Control Eng. Pract.*, vol. 3, no. 8, pp. 1125–1130, Aug. 1995.

[60] C. E. Cohen, E. G. Lightsey, B. W. Parkinson, and W. A. Feess, "Space flight tests of attitude determination using GPS," *Int. J. Satell. Commun.*, vol. 12, no. 5, pp. 427–433, Sep. 1994.

[61] P. Axelrad and L. M. Ward, "Spacecraft attitude estimation using the global positioning system-methodology and results for RADCAL," *J. Guid., Control, Dyn.*, vol. 19, no. 6, pp. 1201–1209, Nov./Dec. 1996.

[62] E. G. Lightsey, E. Ketchum, T. W. Flatley, J. L. Crassidis, D. Freesland, K. Reiss, and D. Young, "Flight results of GPS based attitude control on the REX II spacecraft," in *Proc. ION GPS*, Alexandria, VA, USA, 1996, pp. 1037–1046.

[63] H. Uematsu, L. Ward, and B. Parkinson, "Use of global positioning system for gravity probe B relativity experiment and co-experiments," *Adv. Space Res.*, vol. 25, no. 6, pp. 1199–1203, Jan. 2000.

[64] P. Tortora, "A GPS based attitude determination algorithm for the spin-stabilized microsatellite UNISAT," *Acta Astronautica*, vol. 47, nos. 2–9, pp. 139–146, Jul. 2000.

[65] J. Kuang and S. Tan, "GPS-based attitude determination of gyrostat satellite by quaternion estimation algorithms," *Acta Astronautica*, vol. 51, no. 11, pp. 743–759, Nov. 2002.

[66] M. Unwin, P. Purivigraipong, A. Da Silva Curiel, and M. Sweeting, "Stand-alone spacecraft attitude determination using real flight GPS data from UOSAT-12," *Acta Astronautica*, vol. 51, nos. 1–9, pp. 261–268, Jul. 2002.

[67] E. Rooney, M. Unwin, S. Gleason, M. Adjrad, and T. Ebinuma, "Demonstration of GPS on the ALSAT-1 disaster monitoring satellite," in *Proc. AIAA Guid., Navigat., Control Conf. Exhibit*, Austin, TX, USA, 2003, p. 1.

[68] P. C. Fenton, W. Kunysz, and G. Garbe, "Using GPS for position and attitude determination of the canadian space agency's active rocket mission," in *Proc. ION GPS*, Nashville, TN, USA, 1998, pp. 1791–1800.

[69] K. Chiang, M. Psiaki, S. Powell, R. Miceli, and B. O'Hanlon, "GPS-based attitude determination for a spinning rocket," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 50, no. 4, pp. 2654–2663, Oct. 2014.

[70] N. Lovren and J. Pieper, "Error analysis of direction cosines and quaternion parameters techniques for aircraft attitude determination," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 34, no. 3, pp. 983–989, Jul. 1998.

[71] K. P. A. Lievens, J. A. Mulder, and P. Chu, "Single GPS antenna attitude determination of a fixed wing aircraft aided with aircraft aerodynamics," in *Proc. AIAA Guid., Navigat., Control Conf. Exhibit*, San Francisco, CA, USA, 2005, pp. 1–14.

[72] S. Park and C. Kee, "Enhanced method for single-antenna GPS-based attitude determination," *Aircr. Eng. Aerosp. Technol.*, vol. 78, no. 3, pp. 236–243, May 2006.

[73] J. T. Pinchin, "GNSS based attitude determination for small unmanned aerial vehicles," Ph.D. dissertation, College Eng., Univ. Canterbury, Christchurch, New Zealand, 2011.

[74] S.-H. Oh, D.-H. Hwang, C.-S. Park, and S.-J. Lee, "Attitude determination GPS/INS integration system design using triple difference technique," *J. Electr. Eng. Technol.*, vol. 7, no. 4, pp. 615–625, Jul. 2012.

[75] S. Alban, "Design and performance of a robust GPS/INS attitude system for automobile applications," Ph.D. dissertation, Dept. Aeronaut. Astronaut., Stanford Univ., Stanford, CA, USA, 2004.

[76] C. Chia-chyang and L. Hsing-wei, "Evaluation of GPS-based attitude parameters applied to bathymetric measurements," *Wuhan Univ. J. Nat. Sci.*, vol. 8, no. 2, pp. 685–692, Jun. 2003.

[77] D. Gebre-Egziabher and F. A. P. Lie, "Analysis of GPS-based real time attitude determination system for ITS application," Dept. Aerosp. Eng. Mech., Univ. Minnesota, Minneapolis, MN, USA, Tech. Rep. CTS 12-36, Nov. 2012.

[78] M. L. Psiaki, F. Martel, and P. K. Pal, "Three-axis attitude determination via Kalman filtering of magnetometer data," *J. Guid., Control, Dyn.*, vol. 13, no. 3, pp. 506–514, May 1990.

[79] X.-K. Yue and J.-P. Yuan, "Neural network-based GPS/INS integrated system for spacecraft attitude determination," *Chin. J. Aeronaut.*, vol. 19, no. 3, pp. 233–238, Aug. 2006.

[80] C. Wang, "Development of a low-cost GPS-based attitude determination system," M.S. thesis, Dept. Geomatics Eng., Univ. Calgary. Calgary, AB, Canada, 2003.

[81] A. C. Louro, R. V. F. Lopes, and H. K. Kuga, "Autonomous fault detection on a low cost GPS-aided attitude determination system," in *Proc. ION GNSS*, Fort Worth, TX, USA, 2007, pp. 2658–2667.

[82] J.-K. Shiau and I.-C. Wang, "Unscented Kalman filtering for attitude determination using mems sensors," *J. Appl. Sci. Eng.*, vol. 16, no. 2, pp. 165–176, Jan. 2013.

[83] J. Cheng, J. Wang, and L. Zhao, "A direct attitude determination approach based on gps double-difference carrier phase measurements," *J. Appl. Math.*, vol. 2014, pp. 1–6, Mar. 2014.

[84] Y. T. Chiang, L. S. Wang, F. R. Chang, and H. M. Peng, "Constrained filtering method for attitude determination using GPS and gyro," *IEE Proc.-Radar, Sonar Navigat.*, vol. 149, no. 5, pp. 258–264, Nov. 2002.

[85] J. Garrick, "Investigation of models and estimation techniques for GPS attitude determination," in *Proc. Flight Mech./Estimation Theory Symp.*, Greenbelt, MD, USA, 1996, pp. 89–98.

[86] P. Axelrad, "GPS based attitude determination," Colorado Center Astrodyn. Res., Univ. Colorado, Boulder, CO, USA, Tech. Rep. N00014-93-1-G030, 1995.

[87] S. Yoon and J. B. Lundberg, "An integer ambiguity resolution algorithm for real-time GPS attitude determination," *Appl. Math. Comput.*, vol. 129, no. 1, pp. 21–41, Jun. 2002.

[88] A. Pasetti and L. Giulicchi, "Experimental results on three multipath compensation techniques for GPS-based attitude determination," in *Proc. 21st Annu. AAS Guid. Control Conf.*, Breckenridge, CO, USA, 1999, pp. 1–21.

[89] K. Kyndal and K. Tue, "Aeroplane attitude determination using GPS," Inst. Elect. Syst., Aalborg Univ., Aalborg, Denmark, Rep. Group GPS 1048, 2003.

[90] N. Xia, D. Han, G. Zhang, J. Jiang, and K. Vu, "Study on attitude determination based on discrete particle swarm optimization," *Sci. China Technol. Sci.*, vol. 53, no. 12, pp. 3397–3403, Dec. 2010.

[91] J. Cóias, "Attitude determination using multiple L1 GPS receivers," M.S. thesis, Dept. Aerosp. Eng., Technical Univ. Lisbon, Lisbon, Portugal, 2012.

[92] D. Li, R. Landry, and P. Lavoie, "Low-cost MEMS sensor-based attitude determination system by integration of magnetometers and GPS: A real-data test and performance evaluation," in *Proc. IEEE/ION Position, Location Navigat. Symp.*, Monterey, CA, USA, 2008, pp. 1190–1198.

[93] W.-R. Jin, C.-R. Zhai, L.-D. Wang, Y.-H. Zhang, and X.-Q. Zhan, "Hybrid analytical resolution approach based on ambiguity function for attitude determination," *J. Zhejiang Univ. Sci. A*, vol. 10, no. 7, pp. 1038–1048, Jul. 2009.

[94] C. Chun and F. C. Park, "Dynamics-based attitude determination using the global positioning system," *J. Guid., Control, Dyn.*, vol. 24, no. 3, pp. 466–473, May 2001.

[95] D. Lin, "A study of GPS based attitude determination technique," M.S. thesis, School Elect. Electron. Eng., Nanyang Technol. Univ., Singapore, 2005.

[96] M. E. Cannon, H. Sun, T. E. Owen, and M. A. Meindl, "Assessment of a non-dedicated GPS receiver system for precise airborne attitude determination," in *Proc. ION GPS*, Salt Lake City, UT, USA, 1994, pp. 645–654.

[97] J. Bejeryd, "GPS-based attitude determination," Dept. Elect. Eng., Linköpings Univ., Linköping, Sweden, Tech. Rep. LiTH-ISY-EX–07/4127-SE, 2007.

[98] J. L. Crassidis and F. L. Markley, "New algorithm for attitude determination using global positioning system signals," *J. Guid., Control, Dyn.*, vol. 20, no. 5, pp. 891–896, Sep. 1997.

[99] D. Freesland, K. Reiss, D. Young, J. Cooper, and C. A. Adams, "GPS based attitude determination: The REX II flight experience," in *Proc. ION GPS*, Kansas City, MO, USA, 1996, pp. 1037–1046.

[100] M. Dumaine, "High precision attitude using low cost GPS receivers," in *Proc. ION GPS*, Kansas City, MO, USA, 1996, pp. 1029–1035.

ALMAT RASKALIYEV received the B.Sc. degree in applied mathematics and physics from the Moscow Institute of Physics and Technology, in 2007, the master's degree in information systems from Kazakh British Technical University, in 2011, and the Ph.D. degree in space engineering and technology from Al-Farabi Kazakh National University, in 2017.

From September 2015 to February 2016, he was a Research Scholar with the Robotics, Intelligent Sensing and Control (RISC) Laboratory, School of Engineering, University of Bridgeport. He currently works as a Leading Researcher with the Laboratory of Satellite Navigation Technologies, Institute of Space Technique and Technology, Almaty, Kazakhstan. His research interests include GNSS, sensor data fusion, and digital signal processing. His poster based on the Ph.D. research results won the second position in the graduate research poster category of the American Society for Engineering Education 2016 conference.

SAROSH HOSI PATEL received the B.Sc. degree (Hons.) in electrical engineering from Osmania University, India, and the M.S. degree in electrical engineering and technology management and the Ph.D. degree in computer science from the University of Bridgeport.

He is currently an Assistant Professor with the School of Engineering, University of Bridgeport. His research interests include manipulator prototyping, inverse kinematic solutions, industrial control, and robotic swarms.

TAREK M. SOBH received the B.Sc. degree (Hons.) in engineering in computer science and automatic control from the Faculty of Engineering, Alexandria University, Egypt, in 1988, and the M.S. and Ph.D. degrees in computer and information science from the School of Engineering, University of Pennsylvania, in 1989 and 1991, respectively.

He is currently the Vice President for Graduate Studies and Research and the Dean of the School of Engineering. He is also a Distinguished Professor of engineering and computer science with the University of Bridgeport (UB), CT, USA, the Founding Director of the Interdisciplinary Robotics, Intelligent Sensing, and Control (RISC) Laboratory, and the Co-Founder of the High-Tech Business Incubator, UB. He is also a Fellow of the African Academy of Sciences and a member of the Connecticut Academy of Science and Engineering.

AIDOS IBRAYEV received the B.Sc. and M.Sc. degrees in mechanical engineering and the Ph.D. degree in space engineering and technology from Al-Farabi Kazakh National University, Almaty, Kazakhstan, in 2011, 2013, and 2019, respectively.

From 2012 to 2017, he worked as an Engineer, a Software Developer, and a Scientific Researcher with the Institute of Space Technique and Technology, Almaty. From 2017 to 2018, he was a Senior Lecturer with the Almaty University of Power Engineering and Telecommunication. Since 2018, he has been a Senior Lecturer with Al-Farabi Kazakh National University.

• • •