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Noether-Wald Charges in Critical Gravity

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Abstract. Critical Gravity theory is defined by a particular combination of quadratic couplings in the curvature added on top of 4D Einstein-Hilbert action with negative cosmological constant. As the Lagrangian is given by a Weyl-squared term, the asymptotic form of the curvature is not modified. The coupling of the $Weyl^2$ term is such the massive scalar mode is eliminated and the massive spin-2 mode become massless, rendering the theory consistent around the critical point.

In the present work, we construct the Noether-Wald charges for the action of Critical Gravity. Such construction makes manifest a defining property of this theory: both the energy and entropy for Einstein black holes vanish identically.

1. Introduction

Quadratic-curvature gravity can be seen as a correction to Einstein-Hilbert theory, whose action is given by the general expression

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - 2\Lambda + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2\right) , \tag{1}$$

where α and β are arbitrary couplings, and $\Lambda < 0$ is the cosmological constant. The Riemann-squared term is absent because it can be always traded off by the Gauss-Bonnet (GB) invariant plus the other curvature-squared terms. The GB invariant does not contribute to the bulk dynamics of the theory. Generically speaking, this class of actions produces higher-derivative field equations, which describe a massless spin-2 graviton, a massive spin-2 field and a massive scalar.

For an arbitrary set of couplings, this kind of theories suffers from the appearance of ghosts. That problem can be solved by flipping the sign of the Einstein-Hilbert term. However, this change would produce a negative value for the mass of a Schwarzschild-AdS black hole. Thus, while the energy of the perturbations around a background are negative, the mass of a black hole is positive, or viceversa, what it is clearly inconsistent.

When the couplings take the particular values $\alpha = -3\beta$ and $\beta = -\frac{1}{2\Lambda}$, the massive mode scalar is eliminated and the massive spin-2 mode becomes massless, giving rise to a four-dimensional gravity theory which is free of the inconsistencies mentioned above [1].

Indeed, in [1], it was shown that, using the Ostrogradsky method for higher-order Lagrangians, the on-shell energy of the massive mode in the critical value of the couplings vanishes.

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On the other hand, one can obtain the Abbott-Deser-Tekin mass for a Schwarzschild-AdS black hole [2, 3], which is given by

$$M = m\left(1 + 2\Lambda(\alpha + 4\beta)\right) \tag{2}$$

where m is the usual mass parameter in the solution.
It is easy to see that in (2), for the critical condition mentioned above, the mass for Schwarzschild-AdS black hole becomes zero.
The main aim of this note is to arrive at the same conclusion from the point of view of Noether-Wald charges [4].

2. Noether-Wald Charges

For an arbitrary theory of gravity in which the Lagrangian density depends only on the metric and the curvature $L(g_{\mu\nu}, R_{\mu\nu\alpha\beta})$, one can construct a conserved current for a set of Killing vectors $\{\xi^\mu\}$ which has the form [5]

$$\sqrt{-g}J^\mu = \Theta^\mu\left(\delta_\xi g\right) + \Theta^\mu\left(\delta_\xi \Gamma\right) + L\xi^\mu \tag{3}$$

The first term in (3) is zero due to the fact that $\delta_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\nu = 0$ which corresponds to the Killing equation.

The rest of the current, for a generic gravity theory, is given by

$$\sqrt{-g}J^\mu = 2E^{\mu\nu}_{\alpha\beta}\left(\nabla_\nu \nabla^\alpha \xi^\beta + R^{\alpha\beta}_{\nu\sigma}\xi^\sigma\right) + \xi^\mu L \tag{4}$$

where $E^{\mu\nu}_{\alpha\beta}$ is the functional derivative with respect to the Riemann tensor $R^{\mu\nu}_{\alpha\beta}$

$$E^{\mu\nu}_{\alpha\beta} = \frac{\partial L}{\partial R^{\alpha\beta}_{\mu\nu}}\,. \tag{5}$$

The last two terms on the right side of (4) are proportional to the equation of motion and, as a consequence, they vanish on-shell.
The first term on the right side of (4) satisfies the Bianchi identity, what implies that the conserved current can be written as

$$\sqrt{-g}J^\mu = 2\nabla_\nu\left(E^{\mu\nu}_{\alpha\beta}\nabla^\alpha \xi^\beta\right) \tag{6}$$

We know that, whenever the current J^μ can be written as $J^\mu = \nabla_\nu Q^{\mu\nu}$, the conserved charge is expressed as an integral on the two-dimensional surface Σ

$$Q = \int_\Sigma d\Sigma_{\mu\nu} Q^{\mu\nu} \tag{7}$$

where $d\Sigma_{\mu\nu} = d^2x\sqrt{\sigma}u_\nu n_\mu$. Here, σ is the determinant of the two-dimensional surface metric, u_ν corresponds to a unit timelike vector, normal to the two-dimensional surface Σ and n_μ is a unit spacelike vector, normal to the boundary ∂M.
Finally the conserved charge is written as

$$Q[\xi] = 2\int_\Sigma d^2x\sqrt{\sigma}u_\nu n_\mu E^{\mu\nu}_{\alpha\beta}\nabla^\alpha\xi^\beta. \tag{8}$$

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3. Conserved charges in Critical Gravity

The action for Critical Gravity is given by ($\Lambda = -3/\ell^2$ in terms of the AdS radius)

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[(R+\frac{6}{\ell^2}) - \frac{\ell^2}{2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)\right] \tag{9}$$

An alternative form for the action of Critical Gravity considers the difference between $Weyl^2$ and the GB invariant E_4 [6], as the latter term does not contribute to the field equations

$$I = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\left(R+\frac{6}{\ell^2}\right) + \frac{\ell^2}{64\pi G}\left(E_4 - W^{\mu\nu\alpha\beta}W_{\alpha\beta\mu\nu}\right)\right]. \tag{10}$$

Using the Noether-Wald formula for the current (6), we can split the action in two parts, I_{ren} which correspond to the action of Einstein-Hilbert plus GB term and I_{CG} which correspond to the action of Conformal Gravity.

For the first part, the functional derivative with respect to the Riemann tensor of the Lagrangian in I_{ren} produces

$$E^{\mu\nu}_{\alpha\beta} = \sqrt{-g}\frac{\ell^2}{128\pi G}\delta^{[\mu\nu\gamma\lambda]}_{[\alpha\beta\rho\delta]}\left(R^{\rho\delta}_{\gamma\lambda} + \frac{1}{\ell^2}\delta^{[\rho\delta]}_{[\gamma\lambda]}\right), \tag{11}$$

whereas, for the Conformal Gravity part L_{CG}, we get

$$\tilde{E}^{\mu\nu}_{\alpha\beta} = -\sqrt{-g}\frac{\ell^2}{128\pi G}\delta^{[\mu\nu\gamma\lambda]}_{[\alpha\beta\rho\delta]}W^{\rho\delta}_{\gamma\lambda} \tag{12}$$

Using the Noether-Wald formula (8), the total charge for the theory

$$Q = \frac{\ell^2}{64\pi G}\int d^2x\sqrt{\sigma}u_\nu n_\mu \delta^{[\mu\nu\gamma\lambda]}_{[\alpha\beta\rho\delta]}\nabla^\alpha\xi^\beta\left[\left(R^{\rho\delta}_{\gamma\lambda} + \frac{1}{\ell^2}\delta^{[\rho\delta]}_{[\gamma\lambda]} +\right) - W^{\rho\delta}_{\gamma\lambda}\right]. \tag{13}$$

By definition, the Weyl tensor is

$$W^{\mu\nu}_{\alpha\beta} = R^{\mu\nu}_{\alpha\beta} - \frac{1}{2}R^{[\mu}_{[\alpha}\delta^{\nu]}_{\beta]} + \frac{1}{6}R\delta^{[\mu\nu]}_{[\alpha\beta]}. \tag{14}$$

For Einstein spaces, $R_{\mu\nu} = -3/\ell^2 g_{\mu\nu}$, the Weyl tensor adopts the particular form

$$W^{\rho\delta}_{\gamma\lambda} = R^{\rho\delta}_{\gamma\lambda} + \frac{1}{\ell^2}\delta^{[\rho\delta]}_{[\gamma\lambda]}, \tag{15}$$

where the r.h.s, is referred to as AdS curvature.

Using the above fact, the conserved quantity in Critical Gravity is identically zero for Einstein spaces.

4. Conclusions

In this work, we have provided a more explicit proof of the vanishing mass for Einstein black holes in Critical Gravity. Because the black hole entropy can be obtained as the Noether-Wald charge evaluated at the horizon, it is evident that the entropy for this class of solution is zero, as well. The departure from the Einstein condition in a more generic set of solutions for Critical Gravity should link the energy definition to a higher-derivative tensor, e.g., the Bach tensor [7].

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