Multi-Round Attacks on Structural Controllability Properties for Non-Complete Random Graphs

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Abstract. The notion of *controllability*, informally the ability to force a system into a desired state in a finite time or number of steps, is most closely associated with control systems such as those used to maintain power networks and other critical infrastructures, but has wider relevance in distributed systems. It is clearly highly desirable to understand under which conditions attackers may be able to disrupt legitimate control, or to force overriding controllability themselves. Following recent results by Liu *et al*., there has been considerable interest also in graph-theoretical interpretation of Kalman controllability originally introduced by Lin, *structural controllability*. This permits the identification of sets of *driver nodes* with the desired state-forcing property, but determining such nodes is a $W[2]$-hard problem. To extract these nodes and represent the control relation, here we apply the POWER DOMINATING SET problem and investigate the effects of targeted *iterative* multiple-vertex removal. We report the impact that different attack strategies with multiple edge and vertex removal will have, based on underlying non-complete graphs, with an emphasis on power-law random graphs with different degree sequences.

Keywords: Structural Controllability, Attack Models, Complex Networks

1 Introduction

Structural controllability was introduced by Lin's seminal work [1] as an alternative to the controllability, identifying a graph-theoretical model equivalent to Kalman's control [2] in order to reach a desired state from an arbitrary state in a finite number of steps. Although the Kalman's model enables the use of a general, rigorous, and well-understood framework for the design and analysis of not only control systems but also of networks in which a directed control relation between nodes is required, the model presents some restrictions for complex and large systems. A time-dependent linear dynamical system $\mathbf{A}$ is *controllable* if and only if $\text{rank}[\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \ldots, \mathbf{A}^{n-1}\mathbf{B}] = n$ (Kalman's rank criterion), where $\mathbf{A}$ is the $n \times n$ adjacency matrix identifying the interaction among nodes and the $n \times m$ *input* matrix $\mathbf{B}$ identifies the set of nodes controlled by the *input vector*, which forces the system to a desired state. Whilst straightforward, for large networks the exponential growth of input values as a function of nodes is problematic, giving importance to concept of structural controllability. In this context, the graph-theoretical interpretation would be $G(\mathbf{A},\mathbf{B}) = (V,E)$ as a digraph where $V = V_{\mathbf{A}} \cup V_{\mathbf{B}}$ is the set of vertices and $E = E_{\mathbf{A}} \cup E_{\mathbf{B}}$ is the set of edges. In this representation, $V_{\mathbf{B}}$ comprises nodes able to inject control signals into the entire network.

Moreover, recent work by Liu *et al*. [3] has renewed interest in this approach as it allows the identification of *driver nodes* (n_d corresponding to $V_{\mathbf{B}}$) capable of observing the entire network (graph). This work is relied on a non-rigorous formulation of the maximum matching problem and has been expanded upon multiple times [4, 5]. However, we here focus on the equivalent POWER DOMINATING SET (PDS) problem, originally introduced as an extension of DOMINATING SET by Haynes *et al*. [6], mainly motivated by the structure of electric power networks, and the need of offering efficient monitoring of such networks. A real world scenario related to this field is precisely the current control systems (e.g. SCADA systems) which deploy their elements following a mesh distribution to supervise other critical infrastructures (e.g. power systems), where $\mathscr{G} = (V,E)$ depicts the network distribution with V illustrating the elements (e.g. control terminal units, servers, etc.), and E representing the communication lines. In this context, PDS can be defined using the two following *observation rules* simplified by Kneis *et al*. [7]: **OR1**, a vertex in the power dominating set observes itself and all its neighbours; and **OR2**, if an observed vertex v of degree $d \geq 2$ is adjacent to $d-1$ observed vertices, the remaining unobserved vertex becomes observed as well.

With the omission of **OR2**, this reverts to the DOMINATING SET problem already known to be $\mathscr{NP}$-complete with a polynomial-time approximation factor of $\Theta(\log n)$ as shown by Feige [8]. The approach relies on creating directed acyclic graphs $\mathscr{G} = (V,E)$ to find a sequence of driver nodes (denoted as $N_D / \ \forall \ n_d \in N_D$) such that $N_D \subseteq V$ can observe all vertices in V satisfying **OR1** and **OR2**. Instances of driver nodes from a given $\mathscr{G} = (V,E)$ are not unique and clearly depend on the selection order of vertices $\in V$ to create DS using **OR1**. Here, we follow the three strategies defined in [9]: (1) Obtain the set of driver nodes with maximum out-degree satisfying **OR1** ($\mathbf{N}_D^{\max}$); (2) find the set of driver nodes with minimum out-degree satisfying **OR1** ($\mathbf{N}_D^{\min}$); and (3) obtain the set of random nodes satisfying **OR1** ($\mathbf{N}_D^{\text{rand}}$). Hence, each $\mathbf{N}_D^{strat}$ represents a partial order given by the out-degree ($\leq$ or $\geq$) in case of $\mathbf{N}_D^{\max}$ or $\mathbf{N}_D^{\min}$, respectively; in case of $\mathbf{N}_D^{\text{rand}}$, no such relation exists as its elements are randomly chosen.

These three strategies have already been analysed for non-interactive scenarios, in which a single vertex is exposed to a particular type of attack. Our contribution in this paper is therefore to expand the approach from [9] to multiple-round attacks, studying the robustness of controllability when multiple and combined attacks affect control in several different graph classes, namely random (Erdős-Renyi (ER)), small-world (Watts-Strogatz (WS)), and power-law (both (Barabási-Albert (BA)) and general power-law (PLOD) distributions)[1]. In addition, we also analyse here the interaction between different multi-round attack strategies and the underlying control graph topology on robustness, considering both the earlier work on single attacks [9] and new three multi-round attack scenarios. These scenarios are as follows: (1) Removal of some ran-

[1] For more detail on these distribution networks, please go to [9].

dom edges $\in E$ from a single or several vertices, (2) isolation of some vertices $\in V$, and (3) removal of some random edges and vertices from a dense power-law subgraph.

The remainder of this paper is structured as follows: Section 2 describes the threat model based on three different multi-round attack scenarios and on a set of attack models characterised by the number of targets. Later, in section 3 we proceed to evaluate the impact of exploiting strategic points of the aforementioned topologies and its controllability (given by $\mathbf{N}_D^{max}$, $\mathbf{N}_D^{min}$, or $\mathbf{N}_D^{rand}$), discussing the results obtained on connectivity and observability terms. Finally, our conclusions together with an outlook on our ongoing work are given in section 4.

2 Multi-Round Threat Model

In order to study the robustness of the different types of network topologies, first we consider the different attack types (edge and vertex removal), which may disrupt controllability (e.g. denial of service attacks to communication lines in order to leave parts of a system uncontrolled, unprotected or isolated by n_d), and the resulting effects that such attacks may cause in the control, the network connectivity and observability (as the dual of controllability). The threats studied here are multi-round attacks with prior knowledge, but do not explicitly take mitigating responses of defenders into account.

Algorithm 2.1: ATTACK MODELS ($\mathscr{G}(V,E), \mathbf{N}_D^{strat}, AM, Scenario$)

```
output (Attack of one vertex for a given G(V,E));
local i, target;

if AM == AM_1 (F)
  then {target ← N_D^strat[1];
  else { if AM == AM_2 (M)
           then {target ← N_D^strat[(SIZE(N_D^strat))/2];
           else { if AM == AM_3 (L)
                    then
                    {target ← N_D^strat[(SIZE(N_D^strat))];
                    else { if AM == AM_4 (BC)
                             then
                             {target ← BETWEENNESS CENTRALITY(G(V,E));
                             else {target ← OUTSIDE N_D^strategy(G(V,E), N_D^strat);
if Scenario == SCN-1
  then REMOVE SELECTIVE EDGES(G(V,E), target);
if Scenario == SCN-2
  then ISOLATE VERTEX(G(V,E), target);
return (G(V,E))
```

Targets	Combination of AM$-x$	Num. of Attacks
TG-1	F, M, L, BC, O	5
TG-2	F-M, F-L, F-BC, F-O, M-L, M-BC, M-O, L-BC, L-O, BC-O	10
TG-3	F-M-L, F-M-BC, F-M-O, F-L-BC, F-L-O, F-BC-O, M-L-BC, M-L-O, M-BC-O, L-BC-O	10
TG-4	F-M-L-BC, F-M-L-O, F-M-BC-O, F-L-BC-O, M-L-BC-O	5
TG-5	F-M-L-BC-O	1

Table 1. Five attacks rounds with permuted AM

These threats are based on the combination of five attack models (**AMs**), which have been grouped into three scenarios for the purposes of further analysis: **Scenario 1 (SCN-1)**, it focuses on removing a small number of random edges of one or several vertices, which may compromise controllability of dependent nodes or disconnect parts the of control graph and underlying network. The selection of target nodes depends on the **AM** described below, and the removal of edges avoids spurious node isolation. **Scenario 2 (SCN-2)** destined to isolate one or several vertices from the network by intentionally deleting all the links from these vertices. This threat may result in the isolation of vertices which depend on the compromised node or in the partition of the network into several sub-graphs. **Scenario 3 (SCN-3)** aims to attack one or several vertices of a sub-graph by randomly deleting part of their links (**SCN-1**), or carry out the isolation of such nodes (**SCN-2**) so as to later assess the resulting effect of the threat with respect to the entire graph. For the extraction of the sub-graph, we consider the *Girvan-Newman* algorithm to detect and obtain specific communities within a complex graph [10]. A community structure refers to a subset of nodes with dense links within its community and with few connections to nodes belonging to less dense communities. For this, links between communities are sought by progressively calculating the betweenness of all existing edges and removing edges with the highest betweenness.

Algorithm 2.2: MULTI-ROUND ATTACKS($\mathscr{G}(V,E), \mathbf{N}_D^{\text{strat}}$, **TG**-x, *Scenario*)

```
output (Attack of one or several vertices for a given 𝒢(V,E));
local i, Combination_AM, AM, SCN;

if Scenario == SCN-3
  then { 𝒢_sub(V,E) ← GIRVAN-NEWMAN(𝒢(V,E));
         N_D^strat ← EXTRACT DRIVER NODES FROM SUBGRAPH(𝒢_sub(V,E), N_D^strat);
         SCN ← DETERMINE NEW SCN-1-2(); }
Combination_AM ← COMBINE ATTACKS(TG-x); comment: See table 1;
for i ← SIZE(Combination_AM)
  do { AM ← Combination_AM[i];
       if Scenario == SCN-3
         then { 𝒢(V,E) ← ATTACK MODELSII(𝒢(V,E), 𝒢_sub(V,E), N_D^strat, AM, SCN);
                comment: Algorithm analogous to 2.1, but considering 𝒢_sub(V,E) }
         else { 𝒢(V,E) ← ATTACK MODELS(𝒢(V,E), N_D^strat, AM, Scenario); }
return (𝒢(V,E);)
```

For each scenario, we select a set of attacks in which it is assumed that an attacker is able to know the distribution of the network and the power domination relation (control graph). In real scenarios, these attackers could be insiders who belong to the system, such as human operators, who known the topology and its system itself; or outsiders who observe and learn from the topology to later damage the entire system or sub-parts. The mentioned attacks, summarised in algorithm 2.1, are denoted as **AM-1** to **AM-5**. **AM-1** consists of attacking the first (**F**) driver node n_d in a given ordered set $\mathbf{N}_D^{\text{strat}}$. Depending on the attack scenario, the attacker could randomly delete some edges or completely isolate the n_d from $\mathscr{G} = (V,E)$. In constrast, **AM-2** aims to attack or isolate a vertex n_d belonging to a given ordered $\mathbf{N}_D^{\text{strat}}$ positioned in the middle (**M**) of the set. **AM-3** attacks the last (**L**) driver node n_d in the ordered set given by $\mathbf{N}_D^{\text{strat}}$. **AM-4** compromises the vertex $v \in V$ with the highest *betweenness centrality* (**BC**), whereas **AM-5** randomly chooses a vertex $v \in V$ and $\notin \mathbf{N}_D^{\text{strat}}$ (outside (**O**)).

Combinations of **AM-**x (which are only representative of wider classes), such that $x \in \{\mathbf{1,2,3,4,5}\}$, result in a set of rounds based on multi-target attacks, which are represented in table 1 and described as follows: **1 Target (TG-1)** illustrates a non-interactive scenario in which a single vertex $v \in V$ is attacked according to an **AM-**x, being v a driver node or an observed node. In contrast, **2 Targets (TG-2)** corresponds to a multi-round scenario based on two attacks **AM-**x and **AM-**y $x, y \in \{\mathbf{1,2,3,4,5}\}$ such that $x \neq y$, e.g. the attack **F-BC** identifies multiple attacks of type **AM-1** and **AM-4**, in which one or several attackers compromise two strategic nodes. Note that **3-5 Targets (TG-3-5)** is a multi-round scenario based on 3, 4 or 5 threats with analogous goals and similar features to **TG-2**. All objectives are summarised in algorithm 2.2, which depends on the type of scenario and the number of targets to be attacked. For scenarios of type **SCN-3**, we first extract the sub-graph from $\mathscr{G}(V,E)$ using the Girvan-Newman algorithm and its driver nodes to be attacked. For the attack itself, we not only consider the sub-graph itself but also $\mathscr{G}(V,E)$ to study the effects that attacks on dense sub-graphs may have on the overall network.

Nomenclature	Definition
n_d	Driver node
AM-x	Attack model following a particular attack strategy x, such that $x \in \{$**AM-1**$,\ldots,$ **AM-5**$\}$
TG-x	Number of target nodes such that $x \in \{$**TG-1**,..., **TG-5**$\}$
$\mathbf{N}_D^{strat}$	Set of driver nodes n_d following a particular controllability strategy such as $\mathbf{N}_D^{\text{max,min,rand}}$
$\mathbf{N}_D^{max,min,rand}$	An attack with minor impact on structural controllability $\mathbf{N}_D^{max,min,rand}$
$\mathbf{N}_D^{max_\dagger,min_\dagger,rand_\dagger}$	An attack with intermediate impact on structural controllability, intensifying effect caused by $\mathbf{N}_D^{max,min,rand}$
$\mathbf{N}_D^{max_\ddagger,min_\ddagger,rand_\ddagger}$	An attack with major impact on structural controllability, intensifying effect caused by †
$*$	Symbol stating *for all* the cases
$\mathbf{N}_{D_{s,l,*}}^{\text{strat}}$	Representation of small and large networks
*****,{**AM-x**}	Influence of all attacks, but with a special vulnerability for **AM-x**
{X-**AM-x**}	Any X threat combined with **AM-x**
$x-y$%	Minimum and maximum rate of observability

Table 2. Nomenclature for analyses

3 Attack Scenarios on Structural Controllability

So as to evaluate the structural controllability strategies defined in [9] ($\mathbf{N}_D^{\text{max}}$, $\mathbf{N}_D^{\text{min}}$, $\mathbf{N}_D^{\text{rand}}$) with respect to ER, WS, BA and PLOD distributions, scenarios **SCN-1**, **SCN-2** and **SCN-3** defined in section 2 were studied Matlab simulations. Several topologies and network sizes were generated, giving small ($\leq$ 100) and large ($\geq$ 100) networks with 100, 1000 and 2000 nodes, and with low connectivity probability so as to represent sparse networks. Under these considerations, we assess the robustness from two perspectives: First, the *degree of connectivity* through the diameter, the global density and the local density using the average clustering coefficient (CC). These statistical values should maintain small values in proportion to the growth and the average degree of links per node, and more specifically, after an attack. Second, the *degree of observability* by calculating the rate of unobserved nodes after a threat using **OR1** [9]. Given the number of simulations carried out and results obtained for the analyses[2], we have defined a language to summarize and interpret results shown in table 2.

3.1 SCN-1 and SCN-2: Exploitation of Links and Vertices in Graphs

For **SCN-1** (see table 3), we observe that ER topologies are sensitive in connectivity terms. The diameter for small networks is variable and, particularly, for networks under the control of $\mathbf{N}_D^{\text{max,min}}$, with a special emphasis in scenarios **TG-3** where a complete break up of the network is verified and the observation rate is largely influenced,

[2] Full results and code is available from authors

reaching null values. As for local and global density, it is also variable for all network distributions and for all **TG-x**, where the controllability $\mathbf{N}_D^{min,rand}$ are mainly affected. For WS graphs, the diameter changes for any distribution, but particularly for small networks, and the greatest effect is obtained when launching a **TG-3** attack. For this topology, the density of the network is slightly modified when performing a **TG-2** attack, whereas no relevant effect has been registered for the other cases. This does not, however, hold for local density, since the effects on the network become more and more evident as the number of targets increases, especially when the number of nodes that constitute the network is not high (as expected in small-world networks). The impact on the observability is not very accentuated for this topology, as the effect is more evident when performing an attack to the n_d with the maximum out-degree in small networks.

	Connectivity					Observability		
TGx	**Network**	**Diameter**	**Density**	**CC**	**Attack**	**Observation**	**Attack**	**Rate**
TG-1	ER	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand_\dagger}$	$\mathbf{N}_{Ds}^{max,min_\dagger,rand_\dagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	*	$\mathbf{N}_{Dl}^{max_\dagger,min,rand_\dagger}$	*	96.8-100%
	WS	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*,{**BC**}	$\mathbf{N}_{Ds}^{max_\dagger,min,rand}$	*	84-99%
	BA	$\mathbf{N}_{Ds}^{max,min,rand}$	-	-	*,{**BC**}	$\mathbf{N}_{Ds}^{max_\dagger,min,rand}$	*,{**F**}	16-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D\ast}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max_\dagger,min,rand_\dagger}$	*,{**BC**}	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D\ast}^{max,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*,{**BC**}	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D\ast}^{max,min_\ddagger,rand_\ddagger}$	-	$\mathbf{N}_{Ds}^{min,rand}$	*	-	-	$\simeq$ 100%
TG-2	ER	$\mathbf{N}_{D\ast}^{max,min,rand}$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	*, {**X-BC**}	$\mathbf{N}_{Dl}^{max_\dagger,rand_\dagger}$	*	96.7-100%
	WS	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{min,rand}$	$\mathbf{N}_{D\ast}^{max_\dagger,min_\dagger,rand_\dagger}$	*	$\mathbf{N}_{D\ast}^{max_\dagger,min,rand}$	*	88-97.85%
	BA	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	*	$\mathbf{N}_{Ds}^{max_\dagger,min,rand}$	*, {**F-BC,L-BC**}	4-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	-	$\mathbf{N}_{Ds}^{max_\dagger,min,rand}$	*, {**F-BC**, **M-BC**, **BC-O**}	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*, {**M-BC**, **L-BC**, **BC-O**}	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand_\dagger}$	*	-	-	$\simeq$ 100%
TG-3	ER	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand}$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	*, {**M-BC-O**, **L-BC-O**}	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand}$	*, {**M-BC-O**, **L-BC-O**}	0-100%
	WS	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand}$	-	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	*, {**M-BC-O**, **L-BC-O**}	$\mathbf{N}_{D\ast}^{max_\dagger,min_\dagger,rand}$	*, {**M-BC-O**, **L-BC-O**}	2-98%
	BA	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand}$	$\mathbf{N}_{Ds}^{rand}$	-	*, {**M-BC-O**, **L-BC-O**}	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand}$	*, {**F-M-L**, **M-BC-O**, **L-BC-O**}	0-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*, {**M-BC-O**, **L-BC-O**}	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger}$	*, {**M-BC-O**, **L-BC-O**}	0-100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*, {**M-BC-O**, **L-BC-O**}	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger}$	*, {**M-BC-O**, **L-BC-O**}	0-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand_\dagger}$	*, {**M-BC-O**, **L-BC-O**}	$\mathbf{N}_{D\ast}^{max_\ddagger,min_\ddagger}$	*, {**M-BC-O**, **L-BC-O**}	0-100%
TG-4	ER	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	*	$\mathbf{N}_{Dl}^{max,rand}$	*	96.4-100%
	WS	$\mathbf{N}_{Ds}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	*	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand_\dagger}$	*	86-97.85%
	BA	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{rand}$	-	*, {**F-M-L-O**}	$\mathbf{N}_{Ds}^{max_\dagger,min,rand}$	*, {**F-M-L-O**, **F-M-BC-O**}	4-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D\ast}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max_\dagger,min,rand}$	*	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D\ast}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*	-	-	$\simeq$ 100%
TG-5	ER	$\mathbf{N}_{Ds}^{rand}$	$\mathbf{N}_{D\ast}^{max,min_\dagger,rand_\dagger}$	$\mathbf{N}_{D\ast}^{max_\dagger,min_\dagger,rand_\dagger}$	*	$\mathbf{N}_{Dl}^{max,rand}$	*	96.3-100%
	WS	$\mathbf{N}_{Ds}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	*	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand_\dagger}$	*	86-97.85%
	BA	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{rand}$	-	*	$\mathbf{N}_{Ds}^{max_\dagger,min,rand}$	*	14-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D\ast}^{max,rand}$	-	$\mathbf{N}_{Ds}^{min,rand}$	*	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D\ast}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D\ast}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*	-	-	$\simeq$ 100%

Table 3. SCN-1: Removal of a small number of edges $\in E$ from one or several vertices $\in V$

For BA graphs, the diameter shows a small variation for any $\mathbf{N}_D^{strat}$ and for both single and multiple targets. The difference is made by the **TG-3** strategy, for which the consequences on the network are remarkable both for small and large networks. The global density of the network is influenced mainly when a small network is considered and the links of a random n_d are damaged ($\mathbf{N}_D^{\mathrm{rand}}$). Unlike ER and WS, the CC of the BA does not significantly change, but its observability is heavily compromised for any **TG-x** where the control relies on $\mathbf{N}_D^{\mathrm{max}}$. In contrast, power-law distributions with $\alpha = 0.1$, 0.3 and 0.5 show a high robustness in connectivity and observability terms where observation rate reaches values $\simeq 100\%$. The global density is not affected even if CC mainly varies for small networks and the diameter specially impacts on both $\mathbf{N}_D^{\mathrm{min,rand}}$ for dense distributions with $\alpha = 0.5$ and $\mathbf{N}_D^{\mathrm{max,min}}$ for different exponents in **TG-3** scenarios.

		Connectivity				Observability		
TGs	**Network**	**Diameter**	**Density**	**CC**	**Attack**	**Observation**	**Attack**	**Rate**
TG-1	ER	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\dagger},min_{\dagger},rand_{\ddagger}}$	$\mathbf{N}_{Ds}^{max_{\ddagger},min_{\dagger},rand_{\dagger}}$	***, {F, BC}**	$\mathbf{N}_{Ds}^{max_{\dagger},rand_{\dagger}}$	***, {F,M}**	86-100%
	WS	$\mathbf{N}_{D*}^{max,min,rand}$	-	$\mathbf{N}_{D*}^{max_{\dagger},min_{\dagger},rand_{\ddagger}}$	***, {BC}**	$\mathbf{N}_{Ds}^{max,min,rand}$	*****	89-100%
	BA	-	-	-	-	$\mathbf{N}_{Ds}^{max_{\ddagger},min,rand}$	***, {F, M, L, BC}**	2-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max_{\dagger},min,rand_{\dagger}}$	***, {BC}**	$\mathbf{N}_{D*}^{max,min,rand}$	***, {O}**	99-100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D*}^{max,min,rand}$	-	$\mathbf{N}_{D*}^{max_{\dagger},min_{\ddagger},rand_{\dagger}}$	***, {BC}**	$\mathbf{N}_{D*}^{max,min}$	*****	98-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max_{\dagger},min_{\dagger},rand_{\dagger}}$	***, {BC}**	$\mathbf{N}_{Ds}^{max}$	*****	97-100%
TG-2	ER	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	$\mathbf{N}_{D*}^{max_{\dagger},min_{\ddagger},rand_{\dagger}}$	***, {F, BC}**	$\mathbf{N}_{D*}^{max_{\ddagger},min,rand_{\dagger}}$	***,{F-M,F-BC,F-O}**	70-100%
	WS	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	*****	$\mathbf{N}_{D*}^{max_{\ddagger},min,rand}$	***,{F-O}**	84-98%
	BA	-	$\mathbf{N}_{Ds}^{min,rand}$	-	*****	$\mathbf{N}_{D*}^{max_{\ddagger},min,rand_{\dagger}}$	*****	2-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max_{\dagger},min,rand_{\dagger}}$	***, {F-BC, M-BC, L-BC, BC-O}**	$\mathbf{N}_{Ds}^{max,min,rand}$	*****	99-100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D*}^{max_{\dagger},min_{\dagger},rand_{\dagger}}$	$\mathbf{N}_{Ds}^{max,min,rand_{\dagger}}$	$\mathbf{N}_{Ds}^{max_{\dagger},min_{\ddagger},rand_{\dagger}}$	***,{L-BC, BC-0}**	$\mathbf{N}_{Ds}^{max,min}$	*****	98-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\dagger},min_{\dagger},rand_{\dagger}}$	***, {F-BC, M-BC, L-BC, BC-O}**	$\mathbf{N}_{Ds}^{max}$	*****	97-100%
TG-3	ER	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\dagger},rand_{\dagger}}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\dagger},rand_{\dagger}}$	***,{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\dagger}}$	***,{M-BC-O, L-BC-0}**	0.15-99.90%
	WS	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\dagger}}$	$\mathbf{N}_{Ds}^{max_{\dagger},min,rand}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	***,{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand}$	***,{M-BC-O, L-BC-0}**	2-98%
	BA	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand}$	$\mathbf{N}_{Ds}^{min,rand}$	-	***,{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\dagger}}$	***,{M-BC-O, L-BC-0}**	0-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\dagger},min,rand}$	***,{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand}$	***,{M-BC-O, L-BC-0}**	0.15-100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand}$	$\mathbf{N}_{Ds}^{max,min_{\dagger},rand_{\dagger}}$	$\mathbf{N}_{Ds}^{max,min,rand}$	***,{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger}}$	***,{M-BC-O, L-BC-0}**	0-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{D*}^{max,min_{\dagger},rand_{\dagger}}$	***,{M-L-O, L-BC-0} L-BC-O, M-BC-O}**	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand}$	***,{M-BC-O, L-BC-0}**	0-100%
TG-4	ER	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	*****	$\mathbf{N}_{D*}^{max_{\dagger},min,rand_{\dagger}}$	*****	66-99.90%
	WS	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\ddagger},min_{\dagger},rand_{\dagger}}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	*****	$\mathbf{N}_{D*}^{max_{\dagger},min_{\dagger},rand_{\dagger}}$	*****	82-97.85%
	BA	-	$\mathbf{N}_{Ds}^{max,min,rand_{\dagger}}$	$\mathbf{N}_{Ds}^{min,rand}$	*****	$\mathbf{N}_{Ds}^{max_{\ddagger},min,rand_{\dagger}}$	*****	2-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\ddagger},min,rand_{\ddagger}}$	*****	$\mathbf{N}_{Ds}^{max,min,rand}$	*****	99-100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	*****	$\mathbf{N}_{Ds}^{max,min}$	*****	98-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\dagger},min_{\dagger},rand_{\dagger}}$	*****	$\mathbf{N}_{Ds}^{max}$	*****	96-100%
TG-5	ER	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	*****	$\mathbf{N}_{D*}^{max_{\dagger},min,rand_{\dagger}}$	*****	68-99.85%
	WS	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\ddagger},min_{\dagger},rand_{\dagger}}$	$\mathbf{N}_{D*}^{max_{\ddagger},min_{\ddagger},rand_{\ddagger}}$	*****	$\mathbf{N}_{D*}^{max_{\dagger},min_{\dagger},rand_{\dagger}}$	*****	84-97.85%
	BA	-	$\mathbf{N}_{Ds}^{max,min,rand_{\dagger}}$	$\mathbf{N}_{Ds}^{min,rand}$	*****	$\mathbf{N}_{Ds}^{max_{\ddagger},min,rand_{\dagger}}$	*****	2-100%
	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\ddagger},min,rand_{\ddagger}}$	*****	$\mathbf{N}_{Ds}^{max,min,rand}$	*****	99-99.85%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	*****	$\mathbf{N}_{Ds}^{max,min}$	*****	98-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_{\dagger},min_{\dagger},rand_{\dagger}}$	*****	$\mathbf{N}_{Ds}^{max}$	*****	96-100%

Table 4. SCN-2: Isolation of one or several vertices $\in V$

For **SCN-2** scenarios, we observe that ER topologies continues to be very sensitive in connection terms, and the global and local density drastically vary for any **TG-x**. The observation rate is moderately high, but it presents certain weaknesses to attack models containing **AM-1**, **AM-2**, **AM-4** and **AM-5** aiming to break down $\mathbf{N}_D^{\mathrm{max,rand}}$. The diameter in WS networks slightly changes for any $\mathbf{N}_D^{\mathrm{strat}}$ where the global density remains invariant for **TG-1** and its value notably decreases according to the number of isolated nodes, and specifically for small networks despite the drastic change for CC. The observation rate remains high with exception to multi-interactive threat scenarios based on **TG-3**. As in **SCN-1**, the diameter, density and the CC of BA in **SCN-2** networks remains almost invariant what shows its robustness degree for all types of **AM-s**. Nonetheless, the densities can suffer some changes when three or more nodes are compromised and these nodes belong mainly to $\mathbf{N}_D^{rand}$. Moreover, the rate reaches $\simeq 2\%$ of the observation when driver nodes primarily of the $\mathbf{N}_D^{max}$ are compromised.

	Connectivity					Observability		
TGs	**Network**	**Diameter**	**Density**	**CC**	**Attack**	**Observation**	**Rate**	**Attack**
TG-1	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max,min,rand\dagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{M, L, BC}**	$\mathbf{N}_{Ds}^{max,rand}$	**{F, M, L, BC}**	99.70-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{Dl}^{rand}$	-	$\mathbf{N}_{Ds}^{max,rand}$	**{L}**	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{Dl}^{max,min}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{L, BC}**	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{M, L, BC}**	$\mathbf{N}_{Ds}^{max}$	*	98-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{Dl}^{max,min}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{L, BC}**	$\mathbf{N}_{Ds}^{max}$	*	98-100%
TG-2	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand\dagger}$	*,**{X-BC}**	$\mathbf{N}_{Ds}^{max,rand}$	*	99.70-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{D*}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{F-O, F-L, M-L, M-O}**	-	-	$\simeq$100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{Dl}^{max\dagger,min\dagger,rand\dagger}$	-	$\mathbf{N}_{Ds}^{max\dagger,min\dagger,rand}$	*	-	-	$\simeq$100%
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max\dagger,min\dagger,rand\dagger}$	-	$\mathbf{N}_{Ds}^{max\dagger,min,rand}$	*	$\mathbf{N}_{Ds}^{max}$	*	97-100%
	PLOD $\alpha \simeq 0.5$	-	-	$\mathbf{N}_{Ds}^{max\dagger,min,rand}$	**{L, BC}**	$\mathbf{N}_{Ds}^{max}$	*	98-100%
TG-3	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand\dagger}$	*,**{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	*,**{M-BC-O, L-BC-0}**	0-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{M-L-O,M-BC-O L-BC-0}**	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	* **{M-BC-O, L-BC-0}**	0-100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	-	$\mathbf{N}_{Ds}^{max\dagger,min\dagger,rand}$	*,**{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	*,**{M-BC-O, L-BC-0}**	0-100%
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	-	$\mathbf{N}_{Ds}^{max\dagger,min,rand}$	*,**{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	*,**{M-BC-O, L-BC-0}**	0-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	-	$\mathbf{N}_{Ds}^{max\dagger,min\dagger,rand}$	*,**{M-BC-O, L-BC-0}**	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	*,**{M-BC-O, L-BC-0}**	0-100%
TG-4	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max\ddagger,min\ddagger,rand\ddagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand\dagger}$	*	$\mathbf{N}_{Ds}^{max,rand}$	*	99-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{Dl}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{F-L-BC-O,F-M-L-O F-M-L-BC}**	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{Dl}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max\dagger,min\dagger,rand}$	*	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max\dagger,min\dagger,rand\dagger}$	-	$\mathbf{N}_{Ds}^{max}$	*, **{F-M-BC-O}**	$\mathbf{N}_{Ds}^{max}$	*, **{F-M-BC-O}**	97-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{Ds}^{max}$	-	$\mathbf{N}_{Ds}^{max\dagger,min\dagger,rand}$	**{M-L-BC-O}**	$\mathbf{N}_{Ds}^{max}$	*	98-100%
TG-5	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{Dl}^{max\ddagger,min\ddagger,rand\ddagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand\dagger}$	*	$\mathbf{N}_{Ds}^{max,rand}$	*	99-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{Dl}^{min,rand}$	-	$\mathbf{N}_{Ds}^{rand}$	*	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{Dl}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	*	-	-	$\simeq$ 100%
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{Dl}^{max,min,rand}$	-	-	*	$\mathbf{N}_{Ds}^{max}$	*	97-100%
	PLOD $\alpha \simeq 0.5$	-	-	$\mathbf{N}_{Ds}^{max\dagger,min,rand}$	*	$\mathbf{N}_{Ds}^{max}$	*	98-100%

Table 5. SCN-3: Removal of a few edges (**SCN-1**) of a given sugraph $\mathscr{G}_{sub} = (V,E)$

This does not occur with general power-law networks where the observability degree, except for **TG-3**, reaches the 90% of the observation at all times, in addition to following similar behaviour pattern for any exponent value. While no effect is appreciated in diameter, the density decays only in small networks when two or more nodes are excluded from the graph. The consequences on the CC for small networks are not negligible, but the greatest consequences have been observed in observability when 3 nodes are removed. Lastly, common behaviours in **SCN-1** and **SCN-2** arise. The removal of random links in three vertices or the isolation of three vertices (**TG-3**) using the combination **M-BC-O** and **L-BC-O** can cause the breakdown of the entire graph. These two configurations seem to be the most menacing within the configuration given in table 1, in which the observability is largely influenced for any distribution and the diameter is drastically decreased for $\mathbf{N}_D^{max,min}$. In addition, threats of the type **AM-4** stand out from the rest, underlying the importance of protecting the node with the highest centrality.

3.2 SCN3: Exploitation of Links and Vertices in Power-Law Subgraphs

Tables 5 and 6 show results obtained for attacks on a small number of random edges (**SCN-1**) or isolation of one or several vertices (**SCN-2**) from power-law subgraphs. Varying the exponent, we observe that these types of networks have similar behavioural characteristics to those analysed in section 3.1. Unfortunately, the observation degree decays extremely when the graph is subjected to attacks of type **M-BC-O** and **L-BC-O**, where two n_d of the sub-graph and a vertex of the sub-graph, but outside the $\mathbf{N}_D^{\mathrm{strat}}$, are attacked simultaneously. Moreover, these two attack combinations are also dangerous in connectivity terms. The diameter values radically vary for any $\mathbf{N}_D^{\mathrm{strat}}$ and for any distribution, although the global density remains broadly constant. Obviously, when the sub-graph is subjected to massive attacks to isolate a single or multiple nodes, the diameter, density, and CC of the entire network vary. Table 6 shows this, where the diameter primarily changes for any large distribution, whereas the local and global densities impact on small networks. As in the previous case, the observability is high at all times, even if insignificant variations caused by attacks in $\mathbf{N}_D^{\mathrm{max}}$ arise.

	Connectivity					Observability		
TGs	**Network**	**Diameter**	**Density**	**CC**	**Attack**	**Observation**	**Rate**	**Attack**
TG-1	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max_\dagger,min_\dagger,rand_\dagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand_\dagger}$	**{M, L, BC}**	$\mathbf{N}_{Ds}^{max,rand}$	*	99-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{Dl}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	***{L,BC}**	-	-	$\simeq 100$
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{Dl}^{max_\dagger,min_\dagger,rand_\dagger}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{L, BC}**	-	-	$\simeq 100$
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max_\dagger,min_\dagger,rand_\dagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand_\dagger}$	***{M, L, BC}**	$\mathbf{N}_{Ds}^{max_\dagger,rand}$	*, **{M, BC}**	96-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max,min,rand}$	-	$\mathbf{N}_{Ds}^{max,min,rand}$	**{M, L}**	$\mathbf{N}_{Ds}^{max}$	*	99.60-100%
TG-2	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max_\dagger,min_\dagger,rand_\dagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand_\dagger}$	**{F-O, M-L, X-BC}**	$\mathbf{N}_{Ds}^{max,rand}$	*	98-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_\dagger,min,rand_\dagger}$	***{F-O, M-L, L-O}**	-	-	$\simeq 100\%$
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{Dl}^{max,min_\dagger,rand_\dagger}$	-	$\mathbf{N}_{Ds}^{min_\dagger,rand}$	**{M-L, X-BC}**	-	-	$\simeq 100\%$
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max_\dagger,min_\dagger,rand_\dagger}$	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand_\dagger}$	$\mathbf{N}_{Ds}^{max_\ddagger,min_\dagger,rand_\dagger}$	***{X-BC}**	$\mathbf{N}_{Ds}^{max_\dagger,rand}$	*, **{F-X, X-BC}**	97-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{Ds}^{max_\dagger,min,rand}$	-	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand}$	*,**{F-L, M-L, X-BC}**	$\mathbf{N}_{Ds}^{max}$	*,**{BC-O}**	96-100%
TG-3	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand_\dagger}$	*,**{M-BC-O, L-BC-O}**	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	*	0-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_\dagger,min,rand_\dagger}$	*,**{M-BC-O, L-BC-O}**	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	*,**{M-BC-O, L-BC-O}**	0-100%
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min_\dagger,rand}$	*,**{M-BC-O, L-BC-O}**	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	*,**{M-BC-O, L-BC-O}**	0-100%
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand_\dagger}$	$\mathbf{N}_{Ds}^{max_\ddagger,min_\dagger,rand_\dagger}$	*,**{M-BC-O, L-BC-O}**	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	*,**{M-BC-O, L-BC-O}**	0-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max_\dagger,min_\dagger,rand_\dagger}$	*,**{M-BC-O, L-BC-O}**	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	*,**{M-BC-O, L-BC-O}**	0-100%
TG-4	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand_\dagger}$	*	$\mathbf{N}_{Ds}^{max,min,rand}$	*	99-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{D*}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	*	-	-	$\simeq 100\%$
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{Dl}^{max_\dagger,min_\dagger,rand_\dagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{min_\dagger,rand}$	*	-	-	$\simeq 100\%$
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max_\dagger,min_\dagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max_\ddagger,min_\dagger,rand_\dagger}$	*	$\mathbf{N}_{D*}^{max_\dagger,rand}$	*	96-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max_\dagger,min,rand}$	$\mathbf{N}_{Ds}^{max,min,max}$	$\mathbf{N}_{D*}^{max_\dagger,min,rand}$	*	$\mathbf{N}_{Ds}^{max}$	*	96-100%
TG-5	PLOD $\alpha \simeq 0.1$	$\mathbf{N}_{D*}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	*	$\mathbf{N}_{Ds}^{max,min,rand}$	*	99-100%
	PLOD $\alpha \simeq 0.2$	$\mathbf{N}_{Dl}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{max,rand}$	*	-	-	$\simeq 100\%$
	PLOD $\alpha \simeq 0.3$	$\mathbf{N}_{D*}^{max_\dagger,min,rand}$	$\mathbf{N}_{Ds}^{max,min,rand}$	$\mathbf{N}_{Ds}^{min_\dagger,rand}$	*	-	-	$\simeq 100\%$
	PLOD $\alpha \simeq 0.4$	$\mathbf{N}_{D*}^{max_\ddagger,min,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max_\ddagger,min_\ddagger,rand_\ddagger}$	$\mathbf{N}_{Ds}^{max,min_\ddagger,rand_\ddagger}$	*	$\mathbf{N}_{D*}^{max_\dagger,rand}$	*	96-100%
	PLOD $\alpha \simeq 0.5$	$\mathbf{N}_{D*}^{max_\dagger,min,rand}$	$\mathbf{N}_{Ds}^{max,min,max}$	$\mathbf{N}_{D*}^{max_\dagger,min,rand}$	*	$\mathbf{N}_{Ds}^{max}$	*	96-100%

Table 6. SCN-3: Isolation of vertices (**SCN-2**) of a given sugraph $\mathscr{G}_{sub} = (V,E)$

Given this, we conclude that both the connectivity and observation not only depend on the network topology and construction strategies of driver nodes ($\mathbf{N}_D^{\mathrm{strat}}$), but also on the nature of the perturbation [5], where degree-based attacks (e.g. **AM-1**) and attacks to centrality (**AM-4**) are primarily significant. On the other hand, BA (see table 3) and power-law (PLOD) distributions present analogous behaviours with respect to observability. Both are mainly vulnerable to threats given in $\mathbf{N}_D^{\max}$ for small networks, and they are no only sensitive to **TG-3** attacks, but also to **TG-4** based on a planned **F-M-BC-O** attack in **SCN-1**. This also means that an adversary with sufficient knowledge of the network distribution and its power domination can disconnect the entire network and leave it without observation at very low cost.

4 Conclusions

We have reported results of a robustness analysis on structural controllability through the POWER DOMINATING SET problem, extending the study given in [9] to consider multi-round attack scenarios. We have primarily focused on random (*Erdös-Renyi*), small-word (*Watts-Strogatz*), scale-free (*Barabási-Albert*) and power-law (*PLOD*) distributions, where we have observed that these networks are sensitive in connectivity and observability terms. These weaknesses are mainly notable when nodes with the highest degree distribution and with the maximum value of betweenness centrality are compromised. Moreover, we have shown that combined attacks based on three specific nodes (**M-BC-O** and **L-BC-O**) can become highly disruptive, even if the power-law network has proven to be robust with respect to the rest of topologies. Regarding future work, sub-optimal approximations to repair the controllability when the power dominance relationship might have been partially severed will be considered taking into account the handicap of the non-locality of the PDS and the $\mathcal{NP}$-hardness demonstrated in [6].

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References

1. C. Lin. Structual Controllability. *IEEE Trans. on Automatic Control*, 19(3):201–208, 1974.
2. R. Kalman. Mathematical description of linear dynamical systems. *Journal of the Society of Industrial and Applied Mathematics Control Series A*, 1:152–192, 1963.
3. Y. Liu, J. Slotine, and A. Barabási. Controllability of Complex Networks. *Nature*, 473:167–173, 2011.
4. W. Wang, X. Ni, Y. Lai, and C. Grebogi. Optimizing controllability of complex networks by minimum structural perturbations. *Physical Review E*, 85(2):026115, 2012.
5. C. Pu, W. Pei, and A. Michaelson. Robustness analysis of network controllability. *Physica A: Statistical Mechanics and its Applications*, 391(18):4420–4425, 2012.
6. T. Haynes, S. Hedetniemi, S. Hedetniemi, and M. Henning. Domination in Graphs Applied to Electric Power Networks. *SIAM Journal on Discrete Mathematics*, 15(4):519–529, 2002.
7. J. Kneis, D. Mölle, S. Richter, and P. Rossmanith. Parameterized Power Domination Complexity. *Information Processing Letters*, 98(4):145–149, 2006.
8. U. Feige. A Threshold of $\ln n$ for Approximating Set Cover. *Journal of the ACM*, 45(4):634–652, July 1998.
9. C. Alcaraz, E. E. Miciolino, and S. Wolthusen. Structural Controllability of Networks for Non-Interactive Adversarial Vertex Removal. In *the Eighth International Workshop on Critical Information Infrastructures Security*, LNCS. Springer, 2013. (in press).
10. M. Newman ;M. Girvan. Community structure in social and biological networks. *the National Academy of Sciences of the United States of America*, 99(12):7821–7826, 2002.