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Conserved quantities for a charged rotating black holes in 5D Einstein-Maxwell-Chern-Simons theory

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Abstract. In this work, we compute the conserved quantities of a charged rotating black hole which appears as the solution of Einstein-Maxwell action in five dimensions coupled to a Chern-Simons term for $U(1)$ field. The addition of the Chern-Simons term will modify the Maxwell equations and the definition of charge but not the Einstein field equations. Upon the addition of suitable boundary terms for the pure gravity sector of the theory, which depend on the extrinsic and intrinsic curvatures (Kounterterms), we obtain the correct conserved quantities of the solution.

1. Introduction

A black hole is a solution of General Relativity with nontrivial causal structure, which has been focus of study in fundamental physics in the last decades. It is expected that such objects can give better insight on the understanding of Quantum Gravity. It has been recently found in Ref.[3] the metric of Kerr-AdS black hole for an arbitrary dimension D , with maximal number of rotation parameters $N = [(D - 1)/2]$. Electrically charged generalizations of rotating AdS black holes can be obtained in the form of Kerr-Newman solution in $D = 4$. No higher-dimensional counterparts have been reported in the literature.

On the other hand, non-extremal rotating charged black holes appear as solutions of the bosonic sector of gauged supergravity in five dimensions, which contains a Chern-Simons term for $U(1)$ on top of the Einstein-Maxwell Lagrangian. This is the closest one may be to the concept of Kerr-Newman solution in $D > 4$. In this case, the action adopts the form

$$I = \frac{1}{16\pi G_5} \int_{\mathcal{M}} d^5x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) - \frac{\lambda}{12\sqrt{3}} \epsilon^{\mu\nu\alpha\beta\gamma} A_\mu F_{\nu\alpha} F_{\beta\gamma} + c_4 \int_{\partial\mathcal{M}} d^4x B_4 \quad (1)$$

where the cosmological constant takes the value $\Lambda = -6/\ell^2$, A_μ is the vector potential, $F_{\mu\nu}$ is the field strength and B_4 its the boundary term that regularizes both the Euclidean action and the conserved charges.

As the Chern-Simons term does not depend on the metric, it does not contribute to the Einstein equation, but it does modify the Maxwell equation

$$\nabla_\mu F^{\mu\nu} + \frac{\lambda}{4\sqrt{3}} \epsilon^{\mu\nu\alpha\beta\gamma} F_{\mu\alpha} F_{\beta\gamma} = 0. \quad (2)$$



2. Black Hole Solution

The metric for this black hole solution, with two rotation parameters and electric charge, can be written in BoyerLindquist coordinates $x^\mu = (t, r, \theta, \phi, \psi)$ [1]

$$ds^2 = -\frac{\Delta_\theta \left[\left(1 + \frac{r^2}{\ell^2}\right) \rho^2 dt + 2q\nu \right] dt}{\Xi_a \Xi_b \rho^2} + \frac{2q\nu\omega}{\rho^2} + \frac{f}{\rho^4} \left(\frac{\Delta_\theta dt}{\Xi_a \Xi_b} - \omega \right)^2 + \frac{\rho^2 dr^2}{\Delta_r} + \frac{\rho^2 d\theta^2}{\Delta_\theta} + \frac{(r^2 + a^2)}{\Xi_a} \sin^2 \theta d\phi^2 + \frac{(r^2 + b^2)}{\Xi_b} \cos^2 \theta d\psi^2, \quad (3)$$

with

$$\begin{aligned} \nu &= b \sin^2 \theta d\phi + a \cos^2 \theta d\psi, \\ \omega &= a \sin^2 \theta \frac{d\phi}{\Xi_a} + b \cos^2 \theta \frac{d\psi}{\Xi_b}, \\ \Delta_\theta &= 1 - \frac{a^2}{\ell^2} \cos^2 \theta - \frac{b^2}{\ell^2} \sin^2 \theta, \\ \Delta_r &= \frac{(r^2 + a^2)(r^2 + b^2)(1 + r^2/\ell^2) + q^2 + 2qab}{r^2} - 2m, \\ \rho^2 &= r^2 + a \cos^2 \theta + b^2 \sin^2 \theta, \\ \Xi_a &= 1 - \frac{a^2}{\ell^2}, \quad \Xi_b = 1 - \frac{b^2}{\ell^2}, \\ f &= 2m\rho^2 - q^2 + 2abq\rho^2/\ell^2, \end{aligned} \quad (4)$$

and the gauge potential

$$A = \frac{q\sqrt{3}}{\rho^2} \left(\frac{\Delta_\theta dt}{\Xi_a \Xi_b} - \omega \right). \quad (5)$$

The conserved quantities of this solution were given in Ref.[1], as obtained from the integration of the First Law of Thermodynamics, and not as surface integrals at infinity. Here, following the Kounterterms method, we intend to provide an alternative derivation of the existing results.

3. Kounterterms

A prescription for the boundary terms required by the regularization of AdS action in odd spacetime dimensions was given in Ref.[2]. The boundary term depends on the intrinsic and extrinsic curvatures and a closed expression exists in any dimension in the form of a polynomial with coefficients given by parametric integrations.

We consider a radial foliation of the spacetime, in such a way that the general form of the line element is written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = N^2(r) dr^2 + h_{ij}(r, x) dx^i dx^j \quad (6)$$

where h_{ij} is the induced metric, such that (i, j) denote boundary indices.

In this coordinate frame, the extrinsic curvature adopts the simple form

$$K_{ij} = -\frac{1}{2N} \partial_r h_{ij}, \quad (7)$$

due to the fact that *shift* functions in the radial ADM-like metric (6) are zero.

In five dimensions, the boundary term needed by the gravitational sector of the theory is

$$B_4 = \sqrt{-h} \delta_{[j_1 j_2 j_3]}^{[i_1 i_2 i_3]} K_{i_1}^{j_1} \left(\mathcal{R}(h)_{i_2 i_3}^{j_2 j_3} - K_{i_2}^{j_2} K_{i_3}^{j_3} + \frac{1}{3\ell^2} \delta_{i_2}^{j_2} \delta_{i_3}^{j_3} \right), \quad (8)$$

with a fixed coupling constant

$$c_4 = \frac{\ell^2}{128\pi G}. \quad (9)$$

As it is a boundary term it does not modify the bulk dynamics, but it does modify the conserved charges, leading to a finite conserved quantity. For the electromagnetic part, we are assuming that the potential vector A_μ is kept fixed at the boundary.

A splitting of the charges of the form

$$Q[\xi] = \int_{\Sigma} d^{d-1}y \sqrt{\sigma} u_j \xi^i \left(q_i^j + q_{(0)i}^j \right) \quad (10)$$

is a feature that appears naturally in this method. Indeed, it separates the contributions proportional to the Weyl tensor from the ones that give rise to the vacuum energy $q_{(0)i}^j$. Here, ξ^i is a Killing vector associated to boundary isometries and u_j is a unit vector, normal to a constant-time slice.

In five dimensions the explicit expressions for the conserved charges are

$$q_i^j = \frac{\ell^2}{64\pi G} \delta_{[i_1 i_2 i_4 i_4]}^{[j j_2 j_3 j_4]} K_j^{i_1} \delta_{j_2}^{i_2} \left(R_{j_3 j_4}^{i_3 i_4} + \frac{1}{\ell^2} \delta_{[i_4 i_4]}^{[j_3 j_4]} \right) \quad (11)$$

and

$$q_{(0)i}^j = \frac{\ell^2}{128\pi G} \delta_{[k i_2 \dots i_4]}^{[j j_2 \dots j_4]} \left(K_i^k \delta_{j_2}^{i_2} + K_{j_2}^k \delta_i^{i_2} \right) \left(\mathcal{R}_{j_3 j_4}^{i_3 i_4} - K_{j_3}^{i_3} K_{j_4}^{i_4} + \frac{1}{\ell^2} \delta_{j_3}^{i_3} \delta_{j_4}^{i_4} \right). \quad (12)$$

4. Conserved quantities

Evaluating the charge formula for the charged rotating black hole metric (3), we obtain

$$E = q(\partial_t) = \frac{m\pi(2\Xi_a + 2\Xi_b - \Xi_a\Xi_b) + 2\pi qab(\Xi_a + \Xi_b)/\ell^2}{4\Xi_a^2\Xi_b^2}, \quad (13)$$

$$E_0 = q_0(\partial_t) = \frac{2\pi\ell^2}{32} \left(1 - \frac{(\Xi_a - \Xi_b)^2}{9\Xi_a\Xi_b} \right), \quad (14)$$

where E_0 is the same as in the case of Kerr-AdS₅ black hole. E corresponds to the black hole mass. There is a subtlety regarding this point, though. Commonly, a proper definition of energy for a rotating black hole is associated to a Killing vector that does not rotate at infinity, $\xi = \partial_t - (a/\ell^2)\partial_\phi - (b/\ell^2)\partial_\psi$. As it has been widely discussed in the literature, this is the only notion of energy that satisfies the First Law of Thermodynamics for Kerr-AdS black holes. Being able to reproduce the correct value of the energy from a Killing vector $\xi = \partial_t$ is a reflection of the fact that one does not recover Kerr-AdS metric from the uncharged limit of the solution (3). On the other hand, for the Killing vectors associated to the azimuthal angles ϕ and ψ ,

$$J_\phi = q(\partial_\phi) = \frac{\pi}{4\Xi_a^2\Xi_b} [2am + qb(1 + a^2/\ell^2)], \quad (15)$$

$$J_\psi = q(\partial_\psi) = \frac{\pi}{4\Xi_b^2\Xi_a} [2bm + qa(1 + b^2/\ell^2)], \quad (16)$$

which correspond to the angular momenta of the black hole.

All the conserved quantities computed here are in agreement with Ref.[1], where the charges were obtained from the integration of the First Law.

5. Conclusions

We have calculated the Noether charges associated to the isometries of the charged rotating black hole in five dimensional found in Ref.[1]. When the dilaton fields present in a $5D$ gauged supergravity theory are kept fixed, they act as a cosmological constant. Black holes like the one studied here are solutions to the bosonic sector of this supergravity. The addition of counterterms that depends on the intrinsic and extrinsic curvatures renders the conserved charges both finite and correct. These quantities correspond to the energy of the system, which also involves the electric charge, two angular momenta, and a vacuum energy E_0 , which is the same as in Kerr-AdS₅ (computed in Ref.[4] and Ref.[5] using the standard counterterm method).

The finite action defined by Eq.(1) provides the right starting point to obtain thermodynamic relations directly from the Euclidean action. However, depending on thermodynamic ensemble one is working on, the action may require additional boundary terms, as it was discussed in the case of Kerr-Newman-AdS black hole in Ref.[6].

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