



**A Comparative Analysis of Three High School Textbooks’
Concepts in Algebra in South Africa and Angola**

by

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ABSTRACT

The purpose of this study was to conduct a comparative analysis of algebra sections in three textbooks; one South African grade 9, one South African grade 10, and one Angolan grade 10. Firstly, I compared the textbooks in terms of topics covered. Secondly, I compared the distribution of the text on explanations, examples and exercises, respectively. Thirdly, I used the levels of understanding of algebraic expressions suggested by Sfard and Linchevski (1994) to investigate progression and consistency of the texts. Finally, I looked at the number of steps required to move from task to solution in the examples and exercises, providing a different measure of progression. The findings revealed that the textbook for Angola is more advanced than the textbooks for South Africa in terms of topics, explanations and examples, and contains far fewer exercises. Therefore, in terms of their explanations, the textbooks from Angola are denser and have more detailed clarifications than the South African textbooks. In terms of examples and exercises, the textbooks from Angola have fewer exercises than South African textbooks. The progression both in terms of levels of understanding of algebraic expressions and in terms of number of steps required to solve tasks is swifter in the Angolan textbook. Also, there is less focus on symbol manipulation without conceptual content in the Angolan textbook. Although much depends on the ways in which textbooks are used in the classroom, this suggests that the Angolan textbook offers the learners more opportunities to learn. There were some signs that the South African textbooks had been organized in ways informed by research, and in a few cases the exercises in the South African textbooks were more explorative, allowing learners more opportunities to develop deeper conceptual understanding. This was, however, not a dominant feature.

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DECLARATIONS

I, Isabel Maria Joaquim Borges, declare that:

- i. The research reported in this dissertation, except where otherwise indicated, is my own work.
- ii. This dissertation has not been submitted previously for any degree or examination at any university or other higher education institution.
- iii. This dissertation does not contain other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.

Signed:

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CHAPTER 1 : INTRODUCTION

This thesis aims to contribute to research on textbooks for mathematics used at high school level, focusing on their similarities and differences in the opportunities to learn. The study is limited to textbooks in use in Angolan and South African schools. Sources of data for the study include the algebraic content of three high school textbooks which is analysed in terms of progressive levels of algebraic understanding and consistency. In this chapter, I introduce the study, and focus on its background, motivation, objectives, formulate the problem statement and the research questions and finally I present the significance of this study.

1.1 Introduction and background to the study

The textbook plays an important role in teaching and learning as well as making lists of content, objectives, and the connections between them (Gak, 2011). Textbooks have different purposes in teaching such as providing the content being taught, being sources of variety in the teaching-learning process, concretising the teaching programme/intended curriculum, and standardization of instruction (Richards, 2001). However, textbooks can also contain content and examples which may be irrelevant or inappropriate for both teachers and learners, and the sequence of units, activities, reading and visuals may be considered boring (Gak, 2011). Textbooks' pedagogy generally refers to strategies of instruction designed for teaching and assessments leading to learning outcomes (Railean, 2012). Textbook writers and teachers thus interpret the curriculum document and the scientific topic in a pedagogical recontextualisation process (Bertram, 2012). Therefore, textbooks and their role in education may be understood through their position in the pedagogic device (Bernstein, 2000). The pedagogic device operates following three 'rules', each linked to a field and certain processes. In the field of production, knowledge is produced/constructed – for instance at universities. In the official recontextualisation field, the state and its agents frame how the transmission of knowledge can take place – as in drawing up curricula. This represents an interpretation or recontextualisation of the content for educational purposes, which is why this is where the recontextualising rules operate. The curriculum is further recontextualised by, for instance, textbook authors, in the pedagogic recontextualisation field. Finally, teachers recontextualise the content from textbooks and according to the curriculum, in their classrooms, aim to reproduce the knowledge with their learners. Hence the notion of

evaluative rules, because at this level it is evaluated if the learners have acquired the desired knowledge. The pedagogic device illustrates how textbooks fit into a greater picture, where they themselves are the products of recontextualisation of, for instance, algebraic knowledge, and will be further recontextualised during their use in the classroom. These recontextualisation processes do not take place in isolation, but link to the broader context of society in which they occur.

Since independence, in 1975, Angola has experienced several curriculum reforms. Even so, all these curriculum reforms were affected negatively by the civil war that affected the country during the period 1975-2002. These reforms were always implemented in an immediate way as is usual in conflict contexts (Ferreira, 2005; Nicolai, 2009). In 2001, the Angolan Government created a law called *Law of the Basis for the Educational System*. This law proposed education as a right for every child at the schooling age, the reduction of analphabetism¹, and professional training of teachers (Dunguionga, 2010). In 2002, the armed conflict ended and consequently an opportunity to implement the policies contained in the Law of the Basis for the Educational System emerged. Consequently, the Angolan Government started the new curriculum reform, in 2002. The overall purpose of this curriculum reform was to qualitatively improve the education system and the teaching, to re-adapt the educational system, so that it can face the new challenges of training the necessary human resources to develop the Angolan society (Dunguionga, 2010). During the civil war years after independence in 1975, Angola experienced several influences on its education system, from American and European countries such as the USA, Cuba, Russia, Poland, Bulgaria, and so on. Although Angola is located in the Southern African Development Community (SADC) region, there are no reports from the reviewed literature indicating that the curriculum reforms that have taken place, including the last one, have taken into account the regional context.

The curriculum in a democracy is a curriculum for democracy, and incorporates both a record of its past and a message for its future (Carr, 1998). In South Africa, after the apartheid era, there was a need for its education system to be transformed by phasing out the apartheid curriculum and introducing a new one. Since the advent of democracy in South Africa in April 1994, four national curriculum reform initiatives took place, focusing on schools.

¹ In 2015 the rate of Angolan analphabetism was sitting around 30%, but around 40% for women and only around 20% for men (<http://www.laenderdaten.de/bildung/alphabetisierung.aspx>, accessed 27th July 2016).

The first attempt was to rid of the apartheid curriculum of racially offensive and outdated content. The second was about to introduce continuous assessment into schools. The third, was the most ambitious and is referred to as Outcomes Based Education (OBE) (Jansen, 1998). The Fourth, was to help to implement the National Curriculum and is named the Curriculum and Assessment Policy Statement (CAPS) (DoBE, 2011).

In his critical analysis about OBE, Jansen (1998) considers that this initiative was going to have a negative impact on South African schools. He justified his point of view by stating that it was being driven, in the first instance, by political imperatives, which was not in synergy with the realities of the classroom. Jansen (1998) outlined his reasons by stating that:

- Very complex, confusing and contradictory language of innovation, associated with the outcome based education;
- Involvement of the outcome based education, as curriculum, in problematical and assumptions about relationship between curriculum and society;
- It is based on flawed assumptions about what happens inside the schools;
- Philosophically is strongly questionable in democratic school systems;
- Politically and epistemologically, there are important objections to the outcome based education as curriculum policy;
- OBE enables policy makers to avoid dealing with the central question consisting in what education is for;
- Multiplies the administrative burdens placed on teachers;
- Trivializes curriculum content;

Despite his critical analysis, Jansen (1998), also outlined suggestions for OBE to succeed. According to Jansen (1998) it is necessary for the

- training and retraining of teachers, managers or principals;
- radical new forms of assessment and classroom organization to occur;
- implementation of new forms of learning resources and opportunities for teacher dialogue and exchange;
- a radical revision of the most potent mechanism in schools militating against curriculum innovation, such as the system of assessment.

As it can be seen, in both countries, the curriculum reforms initiatives were motivated by different reasons. In Angola, after several years of post-independence war there was a need for a curriculum reform to reduce the analphabetism and to train teachers. In South Africa, after apartheid era, there was a need to phase out apartheid curriculum and to introduce new curriculum. Despite the different contexts in which the curriculum initiatives took place in each one of the countries, it seems that in both countries there are some common features:

- Political motivation in both main curriculum initiatives;
- Both did not take into account the regional context.

The last feature makes it a challenge the decision to choose books from Angola and from South Africa to compare. For this reason I chose one book from Angola, from grade 10, and two from South Africa, grades 9 and 10.

1.2 Motivation

Textbooks play an important role in teaching and support learners to understand mathematics (Johansson, 2003). Most developing countries use textbooks as a source of both content and pedagogy, and without textbooks used either directly or in teachers' planning, lessons 'cannot' be taught (Fullan & Langworthy, 2014). Similarly, I have been a teacher using textbooks in teaching algebra in high schools in grade 10. However, over time I have started to question the quality of the textbook I used. I therefore have a personal interest in contrasting and comparing the ways in which recontextualisation of mathematical content, and my favourite topic algebra in particular, is done in different countries. As I undertook a Masters in South Africa, I chose to compare the standard Angolan grade 10 textbook to South African textbooks. A comparison plays a fundamental role in concept-formation by bringing into focus suggestive similarities and contrasts among cases (Collier, 1993). The study is also motivated by gaps identified in existing research.

According to Cheung (2003), research on cross-cultural studies suggest that the intended curriculum plays a significant role in the development of learners' mathematics competency. He defined intended curriculum as the intentions, aims and goals of curricula. These are often made available to teachers and learners, albeit recontextualised, through textbooks. This motivates

looking into the content and opportunities to learn provided by textbooks, as a way to interrogate the results of the recontextualisation on this level.²

There is no current research on mathematics textbooks' education in Angola, only suggestions from ethnomathematics educators on the use of Angolan sand drawings in teaching (based on a search in Google Scholar, Education Source, ERIC and Sabinet). There are however studies of mathematics textbooks from other contexts.

There are several comparisons of mathematics textbooks within or across national contexts. However, few of these focus on high school level, and few focus specifically on algebra. Similarly, most studies do not include developing contexts, or Africa in particular. One study which does engage algebra specifically, focuses on the content in relation to the curriculum Cheung (2003), not on the conceptual focus within the textbooks. Thus, Cheung's study addresses the process of recontextualisation but not the resulting opportunities to learn, which my interest is. On the other side of the world, a study of Swedish textbooks revealed that algebra sections in textbooks varied with respect to which topic was the main object of study and point of entry: algebraic expressions or equations; and that over time there has been a move towards seeing mathematics as process or activity and as more of an applied subject (Jakobsson-Åhl, 2006) . This is not a universal move; a recent comparison of two US textbooks on algebra showed a significant difference in the number of tasks which engaged the learners in reasoning or proving (Davis et al., 2014). Huntley and Terrell (2014) recently compared five US textbook series' sections on linear equations with respect to their implied cognitive behaviour (know, apply, reason) (Huntley & Terrell, 2014). Amongst others, Huntley and Terrell (2014) found substantial variations in the cognitive behaviour of problems across the textbooks.

I have not found relevant African studies on algebra in textbooks, although one study focused on opportunities to learn algebra in grade 10 at three Catholic schools (Chabongora & Jita, 2013). The study has as its premise that conversions between representations support learning, and thus that one should not focus on mastery of concepts and procedures only. Thus, it points to an additional aspect to consider in comparisons of textbooks. However, I have chosen to focus on a more

² The recontextualisation in the classroom is as relevant, but is not in focus here. Thus, the study does not concern itself with the ways in which textbooks are used in the mathematics instruction.

ubiquitous component, namely the conceptual understanding of algebraic expressions, informed by research on the development of algebraic concepts (specifically a framework developed by Sfard and Linchevski (1994). As the short summaries above indicate, this is lacking in the field in general, and in the African context in particular.

The current study also has a more personal motivation. Between 1997 and 2009, I taught mathematics in an Angolan high school in grade 10, and I found that many learners have difficulties with the understanding of mathematical concepts in algebra. I believe that comparing and appraising algebraic concepts and how they are presented in different textbooks may help clarify why learners struggle and thus help to ease the learners' problem-solving difficulties. It is my impression that many textbooks, in terms of processes, steps and procedures used in examples and exercises, make it difficult for learners in high school to grasp the actual algebraic concepts. Using Sfard and Linchevski's framework (1994, p.191) for assessing the quality of the explanations of concepts and the progression in content, my research will focus on all four of their levels of algebraic understanding.

1.3 Objective of the study

- To identify the similarities and differences between some Angolan and South African textbooks in terms of:
 - topics coverage;
 - content distribution between explanations, examples and exercises, and
 - conceptual progression in algebra
- To investigate the way in which the conceptual progression is linked to procedural competencies. Do tasks, explanations and examples corresponding in terms of their levels of algebraic thinking required, in order to identify opportunities to learn in the textbooks.
- To test a method of comparing textbooks.

1.4 Statement of the Problem and research question

The problem which drives this study is to interrogate the opportunities to learn of the Angolan mathematics textbook's sections on algebra, in particular to evaluate the balance between the structural and operational aspects of algebraic concepts of grade 10, and to investigate the extent to which these are similar to what are being taught in South Africa.

The following research questions guided the study:

- What are the similarities and differences between the three textbooks in terms of topics coverage and content distribution between explanations, examples and exercises?
- What are the similarities and differences between the three textbooks in relation to conceptual progression in algebra?
- How is the conceptual progression linked to developing procedural competencies? Do tasks, explanations and examples correspond in terms of their levels of algebraic thinking required?

1.5 Significance of the Study

This study is expected to contribute to the findings of previous research about using textbooks in mathematics and hopes to bring new insight about the content of algebra sections of textbooks for both South Africa and Angola. There is no previous research making a comparison of topics coverage and content distribution between explanations, examples and exercises in terms of conceptual progression in algebra and in terms of their level of algebraic thinking, in high school mathematics textbooks from two countries. The research was therefore undertaken in order to provide an empirical case of mathematics textbooks in two educational systems, with a focus on opportunities to learn for learners at high school. At the same time, this is a conceptual case, experimenting with and reporting on an approach to comparison of textbooks.

Instead of trying an overall survey of content coverage in the whole of mathematics textbooks, the research focused on algebraic content area, particularly explanations, examples and exercises of the textbooks. However, this study did not only analyse to what extent the topics are covered, but also how they are presented. The very different histories of Angola and South Africa mean that the study may also, although indirectly, contribute to an understanding of the recontextualisation processes in these contexts.

1.6 Outline of the Thesis

In chapter two, I present a survey of literature related to my research and describe how my research is related to this literature. Chapter three discusses the theoretical framework I have adopted from Sfard and Linchevski (1994) and expanded in order to suit my project. The theoretical framework is presented before the research design and methodology, because in the theoretical framework I

explain the method used to investigate the level(s) of algebraic understanding enabled by the textbooks, which makes it easier for the reader to make sense of these levels as defined in the research design and methodology. The methodology chapter thus follows on the theoretical framework. Here I describe the method used for the selection of textbooks and for data analysis. In chapter five, I analyse consistency and progression in levels of algebraic thinking throughout the three books. In chapter six, I discuss the findings in general and I conclude in chapter seven, where the results of the analyses of the textbooks are compared and contrasted.

CHAPTER 2 : LITERATURE REVIEW

This chapter presents the literature relevant to the current study. During my reading I was particularly interested in studies relating to analysis and comparisons of mathematics textbooks, particularly for the topic of algebra and for grade 9-10, but felt additional background reading would be beneficial. Therefore, this review of literature has five focal points. The first part of this chapter aims to review the literature regarding the use of mathematics textbooks and its role more generally. The second part looks at studies analysing the use of mathematics textbook in teaching. The third part is a more comprehensive review of studies analysing algebra in textbooks. The fourth part deals with studies analysing and comparing mathematics textbooks across countries. The fifth part focuses on the use of textbooks in developing countries. I end the chapter with a summary.

2.1 Understanding the use of textbooks in the classroom in teaching and learning

The study surrounding the usage of textbooks in the classroom received much attention in the early 1980s. During this time, “education researchers had begun to find impressive results from their studies of what “inputs” into students’ education affected their achievements on tests” (Moulton, 1994, p. 8). A World Bank publication in particular, stated that the availability and use of textbooks and other printed materials was one of the more consistent indicators of achievements (Fuller, 1987; Heyneman & Loxley, 1983).

Moulton (1994) states that the availability of textbooks does not necessary mean learners’ achievement, if they do not use them perfectly. Using both availability and use, the early 1990s research showed that there was a positive achievement as an effect of textbooks supplies and utilization in primary schools (Fuller & Clarke, 1994). Studies on the usage of textbooks have also been present in the 21st century, with scholars such as Bierman et al. (2006) arguing that textbooks are not dead but their roles and appearance are changing fast (Bierman, Massey, & Manduca, 2006). Bierman, et al. (2006) add that “the goal is to retain the core stability and authority that make the textbook so valuable while at the same time providing the flexibility, timeliness, and inquiry-focused approach that the web and other electronic resource provide” (see also Mahadi & Shahrill, 2014; Nor & Shahrill, 2014; Yong, 2010).

As an education tool, a textbook can be regarded as an artefact (Gueudet, Pepin, & Trouche, 2012; Johansson, 2005, 2006; Matic & Gracin, 2016). Artifacts can be described as the tools made by

human beings in actions and are applied by (or function as) humans according to different needs. Artifacts may be considered as the production or reproduction of human beings' social activities (Wartofsky, 1971). According to Grevholm (2011) "in the process the teacher shapes the artefact (the curriculum materials), and at the same time the artefact shapes the teacher's use through its affordances and constraints" (2011, p. 97). This provides a model for analysing textbook's use based on activity theory (Rezat, 2006). In this process the teacher plays a role as mediator between learners and mathematical knowledge by using artefacts (Gracin, 2014). The textbook (artefact) is thus placed in the heart of the activity system, which according to Rezat, 2006 as cited in Costa and Matos (2014), "contradicts the notion that human activity is directed to gain control of their artefacts" (p. 180). Textbooks thus act as mediating artefacts that significantly contributes towards the promotion of learners' learning.

Knight takes a different approach to the use of textbooks in learning. Given the changes in the 21st century, where there is easy access to information, digital innovation and new learning techniques and tools (Weisberg, 2011), Knight explores teachers' use of textbooks in the digital age (2015). He notes that "while technology has impacted student (learner) behaviours and distinct preferences for learning, one thing that has not changed is the essential need for credible content" (Knight, 2015, p. 4). While Knight acknowledges the benefit of technology in the learning process, he however believes that "useful educational technology will be the ones that are built on reliable content using technologies to support students (learners) in manipulating the materials and engaging with the teaching" (Knight, 2015). Thus, the reliability of textbooks as tools can provide learners' with creditable information which can enhance their understanding of critical concepts, "and that they present bite-size chunks of information to cement student (learner) learning" (Knight, 2015, p. 1).

Much research has, therefore, undeniably, referred to textbooks as being among the most powerful influences in teaching and learning (Mullis, Martin, & Foy, 2008; Valverde, Bianchi, & Wolfe, 2002). Textbooks play a substantial role in teachers' planning and preparing for instruction as well as being perceived as indicators of the official intended curriculum by teachers, hence topics not included in the textbook are most likely not to be presented by the teacher (Johansson, 2006). Fan (2013) states that teaching and learning materials are necessary to the operative running of an education system and also assert that these resources are an integral part of curriculum

development and well-resourced classrooms that ideally should include learning support materials such as textbooks for each learning programme and other print-based materials including print materials like readers, diagrams, dictionaries, and so on (Fan, 2013).

On the other hand, Mohammad and Kumari (2007) concur with Knight (2015) and notes the new academic challenges faced by teachers and that new strategies need to be employed to reach a wider range of learners:

In this post-modern world of technological advancement, rapidly changing markets and increasing competition, teachers are faced with new academic and pedagogical challenges. In order to prepare students, teachers must teach more challenging and extensive subject areas, develop different instructional strategies and reach a wider range of students (p. 2).

In ensuring that the curricula and the textbook contribute positively to the life of the learner, some argue that, it is important that both the curricula and the textbooks should contain the meaningful and appropriate skills for the life experiences to prepare learners for real life (Mohammad & Kumari, 2007). Moreover, in terms of how learners understand or show their participation in the classroom, scholars report different arguments that “textbooks cause kids to get bored. Textbook does do not get information out of them...Reading textbooks does not prepare kids for much of the authentic nonfiction reading they will do in their lifetimes” (Harvey & Goudvis, 2007, p. 145). Nilsson (2006) makes a study to hear what both the teachers and the learners find useful between textbooks and alternative material. Nilsson (2006) finds that majority of the teachers reported that it good to work with alternative material because they can use current topics which do not exist in textbooks. However, Nilson (2006) believes that it takes too much time to work exclusively with alternative material because the workload becomes too big for them. It also happened that the majority of the learners enjoyed working with textbooks due to the fact that they know exactly what to do from time to time and can work in advance when they like to do so. Nilsson (2006) add that the learners found it boring, saying that textbooks never contain current topics and they preferred to work with alternative material because they possess interesting topics and the possibility of choosing topics themselves (Nilsson, 2006).

Drawing on specific reference to school mathematics, Lepik (2015) notes that “textbooks are equally important resources for both groups – for pupils (learners) to learn mathematics and for the teachers to plan and teach their mathematics lessons” (2015, p. 90). Hutchinson and Torres (1994), Li (2000) and Oakes and Saunders (2002), agree that that textbooks helps teachers to plan their work. Apart from planning, one of them added that textbooks assist in presenting the subject (Hutchinson & Torres, 1994). In as much as Oakes and Saunders (2002) acknowledge that textbooks are instrumental in planning, and give the teacher all the plans and lessons s/he needs in order to cover a topic in some details as well as providing a chronological presentation of information, it appears that some teachers reject a textbook approach to learning because the textbook is out-dated or insufficiently covers a topic or subject area. As a teacher, it is helpful to make many decisions and one of those is how you want to use textbook. Some disadvantages in using textbooks are that they can get old or outdated and then information shared with learners is not current or relevant (Oakes & Saunders, 2002).

Furthermore, textbooks provides material which help to cover and design each lesson carefully and, spell out details of organized units of work, some argue (Fredericks, 2010). Thus, it appears that textbooks are very influential on (or one may say helpful to) teachers’ work, across the varied contexts of the studies discussed so far. They may be considered, as suggested in a study from Cameroon, to save time, give direction to lessons, guide discussion, facilitate giving homework, make teaching “easier, better organized, more convenient”, and provide teachers with confidence and security (Ebong, 2004, p. 152). Teachers often see the textbooks as a “framework” or “guide” which helps them to organize their learning both inside and outside the classroom (McLelland & Dasgupta, 2005). Some claim that textbooks help leaners to teach learners to learn, are resource books for ideas and activities, and provide teachers with a rationale for what they do (Davey, 1988; Gak, 2011; Mohammad & Kumari, 2007). However, Richards (2001) lists their weaknesses. He states that “if teachers use textbooks as the primary source of their teaching leaving the textbook and teachers’ manual to make the major instructional decisions for them the teachers’ role can become reduced to that of a technician whose primary function is to present materials prepared by others” (p.2). Rezat (2009) adds that the pedagogical model only becomes operative when the textbook is actually used. As discussed above, textbooks were declared to serve several purposes

such as being core resource, providing supplementary materials, providing inspiration for classroom activities, as well as serving as curriculum (Garinger, 2002).

In order to serve an effective teaching, Kochhar (1985) quoted in Sunday and Adebowale (2013), suggests that “textbooks should be interesting to learners, well written, and beautifully compiled so that it might win and retain users’ goodwill by virtue of more solid qualities, illustrated with attractive colours, inspiring drawings and photographs, it should be attractive, inviting, a pleasure to look at and read, with well-chosen illustrations, well-connected and carrying through a sequence; it should be up-to-date in content, frequently revised and reprinted when necessary” (p. 161). Besides, these authors emphasize that textbooks must be complete with its table of contents, illustrations, charts or other references. In addition, a textbook must be well-graded i.e. suitable for the capability of the children for which it is intended. Success in different courses depends on learning to use effective reading strategies for a variety of college textbooks (Dembo & Seli, 2008, p.188).

2.1.1 Advantages of textbooks

Every teaching method has advantages and disadvantages. Textbooks and other teachers’ guides need to embrace the various needs of all learners in an extensive range of cultural contexts, economic conditions and educational settings.

Textbooks serve in managing a lesson (Snowman & McCown, 2011). They are known to save time, give direction to lessons, guide discussion, assist in organising homework, making teaching very easier, better systematized and more useful (Ornstein, 1994; Zaphiris & Ioannou, 2014). Good textbooks turn the standard procedures in the official government programs into a rich source of content, texts, and activities that would be beyond the abilities of most teachers to develop on their own. Using textbooks it is not a reflection of a deficiency on the part of the teacher (Richards, 2001) . Textbooks are especially helpful for beginning teachers (Ball & Feiman-Nemser, 1988; Korn, Sikorski, & Stephen, 2012). In textbooks, the material to be covered and the design of each lesson are carefully spelled out in detail (Chirwa & Naidoo, 2016; Fredericks, 2010). A textbook series provides you with a balanced, chronological presentation of information (Domur, 2015).

Other authors state that textbooks are a detailed sequence of teaching procedures that tell you what to do and when to do it (Fredericks, 2010). This means that textbooks offer both administrators

and teachers with a whole program. Klymkowsky (2007) considers good textbooks as excellent teaching aids and resource for both teachers and learners. This means that textbooks are not only teaching materials for teachers, but also learners “own learning and teachers” self-directed learning materials for preparation and revision purposes (Klymkowsky, 2007).

Read (2015) agrees that both printed and electronic textbooks (e-textbooks), support a learner-focused curriculum as well as learning strategies useful for the study of the subject. Read (2015) has also a view that textbooks improve learning for the poorest learners by increasing motivation, performance and opportunity to learn as well as providing an appropriate amount of quality texts for learners to “read to learn” independently (Read, 2015). Thus, learners will then have opportunities for developing diverse skills of learning according to their interests, needs and abilities.

Read (2015) further declares that textbooks are responsible for changes in educational practices such as assigning homework and increasing classroom reading time.

2.1.2 Disadvantage of textbooks

Although various advantages of the textbooks were discussed above, other scholars have different visions. In his study, Harmer (2001) finds that teachers’ views about textbooks use were different. Harmer (2001) reports that some teachers claim that their learners were bored by using textbooks due to the fact that textbooks many times contain material that is not interesting enough (Harmer, 2001). Furthermore, Harmer (2001) claims that there were little variation in textbooks, which makes teaching and learning stifling. Harmer (2001) finds that teachers do not approve of textbooks like to use their own imagination, alternative material such as pieces from books and magazines and ideas that learners give them. Harmer (2001) finds it problematic for both teachers and learners to attach to textbooks for long as they ignore all other sources of material.

Textbooks are helpful but they do not have to control all the teaching and learning in the classroom. It was also claimed that teachers who mostly focus on the use of the textbooks can become de-skilled (Eilam & Gilbert, 2014; Richards, 2001; Sayer, 2015).

Another disadvantage is the lack of representation of real issues in their content, due to the fact that editors exclude topics that are controversial so that the textbooks will be more widely accepted (Nilsson, 2006). In their study about “Improving Students’ (learners) Learning With

Effective Learning Techniques”, Dunlosky, Rawson, Marsh, Nathan, and Willingham (2013) list that many of these textbooks did not provide sufficient coverage, which would limit the up-to-date reviews of their effectiveness and analyses of their generalizability (pag,6). This reflects the weakness of textbooks coverage.

Richards (2001) also lists some limitations of the use of textbooks. He states that textbooks may contain unauthentic language because texts, dialogues, and other aspects of content tend to be specially written to incorporate teaching points and are often not illustrating the real language use. Textbooks often present an idealized view of the world or fail to represent real issues (Richards & Renandya, 2002). They may not reflect learners’ needs because textbooks are often written for global markets, they may not reflect the interests and needs of learners and hence may require adaptation (Richards, 2001).

In his study , Problem-Based Learning, White (1995), emphasise that learners are more likely to work independently of the textbooks’ exercises, because they realize that the knowledge they gain by thinking about their own problems, will be useful in the future.

Other studies list that, if teachers use the textbook as the primary source of their teaching, leaving the textbook and teacher’s manual to make the major instructional decisions for them, the teacher’s role can become reduced to that of a technician whose primary function is to present materials prepared by others (Richards, 2001).

In his study about teachers use textbooks, Moulton (1994) finds that in classes lacking textbooks, teacher education was significantly related to learner achievement whereas in classes with textbooks, teacher education was negligible. Moulton (1994) agrees that teachers who use textbooks do not necessarily use time more effectively and that textbooks do not necessarily encourage teachers to assign more homework. The next section discusses textbook and pedagogy.

2.2 Textbook and pedagogical content knowledge.

Good textbooks are well organized, coherent, unified, relatively up-to-date, accurate, and relatively unbiased and they reflect the relationship between views of learning, learners, knowledge and pedagogy. In exploring the way how mathematics has been taught, Wood, Cobb, and Yackel (1992) find that:

“Teaching mathematics in schools is characterised by heavy reliance on the textbook by teachers both as a source of activities and for explanations of procedures to use in completing the task” (p.179).

During the 19th and 20th Centuries, textbooks were mostly used to transmit the content along with pedagogy. This means that the printed textbook has been the means with which knowledge is organized and distributed. Textbooks should teach learners to learn, be resource books for ideas and activities, for instruction/learning and give teachers rationale for what they do, bring about an effective learning situation (Marble, Finley, & Ferguson, 2000; Swango, 2003). Textbooks are usually part of a pedagogical design, i.e. it can be the center piece of a course syllabus, it can be used for self-study, and teachers can assign just parts for reading. However, learners need textbooks to access information, to practice themselves by doing homework and studying for tests and exams. On the other hand, teachers need textbooks too to capture the subject matter content and plan lessons and to ensure that the right amount of time is spent on each section of work, so that the whole curriculum is covered. Teachers further use textbooks to set homework, tests and exams. In other words, textbooks help teachers with both what to teach and how to teach (Richards, 2001). Textbooks play a role of pedagogic tools since they reflect the objectives of the language programme, the kind of syllabus used, the skills being taught and the methodologies adopted, and might be seen to function as a intermediating object between the teacher and learner (Lee & BathmakeR, 2006; Riet, 2015).

Studies argue that textbook writing contains the instructional design and therefore one might look at textbooks in terms of some instructional design models and methods. Textbooks reflect the most important non-personnel instructional resource in classrooms, heavily influence what teachers teach and how, however, they are limited in terms of supporting the constructivist pedagogy (Al-Abdulkareem & Hentschke, 2014; Oates, 2014). On the other hand, Al-Abdulkareem and Hentschke (2014), add that textbooks can provide a consequential mechanism for fostering the growth and development of constructivist pedagogical practices. Their findings show that textbooks only minimally support constructivist pedagogical practices and textbook emphases on constructivist pedagogical practices do not vary significantly by subject matter area. Joubert (2015), adds that textbooks contain the information required for learners in the form of the

actual text, photos, graphs, cartoons, drawings, paintings, posters and other visual images and they also have activities to keep learners busy.

Textbooks can be very helpful to educators for assessments. The sources can be provided by the textbooks which must then be referenced correctly and then utilised as actual assessment questions or examination questions (Joubert, 2015).

Textbooks often have pedagogical functions when they present knowledge of subjects (Gencturk, 2012; Lebrun et al., 2002; Love & Pimm, 1996). These pedagogical texts “construct authors as transmitters and readers as acquirers” (Barnett & Griffin, 1997, p. 331).

Textbook content includes concepts, principles, rules, and theories of the discipline in a subject. Jaffer (2001), adds that textbooks as a pedagogical text for learners, they play a substantial role in terms of transmitting and attainment of mathematics as well as providing fortunate ways of learning mathematics. However, the textbook, read intertextually with the accompanying teacher’s guide, is potentially a pedagogic text for teachers where the focus is on the transmission of a privileged mode of teaching mathematics. Both teachers and learners can therefore be considered as acquirers (Jaffer, 2001, p. 26). Textbooks include the methods of transforming or teaching the content in order to be understandable to others. In other words, textbooks reflect pedagogical content knowledge which enables teachers to transform the content into accessible learning information through strategies or technics (Shulman, 1987). Therefore developing textbooks as well as the curriculum material can be one way to improve the quality of teaching, given that a textbook recapitulates the goals of learning, such as purposes, content, teaching, assessment strategies, and so on which describe the determination of learners’ learning.

Understanding teachers’ use of textbooks and other relevant curriculum materials provides insight into the contribution of such materials to the classroom learning (Özgeldi & Çakiroğlu, 2011). Many researchers have found that textbooks are closer to the classroom than national curricula because they embody teaching strategies or classroom activities (Fan, 2013; Fan, Zhu, & Miao, 2013; Johansson, 2003; Leung, 1995). Thus, textbooks often play a double role as both curriculum and providing materials as an integral part of teachers’ daily work, offering ongoing support for pedagogy and subject matter content throughout an entire school year (Collopy, 2003). The relation between the official curriculum and the textbook is not straight forward. Some studies suggest that textbooks are one way to promote a common curriculum across diverse setting (Ball & Cohen, 1996). However, in countries where more than one textbook is on offer, textbooks are selected by schools or individual teachers under the guidance of the curriculum, amongst others.

Others foreground that teachers mediate the textbook by choosing, structuring and assigning tasks (Reinhardtson, 2012), and recontextualise textbook content to varying degree, even suggesting that this is the most prevalent finding from early studies on the use of textbooks in classrooms (Moulton, 1997). Thus “the textbooks also reflect options which depend on the multiple relations between different pedagogic fields” (Petersen, 2015, p. 12).

Teachers must understand that the intermediation goes beyond the chosen content and involves the judgments about broader pedagogical issues (Pepin & Haggarty, 2001). In teaching mathematics, some scholars see challenges in using textbooks. One task faced by mathematics teachers is selecting and transposing mathematical content knowledge, for which many teachers utilize a textbook (Das, 2015).

Textbook writers and teacher trainers then interpret the curriculum document in the “Pedagogic Recontextualising Field”, where teachers engage in the pedagogic and assessment practice (Bertram, 2012). However, a textbook which embodies an inductive, exploratory pedagogy, cannot on its own achieve learner's apprenticeship into mathematics, or a teacher's apprenticeship into its privileged mode of teaching mathematics (Jaffer, 2001).

2.2.1 Textbooks and curricula

Textbooks have been designed to drive the curriculum. Studies advocate that when they are used wisely, they can implement a well-designed curriculum (National Research Council, 1997, p. 30). According to Valverde, et al (2002), textbooks have an indispensable position in curriculum reforms and are considered as the most important tool for the application of the new curriculum in many countries.

Textbooks are more likely to improve learner learning when they are based on a curriculum, when they employ a language that is easily understood and at an appropriate level for learners and teachers, and when teachers adapt their pedagogy to achieve effective use (Marble et al., 2000). However, if they contain errors, teachers with poor content knowledge may not be in a position to recognise mistakes, which are thus likely to be transmitted to learners (Sanders & Makotsa, 2016). Textbooks are an instructional assistance in the teaching-learning process and must correspond to curricula so far as objectives, content and methodology of instruction of each subject are concerned (Cramer, 1993; Seguin, 1989).

Phaeton and Stears (2017) agree that it is supportive to conceptualize the curriculum as consisting of different levels so that analysis and understanding of the curriculum can be made more specific. Curriculum theory commonly distinguishes between the intended curriculum (curriculum standards, frameworks, or guidelines that outline the curriculum, teachers are expected to deliver), the interpreted curriculum (by teachers and textbook writers), the implemented curriculum (actual instructional practices) and the attained curriculum (competencies and attitudes learners demonstrate as a result of the teaching and learning process) (Aikenhead, 2006; Phaeton & Stears, 2017).

A textbook therefore constitutes an interpreted curriculum for teachers. How this interpreted curriculum is implemented depends on many factors: the teacher's mathematical-knowledge, the context of the school, the teacher's own background in education and so on. The implemented curriculum potentially differs from teacher to teacher. Bernstein's (1990) notion of fields within educational systems provides a framework for examining the relationship between a curriculum and textbooks. He identifies three fields namely, fields of production of discourse (primary context), fields of reproduction of discourse (secondary contexts) and fields of recontextualizing.

Johansson (2003) suggests that the study of the development of textbooks should be represented in the light of the curriculum development. Consequently, a narrow view of the curriculum development is likely to produce the new syllabuses and tests which are not helpful for the development of textbooks. This means that curriculum must mean more than syllabus, and must incorporate the aims, content, methods and assessments strategies (South African Qualifications Authority, 2000). Johansson (2003) further compares the textbooks as the implemented, and associate to the enacted curriculum.

2.3 Textbook in teaching mathematics

The previous section has looked at the use of textbook in a classroom set up in general, the next section will specifically focus on the general use of mathematics textbooks in the classroom. In mathematics education, different studies discussed about the use of mathematics textbooks in the classroom and focused on role of textbooks in mathematics teaching and learning (Čeretková, Šedivý, Molnár, & Petr, 2008; Gracin & Matic, 2016; M. A. Lambert, 1996; Nicolai, 2009), comparing the similarities and differences of two or more series of mathematics textbooks

(Chandler & Brosnan, 1995; Zhu & Fan, 2006), how textbooks shape the way of teaching and learning of mathematics and the relationship between textbooks and learners' achievement (Fan et al., 2013). Those authors adds that textbooks define school subjects not only for teachers and learners but for the public as well.

Johansson (2003) describes a mathematics textbook as a complex package of materials, books, booklets, work cards and work sheets as well as a combination of teaching guides or computer software. Textbooks play an important role in mathematics education because of their close relation to classroom instruction (Johansson, 2003). They categorize the topics and order them in way learners should discover them. They also endeavor to specify how classroom lessons can be structured with suitable exercises and activities. In some sense, they provide an interpretation of mathematics to teachers, learners and parents (Johansson, 2003, p. 20).

In teaching and learning mathematics, textbooks have both advantages and disadvantages. The following are various advantages of using textbooks in mathematics instruction. They hold various activities which assist learners in learning. These include instructive texts and acquiring a new content, stories, news, analyzing tables and graphs, consulting other worked examples, mathematics tasks, and so on. Textbooks are mostly recognized as the essential artefacts in mathematics education (Gracin, 2014; Rezat, 2006, 2009). In teaching mathematics, studies argue that “the textbook should arouse students’ (learners) interest in learning mathematics, help students (learners) to study mathematics actively, develop students’ (learners)potential in creativity through the process of learning basic knowledge, improve students’ (learners) mathematical thinking when trying to understand the essence of mathematics knowledge, and raise students’ (learners) awareness to apply mathematics knowledge in everyday lives” (Li, Zhang & Ma, 2009, p. 743).

Studies argue that mathematics textbooks not only play an important role in transmitting major mathematics concepts, but also are an independent tool to help learners learn mathematics (Yang & Lin, 2015). Textbooks have been used to identify the mathematics topics covered (or subject matter content) and the complexity of the exercises (or degree of cognitive challenge) (US Department of Education, 2013) as well as serving as an indicator of the intended course curriculum (Schmidt, McKnight, & Raizen, 1998). Haggarty and Pepin (2002) concludes that learners in different countries are offered different mathematics and given different opportunities to learn that mathematics, both of which are influenced by textbooks and by teachers.

In teaching mathematics, the textbook is often considered an important resource for assessing and learning mathematics (Lepik, 2015; Rezat, 2009). Johansson (2003) states that the role of the textbooks in mathematics, is mostly manifested in a reform of mathematics curriculum. Johansson (2003) adds that developing textbooks and curriculum material can be seen as quick and easy way to change teaching. Johansson (2003) agrees that textbooks can also be considered as an obstacle to the development.

Textbooks largely influence how teachers portray a mathematical topic and implement their understanding of learners' learning trajectories in a classroom (Valverde et al., 2002, p. 10). A study carried out in US, discussed that in mathematics discourse, textbooks are an integral part of what is involved in doing school mathematics; they provide frameworks for what is taught, how it might be taught, and the sequence for how it could be taught (Nicol & Crespo, 2006). Textbooks would also be relevant especially for the intermediate phase learners who were expected to work mathematical tasks independently and unsupervised ((DoBE, 2011).

Further studies also discuss that textbooks describe what is school mathematics and also what is mathematics for both teachers and learners (Johansson, 2003, 2006). Johansson (2006) also articulates the idea that textbooks help teachers to organize their teaching, given that the topics, exercises and other learners' activities are arranged in appropriate order. It has to be combined by the teacher with other curriculum materials and activities to give the learners the opportunity to develop a better understanding of mathematics (Kenneman, 2014). Learning mathematics with a textbook comprises activities such as reading explanatory texts and acquiring a new content, analysing tables and graphs, looking through worked examples, solving tasks, and so on (Lepik, 2015). Moreover teachers may or may not use the textbook in the lessons; they may simply use it as a source of exercises or they may utilize the full potential of the materials presented in the textbook (Lepik, 2015). However, textbooks "are not only mediators between intention and implementation, they are also components of opportunities to learn school subjects and have their own characteristic impact on instruction" (Valverde et al, 2002, p. 10). Accordingly, textbooks should be researched from the aspect of the content as well as their use in classrooms.

In a study carried out by Mesa (2004), the conception of function enacted by problems and exercises is investigated. The study was based on 35 mathematics textbooks for seventh and eighth grade from 18 countries and it identified five conceptions of function as promoted in the textbooks:

symbolic rule, ordered pair, social data, physical phenomena, and controlling image. As result of the study, it was found that there was no canonical curriculum for teaching function and there are no established practices of organizing mathematics textbook content on function.

The results indicate that most mathematics textbooks combine the use of symbolic rule and ordered pair approaches to functions whereas physical phenomena and controlling image were used in few mathematics textbooks. According to Mesa (2004), approximately all the 7th and 8th grade mathematics textbooks examined have pictures, drawings and photographs relating to the content. Mesa (2004), similar to Haggarty and Pepin (2002), find that most of the exercises in England appeared without words and they were containing very little or no relevant information on how to solve the task. The literature indicates that teachers with low levels of confidence in their subject matter knowledge in mathematics often hold the belief that textbook authors possess more mathematical expertise (Ball, 1988). Ball (1988) infers that teachers who hold this belief become textbook dependent as they believe the authors of the textbook are better able to develop a mathematics program.

However, it is often claimed that textbooks are boring and offer little variation (Nilsson, 2006). Boaler (1998) reports that learners who followed a traditional approach developed a procedural knowledge that was of limited use to them in unfamiliar situations (Boaler, 1998). On the other hand learners who learned mathematics in an open, project-based environment developed a conceptual understanding that delivered them with advantages in a variety of assessments and situations. Other studies highlight that teachers' heavy dependence on the mathematics textbook, limits the development of the educational outcomes of learners or can be regarded as an obstacle of the development (Johansson, 2003; Jorgensen & Dole, 2011). Thus, textbooks have both positive and negative effects in teaching and learning. From these views, it can be noted that the quality of mathematics instruction would be greatly improved if teachers would become familiar enough with the material that they are capable of teaching it without textbooks, whether they do so or not. Teachers can also increase learners' appreciation for math by teaching with a real-world perspective not limited to textbooks. These next section focuses on the use of textbooks in teaching algebra.

2.4 The use of textbooks in teaching algebra

Algebra can be understood as a generalization of the ideas of arithmetic where unknown values and variables can be found to solve problems (Taylor-Cox, 2003). Algebra contains numbers, fractions, decimals and percentages which can be used to solve problems in everyday life such as money, sports, measurements, election, lottery, newspapers and magazines (Usiskin, 1995). It also contains multiple representations such as symbols, equations and graphs, geometry and calculus which are necessary skills in the workplace (Chabongora, 2011; Star et al., 2015). These authors add that algebra is often the first mathematics topic that involves extensive abstract thinking, provides new skills for learners as well as the key to success in future mathematics courses.

The use of algebra in textbooks has also been thought of as symbolic representations, with exercises generally presented without the use of words other than general instructions to “simplify” or “solve for x ” and so on (Haggarty & Pepin, 2002; Mesa, 2004). Some countries like France and Germany, have textbooks which are denser as compared to English mathematical textbooks (Haggarty and Pepin, 2002). Sood and Jitendra (2007) carry out in US, comparing traditional and “reform-based textbooks”, mentioned that the later textbooks, which emphasize real world and practical activities, are better than traditional textbooks which mostly focus on “teacher-directed instruction” (Sood & Jitendra, 2007, p. 154).

Hodgen et al. (2010) focus on understanding the different ways in which textbooks support the teaching of algebra and identifying ways in which such textbooks can be used to support more effective teaching and learning. Most of these focused on how tasks are presented in textbooks based on different curricula.

In her study, Chabongora (2011) examines different opportunities in learning grade 10 in algebra in three South Africa secondary schools. Chabongora (2011) findings shows that the use of learner-centered approaches and problems solving would be applied during the learning of algebra topics. However, Chabongora (2011) adds that many learners find algebra challenging

The grade 10 syllabus in South Africa aims to prepare learners who can flexibly convert between the different representations of algebra concepts (DoBE, 2011). The focus is on algebraic expressions and exponents, numbers patterns, sequences and series, equations and inequalities, trigonometry, functions, Euclidean geometry, analytical geometry, finance and growth and statistics (DoBE, 2011). However, the main topics taught in grade 10 in Angola are polynomials,

functions and graphs, in equations and equations, real functions, monotonic and extreme relatives, in equations of 2 degrees, model functions and operations for polynomial.

Similarly, working with mathematics textbooks, Cheung (2003) focuses on algebra chapters which constituted a large part of the secondary school mathematics curriculum in Hong Kong. His study was focused on finding out the similarities and differences of the problems appearing in textbooks based on the two different curriculums. The findings indicated that there appears only slight differences in distributions of problem types of to-be-solved problems in the two groups of textbooks, whereas there was no difference in the distributions of problem types of worked examples. Cheung (2003) findings also indicate that all relevant textbook problems were classified into two categories: worked examples or to-be-solved problems. However, Cheung (2003) mainly focuses on the role of textbooks relating to curriculum, not on the conceptual focus within the textbooks.

On the other hand, Sönnerhed (2011) explores the content and PCK in Swedish upper secondary schools. Sönnerhed (2011) focuses on the type of mathematics exercises in the textbooks which are provided for learners to practice the related algebra content and facilitate learning in the embedded teaching trajectories; how they are constructed and what pedagogical aims they have. This includes uncovering pedagogical aims of provided mathematics exercises, activities, and problems in the textbooks. The results show that the analyzed Swedish mathematics textbooks reflect the Swedish mathematics syllabus of algebra. It is found that the algebra content related to solving quadratic equations is similar in every investigated textbook. Besides, Sönnerhed (2011) findings shows an accumulative connection among all the algebra content with a final goal of presenting how to solve quadratic equations by quadratic formula. The study finds that the presentation of the algebra content related to quadratic equations in the selected textbook is organized based on the history of algebra. It can be traced back to the al-Khwarizmi's geometric solution of quadratics, showing that quadratic equations are historically related to the application of mathematics in both real world and pure mathematics.

In addition Sönnerhed (2011) findings illustrates the historically related pedagogy and application of mathematics in both real world and pure mathematics contexts are the pedagogical content knowledge related to quadratic equations.

An analysis of the notion of school algebra was also presented through a study of mathematics textbooks and their revisions during the curricular reform in Sweden in the period 1960-2000 (Jakobsson-Åhl, 2006). The findings of this study divided the mathematics textbooks according to their main subject of study, which concluded that in some of the textbooks the algebraic expressions predominated, in the others the equations predominated, and there was another group where both the algebraic expressions and the equations played this role. This study also concluded that the current Swedish textbooks emphasized mathematics as a process unlike the previous ones where the emphasis was on the literal symbols. Finally, an analysis relating textbooks to curriculum and an analysis about studies comparing textbooks and notebooks were presented (Jakobsson-Åhl, 2006).

Kenneman (2014), carried out a study with the purpose of enhancing the understanding of how classroom discourse supports the students' learning of algebra. Among the assumptions in the study it is found that the textbooks used in school are envisioned as potential means for supporting students' algebraic development.

Kenneman (2014) analysis the progression of algebraic objects in the mathematics textbooks in grade 2, 5 and grade 8. Kenneman (2014) analysis calculated the mean value of the number of words constituting the signifier of algebraic object, signifier length equal to or exceeding two words, signifier length equal to or exceeding six words, and as amount of signifiers of algebraic objects a higher discursive level. The findings showed that there were signifiers of algebraic objects presented in all three mathematics textbooks and in teachers' lesson talks. The number of these signifiers of algebraic objects in the mathematics textbooks grew substantially between grade 2 and 5 with a moderate increase between grade 5 and 8. Moreover, Kenneman (2014), adds that the mean value of the number of words constituting these signifiers of algebraic objects grew between grade 2 and 8, as well as the amount of signifiers of algebraic objects consisting of six or more words. This tells that in his study, it seems that the progression of algebraic objects increases from grade 2 to 5 than grade 5 to 8.

The issue of how algebraic representations are engaged in most school mathematics textbooks has recently attracted the attention of many researchers globally. Most of these studies focus on different aspects such as conceptual learning; problem solving, and the use of textbooks (Fan & Zhu, 2000; Hensey, 1996; Li, 1999; Mesa, 2004; Ng, 2002; Zhu & Fan, 2006). The quality of

textbooks, therefore, has a critical impact on the quality of teaching (Ramnarain & Padayachee, 2015). Furthermore, the access of the quality of textbooks is one of the important factors in the successful implementation of curriculum change (Cheung, 2003).

The studies reviewed above suggest that algebraic representations in school mathematics textbooks can be engaged in different ways. In fact, as illustrated above, a number of authors in different countries have conducted various studies on the role of textbooks relating to curriculum, and others relating to the conceptual focus. The notion of school algebra, as manifested in Sweden in the period 1960-2000, was described and analyzed by Jakobsson-Åhl through the study of mathematics textbooks and their revisions during curricular reforms (Jakobsson-Åhl, 2006). The study was conducted with a historical-epistemological perspective, specifically on the teaching of mathematics. It focused on the treatment of algebra for Natural Science learners in their first year of Swedish upper-secondary school. The study was conducted on two sets of textbooks and the analysis focused on the algebraic content, in particular on the way it was presented by the textbook authors, including definitions, descriptions, and worked examples and exercises for learners. As the outcome, the algebra in school mathematics was characterized over time. The findings showed that in some mathematics textbooks, algebraic expressions were the main object of the study and in some textbooks equations and algebraic expressions were put on an equal level. However, in other textbooks, equations played were the main object. Jakobsson-Åhl also found that the use of a mixture of literal symbols and numbers was of paramount importance in the first decades in the sixteenth century. Therefore, he could conclude that the current Swedish mathematics textbooks are different from previous ones. In the first decades in the sixteenth century there was emphasis on the literal symbols and currently there is more emphasis on mathematics as a process. When this is looked at through Sfard and Linchevski's framework, one notices that this would be the first and second levels of algebraic thinking (expression and computational process) – see chapter 3. His research focused on a procedural understanding.

Jakobsson-Åhl (2006) presents an analysis of two sets of textbooks, concentrated on the algebraic content as it was presented by the textbook authors, including definitions, descriptions, worked examples and exercises for students. The study served to characterize algebra in school mathematics over time. It was found that there were significant changes on the notion of algebra over time, from the simple focus on skills in the manipulation of algebraic expressions to the

gradual integration of algebra with other mathematical subjects and then to abstract algebra. The study contributed to collect the existing knowledge on the teaching and learning of algebra.

Their findings revealed that most learners were not very likely to be able to cope with the problems which require interpreting that the objects presented were functions and not numbers. The results also showed that although none of the learners could see the object (function) behind the symbols, learners with only procedural understanding when faced with unfamiliar problem-solving involving concepts found them incomprehensible.

Bay-Williams (2001) articulate the idea that algebra helps to understand patterns, relations, and functions; represents and analyses situations; uses mathematical models to represent and understand quantitative relationships and analyses change in various contexts (Bay-Williams, 2001). Spielhagen (2006) analyzed long-term academic results for learners who register in and those who did not register in eighth-grade algebra, and found that those who completed algebra in the eighth grade attended college at greater rates than those who did not (Spielhagen, 2006). Further researchers like Clotfelter, Ladd and Vigdor (2012) argue that algebra plays several important outcomes in US middle schools. These authors articulate the idea that learners who do not take Algebra I in middle schools face a mathematics placement decision in high schools. They add that studying of algebra helps them to be selected for the college preparatory track even if the learners' performance in the course is relatively poor (Clotfelter, Ladd, & Vigdor, 2012). Additionally, it is believed that algebra can be used to motivate learners for mathematics. Motivation beliefs can be considered as a psychological mechanism that influences learners' motivation to exert effort on learning tasks (Wigfield & Eccles, 2000). The following section looks at the comparison of mathematical textbooks within and across the countries, by identifying the similarities and differences between them.

2.5 Comparisons of Mathematics Textbooks within and across Countries

There are several studies comparing how textbooks are used in classrooms. (Zhu and Fan, 2006) articulate the idea that comparing the similarities and differences of cross-national textbooks can provide meaningful information for improving the future textbook design, particularly in the representation of problems.

Most of these works focus on aspects of presenting and teaching problem-solving in textbooks and how teachers plan from textbooks (Johansson, 2006). For instance, Fan and Zhu (2007) and Yeap, Ferruci and Carter (2002) analyse problem-solving using a qualitative approach, while Mayer, Sims and Tajika (1995) compared using quantitative data of how textbooks teach addition and subtraction across two countries.

Pepin and Haggarty (2001) carry out a study on content analysis of mathematics textbooks in England, France and Germany associated with their use by teachers in mathematics classrooms at lower secondary level. The analysis in this comparative study considers four areas in terms of textbook content and structure: the mathematical intentions; pedagogical intentions; sociological contexts; and the culture traditions represented in textbooks. Their study aims at exploring the mathematics classroom teaching and learning culture against a background of a textbook analysis and teachers' use of them. Their findings revealed that French textbooks embed pedagogical ideas of encouraging learners' thinking and reasoning mathematics. French textbooks emphasize that everyone should learn the same. German textbooks provide difficult and complicated mathematics tasks while British textbooks emphasize routine exercises without requiring deep level of thinking from learners, while teachers in British schools depend less on mathematics textbooks.

Fan and Zhu (2007) carry out a study to compare how textbooks are used in the US, China and Singapore at the lower secondary grade level (Fan & Zhu, 2007). The findings revealed that the Chinese series adopted the most explicit way, whereas the Singaporean series introduced specific problem-solving heuristics separately from other topics. In the US, more than two-thirds of the problem solutions in the US textbooks modelled at least two problem-solving stages in Pólya's model (Fan & Zhu, 2007). However, recent studies about the comparison of textbooks on algebra in the US showed significant difference in the number of tasks which engaged the learners in reasoning or proving (Davis, Smith, Roy, & Bilgic, 2014).

Cheung (2003) carry out a study to find out the similarities and differences of the problems in textbooks based on the four Hong Kong mathematics textbooks in which two books are based on a curriculum document published in 1985 and the other two are based on a curriculum document published in 2001. The findings revealed that only slight differences were found in distributions of problem types of to-be-solved problems in the two curricula.

One trend of textbook comparative studies aims to discover what mathematics textbooks actually look like, for example the layout (Sönnerhed, 2011). Pepin and Haggarty (2001) compare English with French and German textbooks and found that the layout of English textbooks has fewer questions and the structure is relatively brief and less coherent and more routine than the French and German ones (Pepin & Haggarty, 2001).

Flanders (1994) explores the relationship between intended, implemented, and tested curricula of 84 classes of the US eighth-grade mathematics learners in 1981-82 in the US using data from the Second International Mathematics Study (SIMS) (Flanders, 1994). Flanders (1994) finds that the SIMS test for eighth graders was not representative of the curriculum defined by learners' texts, given that the books were heavily biased toward a review of arithmetic and away from algebra or geometry items. In his study Flanders found that even though the learners also practiced solving items not covered in textbooks, the teachers believe that textbooks remain the major source that shapes the way in which the learners learn.

Nathan and Alibali (2002) analyse how problem solving activities were presented to eighth and ninth grades learners. It showed that the symbolic problems were presented prior to verbal problems (Nathan, Long, & Alibali, 2002). The study examined a corpus of 10 commonly used pre-algebra and algebra textbooks, and it was found that, for the latter category of textbooks, this pattern is stronger than the first one. According to Nathan, Long and Alibali (2002), when it comes to designing a textbook, one has to consider assumptions about learning and further, to implement these assumptions through the educational system, to benefit the learners and teachers. Thus, they can be analyzed with the textbook's content and structure.

Mayer, et al (1995) compare the lessons on addition of numbers across two countries Japan and the US using eleven mathematics textbooks; seven of these were Japanese and four were from the US. Using quantitative data for each content area, they considered the number of exercises, the number of irrelevant illustrations, the number of relevant illustrations, the number of worked examples, the number of words, the area occupied by exercises, the area occupied by irrelevant illustrations, the area occupied by relevant illustrations, and the area occupied by explanation for each topic of each mathematics book. Mayer, et al (1995) find that the Japanese textbooks contained more worked examples, relevant illustrations, and also gave more space to explanation of solution of examples than the US mathematics textbooks. However, the US mathematics

textbooks contained roughly the same number of exercises and irrelevant illustrations, but gave more space to to-be-solved problems, contrary to the Japanese mathematic textbooks. The authors suggested that problem solving in Japanese mathematic textbooks was seen more as a process, whereas in the US problem solving was emphasized more as product.

Yeap, et al (2002) compare arithmetic problems included in elementary schools textbooks in Singapore and the US and how textbooks are used to teach problem solving. Yeap, et al (2002) find that problems appearing in the Singaporean textbooks seem to be difficult and often presented new concepts, whereas the US mathematics textbook provided more opportunities to know heuristic strategies and also presented problem solving more explicitly.

Devetak, et al (2010) carry out a study in Slovenian science textbooks and notebooks on the topic states of matter. They analyzed educational material from two randomly selected publishers for learners aged 6 to 14 in the Slovenian primary and the lower secondary school. Their analysis was focused on exercises and images analysis and content analysis identifying key concepts and connections between them according to the national curriculum recommendations. The findings revealed some differences when comparing the type of images in the educational material. Moreover, the content analysis of the selected textbooks also shows that they retain the content directed by the national curriculum, but the ways (examples, content of the images and so on) in which authors present the topic differ.

Results also show that the exercises presented in the sciences textbooks had very low levels according to Bloom's taxonomy, and that none of them assessed the highest category of Bloom's taxonomy, evaluation. As I mentioned above, just like Cheung, their selected textbooks show that the textbooks retain the content directed by the national curriculum and also that teachers could use both to reach the aims that the national curriculum directs. On the other hand, a comparison of problems presented in four Hong Kong mathematics textbooks found that there were more problems (worked examples and to-be-solved problems) in textbooks C and D (based on a curriculum document published in 1985) than in textbooks A and B (based on a curriculum document published in 2001) (Cheung, 2003).

Stigler et al. (1986) analyse different types of addition and subtraction word problems in four American textbook series in parallel with the presentation in one Soviet textbook series. Stigler et

al. (1986) find that the American textbook series was at the time considered the most widely sold and used, and the Soviet textbook series was officially mandated by their government. Two classifications of word problem solving were compared: (1) one-step word problems and (2) two-step word problems. The comparison between the two countries found that the frequency in Soviet mathematics textbooks of two steps problems was higher than mathematics textbooks in the US (Stigler, Fuson, Ham, & Sook Kim, 1986). Stigler et al. (1986) find that the Soviet mathematics textbooks series had the most problems compared to the American mathematic textbooks series. Soviet books also contained many more two-step problems than the American books. Distribution of both one and two steps word problems solving across grade level in the Soviet textbooks series dropped sharply whereas it increased sharply in the American mathematic textbooks series.

Sood and Jitendra (2007) compare number sense instruction of three first-grade level traditional mathematics textbooks in order to evaluate effective instruction. The findings revealed that traditional textbooks offered more opportunities for linking tasks than reform-based mathematics textbooks. Besides, they found that instruction was more explicit in traditional mathematics textbooks, just as Haggard and Pepin, Sood and Jitendra's selected mathematics textbooks were deemed representative of mathematics books typically adopted in the country, here the United States of America. Devetak et al. (2010) investigate Slovenian science textbooks and notebooks-practical methods for learning, on the topic 'states of matter'. General information of the textbook and notebook, textual material, and pictorial material were classified (Devetak, Vogrinc, & Glazar, 2010).

Stevenson and Stigler (1992) compare textbooks of Japan and US. Both textbooks in reading and in mathematics were analyzed. It was found that the mathematics textbooks in Japan and US included about the same amount of concepts and skills while the difference in number of pages was considerable. The American textbooks tended to be very explicit while the Japanese textbooks are less detailed and the learners must rely on the teacher for an elaboration on the mathematical content. The US textbooks provide many examples of particular tasks and show each step of the solution. In comparison the Japanese textbook writers invites the learners to engage in problem solving to a larger extent by not providing detailed solutions. A result is that the learners come up with different solutions strategies and this activates meaningful discussions in the classroom.

Lastly, the Japanese textbooks are attended to in every detail while parts of chapters and also whole chapters are omitted by the teachers in the US.

A comparative study of mathematics textbooks in grade seven in Japan and US, showed that 81% of the Japanese textbooks were devoted to explaining solution procedure for worked examples, whereas the corresponding amount in the US textbooks were 36%. In addition, The US textbooks devoted 45% of their space to unsolved exercises and 19% to irrelevant illustrations. The corresponding amounts in the Japanese textbooks were 19% and 0% respectively (Mayer, Sims, & Tajika, 1995).

Mouzakitis (2006) analysis textbooks from Italy and Greece focusing on textual analysis, which means analyzing texts as a possible means of instruction (Mouzakitis, 2006). Mouzakitis (2006) study was similar to Jakobsson-Åhl's (2006) but the later has worked on geometry textbooks. Jakobsson-Åhl (2006) develop a theoretical framework based on a modification of the approach elaborated by the Third International Mathematics and Science Study (TIMSS) team and segmented the sample content of the two textbooks into blocks, then classified them into different categories. Similar to Jakobsson-Åhl (2006) and Mouzakitis (2006) analysis textbooks focusing on textual analysis, which means analyzing texts as a possible means of instruction. His results showed that there are a few worked examples in both textbooks. In contrast, there is an abundance of graph blocks but the number of examples in the two textbooks is rather small, especially in the Greek textbook. However, Mouzakitis' theoretical framework provided a frame in order to detect any mathematical errors, ambiguities and inexactitudes. Based on these different comparisons, it is therefore expected that comparison of textbooks is a successful area of mathematics educational research. Such studies may focus on different aspects such as conceptual learning, problem solving, the coverage of reasoning or proving, the use of textbooks in classroom, or more recently - the use of computers encouraged in the textbooks (Fan & Zhu, 2000; Hensey, 1996; Li, 1999; Mesa, 2004; Ng, 2002; Zhu & Fan, 2006).

Yang and Li (2015) examine the Differences of Linear Systems between Finnish and Taiwanese Textbooks for grades 7 to 9, using content analysis. Yang and Li (2015) explore the teaching sequence, application types, representation forms, response types, and level of cognitive demand. The findings revealed the main difference between the Finnish and Taiwanese textbooks was that the Finnish textbooks introduced the topic of linear systems using a graphical approach, while the

Taiwanese textbooks used an algebraic approach. Results also showed that the Taiwanese textbooks had fewer problems, but more challenging problems requiring a higher level of cognitive demand; the Finnish textbooks had more authentic application problems, and even more problems were displayed in visual forms. In addition, the Taiwanese textbooks had more open-ended problems, particularly problems asking learners to explore or explain, whereas the Finnish textbooks did not have exploration problems. The topic of linear systems was also taught earlier in Taiwanese textbooks.

Thirdly, how to present the content is highlighted in studies. After comparing the content presentation of the addition and subtraction of integers between American and Chinese mathematics textbooks, the Chinese textbooks were found to contain more problems with high level mathematics content (Li, 2000, p. 239).

Researchers examined the difference between England and Japan regarding the solution of quadratic equations in junior secondary schools (Whitburn, 1995). The results showed that in England, the approach to this topic is too limited, while in Japan it would be taught both algebraically and graphically.

Gatabi, Stacey and Goova (2012) compare textbooks in Australia and Iran by investigating grade 9 textbook problems for characteristics related to mathematical literacy in those countries (Gatabi, Stacey, & Gooya, 2012). The findings revealed that Australian mathematics textbooks contain many more problems than the Iranian textbook and each chapter includes problems in a diverse range of contexts and the mathematical tasks range from simple applications of formulas to real world modelling although of a constrained nature. Besides, the Iranian textbook has considerably less diversity of context and very little opportunity for learners to engage in mathematical modelling, the key process of mathematical literacy.

Vula, Kingji-Kastrati and Podvorica (2015) compare mathematics textbooks from Kosovo and Albania based on the topic of fractions for grade 1 to 5. Findings showed that Albanian mathematics textbooks covered more lessons on fractions than Kosovar text-books. Textbooks from both countries focus mostly on part whole and operator construction. Also, the majority of problems focused on pure mathematics and single procedures. Few problems in Kosovar textbooks required problem explanation, while problem solving as a cognitive requirement was used more

in Albanian textbooks (Vula, Kingji-Kastrati, & Podvorica, 2015). Based on these different comparisons, it is therefore expected that comparison of textbooks is a successful area of mathematics educational research. Such studies may focus on different aspects such as conceptual learning, problem solving, the coverage of reasoning or proving, the use of textbooks in classroom, or more recently - the use of computers encouraged in the textbooks (Fan & Zhu, 2000; Hensey, 1996; Li, 1999; Mesa, 2004; Ng, 2002; Zhu & Fan, 2006).

In this section I discussed the comparison of the use of textbooks within and across selected countries. I highlighted a comparison of mathematics textbooks across three European countries which found that, for example, in France and Germany the learners have more access to the mathematics textbooks than in England where the textbooks were also found to be less densely packed with examples and exercises. The other study, in the US, found that teachers believe in the efficiency of the textbooks for the learners' success, even though they also practice solving items not covered in textbooks. Besides, an analysis of the complexity of tasks via a method of counting the steps of the solution on the exercises and examples in different textbooks was presented. In this analysis I found that textbooks have similarities and differences. This section was about discussion on the comparisons of the use textbooks in different countries. What can be noted here, is that almost of those comparisons were made in developed countries rather than in developing. Therefore, this study is focused on the comparison of textbooks content of algebra in developing countries such as Angola and South.

2.6 Using textbooks in developing countries

The availability of textbooks has proved repeatedly to be the major factor predicting learning success in developing countries, defining the scope and structuring of the content to be taught, providing guidance on pedagogy, and saving time in providing a record of correct information for learners (Verspoor & Wu, 1990). One study in particular, stated that the availability and the access of textbooks and other instructional material was one of the most significant factor of achievements (Mohammad & Kumari, 2007). Using both availability and access, the rural areas research showed that there was serious implications on the school improvement. On the other hand given the fact in the rural context textbook plays a fundamental role, the school achievement should be based on textbook reform (Mohammad & Kumari, 2007).

However Moulton (1994) reports that there existed a link between textbook availability and learner achievement in developing countries. Taylor and Vinjevold (1999) report that there is a lack of large scale studies of the effects of textbooks or other learning materials on learner learning in South Africa. Although there has been no large-scale study on textbook provision in South Africa, in 1998 the Department of Basic Education (DoBE) established a task team to investigate the problems involved in textbook procurement and distribution processes in South Africa (R. Taylor, 1990, p. 167).

Textbooks are almost ubiquitous in mathematics classrooms across the developed world and are amongst the most influential factors in the implemented curriculum (Hodgen, Küchemann, & Brown, 2010).

Moulton (1997) argues that the use of textbooks in a developing country plays an essential role to learner achievement. Moulton (1997) explore the availability and use of textbooks in primary schools in developing countries and US. His findings revealed that research on textbooks' use in developing countries is smaller and less known. However, US teachers are expected by their mentors, peers, bosses and clients (parents) to use textbooks extensively, which is different in developing countries. Moulton (1997) adds that, in developing countries, if these textbooks are available, teachers either do not use them or do not use them effectively. The failure of not using textbooks mostly apparent in Botswana, Ghana, and Chile. In South Africa a number of studies of the use of textbooks is increasing. For instance, Ramnarain and Padayachee (2015) compare textbooks using three grade 10 Life Sciences textbooks and three Biology textbooks. Ramnarain and Padavachee (2015) find that both Life Sciences and Biology textbooks still overwhelmingly represent the theme "Science as a body of knowledge" (pag, 2). (Mumanyi, 2014) investigates how new primary mathematics grade textbooks were used in Mashonaland Est Province of Zimbabwe. The findings revealed that textbooks increase the teachers' and learners' enthusiasm, generate some opportunities for teaching and develop the learners' performance in mathematics. In South African schools, one history teacher argues that textbooks play an important role in teaching and learning (Joubert, 2015). According to Joubert (2015) textbooks are useful to give teachers an overview of the curriculum and they were very helpful in terms of settling assessments and examinations questions. In terms of learning, they may help learners for reading and other activities that help them in their learning process. Textbooks contain "texts, photos, graphs,

cartoons, drawings, paintings, posters and other visual images” (p.265). Joubert (2015) also adds that she “uses textbooks that has most variety of sources and questions for learners to be able to think critically and so that there is enough content in the text itself to be able to write an essay on that particular topic” (p.265).

In Namibia Van Graan *et al.* (2002) report that only 62% of the learners in senior primary classes have their own textbook for mathematics and English. In neighbouring South Africa, Howie (2001) reports that the head teachers of 45% of secondary schools surveyed indicated that the progress of their learners in science and mathematics is hampered by a lack of textbooks. On the other hand, in industrialized countries in the UK for instance, most learners have access to one of a class set of textbooks provided by the school but, in about a quarter of classrooms, textbooks are shared by learners (M. Johnson, 1999; D. Lambert, 1999).

In this study, I am interested to know the way the content of algebra is covered [*in the textbook*], how it is explained, what are the levels of the examples, what kind exercises are there in the two developing countries (South Africa and Angola). Thus, this study hopes to bring a new insight about the content of algebra topics, their explanations as well as examples and exercises in those countries. I also present a comparison of algebraic concepts between the two grade 10 textbooks one from South Africa and another one from Angola to check the similarities and differences between them.

2.7 Textbooks in South Africa and Angola

Although textbooks studies are extensively increasing in different parts of the world, but few are explored in terms of algebra content is organized particularly in Angola and South Africa. There is no current research on mathematics education in Angola, only suggestions from ethno mathematics educators on the use of Angolan sand drawings in teaching (Christiansen & Borges, 2015).

The use of textbooks is one of the Department of Basic Education’s policies which suggests that each learner must be provided with a textbook for each subject before commencement of the academic year (DoBE, 2011). The South African government’s textbook policy gives priority to ensuring that every school learner has access to textbooks for all subjects (Adonis, 2015; KZN, 2010). The DoBE (2011) added that textbooks, workbooks and lesson plans will provide the curriculum support at all levels.

Historically, South African learners have not enjoyed the adequate access to learning materials including textbooks. In 2007, a study carried out by Moloai and Chetty (2010) reported that 45% of the learners had reading books and 36,4% had mathematics textbooks in grade 6.

Kiggundu and Nayimuli (2009) analyse 24 experiences of learner teachers in the Vaal University of Technology Postgraduate Certificate in Education (PGCE) during their teaching practice in the Vaal area, showed that the schools where these learner teachers teach, did not have resources to facilitate the teaching and learning process. Learners did not have textbooks, which made teaching problematic.

There was no library in the school, and there was a shortage of textbooks. Their findings also revealed by the majority of respondents that learners had to share textbooks in class and that due to the shortage of the textbooks, learners were not permitted to take them home, so it was problematic when they were given homework.

In teaching mathematics, research carried out in South Africa, articulates that textbooks represents a pedagogic text for teachers (Jaffer, 2001). Jaffer analyzed the use of textbooks in mathematics discourse in “Maths (mathematics) for all Grade 7 Learner’s Activity Book” in the province of Western Cape. His findings show that teachers' use of textbooks show that in most cases, the pedagogy contained in the textbooks was considerably different from the favoured pedagogy of the teachers. This was due to the fact that most teachers desired a deductive pedagogy and used the textbook in ways which fragmented the mathematical knowledge presented to learners, abridged the mathematical complexity of the textbook tasks and consequently transformed the pedagogic intentions of the textbook. With regards to school wealth in South Africa, studies report that 4 quartile 4 schools’ learners are far more likely to have their own textbook, receive homework frequently, experience less teacher absenteeism, repeat fewer grades, live in urban areas, speak English more frequently at home, and have more educated parents (Spaull, 2013).

Adler et al. (2001) find that teachers in grades 7 and 9 had access to textbooks. In grade 7 there were sufficient books for all learners across all contexts. In grade 9, however, not all classrooms had sufficient textbooks for all learners. Adler et al. (2001) attribute the difference in availability of textbooks in grade 7 and grade 9 to the current curriculum reform. They contend that since 2005 has been implemented in grade 7 and not grade 9, the provisioning of textbooks by government at grade 7 level has been stimulated. Other findings suggest that textbooks are generally unavailable

in South African schools and where textbooks are available there are insufficient textbooks for all learners (N. Taylor & Vinjevold, 1999). All of these views indicate that using textbook in South Africa and Angola is still underdeveloped and the pedagogy contained in textbooks is different from what they teach.

Research on the availability of textbooks is not directly related to my study but clearly the availability of textbooks forms a necessary condition for learning and teaching in general and mathematics education in particular. This study is not relying on textbook availability, neither to learners' achievement, nor to teachers' use of textbooks. It is based on comparing the content of algebra in grade 10 in two countries such as Angola and South Africa since there are no previous studies about comparison of the use of textbooks in grade 10 in those countries. Therefore, this study is needed in order to compare the algebra sections of textbooks in grade 10 in Angola and in South Africa.

2.8 General Summary

In this literature chapter I have discussed the use of textbooks and their role in general, particularly the use of mathematics textbooks in the classroom. As said in the literature, textbooks are helpful documents to carry out the work of mathematics. What can be noted about textbooks' use, is that they should be considered as one of the many resources teachers can draw upon in generating effective teaching and learning, but teachers need professional course and experience in adjusting and transforming textbooks as well as in using genuine materials as well as in generating their own teaching materials.

Thirdly, I discussed the role and learning of algebra, as noted that algebra help learners to be problem solvers and think mathematically. In the fourth section, the chapter focused on the comparison of textbooks within and across the country. It was noted that, textbooks have similarities and differences. Finally, I discussed the use of textbooks in a developing country and, found that textbooks are less usable in some developing countries than in the US for instance. It was found that understanding textbooks' use and how problem solving is structured are important when considering giving learners access to the content. According to this literature, textbooks' use would be considered by many authors to be a primary source for learners' learning, but with recognition that textbooks recontextualise the curriculum, and teachers recontextualise textbook

content. Moreover, it was noted that although similar studies to this one may have been done, there is still a place for this study when it comes to looking at conceptual progression in algebra.

CHAPTER 3 : THEORETICAL FRAMEWORK

The research questions of this study are: “what are the similarities and differences between the three textbooks in terms of topics’ coverage and content distribution between explanations, examples and exercises?”; “what are the similarities and differences between the three textbooks in relation to conceptual progression in algebra?”; and “how is the conceptual progression linked to developing procedural competencies? Do tasks, explanations and examples correspond to levels of algebraic thinking?” There are three content categories for the analysis: examples, explanations and exercises. Identifiable blocks of these constitute the units of analysis, as discussed in the methodology chapter. Textbooks are coded and analysed systematically in these three aspects according to the conceptual framework described in this chapter.

Before describing the framework for this study, I first discuss the definition of the terms *concept*, *conceptual understanding* and *conceptual progression*, in order to discuss what is different between the learners’ understanding and the ‘social’ concepts as understood in the community. I will also describe the duality process-object in order to make it easier for the reader to understand the framework used.

3.1 Concept, concept image, concept definition, conceptual understanding and conceptual progression

Tall and Vinner (1981) state “Concept” refers to an idea of something that exists in your mind. A concept can be seen as an abstract idea, a mental symbol or a construction of knowledge. Differently, “concept image” is a much wider concept, representing “the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated proprieties and processes” (Tall & Vinner, 1981, p. 152). I further discuss these two terms. This will guide the discussion about conceptual understanding and conceptual progression.

The mathematical concept defined in a formal way is different from the cognitive process by which they are conceived (Tall & Vinner, 1981). Tall and Vinner (1981) indicate that the concepts which we use can be formal or individual. As time passes these concepts may be refined in their meaning to get an accurate definition. According to Tall and Vinner (1981) “in this process the concept is given a symbol or a name which enables it to be communicated and aids in its mental manipulation” (p. 1).

Furthermore, Tall and Vinner (1981) state that learners' concept definitions are a part of their concept images. Tall and Vinner (1981) describe this as the statement that the individual will exteriorize when asked to define a concept. Definitions constructed by individuals are a description of their concept images (Vinner, 1983). Tall and Vinner (1981) observe that the concept image of a concept is built up over the years through all kind of experiences, changing "as the individual meets new stimuli and matures" (p. 152). For example, the concept of subtraction is generally first met as a process that involves integer positive numbers. At this level, a child may define it as an operation where the result is always less than the minuend. As time passes, for a child, this observation will become part of his concept image. However, this may cause some conflict when the subtraction is extended to integer numbers (positives and negatives). Additionally, Tall and Vinner (1981) suggested that "all mental attributes associated with a concept, whether they be conscious or unconscious, should be included in the concept image" (p. 2). Tall and Vinner (1981) made a final argument "as the concept image develops it need not to be coherent at all times" (p. 2).

Tall and Vinner (1998) and Thompson (1984) agree with the above writers; they defined the concept image as the total cognitive structure in an individual's mind that the learner will associate with the concept (Tall & Vinner, 1981; Thompson, 1984). Concept images have aspects which are different from the concept definition. For example, Tall and Vinner (1981) define the expression concept definition as a form of words, that can be written or spoken, used to identify a concept and also to specify meaning and can be given to one or constructed by oneself in the way that an individual's concept definition is different from a formal concept of definition but accepted by the mathematical community at large.

When it comes to mathematics, it is very important for the learner to learn new ideas by connecting them to those already known. This takes us to 'conceptual understanding', which according to Kilpatrick (2001) refers to an "integrated and functional grasp of mathematical ideas" (p.113), which enables the learner to learn new ideas by connecting them to what he/she already knows without necessarily going through all the numerous tasks enshrined in the textbooks. He states that learners must understand what mathematical symbols and procedures mean (that is, develop concept images which are fairly consistent with socially accepted concepts) so they can explain their final answer and support that explanation clearly. Explaining the final answer helps learners

to construct facts (memorizing and understanding) and skills (procedures) for solving new problems. Some of the advantages of building conceptual understanding are that it supports retention and prevents common errors.

Grouws and Cebula (2000) state [“There is evidence that students (learners) can learn new skills and concepts while they are working out solution to problems” (Grouws & Cebulla, 2000).]

According to these authors:

Research suggests that students (learners) who develop conceptual understanding early perform best on procedural knowledge later. Students (learners) with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students (learners) without conceptual understanding are able to acquire procedural knowledge when the skill is taught, but research suggests that students (learners) with low levels of conceptual understanding need more practice in order to acquire procedural knowledge (Grouws & Cebulla, 2000, p. 15).

If the learner does not have a good understanding, he/she can fail in the process of checking if the answer makes sense and this will result in errors and misconceptions. Thus the stronger the background a learner has, the easier it is for him/her to understand the examples and exercises in the textbook even without the teacher’s explanation (Kilpatrick, 2001). Without a solid background in mathematics, the textbook’s content is nothing but isolated facts and methods for the learner.

As explained through an example:

The concept of the derivative is according to its formal definition a limit of a difference quotient. On the other hand, the derivative can be visually interpreted as a slope of the tangent line, or it can be understood as a measure of an instantaneous rate of change. These are three different interpretations concerning the meaning of the concept of the derivative. If an individual has a highly coherent concept image, he/she is able to utilize all these interpretations in a problem solving process regardless the original form of the problem (Vjjholainen, 2008, p. 6).

This is consistent with Peacocke (2005) suggests that “the same concept may be differently represented, and have different computational or associative procedures, operating on its mental representations, in different individual thinkers” (Peacocke, 2005, p. 168).

At this level where the learner is able to learn a new concept by connecting to a concept that he/she already knows it is seen as developing conceptual understanding. As was mentioned above, Kilpatrick (2001) describes conceptual understanding as referring to learners comprehending mathematics concepts, operations and relations in ways which connect ideas to each other and to what they already know.

Expanding on this, Sfard (1991) previously describes the dual nature of mathematical concepts in the sense that a concept can be seen as a process and as an object (Sfard, 1991). Generally, the process aspect precedes the object aspect both socially/historically for the individual learning the concept. For functions and algebraic expressions in particular, this has been expanded in her later work. Skemp (1971) explains it when he discusses a man born blind as a way to understand the concept of ‘redness’. Skemp (1971) discusses exactly this point in a very helpful illustration of how we learn concepts. Considering the hypothetical situation of an adult born blind but given sight by an operation, he suggests that there is no way we can help the adult to understand the concept of ‘redness’ by means of definition (Skemp, 1971). It is only by pointing to a variety of objects which are red that the adult could himself abstract the idea, the property which is common to all of the objects. Clearly, one would also assume that the counter-examples, the objects which were not red, would also help to clarify what was meant by ‘redness’. Skemp (1971) was claiming that the learning of mathematical concepts is comparable. We must not expect children to learn through definitions (Tall, 2004). Based on this, understanding mathematical symbols and procedures are important instruments for the teachers and learners to understand and access new concepts.

3.2 The duality process-object

Regarding the concepts themselves, the duality process-object of a mathematical concept can be seen in two different ways as a process and as an object. For example, Sfard and Linchevski (1994) discuss this with regard to an algebraic expression which can be seen as a process - a description of a computation, and as an object – a number, a function or even a family of functions. When we look at a mathematical concept, an algebraic expression or any representation of a mathematical concept what we see depends on which context or problem we are facing.

Breidenbach et al. (1992) use the words *process* and *object* to describe a dual nature of functions. Breidenbach et al. (1992) develop a process conception to understand the concept of function in which objects and processes are constructed (Breidenbach, Dubinsky, Hawks, & Nichols, 1992). Focusing on function as process Breidenbach et al. (1992) consider that the dual nature of

functions is accessed in three ways: pre-function, action, and process. Similarly following Piaget, processes and objects are said to be constructed by reflective abstractions (see also Dubinsky, 1991). Dubinsky (1991) considered an example of function to describe the mutual dependence between processes and objects. Piaget and Garcia (1989) suggested that reflective abstractions help learners to construct a process conception of function. Thus, going through the reflective abstraction is the advanced process of mathematical thinking where the process is separated from content and that processes are themselves converted to objects (Piaget & Garcia, 1989).

Kaput (1989) articulated the idea that “mental entity building through reification of actions, procedures, and concepts into phenomenological objects which can then serve as the basis for new actions, procedures, and concepts at a higher level of organization” (p. 106). This suggests that in mathematics operational concepts must be engaged before structural concepts (Kaput, 1989). The notion of “reification” of concepts comes from the study of Sfard (1991), where she declares that concept development is acquired through three levels which include (1) where the learner processes operations on lower level mathematical objects; (2) where the learner combines processes to make comparisons and generalize, and lastly (3) where the learner at this level is able to conceive of the mathematical concept as an object on its own. For instance, in the case of linear functions, the learner should thoroughly understand the different representations that a function can take, for example when it is compared with quadratic functions (Cifarelli, 1988).

Sfard (1991) referred to these three levels which transform a concept from process to an object as interiorization, condensation and reification. Sfard (1991) considered an explanation of a word problem involving the concept of linear inequality and explained with regard to learners’ transition from arithmetic to algebra. In their study, problem solving activities were explained using the theory of reification and the theory concerning levels of reflective abstraction. Sfard (1991) suggested that learners need to move through these from operational process to abstract object.

In this regard, (Abadzi, 2006) suggested that textbooks of all levels should have rich, extensive explanations and elaboration of concepts. Textbooks with explicit explanations facilitate teachers and learners to understand concepts. Success in mathematics is accessed by understanding concepts and complete explanations (Leinhardt, 1987). For this criterion providing an extensive explanation is an important skill for helping teachers and learners to clarify their thinking and solutions processes.

3.3 Sfard and Linchevski’s Framework for the Process-Object Duality

The study adopted the theoretical framework of (Sfard & Linchevski, 1994). These authors presented a theoretical framework for the different ways to look at an algebraic expression and tried to answer the question “to what extent is the learner capable of seeing and using the variety of possible interpretations of algebraic constructs?” (p. 192). Sfard and Linchevski (1994) consider

an example of an algebraic expression and explained it in different ways, depending on the level of conceptual understanding (p. 1). As I agreed with Sfard and Linchevski (1994) that competence in algebra can be viewed as “a function of versatility and adaptability in the interpretation of symbols” (p. 202), and that “today, to solve one little problem from a standard textbook, the learner must often resort to all the different perspectives together.” (p. 202), we felt an interrogation of the level of understanding encouraged by the textbooks would be helpful. While Sfard and Linchevski (1994) do not operationalise their discussion into a distinct external language of description, they provide enough examples to warrant using their descriptions as such. Thus, I constructed four ‘levels’ of understanding of algebraic expressions mirroring their descriptions. I note that if an explanation, example or exercise in a textbook engages expressions as functions, this does not exclude also taking an operational view. “Our claim that certain kinds of algebra were operational in their character while others were structural should be understood as referring to what constitutes the *primary focus* of a given type of algebra.” (pp. 195-196).

In their analysis the same expression was perceived as different objects: number, function, family of function and also as a computational process.

Sfard and Linchevski (1994) suggest that learners need to move through these from process to object, and from less to more generalized object.

The algebraic expression is viewed as a:

1. Computational process: $3x^2 + 5x + 7$, is seen as a sequence of the following instructions: Multiply by 3 the square of the number at hand, add to the result the product of 5 and the number at hand and finally add 7.
2. Number: $3x^2 + 5x + 7$ may represent a certain number. Given x it is possible to get a certain number y calculated as $3x^2 + 5x + 7$.
3. Function: Changing the context $3x^2 + 5x + 7$ may become a function, which is a mapping that translates each number x in another. The result is set of ordered pairs (x, y) or $(x, 3x^2 + 5x + 7)$. The function is seen as an object (already mentioned), but a more generalized or abstract object than on level two.
4. Family of functions: If we change the numerical coefficient of x^2 to an unknown, $3x^2 + 5x + 7$ may be a member of a family of functions $ax^2 + 5x + 7$ from R to R . Alternatively $ax^2 + 5x + 7$ represents a function of two variables from R^2 to R , ie. Mapping a and x onto $ax^2 + 5x + 7$.

Looking at consistency and progression of these different levels, the conceptual step must grow gradually; if a learner jumps on the levels we must not expect her/him to be confused and helpless, but it could mean that individual tasks were revisited in ways which were reduced to operational

knowledge and in the level where he/she is faced with a new mathematical concept he/she may be lost.

As Sfard & Linchevski (1994) demonstrate, this framework can be used to interrogate the level of algebraic understanding of learners. I am going to use it to investigate the algebraic level(s) of the textbooks and then describe the progression in terms of movement between these levels. This seems a reasonable extension of the use of the framework in order to gain insight into the algebraic concepts progression presented by three different high school textbooks within their chapters. While attempting to analyse data, I found that all three textbooks gave a substantial amount of space to working with expressions in a way which reflected neither an operational nor a structural understanding. On this basis, I had to include the level symbolic manipulation in order to suit my research. "Symbolic manipulation" has been added as a separate level whenever the textbooks' explanations, exercises and examples could not be classified as belonging in any of the four levels, namely process, a number, a function or set of functions. In the next chapter I will explain this in more detail.

CHAPTER 4 : METHODOLOGY

This chapter aims to present the chosen paradigm, methodological approach, and type of data collection. It also presents the validity of the data collected.

4.1 Paradigm

I worked within an interpretivist paradigm. I wanted to understand how the pedagogic device operates when it comes to reproducing and assessing textbook knowledge. Do textbooks offer algebraic concepts that have been produced, structured, and measured, to be further recontextualized by their use in the classroom? I analysed textbooks with a view to capturing the manner of conceptual progression and consistency. Through a reliable and consistent presentation of concepts, credible knowledge is produced about how meanings are created and processes understood in the pedagogic recontextualization field (Wahyuni, 2012). An interpretivist paradigm entails a study that uncovers meanings in the pedagogic recontextualization field, thus contributing to sound social knowledge (Wahyuni, 2012, p. 71). On the other hand, by applying the interpretative paradigm in the process of acquiring knowledge by investigating the content of textbooks in different contexts, one may reach different conclusions regarding one and the same theme (Clariana & Koul, 2008).

Working within the interpretivist paradigm, I analysed and compared textbooks so as to investigate the algebraic concepts in terms of how problem-solving is represented and to compare degrees to which the different scrutinized textbooks focus on structural and/or operational aspects of algebraic concepts. Within this paradigm, the different approaches of textbooks to providing ways of understanding concepts were analysed. Hence, the qualitative method of analysing data was used for the purpose of gaining insight into algebraic concepts' progression. This was achieved by coding the textbooks to check consistency and progression in levels of algebraic thinking.

4.2 Sample

The sample consisted of three textbooks for teaching algebra, one for grade 10 from Angola and two textbooks (grade 9 and 10) from South Africa. The purpose was to compare the Angolan textbook to another textbook. The SA textbooks chosen were more “traditional” and the closest version of the edition to the Angolan textbook which is still in use so that the comparison would not merely reflect pedagogical differences. These books belong to the old curriculum because the textbook grade 10 from Angola is still used by teachers, and was drawn from the Angolan

curriculum 2006 which has not changed. Sampling was made purposively (Tongco, 2007). It is a non-probabilistic tool used in qualitative research which may be applied on selecting unities based on specific purposes associated with finding the answer to the research questions (Teddlie & Yu, 2007). Tongco (2007) articulates the idea that “choosing purposive sampling is a fundamental to the quality of the data gathered” (p.147).

As I discussed previously, I have been a teacher in high school grade 10 in Angola, which is the first grade in Angolan high schools, and wanted to compare the content of algebra in grade 10 for South Africa and Angola in terms of their topics given that they use in different curricula, to examine which one performs better than the other. I also included grade 9 to identify the progression in levels from grade 9 to 10 in South Africa. I would have explored the progression in levels from grade 9 to grade 10 in Angola however I only focused on the textbooks being used in high schools, which starts from grade 10 in Angola.

The South African textbooks are written in English; the Angolan textbook is in Portuguese. They are the following:

- South Africa: Classroom Mathematics. Grade 9. Learners’ Book (Laridon et al., 2006).
- South Africa: Classroom Mathematics. Grade 10. Learners’ Book (Laridon et al., 2004).
- Angola: Matematica 10 Classe. Textos De Apoio Ao Aluno (Trindade, 2009)

Table 4.1 indicates the content of algebra in all three textbooks. For instance, SA grade 9 textbook is composed of algebra, functions and patterns and equations. For SA grade 10 textbooks, there appears algebraic expression, exponents and equations and inequalities while Angola has polynomials and functions and graphs.

Table 4.1 An overview of the chapters from the different textbooks.

The table lists chapter numbers and titles. When titles are not specific, content of chapter is listed in brackets.

SA grade 9	SA grade 10	Angola grade 10
* Chapter 5. Algebra * Chapter 6. Functions and patterns * Chapter 7. Equations	* Chapter 5. Algebraic expressions * Chapter 7. Exponents * Chapter 8. Equations and inequalities	* Chapter 1. Polynomials * Chapter 5. Functions and graphs. Generalities (inequalities and equations, real functions, monotonic and extreme relatives, inequalities of 2 degrees, model functions, operations for polynomials)

4.3 Data collection

Data collection was made using both qualitative and quantitative approaches which can be referred to as a mixed approach (Creswell, 2013; R. B. Johnson & Onwuegbuzie, 2004; Mertens, 2014). On the one side, qualitative research deals with the phenomena which does not include quantifiable and statistical tools (Corbin & Strauss, 2014). On the other side, quantitative research consists of those studies in which the data can be analysed in terms of numbers (Creswell, 2013; Hughes, 2006).

In this study, the qualitative approach was used to analyse the data from the three sets of textbooks, two from South Africa and one from Angola. It was used to compare the levels of algebraic content between the selected Grade 10 South African and Angolan textbooks. This comparison was ultimately a means to evaluate the algebraic concepts from a structural and operational perspective.

I used Sfard and Linchevski's (1994) approach, as discussed in chapter three, to classify explanations, examples and exercises. Similarly, qualitative was used to examine the progression between two books one from Grade 9 and another from Grade 10 in South Africa.

On the other hand, a quantitative approach was used by quantifying explanations and examples found in three textbooks for both countries to obtain the total length of their treatment in a textbook. The quantitative approach was included in order to check the length of explanations or examples,

where I counted the number of units of analysis in each of the content categories for the three textbooks. This measure has been done page by page and it was measured in centimetres.

Simply counting the number of occurrences within each category would disguise issues of length (which may or may not be indicative of depth), and simply measuring the extent of the occurrences would disguise issues of frequency, therefore, it was decided to do both.

It might be worthwhile to investigate all the units in the selected textbooks. However, due to the time limitation of the current project, this was not possible. On the other hand, the triangulation method refers to the combination of data or methods so that diverse viewpoints or standpoints cast light upon a topic (Olsen, 2004). Olsen (2004) also articulates that triangulation implicates both qualitative and quantitative methods which lead to the validity of the results. Therefore, the triangulation method which is known as the combination of both qualitative and quantitative methods was used in this study (Anney, 2014; Doorenbos, 2014; Olsen, 2004).

In order to code the explanations, examples and exercises, each paragraph or connected block of text was classified as an explanation, example or exercise. A set of related paragraphs that discussed a single identifiable item, concept or procedure was seen as a unit. For instance, for the item products and factors the paragraphs discussing identities, identity sign, and changing the expression to an equivalent form were considered as explanations. If an explanation was 'interrupted' by an example but then continued after the example, it was counted as one explanation only. Finally, the measurements referring to the same category were summed and calculated in percentages, to obtain the total length of their treatment in a book. In the category 'number of pages', the pages were counted page by page.

4.4 Coding approach

As textbooks were the objects of this study, content analysis or document analysis was used as the research method and coding. Content analysis is known as a methodology that has been widely used in the social sciences in recent years. It is recognized as a method for analysing documents to test theoretical issues related to the understanding of data (Elo & Kyngäs, 2008). Today, it is used for studying the content of communication, such as books or websites. Content analysis is considered as fundamental in communication research and was originally introduced in content communication such as occurs in speech and written text (Lombard, Snyder-Duch, & Bracken,

2002). Document analysis has been used in several textbook analysis studies (Schmidt et al., 1998). Researchers in recent years agree that qualitative content analysis is one of the most used methods to analyse text data. However, in the 1950s, researchers involved both qualitative and quantitative methods to analyse data. Later, content analysis was used “primarily as a quantitative research method, with text data coded into explicit categories and then described using statistics” (Hsieh & Shannon, 2005, p. 1278). Therefore, content analysis is defined as a research method to interpret meaning from the context of text data through the systematic classification process of coding and identifying themes or patterns. Interpreting meaning from the context can be assessed by the process of counting to identify patterns and to contextualize the codes (Morgan, 1993, here from Hsieh and Shannon, 2005).

Similar to Morgan, several researchers have interpreted specific content of textbooks by using the quantitative content analysis to evaluate the quality of the content. Mayer et al. (1995) start by counting the pages covered, the number of exercises, the number of relevant illustrations, the number of worked examples, the number of words, the area occupied by exercises, and the area occupied by explanation (Mayer et al., 1995).

To check the progression and focusing on the frequency of how children are exposed to problem solving, Stigler et al. (1986) evaluate how problem-solving were presented in mathematics textbooks by counting the steps required to reach the solution of tasks or examples.

The analysis applied in the current study has both quantitative and qualitative elements. I have analysed the textbooks in terms of the categories exercises, examples and explanation both quantitatively and qualitatively.

4.4.1 Distribution on textual elements

Firstly, I compared the content topics across each textbook. Next, each content category was measured to allow for the fact that the length of an explanation, example or exercise may vary. Each category was measured vertically by a ruler. It must be noted that when an explanation required the use of examples, the examples were measured and counted as examples and the measure was not included in the measure for explanations. This measure has been done page by page and it was measured in cm. Finally, the measurements referring to the same category were

summed and calculated in percentages, to obtain the total length of their treatment in a book. In the category ‘number of pages’, the pages were counted page by page.

As length of explanations or examples vary I also counted the number of units of analysis in each of the content categories for the three textbooks. As will be discussed in chapter five and six, these two quantitative measures gave somewhat different impressions of the texts.

4.4.2 Categorisation on levels of algebraic understanding

Each unit of analysis was then coded on the basis of the five levels of algebraic understanding. This implies analysing and classifying the different items as a process, a number, a function, or a set of functions. To check the progression, the variation across the levels was noted for each content category. This also gave insight into the consistency of explanations, examples and exercises. The number of steps required to move from tasks to solution in the exercises was provided to show a different measure of progression.

Firstly, I coded each unit of analysis as level zero, one, two, three and four. Secondly, I compared the categories of each unit of analysis as being in the same level to check the consistency. It must be noted that I also considered consistency when there were progressions in levels over the unity of analysis in the cases where all the categories change together, for instance, explanations, examples and exercise are classified as levels one and three. Thirdly, I coded the progression between the units of analysis in the same textbook by checking the movements in terms of levels and meaning in terms of the number of steps required to move from tasks to solution in the exercises. Fourthly, I relabelled all by number to facilitate the process of plotting, for instance, when one block of exercises was measured with ten items, it was counted as ten exercises; I counted the number of steps required for each exercise to allow me to check the progression in terms of steps, for instance, in one block of exercises these started with exercises requiring fewer steps and then increased the number of steps needed (see Figure 5-26).

In this analysis and following Sfard and Linchevski’s (1994) theory of reification, I have used the table of representation illustrated below, as an instrument to list the frequency of explanations, exercises and examples at each level in each book. The instrument was used as in Sfard and Linchevski (1994), with one addition, namely ‘expression’.

Each section of the books has been separately coded to better capture progression.

Representation			
Categories Levels	Book 1		
	Explanation	Exercises	Examples
0. expression			
1. process			
2. a number			
3. a function			
4. a set of functions			
Total			

Figure 4-1 Levels of algebraic thinking across categories

4.4.3 Examples of categorization on levels of algebraic understanding

The following examples illustrate the ways in which I categorised content according to the levels of algebraic understanding:

Level zero: Expression (symbolic manipulation)

Book 1- Grade 9: Exercise 5.2

- Work on your own.
- Add the given polynomials.

1. $2x^2 - 5x; x^2 + 4; -7x + 8$

2. $x^2 - 3x + 1; 2x^2 + 7x; -5x - 2$

In this exercise, the expressions can be seen as a string of symbols to be manipulated. In the process the learner needs to add expressions. For instance, by realizing the operation of adding expressions introducing brackets and grouping like terms he/she will get reduced expression. This means that the representation will be only symbolic.

Level one: computational process

Book 1-grade 9: Exercise 5.19

- Work on your own.

1) $\frac{10ab^2}{15a^2b}$

2) $\frac{16x^2y^3}{12xy}$

In this exercise, the learner has to interpret the algebraic fractions in a computational process which is completed by reduced algebraic fractions; that is why it was coded as level one. The problem here has to be seen as nothing more than a sequence of instructions. As the idea of simplifying algebraic fractions is presented it is expected that the learner manipulates these operationally. In this task the learner has to simplify the algebraic fractions by the operator multiply and manipulate them by factorizing and dividing like factors.

Level two: number

Book 3-grade 10: Exercise 8

- Work on your own.

Solve the equations:

a) $(4x - 3)(5 + 2x) = 0$

b) $16 - x^2 = 0$

In this exercise, the expressions can be seen as a string of symbols in the process where the operations are manipulated but at the end of the process $(4x - 3)(5 + 2x) = 0$ or $16 - x^2 = 0$ has to be seen as numbers. For instance by applying the operators factorizing expressions introducing brackets and isolating the x -term by using inverse operations he/she will get one or two unknown numbers. That is not to say that the exercise could not be solved using symbolic manipulation only, if sufficient memorisation of algorithmic knowledge was present, but that is a general concern in mathematics education. Nonetheless, the exercise invites considerations involving seeing expressions as numbers.

Level three: function

Book 2-grade 10: Exercise 8.11

- Solve for x and y :

1.
$$\begin{cases} x + y = 5 \\ x - y = 3 \end{cases}$$

2.
$$\begin{cases} x + 2y = 4 \\ x - 2y = 0 \end{cases}$$

In this exercise, the expression can be interpreted as a single equation representing unknown but fixed numbers but the end of the process has to be seen as function for instance by manipulating the variables substituting values into expressions. In this exercise the learner has to find values of x and y in relationship to each other which will satisfy both equations.

Level four: Family of functions

There were no examples of any such explanations, examples or exercises in any of the three textbooks.

4.4.4 Progression within levels

To get additional information regarding the progression in difficulty level, it was noted that the difficulty can still vary within each of the five levels of algebraic understanding. Therefore, following the work of Stigler et al. (1986), the items categorized as ‘example’ or as ‘exercise’ were chosen for further analysis. Exercises were solved step by step in the way learners are expected to do (with consideration given to the examples or explanations of the chapter if more than one solution method was possible – which it rarely was). However, a two-step problem on the function level may be harder than a five step problem on the number level. A three step problem involving fractions – which many learners are not comfortable with – may be harder than a five step problem involving only natural numbers. To give me an indication of this, I decided to try to determine the number of actions or “steps” required in examples and exercises to go from problem formulation to solution. Determining the number of steps required in examples and exercises was generally not hard to code. For instance, an exercise such as: Factorise completely: $16x^2 - 4b^2$ requires the

learners to take out the common factor and then factorise the difference of square. Thus, it would be categorized as requiring two steps. I did not make a distinction between steps engaging procedures just learned and earlier or easier steps. Cases of visual recognition only (see Figure 5-11 and Figure 5-12) were coded as requiring one step only.

Thus, in analysing the way algebraic concepts are presented in textbooks in terms of steps and algebraic thinking it is useful for teachers to understand the performance by learners in classrooms and to clarify why sometimes learners struggle when accessing new concepts (Zhu & Fan, 2006).

4.5 Reliability and validity

Data validity means the degree to which the data can reflect the phenomenon under study. Qualitative data are recognized to have a high validity given that qualitative focuses on determining the meaning of the phenomenon and requires the detailed explanations (Wippel, 2014). On the other hand, reliability “refers to the absence of differences in the results if the research was repeated” (Hussey et al., 2009, p. 64). Qualitative data has a lower degree of reliability because of the difficulty in repeating the same research. Qualitative methods are, for the most part, intended to achieve depth of understanding while quantitative methods are intended to achieve breadth of understanding (Palinkas et al., 2015).

With regards to validity, I first discussed about the validity in data collection. In this study, I would have included the book from grade 9 from Angola, however, as I highlighted above, the study focused only on the textbooks used in high schools and in Angola that it starts from grade 10 differently of South Africa which stars from grade 8. Thus, two South Africa textbooks were involved in the study because there are used in high school. (Primary schools were not objective of this study). Given that the purposive sampling was used to select the books in the study, and various attempts to ensure consistency in my coding and analysis, I hoped that these results would be internally valid (Tongco, 2007). Besides, a coding scheme on a continuum that was intended to interrogate the different levels of algebraic thinking provided by individual textbooks, was performed by myself and my supervisor who discussed borderline cases with me. Inter-coder reliability is the widely used term for the extent to which independent coders evaluate a characteristic of a message or artifact and reach the same conclusion (Lombard et al., 2010). Thus, inter-coder reliability was only accomplished for part of the coding.

4.6 Summary

In this chapter, I discussed that this study falls into the interpretivist paradigm. Therefore, as interpretivist, I wanted to create knowledge contained in the textbooks from South Africa and Angola. I also discussed purposive sampling as the method selected to get three textbooks based on my research questions. I further discussed that the data used in this study are both qualitative and quantitative. The coding approach was also focused in this chapter where content analysis was used for coding and a coding scheme was made according to the levels of algebraic understanding (Sfard & Linchevski, 1994). Finally, I discussed the reliability of the results where I testify that the results of this study are valid and reliable since both qualitative and quantitative data were collected. This chapter was concerned with methodology. I have discussed the paradigm and sampling methods, data collection, coding as well as the validity and reliability of the study. The analysis and coding was done by me individually; thus, the result depended on my own judgment, although my supervisor did do some control coding. Thus, inter-coder reliability was only accomplished for part of the coding. The next chapter focuses on analysis.

CHAPTER 5 : ANALYSIS

This chapter provides the analysis of data obtained by using the qualitative and quantitative tools in order to make a comparison of the three textbooks and to check progression in levels of algebraic thinking as demonstrated in the selected textbooks in examples, explanations and exercises relating to the five categories which are the focus of this thesis. In this case, I identified a significant difference both in the quality and quantity of exercises between the Angolan and the two South African textbooks. I also compared the textbooks' treatment of the different categories, both in terms of their physical length and of the number of explanations, examples and exercises provided. Furthermore, I checked similarities and differences in the three textbooks and how their conceptual progression is linked to procedural competencies. Finally, I compared the number of steps of the solutions required to solve the examples and exercises in different textbooks in order to analyse complexity of its tasks.

5.1 Topics coverage in the three textbooks

Table 5.1 contains the topics covered in the algebra presented by chapters of the three textbooks.

Table 5.1 The topics of the three textbooks

Topic	SA 9	SA 10	Angolan 10
Basic operations on polynomials	√	√	√
Factorising algebraic expressions	√	√	√
Numerical and geometrical patterns	√		
Linear functions	√	√	√
Non-linear functions	√	√	√
Equations	√	√	√
Simplifying expressions involving algebraic fractions		√	
Exponential notation		√	
Simultaneous equations		√	√
Inequalities		√	√
Injective and bijective applications, monotonicity			√
Modulus function			√

As seen in the table, the textbooks have some similarities and differences. For instance, the topics such as “basic operations on polynomials, factorising algebraic expressions, linear functions, non-linear functions and equations” are common for all three textbooks. Besides simultaneous equations and inequalities are common in both grade 10 textbooks. However, injective and bijective applications, monotonicity, and modulus function appear in the Angola grade 10 textbook but not in the South African grade 10 textbook. Simplifying expressions involving algebraic fractions, and exponential notation appear in the South African grade 10 textbook but not in the Angolan grade 10 textbook. This picture informs us that it seems reasonable to say that the Angolan textbook appears to address more advanced mathematical processes and concepts. From this analysis, the findings indicate that there are 9 algebraic topics in grade 10 in the Angolan and South African textbooks.

This tells us that there is some indication in the ordering of the topics in the South African textbook that they are informed, directly or indirectly, by research on algebraic learning.

5.2 Extent of content categories in the selected textbooks

5.2.1 Distribution on categories per length

I counted as well as measured the extent of the pages in each category (examples, explanations, exercises) to get a better sense of the dimension of each category. The finding indicates that some examples or explanations were longer than others, and the type setting varied between the books. Thus, simply counting the number of occurrences within each category would disguise issues of length (which may or may not be indicative of depth), and simply measuring the extent of the occurrences would disguise issues of frequency. Therefore, I decided to do both.

Table 5.2 shows the total number of cm for each category and the proportion of the different categories of the algebra sections of the three textbooks from the two countries.

Table 5.2 Items from chapter 6 classified according to content categories, in both measurements and page counts

Textbook	Explanations in cm	Examples in cm	Exercises in cm	Total cm	Number of pages
South Africa- Grade 9	158 (12.9%)	347 (28.4%)	718.5 (58.7%)	1223.5	73
South Africa- Grade 10	219 (19.4%)	193 (17.1%)	717.5 (63.5%)	1129.5	66
Angola- Grade 10	482 (51.6%)	281 (30.1%)	172 (18.4%)	935	64

I found that the algebraic content chapters for the South African textbook grade 9 spanned 158 cm for explanations, 347 cm for examples and 718.5 cm for exercises. Moreover, the South African textbook grade 10 contains 219 cm for explanations, 193 cm for examples and 717.5 cm for exercises. However, the Angolan textbook covers 482 cm for explanations, 281 cm for examples and 174 cm for exercises.

The large space dedicated to explanations in the Angolan textbook indicates that the book is either very dense, or that the explanations are very detailed. This indicates that at least in theory learners could do the exercises independently without much support from the teachers. However, the space devoted to exercises in the Angolan textbook may seem very low (around 18.4% of the total text measure), compared to the South African textbooks (58.7% and 63.5% in grades 9 and 10 respectively).

Overall, the smaller number of exercises in the Angolan textbook meant that progression to more complex tasks happened more quickly, with the Angolan textbook favouring exercises with six or more steps (see Figure 5-4), while the South African textbooks textbook grade 10 favoured exercises with one or two steps (see Figure 5-2 and Figure 5-3).

Table 5.2 also shows the number of pages in the three textbooks. It was found that the South African grade 9 and grade 10 textbooks have 73 and 66 pages respectively, while the Angolan textbook contains 64 pages relating to algebraic content. Generally, the number of pages in the three textbooks on algebraic content was quite similar – though the page measure shows that the South African sections on algebra were 20-30% longer than the comparable sections in the Angolan textbook, due to the type set being different. This is illustrated visually in Figure 5-1.

The results displayed in Table 5.2 show a stark difference between the three textbooks regarding the space devoted to the categories of explanation and exercises. In contrast, there was a compatible measure of examples. The South African textbook grade 9 has the least relative amount of space devoted to the item ‘explanations’ (12.9%) and the Angolan textbook the highest (51.6%). When it comes to exercises, the situation is opposite; the Angolan textbook has the least space set aside for exercises, less than a fifth of the total pages, while exercises take up more than half of the pages in the two South African textbooks. This is also illustrated visually in Figure 5-1, and as averages in Table 5.3.

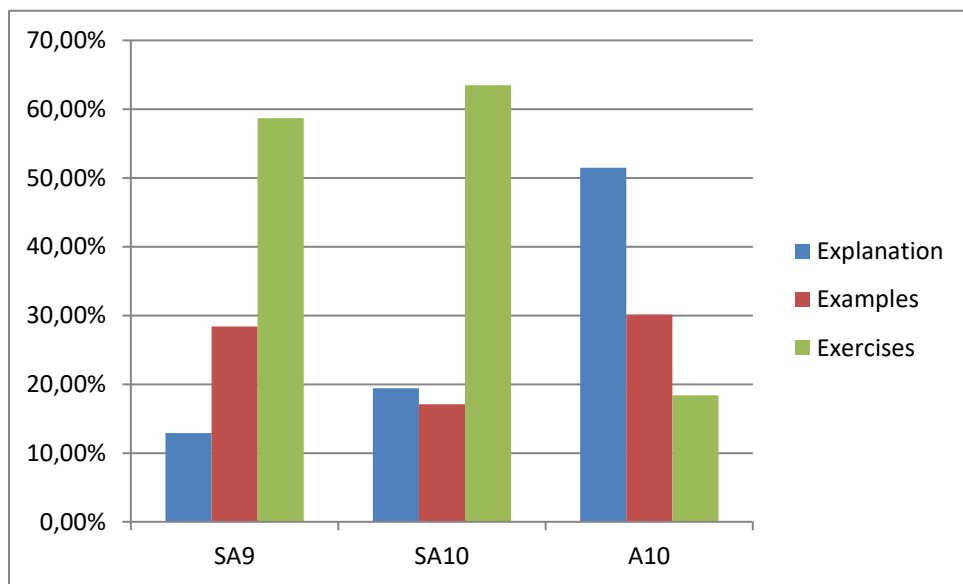


Figure 5-1 The relative frequency of the categories as per length

Table 5.3 Comparison of the average of cm per page for each category

Textbook	Explanation	Examples	Exercises	Total
South Africa - Grade 9	2.16	4.75	9.84	16.75
South Africa – Grade 10	3.32	2.92	10.9	17.14
Angola - Grade 10	7.53	4.39	2.69	14.61

One should take into consideration however, as I will discuss later, that in the Angolan textbook exercises the progression in terms of task complexity was quick, whereas the opposite was the case for the South African textbooks. Thus, the measure of categories is only a first crude indication of the differences between the books.

Exercise 5.9:

Work on your own.

Find the products. Simplify is possible.

1. $3x(x - 5)$

2. $(x + 2)4x$

3. $-3x(x - 2)$

4. $(x - 2) - 3x$

5. $(x - 2)(3x)$

Figure 5-2 Extracted from South African textbook grade 9, pages 92 and 93

Exercise 5.3:

Find the followings Products using the second method.

1. $(2x - 3y)(2x + 4y)$

2. $(2a - 7b)(3a - 2b)$

3. $(7x - 3y)(2x - 4y)$

4. $(9a + 2b)(2a + 3b)$

5. $(3a + 2b)(2a - 5)$

Figure 5-3 Extracted from South African textbook grade 10, page 102

Exercise 2:

Simplify.

2.1. $2k^3z \times \frac{3}{4} k^3z^2$

2.2. $(3ab + 5a^2b - 3ab) \times 4a^2b^3$

2.3. $(6e^3t - 12e^3t - 5e^3t) \div (-10et - 12et)$

2.4. $\frac{-5y^2+3y^2z}{4wy^3z^4} - \frac{2z^3}{3wy}$

2.5. $\frac{5y^2}{4x^2} - 3x^{-2}y\left(\frac{2x^3y^4}{6x^3y^5}\right)$

Figure 5-4 Extracted from Angolan textbook grade 10, page13

5.2.2 Frequency of occurrences of content categories in the selected textbooks

The second measure of the extent of the content categories was a simple count. It was easy to distinguish examples, while an explanation that continued on the other side of an example was counted as one, not two, explanations. Exercises were found in ‘blocks’ of tasks, and I initially counted each exercise. However, I also wanted to examine the frequency with which learners were expected to engage in practice, so I also added to this a count of ‘blocks’ of exercises. To be able to obtain relative frequencies, I added up the number of explanations, examples and exercises. The results are presented in Table 5.4. Looking at it this way, the books appear more similar (see also) than when comparing the extent of content categories using measure of length, though the Angolan textbook still has a higher relative frequency of explanations and a lower frequency of blocks of exercises.

Table 5.4 Frequency count of content categories, with relative distributions

Textbook	Number of Explanations	Number of Examples	Number of blocks of exercises	Total	Number of Exercises
South Africa – Grade 9	15 (10.6%)	74 (52.5%)	52 (36.9%)	141	680
South Africa - Grade 10	29 (16.0%)	80 (44.2%)	72 (39.8%)	181	1050
Angola -Grade 10	42 (22.7%)	90 (48.6%)	53 (28.6%)	185	191

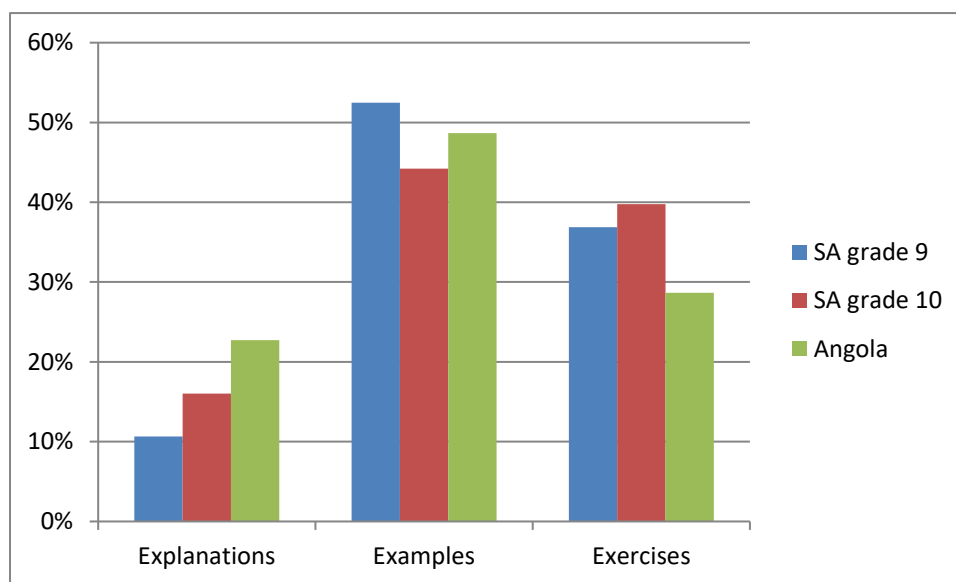


Figure 5-5 The relative distribution on categories of ‘blocks’ of text.

The main difference is caused by the vast number of *individual* exercises in the two South African textbooks compared to the Angolan textbook. The individual exercises did not vary substantially in length (1.1 cm/exercise in the South African grade 9 textbook, 0.7 cm/exercise in the South African grade 10 textbook, and 0.9 cm/exercise in the Angolan textbook), but the blocks of

exercises were on average longer in the South African textbooks (13.1 and 14.6 exercises/block respectively) than in the Angolan textbook (3.6 exercises/block).

5.3 Consistency and progression in levels of algebraic thinking

In this section, I firstly summarise the analysis of each textbook, chapter by chapter. I then discuss the progression within and between chapters.

Secondly, I engage the consistency of the textbooks. I define consistency of a chapter with respect to levels of algebraic thinking as the explanations, examples and exercises are on the same algebraic level; this includes situations where there is progression in levels over the chapter but the explanations, examples and exercises all change ‘together’. I argue that the way algebraic concepts are presented in terms of progression in levels of algebraic thinking and consistency may help learners understand and access new mathematic concepts

All three textbooks had a fairly high consistency of all chapters. What I will show in this chapter is that contrary to the Angolan textbook, the South African textbook does not follow a progression according to the Sfard and Linchevski’s (1994) levels of algebraic understanding between the chapters. Finally, in the Angolan textbook the progression in the exercises and examples in terms of steps required is rapid, contrary to the South African textbooks, where progression in this respect is quite slow.

5.3.1 Book: South Africa, Grade 9

In this book, I have classified the categories (explanation, example and exercises), included in each topic into levels of algebraic understanding. The book was not what I would consider a traditional type of textbook, where explanations are followed by examples followed by exercises. In this textbook, examples are nearly always followed by exercises which are then followed by explanations. The topics covered in grade 9 are algebraic, functions and patterns and equations (see Table 5.5).

Table 5.5 shows that there is consistency in levels of algebraic thinking for the categories (explanations, examples and exercises) within the algebra topic, given that almost all content across categories was classified as falling on level zero, except the 10 (2.8%) exercises on level two. The above mentioned activities were classified as being on level zero, as also exemplified in

chapter four, because they did not refer to the meaning of the expressions but instead focused on manipulation of symbols. Table 5.5 also shows that there is consistency in levels of algebraic thinking for the categories within functions and patterns topic given that almost all content across categories were classified as level three, except the 1 (33.3%) explanation and the 33 (28.7%) exercises on level one. The majority of activities were classified as being on level three, as also exemplified in the methodology chapter, because the variables generally take on infinitely many values in relationship to each other.

Table 5.5 Items and levels from grade 9 South Africa textbook

		Algebra expressions		Functions and patterns		Equations	
		Frequency	Relative frequency (%)	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
Explanation	Total	6		3		6	
	Level 0	6	100.0	0	0	0	0
	Level 1	0	0	1	33.3	3	50.0
	Level 2	0	0	0	0	3	50.0
	Level 3	0	0	2	66.7	0	0
	Level 4	0	0	0	0	0	0
Examples	Total	27		6		40	
	Level 0	27	100.0	0	0	0	0
	Level 1	0	0	0	0	15	37.5
	Level 2	0	0	0	0	25	62.5
	Level 3	0	0	6	100.0	0	0
	Level 4	0	0	0	0	0	0
Exercises	Total	352		115		215	
	Level 0	342	97.2	0	0	0	0
	Level 1	0	0	33	28.7	17	7.9
	Level 2	10	2.8	0	0	192	89.3
	Level 3	0	0	82	71.3	6	2.8
	Level 4	0	0	0	0	0	0

Furthermore, Table 5.5 informs us that there is a consistency in levels of algebraic thinking for the categories within equation topic because almost all content across categories was classified as levels one and two, except the 6 (2.8%) exercises on level three. The above mentioned activities were classified as being on level one and two, as also exemplified in the methodology chapter, in the first case as computational processes because they did not refer to the meaning of variables but

instead focused on the algorithm for solution, and in the second case as numbers because the variables were treated as single values.

In this analysis the exercises on level two in algebra expressions topic, consisted in replacing the variable in the expression by given numbers to get corresponding numeric values. Here the learner will find that for each numeric value replacing the variable in the expression there will be a corresponding numerical value of the expression. In these exercises, the learner has to complete a table by certain procedures such as multiplying or adding, replacing particular variables of the expression (at 0, 1, 2, 3, and 4) and get single values. At this level, the learner begins to see the expression as a function (object). However, the process here is formed by replacing the variables with given numbers and manipulating them operationally. These exercises initially are interpreted as a computational process through which input values are converted into a new mathematical object - a number.

In this topic, it was hard to code two exercises. These exercises (see Figure 5-6) contained a table with three expressions, and five rows requiring the learners to find the numerical values of the three expressions for five different values of the variable. However, the exercises also asked learners to compare the resulting values and explain – though it was not made explicit that the explanation should be with reference to the original expressions. Thus, while a mathematician or mathematics teacher will likely recognize the purpose of the activity as inviting learners to contemplate equivalent expressions, this was not made explicit to the learners by the textbook itself. Coding for the level was tricky, because if these exercises were read by a mathematician, they could be seen as reflecting a function understanding of algebraic expressions, but the partial solutions represented numbers, so the learners will find that the first levels of the exercise are on level two. However, it is possible for the teacher to use this to guide the learners to level three thinking, when they contrast and compare their numerical responses and thus become aware of the equivalence of expressions, which is moving from level two to level three thinking. Thus, coding the exercises as being on levels two was problematic, because even if the expressions are treated as single values the learner will translate every expression into a different value. In the end, I coded the exercise on level two.

Exercise 5.6:

1. Copy and complete the table if x is replaced by the number in the first column.

x	$A = (x + 2)^2$	$B = x^2 + 4x + 4$	$C = x^2 + 4$
0	$A = (0 + 2)^2 = 4$		
1			
2		$B = 2^2 + 4(2) + 4$ $= 16$	
3			
4			$C = 4^2 + 4 = 20$

- a) Compare the values you calculated in the three columns.
- b) Write down your conclusions.

2. Copy and complete the table by working out values of the expressions for different y values.

y	$A = 3y(y^2 + 2)$	$B = 3y^3 + 6y$	$C = 3y^3 + 2$
-1			
1			
2			
3			
4			

- a) Compare the values you calculated in the three columns.
- b) Write down your conclusions.

Figure 5-6 Extracted from South African textbook grade 9, pages 89 and 90

Overall, I find that in terms of levels of understanding algebraic expressions topic, the level zero is static in South African textbook grade 9, with the exception of a few exercises. The following topic of this textbook is dedicated to functions and patterns, which is level three. Thus, these activities may direct the learners to move towards a higher level of algebraic thinking. An exercise such as the one reproduced in Figure 5-6 has the potential to let learners work on one level but,

with support from the teacher, begin to move onto the next level. However, it must be noted that this transition took place in the exercises only, and appears to me to rely on the teacher to list the connections; if the teacher does not ask the learners to work on this exercise or does not draw out the connections, the learning potential is likely reduced.

The exercises on level one in the functions and patterns topic, consisted of describing and discussing in words the rule used to complete the tables. In these exercises, the learner has to complete a table and then discuss with their classmate in words the operations made to get the output-values. The exercises classified as level one were at the beginning of the chapter.

In general, I find that the levels of algebraic thinking in the topic of functions are also fairly static, with the exception of the explanations. In South African textbook grade 9, all the examples were classified as level three. The following chapter of this textbook is dedicated to equations, where the learners will work with equations dealing with brackets and fractions, and so on. Thus, the textbook does not follow a progression according to the Sfard and Linchevski (1994) levels of algebraic understanding. It is worth considering to what extent the levels are truly hierarchical or not, but this is beyond the scope of this thesis.

In a problem like the one presented in Figure 5-7, the learners were supposed to consider numbers, though they were implied functions. To understand the pairs of values as a function, a learner must realize that for each given input value it is possible to get corresponding output values. These tasks were classified as level one although the teachers can use them to step in and challenge the learners, pushing them towards a better understanding of expressions as numbers so that tasks classified as level one can be a way of moving to level two thinking.

Exercise 6.1:

4. a) you are given this set of data. Complete the table.

Input (x)	1	2	3	4	5	7	10	12	15
Output (y)	7	9	11	13					

- b) Describe, in words, the rule you used to complete the table.
- c) Design a flow diagram that will represent the data given in the table.
- d) Represent the table or flow diagram in algebraic language.
- e) Analyse the rate of change in the different representations.
- f) Discuss your observations with your classmate.

Figure 5-7 Extracted from South African textbook grade 9, page113

This exercise displayed in Figure 5-7, is classified as level one and consisted in dealing with the relation between the input values and the output values given to fill the table. In that process the learner will find that in order to complete the table he/she must analyse the given set of data. In this exercise, the learner has to complete the table in the process of multiplying the input value by 2 and adding 5. However, the process here is formed by constructing mathematical relationships and manipulating them operationally. In this case if the exercise was read by a mathematician it could be seen as understanding expressions as functions, but the solutions of the exercise represent a description of word or symbols (formulae), so to the learners this exercise may be attempted on level one. However, it is possible for the teacher to use this to guide the learners to level two thinking, when they get one single output value. Unlike the case discussed in the previous chapter, this example contains more explicit questions guiding the learner from looking at individual number pairs to looking across these to identify a 'rule'. Thus, it could, with fairness, also have been classified on a higher level. This activity comes early in this book, and thus does support some progression in levels, but in other respects, the chapter is consistently working with expressions as functions. The exercises on level three consisted in dealing with the relation between variables to fill the table. In that process the learner will find that for each value substituting the variable in the expression there will be a relationship to each other.

Generally, I find that in categories of function concept, the levels of algebraic thinking moved from level one to level two. The question now is: why did the chapter include exercises on level three? Since the previous chapter is regarding functions some questions can fall on this level as ‘functions’, perhaps to assist topics that have been seen previously or pre-empting future topics.

5.3.1.1 Progression between Chapters 5, 6 and 7 in grade 9 textbooks South Africa

Progression in Explanations

In this category, the findings indicate that the progression starts from level zero to level three. In chapter five grade 9, I found that all the explanations are classified as level zero. However, in chapter six grade 9 the progression is from level one to level three and in chapter seven grade 9, the progression goes from level one to level two. Thus, I can say that the progression between chapters in grade 9 is not consistent with Sfard and Linchevski’s (1994) levels in the item explanation.

Progression in Examples

In this category, I found that the progression is moving from level zero to level three; I can see that in the first chapter grade 9 all the examples are consistent in terms of level zero, in chapter six grade 9 all the examples are classified as level three but in the last chapter grade 9 examples are classified as level one and two. Therefore, I can see that the progression moves from level three to level two.

Progression in Exercises

In this category, I found that the progression is from levels zero to level three, but here I can see that in chapter five grade 9, 97.2% of exercises belong to level zero and in chapter six grade 9, 71.3% are on level three. Therefore, the progression jumps mainly from level zero to three between these two chapters, while in chapter seven grade 9, 89.3% of exercises are on level two.

In general, the analysis showed that there is progression from chapter five to chapter six in South African textbook grade 9, but the progression over the textbook is not consistent with Sfard and Linchevski’s (1994) levels. The function concept is introduced in chapter six grade 9 but chapter seven grade 9 does not appear to build on this understanding of the mathematical concept of function through the levels of algebraic thinking. According to Sfard and Linchevski’s (1994)

levels the expectation would be that if chapter six grade 9 was consistent on level three – function - the next chapter should be classified on the next level - family of function, or alternatively address more complex functions. Instead, in chapter seven grade 9 all the explanations, examples and exercises moved almost entirely between level one and two (number). However, previous studies related to the learning of algebra tell us that learners should start work in algebra with exploration of patterns, leading to structural algebra - variables representing a single number that can vary – not a single solution (Friel & Markworth, 2009; Smith, Hillen, & Catania, 2007). This suggests that solving of equations actually should be classified as more complex than working with functions. If one considers that an equation could be viewed as asking when a function takes on a specific value, then this makes sense. This means that using Sfard and Linchevski’s framework indiscriminately may disguise other forms of progression – in particular the progression from working with variation to solving equations. However, Boaler (1998) argues that the progression should be from using variables to meaningfully expressing relationships to other uses of variables, such as evaluating expressions and solving equations, and while this fits with the progression between chapters six and seven in this textbook, yet it certainly does not fit with chapter five starting at level zero, where the algebraic expressions are treated more or less as meaningless symbols. Thus, it appears that the textbook’s progression between chapters may be a mix of following the traditional South African approach of starting on what I have deemed level zero, and following the ordering of content suggested by research.

5.3.2 Book: South Africa, Grade 10

As I mentioned in discussing textbook grade 9, the textbook grade 10 does not follow a traditional approach. For instance, examples are nearly always followed by exercises followed by explanations. The topics included in this book are algebraic expressions, exponentials and equations and inequalities (see Table 5.6). As I discussed, in this book I also classified the categories (explanations, examples and exercises) of each topic into levels as shown in Table 5.6.

Table 5.6 Items and levels from grade 10 South Africa textbook

		Algebra expressions		Exponents		Equations and inequalities	
		Frequency	Relative frequency (%)	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
Explanation	Total	16		3		9	
	Level 0	16	100.0	3	100.0	0	0
	Level 1	0	0	0	0	0	0
	Level 2	0	0	0	0	7	77.8
	Level 3	0	0	0	0	2	22.2
	Level 4	0	0	0	0	0	0
Examples	Total	54	0	3		21	
	Level 0	54	100	1	33.3	0	0
	Level 1	0	0	0	0	0	0
	Level 2	0	0	2	66.7	16	76.2
	Level 3	0	0	0	0	5	23.8
	Level 4	0	0	0	0	0	0
Exercises	Total	474	0	214		346	
	Level 0	462	97.5	179	83.6	0	0
	Level 1	2	0.4	0	0	0	0
	Level 2	10	2.1	35	16.4	211	61
	Level 3	0	0	0	0	135	39
	Level 4	0	0	0	0	0	0

The findings indicated that there is a consistency in the levels of algebraic thinking for the categories (explanations, examples and exercises) within the topic of algebraic expressions, given that almost all content across categories was classified as level zero, except the 2 (0.4%) exercises on level one and the 10 (2.1%) exercises on level two. The above mentioned activities were classified as being on level zero, as also exemplified in chapter four, because they did not refer to the meaning of the content but instead focused on manipulation of symbols. It is noticeable that this chapter corresponds to chapter five in the grade 9 textbook, in terms of its focus on symbol manipulation, but with more complex cases such as those involving algebraic fractions. This indicates to me the notion of a spiral curriculum where topics are revisited in later years, but with

some progression in difficulty. As I only looked at an Angolan textbook for one grade, I cannot say to what extent the same applies in that country.

Table 5.6 also shows that there is a partial consistency in the levels of algebraic thinking for the categories (explanations, examples and exercises) within exponents topic given that almost all content across categories were classified as level zero, except 2 (66.7%) examples on level two and 35 (16.4%) exercises on level two. The majority of activities were classified as being on level zero, as also exemplified in chapter four, because they did not refer to the meaning of the content but instead focused on manipulation of symbols.

Table 5.6 also indicates that more than 70% of the categories' explanations (77.8%) and examples (76.2%) fall in level two and level three with 22.8% and 23.8% respectively. This tells that there is also a partial consistency in the levels of algebraic thinking of categories within equation and inequalities topic. However, when it moves to the category exercises, only 61% of the exercises fall in level two and the other 39% fall in level three. Thus, the majority of the activities were classified on level two, which clearly gives some consistency to the chapter. These activities were classified on this level as also exemplified in chapter four, because the variables in the algebraic expressions are treated as one or two unknown numbers.

The findings reveal that the exercises on level one in algebraic expressions topic, consisted in manipulating algebraic expressions in different forms by creating combinations of two or more patterns to get the solution.

Here the learner will find, for example, that each match with two matches added will make an additional triangle. In these exercises, the learner has to make a row of n triangles focusing on manipulating algebraic expressions where he/she has to create a series of pattern by a certain procedure such as adding to get the answer. However, it is possible for the teacher to use this to guide the learners to level two and three thinking, when they manipulate algebraic expressions in different forms and thus become aware of general relationships or of solving equations.

However, exercises on level two in algebraic expression topic, consisted in replacing the variable in the expression by given numbers to get corresponding values. In these exercises, the learner has to complete a table by certain procedures, such as multiplying or adding, replacing particular variables of different expressions (with values 1, 2, -1, -2) and getting single values. However, it

is possible for the teacher to use this to guide learners to level three thinking, when they contrast and compare the values obtained and thus become aware of equivalence of expressions, which is moving from level two to three.

Overall, I find that levels of algebraic expressions are fairly static, with the exception of a few exercises. The following chapter of this textbook is dedicated to exponents, where the learners will work with exponents that are integers, and factorize expression involving exponents, and so on.

In this analysis, the exercises on level two in the *exponents'* topic, consisted in factorizing expressions involving exponents and dealing with the laws of exponents to find values of the variable. Here, the learner would find that in order to find the value of the variable he/she must convert the bases to prime factors. However, the process here is formed by manipulating the law of exponents to get single values of the variable.

The following exercise is one exercise classified on level two:

Activity 7.6

1. Solve for x . Give all possible answers ($x \in Z$).

a) $9^2 = 3^x$

Figure 5-8 Extracted from South African textbook grade 10, page 156

Overall, I find that in terms of levels of understanding of algebraic expressions, there is progression from level zero to level two in the chapter.

In this analysis the topic is equations and inequalities in which some items can fall on level three. Thus, it can be seen that the activities in this topic may direct the learners to move towards a higher level of algebraic thinking. An exercise such as the one that was presented (see Figure 5-8) has the potential to let learners work on one level but with support from the teacher to begin to move onto the next level.

In this topic, the categories explanations, examples and exercises which are on level three consist of substituting the variables in the expressions by giving different values to get many values in relationship to each other. This tells us that the learner will find that for each value substituting the variable in the expression there will be a relationship to each other. Especially, in the category of exercises, the learner would be able to solve algebraic expressions by a certain procedure, such as

multiplying or adding, or by substituting values into the expressions to get values of the variables that satisfy both equations.

Overall, I find that there is progression in the levels of algebraic thinking from level two to level three in the topic of equations and inequalities, since it is the last topic of algebra in this textbook and there is a correspondence within levels in the categories. Therefore, it seems that activities in this topic may guide learners' work on these levels and supported by teachers they can begin to move onto the next level in the next grade which according to Sfard and Linchevski's framework is level four - family of functions.

5.3.2.1 Progression between Chapters 5, 7 and 8 in grade 10 textbooks South Africa

Progression in Explanations

In this category, the findings show that there is progression from level zero to level three. In chapters five and seven the analysis shows that all the explanations are classified as level zero. However, in chapter eight, the progression goes from level two to level three. Thus, it seems that the progression between chapters is consistent according to Sfard and Linchevski's (1994) levels in the item explanation.

Progression in Examples

In this category, I found that the progression is moving from level zero to level three. In chapter five, all the explanations are classified as level zero, but in chapter seven the progression is from level zero to level two, and in chapter eight the progression goes from level two to three.

Progression in Exercises

In the category of exercises, the progression is from level zero to level three. Nevertheless, the progression in chapters five and seven goes from level zero to two whereas in chapter eight the progression is from level two to three.

In general, the analysis showed that there is consistent progression over the textbook according to Sfard and Linchevski's (1994) definition of the levels. Also, there is consistent progression in level zero and two in chapters five and seven with the majority of categories on level zero, and a consistent progression in levels two and three with predominance on level two.

However, the progression between grade 9 and 10 South African textbooks is not consistent with Sfard and Linchevski's (1994) definition of levels. The function concept introduced in grade 9 textbook does not appear to build the understanding of the mathematical concept of function through the levels of algebraic thinking in the grade 10 textbook, which is consistent with the level zero-expression. However, other research on the learning of algebra suggests that learners must first work with patterns and then with equations. Thus, according to Sfard and Linchevski, it seems to be the progression from the grade 9 book.

5.3.3 Book: Angola, Grade 10

Conversely to the grade 9 and 10 textbooks of South Africa, the book grade 10 textbook in Angola is a traditional type of textbook in the sense that explanations were followed by examples followed by exercises.

In this book, the topics covered were polynomials and functions and graphs (see Table 5.7).

As I discussed above, in Angolan textbook I also classified the categories (explanations, examples and exercises) of each topic into levels as is shown in Table 5.7.

Table 5.7 Items and levels from grade10 from the Angolan textbook

		Polynomials		Functions and graphs Generalities	
		Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
Explanation	Total	16		27	
	Level 0	10	62.5	4	14.8
	Level 1	0	0	0	0
	Level 2	2	12.5	5	18.5
	Level 3	4	25.0	18	66.7
	Level 4	0	0	0	0
Examples	Total	31		51	
	Level 0	18	58.1	6	11.8
	Level 1	0	0	0	0
	Level 2	4	12.9	122	3.5
	Level 3	9	29.0	33	64.7
	Level 4	0	0	0	0
Exercises	Total	56		135	
	Level 0	34	60.7	19	14.1
	Level 1	0	0	0	0
	Level 2	13	23.2	27	20.0
	Level 3	9	16.1	89	65.9
	Level 4	0	0	0	0

The analysis indicates that there is consistency in the levels of algebraic thinking of categories (explanations, examples and exercises) within the topic of polynomials given that almost all content across categories was classified as falling on level zero, two and three, with similar proportions of the categories on the different levels. It is interesting to note that the items classified as level zero ‘expression’ in this chapter were high; more than 58% were attached to this level.

In this analysis, the activities on level three consisted in treating two or more variables as many values in correspondence to each other. It was interesting to note that there were no items classified on level one. Does it mean that no explanation, example or exercise was interpreted as computational process? In a problem like those presented in this chapter, the expressions that at some level were operated on level one were converted into mathematical objects - a single variable

representing one or two unknown numbers; two or more variables representing infinitely many values relating to each other, for example typically exercises like “solving systems of two and three equations or solve equations using the quadratic formula”.

The two problems below show why these problems were classified as number and functions and not as computational process.

Exercise 8:

Solve the following equations;

$$8.1. \quad (x - 3)(5 + 2x) = 0$$

$$8.2. \quad 16 - x^2 = 0$$

Figure 5-9 Extracted from Angolan textbook, page 21

The exercise in Figure 5-9 consisted in finding values for x which will satisfy each of the equations. Here the learner will find that for each value found in the expressions they will make the original algebraic expressions true. However, the process here is formed by finding values that make the original expressions true. This exercise that in the same level is interpreted in a computational process was converted into an object - a number.

Exercise 11:

Solve the following systems of equations:

$$11.1. \quad \left\{ \begin{array}{l} x - \frac{x+y}{2} + z = 3 \\ 3x - 2y + 4z = 12 \\ \frac{x+y}{4} + 2z = 6 + x \end{array} \right.$$

Figure 5-10 Extracted from Angolan textbook, page 31

The exercise in Figure 5-10 consisted in finding values of x , y and z which will satisfy all equations simultaneously. Here the learner has to manipulate the variables substituting values into expressions. However, the process here is formed by reducing equations by repeatedly eliminating variables. This exercise that at some level is interpreted as a single equation representing unknown but fixed numbers was converted into a mathematical object - a function.

Overall, I found that in terms of levels of function concept, the progression is fast from level zero to three given that this is chapter one of this textbook and the levels of algebraic thinking are progressing across activities and also all the items are classified on the corresponding levels and the following chapter is dedicated to functions, which is level three. It seems that activities may direct the learners to move towards a higher level of algebraic thinking. Thus, I can say that this chapter is consistent with regard to Sfard and Linchevski's (1994) definition of levels.

As I discussed above, in Angolan textbook I also classified the categories (explanations, examples and exercises) of each topic into levels as is shown in Table 5.7.

Table 5.7 also shows that there is consistency in the levels of algebraic thinking of categories (explanations, examples and exercises) within the topic of functions and graphs given that all the content across categories was classified as falling on level zero, two and three, in corresponding proportions. Even if all the items are not classified in the same level, I considered it consistent because when it comes to progression in terms of levels of algebraic thinking all the explanations, examples and exercises change 'together', with minor exceptions which may be explained in terms of review of previous topics or pre-empting future topics. Thus, the majority of activities were classified on level three (more than 60%), which clearly gives consistency to the chapter.

Overall, the finding indicates that the levels of algebraic thinking progress across the activities, since that is the last chapter of algebra in this textbook and the items, are classified on the corresponding levels. This informs us that activities may guide learners' work on these levels and supported by teachers, learners begin to move onto the next level in the next grade which according to Sfard and Linchevski's (1994) framework is level four - family of functions. However, it must be noted that more than 64.7% of the activities were consistent in terms of level three - function.

5.3.3.1 Progression between Chapters 1 and 5 in grade 10 Angola textbook.

Progression in Explanations

In this category, the findings show that there is progression from level zero to three, but with substantially less content on level zero and level two in the latter chapter. As I can see, in both chapters the progression is from level zero to level three, with no occurrences of level one activity which may be an indication that the activities previously treated in an operational view of algebra (level zero) developed and became a concept through numerical processes.

Progression in Examples

In this category, the analysis showed that there appears a progression from levels zero to three since the progression is from level zero to level three in both polynomials and functions and graphs topics.

Progression in Exercises

In addition, I found that that there is progression within exercises, from level zero to level three given that the progression is from level zero to level three seen here in both topics.

Generally, the findings indicate that there is a progression from chapter one to chapter five, with the presence of levels zero, two and three with no occurrence on level one. According to Sfard and Linchevski (1994) level one activity is an operational view on algebra, where expressions are seen as a series of operations to carry out. Thus, the lack on level one may not be indicative of a jump in progression, but could mean that individual tasks were revisited in ways which were reduced to operational knowledge (level zero), and it may indicate that the manipulation of symbols becomes a bridge to access new concepts. According to Tall and Vinner (1981), concept is generally first met as a process where the learner will define it as an operation.

5.4 Comparison of the selected textbooks in terms of how many steps were required in the categories examples and exercises

In this section, I first summarise the analysis of each textbook, chapter by chapter in the categories examples and exercises by identifying the number of steps needed to complete each exercise. It did not make sense to use this approach to identify the complexity of explanations, which therefore have been omitted from the discussion in this section. I then compare the textbooks in terms of the

number of single step and multiple step questions. Finally, I separate and combine the steps between one and two steps, three and four steps and six or more steps for both categories.

Table 5.8 Step Question in the different categories.

	Examples			Exercises		
	<i>South Africa - Grade 9</i>	<i>South Africa - Grade 10</i>	<i>Angola - Grade10</i>	<i>South Africa - Grade 9</i>	<i>South Africa - Grade 10</i>	<i>Angola - Grade 10</i>
Single Step Question	2 (6.7%)	22 (38.6%)	8 (20.5%)	93 (26.5%)	126 (33.1%)	5 (8.2%)
Multiple Step Question	28 (93.3%)	35 (61.4%)	31 (79.5%)	258 (73.5%)	255 (66.9%)	56 (91.8%)
Total	30	57	39	351	381	61

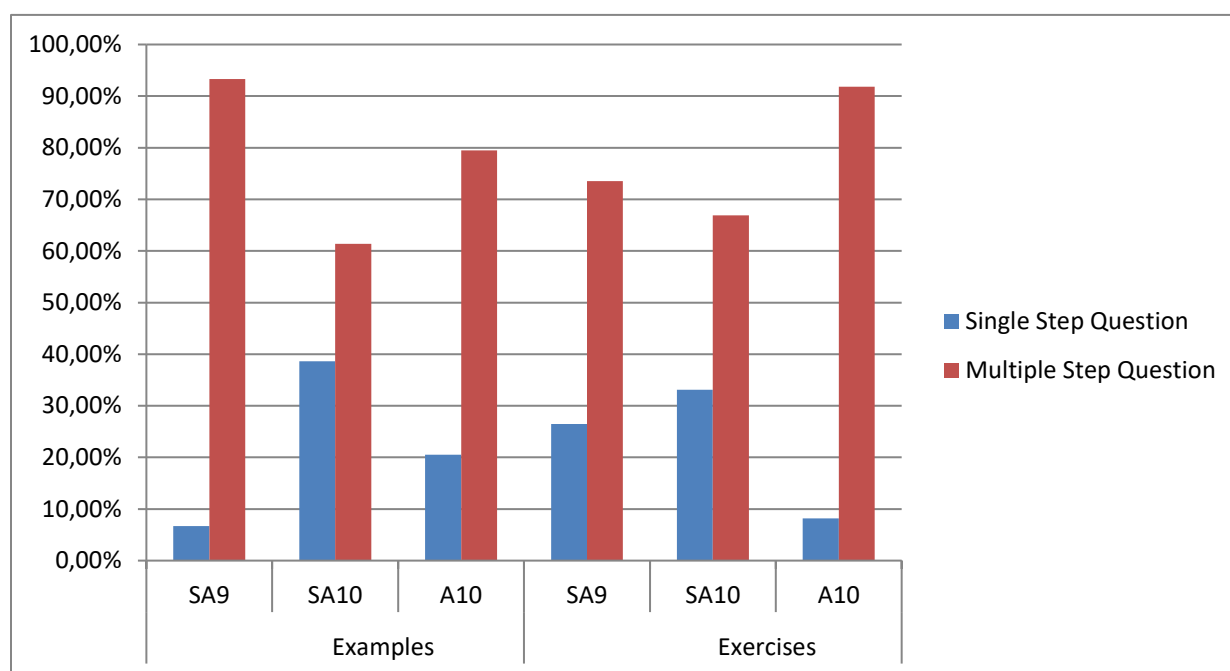


Figure 5-11 Comparison of the numbers of single and multiple step questions

The results displayed in Table 5.8, show a marked difference between the number of single step question and the number of multiple step questions in all textbooks. The number of examples and exercises classified as multiple step questions is higher compared to a single step question in the categories examples and exercises. At the end of the chapter, I will show progression over

exercises in terms of number of steps required, in order to check the difference between the three textbooks.

As shown in the Table 5.8, most of the examples and exercises in the three textbooks are multiple steps question.

Multiple step questions are usually more challenging than single step questions, although this also depends on other factors.

The following exercises shows two cases classified as one step:

Question 1:

Exercises 5.16:

3. Factorize completely: $16x^2 + y^2$.

Figure 5-12 Extracted from South African textbook grade 9, page 100

In this case it was easy to classify only as one step since we could not factorize because it was not a difference of square and there is no common factor. That was just a question which checks if the learner has understood that only a difference not a sum of square can be factorized. It is a question which does not require any obvious doing, and thus was a coding challenge because it does not have ‘visible’ steps.

Question 2:

Exercise 5.24(1): which expression is larger and by how much?

a) $12x + 6x - 4x - 2x + 8x$

b) $-50x + 44x - x + 4x - 7x$.

Figure 5-13 Extracted from South African textbook grade 9, page 108

In this exercise it was actually challenging because, mathematically, it does not have an answer - it depends on the value of x . It is an example of a question which is mathematically challenging and where teachers can step in and challenge learners, pushing them towards a better understanding of function, making this the kind of exercise which can be used to challenge learners to move from

level two to level three of algebraic understanding. However, it is not an exercise which can be characterized as having a number of steps. There were, however, very few such explorative or open-ended exercises.

However, there were many more single step examples in the South African grade 10 textbook (38.6%) compared to the two other textbooks (6.7% and 20.5% respectively). When it comes to exercises, the Angolan textbook had fewer single step tasks (8.2%) compared to the South African textbooks (26.5% and 33.1% respectively). Thus, exercises with single step question are a problem that the majority of learners are expected to solve but multiple-step questions are more challenging and important to access new concepts (Cheung, 2003).

For a better illustration, I present another table below which illustrates the frequencies of examples and exercises requiring one, two, three, four, five and six or more steps. The large number of multiple step questions in the three textbooks indicates that all textbooks place less emphasis on examples and exercises with single-step questions.

Table 5.9 Comparison of the selected textbooks in terms of one, two three, four, five and six or more steps question.

	Examples			Exercises		
	<i>South Africa - Grade 9</i>	<i>South Africa - Grade 10</i>	<i>Angola - Grade 10</i>	<i>South Africa - Grade 9</i>	<i>South Africa - Grade 10</i>	<i>Angola - Grade 10</i>
<i>One Step Questions</i>	2 (6.7%)	22 (38.6%)	8 (20.5%)	93 (26.5%)	126 (33.1%)	5(8.2%)
<i>Two Step Questions</i>	18 (60%)	19 (33.3%)	7 (17.9%)	121 (34.5%)	147 (38.6%)	13 (21.3%)
<i>Three Step Questions</i>	6 (20%)	14 (24.6%)	8 (20.5%)	80 (22.8%)	33 (8.7%)	9 (14.8%)
<i>Four Step Questions</i>	4 (13.3%)	2 (3.5%)	4 (10.3%)	34 (9.7%)	43 (11.3%)	6 (9.8%)
<i>Five Steps Question</i>	0 (0%)	0 (0%)	1 (2.6%)	12 (3.4%)	18 (4.7%)	2(3.3%)
<i>Six or more Step Questions</i>	0 (0%)	0 (0%)	11 (28.2%)	11 (3.1%)	14 (3.7%)	26 (42.6%)
<i>Total</i>	30	57	39	351	381	61

As shown in Table 5.9, the number of exercises found in the South African textbooks was larger compared with the number of examples. The number of examples in the South African textbook grade 9 was 30, while the number of exercises was 351. In the South African textbook grade 10, there were 57 examples and 381 exercises. In contrast, the number of examples in the Angolan textbook was 39 with 61 exercises.

The results show that the relative frequency of the number of one step questions and of two steps questions in the category example is higher in the South African textbooks (about 39-60%) compared to the Angolan textbook, in which the largest category of examples are classified as six or more steps questions (28,2%). This is illustrated visually in Figure 5-14 and as averages in Table 5.9.

Similarly, for the category exercises where the frequency of the number of one step questions and two steps questions in the South African textbooks was high (about 27-39%), compared to the Angolan textbook where the largest category of exercises were classified as six or more steps (42.6%). This is illustrated also visually in Figure 5-14 and as averages in Table 5.9.

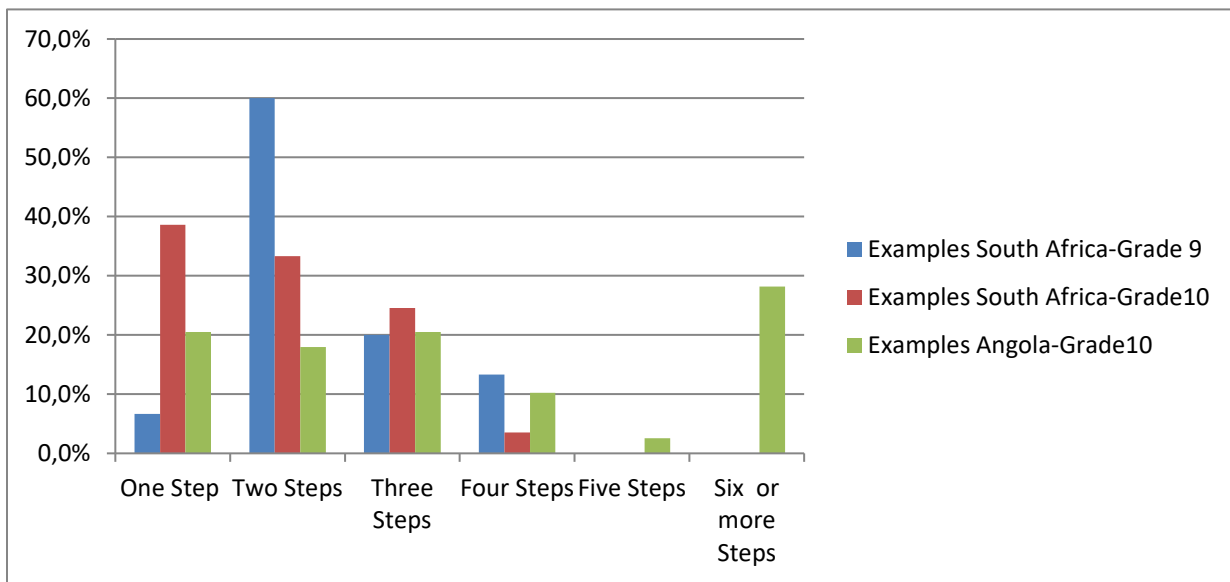


Figure 5-14 Distribution of examples in the three textbooks, according to number of steps

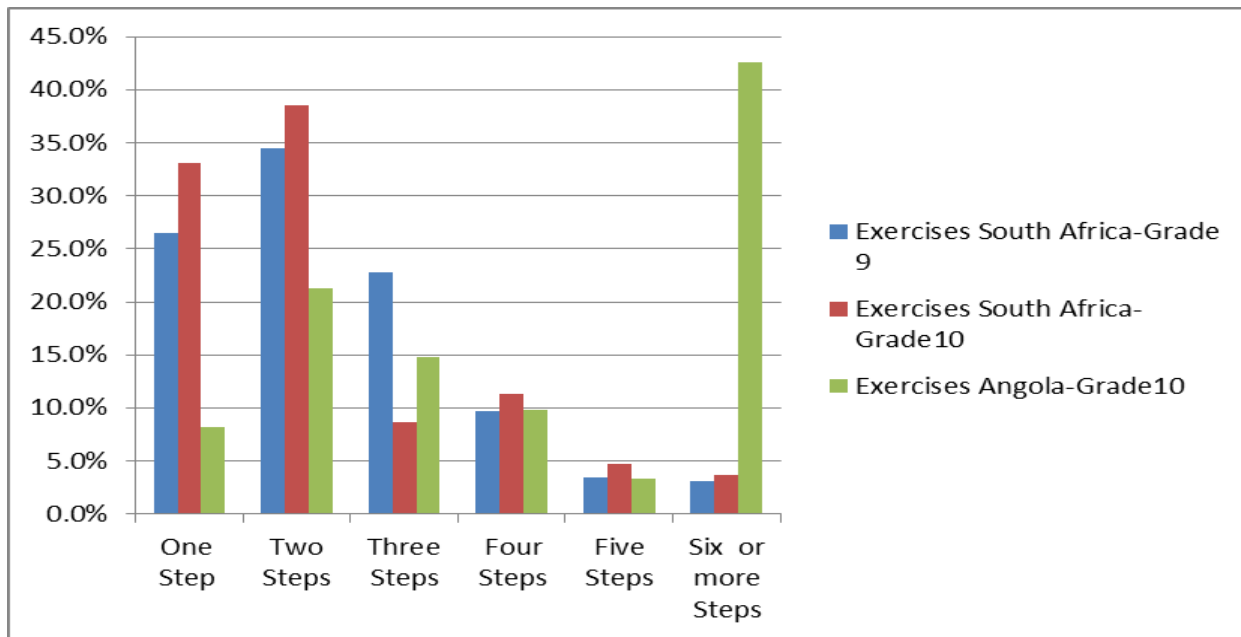


Figure 5-15 Distribution of exercises in the three textbooks, according to number of steps

Table 5.9 also shows that none of the examples in both South African textbooks were classified as five steps questions or six or more steps questions. If I consider that the South African textbooks follow a non-traditional format where different examples are presented between exercises I do think that the absence of five steps questions and six or more steps questions under the category examples should not be a problem. Some examples also serve as consolidation rather than demonstration of methods of exercises. Since learners have learnt the methods of solving these problems in previous grades, the example is sometimes for consolidation, instead of demonstrating methods of solving (new) problems.

In some chapters, there are some examples contained in the concepts presentation notes and they were coded as example. The purpose of these examples is to help the explanations of content.

The following is an example extracted from the Angolan textbook.

If the product of two or more factors is zero, at least one of the factors must be zero. We usually use in symbol if $a \times b = 0 \leftrightarrow a = 0 \text{ or } b = 0$

Take for example the equation $4x + (5 - 2x)(x + 1) = 10$ to solve this equation; I need to make the right-hand side equal to 0.

An example follows to illustrate this concept.

If I make the right-hand side equal to zero, I have

$$4x + (5 - 2x)(x + 1) - 10 = 0 \leftrightarrow 4x - 10 - (2x - 5)(x + 1) = 0 \leftrightarrow$$

$$\leftrightarrow 2(2x - 5) - (2x - 5)(x + 1) = 0 \leftrightarrow$$

$$\leftrightarrow (2x - 5)[2 - (x + 1)] = 0 \leftrightarrow$$

$$\leftrightarrow 2x - 5 = 0 \text{ or } 2 - (x + 1) = 0 \leftrightarrow$$

$$\leftrightarrow 2x = 5 \text{ or } 1 - x = 0 \leftrightarrow x = \frac{5}{2} \text{ or } x = 1 \quad S = \left\{ 1, \frac{5}{2} \right\}$$

I see that:

After making the right-hand side equal to zero, the term $4x - 10$ was factorised, before the left-hand side was factorised. By using the fact that the entire left-hand side was factorised, the fact that at least one of the factors was equal to zero was used.

Figure 5-16 Extracted from Angolan textbook grade 10, page 21

From this extract I can see that the examples are used to illustrate the mathematical concepts to be learnt and to demonstrate how to apply the concepts in solving exercises.

However as shown in Table 5.9 and as I mentioned above, none of the examples in both South African textbooks were classified as five steps or more, contrary to the Angolan textbook which included examples classified on these levels. The two South African textbooks provided examples and exercises with two steps, while the Angolan textbook provided examples and exercises with six or more steps. In traditional textbooks, mathematical concepts are discussed and related

examples follow. Thus, textbooks with a reasonable proportion of examples help learners in the purpose of demonstrating methods and procedures of solving exercises (Cheung, 2003). Stigler, Gallimore and Hieber (2000) agree that comparing textbooks across cultures may be contributing to developing a better understanding of the best ways of teaching. However, these authors do not state that “mathematical concepts and procedures either can be simply stated by the teacher or developed through examples, demonstrations, and discussions” (Stigler, Gallimore, & Hiebert, 2000, p. 92). However, it may just reflect national differences in the expectation of what teachers and learners do in classrooms. As I stated previously, I will now interrogate the progression in the number of steps required in exercises to further understand the differences between the three textbooks.

5.4.1 Progression in number of steps required within blocks of exercises

As discussed, with the three textbooks I analyzed the levels of progression over exercises regarding the number of steps necessary to find a solution. This analysis was done throughout the topics relating to algebraic expression from each textbook. My aim was to find the differences regarding the way each textbook makes progress over exercises when it comes to the number of steps needed to reach a solution.

To illustrate these progression differences between the textbooks I present the following figures.

The graphics below show the steps needed to solve the exercises in the different sections. I re-labelled all the exercises by numbers to facilitate the process of plotting. For instance, in one section question 1a) was re-labelled 1, 1b) became 2, 1c) became 3, 2a) became 4, and so on up to 2e), and 3 was re-labelled 9. In some blocks the number of steps decreases. This is illustrated with a star on the graph, see for instance Figure 5-18, Figure 5-20, Figure 5-21 and Figure 5-26.

5.4.1.1 The South African grade 9 textbook



Figure 5-17 The number of steps required for exercise in a 'block' in the SA grade 9 textbook

As it can be observed, in Figure 5-17, all exercises needed only one step. Therefore it seems reasonable to suggest that there is no progression in difficulty levels within the exercises.

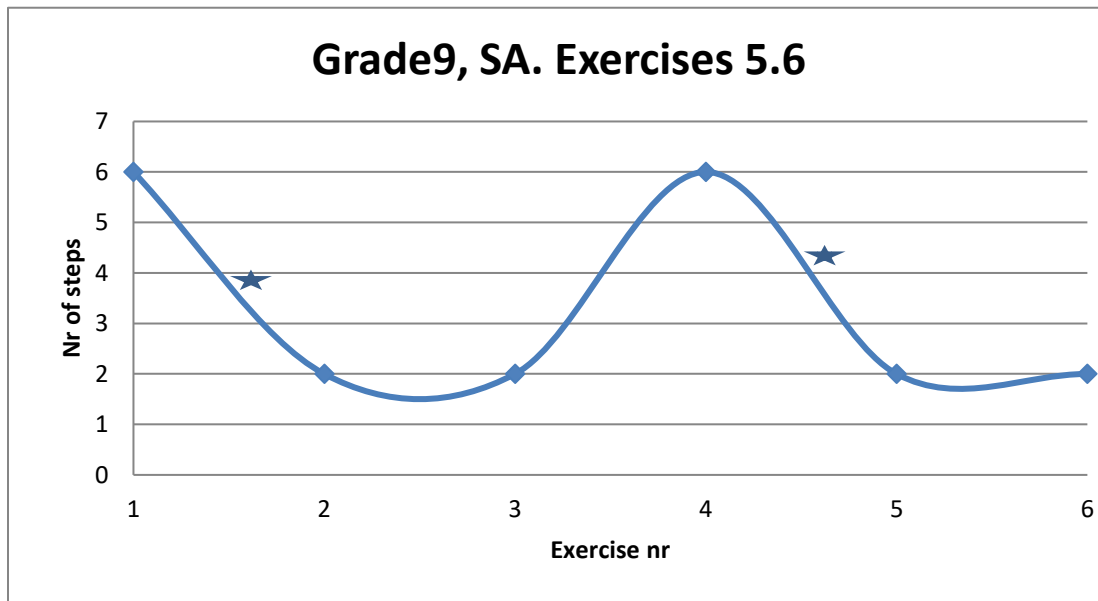


Figure 5-18 The number of steps required for exercise in a 'block' in the SA grade 9 textbook

Figure 5-18, shows that exercise 1 needed six or more steps, and then the numbers of steps dropped to two for the exercises 2 to 3, and again go up to six or more steps for the exercise 4 and again the number of steps dropped to two for the exercises 5 to 6. It is important to see that the number

of steps required within the exercises started out high for the first task, and then dropped to two steps at the end.

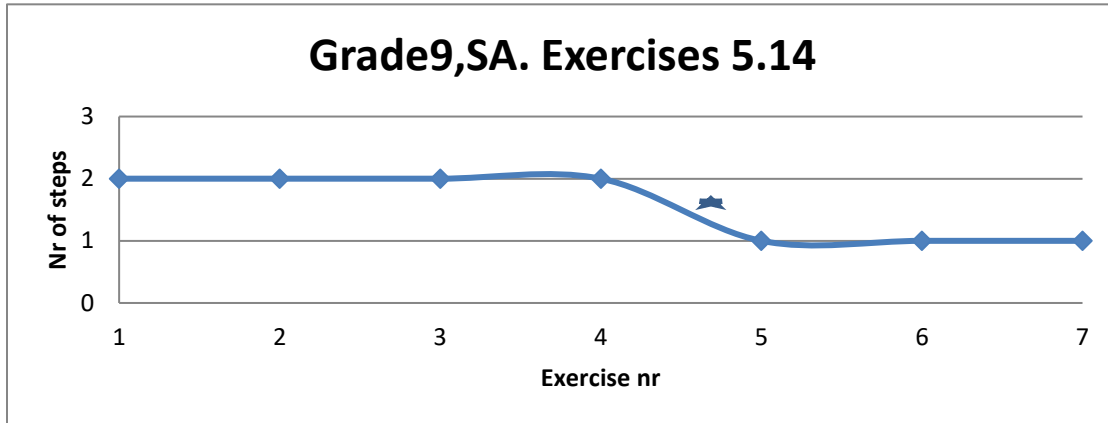


Figure 5-19 The number of steps required for exercise in a 'block' in the SA grade 9 textbook

Figure 5-19, shows that exercises 1 to 4 needed two steps and dropped to one step in the exercises 5 to 7. Therefore it seems reasonable to suggest that the progression in difficulty levels within the exercises decreases.

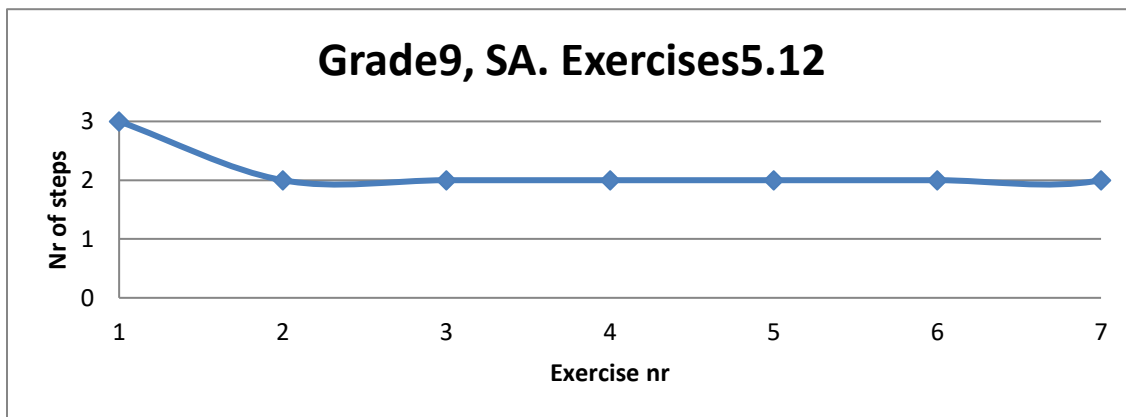


Figure 5-20 The number of steps required for exercise in a 'block' in the SA grade 9 textbook

Figure 5-20, shows that exercise one starts with three steps, then dropped to two steps in exercise 2 and keeps on that level until exercise 7. Therefore it seems reasonable to suggest that there is no progression in difficulty levels within the exercises.

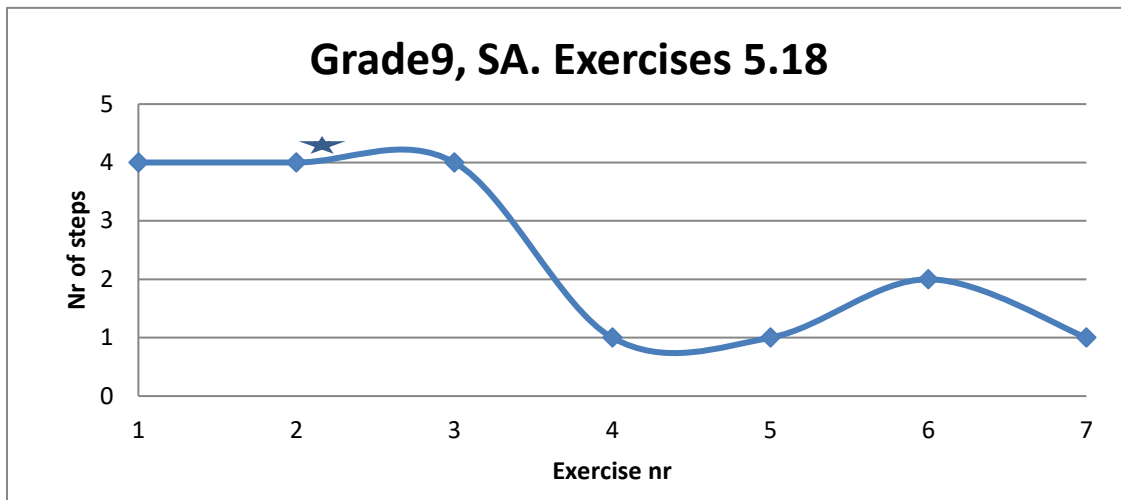


Figure 5-21 The number of steps required for exercise in a ‘block’ in the SA grade 9 textbook

Figure 5-21, shows that exercises 1 to 4 needed four steps, the exercises 4 and 5 needed only one step, then the number of steps is increased to two in the exercise 6, followed by another drop to one in the next exercise. The number of steps required in within the exercises started out high for the first three tasks then at the end dropped to one-step in the last exercise. Therefore it seems reasonable to suggest that there is little progression in difficulty levels within the exercises.

The exercises in the South African grade 9 textbook are predominately one step (about 26.5%), two steps (about 35.4%) and three steps (about 23.8%).

In general, the number of steps required in the exercises does not increase, and also the level of algebraic thinking generally stays the same (see Table 5.5, and Table 5.6), therefore it seems reasonable to suggest that there is little progression in difficulty levels within the blocks of exercises.

5.4.1.2 The South African grade 10 textbook

The exercises in the South African grade 10 textbook are predominately one step (about 26.8%), two steps (about 30.9%) and three steps (about 28.5%), which is very similar to the grade 9 textbook. The progression is also similar.

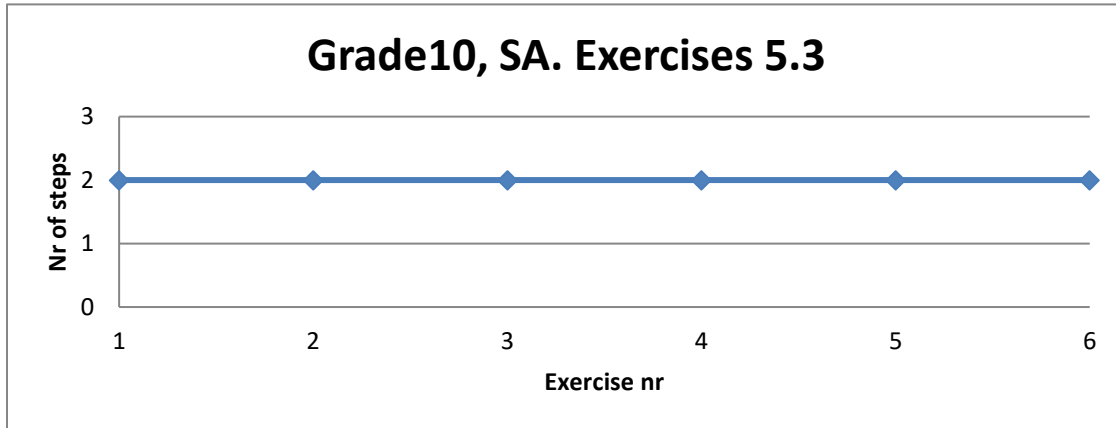


Figure 5-22 The number of steps required for exercise in a 'block' in the SA grade 9 textbook

Figure 5-22, shows that all the exercises needed two steps. Therefore it seems reasonable to suggest that there is no any progression in difficulty levels within the exercises.

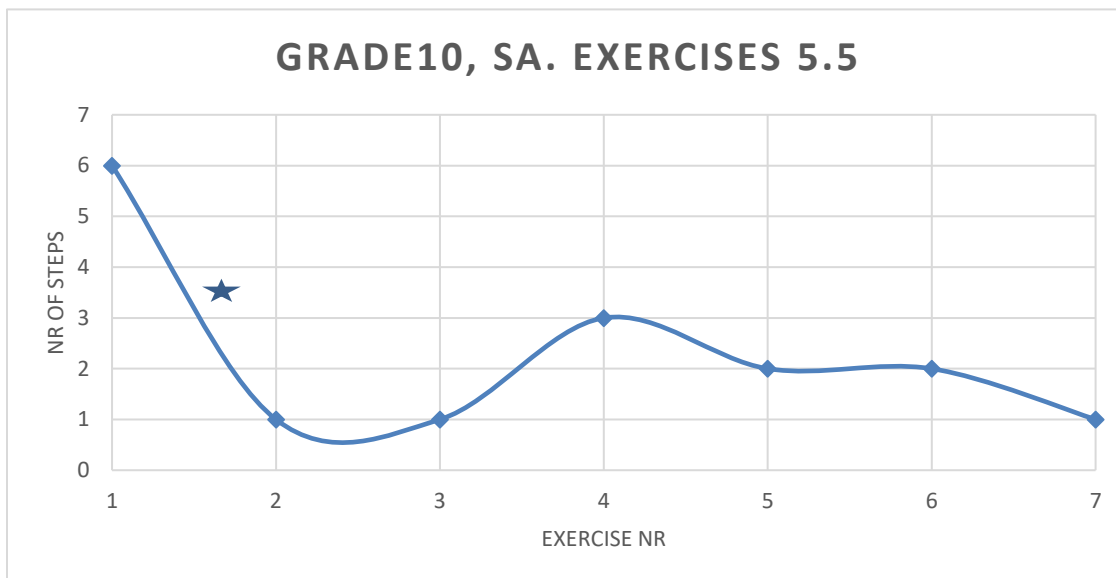


Figure 5-23 The number of steps required for exercise in a 'block' in the SA grade 10 textbook

The number of steps required in the exercises in block 5.5 (see Figure 5-23) started out high for the first task, then dropped to one step in exercise 7.

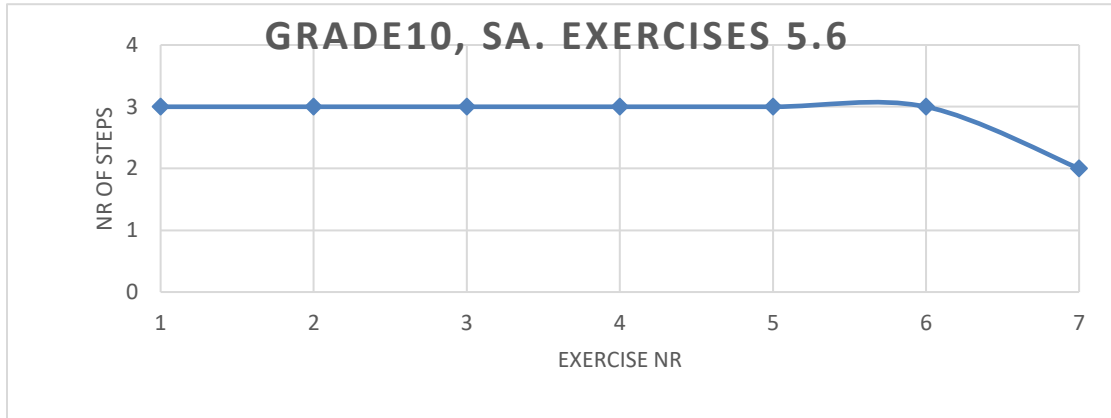


Figure 5-24 The number of steps required for exercise in a 'block' in the SA grade 10 textbook

Figure 5-24 illustrates that the number of steps required for exercises 1 to 6 are three and for exercises 7, are two. Therefore it seems reasonable to suggest that there is no any progression in difficulty levels within the exercises.

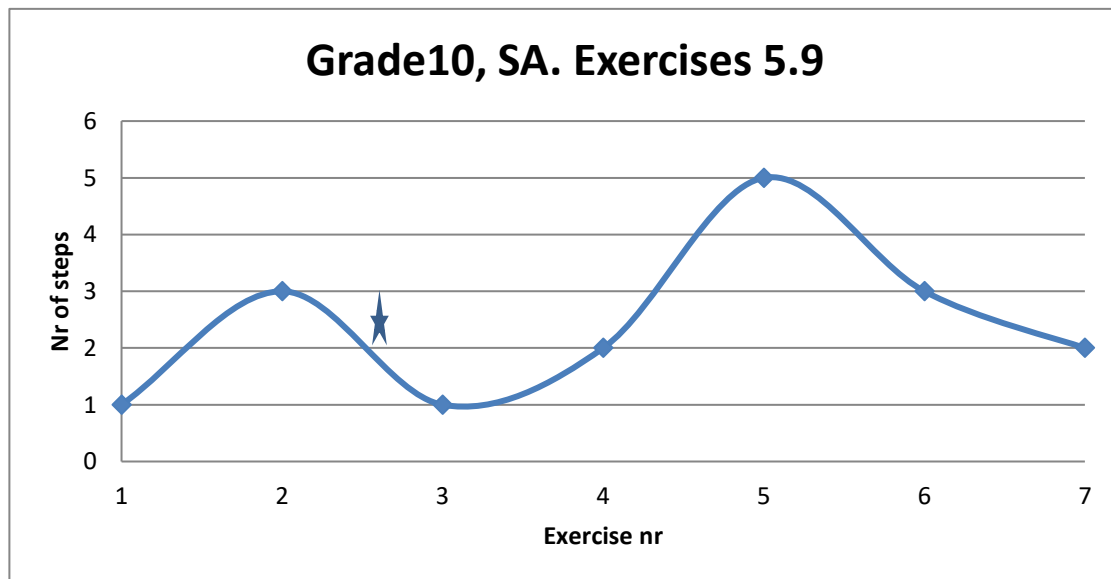


Figure 5-25 The number of steps required for exercise in a 'block' in the SA grade 10 textbook

Figure 5-25, shows oscillations on the number of steps needed for exercises 1 to 7. As shown, the number of steps go up from exercise 1 to exercise 2, and then, going down from exercise 2 to 3, going again up, from 3 to 5, and finally go down from 5 to 7. The number of steps required in the exercises does increase at some stage, there are some fluctuations.



Figure 5-26 The number of steps required for exercise in a 'block' in the SA grade 10 textbook

Figure 5-26, illustrates that exercise 1 needed two steps, the exercise 2 three steps and continue to go up to five steps in the exercises 3 and 4 and then dropped to two in the exercises 5, 6 and 7. The number of steps required in the exercises does increase, at some stage.

In general, the number of steps required in the exercises does not increase, and there are some fluctuations, showing that the progression factor, if any was indeed intended, is neither levels of algebraic understanding (see Table 5.5 and Table 5.6) nor the number of steps required.

5.4.1.3 The Angolan grade 10 textbook

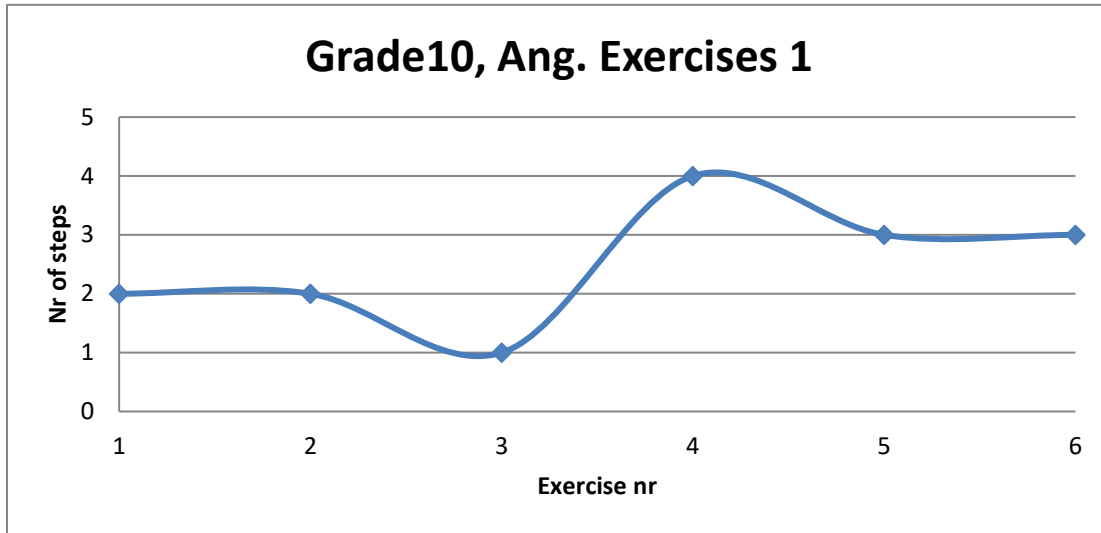


Figure 5-27 The number of steps required for exercise in a 'block' in the Angolan textbook

Figure 5-27, shows that exercises 1 and 2 needed two steps, and the numbers of steps dropped to one for exercise 3, then go up to four steps in the exercise 4, the exercises 5 and 6 needed three steps. The number of steps required in the exercises does increase, at some stage.

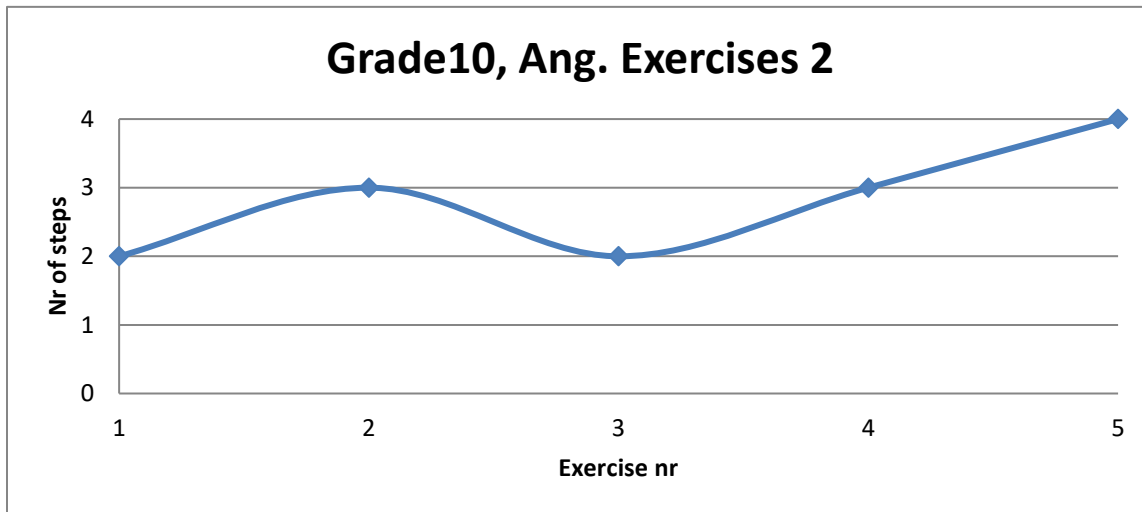


Figure 5-28 The number of steps required for exercise in a 'block' in the Angolan textbook

Figure 5-28, shows, the exercise 1 needed two steps, the exercise 2 needed three steps, and then the numbers of steps dropped to two again for the exercise 3, then go up to three steps in the next exercise and continue to go up to four steps in the exercise 5. The number of steps required in the exercises does increase, and there are some fluctuations, showing that the progression factor, was indeed on levels of algebraic understanding (see Table 5.7).

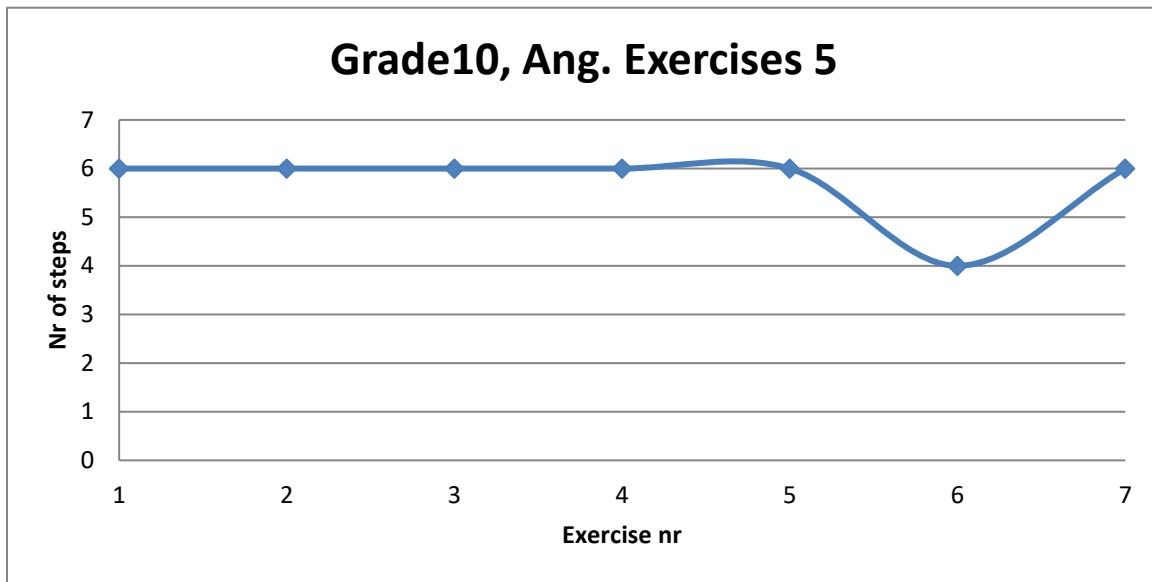


Figure 5-29 The number of steps required for exercise in a 'block' in the Angolan textbook

Figure 5-29, shows that exercises 1 to 5 needed six or more steps, the exercise 6 needed four steps and then the numbers of steps go up to six or more steps again in the exercise 7,

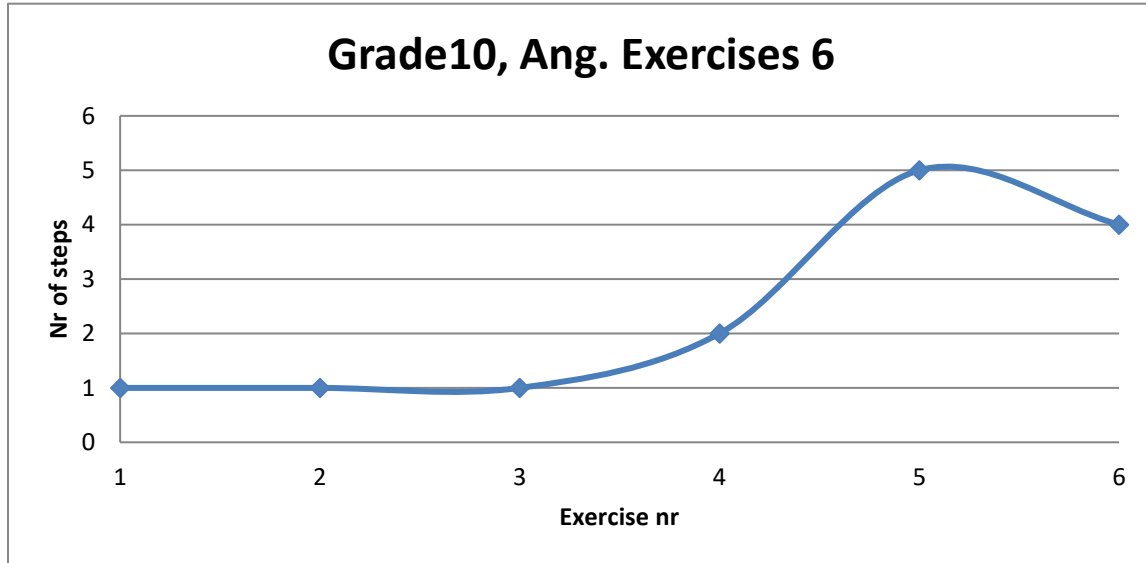


Figure 5-30 The number of steps required for exercise in a 'block' in the Angolan textbook

Figure 5-30, shows that exercises 1 to 3 needed only one step, the exercise 4 needed two steps, the exercise 5 needed five steps and then drop to four steps in the exercise 6. The number of steps needed to reach a solution in the exercises started with few steps and then increased the number of steps.

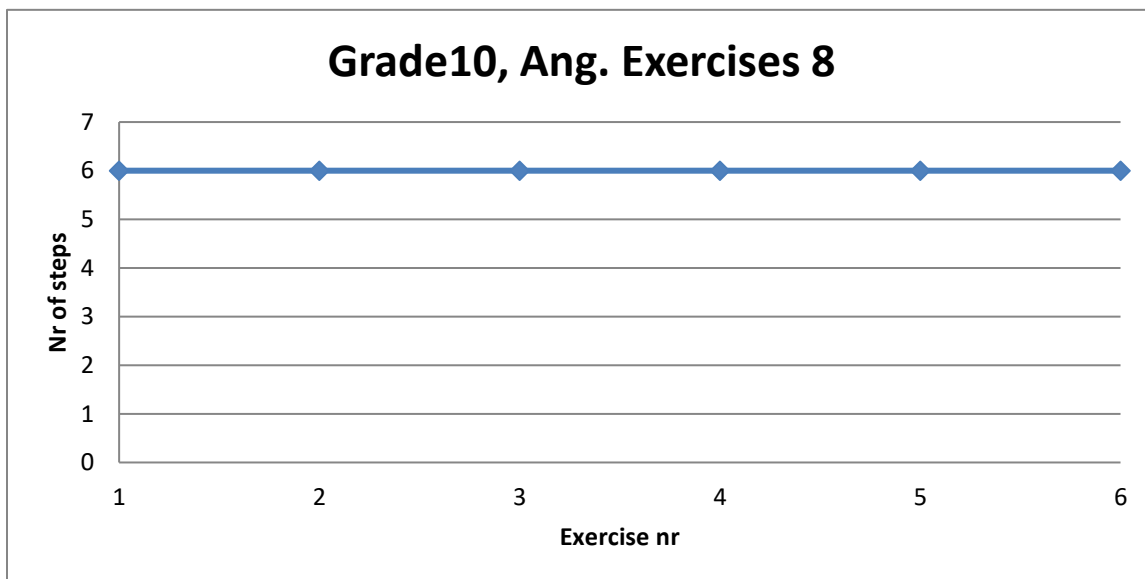


Figure 5-31 The number of steps required for exercise in a 'block' in the Angolan textbook

Lastly in Figure 5-31, all the exercises needed six or more steps. The number of steps needed to sort out the exercise increases starting from six steps. Therefore it seems that the levels of difficulties to sort out an exercise increases with the solutions (see Table 5.7).

The results displayed in the graphs above did not show any noteworthy difference, and thus it would be difficult to measure progression in terms of levels of algebraic thinking or number of steps required within the exercises between the South African and the Angolan textbooks. The South African blocks of exercises generally did not start with exercises requiring fewer steps and then increasing the number of steps needed (see Figure 5-19 and Figure 5-23). The blocks of exercises in the Angolan textbook had far fewer exercises in them, and thus it would be unreasonable to make claims about any progression in the number of steps required, though in many cases it appeared rather static (see Figure 5-27 and Figure 5-28). I observed that the number of steps needed to reach a solution in the Angolan textbook generally started with few steps and then increased the number of steps; perhaps this can be explained by the lesser amount of space dedicated in the category exercises in the Angolan textbook.

5.5 General summary of the analysis

In this chapter, I compared all the topics covered in grade 9 and 10 South African textbooks, and grade 10 Angolan textbooks within the topic of algebra. The findings indicate that the number of the topics covered in the grade 10 Angolan textbook is greater than the number of the topics covered in each grade in the South African textbooks. However, there appeared some similarities and differences with the topics of algebra. For similarities, I found that the topic such as “basic operations on polynomials and factorising algebraic expressions” (see Table 5.1) are common for all three textbooks. Besides equations, simultaneous equations and inequalities are common in both grade 10 textbooks. However, non-linear functions, injective and bijective applications, monotonicity, and modulus function appeared in the Angolan grade 10 textbook and not in the South African grade 10. Besides, simplifying expressions involving algebraic fractions, and exponential notation appeared in the South African grade 10 and not in the Angolan grade 10 textbook. As a result, Angolan textbook appeared to address more advanced mathematical processes and concepts.

Moreover, it appeared that in the Angolan textbook, the explanations are very detailed. This indicates that at least in theory learners could do the exercises without any additional support. However, the space devoted to exercises in the Angolan textbook may seem very low, compared

to the South African textbooks. Furthermore, it appeared that examples were more detailed in the grade 10 Angolan textbook than in the grade 10 South African textbook.

In terms of consistency in the levels, I found that there is a consistency in the levels of algebraic thinking of the categories (explanation, examples and exercises) within the topics in the grade 9 South African textbook and the grade 10 Angolan textbook whereas it appeared a partial consistency of the levels of algebraic thinking in grade 10 South African textbooks.

Analysis of the progression of the chapters showed that there is a progression within the categories in chapters five, six and seven in the grade 9 South African textbook. The findings revealed that there is also a progression with the categories in chapter five, seven and eight in the grade 10 South African textbook. However, the progression between grade 9 and 10 South African textbooks appeared not to be consistent according Sfard and Linchevski's (1994) definition of levels.

CHAPTER 6 : CONCLUSION

In this chapter, I will discuss the findings and relate them back to the original research questions. Limitations of the study and recommendations for further study are then discussed. Lastly a critical reflection on the framework will be provided.

6.1 Research question 1: Composition of the textbooks

What are the similarities and differences between the three textbooks in terms of topics' coverage and content distribution between explanations, examples and exercises?

The Angolan textbook included more advanced topics, to which it progressed quickly.

The three selected textbooks, two from South Africa and one from Angola all had text in the categories explanation, examples and exercises. However, the textbook from Angola was predominately explanatory while the two textbooks from South Africa were dominated by the category exercises. This, in my view, positions the textbooks as different types, supposedly reflecting different views on teaching and learning, in line with national differences between South Africa and Angola.

6.2 Research question 2: Conceptual progression

What are the similarities and differences between the three textbooks when it comes to conceptual progression in algebra?

The South African grade 9 textbook had an unexpected progression in the levels of algebraic understanding evoked. In the first chapter, almost all the explanations, examples and exercises were classified as level zero - expression, except 10 (2.8%) of the exercises which were on level two. In the second chapter, the categories were classified as level one and three with the majority of items on level three. Finally, in the third chapter, almost all categories were classified as levels one and two, except 2.8% of exercises on level three. In this textbook the function concept was introduced in the second chapter but the following chapter did not build on this level of algebraic thinking. Thus, the progression did not follow the expected move from level one to level three.

The South African textbook grade 10 shows a very slow progression in levels of algebraic understanding. In the first chapter, almost all the items were classified as level zero, except 0.4% of exercises classified as level one and 2.1% exercises classified as level two. In the second chapter, almost all the explanations and exercises were classified as level zero, except 16.4% of

the exercises which were classified as level two. Finally, in the last chapter of this textbook, the categories were classified as level two and three with the majority on level two. Thus, the progression did not follow the expected move from level one to level three; perhaps this is in part because of the larger number of exercises per topic in this textbook. In this textbook the function concept is introduced in the last chapter.

The Angolan textbook shows progression through the levels two and three of algebraic thinking, but with no level one content. The first chapter is consistent in terms of levels zero, two and three with the majority of content on level zero. The second chapter is also consistent in terms of levels zero, two and three with the majority of the text on level three. The function concept is introduced in the first chapter and the second chapter builds on this understanding of mathematics concept through the levels of algebraic thinking.

In summary, the progression in the South African textbook did not follow the levels of algebraic understanding proposed by Sfard and Linchevski (1994); the South African grade 9 textbook moved from a chapter on level zero to a chapter that combined level one and level three, and only then a chapter combining level one and level two. There was barely any level on content in the South African grade 10 textbook and none in the Angolan grade 10 textbook.

It appears that the Angolan textbook moves quickly with heavy content, whereas the South African textbook moves more slowly and with somewhat lighter content. What of course has not been discussed here is the extent to which the fast progression of the Angolan textbook generally falls within the zone of proximal development (Vygotsky, 1980) of the Angolan learners, and the extent to which the progression and spiralling curriculum in the South African textbooks is suited to the present group of learners.

6.3 Research question 3: Procedural competencies

How is the conceptual progression linked to developing procedural competencies? Do tasks, explanations and examples correspond in terms of their level of algebraic thinking required?

All the textbooks had substantial representation on what I called level zero, ranging from 37-51% across categories in the South African grade 9 textbook, over 62-71% in the South African grade 10 textbook, to 28-33% in the Angolan textbook. This may well indicate a higher conceptual focus in the Angolan textbook, which is also reflected in the range of topics included in the book.

There was consistency in the three textbooks in terms of levels of algebraic understanding, in the sense that explanations, examples and exercises generally were on the same level of algebraic understanding. However, when it came to progression in levels, the textbook from Angola presented a quicker progression than both South African textbooks.

The Angolan textbook also provided more exercises requiring learners to perform six or more steps of calculations, while the South African textbooks provided exercises requiring only a few steps. However, the South African textbooks emphasized procedural manipulation of symbols more than the Angolan textbook. This, in my view, positions the textbooks as different types, supposedly reflecting content organization on teaching and learning in line with national differences between South Africa and Angola. In summary, the South African textbooks indicate a strong procedural practice focus without conceptual understanding to balance it and few relevant representations of mathematical connections; this is seen in the analysis with the large number of exercises with very slow progression across these in the South African textbooks. It appears that the Angolan textbook offers more opportunities to the learners to learn mathematics. I suggest that it is necessary to investigate the way the teachers engage their learners and textbooks in the classroom.

6.4 Limitations of this study

The method of sampling used was convenience, as it was beyond the scope of this study to first investigate the use of textbooks on a national level. This study was limited to only one book from Angola which makes generalizing the results about the textbooks across grades and between countries impossible. The sample size was altogether limiting and could have been extended to include more textbooks. . Moreover, given that the study did not include the book of grade 9 from Angola, this limited me to study the progression in the levels from grade 9 to grade 10 in Angola. Thus, the study has not been able to compare the progression from grade 9 to 10 for the two countries.

The addition of grade 9 from Angola could have assisted me in drawing broader conclusions about the progression between grades in different countries. The approach used provided, opportunities for looking at how algebraic concepts are presented in different textbooks, but obviously it could not be linked to how algebraic concepts are taught in classrooms. I think that an investigation of how teachers engage their learners in the process of learning could be a possible extension to this

study. While such studies exist in South Africa, there have so far been no such studies conducted in Angola. Thirdly, there are limited studies analysing textbooks in terms of progression. While this means that this study was an exploration of different methods for interrogating progression, it sets limitations as to the depth and theoretical insights which could be reached.

Fourthly, there are no studies analysing and comparing textbooks based on Sfard and Linchevski's (1994) levels. Therefore, a study analysing textbooks using Sfard and Linchevski's (1994) theoretical framework is needed. Finally, not analysing how explanations, examples and exercises are used by teachers in the classroom limits what can be inferred about the differences in opportunities to learn. Therefore, research analysing how textbooks are used in classrooms by teachers is needed to understand the difficulty in the teaching and the learning processes.

I suggest that it is necessary to investigate the way the teachers engage their learners and textbooks in the classroom in order to say anything conclusive about learning, but the analysis of textbooks gives some indication of the opportunities to learn. Overall, our analysis suggests that learners are given more opportunities to learn mathematics if they engage with the Angolan textbook, despite the sequencing of topics in the South African textbooks. The only objection could be that it moves relatively quickly through the content, and thus pushes the teacher into "forced autonomy" around how to facilitate learning.

6.5 Suggestions for further studies

This thesis is a contribution based mainly on a comparison of algebraic concepts in mathematics textbooks by analysing the conceptual progression in explanations, examples and exercises in three different high school textbooks. It therefore opens the way for future researchers interested in extending the scale of textbook analysis. Future works can involve the following: Firstly, extension to all topics in selected textbooks. This can provide a better picture of problem-solving opportunity provided by high schools' textbooks. Secondly, expanding the number of textbooks from the different countries to capture the progression between grades would be useful. The third suggestion is to examine how the books are used during instruction. Finally, special attention have to be taken to analyse the levels of algebraic understanding.

6.6 Critical Reflections on the Theoretical Framework

In general, I observed that the progression was not consistent with Sfard and Linchevski's levels across the South African textbook grade 9. The function concept is introduced in chapter six but the following chapter does not appear to build on this understanding of mathematical concepts through the levels of algebraic thinking. According to Sfard and Linchevski's (1994) levels, the expectation is that once chapter six was consistent on level three – function - the next chapter should be classified on the next level - family of function, or alternatively address more complex functions.

A problem with the framework becomes evident, for example, when one looks at the South African textbooks' results. It is not clear how to classify the progression in the textbook. The South African textbooks grade 9 and 10 have a chapter with mostly level three content before a chapter with mostly level two content and this appears to contrast the claim by Sfard and Linchevski (1994) that the process level precedes the structural level. However, it seems that the progression between the referred chapters is more compatible with Boaler's (1998) thinking, which is based on research with evidence (Friel & Markworth, 2009; Smith et al., 2007). According to this thinking, working with functions should precede working with equations.

Subsequently, it can be deduced that the theoretical framework may fall short on the levels of progression, once it is clear if the textbook provides an expected good progression. In general, looking to the South African textbooks' results, this framework did not offer enough insight into the progression in the content, so it becomes difficult and unclear to classify which textbook offers a good option regarding the progression levels. Despite this finding, this theory provides insight on how algebraic concepts have been engaged in the textbooks; however, textbooks tend to emphasize and start with algebraic expressions in a way which reflect neither an operational nor a structural understanding. As listed by Sfard and Linchevski (1994):

There is, of course, a much simpler way of looking at $3(x + 5) + 1$: it may be taken at its face value, as a mere string of symbols which represents nothing. It is an algebraic object in itself. Although semantically empty, the expression may still be manipulated and combined with other expressions of the same type, according to certain well-defined rules (Sfard & Linchevski, 1994, pp. 191-192)

It must be a problem that all three textbooks still work with an approach to algebra where we start by considering the symbols as ‘empty’ shells to be manipulated. Also, the framework provides me with insight into the consistency of the explanations, examples and exercises according to the levels of algebraic thinking in the textbooks.

6.7 General Discussion

The purpose of this research was to analyse content of algebra in three high school textbooks - two from South Africa and one from Angola - in order to gain insight into the algebraic concepts of consistency and progression. In this study, the selected textbooks were analysed in terms of the extent of content of each category; in terms of progression in levels of algebraic thinking and consistency in this respect as well as in terms of number of steps of the solutions required to solve the examples and exercises. I found that the Angolan textbook contained more explanations and fewer exercises than the two South African ones. All the textbooks were fairly consistent within chapters when it came to the levels of algebraic understanding represented in the three textual categories. The progression in the Angolan textbook tended to be faster, but this was in part also because there were fewer exercises per topic. The number of steps required in the examples and exercises differed substantially between the two sets of books, with the Angolan textbook providing examples and exercises with six or more steps, while the two South African textbooks provided examples with two steps and exercises with one or two steps. Thus, the Angolan textbook appeared more dense, and with more explicit instruction than the South African textbooks, in line with the findings about reform-based textbooks elsewhere (Sood & Jitendra, 2007).

The differences in distribution of text on the three categories (explanations, example and exercises) discussed above may reflect national differences in the expectations of what teachers and learners do. In the experience of the second author, Angolan teachers are expected to construct their own lessons explaining mathematical concepts and procedures. However, the current curriculum reform in South Africa has moved away from leaving this open to the teachers, replacing it with very detailed prescriptions for what must be taught, and when, down to providing weekly topic plans (personal communication with subject advisors and teachers). It would seem in line with this practice for textbooks to provide not only explanations and examples but also exercises which the teacher can use and for which answers are provided.

On the other hand, South Africa has been through a turbulent curriculum history over the past 20 years since the introduction of democracy. It has gone from a highly prescribed curriculum, with strong control over content, sequencing and pacing, over a learner centered approach (reflected in Curriculum 2005 and OBE) with weaker control over content, sequencing and pacing, and back to a highly prescribed curriculum (Hugo, 2013). The inclusion of many exercises in textbooks could be reflective of the interim period, where it would make sense to provide teachers and learners with a broad range of exercises through which to learn, allowing for variations in the levels of the learners' understanding. It could also be linked to the nature of the exercises as part of letting learners construct meaning for themselves. Even so, that would still reflect the expectations that teachers need not make their own exercises, only to select from those provided.

It is interesting to compare this result to the works of Mayer et al. (1995) and Stevenson and Stigler (1992) who compared US and Japanese textbooks and teaching, respectively. Mayer and colleagues found that Japanese books provided more worked-out examples and relevant illustrations compared to US books, whereas the US books contained a higher number of exercises and many more irrelevant illustrations than the Japanese textbook. Stevenson and Stigler found that Japanese teachers tend to ask questions which call for more elaborate answers and focus on fewer problems in more depth. Without being able to make any conclusive claims, this suggests that the Angolan textbook offers more opportunity to learn mathematics, in line with the findings by an author (Stevenson & Stigler, 1992).

This agrees with the second analysis which indicates that procedural manipulation of symbols is emphasized in the South African textbooks to a much larger extent than in the Angolan textbook – here indicated by the prevalence of level zero content. Sadly, this is in line with much classroom research from South Africa across grade levels, which indicates a strong procedural focus without conceptual knowledge to balance it, and few relevant representations of mathematical connections (Ally & Christiansen, 2013; Hoadley, 2012; Mhlolo, Venkat, & Schäfer, 2012; Noubouth, in progress) – although more is known about primary than high school teaching. Such research also suggests that much time in South African classrooms is spent practicing routine type activities, consistent with the finding that the two South African textbooks contain a large number of exercises with very slow progression across these.

More challenging exercises in terms of number of steps can help learners to perform positively, suggests Yan and Lianghuo (2006; Zhu & Fan, 2006), who found that US learners did not perform well in solving multiple-step problems because of the limited exposure to this type of tasks in the textbooks. Cheung (2003) also argued that good textbooks rely on the exposure of single and multiple step question, where the latter is more important to access new concepts.

Furthermore, my study shows that when comparing the textbooks, the frequency of one step questions in the category exercise in the Angolan textbook is fewer than in the South African textbooks, whereas the frequency of tasks requiring six or more steps were substantially higher in the Angolan textbook. The question is, does the number of steps affect learners' opportunity to learn? Zhu and Fan (2006) claimed that including more challenging exercises in terms of number of steps can help learners to perform positively in solving exercises. Zhu and Fan (2006) found that US learners did not perform well in solving multiple-step problems because of less exposure to this type of problem in the textbooks. Cheung (2003) found that the way problems are presented in textbooks in terms of steps may influence the quality of the textbooks. For instance, he argued that good textbooks rely on the exposure of single and multiple step question, of which the latter is more important to access new concepts. This suggests that at least in this respect the Angolan textbook offers better opportunities to learn algebra.

In some ways, the South African textbooks reflected content organization which is in line with other research on the teaching and learning of algebra, whereas the Angolan textbook appears to have taken a more traditional approach. For one, functions were introduced before equations in the South African textbook. Secondly, equations with the variable on both sides of the equal sign were treated as a later and separate topic. Thirdly, there were some exercises which directed learners to realize connections or principles, without having to be told.

Again, I suggest that it is necessary to investigate the way the teachers engage their learners and textbooks in the classroom in order to say anything conclusive about learning, but the analysis of textbooks gives some indication of the opportunities to learn. Overall, this analysis suggests that learners are given more opportunities to learn mathematics if they engage with the Angolan textbook, despite the sequencing of topics in the South African textbooks. The only objection could be that it moves relatively quickly through the content, and thus pushes the teacher into 'forced autonomy' around how to facilitate learning.

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Appendix

A: Exercise 5.2, South African Book, Grade 9



Exercise 5.2

LO1 AS9.1.6; LO2 9.2.7

- Work on your own.
- Add the given polynomials.
 1. $2x^2 - 5x$; $x^2 + 4$; $-7x + 8$
 2. $x^2 - 3x + 1$; $2x^2 + 7$; $-5x - 2$
 3. $a^2 - 4a$; $-a - 5$; $2a + 7$
 4. $2b^2 - 4b + 1$; $-b^2 - b$; $-4b^2 - 2b + 6$
 5. $-2b + 5$; $b^2 + b + 4$; $2b^2 + 2b - 8$
 6. $x^2 - 4x + 2$; $-2x^2 - 3x - 5$; $x^2 + 7x + 3$
 7. $p^2 - 8p - 1$; $2p^2 - 4p - 2$; $-3p^2 - 10p + 3$
 8. $2y^2 + 3y$; $-y^2 + y$; $-3y + 8$
 9. $a^3 + 4a$; $-2a^2 + a$; $2a^3 - 5a^2 - a$
 10. $x^2y + xy$; $3xy - xy^2$; $2x^2y - 4xy$

B: Exercise 5.6, South African Book, Grade 9



Exercise 5.6

L01 AS9.1.6; L02 AS9.2.4, 9.2.7, 9.2.8

- Work on your own.
- 1. Copy and complete the table if x is replaced by the number in the first column.

x	$A = (x + 2)^2$	$B = x^2 + 4x + 4$	$C = x^2 + 4$
0	$A = (0 + 2)^2 = 4$		
1			
2		$B = 2^2 + 4(2) + 4 = 16$	
3			
4			$C = 4^2 + 4 = 20$

- Compare the values you calculated in the three columns.
- Write down your conclusions.

2. Copy and complete the table by working out values of the expressions for different y values.

y	$A = 3y(y^2 + 2)$	$B = 3y^3 + 6y$	$C = 3y^3 + 2$
-1			
1			
2			
3			
4			

- Compare the values you calculated in the three columns.
- Write down your conclusions.

Exercise 5.16

L01 AS9.1.6; L02 AS9.2.7, 9.2.8, 9.2.9

Work on your own.

This exercise equips you with the tools to handle more difficult expressions.

Factorise completely.

1. $5a^2 + 20$

2. $5a^2 - 20$

3. $16x^2 + y^2$

100

Alge

- To d
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D: Exercise 7.6, South African Book, Grade 10

Exercise 7.6

(AS 10.1.2)

1. Solve for x . Give all possible answers ($x \in \mathbb{Z}$).

Example

$$2^{-3} = \frac{1}{x}$$

$$\therefore \frac{1}{2^3} = \frac{1}{x}$$

$$\therefore 2^3 = x$$

$$\therefore x = 8$$

a) $1^6 = x^6$

b) $4^x = 1$

c) $9^2 = 3^x$

d) $2^8 = 4^x$

e) $1^{10} = 1^x$

f) $9 = 3^x$

g) $9 = (-3)^x$

h) $9 = x^2$

i) $-9 = -3^x$

j) $27 = x^3$

k) $-27 = x^3$

l) $10^{-2} = 100^x$

m) $\frac{1}{8} = 2^x$

n) $(-5)^0 = x$

o) $0^{-5} = x$

p) $\frac{1}{81} = x^{-2}$

q) $5^{-2} = \frac{1}{x}$

r) $2^x = 16$

s) $x^4 = 16$

t) $-2^x = -32$

u) $x^5 = -32$

v) $\frac{1}{2^x} = 8$

E: Exercise 5.19, South African Book, Grade 9

••• Classroom Mathematics Grade 9



Exercise 5.19

L01 AS9.1.6; L02 AS9.2.7; AS9.2.8

- Work on your own.

1. $\frac{10ab^2}{15a^2b}$

2. $\frac{16x^2y^3}{12xy}$

3. $\frac{9abc^2}{12a^2bc}$

4. $\frac{8pq}{10p^2}$

5. $\frac{22x^2y^3}{11x^3y^5}$

6. $\frac{5a^7b^5c^4}{10a^3b^3c^3}$

7. $\frac{2rs^2}{8r^2s^4}$

8. $\frac{18e^3fg^2}{9e^2fg}$

9. $\frac{32x^2y^3z}{64x^3y^3z^2}$

10. $\frac{12kmx^2}{20k^2mx}$

11. $\frac{a}{b \cdot c}$

12. $a \div (b \div c)$

13. $a \div b \div c$

14. $\frac{a}{\frac{b}{c}}$

15. $\frac{\frac{5}{2}}{\frac{1}{2}}$

16. $\frac{2}{3} \times \frac{6}{8}$

17. $\frac{3a}{b} \div \frac{6}{b^2}$

18. $\frac{2x^2}{y} \times \frac{y^2}{6x}$

19. $\frac{2ab}{c} \div 4ac$

20. $\frac{x^2}{5} \times \frac{10}{3x}$

21. $\frac{2a}{3} \times \frac{9b}{a}$

22. $\frac{3x^2y}{2} \div \frac{6x^3}{8y}$

23. $\frac{6a^2b^3}{c^4} \times \frac{c^2}{3a^3b}$

24. $\frac{2x^5y^3}{b^2} \times \frac{4by^2}{x^2} \times \frac{b}{8y^4}$

25. $\frac{x^2}{y} \times \frac{xy}{z^2} \div \frac{x^3}{zy}$

26. $\frac{a^2}{bc^2} \times \frac{bc}{a^2} \times \frac{b^2c}{a^2}$

27. $\frac{a^2}{bc^2} \div \frac{bc}{a^2} \times \frac{b^2c}{a^2}$

28. $\frac{a^2}{bc^2} \div \left(\frac{bc}{a^2} \times \frac{b^2c}{a^2} \right)$

29. $\frac{a^2}{bc^2} \times \frac{bc}{a^2} \div \frac{b^2c}{a^2}$


30. $\frac{a^2}{bc^2} \div \frac{bc}{c^2} \div \frac{b^2c}{a^2}$

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F: Exercise 5.24(1), South African Book, Grade 9

••• Classroom Mathematics Grade 9

CHECK YOUR SKILLS

 **Exercise 5.24** L01 AS9.1.6; L02 AS9.2.6, 9.2.7, 9.2.8, 9.2.9

- Work on your own.
- 1. Which expression is larger and by how much?
a) $12x + 6x - 4x - 2x + x + 8x$ b) $-50x + 44x - x + 4x - 7x$


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G: Example, Angolan book, Grade 10

4. a) You are given this set of data. Complete the table.

Input (x)	1	2	3	4	5	7	10	12	15
Output (y)	7	9	11	13					

b) Describe, in words, the rule you used to complete the table.
c) Design a flow diagram that will represent the data given in the table.
d) Represent the table or flow diagram in algebraic language.
e) Analyse the rate of change in the different representations.
f) Discuss your observations with your classmate.



The rate of change is the amount by which the output variable changes when the input variable is increased by 1.

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H: Example, Angolan textbook, Grade 10

A equação $4x + (5 - 2x)(x + 1) = 10$ ainda não tem qualquer dos seus membros igual a zero pelo que, na sua resolução, teremos que passar o 10 para o primeiro termo.

$$4x + (5 - 2x)(x + 1) - 10 = 0 \Leftrightarrow 4x - 10 - (2x - 5)(x + 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2(2x - 5) - (2x - 5)(x + 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow (2x - 5)[2 - (x + 1)] = 0 \Leftrightarrow$$

$$\Leftrightarrow 2x - 5 = 0 \vee 2 - (x + 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2x = 5 \vee 1 - x = 0 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{5}{2} \vee x = 1 \quad S = \left\{ 1, \frac{5}{2} \right\}$$

Exercise 8.11

(AS 10.2.5)

Solve for x and y :

1. $x + y = 5$

$x - y = 3$

2. $x + 2y = 4$

$x - 2y = 0$

3. $x + y = 8$

$3x + 2y = 21$

4. $x + 4y = 14$

$3x + 2y = 12$

5. $x + 3y = 5$

$2x - 6y = 2$

6. $3x + y = 4$

$x + y = 2$

7. $3x + y = 2$

$6x - y = 25$

8. $2x = 3 + y$

$x = 3y + 9$

9. $x + y = 1$

$x - 2y = 1$

10. $x + 2y = 5$

$x - y = \frac{1}{2}$

11. $y = x - 1$

$y = 2x + 3$

12. $x - 2y + 3 = 0$

$y + 2x + 1 = 0$

J: Exercise 11, Angolan book, Grade 10

POLINÓMIOS CAPÍTULO 1

Exercício 11.
Determinar a solução de cada um dos sistemas:


11.1.
$$\begin{cases} x - \frac{x+y}{2} + z = 3 \\ 3x - 2y + 4z = 12 \\ \frac{x+y}{4} + 2z = 6 + x \end{cases}$$

11.2.
$$\begin{cases} \frac{5+x}{2} - 3y + 2z = x+y \\ 2x + 3y + 7z = 0 \\ 4x - 2(x+z) + 3y = x \end{cases}$$

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K: Exercise 5.9, South African book, Grade 9

••• Classroom Mathematics Grade 9

 **Exercise 5.9** L01 AS9.1.6; L02 AS9.2.7, 9.2.8, 9.2.9

• Work on your own.

Find the products. Simplify if possible.

1. $3x(x - 5)$ 2. $(x + 2)4x$
3. $-3x(x - 2)$ 4. $(x - 2) - 3x$

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5. $(x - 2)(3x)$
7. $(p^2 - 2p - 1)3p$
9. $5a(2 + b) + (2 + b)5a$
11. $-2x(x + 1) + (x + 1) - 2x$
13. $2x - (x + 3) - x$
15. $2b(b - 3) - (b^2 - 5b) + 2b$
17. $x^2(x^3 - 2x^2) - 2x^2(3x^3 - 8x^2) + 10x^4$
19. $ab + a(b - 3) - (5a + 2ab) + 7a$
21. $5a - [3a - 4\{a - 2(a + 5)\} - (2a + 3)]$
23. Find the sum of the first hundred terms of the series
 $3 + 7 + 11 + 15 + \dots$
 Use the formula $\frac{n}{2}[2a + (n - 1)d]$, where $n = 100$ is the number of terms being added, $a = 3$ is the first term of the series and $d = 4$ is the difference between two successive terms.
24. If $y = 2x - 1$, find $2x - [3x - 2(y + 1)] - 5$ in terms of x . Your answer should contain only x and numbers, no y .
25. Ms Mthembu buys oranges for the athletics team. The delivery arrives in two large baskets and 30 oranges. The delivery note explains that each basket contains five packets and two boxes. There are eight oranges in each packet and 15 in each box. Does Ms Mthembu have enough oranges for 120 students?
6. $-3x - (x - 2)$
8. $p(p - 1) - 2p^2 - 2p$
10. $3(x + 2)x - x(x - 4)$
12. $2(y + 3)y - y(y - 4)$
14. $a(a + 2) - (2a - 1)(-3a)$
16. $x - [3(x - 2) + 2x]$
18. $(2x - 3y)(-2x) - 2x(x + y) - 2xy$
20. $4[2x - 3\{7 - (5 - x)\}]$
22. $x - [2x - 5\{x - 3(x - 1)\} - 4(-x - 2)]$

L: Exercise 5.3, South African book, Grade10

••• Classroom Mathematics Grade 10

Exercise 5.3

(AS 10.2.4)

Find the following products using the second method.

1. $(2x - 3y)(2x + 4y)$ 2. $(2a - 7b)(3a - 2b)$ 3. $(7x - 3y)(2x - 4y)$
4. $(9a + 2b)(2a + 3b)$ 5. $(3a + 2b)(2a - 5b)$ 6. $(3x - 5y)(3x + 5y)$

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M: Exercise2, Angolan book, Grade10

POLINÓMIOS CAPÍTULO 1

Exercício 2.

Efectuar:

2.1. $2k^3z \times \frac{3}{4}k^3z^2$ 2.2. $(3ab + 5a^2b - 3ab) \times 4a^2b^3$
2.3. $(6e^3t - 12e^3t - 5e^3t) : (-10et - 12et)$ 2.4. $\frac{-5y^2z + 3y^2z}{4wy^3z^4} - \frac{2z^3}{3wy}$
2.5. $\frac{5y^2}{4x^2} - 3x^2y \left(\frac{2x^3y^4}{6x^3y^5} \right)$

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