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Why numerical symbols count in the development of mathematical skills: evidence from brain and behaviour

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Why numerical symbols count in the development of mathematical skills: evidence from brain and behavior

Rebecca Merkley and Daniel Ansari



Numerical skills measured prior to school entry are predictive of mathematics achievement longitudinally. It is therefore important that young children start school with strong mathematical foundations. Here we review evidence from behavior and neuroimaging that highlights numerical symbol knowledge as a key mediator between informal and formal mathematical competencies. We argue that future research should aim to elucidate cognitive and neuronal mechanisms underpinning the acquisition of symbolic knowledge. Furthermore, multiple aspects of numerical symbol knowledge, such as identification, cardinality, and ordinality, should be emphasized in preschool childcare environments.

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Introduction

Numeracy skills prior to school entry are strong predictors of subsequent academic achievement over and above other cognitive skills, including literacy and attention [1]. Furthermore, a recent study found that preschool counting and cardinality skills were significantly predictive of fifth grade mathematics achievement [2]. Multiple factors contribute to the development of preschool mathematical abilities, including linguistic abilities, executive functions, home environment, and both formal and informal numeracy competencies [3,4]. Informal mathematics encompasses abilities that are acquired prior to or outside of school, such as a rudimentary understanding of counting. In contrast, formal mathematics knowledge designates skills that are explicitly taught in school, such as competence with numerical symbols and operations (e.g., understanding the meaning of the equals sign) [5]. A recent study demonstrated that the longitudinal relationship between informal

and formal mathematics knowledge in preschoolers was completely mediated by knowledge of numerical symbols $[6^{\bullet\bullet}]$.

Mathematics learning opportunities in early childhood can also be categorized as formal activities, such as explicit lessons about quantity, and informal activities, such as playing a board game that includes numbers. It is important to note that the distinction between informal and formal in the context of home and preschool environment refers to the nature of the activity rather than the mathematical abilities involved. Young children encounter both types of experiences in home and preschool childcare environments, but government-mandated formal mathematics instruction typically does not begin until school entry. Therefore, even prior to the start of formal schooling, there are marked differences in the amount and quality of children's early mathematical experiences, which are predictive of mathematics achievement longitudinally [7^{••},8]. Indeed, the influence of childcare settings on early numeracy skills has been investigated in Canada [8], the UK [9], and the US [10], with all three suggesting that improving early childcare provisions could increase school readiness and in turn lead to higher school mathematics achievement.

In this review, we present evidence from both behavioral as well as brain imaging studies that highlight the importance of numerical symbols in the early development of mathematical competence. To date there is little evidence for a direct, causal link between informal, nonsymbolic mathematical skills and formal mathematics achievement [11]. Symbolic number competence, in contrast, has been more reliably associated with achievement. For example, numeral knowledge assessed near the start of the first year of school was a powerful predictor of longitudinal growth in arithmetic skills over eleven months [12[•]]. We therefore argue that it is necessary for future research to investigate more widely how to better support number learning in early childhood. In turn, it is critical to integrate both formal and informal activities that foster symbolic number knowledge in preschool classrooms in order to strengthen children's mathematical foundations prior to school entry.

Numerical symbol acquisition

Children learn the count sequence by rote before understanding the numerical meaning of number words and Arabic numerals [13]. Importantly, children are only considered to know the exact meanings of symbols once

Box 1 Glossary

Numeracy: ability to grasp and apply simple numerical concepts

Numerical symbols: number words (e.g., 'three') and Arabic numerals (e.g., 3) that designate an exact number of items

Nonsymbolic numerical representations: ways of representing numbers without using symbols, typically via arrays of objects (e.g., ***), but could also be represented in other modalities (e.g., sequential sets of sounds)

Subitizing: in contrast to approximate estimation, subitizing is the ability to quickly and accurately enumerate the precise number of items in a nonsymbolic set of fewer than 5 items [50]

Discrete quantity: the number of items in a set (also termed numerosity)

Continuous quantity: in contrast with discrete quantity, continuous quantity encompasses other properties of nonsymbolic sets, such as size and density of items

Cardinality principle: the understanding that the last number word used when counting a set indicates the number of objects in the set [51]

CP-knower: a child who demonstrates that they can apply the cardinality principle to numbers greater than five is considered to be a CP-knower [52]

Subset-knower: young children gradually learn the cardinality of numbers 'one' to 'four' sequentially, and are deemed subset-knowers during this period (i.e., one-knowers reliably understand the cardinality of 'one', two-knowers understand the cardinality of 'two', etc.) [52]

SFON: spontaneous focusing on numerosity, defined as the frequency with which children attend to the numerical magnitude of sets without being instructed to do so [53]

they have acquired the cardinality principle (see Box 1) in addition to being able to identify the symbols (see [14[•]] for a review of young children's acquisition of number knowledge). The pervasive view on how the meanings of numerical symbols are acquired is illustrated by this quote: "when we learn number symbols, we simply attach their arbitrary shapes to the relevant nonsymbolic quantity representations" [15, p. 552]. Preschoolers' nonsymbolic abilities have been linked to numeral naming [16] and cardinality knowledge [17], which has further bolstered the suggestion that nonsymbolic abilities play a fundamental role in learning about symbols [15]. However, the increasing contradictory evidence for this proposal is reviewed in [11]. For example, some studies have failed to find significant relationships between symbolic and nonsymbolic magnitude comparison performance concurrently [18,19] or longitudinally six months later [18]. Moreover, young children who had not yet acquired the cardinality principle failed to accurately choose the more numerous of two nonsymbolic arrays under some conditions [20,21[•]]. Specifically, children who did not know cardinality did not succeed at nonsymbolic magnitude comparison when continuous quantity conflicted with discrete quantity (e.g., the numerically larger array of dots in a comparison task occupied, on average, a smaller area than the numerically smaller dot array) [20,21[•]]. This adds to a growing body of evidence showing that non-numerical variables, such as density and size, influence judgments of nonsymbolic numerosity [22]. Furthermore, these data suggest that rather than nonsymbolic abilities scaffolding the acquisition of cardinality. acquiring the cardinality of symbols may instead facilitate children's understanding that the more numerous array is the one with more discrete items. This could explain the observed correlations between nonsymbolic comparison and number knowledge [21[•]]. This notion is further supported by the finding that preschoolers' cardinality knowledge and numerical symbol identification was strongly related to their nonsymbolic comparison performance seven months later, but the reverse relationship between nonsymbolic comparison at the first time point and subsequent number knowledge was less strong [23[•]].

Children who have acquired the cardinality principle are also more likely to succeed on other nonsymbolic numerical tasks than children who have not yet gained this understanding [24–27]. For example, CP-knowers (please refer to Box 1) were more accurate than subset-knowers at performing cross-format comparisons across symbolic and nonsymbolic notations [24]. Furthermore, in a cardmatching task, CP-knowers performed above chance, but subset-knowers did not, when selecting cards on the basis of the number of items when the set size was greater than four [26]. In addition to numerosity, cards varied in the color and mood of the characters (e.g., whether they were smiling) and children were also asked to match along those dimensions. In contrast with numerosity, all children performed above chance when selecting cards on the basis of the color and mood of the items [26]. This suggests that acquisition of the cardinality principle is associated with changes in their ability to use numerical symbols to enumerate nonsymbolic representations. Thus, it could be that acquiring symbolic knowledge influences nonsymbolic skills (see [28] for a review of the relationship between symbolic and nonsymbolic number knowledge). Taken together, there is a lack of evidence for a unidirectional and causal relationship between informal, approximate nonsymbolic numeracy competencies and the acquisition of the meaning of numerical symbols. It is therefore necessary to emphasize numerical symbols and their relations to exact quantity in early childhood in order to scaffold the acquisition of numerical symbols.

Symbolic number knowledge predicts mathematics achievement

Emphasizing symbolic skills in early childhood could have long-term implications for mathematical development. Evidence from preschool children, as well as neural and behavioral evidence from school-age children, has revealed important links between symbolic number knowledge and mathematics achievement and it is to





Relationships between numerical symbol knowledge, informal, and formal mathematical abilities. Numerical symbol knowledge encompasses multiple aspects of symbolic understanding.

this literature we now turn. Preschoolers' symbolic number knowledge mediated the relationship between their informal numeracy skills and subsequent formal mathematics achievement [6^{••}]. Specifically, naming Arabic numerals and linking them to their corresponding nonsymbolic quantity together fully mediated the longitudinal relationship between informal competencies, such as counting, and measures of formal mathematics ability one year later [6**]. Crucially, each symbolic number ability separately only partially mediated the relationship, which highlights the importance of being able to map proficiently between all three representations of number: words, numerals, and nonsymbolic arrays [6^{••}]. Therefore, multiple aspects of symbolic knowledge play a role in mathematical development (see Figure 1). Additionally, preschoolers' cardinality knowledge mediated the relationship between approximate nonsymbolic abilities and early mathematics achievement [29,30], and kindergartener's symbolic approximation abilities mediated the relationship between nonsymbolic approximation and math achievement [31]. This further supports the notion that the relationship between informal numeracy skills and mathematics achievement is mediated by the understanding of numerical symbols.

In a related vein, individual differences in another informal skill, spontaneous focusing on numerosity (SFON, please refer to Box 1), were predictive of math achievement 6 years later [32]. However, in another study investigating concurrent relationships in 5-year-olds' numeracy skills, SFON accounted for less variance in arithmetic when performance on a numeral identification and a numerical mapping (i.e., matching numerals to corresponding quantities) task were also entered into the model [33]. Therefore, the correlation between SFON and formal math can be explained, in part, by symbolic knowledge. Taken together, the available evidence suggests that expertise with numerical symbols can account for observed relationships between informal and formal mathematics skills. Importantly, as symbol knowledge is associated with changes in performance on informal math tasks and formal mathematics competence, it is not possible to determine the direction of these relationships from existing research. Further research is therefore necessary to explore causal relationships underlying the acquisition of formal mathematical competencies.

A recent systematic review of research on school-age children suggested that symbolic numerical comparison has been found to be a more reliable predictor of mathematics achievement than nonsymbolic magnitude comparison [34]. Furthermore, a recent quantitative metaanalysis confirmed that effect sizes were stronger for the correlation between symbolic comparison and math competence than for nonsymbolic comparison and math competence [35]. Similarly, a large cross-sectional study revealed that symbolic comparison was the strongest predictor of arithmetic skill at the start of primary school [36]. This was further corroborated by a longitudinal study that revealed numeral identification was a better predictor of arithmetic growth in the first year of school than nonsymbolic comparison [12[•]]. Moreover, another longitudinal study demonstrated that symbolic comparison is as powerful a longitudinal predictor of math ability

as phonological awareness is a predictor of reading ability [37]. Of note, recent studies have shown that individual differences in performance on an ordinality judgment task with numerical symbols are also strongly related to arithmetic ability [36,38]. In sum, symbolic numerical abilities, including understanding of both cardinality and ordinality, are more strongly and consistently related to mathematics achievement than nonsymbolic abilities.

Turning to the neural level of analysis, there has been a growth in functional neuroimaging studies that investigate how neural representations of numbers change with increased experience with symbols. Increases in the magnitude of neural activation during a symbolic numerical comparison task in the left IPS, but not the right IPS, were positively associated with age, suggesting that the left IPS becomes increasingly specialized for processing numerical symbols with experience [39]. Similarly, developmental changes in neural responses in the left, but not right, IPS were associated with behavioral changes in matching symbolic to nonsymbolic representations of number [40[•]]. Right parietal activation in six-monthold infants was modulated by changes in nonsymbolic numerosity [41], further supporting the notion that the role of the right IPS in numerical representations is stable over development, whereas activation of the left IPS changes with increased experience with numerical symbols [40[•]]. Furthermore, individual differences in eightyear-old children's activation in the left intraparietal sulcus (IPS) during symbolic comparison were correlated with arithmetic competence (see Figure 2) [42]. In a related vein, individual differences in 4-11-year-old children's functional connectivity in a frontal-parietal network associated with mapping between representational formats of number were correlated with performance on a standardized mathematics assessment [43]. To summarize, competence with number symbols is associated with changes in the neurobiology of numerical representation,

Figure 2

as well as with individual differences in mathematics performance over development.

Promoting number symbol learning in early childcare settings

Given that knowledge of numerical symbols is a powerful predictor of mathematical achievement, it is necessary to emphasize symbolic knowledge in young children's early numeracy experiences. Crucially, this encompasses multiple aspects of number knowledge: identification, cardinality, and ordinality. This is a potential area in which cognitive science evidence could inform education policy. A study investigating the relationships between parent-led numeracy activities and early mathematics skills revealed informal home numeracy activities were uniquely associated with 5-6-year-olds' nonsymbolic arithmetic (i.e., arithmetic with sets of objects rather than numerals) performance, whereas formal home numeracy activities were associated with their symbolic number knowledge [7^{••}]. It is possible for informal activities, such as games, to incorporate numerical symbols (numerals or words), and therefore symbol knowledge could be fostered through informal activities (see Figure 3). The distinction between formal and informal as applied to both learning opportunities and mathematical knowledge could be counterproductive and limit the types of activities aimed at preschoolers. Formal knowledge, in particular the learning of numerical symbols, should be introduced in preschool in order to prepare students for school-level mathematics.

Preschool numeracy intervention effectiveness has been the focus of recent research from both cognitive and educational perspectives, but a full discussion of these is beyond the scope of the current review (see [44] for a review of early mathematics interventions). While a playbased math curriculum encompassing multiple aspects of numeracy was shown to be effective [45], more research is necessary to evaluate the relative contributions of specific



Correlation between activation in the left IPS during symbolic comparison and children's arithmetic performance. *Source*: Adapted from [39].





Informal activity incorporating numerical symbols in a preschool classroom. This illustrates an activity that fosters children's understanding of multiple representations of number. Photo credit: Anamaria Ralph, Full-Day Early Learning-Kindergarten Program, Maurice Cody Junior Public School, Toronto, Canada.

numerical abilities. In one intervention study, kindergarteners demonstrated improved place-value understanding following researcher-led explicit training using Arabic numerals to represent multi-digit numbers, but training using base-10 blocks to represent multi-digit numbers was not as effective [46[•]]. This highlights that preschoolers can show increased competency with symbolic representations of number following instruction that focuses on symbols rather than manipulating objects. In another intervention study, playing a numerical board game led to improvements in early numeracy in preschoolers from low-income backgrounds [47], and this was adapted in a classroom setting [48]. This is an example of successfully scaling-up from cognitive science findings to educational practice [44]. Combining insights from cognitive science with educational practitioners' classroom experience could lead to the development of informal ways to promote numerical symbol acquisition in young children. As teachers have extensive experience implementing activities promoting early numeracy, they likely have valuable insights into the learning process. Therefore, targeted research questions about cognitive processes should be derived from existing educational practices, known as scaling down from practice to research [49]. Future work should focus on designing and testing informal teacher-led interventions incorporating numerical symbols for preschool childcare settings as well as on elucidating cognitive mechanisms underlying observed improvements.

Conclusions and future directions

In conclusion, developmental cognitive neuroscience research has highlighted knowledge of numerical symbols as a particularly robust longitudinal predictor of math achievement. Crucially, this knowledge mediates the transition between informal and formal mathematics learning, and may promote school readiness. We therefore argue that parents and early childhood care practitioners should incorporate numerical symbols into informal play activities as well as more explicit lessons and not leave it to the formal school context to imbue children with a solid understanding of numerical symbols. Critically, we do not advocate for replacing informal activities with more formal instruction, but instead argue that formal concepts can be introduced in informal contexts earlier to allow children to build stronger foundations. Further work is needed to elucidate cognitive mechanisms underlying the acquisition of mathematical competencies and the specific role of numerical symbols in this process. Thus, future work should focus on implementing and empirically testing classroom-based and home numeracy activities in order to investigate cognitive and neuronal mechanisms that underlie children's developing understanding of symbols. Such work will inform the design and implementation of evidence-based educational practices for promoting early numeracy development.

Conflict of interest

Nothing declared.

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