A Proxy Bidding Mechanism that Elicits all Bids in an English Clock Auction Experiment*

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Abstract

This paper reconsiders experimental tests of the English clock auction. We point out why the standard procedure can only use a small subset of all bids, which gives rise to a selection bias. We propose an alternative yet equivalent format that makes all bids visible, and apply it to a "wallet auction" experiment. Finally, we test the theory against various alternative hypotheses, and compare the results with those that would have been obtained if one had used the standard procedure. Our results confirm that the standard tests are subject to a significant selection bias.

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1 INTRODUCTION

In their seminal contribution, Milgrom and Weber (1982) introduce the affiliated values model, which contains private and common value auctions as special cases, and employ it to rank the three standard single-unit auction formats (first-price, Vickrey, and English clock) in terms of strategic simplicity and sellers' expected revenue. Among other results they find that the open, ascending-bid (English) clock auction yields the highest expected revenue to the seller. Moreover, they show that this auction format is strategically easy to play, because strategies are simple stop rules, and equilibrium strategies are not overly sensitive to bidders' beliefs. Subsequently, that auction format has become preferred in proposed applications, and subjected to extensive experimental testing.

Experimental tests of the English clock auction have been conducted for pure private, pure common as well as for the general affiliated values model (see the surveys by Kagel, 1995, and Kagel and Levin, 2002). Typically, these experiments observe significant deviations from equilibrium, convergence towards a naive strategy, and conclude that bidders tend to fall prey to the winner's curse.

Unfortunately, if one employs a standard English clock auction in a laboratory, one can observe only a small subset of players' bidding strategies. Basing the experimental test of the theory on that partial information may introduce a significant selection bias.

In an English clock auction bidders can only decide whether to stay active or quit the auction, while they observe the prices at which others have already quit the auction. Therefore, a bidding strategy is a collection of quitting rules, each contingent on the prices at which other players have quit the auction.

In the first stage of the auction, each bidder has a stop-rule that instructs him when to stop as long as no other bidder stops before him. The first stage ends as soon as one bidder quits the auction. Thereafter, bidders revise their quitting rules, using the information revealed by that one bidder's quitting price. This continues until only two bidders remain active, and one of them stops, at which point the auction ends.

After the first stage of the auction, only the one (first–round) quitting rule of the bidder who quits is observed, whereas the quitting rules of all other bidders remain invisible. Similarly, in all later stages of the auction, only one of the quitting rules of all remaining bidders is observed, whereas those of all other active bidders remain invisible.

This indicates that using a standard English clock auction in an experimental test of the theory gives rise to a selection bias, because the experimenter sees only the losing bids that terminate each stage of the auction. And it suggests that one should find an equivalent design of the English clock auction that makes all bids in all stages visible, and then carry out a more reliable test of the theory that uses the data of all bids by all bidders.

The present paper is motivated by this critical assessment of the standard practice. In a first step, we propose an equivalent format of the English clock auction that makes the quitting rules of all bidders visible, and this in all stages of the auction. This format will be described in detail in Section 2. In a second step, we apply that proposed auction format to a three player version of the well–known "wallet auction" game, introduced by Klemperer (1998) for two players. That auction is a pure common value auction. Third, we put that wallet auction to an experimental test. During that experiment, we observe all quitting rules of all players that are active in each stage of the game, starting from the first round, where all bidders state a quitting rule, until the last round, when the two remaining bidders state their final quitting rule, contingent upon the observed prices at which others have quit already. These complete data are then used to test the theory, and also to examine to what extent the standard testing procedure is subject to a selection bias that distorts the results of the test. The latter is done by a counterfactual analysis of the auction results, in which we use only those bids that would have been observed if one had used the standard format of the English clock auction. Finally, we compare the factual with the counterfactual tests and assess the nature and extent of the selection bias.

The article is organized as follows. In Section 2 we introduce our "proxy bidder augmented English clock auction" that is equivalent to the standard English clock auction. In Section 3 we state all equilibria in linear strategies of Klemperer's "wallet auction" for three players. In Section 4 we introduce three "naive" strategies that may appeal to players if they do not fully understand the game. Both the equilibrium and the naive strategies are used as reference points in the subsequent experimental test. In Sections 5 and 6 we describe the design and the results of the experiment. The paper closes with a discussion in Section 7.

2 THE PROXY BIDDER AUGMENTED ENGLISH CLOCK AUCTION

In an English clock auction, a strategy is essentially a complete book of instructions, each page of which stipulates at which price a bidder wishes to quit the auction (provided no other active bidder stops before), contingent upon the prices at which the bidders who are no longer active have quit. On the first page of that book one finds the instruction for the first stage of the auction, in which all bidders are active. The first stage ends as soon as one bidder has quit the auction. The second page states the stop rule that applies when one bidder has stopped, contingent upon the price at which that bidder stopped; it ends when the second bidder has stopped. This continues in a similar fashion on the third, fourth and higher numbered pages. The last page states the stop rule that applies in the event that only one other bidder is active, contingent upon the history of quits and associated prices of all the others who are no longer active. Of course, all pages of the book of strategy are contingent upon the respective bidder's private signal, in addition to the other information that is revealed in the course of the auction.

If one runs such an English clock auction in real life or in a laboratory one cannot observe bidders' books of strategies. One cannot even observe all the stop rules that are played at given private signals and given history of quits. The only bids that one can observe are the bids of those who have quit the auction. Indeed, in each round one observes only the lowest bid. Therefore, if there are n bidders one sees altogether only $n-1$ bids out of a total of $\sum_{i=2}^{n} i = \frac{(n-1)(n+2)}{2}$ bids.

We modify the English clock auction by adding a neutral proxy bidder to whom the actual bidding is delegated. The purpose of this modification is to make all the $\frac{(n-1)(n+2)}{2}$ bids visible, without affecting behavior. In each stage of the auction, bidders must instruct that proxy bidder at which reading of the price clock he shall stop, provided no other active bidder has stopped before. We refer to these bids as proxy bids. As in a standard English clock auction, a public price clock is then increased until it reaches the lowest of the proxy bids and the corresponding bidder quits the auction. Then the next stage begins, where the remaining bidders instruct their proxy bidders afresh, and this continues, until the auction ends. If two (or more) bidders state the same (lowest) proxy bid they all leave the auction at this price. If in some stage all remaining active bidders state the same proxy bid, the auction ends and the winner is chosen at random.

This auction mechanism is strategically equivalent to the standard English clock auction. In an experiment, however, it allows us to observe the bids of all active players in each stage and not only the losing bids.[1]

A crucial assumption on which our analysis of the bias in the standard design rests is that subjects indeed use the same strategy in this format as in the standard English clock auction. To maximize the likelihood that this assumption is warranted, we preserve the ascending clock frame. In this respect the proxy bidder augmented English clock auction differs from so-called "survival auctions" (see Kagel et al., 2004). In a survival auction, in each stage bidders place bids and the lowest bidder is eliminated. Both formats are strategically equivalent and extract the same information, but the survival auction does not preserve the ascending clock frame. Hence it might be more difficult for bidders to realize that it is equivalent to a standard English clock auction, which makes it appear less likely that they follow the same strategy. Kagel et al. (2004) find, however, in a multi-unit auction that with experienced bidders efficiency, revenues, and bidder profits do not differ between survival auctions and standard ascending-clock auctions. This lends support to our assumption that bidders use the same strategy in the proxy bidder augmented English clock auction as in the standard English clock auction since our design is even closer to the standard design than the survival auction.

Our approach further differs from Kagel et al. (2004) by focusing on the additional information gathered in the auction while they focus on practical advantages of the survival

[1]It would be preferable to obtain the complete strategies of all bidders, i.e., to extract also the bids that losing bidders would have stated in subsequent stages if another bidder had dropped earlier. This would require to employ the strategy method. This method is, however, cognitively quite demanding. Moreover, if prices increase in small steps and valuations are drawn from a fine grid it is practically not feasible.

compared to an ascending-bid auction. Thus, they advocate survival auctions as a practical tool, whereas we see the main purpose of the proxy bidder augmented auction as a research tool.

3 EQUILIBRIA OF THE WALLET AUCTION

Consider the wallet auction, with three bidders. That auction is a pure common value auction. Each bidder $i \in \{1, 2, 3\}$, privately observes a signal of the object's value, denoted by X_i, and the value of the object for sale is the sum of bidders' signals, $V(X_1, X_2, X_3) := \sum_{i=1}^{n} X_i$. In one interpretation, the value of the object is equal to the sum total of all bidders' wallets, and a bidder's signal is the content of his own wallet (thus, the name "wallet auction"). However, one can also interpret it as the sale of an estate or a chain store that consists of n assets, where each bidder has private information concerning a different asset.

We denote the initial proxy bid function by $\beta(x)$ and the adjusted proxy bid, adjusted after observing one exit at price p, by $b(x \mid p)$.

PROPOSITION 1 (SYMMETRIC EQUILIBRIUM) *The English clock auction has a unique symmetric equilibrium:*

$$\beta(x) = V(x, x, x) = 3x \tag{1}$$

$$b(x \mid p) = V(x, x, \beta^{-1}(p)) = 2x + \frac{p}{3}. \tag{2}$$

PROOF The wallet auction is a special case of the symmetric affiliated values model by Milgrom and Weber (1982). There it was shown that the English clock auction has a symmetric equilibrium. In that equilibrium, a bidder who has drawn signal x bids the value of the object that would apply if all other active bidders had drawn the very same signal x, and the signal of a bidder who has quit at price p is computed by using the inference rule $x = \beta^{-1}(p)$. ◇

However, the wallet auction has also many asymmetric equilibria $(\beta_1, \beta_2, \beta_3, b_i, b_j)$, where $b_i(p_{1k}), b_j(p_{1k})$ denote the strategies of the two bidders who are still active in the last round if the auction does not end in the first round, and p_{1k} denotes the price at which bidder k quits the auction in its first round.

PROPOSITION 2 (ASYMMETRIC EQUILIBRIA) *The English clock auction has the following asymmetric equilibria,* $(\beta_1, \beta_2, \beta_3, b_i, b_j)$*:*

$$\beta_i(x_i) = \alpha_i x_i, \quad \alpha_i > 0, \quad i \in \{1,2,3\} \tag{3}$$

$$\textit{where} \quad \alpha_1\alpha_2 > \alpha_1 + \alpha_2, \quad \alpha_3 := \frac{\alpha_1\alpha_2}{\alpha_1\alpha_2 - \alpha_1 - \alpha_2} \tag{4}$$

$$b_i(x_i \mid p_{1k}) = \gamma_i x_i + \frac{p_{1k}}{\alpha_k}, \quad \gamma_i > 1 \tag{5}$$

$$b_j(x_j \mid p_{1k}) = \gamma_j x_j + \frac{p_{1k}}{\alpha_k}, \quad \textit{where} \quad \gamma_j = \frac{\gamma_i}{\gamma_i - 1}. \tag{6}$$

PROOF To prove that $\beta := (\beta_1, \beta_2, \beta_3)$ are mutual best replies, consider first bidder 3 and assume bidders 1 and 2 play the candidate equilibrium strategies β_1, β_2. Suppose bidder 3 remains active in the first round until price p_1, and wins in the first round at price p_1, which happens if and only if both rivals quit at p_1. Then, his payoff is nonnegative, if and only if p_1 satisfies the following requirement:

$$\begin{aligned} \pi_3(x_3, p_1) &= x_3 + \frac{p_1}{\alpha_1} + \frac{p_1}{\alpha_2} - p_1 \\ &\geq 0 \quad \iff \quad p_1 \leq \frac{\alpha_1\alpha_2}{\alpha_1\alpha_2 - \alpha_1 - \alpha_2} x_3 = \alpha_3 x_3. \end{aligned} \tag{7}$$

This proves that β_3 is a best reply to β_1, β_2.

Similarly, one can show that β_1 is a best reply to β_2, β_3 and β_2 to β_1, β_3.

Now consider the second stage game, played between two bidders, say i and j, in the event when one and only one bidder, to whom we refer as bidder k, has quit during the first round at some price p_{1k}.

This is shown to be an equilibrium as follows: without loss of generality, we set $k = 1$ and $i = 2$, $j = 3$; suppose bidder 2 remains active until bidder 3 quits at some price p_{23}; then, his payoff, π_2, is nonnegative, if and only if p_{23} satisfies the following requirement:

$$
\begin{aligned}
\pi_2(x_2, p_{11}, p_{23}) = & x_2 + \frac{p_{11}}{\alpha_1} + \frac{p_{23}}{\gamma_3} - \frac{p_{11}}{\alpha_1\gamma_3} - p_{23} \geq 0 \\
& \iff p_{23}\left(1 - \frac{1}{\gamma_3}\right) \leq x_2 + \frac{p_{11}}{\alpha_1}\left(1 - \frac{1}{\gamma_3}\right) \\
& \iff p_{23} \leq \left(\frac{\gamma_3}{\gamma_3 - 1}\right) x_2 + \frac{p_{11}}{\alpha_1} \\
& \iff p_{23} \leq \gamma_2 x_2 + \frac{p_{11}}{\alpha_1}
\end{aligned}
\tag{8}
$$

Hence player 2 should quit at price $p = \gamma_2 x_2 + \frac{p_{11}}{\alpha_1}$, as asserted. Similarly, one can show that player 3 should quit at the asserted price. $\diamond$

Not all of these equilibria are plausible. Therefore, in the following we require that an equilibrium be "dynamically consistent", in the sense that if two bidders survive the first round of the auction, they do not change their relative aggressiveness, i.e.

$$\frac{\gamma_i}{\gamma_j} = \frac{\alpha_i}{\alpha_j}. \tag{9}$$

This dynamic consistency requirement selects equilibria that have the following appealing properties:

1. Bidders never raise their proxy bid after they observe the exit by another bidder (this is plausible because if a weak bidder dropped out the remaining players have to become less aggressive or if an aggressive player dropped out then his drop-out is really bad news).

2. Dynamic consistency of equilibrium strategies in the sense that players' strategies in the second round never give rise to a proxy bid below (or equal to) the observed quit price in the first stage;

3. As the price goes up in the second stage, observing that the rival has not yet quit, is always informative about his underlying signal. From the fact that a player i has

not quit in stage 1 until price p_{1k} we already know that $X_i \geq \frac{p_{1k}}{\alpha_i}$, so it could happen that player j knows for sure that player i will not drop out in some interval $[p_{1k}, p]$, but as will be shown below, assumption (9) assures that this does not happen.

In order to prove the first property, note first that (9) implies, since $\gamma_j = \frac{\gamma_i}{\gamma_i - 1}$

$$\alpha_j = \frac{\alpha_i \gamma_j}{\gamma_i} = \frac{\alpha_i}{\gamma_i - 1} \tag{10}$$

Now after bidder k dropped out at p_{1k} bidder i's equilibrium proxy bid is (using $\alpha_k = \frac{\alpha_i \alpha_j}{\alpha_i \alpha_j - \alpha_i - \alpha_j}$)

$$\begin{aligned} b_i(x_i \mid p_{1k}) &= \gamma_i x_i + \frac{p_{1k}}{\alpha_k} \\ &= \gamma_i x_i + p_{1k} \frac{\alpha_i - \gamma_i}{\alpha_i} \\ &< \gamma_i x_i + \alpha_i x_i \frac{\alpha_i - \gamma_i}{\alpha_i} \\ &= \alpha_i x_i = \beta_i(x_i) \end{aligned} \tag{11}$$

where the inequality results from fact that we know that bidder i's initial proxy bid $\alpha_i x_i$ was higher than p_{1k}, the observed drop out price of bidder k. Hence after observing the drop out of bidder k, the other bidders will always reduce their proxy bid, as in the symmetric equilibrium.

Concerning 2), since i's initial proxy bid $\alpha_i x_i$ was higher than k's drop-out price p_{1k}, we know that $x_i > \frac{p_{1k}}{\alpha_i}$. Now (11) yields for $x_i = \frac{p_{1k}}{\alpha_i}$ that $b_i(x_i \mid p_{1k}) = p_{1k}$. Since $b_i(x_i \mid p_{1k})$ is strictly increasing in x_i this implies that $b_i(x_i \mid p_{1k}) > p_{1k}$ for all $x_i > \frac{p_{1k}}{\alpha_i}$ and hence bidder i's updated proxy bid after observing that bidder k dropped out at p_{1k} will always be strictly larger than p_{1k}. Note that we know that $\alpha_i x_i$ is strictly larger than p_{1k}, because if two bidders had chosen the same initial proxy bid, they would both drop at the same price (in an alternative model, where only one of these bidders would be randomly selected to drop out at this price, we would only obtain a weak inequality).

Concerning 3), note that after k dropped at p_{1k}, bidder j knows that bidder i's signal x_i is larger than $\frac{p_{1k}}{\alpha_i}$, but it can be arbitrarily close. From (11), we know that $b_i(x_i, p_{1k}) < \alpha_i x_i$, so i's updated proxy bid can be arbitrarily close to p_{1k} and hence there is no interval where bidder j can be sure that bidder i will not drop out. Thus when the price increases and bidder i does not drop out this will always be informative to bidder j (who can, however, not use this information, because he has already set his proxy bid).

4 PLAUSIBLE ALTERNATIVE STRATEGIES AS REFERENCE POINTS

It is frequently observed that in experiments subjects do not play Nash-equilibrium strategies. In particular in common value auctions bidders often ignore crucial aspects of the game and fall prey to the winner's curse. Therefore, if one tries to identify a pattern of behavior one should not only consider the Nash-equilibrium strategy, but also other strategies that appear plausible for more or less sophisticated players to follow. These include naive strategies that ignore the implications both of winning the auction and of the other players not dropping out until a certain price, as well as strategies that take into account the former but not the latter. We discuss here those that we consider most relevant.

We use these strategies as additional benchmarks to compare our experimental results to and in particular to study whether bidding over time approaches any of these strategies. We will in all cases denote the first stage proxy bid by β_1 and the second stage proxy bid by β_2 and the price where the first stage ended by p_1. Note that β_2 depends on p_1. For notational simplicity we will ignore this below.

In the experiments, signals were independently drawn from a uniform distribution on [1,60]. This will be applied below to derive explicit predictions (and we round the expected value to 30, which subjects who apparently wanted to use the expected value did as well).

4.1 *Naive Strategy — Minimal Information Processing*

The naive strategy ignores the implications of winning the auction and of the fact that at any price the other bidders have not dropped out. The naive bid hence simply equals the expected value of the object given the information at the beginning of the stage. This is the type of naive bidding that typically leads to a winner's curse in a common value auction. In an open ascending auction, however, such a bid is not necessarily too high, because it does not only ignore that winning means that other bidders have a lower signal (if strategies are symmetric) but also that the fact that they have not yet dropped out at the current price implies a lower threshold for their signal. Obviously, a bidder should not conclude that the signal of the bidder who dropped out first is smaller than 0 or larger than 60.

$$\begin{aligned}\beta_1(x) =& x + E[2X] = x + 60\\ \beta_2(x) =& x + E[X] + \beta_1^{-1}(p_1)\\ =& x + 30 + \min\{60, \max\{0, (p_1 - 60)\}\}\end{aligned}$$

We call this strategy N1.

Apart from not being optimal, strategy N1 is not consistent, because if the first stage ends at a price $p_1 > 90$, the remaining bidders will conclude that the bidder who dropped out has a signal $p_1 - 60 > 30$, while at the same time estimating the remaining rival's signal to equal 30. It seems implausible to infer that the remaining opponent has a lower signal than the opponent who quit. More plausible versions of the naive strategy would reevaluate after the first stage and estimate the remaining opponents signal to be either the maximum of $E[X]$ and $\beta_1^{-1}(p_1)$, or, even more reasonable, to be the expected value conditional on being larger than $\beta_1^{-1}(p_1)$. While the latter is far too aggressive, it seems at least consistent in its too extreme aggression. Hence we consider the alternative naive strategies

$$\begin{aligned}\beta_1(x) =& x + E[2X] = x + 60\\ \beta_2(x) =& x + \max\{E[X], \beta_1^{-1}(p_1)\} + \beta_1^{-1}(p_1)\\ =& x + \min\{60, \max\{0, (p_1 - 60), 30\}\} + \min\{60, \max\{0, (p_1 - 60)\}\}\end{aligned}$$

which we call N2, and strategy N3

$$\begin{aligned}
\beta_1(x) =& x + E[2X] = x + 60 \\
\beta_2(x) =& x + E[X|X > \beta_1^{-1}(p_1)\} + \beta_1^{-1}(p_1) \\
=& x + \frac{1}{2}\left(60 + \min\{60, \max\{0, (p_1 - 60)\}\}\right) + \min\{60, \max\{0, (p_1 - 60)\}\} \\
=& x + 30 + \frac{3}{2}\min\{60, \max\{0, (p_1 - 60)\}\}
\end{aligned}$$

All these strategies appear somewhat inconsistent because for small x they imply $\beta_2(x) < p_1$, and hence that a bidder at the beginning of the second stage would wish to have dropped out even earlier. A player who follows such a strategy would then obviously state $\beta_2(x) = p_1$ as his second stage proxy. Such a strategy is inconsistent, because it implies (for $x < 30$ in N1 and N2 and for $x < 20$ in N3) that for all $p_1 > 60$ the bidder would want to quit the auction as soon as possible after the end of the first stage. Then it would obviously have been better to state a lower first-stage proxy.

4.2 *Slightly More Sophisticated Timid Strategy*

Now suppose a bidder is sophisticated enough to know that his bid matters only in the event of winning. This strategy still ignores that the game is dynamic, i.e. that at any price a bidder can infer a lower bound for the signal of the other bidders. In essence this strategy is thus too careful, it manages to avoid the winner's curse, but it does not exploit the information obtained in the course of the auction. Being the most careful strategy, we call this the timid strategy T1. (One might also call this loss avoidance or winner's curse avoidance strategy, because the additional information that it takes into account is just that which, if neglected, causes the winner's curse).

$$\begin{aligned}
\beta_1(x) =& x + E[2X \mid X < x] = 2x \\
\beta_2(x) =& x + E[X|X < x] + \beta_1^{-1}(p_1) \\
=& x + \frac{x}{2} + \frac{p_1}{2} = \frac{3}{2}x + \frac{p_1}{2}
\end{aligned}$$

As for the naive strategy, this strategy appears inconsistent, because $\frac{p_1}{2}$ could be larger than $\frac{x}{2}$, suggesting that the bidder who had dropped out has a higher signal than the remaining bidder. Hence we consider as above the variants

$$\begin{aligned}\beta_1(x) =& x + E[2X \mid X < x] = 2x \\ \beta_2(x) =& x + \max\left\{E[X|X < x], \beta_1^{-1}(p_1)\right\} + \beta_1^{-1}(p_1) \\ =& x + \max\left\{\frac{x}{2}, \frac{p_1}{2}\right\} + \frac{p_1}{2}\end{aligned}$$

which we call T2 and strategy T3

$$\begin{aligned}\beta_1(x) =& x + E[2X \mid X < x] = 2x \\ \beta_2(x) =& x + E[X|\beta_1^{-1}(p_1) < X < x] + \beta_1^{-1}(p_1) \\ =& x + \frac{x + \frac{p_1}{2}}{2} + \frac{p_1}{2} = \frac{3}{2}x + \frac{3}{4}p_1\end{aligned}$$

4.3 *Sealed-Bid Style Strategy*

In this case, bidders use in the first stage the strategy that would be appropriate in a sealed-bid auction. Hence they take into account, that in winning the auction they can bid as if the second highest signal equals their own (because they pay only the second highest bid and hence in an equilibrium if the price equals the winner's own bid this means that the second highest signal equals his own). But the strategy is still too careful, because it ignores that the dynamic price provides information about the lower bound of the third players signal. In the second stage, the bid is, as in equilibrium, $2x + \beta_1^{-1}(p_1)$, but it is actually more aggressive than the equilibrium bid, because $\beta_1^{-1}(p_1) = \frac{2}{5}$ as opposed to $\beta_1^{-1}(p_1) = \frac{1}{3}$ in equilibrium. Since bids in the first stage are less aggressive than in equilibrium, bidders infer a higher signal for the bidder who dropped out and hence bid more aggressively than in equilibrium in the second period. We will refer to this strategy as strategy SB

$$\beta_1(x) = 2x + E[X \mid X < x] = \frac{5}{2}x$$

$$\beta_2(x) = 2x + \beta_1^{-1}(p_1)$$

$$= 2x + \frac{2}{5}p_1$$

5 DESIGN OF THE EXPERIMENTS

Computerized experiments were run at CERGE-EI with students in the preparatory semester for the PhD program. The experimental software was developed using z-Tree (Fischbacher, 1999).

We conducted two treatments: the non-robot treatments, where three subjects interacted repeatedly and the robot-treatments where each subject was matched with two simulated bidders that played the symmetric equilibrium strategy (in the latter case the bidders were informed that the simulated bidders followed the same strategy and that this strategy could be part of an equilibrium without being informed that this was the symmetric equilibrium strategy). In the non-robot treatment, 21 subjects participated in two sessions. The 12 participants in the robot treatment were recruited from the 21 original participants. At the time of recruitment, they were informed that a similar auction would be played, without giving details at this point. The purpose of the robot treatment was to study whether experienced subjects manage to find the equilibrium strategy in a much simpler setting after we observed that no group was able to reach the equilibrium in the non-robot treatment.

In both treatments 40 auctions were conducted. In the non-robot treatment we employed a fixed-matching scheme, the three bidders in one group stayed together for the whole 40 auctions.

In the instructions (see the corresponding author's homepage) we used the frame of an estate consisting of three assets to be auctioned off. Signals denominated in the experimental currency "points" were drawn independently from a uniform distribution on the integers in [1,60], hence the value of the estate was restricted to [3,180].[2]

Each auction was conducted in two stages. First all bidders stated their first-stage proxy bids, restricted to be integers in [0,180]. Then a price clock, starting at 0 increased in integer steps of 1 per 0.3 seconds until the lowest of the first-stage proxy bids was reached. The other two bidders were then informed about the drop-out price p of the first bidder. Then they entered their second-stage proxy-bids, restricted to the integers in $[p, 180]$. The price clock increased again from p until the lower of the second-stage proxy-bids was reached. Then the auction was over and the remaining bidder obtained the estate for the last reading of the price clock. Hence he made a profit equal to the value of the estate (i.e. the sum of the three bidders' signals) less the price where the auction ended. If two bidders stated the same proxy bid in the first stage, they both quit the auction when the price reached that bid and the auction was over. If all remaining bidders in either stage entered the same proxy bid, the winner was randomly selected. Only the winner of the auction was informed about the value of the estate and his profit.

The sessions took altogether between 150 and 180 minutes. At the end of the experiments, earnings were converted into Czech Crowns at a rate of 1 point = 1 CZK and paid in cash on site. Average earnings were 461 CZK (about $ 16.50 or 14.40 Euro at the time of the experiments) in the non-robot treatment (including a show-up fee of 200 CZK and an additional flat payment of 200 CZK that was paid because the sessions took longer than estimated) and 473 CZK (about $ 16.70 or 14.70 Euro at the time of the experiments) in the robot treatment (including a show-up fee of 200 CZK and an additional flat payment of 100 CZK) with minimum earnings of 140 and 380, respectively, and maximal earnings of 850 and 570, respectively.[3]

[2]We choose a minimal signal of 1 instead of 0 in order to be able to distinguish an equilibrium bid from a "friendly" drop-out at 0 in case a bidder draws the minimal signal.

[3]Note that in the Czech Republic these corresponds to rather substantial hourly wages. The subjects were living on stipends of about 6500 CZK per months at the time of the experiment.

6 RESULTS OF THE EXPERIMENTS

6.1 *Non-Robot Treatment*

Table 2 shows descriptive statistics for the non-robot treatment, namely for both stages the equilibrium bid for the given signals, the average actual bid and the average bid relative to the equilibrium bid as well as the shares of bids equal to, larger, and smaller than the equilibrium bid. We see that average first-stage bids are substantially larger than the equilibrium bid, while average second-stage bids are only slightly above the equilibrium.[4] Furthermore, we note that in both stages hardly any bids equal the equilibrium bid. In line with observations for the average bids, first-stage proxy bids are more frequently above the equilibrium than below, while second-stage proxy bids are equally often larger or smaller than the equilibrium bid.

	Stage 1	Stage 2
Average Equilibrium Bid	90.22	92.52
Average Bid	107.07	95.26
Average Bid Relative to Equilibrium Bid	118.7%	103%
bid = equilibrium bid	1.2%	0.2%
bid > equilibrium bid	59.5%	49.8%
bid < equilibrium bid	39.3%	50%

Table 1: Descriptive Statistics for Non-Robot Treatment

In the detailed analysis below, we start by analyzing whether subjects exhibit learning trends towards the (symmetric) equilibrium prediction. Then we study learning trends towards other plausible strategies. Finally, we compare our results to the results we would

[4]Note, however, that second-stage equilibrium bids are computed under the assumption that first stage bids equal equilibrium bids. Given that actual first-stage bids are higher (this holds also for the minimal first-stage bids which are the only ones that the remaining bidders could observe), second-stage bids should actually be smaller.

have obtained in a standard English clock auction, i.e. if we had observed only the lowest of the first- and second-stage proxy bids. This comparison rests on the assumption that our subjects would have indeed followed the strategy corresponding to their proxy bids also in a standard English clock auction without bidding agents, i.e. that they would have dropped out in each stage at that price at which they instructed their proxy bidders to drop out in our setting.

It can happen, either as a result of the inconsistencies of the naive strategies or as a result of very aggressive bids in the first stage, that a strategy predicts $\beta_2(x) < p_1$ (note that except for the naive strategies, this cannot happen if the bidder followed the respective strategy in the first stage). Hence a bidder would choose his second-stage proxy bid equal to p_1. When we study learning trends towards the different strategies, we exclude all such observations, because if we include them a combination of a very aggressive bid in the first stage and an immediate drop-out in the second stage would support that the bidder plays consistently with these strategies in the second stage. Hence if bidders just become more aggressive over time in the first stage and then drop immediately (or at least early) in the second stage, this would appear like a learning trend towards various second stage strategies. In general, the analysis of second-stage bids with respect to trends towards any strategy should be considered with care, because these strategies crucially depend on beliefs about the other bidders' strategies in both the first and second stages and all strategies above are based on the assumption that bidders believe that others follow the same strategy. It does not appear completely implausible, however, that a bidder assumes that other bidders are following, for example, the naive strategy while he plays a best reply towards these strategies. This leads to a large amount of possible strategies that we cannot all analyze.

We run multi-level random-effects panel regressions. These take into account the dependence of the observations on the different levels (subject and group). Logistic regressions are performed when the regressed variable is a dummy.[5]

[5] Standard panel regressions that take the subject as independent observation and ignore the dependence of observations within a group yield nearly identical results. If we take the group as independent observation,

6.1.1 *Learning Trends Towards Plausible Strategies*

Result 1 *Both first- and second-stage proxy bids do not converge towards the equilibrium bids. Subjects do, however, clearly learn qualitative equilibrium behavior.*

Result 1 is supported by the panel regressions. The absolute difference between the observed and predicted first-stage proxy bid increases over time, though not significantly ($p = 26.7\%$). The same holds for the second-stage proxy bids ($p = 58.9\%$). Hence if anything, there is a learning trend away from the equilibrium. The overall number of bids that conform exactly to the equilibrium prediction is with 10 out of 840 in the first stage and only 1 in the second stage very low.

An important qualitative property of the (symmetric as well as asymmetric) equilibria is that second-stage proxy bids are always smaller than first-stage proxy bids. Subjects clearly learn this property. The incidence of bid pairs where the second-stage proxy bid is smaller than the first-stage proxy bid increases significantly over time ($p < 0.1\%$), and that of bid pairs where the second-stage proxy bid is larger decreases significantly ($p < 0.1\%$). In fact, in the first 10 auctions 22.8% bid pairs are increasing, 72.1% decreasing, and 5.1% constant, whereas in the last ten auctions the percentages are 5.7%, 91.4%, and 2.9%, respectively. Indeed the share of decreasing bid pairs increases in all seven groups from the first ten to the last ten auctions and hence this increase is significant according to a Wilcoxon signed-rank test ($p = 1.8\%$, two-sided).

We also analyzed whether the relative bid decrease ($= (\beta_1 - \beta_2)/\beta_1$), given that the bidder lowers his or her bid from the first to the second stage at all, or the difference between the relative bid decrease and the corresponding value for the equilibrium bids changes over time. There is no discernible time trend.

Result 2 *Bidding in the first stage becomes more aggressive over time.*

the results are qualitatively the same.

The incidence of first-stage bids above the equilibrium increases significantly over time ($p = 0.1\%$), while that of bids below the equilibrium decreases ($p = 0.4\%$). Furthermore, the difference between the observed and equilibrium first-stage proxy bids increases over time ($p < 0.1\%$). In the second stage, however, time trends concerning over- and underbidding of the equilibrium are far from significant.

RESULT 3 *First-stage proxy bids tend away from the timid strategy as well as from the sealed-bid strategy over time.*

The absolute difference between the observed and timid first-stage bids increases significantly over time ($p < 0.1\%$) as does the absolute difference to the sealed-bid strategy ($p = 0.3\%$). This effect is clearly consistent with result 2, given that bids are on average already higher than the timid and sealed-bid strategy in the first auctions and that these strategies are more timid than the equilibrium strategy.

RESULT 4 *Second-stage proxy bids tend towards the naive strategy N1.*

The absolute difference between the observed and naive second-stage bids decreases significantly over time ($p = 4.3\%$). In contrast, in the first stage it increases insignificantly ($p = 26.5\%$). It appears doubtful that bidders really "learn" strategy N1 in the second stage, given that behavior in the first stage is rather far from, and does not approach, N1. The behavior of the bidders who dropped out in the first stage, however, was approaching the naive strategy (see Result 7 below). Hence bidding in the second stage according to N1 is less absurd than it may appear at a first glance (since a crucial input in the second stage is not the bidder's own first-stage strategy, but whether the other bidders have followed N1 in the first stage).

RESULT 5 *Second-stage proxy bids tend away from the timid strategies T2 and T3.*

The absolute difference between observed second-stage bids and T2 or T3 increases significantly over time ($p = 0.2\%$, $p = 3.3\%$, respectively). Given that first-stage proxy bids tend away from the timid strategies, this appears to be reasonable, because these strategies are based on the assumption that other bidders follow the timid strategy in the first stage, a belief that is less and less justified over time. Initial play, however, is quite close to the timid strategy.

Due to the increasing aggressiveness of bids, profits are decreasing and losses become more likely over time, though these effects are not significant. Incurring a loss has an insignificant negative effect of the incidence of bidding above the equilibrium in the next period.

Finally, one might wonder whether groups tend to move qualitatively towards asymmetric equilibria. An important property of asymmetric equilibria is that in the first stage one or two bidders bid above the symmetric equilibrium prediction and the other(s) below the symmetric equilibrium and in the second stage one bidder bids above and one bidder below the symmetric equilibrium bid. The incidence of groups satisfying these criteria even decreases marginally over time both in the first and in the second stage. Hence there is no learning trend towards qualitative asymmetric equilibrium behavior.

6.1.2 *Counterfactual Analysis*

We now compare our above results to the results we would have obtained in a standard English clock auction. In order to do this, we restrict the analysis to the lowest of the first- and second-stage proxy bids, which would be the observed drop-out prices in a standard clock design.

While in a standard English clock auction one could also use the observed drop-out price as a lower threshold of the bid of the remaining bidders when estimating bidding functions, this would still not allow us to detect crucial results. Most importantly, with a standard design we could never observe whether a bidder really lowers his intended drop-out price from the first to the second stage within an auction. We could only indirectly address this issue by trying to estimate bidding functions and then inferring whether bidders learn to place relatively lower bids in the second stage.

To be able to detect any clear learning trend would require a very large number of observations. Our design, in contrast, allows for a straightforward test, because we can observe the actual adjustment of intended drop-out prices within an auction.

Result 6 *In a standard English clock auction, learning of qualitative properties of the equilibrium strategy could not have been observed.*

As discussed above, a crucial qualitative property of the equilibrium strategy is that second-stage proxy bids are smaller than first-stage proxy bids and our subjects increasingly conform to this property. In a standard English clock auction, this learning trend could not have been observed, because first-stage and second-stage bids are never both observed for the same subject.

If we restrict the statistical analysis to the lowest proxy bids, which correspond to the bids that would have been observed in a standard English clock auction without proxy bids, the absolute difference between the observed and the equilibrium bids in the first stage is decreasing over time, though far from significantly ($p = 45.8\%$). Including in the regression for the whole data set an interaction term period $\times$ "lowest" where "lowest" is a dummy for the lowest proxy bid, yields a deeper understanding. The effect of the period is (marginally) significantly positive ($p = 5.2\%$), while the interaction effect is (marginally) significantly negative ($p = 8.0\%$). Hence for those bids that would not have been observed in a standard English clock auction, there is a significant trend away from the equilibrium, while this effect is significantly weaker (to an extent that the trend is reversed) for the bids that would have been observed.[6]

Figure 1 shows the development of the average of all first-stage proxy bids as well as the average of the losing (i.e. lowest) first-stage proxy bids relative to the symmetric equilibrium. Neither all nor the lowest bids appear to approach the equilibrium.

[6]Note, however, that even the lowest first-stage proxy bids are substantially (10.6%) higher that the equilibrium bids for the given signals.

Result 7 *The first-stage bids that would have been observed in a standard English clock auction tend towards the naive strategy.*

Figure 2 shows the development of the average of all first-stage proxy bids as well as the average of the losing (i.e. lowest) first-stage proxy bids relative to the naive strategy. We see that while the average of all bids are consistently above the naive strategy, the lowest bids appear to approach the naive strategy from below. This picture would have been obtained in a standard English clock auction given the same strategies.

This result is supported by a regression restricted to the lowest first-stage proxy bids, where the absolute difference between these bids and the naive strategy bids decreases (marginally) significantly ($p = 7.8\%$) over time. Including in the regression for the whole data set the interaction term period $\times$ lowest yields a significantly positive effect of the period ($p = 2.3\%$), while the interaction effect is significantly negative ($p = 2.1\%$). Hence the proxy bids that would not have been observed in a standard English clock auction tend away from the naive strategy, while the interaction effect is significantly negative to an extent that the effect is completely reversed. Furthermore, in a standard English clock auction the tendency of first-stage bids away from the timid strategy would be underestimated in a standard English clock auction, since the effect would have been only marginally significant ($p = 8.9\%$) and the tendency away from the sealed-bid strategy would be missed completely, being insignificant ($p = 57.1\%$).

Result 8 *The second-stage bids that would have been observed in a standard English clock auction do not show the clear tendencies towards N1 or away from T2 and T3.*

Restricting the analysis to the lower second-stage bid, there is no time trend in the difference between observed strategies and N1 ($p = 82.4\%$). Similarly, the tendency of second-stage bids away from T2 would be underestimated, being only marginally significant ($p = 6.2\%$), and that away from T3 would be completely missed, being insignificant ($p = 79\%$).

To summarize, under the assumption that bidders would have bid according to the proxy bids that they placed in our experiment also in a standard English clock auction, the results in the latter would have been misleading because they would have suggested that bidders "learn" the naive first-stage strategy whereas the analysis for the complete data set shows that they clearly do not. Interestingly, in the second stage, this result is reversed, i.e. our results suggest learning of the naive strategy, which would not have been observed in the standard format. Most importantly, we observe clear learning of qualitative aspects of the equilibrium that could not be observed in the standard format. Hence our subjects appear to be more rational than a standard English clock auction might have suggested.

One potential reason why subjects have learned to lower their second-stage proxy bids compared to their first-stage proxy bid might be that placing high first-stage proxy bids is relatively risk-free because the probability that the auction ends after the first stage (if both other bidders place the same first-unit proxy bid) is very low and dropping out at the beginning of the second stage possible. Hence subjects could at a low expected cost stay in until the second stage to gather additional information.[7] Behaving along these lines leads to overall more aggressive bids in the first stage and to lowering of bids in the second stage. Two subjects followed this strategy to the extreme, by usually stating the maximal admissible bid of 180 in the first stage and often placing a second-stage bid equal to the price where the first-stage ended, attempting to drop out immediately. This strategy did not do well and hurt the other bidders in the group.[8]

[7]If, however, two bidders are following this strategy, then the likelihood that they will both try to drop out at the beginning at the second stage and thus one of them makes a loss is quite high. The problem is obviously more severe if all three do.

[8]One might suspect that these two subjects drive the results of the increased aggressiveness of bidding. This is, however, not the case. If we include the two groups from the analysis this does not change the significance of the results with respect to increased overbidding on the first unit or the increase in the frequency of decreasing bid pairs or decrease of increasing bid pairs.

6.2 *Robot Treatment*

Table 2 shows descriptive statistics for the robot treatment. We see that again average first-stage, but not second-stage proxy bids are substantially higher than equilibrium bids for the given signals, although overbidding in the first stage is less severe than in the non-robot treatment. We also note that the number of bids equal to the equilibrium is remarkably high in this treatment, an observation we get back to below.

	Stage 1	Stage 2
Average Equilibrium Bid	95.79	86.49
Average Bid	106.13	89.83
Average Bid Relative to Equilibrium Bid	110.8%	103.9%
bid = equilibrium bid	26%	23.1%
bid > equilibrium bid	44.6%	46.2%
bid < equilibrium bid	29.4%	30.8%

Table 2: Descriptive Statistics for Robot Treatment

The 12 subjects who participated in the treatment where they were matched with two robot bidders had taken part in the non-robot treatment before. Hence some of them have interacted in a group before and they might have discussed the experiment in the days between experiments. Hence the observations in the robot treatment are not statistically independent and our statistical analysis that treats them as independent should be considered with care and rather be seen as illustrative. As far as learning within the robot treatment is considered, the assumption of independence, might however, not be considered too problematic, because bidders did not interact within this treatment and hence learning occurred independently. Since each "group" contains only one human subject, each subject constitutes an independent observation and hence we use standard panel regressions taking the dependence of observations for each subject into account.

Had we initially planned to run the robot treatments, we would have provided subjects with code numbers to be able to match their behavior in the no robot treatment with their behavior in the robot treatment. Unfortunately, this was not done as the robot treatment was inspired by the results from the first treatment.

6.2.1 *Learning Trends Towards Plausible Strategies*

Among the 12 bidders, 5 learned to play the equilibrium strategy after several auctions. As was revealed in discussions after the experiment, however, this learning was partly due to a coincidence. Since signals were integers, all first-stage proxy bids by the robot bidders were divisible by three. Some bidders noticed this, inferred correctly that this might indicate that robots bid three times their signal in the first stage, calculated the strategy that the robots must have used in the second stage given the feedback they received about the total valuation and the drop-our prices and confirmed their hypothesis in the next auctions. They then copied this strategy because they were informed that the robot's strategy formed part of an equilibrium (though they were not informed that it was a symmetric equilibrium, this seemed a reasonable guess).[9] These subjects who copy the equilibrium strategy, drive the results towards equilibrium. It is, however, potentially more interesting to study the behavior of the remaining subjects (and of the copying subjects before they find the equilibrium strategy) to see whether they exhibit learning trends towards the equilibrium or away from the equilibrium. Hence we also present an analysis restricted to the bids that are not exactly equal to the equilibrium.

Result 9 *The incidence of exact equilibrium bids in both stages clearly increases over time. However, for those subjects who do not copy the equilibrium strategy, there is no clear trend towards the equilibrium.*

[9] That 5 out of 12 subjects inferred the strategy of the robot bidders correctly in reasonably short time is quite impressive even in spite of the integer bids. Our subjects were, however, clearly mathematically more skilled than the average subject in an economics experiment.

The incidence of exact equilibrium bids increase over time ($p < 0.1\%$) in both stages and the absolute difference between observed and equilibrium bids decreases over time both in the first ($p < 0.1\%$) and in the second stage ($p = 3.5\%$). However, the effect is far from significant ($p > 15\%$) if bids that exactly match equilibrium bids are excluded. Hence the learning trend seems to be almost exclusively driven by the players who understand and copy the strategy of the robot bidders and not by gradually learning to play a best reply to the robots' strategy. Similarly, qualitative equilibrium behavior, i.e. lower second-stage than first-stage bids, increases significantly over time ($p = 0.6\%$) but this effect is not significant in the set of bid pairs that do not match the equilibrium ($p = 15.3\%$). Note, however, that subjects had learned to lower their second-unit bids already in the first treatment, so that there was only little room for learning in this respect. In fact, already in the first 10 auctions in 82.9% of bid pairs the second stage bid was lower than the first-stage bid, while the reverse was true in only 9.8% of bid pairs and in the last 10 auctions the respective shares are 95.3% and 0% (the incidence of larger second-stage bid indeed decreases significantly, $p = 3.0\%$, even in the data set restricted to non-equilibrium bids). The share of decreasing bid pairs is already 1 for 5 out of 12 subjects in the first 10 auctions and increases in the last 10 auctions only for 5 subjects. Hence according to a Wilcoxon signed-rank test the increase in the share is not significant ($p = 23.4\%$, two-sided).

We also analyzed whether the relative bid decrease ($= (\beta_1 - \beta_2)/\beta_1$), given that the bidder lowers his or her bid from the first to the second stage at all, or the difference between the relative bid decrease and the corresponding value for the equilibrium bids changes over time. The latter decreases significantly over time ($p < 0.1\%$), but this effect is again exclusively driven by the subjects who do found the equilibrium strategy. Excluding these, the effect disappears ($p = 76.7\%$).

Result 10 *Bidding in both stages becomes more aggressive over time. Bidding above the equilibrium becomes less frequent over time. The latter effect, however, is driven by the bidders who copy the equilibrium strategy.*

In the first stage, the incidence of proxy bids below the equilibrium significantly decreases over time, both for the whole data set ($p < 0.1\%$) and excluding bids that are exactly equal to the equilibrium ($p = 1.7\%$). Also, the difference between the bids and the equilibrium bid increases over time ($p < 0.1\%$, both including and excluding equilibrium bids). The incidence of proxy bids above the equilibrium, however, decreases significantly ($p = 0.2\%$), but increases among the bids not equal to equilibrium bids ($p = 1.7\%$). Hence while all bidders become less timid (underbid less), those who did not copy the equilibrium strategy become more aggressive (overbid more).

The results are comparable in the second stage. Bidding below equilibrium decreases significantly both including ($p < 0.1\%$) and excluding equilibrium bids ($p = 6.2\%$). The difference between the bids and the equilibrium bids increases over time ($p < 0.1\%$, both including and excluding equilibrium bids). The incidence of proxy bids above the equilibrium, however, decreases significantly ($p = 7.6\%$), but increases among the bids not equal to equilibrium bids ($p = 6.2\%$). Hence also in the second stage all bidders become less timid (underbid less), and those who did not find the equilibrium become more aggressive (overbid more).

RESULT 11 *In the first stage bids move away from the naive strategy and the timid strategy over time.*

The absolute difference between the first-stage proxy bids and the naive bids clearly increases over time ($p < 0.1\%$), also if equilibrium bids are excluded ($p = 0.8\%$). The absolute difference between first-stage bids and the timid strategy increases significantly over time ($p = 4.5\%$ if all bidders are included, $p < 0.1\%$ if those exact equilibrium bids are excluded).

RESULT 12 *In the second stage bids move away from the naive strategies and the timid strategies T1 and T2.*

Similarly in the second stage, the absolute difference between bids and all naive bids (N1, N2, and N3) clearly increases over time ($p < 1\%$), also if equilibrium bidders are excluded ($p < 1\%$). The absolute difference between bids and the timid strategy T1 increases over time ($p = 2.2\%$), also if equilibrium bids are excluded ($p = 0.1\%$). The increase in the difference between bids and T2 is significant only if equilibrium bids are excluded ($p = 1.1\%$, otherwise $p = 25.8\%$).

The probability of losses is increasing over time, though far from significantly. Bidding above the equilibrium in the first stage decreases significantly if a loss has been incurred before ($p < 0.1\%$), but losses have virtually no effect on overbidding in the second stage. Both profit and the deviation of the profit from the equilibrium profit virtually do not change over time.

6.2.2 *Counterfactual Analysis*

As in the no-robot treatment, the qualitative learning of the equilibrium in the sense of lower second-stage bids could not have been observed in a standard English clock auction. However, in both stages the incidence of equilibrium bids increases significantly ($p < 0.1\%$) also among the lowest proxy bids. Learning trends towards the equilibrium might have been even overestimated because if the analysis for the absolute difference between the bid and the equilibrium bid is restricted to the lowest proxy bids, the coefficient for *period* is about twice as large as for the complete data set.

Result 13 *In a standard English clock auction, the increased aggressiveness of bidders in the second stage would not have been observed.*

Restricting the analysis to the lowest second-stage proxy bids, the incidence of bids above the equilibrium decreases significantly for the whole data set ($p = 0.2\%$), but it also decreases, though insignificantly ($p = 49\%$) among the non-equilibrium bids. Including an interaction term for the period and a dummy for the lowest proxy bid yields no effect of period but a (marginally) significantly negative interaction effect ($p = 6.9\%$). Excluding the equilibrium bids, period has a clear positive effect ($p = 0.7\%$), but the interaction effect is negative ($p = 1.6\%$). Hence the bids that would not be observed in a standard ascending auction become clearly more aggressive, while those that would be observed, do not at all. Similarly, for the lowest proxy bids, the increase in the difference to the equilibrium bid is only marginally significant ($p = 7.5\%$) and not significant if equilibrium bids are excluded ($p = 19\%$).

RESULT 14 *In a standard English clock auction, it would not have been observed that bids in the first stage move away from the naive strategy and the timid strategy. In addition, a trend towards the sealed-bid strategy would have been observed.*

In the first stage the increase in the absolute difference between the lowest proxy bids and the naive bids is far from significant ($p = 71.7\%$) and the absolute difference even decreases if equilibrium bids are excluded ($p = 43.5\%$). The difference between the lowest proxy bid and the timid strategy even decreases insignificantly over time ($p = 24.7\%$) and increases only insignificantly if equilibrium bids are excluded ($p = 24.6\%$).

The difference between the lowest proxy-bid and the sealed-bid strategy decreases significantly over time ($p = 0.5\%$), but there is no clear trend in the difference between all first-stage proxy-bids and the sealed-bid strategy ($p = 24.4\%$). If equilibrium bids are excluded, then there is a slight tendency away from the sealed-bid strategy ($p = 9.9\%$), but this effect would not have been observed in a standard English clock auction ($p = 74.5\%$).

RESULT 15 *In a standard English clock auction, it would not have been observed that bids in the second stage move away from the timid strategies.*

There is no time trend in the difference between the lowest second-stage bids and T1, both including ($p = 95.4\%$) and excluding ($p = 44.2\%$) equilibrium bids. The increase in the difference between the lowest second-stage bids and T2 is marginally significant only if equilibrium bids are excluded ($p = 9.5\%$, otherwise $p = 81.8\%$).

7 DISCUSSION

We have proposed an equivalent format of the English clock auction, the proxy bidder augmented English clock auction. This auction format makes the quitting rules of all bidders visible, and this in all rounds of the auction. In an application to a three player wallet auction, we have demonstrated that it allows us to detect learning of qualitative equilibrium behavior that could not have been observed in a standard English clock auction. Furthermore, we show by comparison with a counterfactual analysis that the standard design would have implied wrong conclusions based on our data. A critical assumption on which our conclusions rest is that subjects recognize that the proxy bidder augmented English clock auction is strategically equivalent to a standard English clock auction and hence follow the same strategy in both formats. The results by Kagel et al. (2004) suggest that this assumption is justified, but further research should provide evidence on this. Given that the assumption is supported, we advance the proxy bidder augmented English clock auction as an alternative to standard English clock auctions that yields more, and less biased, information.

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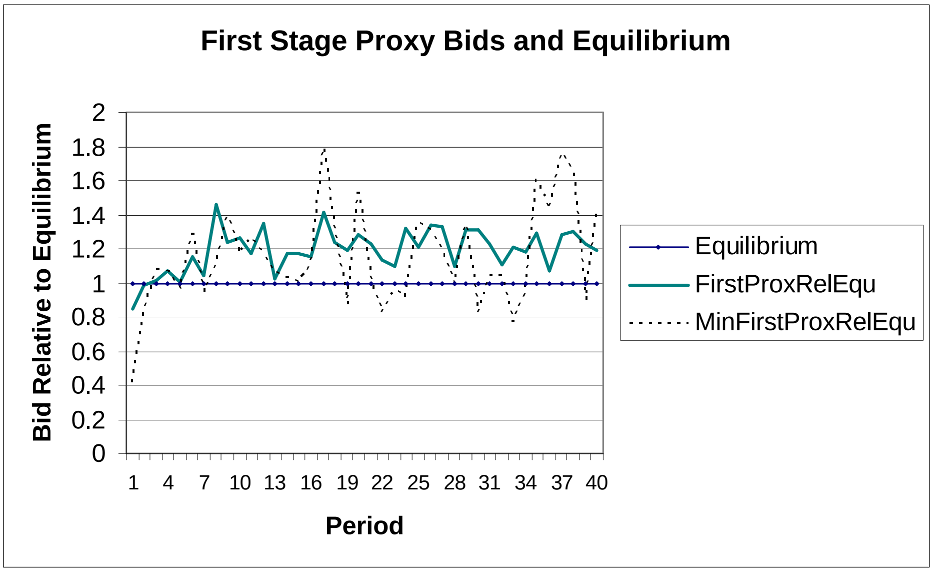


Figure 1: Development over time of the average of all first-stage proxy bids (FirstProxRelEqu) as well as the losing (i.e. minimal) first-stage proxy bids (MinFirstProxRelEqu), both relative to the symmetric equilibrium strategy bid.

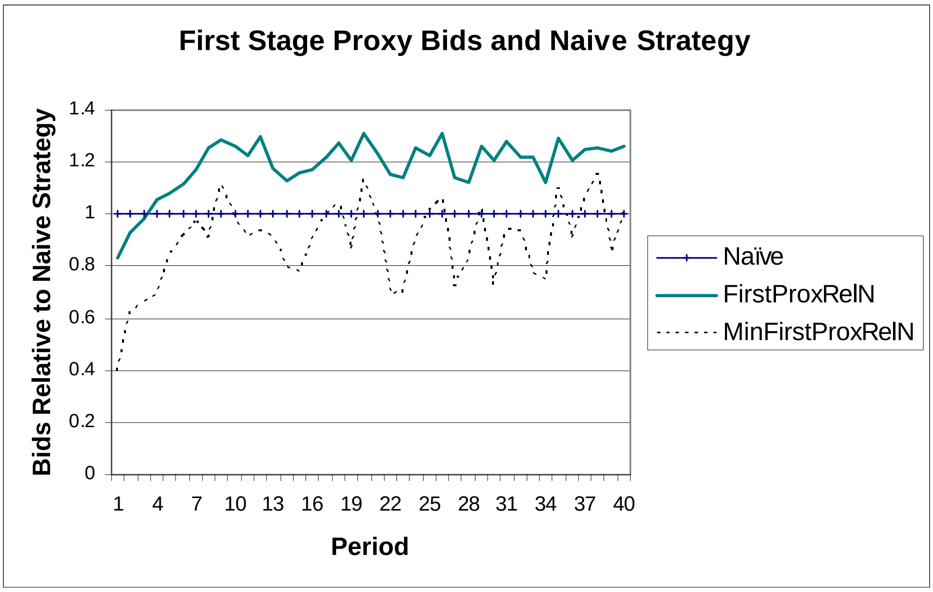


Figure 2: Development over time of the average of all first-stage proxy bids (FirstProxRelN) as well as the losing (i.e. minimal) first-stage proxy bids (MinFirstProxRelN), both relative to the naive strategy bid.

Appendix (Not for Publication)

Instructions (Non-Robot Treatment)

You and two other participants are bidding for an estate that shall be liquidated, in an open, ascending bid, clock auction with bidding agents.

The estate consists of three assets: A, B, and C. The total value of the estate is equal to the sum of the values of these three assets.

At the time of the bidding, no bidder knows the value of the estate. However, each bidder is familiar with one of these assets, and knows its value, but does not know the value of the other two assets. This information is exclusive, i.e. each bidder knows the value of a different asset. All of this is common knowledge, i.e. all bidders have the same instructions.

For example, you may know the value of asset A, but not that of B and C; whereas another bidder knows only the value of B, and the third bidder knows only the value of C. Each bidder thus receives one **signal** of the value of the estate and **the total value of the estate is equal to the sum of the three signals**.

The auction is a "clock" auction in the sense that a price clock indicates that you can either bid the current reading of the clock or quit the auction. If a bidder quits the auction, he cannot return.

The auction is "open" in the sense that you are informed if one of the other bidders has quit the auction, and at which clock reading this had occurred.

The auction is "ascending bid" in the sense that the price clock moves up, at a fixed rate of 1 point per 0.3 seconds, starting at a price of 0.

Instead of bidding incrementally, you will use a bidding agent, who does the bidding for you. All you need to do is to instruct your bidding agent at which reading of the price clock he should stop bidding up.

The auction proceeds in **two rounds**. **In the first round**, after being informed about your signal (the value of one of the assets A, B, or C), you are asked to instruct your bidding agent to stop bidding up for you at a certain reading of the price clock. This price is called your **first-round proxy bid**. When all three bidders have chosen their first-round proxy bids, the price clock starts increasing. Your agent will keep on bidding until the price reaches your proxy bid and will then quit the auction.

When the price clock has reached the lowest of the proxy bids of the three bidders, the bidder who had placed this proxy bid has quit the auction, and the first round is closed. The auction will then proceed to the **second and final round**. The two remaining bidders are informed that the first bidder quit the auction and at which price he quit. They will then place new proxy bids, called their **final-round proxy bids**. These can be equal to or different from their first-round proxy bids. The first-round proxy bids of the two remaining bidders become irrelevant. After both remaining bidders have chosen their final-round proxy bids, the price clock will resume increasing, starting from the price at which the first round ended. Hence the final-round proxy bids cannot be lower than the lowest of the three first-round proxy bids.

The final round of the auction ends as soon as one of the remaining bidders quits the auction, that is when the price reaches the lower of the two final-round proxy bids. The price clock stops when this bidder quits; the last reading of the price clock is called the winning bid. The remaining active bidder is the winner; he or she receives the value of the estate (the sum of the values of the assets A, B, and C, or the sum of the signals of all three bidders), and pays a price equal to the winning bid.

If two bidders choose the same proxy bid in the first round, both will quit the auction when the price reaches this proxy bid and the remaining bidder will be awarded the estate for this price. If all three bidders choose the same proxy bid, they will all quit the auction when the price reaches this proxy bid. One bidder will then randomly be chosen and will be awarded the estate for this price. If the two remaining bidders choose the same proxy bid in the final round, they will both quit the auction when the price reaches this proxy bid. One bidder will then randomly be chosen and will be awarded the estate for this price.

The following round-by-round description summarizes the procedure.

Round 0: Each bidder is privately informed about the value of one of the assets A, B, or C; each bidder is informed about the value of a different asset. This information is displayed on the computer screen of each bidder, and not visible to the other bidders.

Round 1: Each bidder chooses his or her first-round proxy bid. The price clock starts at 0. The price clock increases until it reaches the lowest of the three proxy bids. This bidder is eliminated.

Round 2: The remaining two bidders are informed that the first bidder quit the auction. They choose their final-round proxy bids. The price clock starts at the price at which the first bidder quit. The price clock increases until it reaches the lower of the two final-round proxy bids. The auction ends and the remaining active bidder is awarded the estate for the price at which the auction ended.

When the auction ends, the winning bidder will be informed about the price at which the second bidder quit, which is equal to the price he or she has to pay for the estate, the total value, his or her resulting profit and his or her total profit so far. The other bidders will not be informed about the total value of the estate.

The parameters of the Auction

Overall, **40 auctions will be conducted**. You will always interact with the same two other bidders. You will not know who these two other bidders are and you will also not be informed about this after the experiment. No other participant will be informed about your earnings.

You will start with an **initial capital of 200 points**. Note that **you can make losses** during the course of the experiment. You can, however, always bid in a way that prevents losses.

In each auction, each bidder's signal will be drawn *independently* from the integers in the interval [1,60] points. Each integer is equally likely. Note that since the other bidders' signals are drawn from the same distribution, if your signal is x then you know the minimal possible value of the estate is $x + 2 * 1 = x + 2$ and the maximal possible value of the estate is $x + 2 * 60 = x + 120$.

Since the maximal possible value is $3 * 60 = 180$, the maximal permitted proxy bid is 180. Negative or non-integer proxy bids are not allowed.

At the end of the experiment, your points will be converted into Czech crowns at a rate of

1 point = 1 CZK

and will be paid in cash immediately.

Examples

A few examples to illustrate (note that these examples shall not suggest any reasonable behavior of the bidders, they only illustrate the mechanism):

1. Let the signals of the three bidders be $x_1 = 23, x_2 = 45, x_3 = 12$. Assume they choose first-round proxy bids $b_1 = 88, b_2 = 36, b_3 = 39$.

   Hence the first round ends at $p = 36$. Bidder 2 is eliminated. Bidders 1 and 3 place their final-round proxy bids, which are not allowed to be smaller than 36. Assume the bids are $b_1^f = 77, b_3^f = 45$. The final stage hence ends at $p = 45$ and bidder 1 is awarded the estate for $p = 45$. Since the total value of the estate is $x_1 + x_2 + x_3 = 23 + 45 + 12 = 80$, bidder 1 makes a profit of $80 - 45 = 35$.

2. Let the signals of the three bidders be $x_1 = 15, x_2 = 2, x_3 = 18$. Assume they choose first-round proxy bids $b_1 = 48, b_2 = 16, b_3 = 32$.

   Hence the first round ends at $p = 16$. Bidder 2 is eliminated. Bidders 1 and 3 place their final-round proxy bids, which are not allowed to be smaller than 16. Assume the final-round proxy bids are $b_1^f = 41, b_3^f = 41$. The final stage hence ends at $p = 41$. The estate is then randomly allocated to one of bidders 1 and 3 for $p = 41$. Since the total value of the estate is $x_1 + x_2 + x_3 = 15 + 2 + 18 = 35$, this bidder makes a loss of $35 - 41 = -6$.

3. Let the signals of the three bidders be $x_1 = 45, x_2 = 55, x_3 = 12$. Assume they choose first-round proxy bids $b_1 = 50, b_2 = 100, b_3 = 50$.

   Hence the first round and the whole auction ends at $p = 50$. Bidder 2 is awarded the estate for $p = 50$. Since the total value of the estate is $x_1+x_2+x_3 = 45+55+12 = 112$, bidder 2 makes a profit of $112 - 50 = 62$.

4. Let the signals of the three bidders be $x_1 = 4, x_2 = 33, x_3 = 21$. Assume they choose first-round proxy bids $b_1 = 99, b_2 = 99, b_3 = 99$.

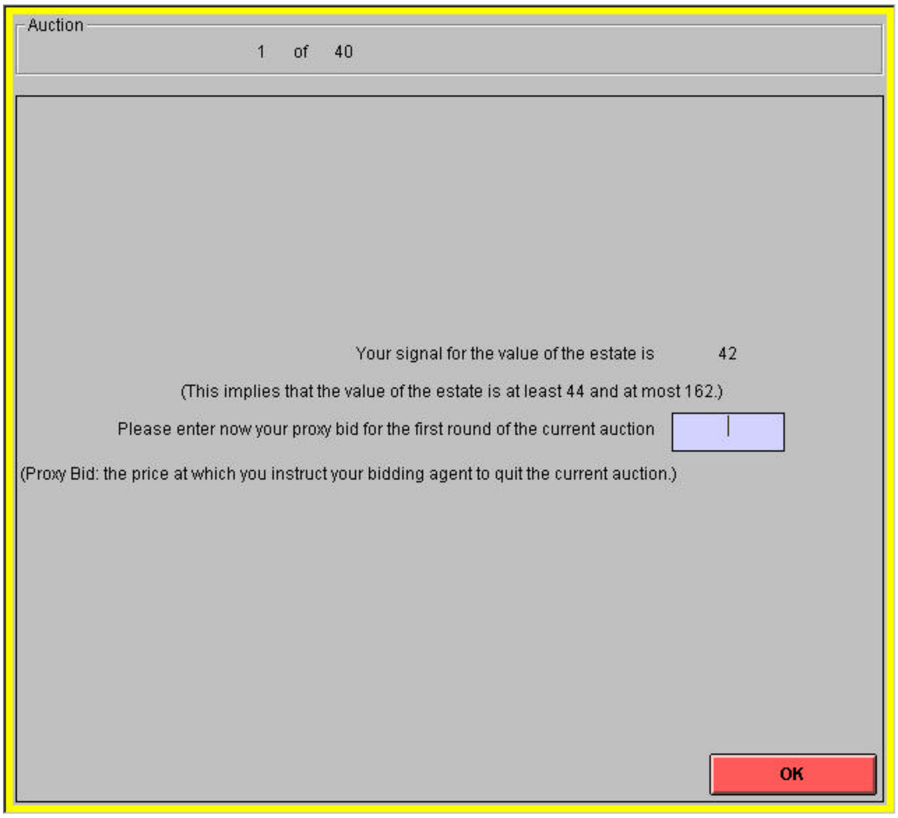
   Hence the first round and the whole auction ends at $p = 99$. The estate is then randomly allocated to one of the bidders for $p = 99$. Since the total value of the estate is $x_1 + x_2 + x_3 = 4 + 33 + 21 = 58$, this bidder makes a loss of $58 - 99 = -41$.

Screen shots

The following screen shots show examples of what you will see during the course of the auction.

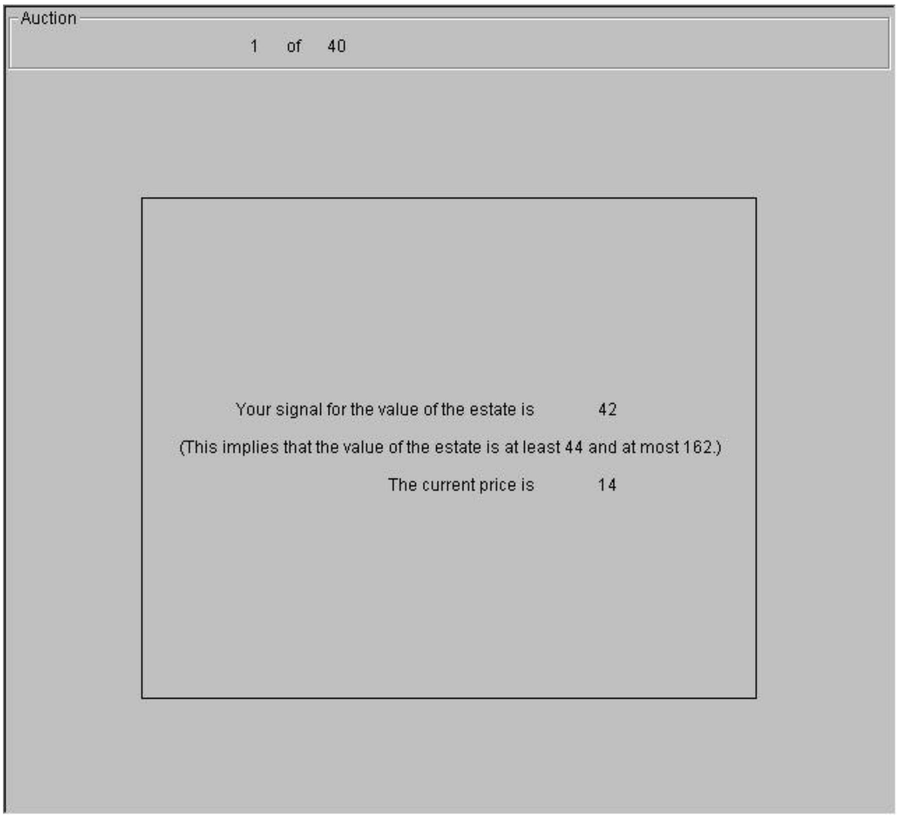
Shot 1: Receiving the Private Signal and Entering the First-Round Proxy Bid

You learn that one of the three assets of the auctioned estate has value 42. Hence you know that the total value of the estate is at least 44 and at most 162. You are asked to enter your first-round proxy bid.



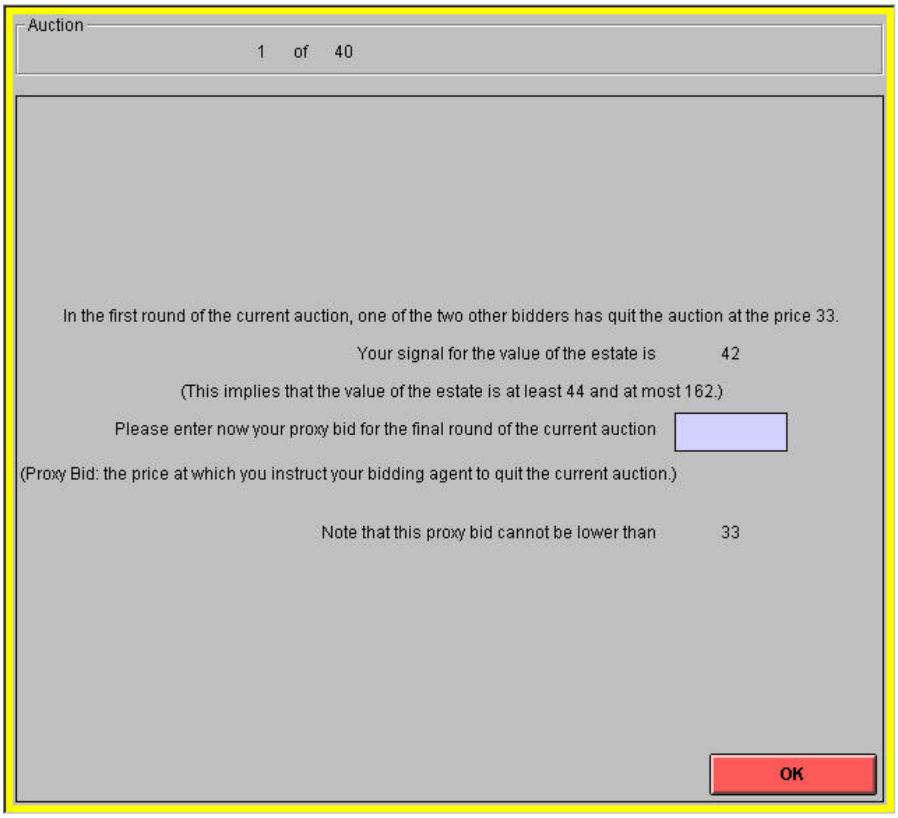
Shot 2: Price Clock in the First Round

You are informed again about your private signal while the price increases. The current price is 14. In the final round of the auction, the screen looks the same.



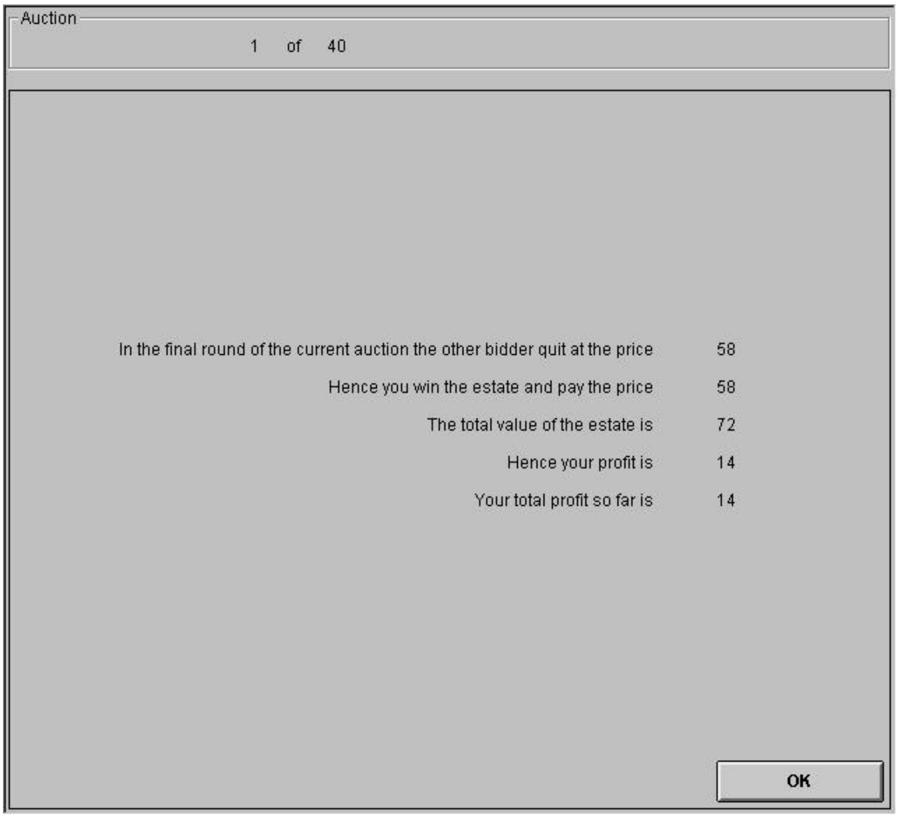
Shot 3: Feedback after the First Round and Entering the Final-Round Proxy Bid

You are informed that one bidder has quit the auction at a price 33, i.e. the lowest proxy bid in the first round was 33. You are asked for your final-round proxy bid, which cannot be lower than 33.



Shot 4: Feedback after Winning the Auction

You are informed that the other remaining bidder in the final round quit the auction at a price 58. Hence you win the estate for a price 58. You are also informed about the total value of the estate which is 72 and your resulting profit $72 - 58 = 14$. Your total profit after the first auction of course equals the profit from this auction. Note, however, that your total capital now is 214, because you start with an initial capital of 200.



Quiz

Please answer the following questions. They serve only to check that you completely understood the instructions. Wrong answers have no consequences.

1. Assume that bidders 1,2, and 3 submit the first-round proxy bids

$$b_1 = 127, b_2 = 59, b_3 = 19.$$

   (a) What happens when the price reaches 19?

   (b) Which is the smallest possible proxy bid in the final round?

   (c) Assume that the final-round proxy bids are

$$b_1 = 133, b_2 = 41.$$

   What happens when the price reaches 41?

   (d) Suppose the bidders' signals were

$$x_1 = 23, x_2 = 32, x_3 = 36.$$

   Compute bidder 1's profit

2. Assume that the bidders submit first-round proxy bids

$$b_1 = 42, b_2 = 42, b_3 = 68.$$

   (a) what happens when the price reaches 42?

   (b) Suppose the bidders' signals were

$$x_1 = 23, x_2 = 2, x_3 = 14.$$

   Compute bidder 3's profit

Instructions (Robot Treatment)

The experiment you will participate in today is **identical** to the experiment which you participated in last week, **with one substantial difference.** Attached find the instructions for last week's experiment if you need to refresh your memory about the rules of the auction.

The difference from last week's experiment is that this time the two other bidders are not other participants in the experiment, but **two simulated bidders**. These bidders are programmed to **follow a particular strategy throughout the experiment**. Both will follow the same strategy. They will not change their strategy during the course of the experiment.

These strategies are not sophisticated in the sense that they do not try to understand the strategy you are playing and to choose a best reply to your strategy. Hence you cannot "teach" the simulated bidders. The strategies are, however, "reasonable", in particular they form part of an equilibrium. That is, if you choose a best reply to these strategies, they will also be best replies to your strategy.

The auction will be run exactly in the same way as last week's auction. In particular, **signals for the bidders will be randomly drawn** and the strategies of the simulated bidders condition only on their own signal and what they observe during the course of the auction. Since signals will be randomly drawn for the simulated bidders, their proxy bids can differ from one auction to another although they follow the same strategy in each auction.