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The Mechanical Waves Conceptual Survey: An Analysis of University Students' Performance, and Recommendations for Instruction

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ABSTRACT
The Mechanical Waves Conceptual Survey (MWCS), presented in 2009, is the most important test to date that has been designed to evaluate university students' understanding of four main topics: propagation, superposition, reflection, and standing waves. In a literature review, we detected a significant need for a study that uses this test as an assessment tool and presents a complete analysis of students' difficulties on the test. This article addresses this need. We administered the MWCS at a private university in Mexico to 541 students. In this article, we present a complete description of these students' performance on the test, a description of their main difficulties, an elaboration of these main difficulties in terms of students' inappropriate conceptions, and recommendations for instruction based on the results obtained by the test. Our analyses may be used by instructors and researchers who intend to use the MWCS or create new instructional material.
Keywords: mechanical waves, students' understanding, propagation, superposition and reflection, standing waves

INTRODUCTION

The physics of mechanical waves is an important topic in most introductory physics curricula at the university level. Many areas of physics depend on a solid understanding of mechanical waves. This explains the importance of assessing how well students understand this topic.

Much research has been done on the subject of mechanical waves (Maurines, 1992; Linder, 1993, Whittmann, 2002; Eshach & Schwartz, 2006; Hrepic, Zollman & Rebello, 2010; Caleon & Subramaniam, 2010; Bhathal, Sharma & Mendez, 2010; Kennedy & De Bruyn, 2011; Kryjevskaia, Stetzer, & Heron, 2011; 2012; Pejuan, Bohigas, Jaén, & Periago, 2012; Eshach, 2014; Zeng, Smith, Poelzer, Rodriguez, Corpuz, & Yanev, 2014). Researchers have identified cognitive aspects of students' difficulties in this topic. For example, Whittman (2002) mentions

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State of the literature

- In 2009, the Mechanical Waves Conceptual Survey (MWCS) was presented in the literature.
- This is the most important test to date that has been designed to evaluate university students' understanding of four main topics: propagation, superposition, reflection, and standing waves.
- No study has used this test as an assessment tool and has presented: a complete description of university students' performance, a description of their main difficulties on the test, an elaboration of these main difficulties in terms of students' inappropriate conceptions, and recommendations for instruction based on results obtained by the test.

Contribution of this paper to the literature

- This study uses the MWCS as an assessment tool and presents all the analyses and recommendations for instruction needed in the area (as established in the State of literature section).
- This article is the first of this kind in the area and offers a synthesis of the studies conducted so far on the issue of mechanical waves.
- The analyses and recommendations presented here may be used by physics instructors who are teaching the topics that are tested on the MWCS, and by physics education researchers who intend to use the MWCS and/or create new instructional material for teaching about waves.

that students approach the topic of wave physics using object-like descriptions of wave pulses. In a previous article, Tongchai, Sharma, Johnston, Arayathanitkul & Soankwan (2009) introduced the Mechanical Waves Conceptual Survey (MWCS) that evaluates university students' understanding of four main topics: propagation, superposition, reflection and standing waves. This is the most important test of its kind to date. The authors presented the test's development and evaluation in detail, focusing on validity and reliability. They briefly showed how the test had been used with diverse populations of students in Thailand and Australia. Its design was primarily based on an existing open-response instrument that was previously designed by Wittmann (1998).

As a preliminary step, we undertook a review of the literature that focuses on the use of the MWCS as an evaluation instrument. This review included the article in which the test was initially introduced and the research articles that cited the original paper. From these studies, only one -a second article by the authors that designed the test (Tongchai, Sharma, Johnston, Arayathanitkul & Soankwan, 2011) - analyzes test results obtained by the MWCS. In this article, the authors analyze the same data presented in the original article; however, they focus on the consistency of students' conceptions of items under the main topic propagation, which is the first of the four main topics on the test. As a result of this review literature, we detected four specific needs. First, we observed that to date, a study that presented a complete analysis of university students' overall performance on the MWCS had not been conducted. Second, there had not been an analysis of students' main difficulties with the test. (Note that the original article did not offer an analysis of the percentages of the various answers chosen for each question.) Third, there had not been an elaboration of these main difficulties in terms of students' inappropriate conceptions based on previous studies. The fourth need we identified is to develop specific recommendations, based on the test results, for instruction of the four main topics evaluated by the MWCS.

To address these needs, we conducted a research study with four objectives: (1) to investigate university students' performance on the MWCS; (2) to investigate students' primary difficulties with the MWCS topics; (3) to elaborate these difficulties in terms of students' inappropriate conceptions; and (4) to establish recommendations for instruction based on the results obtained by the MWCS. In this article we also discuss some deficiencies in the design of several of the test questions.

PREVIOUS RESEARCH

To date there are three multiple-choice tests that assess student understanding of waves: (1) a test for secondary students (Caleon & Subramaniam, 2010), (2) a test for university students at the introductory level: the MWCS, and (3) a test for university students at the advanced level (Rhoads & Roedel, 1999).

Both prior to and following the design of the MWCS, numerous researchers have analyzed the difficulties that university students face with regard to the topics on this test:

(1) Propagation (Linder & Erickson, 1989; Linder, 1992; 1993; Maurines, 1992; Wittman, 1998; 2002; Wittmann, Steinberg & Redish, 1999; 2003; Hrepic, Zollman & Rebello, 2010; Tongchai et al., 2011; Kennedy & De Bruyn, 2011; Pejuan, Bohigas, Jaén & Periago, 2012; Kryjevskaia, Stetzer, & Heron, 2012)

(2) Superposition (Wittman, 1998; 2002; Wittmann, Steinberg, & Redish, 1999; Grayson, 1996; Sengören, Tanel & Kavcar, 2006; Kennedy & De Bruyn, 2011; Kryjevskaia, Stetzer & Heron, 2011)

(3) Reflection (Kryjevskaia et al., 2011)

(4) Standing waves (Zeng, Smith, Poelzer, Rodriguez, Corpuz, & Yanev, 2014)

Note that some studies have proposed new instructional materials or curricular modifications (Wittman, 1998; Wittmann et al., 2003; Kennedy & De Bruyn, 2011; Kryjevskaia et al., 2011; Zeng et al., 2014; Bhathal, Sharma & Mendez, 2010).

This study is the first to offer an analysis of overall student performance on the MWCS, as well as to describe the main difficulties that students encounter with it. In these analyses we compare our results to those reported by the designers of the test in their two previous articles, and we make comparisons with other related articles. We also elaborate these main difficulties in terms of students' inappropriate conceptions based on previous studies.

METHODOLOGY

Context of Research and Participants

This research study was carried out at a private university in Mexico. The campus has 15,000 undergraduate students, half of whom are engineering majors in various fields. Their curriculum includes four one-semester introductory physics courses. In the third course, "Fluids, waves and thermodynamics", the students study the four main topics evaluated by the MWCS.

Table 1. Main topic, subtopic and description of the concept evaluated by each question

Main topic	Subtopic	Question	Concept evaluated in the question
Propagation	Sound variables	1	Interpretation of amplitude and frequency
	Speed of sound waves	2	Speed in air independent of frequency
		3	Speed in air independent of frequency and amplitude
	Speed of waves on strings	4	Speed independent of the changes in hand movement
		5	Speed proportional to density and tension
	Displacement of medium in sound waves	6	Longitudinal oscillation of air particles perturbed
		7	Increase of frequency: Oscillation is faster
		8	Increase of amplitude: Oscillation is wider
Superposition	Superposition-Construction	9	Superposition of two waves in the overlap
		10	Superposition of two waves after the overlap
	Superposition-Destruction	11	Superposition of two waves in the overlap
		12	Superposition of two waves after the overlap
Reflection	Reflection-Fixed end	13	Complete reflection of an asymmetric pulse
		15	Half reflection of a symmetric pulse
	Reflection-Free end	14	Complete reflection of an asymmetric pulse
		16	Half reflection of an asymmetric pulse
Standing waves	Transverse standing waves in strings	17	Increasing frequency in the string, the wavelength of the new standing wave decreases
		18	Increasing tension in the string, the wavelength of the new standing wave increases
		19	Increasing density of the string, the wavelength of the new standing wave decreases
	Longitudinal standing waves in sound	20	Pattern of displacement of air molecules in the first harmonic inside a cylinder with one open end
		21	The fundamental frequency of a tube open at both ends is greater than the same tube with one open end
		22	The pitch generated by air blown across the top end of a bottle will be higher when it contains a greater volume of water

The textbook for this course is "Physics for Scientists and Engineers" by Serway and Jewett (2008). The students also attend corresponding laboratory sessions of which four are on the topic of waves. During the first two sessions, they work with two of the "Tutorials in Introductory Physics" by McDermott and Shaffer (2001): "Superposition and reflection of pulses" and "Reflection and transmission". Then, in the last two laboratory sessions, the students study stationary waves in both strings and sound.

The complete MWCS was administered to 541 students who were completing this course as a diagnostic test and did not count towards the final course grade. Since Spanish is the language of Mexico, three physics instructors with high proficiency in both languages translated the MWCS from English to Spanish.

Description of the MWCS

The test has 22 multiple-choice questions, 17 of them have a traditional multiple-choice format with different numbers of options (**Figure 1** shows Question 4, which is an example of this type of question); and five have a "two-tier" format: Questions 17 (see **Figure 2**), 18, 19, 21 (see **Figure 3**) and 22. As mentioned before, the designers of the test presented the test's development in detail, focusing on validity and reliability. **Table 1** presents a description of the subtopics evaluated within each of the main topics, and a description of the concept evaluated in each questions.

STUDENTS' PERFORMANCE ON MWCS

In this section we address the first objective: to investigate university students' performance on the MWCS. **Tables 2** & **3** show the proportion of students correctly answering all questions on the MWCS. **Table 2** presents the results for propagation, superposition and reflection, while **Table 3** displays the results for standing waves. We decided to divide the information into two tables because the format of the questions in the last topic is different from that of the first three topics.

Students' Scores Obtained on the MWCS

The average score on the MWCS is 9.86 correct answers out of 22 questions. Note that the two-tier format questions were graded as correct only if the answer and the justification were both correct. The distribution of scores was significantly non-normal (Shapiro-Wilk test, W (541) = 0.977, $p<0.001$). The skewness of the distribution of scores is 0.338 (SE=0.105), indicating a pile-up to the right, and the kurtosis of the distribution is -0.581 (SE=0.210), indicating a flatter than normal distribution. The positive skew indicates that the test is difficult for the students. For this type of distribution, it is more useful to use quartiles as measures of spread. The median of the distribution is 9, the bottom quartile (Q1) is 6, and the top quartile (Q3) is 13, so the interquartile range is 7. In this overall analysis, it is interesting to note that the students at the median (9) had difficulty answering 13 questions (out of 22) correctly on the MWCS.

Table 2. Results obtained for the three first main topics of the MWCS. The correct answer is in boldface. N is for students who did not respond

Main topic	Subtopic	Question	Options (%)								
			A	B	C	D	E	F	G	H	N
Propagation	Sound variables	1	20	**65**	2	13					0
	Speed of sound waves	2	**40**	46	10	4					1
		3	13	41	**37**	8					1
	Speed of waves on strings	4	14	34	10	11	3	**28**			0
		5	**70**	11	8	10					0
	Displacement of medium in sound waves	6	1	**40**	19	18	22				0
		7	6	8	**32**	8	17	7	12	9	1
		8	5	**28**	9	19	7	14	11	6	1
Superposition	Superposition-Construction	9	27	13	16	**30**	9	5			1
		10	**84**	9	7						0
	Superposition-Destruction	11	9	16	**50**	20	3	1			0
		12	**69**	10	11	9					0
Reflection	Reflection-Fixed end	13	5	10	4	**61**	20				0
		15	19	**41**	10	28	2				0
	Reflection-Free end	14	8	**59**	15	11	7				0
		16	**42**	9	5	35	9				0

Table 3. Results obtained for Standing waves. The correct answer is in boldface. Note that questions 17, 18, 19, 21 and 22 are in a two-tier format and question 20 is in a traditional multiple-choice format. For the former, we present the correct combination of answers and the four most frequent incorrect combinations. The less-frequent combinations are clustered in the group "Others"

Main topic	Subtopic	Question	Options (%)					
Standing waves	Transverse standing waves in strings	17	**B-4**	**B-2**	**A-2**	**B-1**	**C-3**	**Others**
			58	19	8	3	3	9
		18	**A-3**	**B-3**	**B-4**	**A-4**	**C-1**	**Others**
			26	16	14	10	7	27
		19	**B-3**	**A-3**	**C-1**	**C-2**	**C-3**	**Others**
			42	20	6	6	6	20
	Longitudinal standing waves in sound	20	**A**	**B**	**C**	**D**	**E**	**F**
			18	13	5	12	11	**38**
		21	**C-4**	**C-5**	**B-4**	**B-1**	**B-5**	**Others**
			16	18	16	13	11	26
		22	**B-2**	**C-2**	**C-3**	**B-3**	**B-4**	**Others**
			30	21	17	10	7	15

Table 4. Classification of questions by difficulty level

Difficulty level	Range of correct answer percentages	Questions
High	[0%, 30%)	4, 8, 18, 21
Medium high	[30%, 40%]	2, 3, 6, 7, 9, 20, 22
Medium	(40%, 50%]	11, 15, 16, 19
Medium Low	(50%, 70%)	1, 12, 13, 14, 17
Low	[70%, 100%]	5, 10

Clustering the MWCS Questions by Difficulty Level

To analyze the students' performance on each of the test questions, we decided to cluster the questions based on the range of proportion of the correct answer. We classified the questions by five difficulty levels, as shown in **Table 4**.

The 11 most difficult questions are in the "high" and "medium high" difficulty levels. The high difficulty questions are those with a percentage of correct answers that is lower than the recommended lowest value of 30% (Ding, Chabay, Sherwood & Beichner, 2006). The medium-high difficulty questions are those with a percentage of correct answers that is very close to 30%.

Identification and Analysis of the Most Difficult Main Topics and Subtopics of the MWCS

Analyzing the 11 most difficult questions for students from **Table 4**, we note that they come primarily from two main topics. Questions 2, 3, 4, 6, 7, 8 are from propagation and questions 18, 20, 21, 22 are from standing waves. Question 9 is the only one of the 11 that falls under the topic of superposition, and none of these most difficult questions comes from the topic of reflection. As a result of this classification, we can establish that propagation and standing waves are the two most difficult main topics for students. These topics both refer to waves' phenomena in string and sound separately (see **Table 1**).

An additional result of classifying the questions is that we are able to determine which subtopics are the most difficult for students. We observed that the proportion of correct answers for all questions associated with three specific subtopics was less than or equal to 40%. They are "Speed of sound waves" and "Displacement of medium in sound waves" in propagation, and "Longitudinal standing waves in sound" in standing waves. These subtopics all refer to sound, not strings. We can conclude that students have more difficulty with the topics of propagation and standing waves, and especially with the subtopic of sound.

Superposition and reflection are less challenging for students. Reflection is conceptually related to superposition, i.e., the reflection questions can be solved using a superposition model, as is done in the Tutorials (McDermott & Shaffer, 2001; Kryjevskaia et al., 2011). This model shows that the string extends beyond the boundary and as the incident pulse passes through the boundary, it overlaps (this is the superposition) with a "virtual" pulse that is traveling along the imaginary string toward the real one.

STUDENTS' MAIN DIFFICULTIES WITH MWCS

In this section we address the second objective, to investigate students' main difficulties with the MWCS topics. For all topics, we study each of the subtopics separately. In addition, for the second and third main topics we study and compare the two main topics as a whole, because they are closely related. All of these analyses are based on the most frequent error for each question.

Propagation

Sound variables. Question 1 is the only question from this subtopic. It evaluates the interpretation of amplitude and frequency in sound waves. The question tests whether a student understands that a person, who sings at the same volume as another person, but at a higher pitch, will generate a sound wave with the same amplitude but a different frequency. 65% of the students answered this question correctly (option B). The most frequent error (option A, 20%) was to confuse frequency with amplitude.

Speed of sound waves. Questions 2 & 3 fall under this subtopic. Question 2 evaluates whether students understands that the speed of sound waves in air is independent of the frequency of the waves. In this question, students have to compare the velocity of two sound waves with different frequencies and the same amplitude in air. The correct answer is that both speeds are equal, since sound speed depends only on air properties. Only 40% of the students answered this question correctly (option A). The most frequent error selected by the students (option B, 46%) indicates that the velocity of the wave with the higher frequency is faster, using the equation $v = f\lambda$. These students did not realize that the speed of sound in air is independent of the frequency of the wave. The multiplication of the frequency by the wavelength is the speed of the wave. Since the frequency is different, students think that the speed will be different because (according to the equation) it depends on frequency. However, they don't realize that when the frequency is different in the same medium, (in this case, air) the wavelength also changes accordingly to produce the same speed.

Question 3 evaluates whether students understand that the speed of sound waves in air is independent of the waves' frequency and amplitude. This question is very similar to the previous one, since it asks the test-taker to compare the velocities of two sound waves in air with different amplitudes but the same frequencies. Again, the correct answer is that both speeds are the same. Only 37% of the students answered this question correctly (option C). The most common error (option B, 41%) was due to the incorrect belief that both velocities were the same because both waves had the same frequency, using the equation $v = f\lambda$.

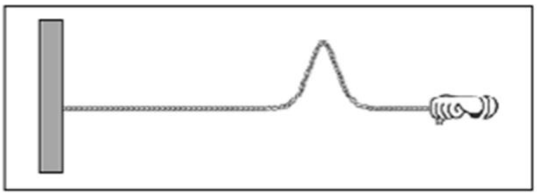
As mentioned before, this subtopic is one of the three most difficult for students. Analyzing the frequent errors in both questions, we found that the most frequent error for was to believe or assume that the speed of sound waves depends on their frequency. This tendency was pointed out by Tongchai et al. (2011), who studied the consistency of students' answers within this subtopic. Upon carrying out a cross analysis of these questions, we found that 31% of the students chose answers that were based on this incorrect assumption (selecting option B for both). This percentage is very similar to what Tongchai et al. reported in their second article (25%).

Speed of wave on strings. Questions 4 & 5 test knowledge of this subtopic. Question 4 evaluates whether students understand that the speed of waves on a string is independent of the changes in the hand movement. **Figure 1** presents Question 4. The correct answer is option F, which demonstrates that the velocity of a pulse on a string depends only on tension and mass density according to the equation $v = \sqrt{T/\mu}$. Only 28% of the students answered this question correctly. The most common error (option B, 34%) was to assume that moving the string faster with a higher frequency would produce a faster pulse. These students, probably thinking of the equation $v = f\lambda$, held the incorrect conception that the speed of waves on strings depends on frequency.

Question 5 evaluates the degree to which students understand that the speed of waves on a string is proportional to the density and the tension of the string. This question refers to the same situation as question 4, but asks students to identify which change in the string's properties will produce a faster pulse. The correct answer is that one should use a lighter string under the same tension (option A). 70% of the students answered this question correctly. In this item we found two most common incorrect options with very similar percentages (option D: 10% and option B: 11%). The first (option D) asserts that "none of the above would produce a pulse that takes a shorter time because the speed is determined by frequency and wavelength according to $v = f\lambda$." In this error, students mistakenly believed that the speed of waves on strings depends on frequency (as in the most common error in question 4). The second (option B) asserts that one should use a heavier string under the same tension.

Analyzing the two questions from this subtopic, we observe a large difference between the proportions of correct answers (70% in question 5 vs. 28% in question 4). Tongchai et al. note in their second article that a large proportion of students (approximately 80%) answered both questions without having a complete understanding, i.e., either by using an alternative conception or by guessing. This was also borne out by our data. In a cross analysis of questions 4 and 5, we observe that 74% of our students belong to this latter group.

Consider the following description and answer questions 4 – 5.
One end of a long taut string is tied to a distant pole while the other end of the string is held by a girl (see figure below). This girl quickly flicks her hand up and down to create a pulse moving towards the pole.



___ **4)** She now wants to produce a pulse that takes a shorter time to reach the pole. How can she do this?

A. Flick the string harder to push more force into the pulse.
B. Flick the string faster to create a pulse with higher frequency.
C. Flick the string further up and down to create a pulse with larger amplitude.
D. Flick the string a shorter distant up and down to create a pulse with smaller amplitude.
E. Wait until the first pulse is reflected back then flick again to add the pulses together.
F. None of the above would produce a pulse that takes a shorter time to reach the pole.

Figure 1. Question 4 of the MWCS

The percentage of correct answers for question 4 is lower than the 30% recommended by researchers (for example see Ding et al., 2009). Two issues that might have led to the low performance should be considered. The first is that the correct answer is “none of the above”. Some researchers (Frey, Petersen, Edwards, Pedrotti, & Peyton, 2005; DiBattista, Sinnige-Egger, & Fortuna, 2013) recommend that this option not be used in multiple-choice questions. The fact that this option was actually the correct answer makes it even less reliable. The second issue is related to the incorrect option A which states: “flick the string harder to push more force into the pulse”. We believe that this option did not use the standard physics terminology for the phenomena of waves on a string. Some students might have interpreted this option as meaning “increasing the tension on the string”, which actually would produce a faster pulse ($v = \sqrt{T/\mu}$). This notion seems to be confirmed by the fact that most of the students who selected this choice for question 4 went on to correctly answer the following related question, in which they needed to apply the equation $v = \sqrt{T/\mu}$. There is some evidence that these two issues contributed to the students’ low performance.

Displacement of medium in sound waves. Questions 6 to 8 fall under this subtopic. Question 6 evaluates the understanding of longitudinal oscillation of an air particle perturbed by sound waves. This question asks students to describe the motion of a particle perturbed by a sound wave in front of a loudspeaker. Only 40% answered this correctly (option B), demonstrating that they understood that the particle would oscillate longitudinally from side to side: "It will move back and forth [in] about the same position". The most common error (option E, 22%) maintained that the particle will move away as a sine curve. Another frequent error (option E, 18%) stated that the particle would also move away, without specifying how it would move. Adding these two percentages, we can state that 40% of the students thought that the particle would move away from the speaker. Another incorrect choice (option C, 19%) stated that the particle would oscillate transversally. It is interesting to note that these percentages are similar to those reported by Wittmann et al. (2003) using a similar open-ended question and administering it to a similar population of students who had attended a lecture on the topic.

Question 7 tests students' understanding of whether an increase in the frequency of the sound waves will produce a faster oscillation of an air particle. This question is a continuation of the previous one, and asks if the change in the movement of the particle would produce a sound wave with a higher frequency. The correct answer is that the motion will be the same, but faster, as expressed by option C: "It will move back and forth faster". Only 32% answered this correctly. The most frequent error (17%, option E) was to believe that the particle will "move up and down faster". These students incorrectly thought that the movement of the particle was the motion with a transversal wave. It's interesting to note that the majority of students selecting this incorrect option also chose a movement related to a transversal wave in question 6. Therefore, these students seem to be consistent in their conclusions.

Question 8 evaluates whether students understands that an increase in a sound wave's amplitude will produce a wider oscillation of an air particle. This question is a continuation of the previous one, and asks if change in the motion of the particle would produce a sound wave with higher amplitude. The correct answer is that the movement will produce a wider oscillation (option B): "It will move back and forth further". Only 28% of the students answered this question correctly. The most frequent error (option D, 17%) was to view the movement of the particle as a wider version of the movement in a transversal wave. As would be expected, the majority of the students who selected this incorrect option also erred by choosing the related transversal wave options in questions 6 and 7.

As mentioned before, this subtopic is one of the three most difficult for students. The questions are all modifications of the same physical situation. In analyzing the frequent errors in this subtopic, we found that two stood out. The first error (10%) was to consistently choose the incorrect answer for each of the three questions in which students thought sound waves were transversal waves rather than longitudinal waves (answer combination C, E, D). The second error (7%) was to answer incorrectly (again, consistently) based on the belief that the particle does not oscillate but instead moves along a line (answer combination D, G, F). These findings agree with those reported by Tongchai et al. (2011). However, there is a consistency issue that needs to be pointed out. As shown in the analysis of question 6, the most common error was to select the option that asserts that the particle "will move away as a sine curve". If we analyze the possible responses to the next two questions, we observe that they also include an incorrect option that is similar to the frequent error in question 7. That error indicates that the particle "will move away faster as a sine curve". However, question 8 does not have an incorrect option that indicates that the particle "will move away as a sine curve with greater amplitude". This fact is important because the absence of this option necessarily affects the consistency analysis. Due to the actual design of these questions, those students cannot be consistent when responding to questions 6, 7 and 8. This problem has not been pointed out before.

Additionally, Question 8 has a correct answer proportion that is lower than the recommended value (30%) and Question 7 has a correct answer proportion very close to this value. We believe that this is due to the high number of multiple choices (8 options) and also to their design. We note that these questions are not independent, since they share the same options. There are some authors, among them Frey el al. (2005), who state in the consensus list of item-writing rules that "items should be independent of each other". We believe that the connection between questions 7 and 8 may be affecting the students' performance.

Superposition

Superposition-Construction. Questions 9 & 10 are under this subtopic. Question 9 asks students to choose the correct sketch of the constructive superposition of two waves at the moment of overlap. Only 30% (option D) answered this correctly by choosing an option that shows the addition of displacements due to each wave pulse on a point-by-point basis. The most common error was to choose a sketch that shows no superposition except for the peaks of the overlapped pulses (option A, 27%).

Question 10 is a continuation of question 9 and asks students to choose the correct sketch of the constructive superposition of two waves after the overlap moment. 84% of the students selected the correct answer, which shows two waves that have passed through one another and retained their shape. The most frequent error was to choose a sketch that shows smaller waves with the legend "waves have become smaller because they have collided and therefore have lost energy" (option B, 9%).

We noted a significant difference in the percentages of correct answers for questions 9 and 10. For question 9, which evaluates the constructive superposition of two waves at the moment of overlap, the correct answer percentage was 30%. For question 10, which evaluates understanding of this concept after the overlap moment, we observed a much higher percentage (84%). In the first article by the designers of the test, a similar difference in the correct answer proportions for the overall population can be noted for questions 9 & 10, but curiously this is not addressed by the investigators in their article.

Superposition-Destruction. Questions 11 & 12 fall under this subtopic. Question 11 is very similar to question 9, but asks about the destructive superposition at the moment of overlap. 50% answered correctly by choosing a sketch that shows the addition of displacements due to each wave pulse on a point-by-point basis (option C). The most frequent error was to choose a wave having an approximately correct form but that lacked the precision needed to be considered correct (option D, 20%).

Question 12 is related to the context indicated in question 11. It asks about a destructive superposition after the overlap moment. In this case, 69% answered correctly by choosing the option that shows that the waves have passed through one another and retained their shapes. The most common error was to choose an option that shows smaller waves, implying that the waves have become smaller because they have lost energy.

In these questions we also observe a considerable difference in the selection of the correct answer. In question 11, which evaluates students' understanding of the destructive construction at the overlap moment, the percentage of correct answers is 50%, while in question 12, which tests knowledge of this concept after the overlap moment, we observe a much higher percentage (69%). As in the latter subtopic, we note that a similar difference in the correct answer proportions for those questions is evident in the first article by Tongchai et al., but again, this difference is not mentioned in that article.

Overall analysis. The questions in this main topic are closely related and allow us to perform an analysis of the topic as a whole. We observe two patterns that are worth noting. The first is that in both subtopics, we detected much higher proportions of correct answers in those questions that ask for the moment after the overlap than in those that ask about the moment of overlap. The second is a pattern of similar frequent errors in both subtopics. In the questions that ask about the moment of overlap, we observe that the most frequent error is an answer that lacks the precision to be considered correct. On the other hand, in those questions that ask about the moment after the overlap, the most common error (again, in both subtopics) is to consider that the waves will become smaller because they have collided and have lost energy as a result.

Reflection

Fixed-end. Questions 13 & 15 are part of this subtopic. Question 13 involves choosing the correct sketch that shows the complete reflection of an asymmetric pulse moving along a string to the right toward a fixed-end. The correct answer is a sketch that shows a pulse on the opposite side of the string and vertically inverted, since its right tail will be reflected as the left tail of the new pulse. 61% of the students answered this correctly (option D). The most common error was option E (20%), which shows a reflected pulse in the correct side of the string (opposite side) but with no vertical inversion (which means a pulse with incorrect leading edge).

Question 15 requires selecting the sketch of a half reflection of a symmetric pulse in a string moving toward a fixed-end. In this case the correct answer is the option that shows a string in its original form (as if there were no pulse) because the half- reflected pulse cancels the half-pulse that has not yet been reflected. 41% of the students answered this correctly (option B). The most common error was to choose option D (28%), which represents a half-reflected pulse on the same side of the string as the complete reflected pulse would have after a complete reflection which is on the opposite side of the original pulse.

In this subtopic we observe an interesting pattern: students have more difficulty with the half reflection of a symmetric pulse on a fixed-end string than with a complete reflection of an asymmetric pulse on the same type of string (41% vs. 61%). This pattern is similar to the one reported in the first article by the test's designers for questions 13 & 15 for the overall population.

Free-end. Questions 14 & 16 fall under this subtopic. Question 14 has the same structure as question 13 but refers to a free-end string. In this case the correct answer is a pulse on the same side of the string but with a vertical inversion, for the same reason as in question 13. 59% of the students answered this correctly (option B). The most common error (option C, 15%) was to select the option which shows a reflected pulse in the correct side of the string (same side) but with no vertical inversion (which means a pulse with incorrect leading edge).

Question 16 is the same as question 15, except that it concerns a free-end string. In this case the correct answer is a pulse with double amplitude, since the half-reflected pulse is on the same side of the string as the half-pulse that has not yet been reflected. 42% answered this question correctly (option A). The most common error (option D, 35%) was to choose the option which represents a half-reflected pulse on the same side of the string as the complete reflected pulse would have after a complete reflection which is on the same side of the original pulse.

In this subtopic, we observe the same pattern: students had more difficulty with the half-reflection of a symmetric pulse on a free-end string than with a complete reflection (42% vs. 59%). Unlike the other patterns of superposition and reflection, in this case we did not find the same tendency in the overall population that had been reported by Tongchai et al. (2009). In their study, they reported a higher percentage of correct answers for question 16 than question 14. However, it is interesting to note that we did observe this pattern (as well as the others) in the population of Australian second-year university students who were studying advanced physics.

Overall analysis. Under the main topic of reflection, we found two interesting patterns. The first is that we observed more difficulty with half-reflections on both types of strings. The second pattern refers to the most frequent errors. In the complete reflections (both types), the most frequent error was to choose a reflected pulse in the correct side of the string but with no vertical inversion. On the other hand, in the half-reflections (both types) the most frequent error was to choose a half-reflected pulse on the same side of the string as the complete reflected pulse would have after a complete reflection.

Overall Analysis of Superposition and Reflection

The second and third main topics (superposition and reflection) are conceptually related. When analyzing the proportions of correct answers in both main topics, an interesting pattern emerges. We observe lower performances on questions in which the physical phenomena (superposition and reflection) are incomplete, i.e., during the overlapping moment in superposition or in the case of half-reflection rather than complete. On the other hand, we observe higher performances when the phenomena are complete, that is, after the overlapping moment in superposition, or when reflection is complete.

Standing Waves

Transverse standing waves on a string. Questions 17 to 19 fall within this subtopic. Question 17 evaluates whether students understand that increasing the frequency on a string will cause the wavelength of the new standing wave to decrease. **Figure 2** shows question 17, which is the first question on the test with a two-tier format. 58% correctly answered that the wavelength of the new standing wave will decrease (option B) and that this is because "the wavelength is inversely proportional to the frequency, since the velocity doesn't change" (option 4). Recall that the physics for this situation is represented by equations:

$$v = \sqrt{T/\mu}, f = v/\lambda.$$

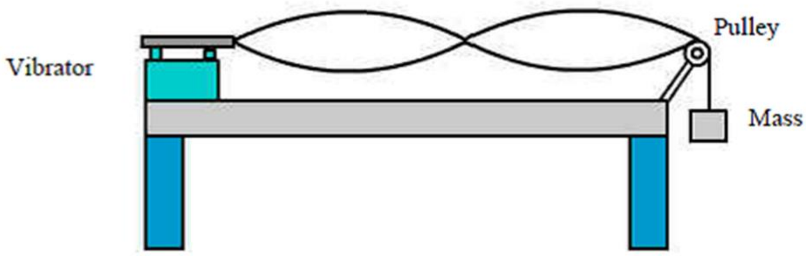
The most common error (19%) was to choose the combination of answers B-2. These students first answered correctly that the wavelength decreases (option B) but then justified it by choosing: "the wavelength is proportional to the frequency since the velocity doesn't change" (option 2).

Question 18 tests whether students understand that increasing the tension on a string will result in an increase in the wavelength of the new standing wave. This question is a follow-up to question 17. The correct answer is to state that the wavelength will increase (option A) and to justify that response by choosing option 3: "as the tension increases, the speed of the wave increases". The physics of this situation is represented by the same equation as in question 17. 26% of the students answered correctly. The most frequent error (16%) was to choose the wrong option (that the wavelength will decrease, option B) but to then choose the correct justification (option 3).

Question 19 evaluates whether students understand that increasing the density of a string will produce a decrease in the wavelength of the new standing wave. This question is a follow-up to the two previous questions. It asks about the change that will be caused by increasing the density of the string. The correct answer is to state that the wavelength will decrease (option B) and justify it by selecting option 3: "As the rope becomes heavier, the speed of the wave decreases". The equations needed are the same as in the previous question. 42%

Consider the following description and answer questions 17-19.

A standing wave is produced with a fixed length string, one end of which is attached to a vibrator, the other end of which is placed on a pulley and hung with a mass. Using the vibrator, the second harmonic standing wave is created (see figure below). The length between the vibrator and the pulley does not change.



17) If the frequency of the vibrator is doubled while everything else stays the same, a different harmonic standing wave is created. How would the wavelength of the new harmonic standing wave change?

___ **Answer:**

A. Increase
B. Decrease
C. Stay the same

___ **Reason:**

1. The wavelength depends on the amount of energy flowing in the waves.
2. The wavelength is proportional to frequency since the velocity doesn't change.
3. The wavelength depends on the length of the rope the frequency doesn't affect the wavelength.
4. The wavelength is inversely proportional to the frequency since the velocity doesn't change.

Figure 2. Question 17 of the MWCS (Question has a two-tier format)

answered this question correctly. The most common error (20%) was to answer that the wavelength will increase (option A) and then choose the correct justification (option 3).

In this subtopic, the three questions are strongly related since they deal with the same physical situation but with changes caused by different variables (frequency, tension and density). All of the questions ask the test-taker to predict the changes in the wavelength of the harmonics and then to state the justification. Analyzing the most frequent errors in the three questions, we observe that in the first question, in which the frequency of the vibrator changes, the most common error was a justification error. By contrast, in the next two questions, we observe that the most frequent error was to predict the opposite of the correct answer, but remarkably, to select the correct justification.

Next we discuss some shortcomings in the actual design of questions 17 and 18. The most common error in question 17 was to choose the combination of answers B-2. These students first answered correctly that the wavelength decreases (option B) but then justified it by choosing: “the wavelength is proportional to the frequency since the velocity doesn’t change” (option 2). It is important to note that this answer is partially correct, since the wavelength is, in a certain way, *proportional* to the frequency. A completely incorrect justification is to state that the wavelength is *directly proportional* to the frequency.

The correct answer percentage for question 18 is lower than the recommended value (30%). We believe that this could be due in part to its design. The formulation of the question doesn’t explicitly describe the increase of tension; it refers to the mass hanging from the string, saying “If the mass is increased by a factor of four while everything else stays the same, a different harmonic standing wave is created. How would the wavelength of the new harmonic standing wave change?” The problem is that the question doesn’t explicitly explain the mass “hanging from the string”. We believe that this fact may strongly affect student performance because they might interpret this “mass” as the mass of the string, not the mass hanging at the end of the string. Note that a change in the mass of the string will actually produce an inverse change of wavelength in relation to the increased tension.

Longitudinal standing waves in sound. Questions 20 to 22 come under this subtopic. Question 20 evaluates the understanding of the pattern of displacement of air molecules inside a closed-ended/open-ended cylinder when the first harmonic is generated. This question has a traditional multiple-choice format and asks students to select the image with the correct pattern. The first harmonic pattern extends from an antinode on the open side to an adjacent node on the closed side. 38% of the students answered this correctly (option F). The most frequent error (18%, option A) was to choose a pattern that extends from a node on the open side to an adjacent node on the closed side.

Question 21 tests students’ understanding that the fundamental frequency of a tube with two open ends is greater than that of the same tube with one open end. **Figure 3** shows Question 21. The correct answer is to choose the option that states that the fundamental frequency is higher in the tube with two open ends (option C) and then justify this selection by stating that this is due to the fact that “the wavelength in the tube with one open end is longer than in the other one” (option 4). In a same-length tube of longitude L, the wavelength of the fundamental frequency is 2L in the tube with two open ends, and 4L in the tube with only one open end. Only 16% answered this question correctly, which indicates that it is the most difficult question on the test. The most common incorrect answer (18%) was to correctly answer the first question (option C) but then state incorrectly that it is due to the fact that “the wavelength in the tube with one open end is shorter than in the other one” (option 5).

21) Two tubes have the same diameter and length. One has two open ends, and the other has only one open end. The fundamental frequencies of these two tubes are…

___ **Answer:**

A. the same
B. greater in the tube with one open end
C. greater in the tube with two open ends

___ **Reasons:**

1. There is more air pressure in the tube with one open end.
2. The two tubes contain the same number of air molecules.
3. The two tubes have the same size and the ends do not matter.
4. The wavelength in the tube with one open end is longer than in the other one.
5. The wavelength in the tube with one open end is shorter than in the other one.

Figure 3. Question 21 of the MWCS (Question is in two-tier format)

Question 22 tests students' understanding that the pitch generated by air blown across the top end of a bottle will be higher when it contains a greater amount of water. This question asks students to compare the pitch of the sound generated when air is blown across the top end of a bottle filled to one-third capacity with water, to the pitch generated when the bottle is half-filled with water. The correct response is to select the option that indicates that the pitch will become higher (option B) and then state that this is due to the fact that "the air column becomes shorter and the wavelength changes" (option 2). The bottle behaves like a tube with one open-end, in which the wavelength is directly proportional to the length of the tube. 30% of the students answered this question correctly. The most frequent incorrect answer (21%) was to assert that the sound will become lower (option C) and then select the correct justification (option 2).

As mentioned before, this subtopic is one of the three most difficult subtopics for students. Analyzing the frequent errors for these questions, we observe that all of the difficulties have to do with describing the wavelength of the harmonics in open and closed tubes. In question 20, students had difficulties with the shape of the wavelength; in question 21 they had difficulties comparing the wavelengths of the same harmonic in the same tube (open and closed); and finally, in question 22 they knew that the wavelength changed but did not understand the way in which it had changed.

Question 21 has a correct answer proportion that is lower than the recommended value (30%). We believe that this could be due in part to its design. If we consider the entire process of reasoning required to answer this question, we observe that it is very elaborate and involves many variables, some of which have similar names ("tube with one open end", "tube with two open ends"). It also has many relationships, some of which are inverse (frequency is inversely proportional to wavelength; greater, smaller, longer). We believe that this complexity probably affected the students' performance on this question.

ELABORATION OF STUDENTS' MAIN DIFFICULTIES IN TERMS OF STUDENTS' INAPPROPRIATE CONCEPTIONS

In our previous analysis of students' main difficulties, we present an overall study of students' main difficulties with each subtopic. In this section we address the third objective, to elaborate these difficulties in terms of students' inappropriate conceptions. Next, we present a synthesis of these main difficulties and elaborate them briefly in terms of students' inappropriate conceptions based on previous studies. Readers may consult these studies to learn more about these conceptions.

- *Sound variables*: confusing frequency with amplitude in sound waves. A previous study pointed out this confusion (Menchen & Thompson, 2003) and two previous studies found an inappropriate conception in which students link the two parameters usually claiming that the greater the amplitude, the higher the frequency (Kelly & Chen, 1999; Pejuan et al., 2012).
- *Speed of sound waves*: believing that the speed of sound waves depends on frequency using the equation $v = f\lambda$. In the test the incorrect options related to this inappropriate conception explicitly use this equation. Therefore, there is evidence that students who have this conception use this equation as a resource. It is noteworthy that Pejuan et al. (2012) found that students consider sound speed as dependent on frequency based on reasoning that suggest object-like properties of sound.
- *Speed of waves on strings*: believing that the speed of waves on strings depends on frequency. In the test there are two incorrect options related to this inappropriate conception. The first option (option B in question 4: "Flick the string faster to create a pulse with higher frequency") does not explicitly use the equation $v = f\lambda$; while the second option (option D in question 5) does explicitly use this equation. Therefore, in an overall analysis we can only say that students having this conception might use this equation as a resource in both questions. It is important to mention that, in questions similar to question 4, Wittmann (2002) found that students who choose an option similar to option B ("flick the string faster"), use object-like descriptions of wave pulses.
- *Displacement of medium in sound waves*: confusing longitudinal vs transversal motion. The most consistent frequent incorrect answer is to consider that an air particle perturbed by sound waves oscillate transversally (up and down along a line) instead of longitudinally (forward and backwards along a line). Wittmann et al. (2003) found that these students tend to interpret the common sinusoidal graph used to describe sound waves as a picture rather than a graph.
- *Superposition (Construction and Destruction in the moment of overlap)*: choosing an answer which demonstrates the lack of a complete understanding. Wittmann (2002) found that students with this kind of errors in constructive interferences often use only a single point on the wave when describing the physics of wave superposition.

- *Superposition (Construction and Destruction in the moment after the overlap)*: considering that waves will become smaller because they lose energy when colliding. Wittmann (2002) found that these students tend to describe the superposition as if it was a collision between objects.
- *Reflection (Fixed and Free ends)*: choosing a reflected pulse in the correct side of the string but with incorrect leading edge (with no vertical inversion) in the complete reflections, and choosing a half-reflected pulse on the same side of the string as the complete reflected pulse would have after a complete reflection in the half-reflections. On this issue, Kryjevskaia et al. (2011) analyzed in detail students' difficulties with reflections of a sequence of two or more simple pulses. They found that students tend to use simple rule-based approaches instead of applying reflection models based on the superposition principle. In our analysis we observe that the most frequent errors have some elements of appropriate responses. More specifically, in all cases, students think of a pulse reflected on the right side of the string. This suggests that students may be using rule-based approaches as stated by Kryjevskaia et al.
- *Transverse standing waves in strings*: having difficulties predicting and giving justifications regarding the changes in the harmonics wavelength when, in the physical situation, the frequency, the tension and the density is changed. The most common error by increasing the frequency in the string is a justification error. The most common error by increasing the tension in the string or increasing the density of the string is incorrectly predicting the opposite to the correct answer, but remarkably, selecting the correct justification. It is noteworthy that no study has analyzed in detail students' inappropriate conception in transverse standing waves in strings. Bhathal et al. (2010) recommend new instructional material on this subject, but that is not designed based on students' difficulties. More studies are needed in this area.
- *Longitudinal standing waves in sound*: having difficulties describing the wavelength of the harmonics in open and closed tubes. In question 20, students have difficulties with the shape of the wavelength in a closed tube; in question 21 they have difficulties comparing the wavelengths of the same harmonic in the same tube (open and closed); and finally, in question 22 they know that the wavelength in a closed tube changes when the length of the tube decreases but do not understand how. Only Zeng et al. (2014) have analyzed students' understanding of displacements nodes and antinodes in open and closed tubes. They found that the most frequent error is confusing displacement nodes with pressure nodes and displacement antinodes with pressure antinodes. Recall that pressure antinodes are displacement nodes, and pressure nodes are displacement antinodes. Question 20 that evaluates students' understanding of the displacement pattern in a closed tube is the most related question to Zeng et al. study. In this question we found that in the most frequent error, students incorrectly consider that there is a node on the

open side instead of an antinode. It is possible that this error is related to the inappropriate conception identified by Zeng et al. More studies are needed to identify the possible relationship between this conception and the frequent errors identified in this subtopic.

RECOMMENDATIONS FOR INSTRUCTION BASED ON RESULTS OBTAINED BY THE TEST

McDermott (2001) suggests that every curricular change should originate from research on students' understanding. The analysis of student performance, the analysis of the main difficulties with the MWCS topics, and the elaboration of these difficulties in terms of students' inappropriate conceptions presented in this article comprise part of such research on students' understanding of waves (objectives 1, 2 & 3). Also, it allows us to establish specific recommendations for instruction on the four topics (objective 4). Next, we summarize the most important findings derived from our analyses and establish recommendations for instruction.

Since the distribution of the students' scores shows a positive skew, we can state that the test presents numerous challenges for students. We noticed that students who are at the median of the distribution (9) had difficulty correctly answering 13 out of 22 questions on the test. Since the topics covered on the test are concepts that the students should have acquired in the course, this result shows the need to modify instruction in order to increase students' conceptual understanding of waves.

According to the classification of questions by difficulty level, the 11 most difficult questions for students are: 2, 3, 4, 6, 7, 8, 9, 18, 20, 21 & 22. Therefore, the most difficult main topics are the first (propagation) and the fourth (standing waves). Moreover, the most difficult subtopics in propagation are "Speed of sound waves" and "Displacement of medium in sound waves"; and "Longitudinal standing waves in sound" in standing waves. These subtopics both refer to waves in sound instead of waves in strings. The analyses offered in section "Students' main difficulties with the MWCS" offer a general view of the students' performance on each question, each subtopic and each main topic. They can be used to guide any modification in instruction, in an effort to increase student comprehension and learning. Our first recommendation is to emphasize the instruction of those concepts that fall under the three most difficult subtopics. In this regard, it is very important to note that Wittmann presented a tutorial for the subtopic "Displacement of medium in sound waves" (Wittmann, 1998; Wittman et al., 2003), and Zeng et al. (2014) presented a new air molecule motion illustration approach for the subtopic "Longitudinal standing waves in sound". Both of them have proven to increase students' understanding of these subtopics.

McDermott (2001) proposes that persistent conceptual errors must be explicitly addressed in instruction. In the previous section we present a synthesis of students' main difficulties and elaborate them briefly in terms of students' inappropriate conceptions based on previous studies. This synthesis can be considered by physics teachers as an overview of students' difficulties and inappropriate conceptions that offers global guidelines for modifying instruction or creating new instructional material. Finally, note that in the previous research section we identified a number of studies that have recommended new instructional material or curricular modifications. We believe that physics teachers should also take these studies into consideration.

CONCLUSIONS

Much research has been done on the subject of mechanical waves, as shown in the section on previous research. The MWCS is the most important test to date that has been designed to evaluate university students' difficulties in propagation, superposition, reflection, and standing waves. However, in a complete literature review, we found that there was a significant need for a study that used this test as an assessment tool and presented a complete analysis of students' difficulties. This article addresses this need and offers a synthesis of the studies conducted so far on the issue of mechanical waves. The analyses, the recommendations for instruction and the discussions about the actual design of some of the test questions presented in this article may be used by physics instructors who are teaching the topics that are tested on the MWCS, and by physics education researchers who intend to use the MWCS and/or create new instructional material for teaching about waves. Finally, it is important to mention that in a future study we will undertake a research project with the objective of converting this test into a standard multiple-choice test with five options for each question, which is the common number of options used in physics education research.

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