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Spiky ice and penitente tilting

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Abstract. Under certain conditions, at high altitude, the surface of snow develops spike-like structures known as *penitentes*. This is a rather counterintuitive phenomenon, which is a consequence of surface sublimation at a given point as a result of the incidence of light scattered by the surrounding region.

Following the existing literature, we model the time evolution of the phenomenon described above as a 1D diffusion equation with a non-local source term, as it represents the light coming from all the line of sight defined for a point of the curve. For small initial perturbations in the surface, the system undergoes a thermodynamic instability which triggers the formation of spikes.

For sunlight coming in at a given angle, numerical simulations account for a feature observed in the real system: penitentes get tilted in the direction of the sunlight.

1. Introduction

Structure formation in snow is controlled by many factors: surface reflectance, wind, daily melt-freeze cycle on the surface, incidence angle of the sunlight, etc. This paper models penitente formation in snow fields as a purely radiation-driven phenomenon. The key condition for the differential ablation is sublimation, what requires that the dew point is lower than the freezing point [1]. On the contrary, if the snow melts, the water weighs down the surface and smooths out any spiky features.

2. Toy model for penitente formation

The ablation at any point is proportional to the heat absorbed due to either direct or scattered sunlight. Part of the heat is lost by convection to the air, while another fraction is transmitted to the neighbouring points. We will restrict ourselves to heat transmission within the surface which is governed by the diffusion equation

$$\frac{\partial U}{\partial t} = \Upsilon(x) + D\frac{\partial^2 U}{\partial x^2} \tag{1}$$

$$\frac{\partial h}{\partial t} = -K\frac{\partial U}{\partial t} \tag{2}$$

where $U(x,t)$ is the thermal energy at a point x for the time t, $h(x,t)$ is the profile height, $D\frac{\partial^2 U}{\partial x^2}$ is the diffusion term and $\Upsilon(x,t)$ is the total energy received at a given point.

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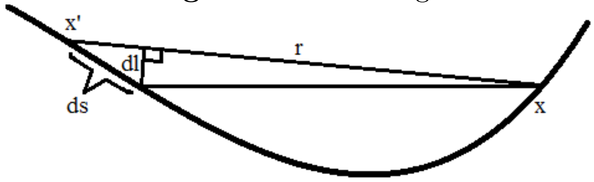
2.1. Geometry of the model
The fact that the snow looks white is a consequence of a diffuse reflection of incident sunlight. The light is partially reflected beneath the surface such that all colors are scattered roughly equally in all directions. Every single point of the curve acts as a source of equal intensity per unit of length, which is spread out over the tangent line to that point. This assumption simplifies the discussion, as surface geometry is the only input required to compute the amount of absorbed energy on a particular point of the curve.

The contribution of an infinitesimal length ds in the emitting surface (at a point x') to the absorbed energy on another point x is proportional to the sustained angle $d\theta$ (See figure).

Figure 1. Line of sight.



Thus, the heat source at any point $\Delta(x)$ comes as the sum over all infinitesimal contributions coming from the *line of sight* (LoS)

$$\Delta(x) = \frac{1}{\pi}\int_{LoS} d\theta = \frac{1}{\pi}\int_{LoS} \frac{\|\vec{r}\times d\vec{s}\|}{r^2}\,. \tag{3}$$

The arc element $d\theta$ in terms of x and $h(x)$, from the above figure, is simply given by $d\theta = \frac{dl}{r}$. At the same time, dl is the height of the parallelogram formed by $\vec{r}$ and $d\vec{s}$. Plugging this in eq.(3)

$$\Delta(x) = \frac{1}{\pi}\int_{LoS} \frac{xh'(x) - h(x)}{x^2 + h(x)^2}dx \tag{4}$$

Here $\Delta(x)$ is a portion of the angle π that measures the amount of light scattered within the surface. The total energy absorbed is then

$$\Upsilon(x) = I\alpha\,(1 + (1-\alpha)\Delta(x)) \tag{5}$$

Where α is the portion of energy absorbed by the surface in a single bounce and I is the energy of the beam. This is a non-local expression that acts as a source of the diffusion eq. (1).

2.2. Line of sight
For given point $P = (x, h(x))$ on the curve, the line of sight in the neighboring peaks is defined as the interval $x' \in [a, b]$ such that $Q = (x', h(x'))$ can be connected by a straight line to P without intersecting the surface. The extreme points in $x' = a, b$ one can see from P are given by the condition that the slope of the connecting segment coincides with the derivative there.

The line of sight must include portions of more distant peaks to P, if they are tall enough. The numerical resolution of the PDE (1) must take this issue into account in the non-local source term Υ by summing over the visible points of all peaks around.

2.3. Shady zones

At intermediate latitudes, the sunlight does not fall in a vertical line. The geometric description below is a model to account for the tilting of the icy spikes in the direction of the sun, which can be observed in penitente fields.

If the light comes in at a given angle ϕ, we suppress the contribution to the scattered light from the portion of the surface which is under the shadow of a peak.

The last lightened point, namely $x = c$, must satisfy

$$h'(c) = \tan(\phi) \tag{6}$$

and such that the curve is convex at that point, i.e., in other words $h''(c) < 0$.
Thus, the shady zone is given as an interval $[c, d]$, where the other end $x = d$ obeys

$$h(d) = \tan(\phi)(d - c) + h(c)\,. \tag{7}$$

3. Results

The set of partial differential equations is solved numerically, using a code in C++ language that is developed to show the time evolution of the system. For an initial surface that contains micro-features, the surface will develop spikes as a result of an instability of the PDEs. The program uses forward difference at t and second-order central difference for the position x corresponding to a finite difference method.

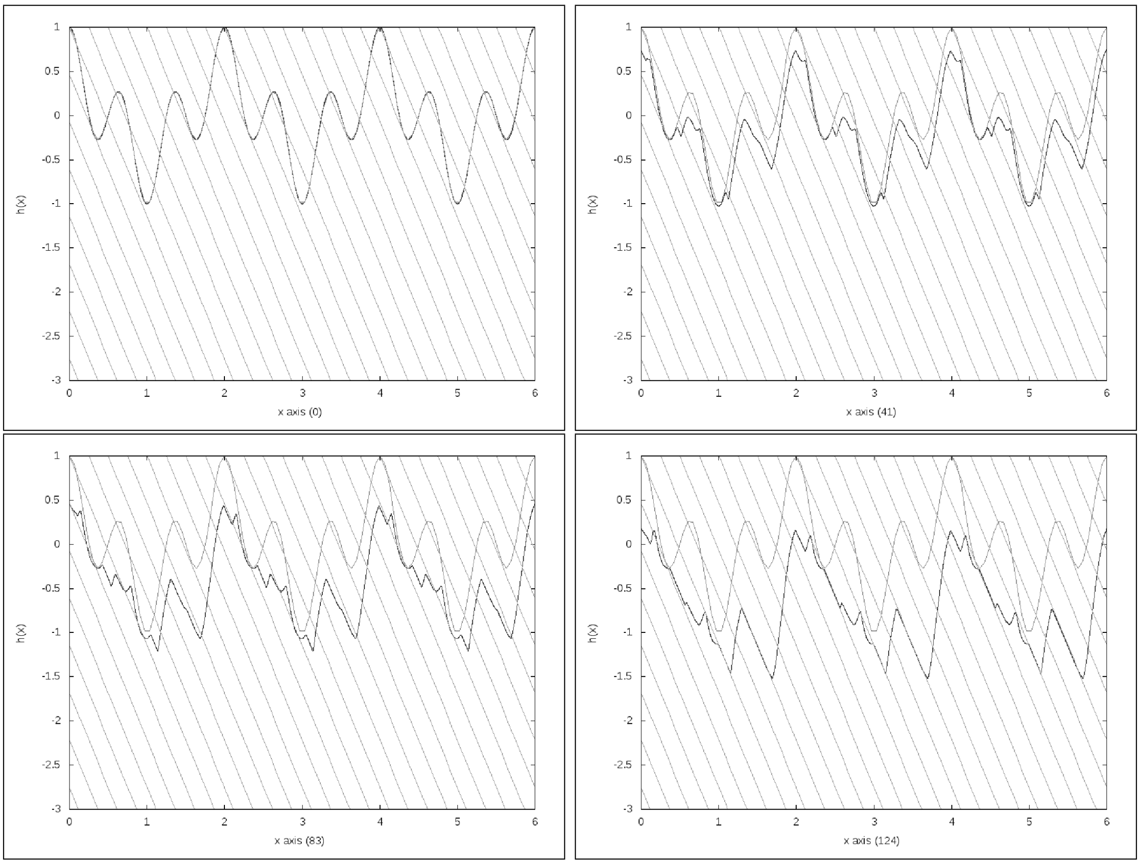


Figure 2. Time evolution

In the images, the time evolution of the penitentes is depicted. The parameters $D = 1$ and $K = 1$ in eq.(1) and eq.(2), respectively, have been taken. We consider $\alpha = 0.5$ in eq.(5), as

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well. The transversal lines correspond to the incident light rays. Initial and final shapes of the surface profile are given as a reference. Notice that the zones which are lower undergo more ablation than the upper ones. It is clear that, due to the presence of shady zones in the line of sight, the structures tilt in the direction of the coming sunlight. We will provide an extended version reporting on this issue in a forthcoming publication [4].

4. Acknowledgments

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