



# Documentos de trabajo

## **Economía y Finanzas**

**N° 19-09**

2019

---

### **Colombian Economic Growth, Investment and-Saving: From 1954 to 2019 and Beyond**

Carlos Esteban Posada

# Colombian Economic Growth, Investment and Saving: From 1954 to 2019 and Beyond

Carlos Esteban Posada

Agosto 20, 2019

## Abstract

*This paper presents an interpretation of post-1953 Colombian economic growth and a discussion on future outcomes. The interpretation takes the form of a data playback guided by the decentralized equilibrium version of the Cass-Koopmans-Ramsey model. The role of technical change as a driver of GDP growth, household income and average wage is highlighted. The model leads to an unusual conclusion when it is applied to a small open economy like the Colombian one: the higher the rate of expected technical change, the higher the firms' investment rate and the lower the households' savings rate, remaining constant other things.*

**Key Words:** Colombian Economic Growth; Cass-Koopmans-Ramsey Model; Small Open Economy; Technical Change; Interest Rate; Investment; Households Savings.

**JEL classification system:** E13, E21, E22, F41, F43, O11, O41, O54.

---

<sup>1</sup> Professor. School of Economics and Finance, Universidad EAFIT, Colombia. E-mail Address: cposad25@eafit.edu.co.

## I. Introduction

Colombian economic growth performance after 1953 did not reach, in an international perspective, the category of "miracle" but it was significant: the average annual growth rates of per worker and per capita product exceeded (slightly) 2% between 1954 and 2018. Notwithstanding, in the last 4 years (and also in 1997-2002, and 2008-2009) there have been growth rates of these variables substantially less than 2% per year. This has led some analysts to consider it probable that the "golden" epoch of economic growth has already passed, and that Colombia have entered a new era in which we would normally expect a smaller increase, say 1%, for real per capita GDP growth. This document arose from concerns about what could be expected for the future in terms of economic growth and financing of capital formation.

In what follows (section II) I present estimates from a macroeconomic point of view. The estimates allow a description and interpretation of the economic growth process over the period 1954 - 2018. The basis of the estimates is the decentralized equilibrium version of the *CKR* (Cass-Koopmans-Ramsey) model<sup>2</sup>.

Such estimations rest on the calibration of the model for two years: 1954 and 2019. It is assumed that the relevant data for each of these years represents a steady state of the economy except for several not too big disruptions associated to transient shocks, and each state could differ from the other one by the fact of having changed some parameters.

Section III provides an examination of what may happen after 2019, and is done by constructing three scenarios related to saving and investment rates associated to some likely economic growth rates. These scenarios can be considered alternative steady states. In these section we discuss a possible problem: household's saving would be a bottleneck for a high economic growth rate. Section IV presents a summary and conclusions.

## II. The model

The theoretical model is simple. I used it because of its advantage to understand the evolution of the economy's productive capacity. It describes a small open economy producing output (= *GDP* measured without indirect taxes, that is, added gross value) with capital and labor force. The production function has the conventional properties. The representative agents have perfect foresight, they are optimizers, and the markets are continually cleared (I suppose the unemployment rate is an unexplained constant).

---

<sup>2</sup> See Wickens (2011).

### The Two Steady States

The next equations were used to estimate the steady state values (1954 and 2018/9) of the model's variables:

$$\begin{aligned}
 (1) \quad & Y = K^\alpha (AN)^{1-\alpha}; \quad 0 < \alpha < 1 \\
 (2) \quad & \frac{\partial Y}{\partial N} = w \\
 (3) \quad & \frac{\partial Y}{\partial K} (1 - \tau) - \delta = r \\
 (4) \quad & r^* + \rho = r
 \end{aligned}$$

Where  $Y, K, A, N, w, \tau, \delta, r, r^*, \rho$  stand for real GDP real (without indirect taxes), capital, a technological (and scale) parameter, labor force (employed), real wage (including all benefits to employees, and taxes to business on the payrolls), profit income marginal tax, capital depreciation rate, basic real interest rate, and the spread associate to default risk on the external debt.

Equation 1 is a conventional hypothesis about the origin of the aggregate value. Equations 2 and 3 are first order optimizing conditions. The last equation corresponds to the case of a small open economy but it supposes the spread does not depend on the external debt level. In another section I will discuss a problem related to this hypothesis.

From equations 1, 2 and 3 we obtain<sup>3</sup>:

$$\frac{K}{AN} = \left( \frac{\alpha}{\frac{\delta + r}{1 - \tau}} \right)^{\frac{1}{1-\alpha}}$$

$$\frac{K}{N} = \frac{\alpha \left( \frac{w}{1 - \alpha} \right)}{\left( \frac{r + \delta}{1 - \tau} \right)}$$

With these two additional equations, plus the ones presented before, we can estimate all values of the endogenous variables we are looking for.

Tables 1 and 2 show numerical values of the parameters and exogenous variables we need to estimate the endogenous variables; Table 3 shows these ones.

---

<sup>3</sup> According to equation 3, there are not adjustment costs related to capital stock changes. In the present analysis those ones do not matter.

|          | 1954   | 2019   | Notes, sources and comments  |
|----------|--------|--------|--|
| $\alpha$ | 0.425  | 0.425  | 0.42 between 1925 and 1981; after (1982-94): [0.36 ; 0.41] (GRECO, 2002, Chap. 3, Table 7, p.52).  |
| $\delta$ | 0.05   | 0.05   | Per year. Colombian National Accounts.   |
| $r^*$    | 0.0276 | 0.0276 | Per year. The median for USA between 1905 and 1997 was 2.76% (GRECO, 2002, Chap. 6, p.180).  |
| $\rho$   | 0.0224 | 0.0224 | Per year. The difference between the (medians) of Colombia and the USA interest rates was 3.3% per year for 1905-1997. (GRECO, 2002, Chap. 6, p. 183). |
| $\tau$   | 0.25   | 0.23   | Profit income marginal tax. Author's estimations   |

|  |                       |  |
|--|-----------------------|--|
| $N_{2019}^S$<br>= $N_{2019}^D$<br>= $N_{2019}$ | 21'621,505<br>persons | = Population x Working Age Rate x Participation Rate x (1-Unemployment Rate). DANE (Colombian Bureau of Statistics); 2018 Census, and Households Survey, April/2019. |
| $N_{1954}^S$<br>= $N_{1954}^D$<br>= $N_{1954}$ | 4'931,394<br>persons  | $= \frac{N_{19}}{(1 + g_{Ns})^{65}}$   |
| $g_{Ns}$                                       | 0.023 per year        | Author's estimations.  |

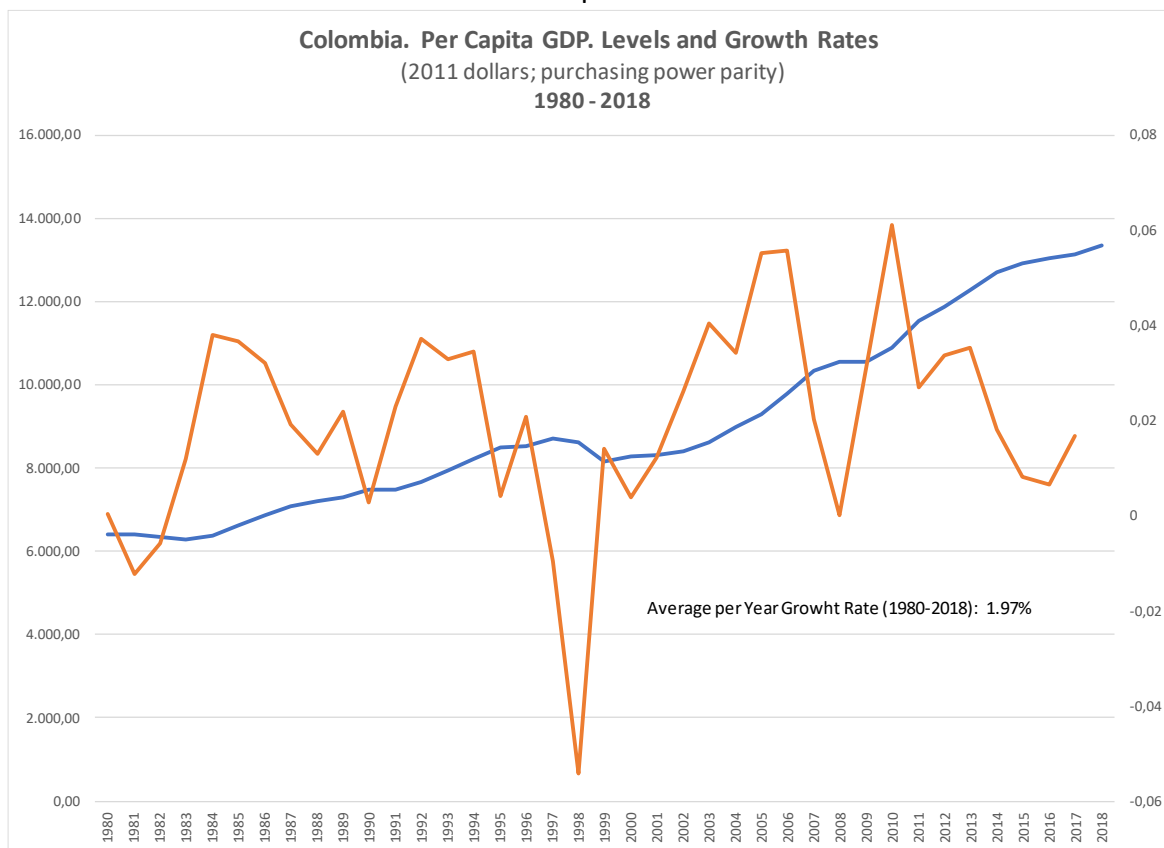
| Variable       | 1954           | 2018/9                  | Annual Growth Rate |
|----------------|----------------|-------------------------|--------------------|
| $r$            | 0.05           | 0.05                    |                    |
| $w/\text{año}$ | 1600.8         | 6031                    | $g_w = 2.06\%$     |
| $A$            | 1181.83        | 4366.74                 | $g_A = 2.03\%$     |
| $K/N$          | 8874           | 34324.18                | $g_{K/N} = 2.1\%$  |
| $Y$            | 13,729'001,560 | $2.2678 \times 10^{11}$ | $g_Y = 4.41\%$     |
| $K/AN$         | 7.5087         | 7.8604                  |                    |
| $Y/N$          | 2784           | 10488.67                | $g_{Y/N} = 2.06\%$ |
| $K/Y$          | 3188           | 3273                    |                    |

Notes: a) The industrial real wage per year growth rate (mean) was 2.4% between 1950 y 2004 according to Echavarría & Villamizar (2007, graph. 11, p. 200).  
b) 1 peso of 1954 equals to 3917.27 pesos of 2019 (April) if we measure the inflation rate by the increase of the Consumer Price Index (IPC). The annual inflation rate was 13.56% between July/1954 and April/2019. The (mean) nominal wage of (all the year) 2019 is 23'625000 pesos (estimated by the author).  
c)  $A$  is not an endogenous variable but its numerical value results from the calibration exercise.  
d) The official estimation (DANE) for the GDP (without indirect taxes) of 2018 measured in prices of 1954 is  $2,26797 \times 10^{11}$ . That means the estimation for  $Y$  in 2018/9 with the model has an error equal to -0,01%.

We can highlight the most important results from Table 3. The real wage (including benefits for employees but also payroll taxes), the capital/labor ratio and the product per worker all increased significantly (and at similar rates) because of the technical change, the long run engine of growth.

The graph 1 shows the path of the Colombian per capita GDP between 1980 and 2018, measured in dollars (2011 prices; purchasing power parity). The data plotted in this graph are (basically) coherent with the estimations from Table 3.

Graph 1.



Source: *World Economic Outlook*; IMF.

### III. Looking Forward: Future alternatives sceneries

In this section I try to answer this question: it will be enough domestic savings in Colombia to sustain a per capita GDP annual growth rate around 2% for the next ten years? To accomplish this task, I designed three alternative sceneries about economic growth, investment and savings using the *CKR* model.

The first scenery is called *basic* (Table 4). It reproduces some principal traits of the Colombian economic performance of the last five years as a simulation for the next ten

years. It looks like the scenery with the greater probability. One of the traits of this scenery is a relatively small technical change rate, only 1% per year.

| <b>Table 4. Basic scenery. Investment and Savings Rates (annual)</b> |        |  |         |  |       |
|--|--------|--|---------|--|-------|
| <b>2020-2029</b>   |        |  |         |  |       |
| <b>Technical Change Rate: 1% (per year)</b>                          |        |  |         |  |       |
| <b>Parameters</b>  |        | <b>Exogenous Variables</b>                     |         | <b>Endogenous Variables</b>                                      |       |
| $\frac{GDP}{GDP_c}$  | 1.1014 | $r$  | 0.05    | $\frac{\text{Gross Investment}}{GDP}$                            | 0.219 |
| $\alpha$   | 0.425  | $g_A$  | 0.01    | $\frac{\text{Households Savings}}{\text{Households Net Income}}$ | 0.085 |
| $\delta$   | 0.05   | $g_{N^s}$                                      | 0.0135  | $\frac{\text{Households Saving}}{GDP}$                           | 0.063 |
| $\tau$   | 0.23   | $\frac{\text{Households Net Income}}{GDP_c}$   | 0.813   | $\frac{\text{National Saving}}{GDP}$                             | 0.167 |
|  |        | $\frac{\text{Public Saving}}{GDP}$             | -0.0063 | $\frac{\text{External Net Borrowing}}{GDP}$                      | 0.024 |
|  |        | $\frac{\text{Firms's Gross Savings}}{GDP}$     | 0.1105  |  |       |
|  |        | $\frac{\text{Direct Foreign Investment}}{GDP}$ | 0.028   |  |       |

Notes and Sources:  
A)  $GDP_c$  is the gross added value or GDP minus indirect taxes (minus subventions) charged on producers.  $Y$  is the proxy for  $GDP_c$ . B) "Firms" is (in this paper) the equivalent for the so called "Societies" in Colombian National Accounts. C) Household Net Income is my estimation based on Colombian National Accounts (current prices). D) Data source on public and firms 's savings: Colombian National Accounts (current prices) year 2017. D) The figure for Direct Foreign Investment/GDP is the average of estimations for the last seven years coming from the Colombian central bank. E) The figures for gross investment/GDP and households 's savings are estimated according to the equations and Tables A – 1 y A- 2 (see Annex). G) National saving = households saving + public saving + firms 's savings. H) External Net Borrowing + Direct Foreign Investment = Gross Investment – National saving.

The other sceneries (1 and 2) depict outcomes with (it seems) smaller probabilities. Notwithstanding, those ones are interesting to economic policy discussions. Scenery 1 (Table 5) is based on the hypothesis of a relatively high technical change rate: 2% per year. Therefore, the investment rate (gross investment/GDP) is higher than the estimate we did it for the basic scenery. What about the savings rates? I suppose no changes concerning public saving and firms 's savings. But we can predict a lower household saving rate as a consequence of the household prediction of a higher technical change rate (see Annex for an explanation about it). So, the national saving rate is smaller, and, because of that, the net external borrowing is higher (by far) than the corresponding estimates done for the basic scenery.

| <b>Table 5. Scenery 1</b>   |       |
|---|-------|
| <b>Technical Change Rate: 2%</b>  |       |
| Endogenous Variables  |       |
| $\frac{\text{Gross Investment}}{\text{GDP}}$  | 0.245 |
| $\frac{\text{Households Savings}}{\text{Households Net Income}}$                    | 0.023 |
| $\frac{\text{Households Saving}}{\text{GDP}}$                                       | 0.017 |
| $\frac{\text{National Saving}}{\text{GDP}}$   | 0.121 |
| $\frac{\text{External Net Borrowing}}{\text{GDP}}$                                  | 0.096 |
| The parameters and the other exogenous variables are the same for the basic scenery |       |

Scenery 2 (Table 6) shows the consequences of a possible reaction of the foreign lenders to expectations about a higher external debt: a bigger interest rate charged to Colombian borrowers<sup>4</sup>, a smaller investment rate, and a larger household saving rate.

| <b>Table 6. Scenery 2</b>   |       |
|---|-------|
| <b>Technical change rate: 2%; Real interest rate: 6.5%</b>                          |       |
| Endogenous Variables  |       |
| $\frac{\text{Gross Investment}}{\text{GDP}}$  | 0.213 |
| $\frac{\text{Households Savings}}{\text{Households Net Income}}$                    | 0.085 |
| $\frac{\text{Households Saving}}{\text{GDP}}$                                       | 0.063 |
| $\frac{\text{National Saving}}{\text{GDP}}$   | 0.167 |
| $\frac{\text{External Net Borrowing}}{\text{GDP}}$                                  | 0.018 |
| The parameters and the other exogenous variables are the same for the basic scenery |       |

<sup>4</sup> This possibility corresponds to the case called “external debt-elastic interest rate” (Uribe and Schmitt-Grohé, 2017, p. 76).



And we could imagine other sceneries with a higher interest rate as a consequence of the lagged reaction of the lenders to biggest external debts/GDP ratios<sup>5</sup>. In cases like this one the economy would risk to gain instability associated to the greater intensity of the response of the interest rate only because of the lags<sup>6</sup>.

#### **IV Summary, conclusions and a final note on how to sustain a high economic growth rate**

This paper presents an interpretation of post-1953 Colombian economic growth and a discussion on future outcomes. The role of technical change as a driver of GDP growth, household income and average wage is highlighted. The model used to this task leads to an unusual conclusion when it is applied to a small open economy like the Colombian one: the higher the rate of expected technical change, the higher the firms' investment rate and the lower the households' savings rate. In the small open economy case, because of the imperfect or lagged statistical data, we can expect the consequences of the eventual gap between investment and saving could be observed in two different moments: the first one is when the gap between investment and saving is growing up; the second one is when the interest rate is pulling up as a (lagged) sequel of the growing external debt<sup>7</sup>.

So, for a small open economy, like the Colombian one, an increase of the technical change rate is no for free; it need to accept (with a lag) a greater interest rate, and, sooner or later, a smaller investment rate to pay for the additional technical change.

But, of course, if the technical change was tigh related with foreign direct investment (like it was in the China case from 1980 to 2005<sup>8</sup>), we would no see the negative effect before mentioned. In other words, an economic growth strategy without a narrow relation between technical change and foreign direct investment risks to fail because of the problems associated with growing external indebtedness.

---

<sup>5</sup> This ratio was 41% in march/2019 according to Banco de la República (Colombian central bank).

<sup>6</sup> The existence of the reactions lags could be a driver of economic cycles (as the "multiplier-accelerator" Samuleson's model shows it; see: Sargent, 1987, pp. 189-90).

<sup>7</sup> In the big economy case the expectation of a higher technical change drives up the interest rate from the beginings, so the more likely outcome will be not an observed gap between investment and saving.

<sup>8</sup> Cai *et al.* (2011).

## Annex. Investment Rate, and the Household Saving Rate

### 1. Steady State Investment Rate

From equations 1 and 3 (Section 1) we deduce  $K/Y$  for the steady state:

$$\frac{K}{Y} = \frac{\alpha(1-\tau)}{\delta+r}$$

But investment is:

$$I_t = K_{t+1} - K_t + \delta K_t$$

Henceforth:

$$\frac{I_t}{K_t} = \frac{K_{t+1}}{K_t} - 1 + \delta$$

So (in the steady state):

$$\frac{I}{Y} = \left(\frac{I}{K}\right) \left(\frac{K}{Y}\right) = [(1 + g_{ANS}) - 1 + \delta] \left[\frac{\alpha(1-\tau)}{\delta+r}\right] = (g_{ANS} + \delta) \left[\frac{\alpha(1-\tau)}{\delta+r}\right]$$

Where  $g_{ANS}$  stands for the sum of (labor augmenting) technical change plus the growth labor force rate.

In consequence, the so called steady state investment rate (Tables 4, 5 and 6) is:

$$\frac{I}{GDP} = \frac{I}{Y} \cdot \frac{Y}{GDP}$$

Where  $Y$  is domestic aggregate value and  $GDP$  is the former plus indirect taxes (minus subventions) to producers.

### 2. The Saving Rate

*Saving rate* is the name assigned to the average of all annual saving rates of the representative household head during his (her) life cycle. The saving rate depends positively of 2 factors: 1) the net assets target for the (final moment of the) last year of the life cycle, and 2) the interest rate; and it depends negatively of 4 factors; 1) the initial net assets (at the beginning of the first period to fix the consumption plan), 2) the subjective discount rate of the future utility,  $\theta$ , 3) the coefficient of relative risk aversion (the inverse of the intertemporal consumption substitution elasticity),  $\sigma$ , and 4) the expected nonfinancial income growth rate<sup>9</sup>.

Those affirmations are based upon the results of this perfect foresight program (used in numerical exercises) executed by the representative agent<sup>10</sup>:

$$\text{Maximise } \omega = \sum_{s=0}^T \left(\frac{1}{1+\theta}\right)^s \frac{C_s^{1-\sigma} - 1}{1-\sigma}, \quad \text{subject to:}$$

<sup>9</sup> See Solow (1970), and Carroll *et. al.* (2019) on the importance of net assets target, and net assets at the beginning of the program.

<sup>10</sup> See Carroll *et al.* (2018) for a defense of this approach (with shocks and expectations).

$$P_0(1+r) + \sum_{s=0}^T \left(\frac{1}{1+r}\right)^s y_s(1-t) = \sum_{s=0}^T \left(\frac{1}{1+r}\right)^s C_s + \left(\frac{1}{1+r}\right)^T P_T$$

And to the budget constraint for each period:

$$P_s(1+r) + y_s(1-t) - C_s = P_{s+1}$$

Where:

$$Saving_s \equiv P_{s+1} - P_s; \quad s \text{ period Saving rate} = \frac{Saving_s}{y_s(1-t) + rP_s};$$

$r$  = real interest rate;  $t$  = nonfinancial income tax rate;  $P_s$   
= Net assets at the beginning of the  $s$  period;

$P_T$ : Net assets target for the end of the final period (exogenous variable).

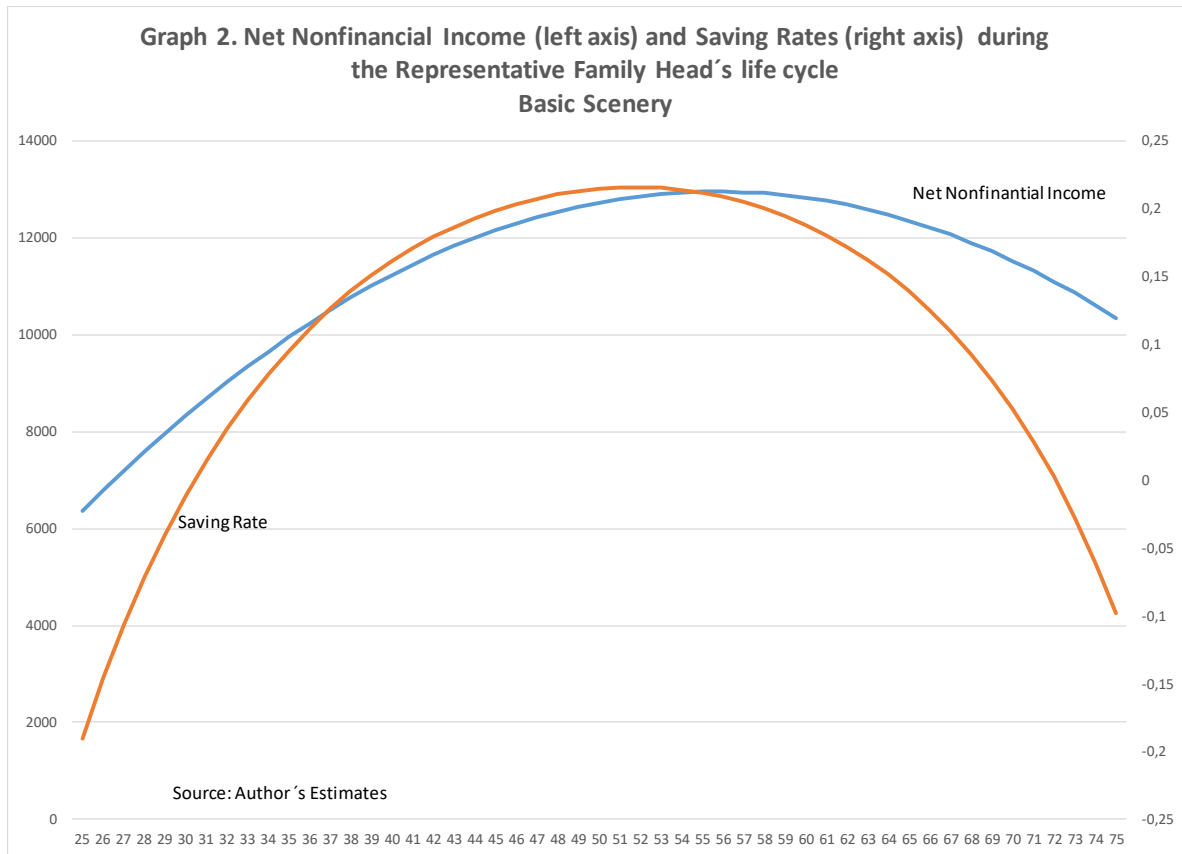
**Table A-1. The Principal hypothesis for the numerical simulation of the Saving Rate Years 2019 and beyond**

The case of the representative agent. His (her) Household utility,  $U$ , depends on consumption,  $c$ :  
 $U = \frac{c^{1-\sigma}-1}{1-\sigma}$ ; life cycle years with incomes (ages from 25 to 75 years old): 51.  
Household net income in the first year = 61% \* *per capita Y (for 2019)*: see Table 3  
The average of the nonfinancial net income growth rate = technical change growth rate:  $g_A$ . In the basic sceneries (Table A- 2; and Graph 2) is 1% per year (in sceneries 1 and 2 is 2%).  
Net assets at the beginning of the life cycle = net income for the first year.  
Net assets at the end of the final year (the target) = 8.7 times the last year income.  
Real interest rate: 5% per year;  $\theta = 2\%$  (Uribe and Schmitt-Grohé, 2017);  $\sigma = 2$  (Uribe and Schmitt-Grohé, 2017).

**Table A-2. Family Head Age Ranks and Household Saving 2019 - Basic sceneries**

| Age ranks (years old)<br>(1) | Mean Saving Rate<br>(In each rank)<br>(2) | Family Head Participation Rates (In each rank)<br>(3) | Saving Rates (Weight averages)<br>(4) = (2) x (3) |
|------------------------------|---|---|---|
| 25-35                        | -0,0252                                   | 0,451   | -0,01134  |
| 36-45                        | 0,1657                                    | 0,233   | 0,03861   |
| 46-55                        | 0,2129                                    | 0,184   | 0,03918   |
| 56-65                        | 0,1627                                    | 0,116   | 0,01887   |
| 65-75                        | 0,0083                                    | 0,016   | 0,0013  |
| Total                        |   | 1   | 0,08545   |

Notes: a) The source for families head participation rates is: Arango and Cardona, 2019); b) the saving rate is the sum of the weight averages rates.



## References

- Arango, L. E., and L. M. Cardona. 2019. "Tarjetas de crédito en personas de ingresos medios y bajos en Colombia: ¿qué determina su uso?", mimeo, Banco de la República.
- Cai, F., H. Cheng, L. Xu, C. K. Leung. 2011. "Economic Growth and FDI in China". *International Business & Economics Research Journal*, Vol. 3, No. 5.
- Carroll, C. D.; J. Slacalek; M. Sommer. 2019. "Dissecting saving dynamics: measuring wealth, precautionary, and credit effects". Working Paper 26131. NBER.
- Carroll, C. D.; E. Crawley; J. Slacalek; K. Tokuoka; M. N. White. 2018. "Sticky expectations and consumption dynamics"; Working Paper 24377. NBER.
- GRECO (Grupo de Estudios del Crecimiento Económico). 2002. *El crecimiento económico colombiano en el siglo XX*. Banco de la República – Fondo de Cultura Económica, Bogotá.
- Echavarría, J. J., and M. Villamizar. 2007. "El proceso colombiano de desindustrialización" (chapter 4 of *Economía colombiana del siglo XX*; J. Robinson y M. Urrutia, editores. Fondo de Cultura Económica - Banco de la República, Bogotá.
- Sargent, T. *Macroeconomic Theory (Second Edition)*. Academic Press, Orlando (Fl.), 1987.
- Solow, R. 1970. *Growth Theory. An Exposition*. Oxford University Press.
- Uribe, M., and S. Schmitt-Grohé. 2017. *Open economy macroeconomics*. Princeton University Press, Princeton, N. J.
- Wickens, M. *Macroeconomic Theory (Second Edition)*. 2011. Princeton University Press, Princeton, N. J.