AN EFFICIENT TECHNIQUE FOR THE RIGOROUS ANALYSIS OF SHIELDED CIRCUITS AND ANTENNAS OF ARBITRARY SHAPES

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Abstract— **This paper presents an efficient technique for the analysis of arbitrary shaped circuits and antennas when embedded in metallic cavities. The technique uses the integral equation formulation and is based on the representation of the rigorous spatial domain boxed Green's functions in terms of modal series expansions. A new analytical integration scheme extended to arbitrary triangular domains is derived and asymptotic extraction procedures are used to enhance the convergence of the integral equation kernel. The technique thus derived is very efficient computationally and the simulated results show good agreement with measurements.**

I. Introduction

The analysis of shielded circuits and antennas is a subject that has always attracted much attention and numerous numerical models have been developed in the past [1]--[3]. Among the techniques commonly used, the Integral Equation (IE) approach has become very popular because of its computational efficiency. The main difficulty, however, of the integral equation approach when formulated in the space domain, is the slow convergence behavior of the modal series involved. Traditionally, the use of the Fast Fourier Transform (FFT) to accelerate the series convergence has been successfully used [1] but this method restricts the subsequent application of the Method of Moments (MoM) to uniform meshes. In consequence, the discretization of arbitrary circuits of complex shapes becomes a difficult task. Recently, two more contributions on series acceleration without the FFT have been reported [2], [3] but a common feature of all above techniques is that they are combined with MoM algorithms based on subsectional roof-top functions, so that the discretization of the circuit's geometries is only accurate when rectangular shapes are involved.

In this paper we provide a new contribution to the analysis of arbitrary shaped shielded circuits and antennas. The technique is based on the space domain integral equation and a new analytical integration scheme on arbitrary triangular domains have been derived, thus allowing the discretization of the geometries involved using general triangular cells based MoM algorithms [4]. To alleviate the problems of the modal series convergence, an asymptotic extraction procedure similar to the one described in [3] is used. The static part of the resulting kernel is frequency independent and convergence is aided with the use of the analytical integration technique derived on the MoM basis and test functions. On the other hand, the dynamic part of the kernel is frequency dependent but convergence is faster due to the asymptotic extraction technique used. The technique derived has shown to be computationally efficient and numerically accurate when compared with measurements.

II. Theory

Following the IE approach in the space domain, the dyadic electric and magnetic field Green's functions are written using the well known modal expansions [5] as

$$\overline{\overline{G}}_{E_J} = \sum_m V_m(z,z')\,\bar{e}_m(x',y')\,\bar{e}_m(x,y), \tag{1a}$$

$$\overline{\overline{G}}_{H_J} = \sum_m I_m(z,z')\,\bar{e}_m(x',y')\,\bar{h}_m(x,y), \tag{1b}$$

$$\overline{\overline{G}}_{E_M} = \sum_m V_m(z,z')\,\bar{h}_m(x',y')\,\bar{e}_m(x,y), \tag{1c}$$

$$\overline{\overline{G}}_{H_M} = \sum_m I_m(z,z')\,\bar{h}_m(x',y')\,\bar{h}_m(x,y), \tag{1d}$$

where $V_m(z,z')$, $I_m(z,z')$ are voltages and currents computed in the transverse transmission line equivalent networks of the specific structure under analysis [6], $\bar{e}_m$, $\bar{h}_m$ are the vector mode functions of electric and magnetic types and the index m runs to all $\text{TE}_{\mathtt{m},\mathtt{n}}$ and $\text{TM}_{\mathtt{m},\mathtt{n}}$ modes in the cavity.

The main difficulty in the implementation of this approach is in the summation of the infinite series shown in (1). Indeed, the associated series exhibit a very slow convergence behavior and there is lack in numerical techniques to efficiently perform the required summations. A way to circumvent this problem is to use the basis and test functions of the MoM algorithm to accelerate the convergence behavior of the series. The problem of using this technique, however, is that standard numerical integration on triangular domains can not be used, because it would require prohibitive large number of integration points, specially when the order of the mode increases. Fortunately, for the case of a rectangular cavity, analytical integration is possible and a simple procedure has been derived. The analytical details are straightforward but cumbersome and hence they will not be reported here for the sake of space. An overview of the integration algorithm will be given in the conference.

Using this analytical integration technique, the convergence of the integral equation kernel is greatly improved and in order to further enhance the convergence properties of the series involved, an asymptotic extraction procedure can, in addition, be implemented [3]. To start first note that when the Green's functions in (1) are introduced in a standard integral equation formulation any MoM matrix coefficient inside a cavity can be easily cast in the following general form

$$R(i,k) = \sum_m t_m(r,s)\, I_f(m,i)\, I_f(m,k), \qquad (2)$$

where I_f is an overlapping integral computed with the analytical integration scheme derived and $t_m(r,s)$ is the voltage or current coefficient shown in (1), and evaluated at interface s_r when the source is placed at interface s_s.

The key step in the formulation is to add and subtract to (2) the asymptotic term t_{0_m} of the spectral domain quantity. Equation (2) can then be written as

$$R(i,k) = \sum_m \Big[t_m(r,s) - t_{0_m}(r,s)\Big] I_f(m,i)\, I_f(m,k) + R_0(i,k), \qquad (3a)$$

$$R_0(i,k) = \sum_m t_{0_m}(r,s)\, I_f(m,i)\, I_f(m,k). \qquad (3b)$$

Note that if the source and observer points are at different interfaces, the energy excited by the source can never reach the observer point in the asymptotic limiting case. In this case the interaction for the asymptotic term is negligible and we directly write: $t_{0_m}(r,s) = 0;\ \forall r \neq s$. For all other cases, the asymptotic term is split into TE and TM parts and after few simple manipulations we obtain

$$R_0(i,k) = j\, d_w^{TE}\, R^{TE}(i,k) + \frac{1}{j\, d_w^{TM}\, R^{TM}(i,k)}, \qquad (4)$$

where we have defined

$$d_w^{TE} = \omega\, \mu_0\, M(s), \qquad M(s) = \frac{\mu_r^{(s)}\, \mu_r^{(s-1)}}{\mu_r^{(s)} + \mu_r^{(s-1)}}, \qquad (5a)$$

$$d_w^{TM} = \omega\, \epsilon_0\, E(s), \qquad E(s) = \epsilon_r^{(s)} + \epsilon_r^{(s-1)}, \qquad (5b)$$

and the following static series

$$R^{TE}(i,k) = \sum_m \frac{1}{k_{\rho_{\mathtt{m},\mathtt{n}}}}\, I_f^{TE}(m,i)\, I_f^{TE}(m,k), \qquad (6a)$$

$$R^{TM}(i,k) = \sum_m k_{\rho_{\mathtt{m},\mathtt{n}}}\, I_f^{TM}(m,i)\, I_f^{TM}(m,k), \qquad (6b)$$

where the transverse wavenumber takes the usual expression in a rectangular waveguide, namely

$$k_{\rho_{\mathtt{m},\mathtt{n}}} = \sqrt{\left(\frac{\mathtt{m}\,\pi}{a}\right)^2 + \left(\frac{\mathtt{n}\,\pi}{b}\right)^2}. \qquad (7)$$

The interesting feature of (6) is that all quantities depend only on geometrical parameters of the antenna and are therefore frequency independent. Consequently the series in (6) are computed once for a given geometry and are not recomputed for each frequency point. On the other hand, the final MoM coefficients are computed with (3a), this time frequency dependent. It is important to note however, that since the asymptotic term is extracted the resulting series will converge much faster than the original ones. Following the proposed method, we have managed to combine a general triangular based MoM formulation with a rigorous modal solution of general cavity backed circuits and antennas.

III. Results and Conclusions

Based on the theory derived in this paper a software tool has been built for the analysis of a structure which can be composed, in the most general case, of an arbitrary number of patches and slots embedded or not in metallic cavities. For those antenna elements outside metallic cavities, a traditional Sommerfeld treatment has been used together with a Mixed Potential Integral Equation

(MPIE) [7], and it is properly combined with the technique described in this paper for the treatment of shielded elements.

All the capabilities of the software developed are fully exploited with the analysis of a circular polarized cavity backed antenna containing two stacked patches of complex shapes. In Fig. 1 we show the basic geometry of the antenna and the triangular meshes used in conjunction with the developed algorithms.

The first interesting property of the method is the convergence behavior of the integral equation kernel for shielded elements. In Fig. 2 we present the convergence plots for the static and dynamic parts of the kernel for a single MoM coefficient with respect the number of modes included in equation (1). As we can observe, for this particular geometry the convergence of the static part is rather slow (5000 modes are needed) but it is computed only once at the beginning and it is therefore not repeated in each subsequent frequency point. On the other hand, the dynamic part of the kernel reaches good convergence with only 1500 modes.

Finally, in Fig. 3 we present the measured versus simulated results for the input impedance of the antenna and in Fig. 4 for the axial ratio, showing in both cases good agreement. Only in the axial ratio a difference of about 2dB can be observed between predicted and measured results. For this analysis the software takes an initial 5 min. 22 sec. to perform the frequency independent calculations plus 53 sec. per each subsequent frequency point on an HP-712/80 platform. Results show that the derived approach is accurate and with very reasonable computational times, even for very complex radiators.

IV. Acknowledgment

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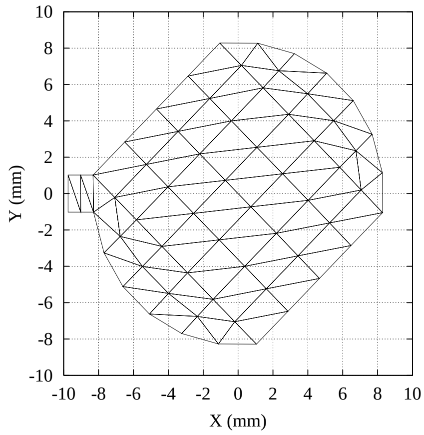
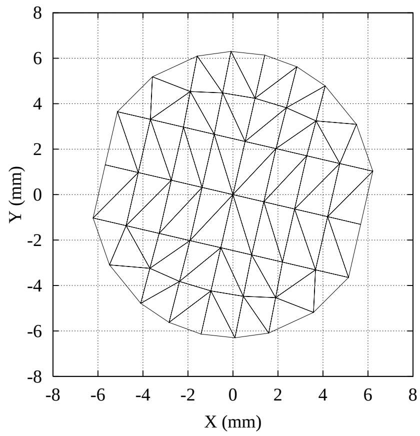
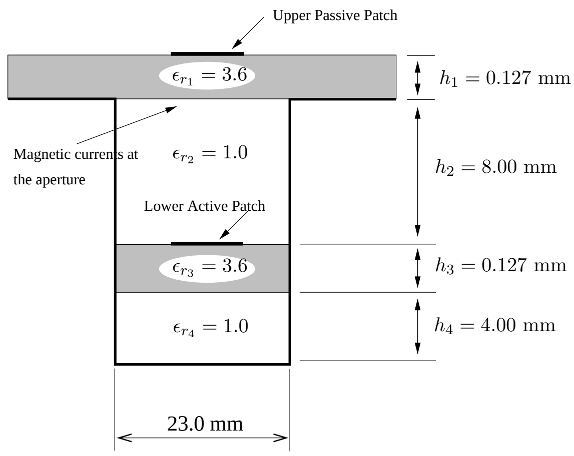


(a)Mesh of lower patch.



(b)Mesh upper patch.



(c)Antenna structure.

Fig. 1. Geometry of the circular polarized antenna analyzed in this paper.

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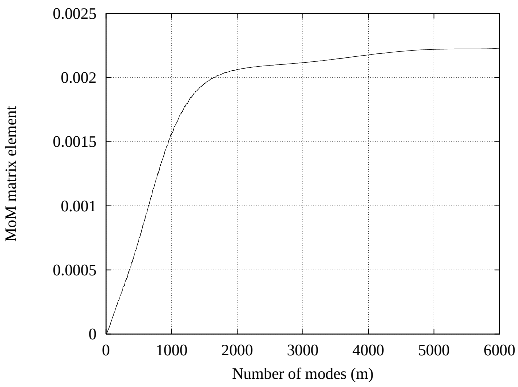
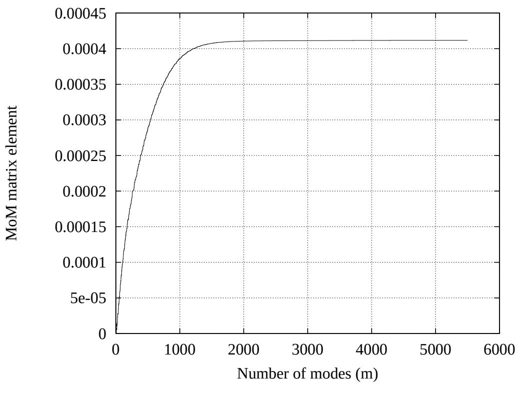
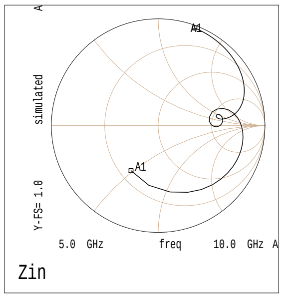


(a)Static part of kernel.

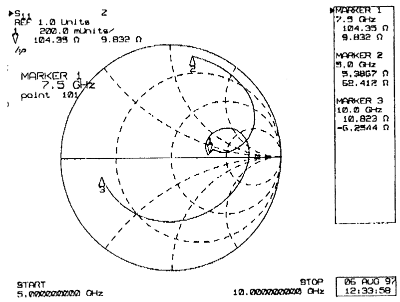


(b)Dynamic part of kernel.

Fig. 2. Typical convergence behavior of the static and dynamic parts of the integral equation kernel.

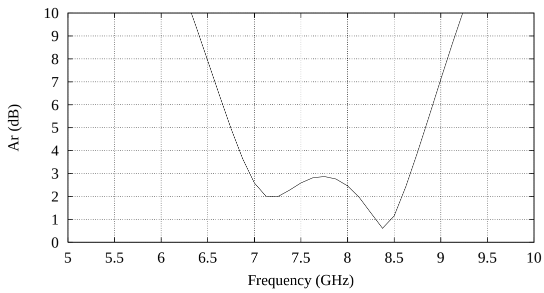


(a)Simulated response.

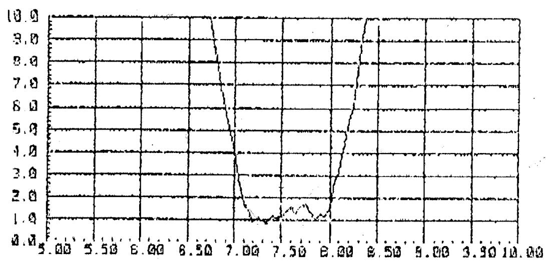


(b)Measured response.

Fig. 3. Measured versus simulated results for the input impedance of the antenna in Fig 1.



(a)Simulated response.



(b)Measured response.

Fig. 4. Measured versus simulated results for the axial ratio of the antenna in Fig 1.