An agent based early warning indicator for financial market instability

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Abstract Inspired by the Bank of America Merrill Lynch Global Breath Rule, we propose an investor sentiment index based on the collective movement of stock prices in a given market. We show that the time evolution of the sentiment index can be reasonably described by the herding model proposed by Kirman on his seminal paper "Ants, rationality and recruitment" (Kirman, 1993). The correspondence between the index and the model allows us to easily estimate its parameters. Based on the model and the empirical evolution of the sentiment index, we propose an early warning indicator able to identify optimistic and pessimistic phases of the market. As a result, investors and policymakers can set different strategies anticipating financial market instability. The former, reducing the risk of their portfolio, and the latter, setting more efficient policies to avoid the effect of financial crashes on the real economy. The validity of our results is supported by means of a robustness analysis showing the application of the early warning indicator in eight different stock markets.

JEL: G10 - C61 - D84

Keywords Herding behaviour · Kirman model · Financial market

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"I define a speculative bubble as a situation in which news of price increases spurs investor enthusiasm, which spreads by psychological contagion from person to person, in the process amplifying stories that might justify the price increases and bringing in a larger and larger class of investors, who, despite doubts about the real value of an investment, are drawn to it partly through envy of others' successes and partly through a gambler's excitement". (Robert J Shiller, Irrational Exuberance, 2015)

1 Introduction

Traditional economic literature, based on Efficient Market Hypothesis (Fama, 1965, 1991; Malkiel and Fama, 1970), highlights the irrelevant role of irrational traders in the long run since they would disappear from the stock market due to their poor performance (Friedman, 1953). A popular definition of investors acting irrationally is "noise traders" (Kyle, 1985; Black, 1986), whose trading might generate crashes and bubbles due to social contagion. This phenomenon is emphasized at the beginning of this article with Shiller's quote, which underlines the importance of social interactions in the stock market as a source of price deviations from the fundamental value. Thus, the presence of noise traders creates situations in which rational investors with risk aversion cannot maintain their position given that "price divergence can become worse before it gets better" (Shleifer, 2000). In other words, noise traders are able to create a particular place in the stock market in which survive (Lux, 2011) to detriment of rational traders.

Agent-Based Models (ABMs) in finance shed more light on the connection between price fluctuations and social interactions since they are able to include herding and contagion phenomena as main features of their dynamics (Lux and Alfarano, 2016). The herding model of Kirman (1991, 1993) represents one of the first references in this field with the introduction of a stochastic model of information transmission initially designed to explain behaviour in ant colonies in the presence of two sources of food. His seminal work was adapted into a financial perspective in which foreign exchange dealers choose their strategy (chartist or fundamentalist) under the influence of social interactions. Most of these models (Lux, 1995, 1996, 1998; Aoki and Yoshikawa, 2002; Wagner, 2003; Alfarano et al., 2005, 2008) have been proposed to provide an explanation of empirical regularities in financial markets, namely, heteroskedasticity in the fluctuations of financial returns, their unpredictability, the fat tails of their unconditional distribution and the long-term dependence in the volatility. The outcomes of these studies show mainly that these statistical regularities can be considered as an emergent property of internal dynamics governed by the interaction of different groups of traders and not just a mere reflection of the incoming new information hitting the market.

One of the main problems in the empirical application of agent based models to financial data lies in the hidden nature of some of the quantities responsible for the internal dynamics of the market. In particular, the fraction of chartists/fundamentalists or pessimist/optimist investors is an unobservable quantity, which should be estimated indirectly from the time series of returns. Gilli and Winker (2003) were the first to estimate some parameters of Kirman model by means of a global optimization heuristic, whose main outcome underlines that the DM/USD foreign exchange market is characterised by switching moods of the agents. Franke and

Westerhoff (2011) employ the method of simulated moments (SMM), along with bootstrap and Monte Carlo techniques, to estimate a structural stochastic volatility model of asset pricing. Their main empirical finding underlines the different behaviour of agents in the S&P 500 index and USD/DM exchange rate. Chen and Lux (2015) use the SMM to estimate the model of Alfarano et al. (2008), which is applied to three stock market indexes (DAX30, S&P 500, Nikkei225), three foreign exchange rates (USD/EUR, YEN/USD, CHF/EUR) and gold price, showing different behaviour of traders in those markets.

The aim of this paper is to shed more light on the empirical research of agent based models that has been previously mentioned. In particular, we employ the herding model introduced by Kirman (1991, 1993) to describe the evolution of a sentiment index that is inspired by the Bank of America Merrill Lynch (BofML) Global Breadth Rule (GBR). Instead of estimating indirectly the fraction of pessimists/optimists from a financial time series of returns, we consider the evolution of the breadth of the entire market as a direct measure of the optimistic/pessimistic mood of traders. We implicitly assume that the collective behaviour of the stocks in a given market is the reflection of the collective mood of traders and the idiosyncratic shocks to the individual firms. Our first contribution of the paper is to notice that the evolution of the market breadth clearly resembles a time series generated by the herding model of Kirman. In our paper, we show that the Kirman model can reproduce accurately the main statistical properties of the empirical sentiment index, namely, unconditional distribution, heteroskedasticity of the fluctuations of the index changes and the exponential decay of the autocorrelation function. Moreover, we can easily estimate the three parameters of the model with a straightforward application of the maximum likelihood method. Our second contribution of the paper relies on the empirical use of our model applied to the sentiment index in order to detect the transition of economic states, from bull markets to bear markets. We use a measure of asymmetry of the Beta distribution, which aggregates the sentiment index z_t, as an early warning indicator. By means of this tool, we identify bull markets in which most of the stocks are simultaneously moving together due to the optimism of investors. Hence, this indicator allows investors and policy-makers to anticipate pessimistic phases in the stock market. The former is able to reduce their financial exposure through the progressive sale of stocks or purchasing put-options, while the latter can control the mood of investors and apply efficient policies to minimise the contagion from financial markets to the real economy.

Finally, the robustness of our results is demonstrated using 2 different stock indices in U.S. (S&P 400 midcap and Nasdaq 100) and 6 different worldwide stock indices: ASX 200 (Australia), TSX (Canada), Euro Stoxx 600 (Europe), Nikkei 225 (Japan), JSE (South Africa) and FTSE 100 (UK). We observe generally the same features as S&P 500 in all stock indices, thus we can apply our sentiment index and early warning indicator in any market regardless of the country specific characteristics.

The rest of the paper is organised as follows. We describe the data in Sec.2 along with the introduction of the sentiment index, z_t, in Sec.3. The herding mechanism is described in Sec.4 and the estimation of the parameters is shown in Sec.5 along with the validation of the model. The definition of our early warning indicator and its empirical application are described in Sec.6. The estimations for the U.S. and worldwide stock markets are shown in Sec.7. Finally, we summarise the main results of this paper with the conclusion in Sec.8.

2 Data

Our data sample is constituted by stock prices that are sourced from Thomson Reuters Datastream at daily frequency. More specifically, we employ all the S&P 500 stocks that have been trading from 01/01/1981 to 01/06/2018. The sample counts a total of 208 stocks.

To ensure the robustness of our results, we employ other two groups of stock markets. On the one hand, we study S&P 400 midcap and Nasdaq 100 to observe whether firm size and sectoral characteristics give rise to different results. On the other hand, we analyse 6 stock markets from different countries to examine whether investors' sentiment can be described by the same model regardless of the different market structure, trading mechanism or country specific characteristics (Australia, Canada, Europe[1], Japan, South Africa and UK). Compared to S&P 500 stocks, we examine these indexes from 01/01/1993 to 01/06/2018 due to the availability of data.[2] Table 1 shows all the stock markets that have been used in this study, reporting the corresponding index, sample period, country, and number of stocks.

Table 1: Stock indices that have been used in the analysis with their corresponding sample periods, countries and number of stocks.

Number	Stock index	Sample period	Country	Stocks
1	S&P 500	01/01/1981 - 01/06/2018	U.S.	208
2	S&P 400 midcap	01/01/1993 - 01/06/2018	U.S.	188
3	Nasdaq 100	01/01/1993 - 01/06/2018	U.S.	48
4	ASX 200	01/01/1993 - 01/06/2018	Australia	53
5	TSX	01/01/1993 - 01/06/2018	Canada	81
6	Euro Stoxx 600	01/01/1993 - 01/06/2018	Europe	282
7	Nikkei 225	01/01/1993 - 01/06/2018	Japan	183
8	JSE All-Share	01/01/2000 - 01/06/2018	South Africa	86
9	FTSE 100	01/01/1993 - 01/06/2018	U.K.	57

Given that we are analysing stock indexes for a time span of 37 years (in the case of S&P 500) and 25 years (for the rest of stock markets with the exception of JSE index), we de-trend stock prices in order to focus on the behaviour of fluctuations around the long-term trend. The latter is mainly affected by the general evolution of the economy and not by the short term fluctuations in the sentiment of the investors.[3] In order to do so, the first step is to compute the cumulative returns of each stock index calculated as:

$$r_{j,t} = \ln\left(\frac{S_{j,t}}{S_{j,1}}\right), \quad j = 1, ..., 9, \tag{1}$$

where $S_{j,t}$ is the stock index of the corresponding market in Table 1. Afterwards, we employ an Ordinary Least Square regression to obtain the corresponding average daily return for each index, defined as β_j. We then compute the de-trended prices for each one of the stocks in the corresponding market, defined as

[1]Given the similarities in relation to the Euro Stoxx 600, we do not report results related to countries like Germany (DAX), France (CAC) and Spain (IBEX). Hence, we focus on the Euro Stoxx 600 to include the European countries (material available upon request).

[2]The only exception is JSE All-Share index. In this case our initial year is 2000.

[3]If we consider the original time series of prices, our results are not significantly affected.

$$
{}_j p_{i,t} = \frac{{}_j P_{i,t}}{e^{\beta_j t}}, \tag{2}
$$

where ${}_j P_{i,t}$ are the raw prices while ${}_j p_{i,t}$ denotes the de-trended prices of the stock i in market j.

3 Sentiment Index

Our paper is inspired by the Bank of America Merrill Lynch (BofML) Global Breadth Rule (GBR) (Hartnett et al., 2015), which is defined as a contrarian indicator of equity markets. More specifically, GBR indicates an extreme pessimistic scenario in which most of the stock indices around the world are oversold, thus triggering a buy signal to take advantage of possible rebounds. The GBR is based on the market breadth technical trading analysis, which employs a comparison of moving averages as a classic chartist technique. Specially, the GBR triggers a buy signal when 88% of stock indices included in the MSCI All Country World Index[4] are simultaneously below their 200-day moving average and 50-day moving average. Such collective movement of all the markets in the same direction is considered as an imprint of a pessimistic market mood affecting all the global financial markets. The origin of this collective movement can be found in a pessimistic persistent mood of investors, overreacting to some exogenous factors. Hence, the main idea of this trading rule is to identify a period of overall pessimistic sentiment in the world financial markets, and consequently, the proper timing to enter in the market.

Given that the GBR is a tool that relies inherently on the sentiment of agents in the worldwide stock market, we similarly create sentiment indexes based on the individual stocks that belong to the stock indices reported in Table 1. We construct an index like GBR based on the stocks of a single financial market instead of indices included in the aggregated international financial index, i.e. the MSCI All Country World Index. In order to construct our sentiment index, we categorise the fixed number of stocks in the market (N) according to two possible states: n_t (state 1) and $N - n_t$ (state 2), where n_t is defined as the number of stocks whose prices are affected by a negative mood of investors, and $N - n_t$ as the number of stocks whose prices are, instead, influenced by an optimistic mood.

As a proxy for the impact of optimism and pessimism on a given stock, we compute the relative price in a given day with respect to its 100-day Exponential Moving Average (EMA, hereafter). We compare the EMA with respect to the current price, so that we construct two mutually exclusive possible states for a single stock.[5] Differently from the GBR index, we only use the price and the moving average, instead of two moving averages. This choice allows us to define a binary characterisation of the state of each stock that could not be possible with the methodology applied to GBR sentiment index. In particular, the EMA is calculated by applying a weight of today's closing price to yesterday's EMA value. We define the weight W to compute the EMA as

$$
\mathrm{W} = \frac{2}{\mathrm{L}+1}, \tag{3}
$$

[4]MSCI All Country World Index aggregates 46 indexes: 23 developed and 23 emerging markets.

[5]The index dynamics is essentially similar regardless of the moving average from 50 to 200 days, instead of the 100 day-EMA that we consider (material upon request).

where L is the length of the considered time window measured in days. Given that in our case we are using a 100-day moving average, the corresponding weight (W) is equal to 1.98%. We denote the EMA of stock i at time t as $\bar{p}_{i,t}$, recursively defined as:

$$\bar{p}_{i,t} = p_{i,t} \cdot W + (1 - W) \cdot \bar{p}_{i,t-1} \quad \text{for } t \geq 1, \tag{4}$$

where the first term denotes the percentage of today price that is added to yesterday's exponential moving average.[6] The starting value of $\bar{p}_{i,0}$ is equal to the simple moving average over a window of length L, i.e.

$$\bar{p}_{i,0} = \frac{1}{L} \sum_{\tau=1}^{L} p_{i,\tau}. \tag{5}$$

We consider that the stock i at time t is in a "pessimistic state" when the price of the stock, $p_{i,t}$, is below $\bar{p}_{i,t}$, while we consider that the stock is in the "optimistic state" when the price of the stock is above $\bar{p}_{i,t}$. We denote the state of each stock i as $n_{i,t}$, which takes the values:

$$n_{i,t} = \begin{cases} 1 \text{ if} & p_{i,t} < \bar{p}_{i,t}, \\ 0 \text{ if} & p_{i,t} \geq \bar{p}_{i,t}. \end{cases} \tag{6}$$

This binary characterisation of the state of a given stock is a rough approximation of the mood of the investors since the state of the single stock might change abruptly due to small fluctuations of the current price with respect to its exponential moving average. It is the aggregate index, in fact, which carries the information on the collective dynamics of the sentiment of the investors in the market. We define n_t as the sum of the individual state of each stock at time t, i.e.

$$n_t = \sum_{i=1}^{N} n_{i,t}. \tag{7}$$

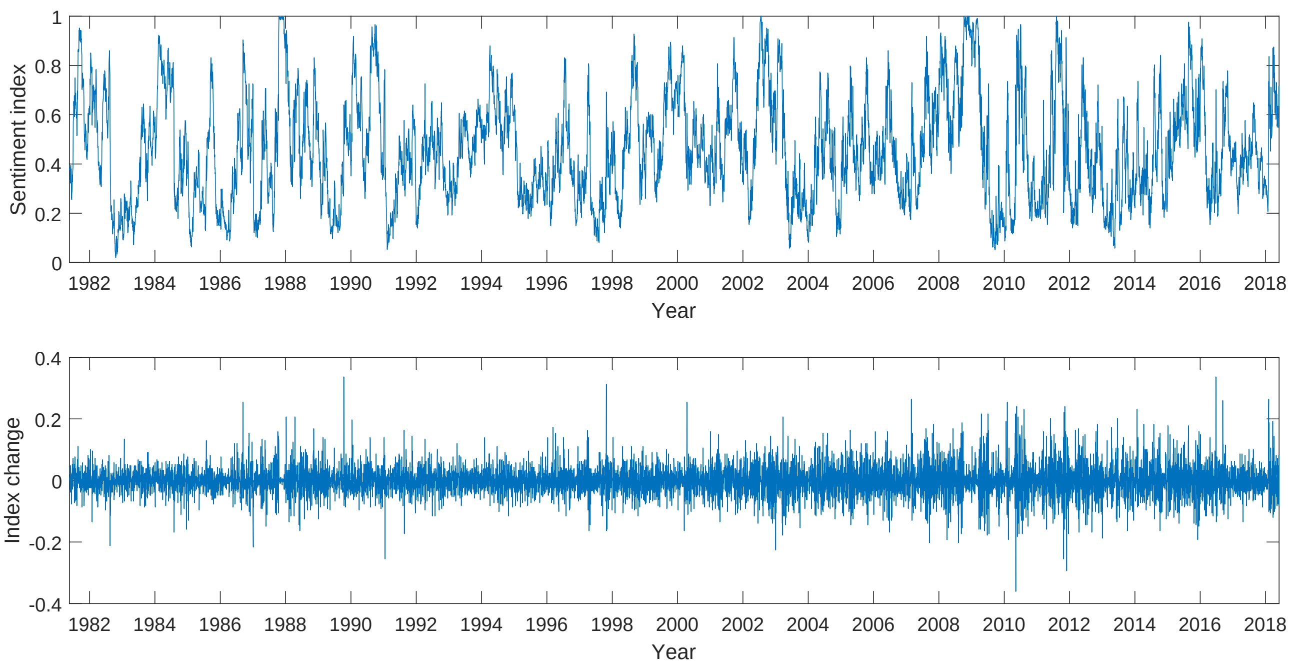
We define the normalised sentiment index, z_t, which represents the collective behaviour of the market at time t in relation to the size of the market N as:

$$z_t = \frac{\sum_{i=1}^{N} n_{i,t}}{N} = \frac{n_t}{N}. \tag{8}$$

A bear market is characterised by a value of the variable z_t close to 1, conversely, a bull market is associated to a value of z_t close to zero. So, when the vast majority of stocks have a price lower than their corresponding moving average, we are in the presence of a wide-spread pessimistic sentiment of the market, or, possibly, of a negative exogenous event, affecting all the stocks at the same time. In order to distinguish between those two cases, we consider the dynamics of the sentiment index and its evolution over time. Fig. (1) shows the resulting sentiment index z_t for the S&P 500 along with the time series of daily changes, which is calculated as the difference between z_t and z_{t-1}. We observe swings of sentiment of the market between the two extreme values, $z = 0$ and $z = 1$. Such sentiment index resembles the behaviour of the GBR index. Moreover, a visual

[6] Hereafter, we omit the counter indicating the particular stock market in order not to deal with a too heavy notation. We are implicitly assuming that all stocks refer to the corresponding market.

Fig. 1: Sentiment index and index change of S&P 500.



inspection of the time series of index changes shows the presence of volatility clusters, a phenomenon shared with the time series of price returns (Cont, 2001). Our simple filtering technique might be useful to characterise the overall behaviour of investors in the market. Fig. (1) shows a remarkable visual similarity to the time series generated by the herding model proposed by Kirman (Kirman, 1991; Kirman, 1993), which is inspired by an entomological experiment on the behaviour of ants. In this paper, we analyse to what extent such visual similarity can be translated into a quantitative correspondence.

4 Kirman Herding Model

Kirman (1991, 1993) has popularised an entomological experiment on the behaviour of an ant colony where the ants can choose between two identical sources of food near their nest. Surprisingly, at a given instant of time, most of the ants are in the proximity of a source of food instead of observing an equal allocation of ants between the two sources, and, even more surprisingly, such majority switches over time between the two sources. This experimental observation has been explained through a combination of recruitment interactions among ants and an autonomous switching of individual ants, as a result of its stochastic search for food. In particular, the pairwise recruiting interaction of two ants, due to the exchange of pheromones to communicate to each other, can be interpreted as a herding behaviour when one ant decides to follow the another regardless of its own information on the location of the source of food.

Based on this entomological experiment, Kirman has introduced a simple stochastic model in order to formalise the herding interaction among ants and to explain the aggregate asymmetric behaviour of the colony and the switching of the majority of ants over time. His model has been successfully applied to financial markets in order to model the switching mechanism among different strategies used by agents to trade an asset.[7] In line with a large number of contributions in the agent based finance and based on a visual similarity of the time series, we apply the ant model of Kirman to formally describe the dynamics of the sentiment index defined in Eq. (8). This allows us to have a formal model of the function and evolution of the sentiment of investors in a financial market.

As previously described, the stocks are categorised into two states within the optimism/pessimism dichotomy. The formalisation of the Kirman herding model starts assuming that the stochastic population dynamics, determined by the aggregate variable n_t from Eq. (7) evolves according to the Poisson probabilities governing the change of one stock from n_t at time t to some n_t' at time t+Δt_0, where $n_t' = n_t \pm 1$. These conditional probabilities are denoted by $\rho(n',\, t + \Delta t_0 \mid n,\, t)$. For sufficiently small time increments Δt_0 the probabilities are linear in the time interval and are defined as:

$$\begin{aligned} \rho(n+1, t+\Delta t_0 \mid n, t) &= (N-n)(a_1 + bn) \cdot \Delta t_0, \\ \rho(n-1, t+\Delta t_0 \mid n, t) &= n(a_2 + b(N-n)) \cdot \Delta t_0, \end{aligned} \tag{9}$$

with the further constraint that:

$$\rho(n, t+\Delta t_0 \mid n, t) = 1 - \rho(n+1, t+\Delta t_0 \mid n, t) - \rho(n-1, t+\Delta t_0 \mid n, t). \tag{10}$$

Moreover, given that the conditional probabilities $\rho(n',\, t + \Delta t_0 \mid n,\, t)$ must be bounded to 1 and must be positive, an upper limit can be computed for the elementary time step, Δt_0:

$$\Delta t_0 \leq \frac{2}{bN(N + \frac{a_1}{b} + \frac{a_2}{b})}. \tag{11}$$

The probabilities of Eqs. (9) and (10) define a Markov chain that can be more precisely classified within the class of non-linear one-step processes (Van Kampen, 1992).[8] The constants a_1 and a_2 account for the change of state due to idiosyncratic external events while the term proportional to b represents the market pressure. An individual stock might change its state due to an idiosyncratic external event, for instance new information affecting the prospects of future cash flows. A new information affects the trading attitude of investors, triggering their buy or sell signals on that particular stock, which, in turn, might change the current stock price, possibly modifying its relative position with respect to EMA and eventually its state. The terms a_1 and a_2 trigger random switches among states, regardless of the state of the other stocks. Notice, however, that this sensitivity

[7]Two main examples of this literature are Lux and Marchesi (2000) and Alfarano et al. (2005). The former relates volatility clustering, fat tails and the unit root property of assets to the interaction of chartists, who can be optimistic and pessimistic, and fundamentalists. In fact, chartists change their mood not only due to the price trend but also because of the majority opinion. The latter also shows that fat tails and volatility clustering can be considered as an emergent property of the interaction of traders, whose main contribution relies on the direct estimation of the underlying parameters of their herding model given a closed-form solution for the distribution of returns.

[8]The Markov chain described in Eq. (9) and (10) is characterised by being homogeneous, irreducible, aperiodic, ergodic, with a unique equilibrium distribution (Feller, 1968). In particular, the transition probabilities are homogeneous since they are time independent. Each state can be reached from the rest of the states in a finite number of states, thus the chain is irreducible. Given that the probability $\rho(n',\, t + \Delta t_0 \mid n,\, t) \neq 0$, the chain is aperiodic. Hence, the Markov chain is ergodic since it is aperiodic and irreducible. Finally, the equilibrium distribution exists and is unique. Garibaldi and Scalas (2010) gives a very precise account of the properties of such process.

to change the state depends on the direction of the transition, given the asymmetry of the two coefficients (a_1 and a_2). The state of a stock might also change under the market pressure modelled by the dependence of the probabilities on the overall number of stocks in the opposite state. In other words, with this term we account for the global coupling among all stocks in a given market, what is captured within the CAPM by the dependence of the individual stock return on the return of the index. Collective changes of the mood of investors due to social interactions based on herd behaviour are reflected into the corresponding changes of states of the stocks. Our sentiment index can be considered, therefore, a devise to detect indirectly the unobservable movement of the sentiment of investors. The non-linear term in Eq. (9) accounts for the impact that the mutual influence in the behaviour of traders has on the collective movements of the stocks.

In Alfarano (2006) and Garibaldi and Scalas (2010), the finitary equilibrium distribution of the Markov chain of Eq. (9) is derived, which turns out to be a Polya distribution. In order to approximate the discrete stochastic process by a continuous diffusion process (Alfarano et al., 2005, 2008, 2013; Alfarano and Milaković, 2009), we define the collective behaviour of the whole market with respect to the intensive variable z_t. Alfarano et al. (2005) shows that the Markov chain of Eq. (9) can be approximated by the dynamics of the variable z_t within the framework of the Fokker-Planck equation (FPE),

$$\frac{\partial \rho(z,t)}{\partial t} = -\frac{\partial}{\partial z}[A(z)p(z,t)] + \frac{1}{2}\frac{\partial^2}{\partial z^2}[D(z)p(z,t)], \tag{12}$$

where $A(z)$ represents the drift term

$$A(z) = a_1 - (a_1 + a_2)z, \tag{13}$$

while the diffusion term $D(z)$ is given by

$$D(z) = 2b(1-z)z. \tag{14}$$

The resulting equilibrium distribution, obtained by Alfarano et al. (2005) is

$$p_0(z) = \frac{1}{B(\varepsilon_1, \varepsilon_2)} z^{\varepsilon_1 - 1}(1-z)^{\varepsilon_2 - 1}, \tag{15}$$

where

$$B(\varepsilon_1, \varepsilon_2) = \frac{\Gamma(\varepsilon_1)\Gamma(\varepsilon_2)}{\Gamma(\varepsilon_1 + \varepsilon_2)}, \tag{16}$$

being $\Gamma(\cdot)$ the Gamma function.[9,10] Interestingly, it turns out that it depends only on the ratios $\varepsilon_1 = a_1/b$ and $\varepsilon_2 = a_2/b$ but not on the size of the constants a_1, a_2 and b. The resulting distribution of Eq. (15) is known as the Beta distribution which is characterised by being a flexible distribution in a bounded domain.

[9]Despite the fact that our sentiment index z_t is not a continuous variable given the finite number N of stocks in the system, for simplicity of the calculus we examine the Beta as equilibrium distribution instead of the Polya distribution (Alfarano et al., 2005).

[10]For values of ε_1 and ε_2 smaller than one, we can observe a clear temporary majority of stocks persistently fluctuating in one state, eventually changing to the other state.

Instead of simulating the stochastic process of Eq. (9) at the microscopic level with a single transition at a time or solving the FPE of Eq. (12) (see Alfarano, 2006), we can also describe the herding mechanism, at the mesoscopic level, by means of a stochastic equation known in the physics jargon as Langevin equation. This approach allows to approximate the conditional distribution of the discrete process of Eq. (9) to a Gaussian distribution. In other words, instead of following the herding dynamics at the microscopic time scale Δt_0, when we observe at most a switch of a single asset, we considered a mesoscopic time scale Δt, during which we aggregate several variations of the variable z_t in order to obtain a simpler description of its dynamics. Alfarano et al. (2005) derive the following approximation for the stochastic process of Eq. (9):

$$\begin{aligned} z_{t+\Delta t} &= z_t + (\varepsilon_1 - (\varepsilon_1 + \varepsilon_2) z_t) b \Delta t + \sqrt{2b\Delta t (1-z_t) z_t} \cdot \lambda_t, \\ &= z_t + (\varepsilon_1 + \varepsilon_2)(\bar{z} - z_t) b \Delta t + \sqrt{2b\Delta t (1-z_t) z_t} \cdot \lambda_t, \end{aligned} \tag{17}$$

where λ_t is a iid normally distributed random variable and $\bar{z}_t$ is defined as $\varepsilon_1/(\varepsilon_1+\varepsilon_2)$, which is the mean of the process itself. The mesoscopic time scale Δt is proportional to N^2, i.e. $\Delta t \sim N^2 \Delta t_0$.[11] The process of Eq. (17) is characterised by a linear mean reverting component and a heteroskedastic random term, conditional to the value of z_t. Given the parabolic dependence of the diffusion function, values of z_t close to 0.5 generate higher fluctuations than when z_t is close to the boundaries of its range. This dependences generates heteroskedastic fluctuations in the time series of Δz_t, which may resemble those illustrated in Fig. 1. Obviously, Eq. (17) is an approximation for large N of the process in Eq. (9), with the further restriction that the variable z_t cannot be close to the boundaries $z_t = 0$ and $z_t = 1$. In those regions, the continuous approximation is no longer valid since it may violate the boundaries. Hence, the natural boundaries implemented in Eq. (9) must be exogenously added to Eq. (17). Consequently, in order to simulate the process of Eq. (17), we have to add reflecting boundaries at $z_t = 0$ and $z_t = 1$ by hand:

$$\begin{aligned} &\text{if} \quad z_t > 1 \quad \text{then} \quad \frac{z_{t+\Delta t} + z_t}{2} = 1, \\ &\text{if} \quad z_t < 0 \quad \text{then} \quad \frac{z_{t+\Delta t} + z_t}{2} = 0, \end{aligned} \tag{18}$$

which are equivalent to a reflection around the edges of the domain of z_t, $z_t = 1$ and $z_t = 0$, respectively. One of the advantages of the Langevin equation is that it is relatively simple to estimate via maximum likelihood, since the conditional probability density function of $z_{t+\Delta t}$ given z_t is a Gaussian with mean $z_t + (\varepsilon_1 - (\varepsilon_1 + \varepsilon_2) z_t) b \Delta t$ and standard deviation $\sqrt{2b\Delta t(1-z_t)z_t}$.

Finally, Alfarano et al., 2005 derive the autocorrelation function (ACF) through the Langevin equation by using a recursive method[12], leading to an exponential autocorrelation function:

$$C_z(t) = e^{-b(\varepsilon_1+\varepsilon_2)t}. \tag{19}$$

[11] See Alfarano et al. (2008) for the details of the derivation.

[12] Eq. (19) can be also computed using the FPE or the Markov chain of Eq. (9).

Summarising, the model of Eq. (17) is characterised by a Beta equilibrium distribution, a mean reverting property with heteroskedastic persistent fluctuations and an exponential autocorrelation function. Moreover, a crucial chracteristic of the model is that can be easily estimated using the maximum likelihood estimator.

5 Validation of the model and estimation of its parameters

We compare the empirical properties of the sentiment index from Eq. (8) to the theoretical properties of the index as described in the previous section. In particular we confront the unconditional distribution of the index z_t, the time series of daily changes and the autocorrelation function. The sentiment index, z_t, computed based on the 208 stocks that have been present in the stock market from 01/01/1981 to 01/06/2018, while the simulated sentiment index, z_t^s, has been computed by using the Langevin equation (Eq. (17)) with estimated parameters from empirical data. First and foremost, we estimate the three parameters which characterise the model by means of the Langevin equation as an aggregate law of motion using the maximum likelihood method. Alternatively, we estimate the parameters ε_1 and ε_2 using the maximum likelihood method but with a likelihood based on the unconditional distribution from Eq. (16) (see Alfarano et al. (2005) for the details of the method). The parameter b, which governs the time scale of the process, does not enter in the determination of the unconditional distribution.

In Table 2 we show the estimations of ε_1, ε_2, b, and the corresponding fit of the unconditional distribution in Fig. (2). We can observe that the parameters estimated using the Beta distribution give rise to a better fit of the empirical probability density function of the sentiment index than the corresponding parameters estimated using Eq. (17). We observe, in fact, the presence of a clear asymmetry in the empirical distribution, while the estimated values point to a symmetric distribution. Such poor performance of the Langevin equation, compared to the estimation based solely on the unconditional distribution, signals a certain degree of incoherence between the conditional property of the process in Eq. (17) and the stimated unconditional distribution from the data. Such discrepancy does not come as a surprise. The Langevin dynamics, in fact, does not constitute a good approximation of the continuous process of Eq. (12) at the boundaries, and therefore, it cannot be estimated with the second approach.

Table 2: Estimated parameters, ε_1, ε_2 and b for the sentiment index.

Method	ε_1	ε_2	b
Unconditional distribution	1.99 ± 0.07	2.25 ± 0.06	-
Langevin equation	1.64 ± 0.16	1.74 ± 0.14	0.0055 ± 0.0001

Given the poor performance of the Langevin, we analyse whether there are a few observations at the boundaries heavily affecting the estimation of the parameters. We observe that the values of $z_t > 0.95$ are 141 daily events, i.e. 1% of the data at our disposal.

Simulating the stochastic process with the estimated parameters of S&P 500 index ($\varepsilon_1 = 1.99$ and $\varepsilon_2 = 2.25$) we notice a clear discrepancy with respect to the empirical data. If we count the days with $z_t > 0.95$ for the

Fig. 2: Probability density function of the sentiment index compared to the theoretical distribution given the estimates of each method.

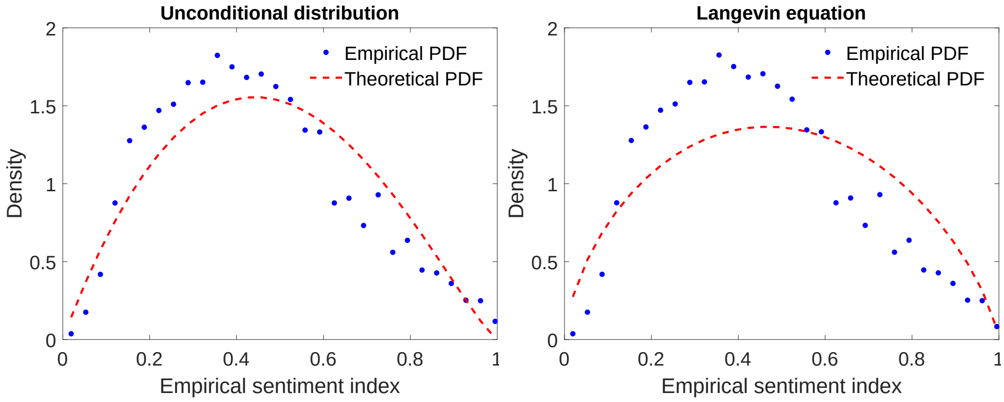
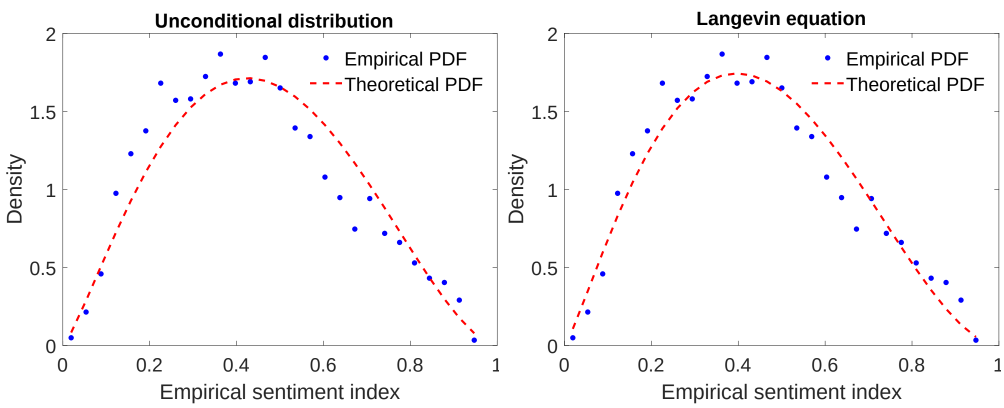


Table 3: Estimated parameters, ε_1, ε_2 and b for the sentiment index, excluding extreme negative events ($z_t > 0.95$).

Method	ε_1	ε_2	b
Unconditional distribution	2.28 ± 0.09	2.72 ± 0.10	-
Langevin equation	2.22 ± 0.18	2.86 ± 0.22	0.0054 ± 0.0001

simulated data, we obtain 42.[13] In other words, we have observed more than three times extreme events than theoretically expected. Excluding those events from the estimation we obtain the parameters in Table 3 and the fit of the corresponding unconditional distribution in Fig. (3). Both methods improve the fit of the empirical data. Interestingly, we cannot reject that the estimates from the Langevin and the unconditional distribution are equal considering the standard error. Fig. (1) and Fig. (4) shows the empirical and simulated index with their corresponding daily changes. Both time series exhibit swings of pessimism and optimism with visually similar characteristics. The time series of the empirical changes of the index and the corresponding simulated time series exhibit heteroskedastic fluctuations. In particular, the higher level of fluctuations is associated to a value of the sentiment index around an equilibrated population of stocks ($z_t = 1/2$), while the lower level of fluctuations is concentrated in regions far from $z_t = 1/2$.

Fig. 3: Probability density function of the sentiment index compared to the theoretical distribution given the estimates of each method and excluding extreme negative events ($z_t > 0.95$).



[13]We have computed 500 Montecarlo simulations. The median number of extreme events is 42 with a mean equal to 44 and a standard deviation of 23. A value of 141 is, therefore, extremely improbable to be observed.

Fig. 4: Simulated sentiment index and index change with the following parameters: $\varepsilon_1 = 1.99$, $\varepsilon_2 = 2.25$, $N = 208$.

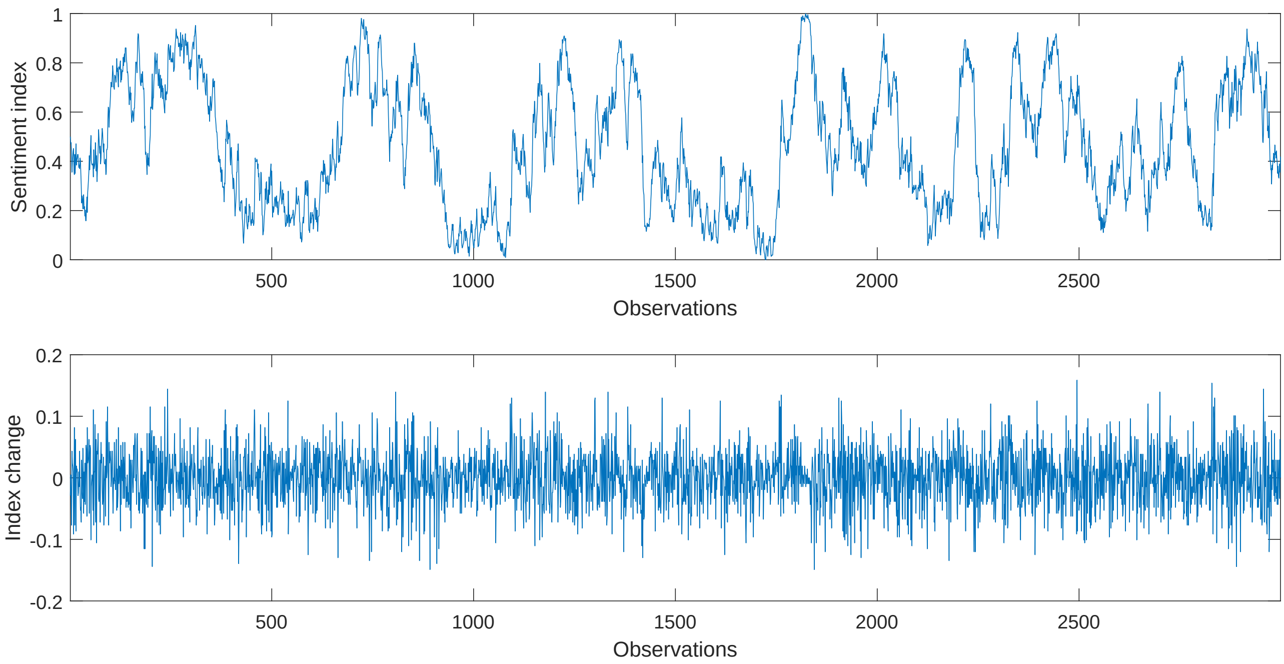
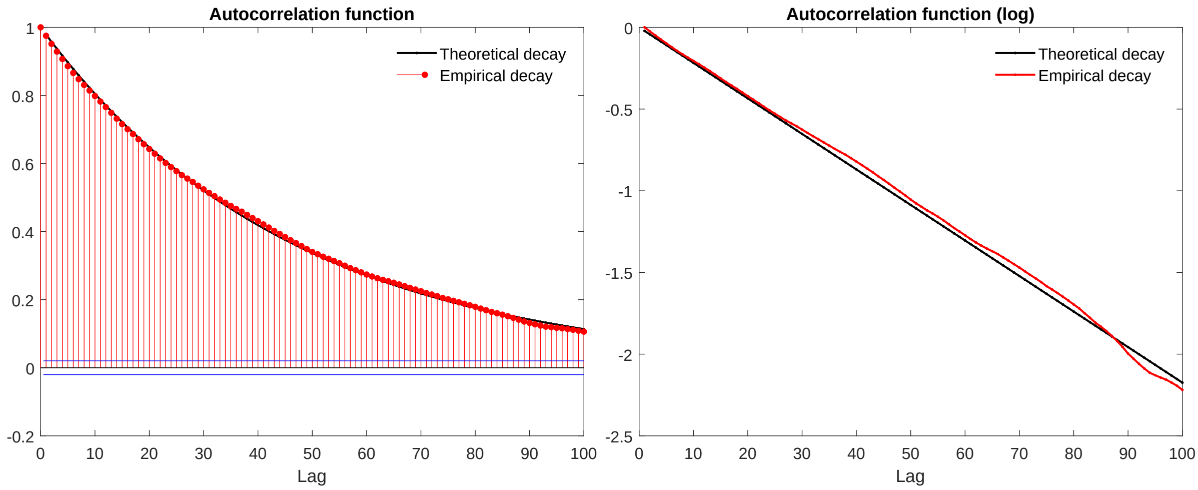


Fig. 5: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function.



Once we have analysed the goodness of fit of the unconditional properties of the model in describing the distribution of the sentiment index and its changes, we focus our attention on the time series properties.

In order to do so, we consider the autocorrelation function. Fig. (5) shows the fit of the autocorrelation of the empirical distribution as compared to the theoretical autocorrelation function from Eq. (19), which turns out to be particularly accurate. We can conclude that the calibrated herding model can satisfactorily describe the degree of persistence of the time series of the sentiment index z_t.

6 Empirical application: Early warning indicator

The sentiment of investors in financial markets has been analysed from many different approaches in a sizeable empirical literature often with the objective of detecting bubbles with their sub-sequent financial crises. Some examples are the Index Cohesive Force (Kenett et al., 2011; Kenett et al., 2012), power law distributions (Kaizoji, 2006; Mizuno et al., 2016), the leverage of the banking sector (Adrian and Shin, 2009), cross-correlations (Podobnik et al., 2009), CAPE ratio (Shiller, 2015) and regime switching approaches (Preis et al., 2011), among other studies. We aim now to employ the sentiment index developed in the previous sections to detect potential optimistic phases in the market, with the objective of providing an early warning indicator of possible downturn periods.

The dynamics of the variable z_t itself is too volatile to be employed as a direct measure of the phase of the market, as we can see from Fig. (1). We have, then, to find a meaningful filtering technique in order to extract a more readable signal from the raw time series of the sentiment index z_t. In order to aggregate the information of the sentiment index during a given time interval and identify the phase of the market, we estimate the parameters $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ through a rolling window of 750 days (three years). We implement a recursive estimation for $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ using the observations of z_t from $t-750$ to t. Under the assumption that the Eq. (17) is the law of motion of the sentiment index, we estimate the parameters $\varepsilon_{1,t}$, $\varepsilon_{2,t}$ and b_t using the maximum likelihood method for each rolling window.[14] Given that the relaxation time scale[15] of the process of Eq. (17) is $\tau_c = 1/[(b(\varepsilon_1+\varepsilon_2)] \approx 20$ days, we can reasonably assume that the Beta distribution with parameters $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ is a good approximation of the unconditional distribution of the process within a given rolling window. We can, then, employ some characteristics of the Beta distribution as a summary indicator for the phase of the market. In particular, we identify in a measure of the asymmetry of the Beta distribution a proper summary indicator. As a measure of asymmetry, we use the difference between the value of the two estimated parameters[16]:

$$\Lambda_t = \varepsilon_{2,t} - \varepsilon_{1,t}. \tag{20}$$

We contemplate three different scenarios: $\varepsilon_{2,t} \approx \varepsilon_{1,t}$, so that $\Lambda_t \approx 0$, which represents a "symmetric" market with approximately the same number of stocks in a pessimistic and optimistic phase; $\varepsilon_{2,t} > \varepsilon_{1,t}$ and $\Lambda_t > 0$ represent a bull market, when most of the stock prices are, on average, raising at the same time; finally $\varepsilon_{2,t} < \varepsilon_{1,t}$ and $\Lambda_t < 0$ underline the existence of a bear market, when most of the stock prices are, on average, falling simultaneously (see Fig. (6)).

Fig. (7) shows the evolution of the indicator Λ_t using a 100-day EMA and a time interval of 750 days for the estimation of parameters. To obtain a sharper characterisation of the phase of the market, we consider certain levels of "excess asymmetry", establishing a threshold equal to 90^{th} percentile, which is represented by dark grey areas, for bull phases, and a threshold equal to 10^{th} percentile, which is represented by light grey areas, for

[14] We use all data in each window without removing extreme values. In this case, using short running windows, we do not observe that extreme events affect the fit of the distribution. In fact, we observe similar results even excluding those values of $z_t > 0.95$.

[15] The characteristic time scale τ_c can be derived considering the autocorrelation function from Eq. (19).

[16] Note that the skewness of a Beta distribution with parameters ε_1 and ε_2 is proportional to $(\varepsilon_2 - \varepsilon_1)$.

Fig. 6: Three possible scenarios for the stock index based on ε_1 and ε_2. In the first one ($\varepsilon_1 < \varepsilon_2$), there is optimism in the market. In the second one ($\varepsilon_1 \approx \varepsilon_2$), there is no dominant mood. In the last one ($\varepsilon_1 > \varepsilon_2$), there is pessimism in the market.

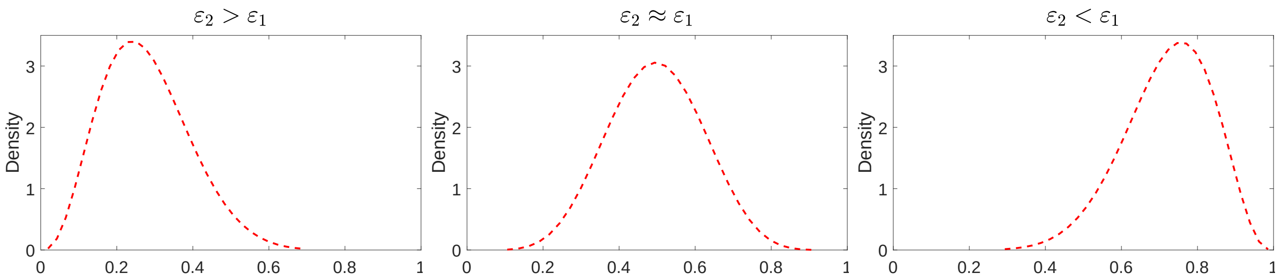
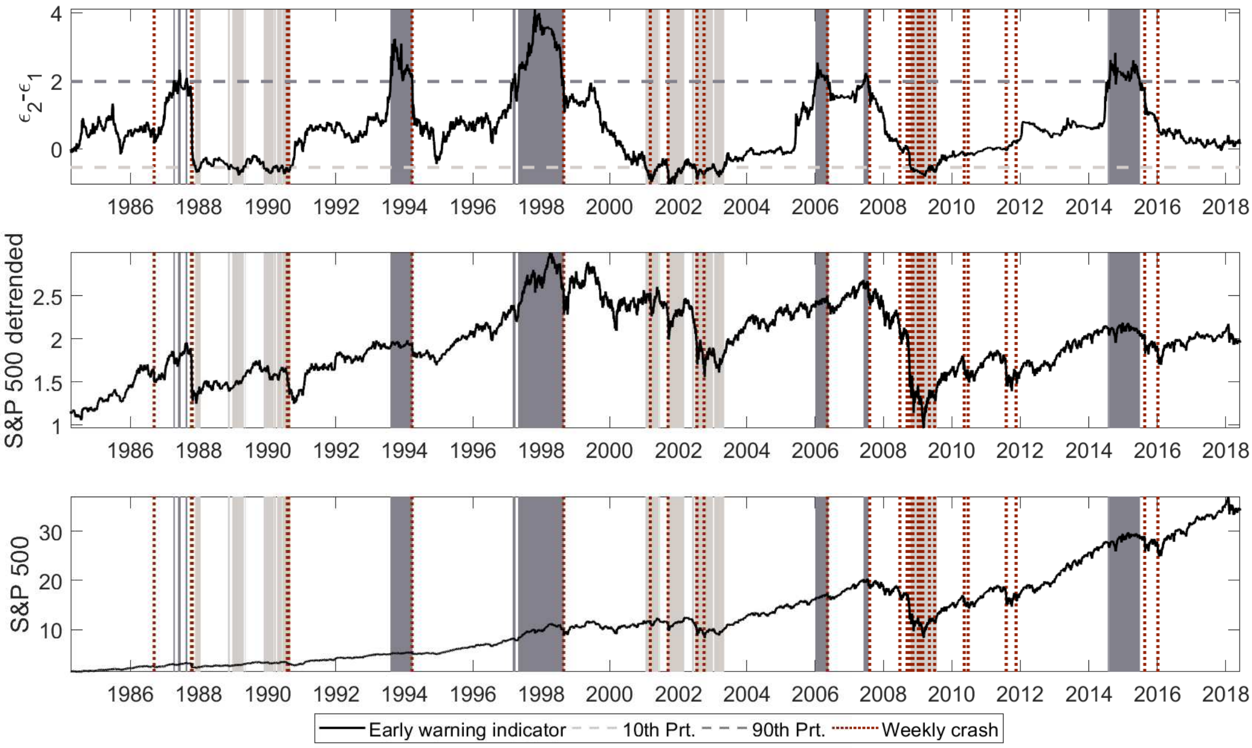


Fig. 7: Early warning indicator using 100-day EMA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. S&P 500 index.



Note. Bull market phases and the subsequent negative events: 1987 (black monday), 1994 (tightening monetary policy), 1997-1998 (Russian financial crisis), 2006 (tightening monetary policy), 2007 (sub-prime mortgage crisis) and 2015-2016 (weakness of China economy). Bear market phases: 1987-1990 (black monday and Gulf war), 2001-2003 (burst of the dot com bubble), 2008-2009 (burst of the housing bubble).

bear markets.[17] Moreover, we plot in Fig. (7) the sequence of the 30 most negative weeks in S&P 500 history. We additionally show the sequence of the S&P 500 index and its de-trended series.[18]

[17] We compute the two thresholds using the entire data set at our disposal. However, as can be observed in the Appendix, we obtain very similar results using past data to compute recursively the value of the two thresholds.

[18] Given that we are not analysing exactly S&P 500, since we only have 208 stocks at our disposal, we show the corresponding index of the available stocks ($\bar{S}_t$), defined as the exponential cumulative sum of market returns at each time t,

$$\bar{S}_t = e^{mr_t \cdot t}, \tag{21}$$

where mr_t denotes the market return at t defined as:

$$mr_t = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}, \quad N = 208, \tag{22}$$

and $r_{i,t}$ denotes the return of each stock i. We also report the S&P 500 detrended index, which is calculated following the same procedure using the returns calculated by means of the de-trended prices ($p_{i,t}$).

Fig. (7) shows the different periods in which we observe the sequence of optimistic and pessimistic phases of the market, along with extreme negative weekly events. The objective of this figure is to evaluate the performance of the early warning indicator with empirical data, that is:

– we expect to observe at least a down-trend period (underlined by negative weeks in the stock index) shortly after the early warning indicator has detected a bull market phase (around its 90^{th} percentile).

As a second hypothesis:

– we expect higher prices after bear market phases (around its 10^{th} percentile) due to the low prices of stocks during extreme pessimistic phases.

Therefore, the detection of the bull and the bear market phases can be exploited by investors to set a long-term trading strategy based on the prospect of future prices. In particular, a possible trading strategy detecting a bull market phase would be based on the progressive sale of stocks, or the purchase of put-options, in order to avoid the down-trends or persistent downturns. In the opposite scenario, a bear market would give the opportunity to accumulate stocks due to the low stock prices as a consequence of the pessimism in the market. The early warning indicator can be also considered as a valid instrument for policy-makers to set more efficient policies in order to avoid the contagion from the financial side to the real side of the economy.

In Fig. (7) we identify the following optimistic market phases: 1987, 1994, 1997-1998, 2006, 2007 and 2014-2015. In favour of the proper performance of the indicator, all these optimistic phases of the market precede at least an extreme negative weekly return, which are triggered by well-identified events. In chronological order, the first optimistic period appears some months before the black monday (October, 1987), from April to August 1987. In fact, we observe in our sample the most negative event with a weekly return equal to -18.92% due to the errors of the computerised trading system, which triggered sell orders of an enormous block of stocks as prices fell (Waldrop, 1987). The second optimistic period is detected from March 1997 to August 1998, whose end is found on the last week of August 1998 as a result of the Russian financial crisis (Buchs, 1999) with a negative weekly return of -7%. Another extreme optimistic period in the S&P 500 can be identified, from January to May 2006, in which a down-trend stopped the increase in prices due to a new prospect of further tightening of the monetary policy in USA, Europe and Japan (IMF, 2006). Despite this small decrease in prices, the optimism remained in the market with another peak of optimism from June to July 2007. Interestingly, our indicator includes the maximum price previously to the United States bear market (2007-2009), triggered by the subprime mortgage crisis in October 2007 (Demyanyk and Van Hemert, 2009). The last optimistic period observed in our sample is detected during 2014-2015, which arise from the unconventional expansionary monetary policy implemented by the central banks, known as quantitative easing, generating an increase in prices without precedence that still remains nowadays. This surprising scenario has only been interrupted by the stock market sell-off in 2015, as a result of fears about China economy and investors concerns over the end of the quantitative easing (Bendini, 2015; Sornette et al., 2015). Our empirical analysis stresses the validity of the early warning indicator since the six extreme bull market phases identified by the early warning indicator systematically precede a down-trend of the market.

Focusing on the second hypothesis, Fig. (7) shows generally three extreme pessimistic periods (levels around the 10^{th} percentile): after the black monday (1987-1990), the burst of the Dot com bubble (2001-2003) and the burst of the housing bubble (2008-2009). As can be observed from the S&P 500 index, there is a considerable increase in prices after the detection of the pessimistic phase. In particular, we can mention three increases in prices: after the Gulf war (1990), after the burst of the dot com bubble (2003), and after the burst of the housing bubble (2005). Thus, the persistent downturns can be used to progressively accumulate call-options or stocks since it is expected the end of the pessimistic phase of the market.

7 Robustness analysis

So far, we have examined the statistical properties of our proxy for the mood of investors estimated using the collective movement of S&P 500 stocks. However, any stock market can be potentially driven in some measure by agent's sentiment. Different statistical properties of the evolution of the sentiment index could characterise diverse stock markets due to factors like country's features (Anderson et al., 2011, Karlsson and Nordén, 2007) proportion of institutional investors, firm size (Ferreira and Matos, 2008) or reports quality (Biddle et al., 2009) among other aspects. Given these differences among markets, it is interesting to study whether the sentiment index and the early warning indicator can be meaningfully applied to other financial markets regardless of the evident differences among countries and indexes.

In order to do so, we repeat the procedure to compute the sentiment index for two alternative data sets: US stock markets and worldwide stock markets. On the one hand, examining different stock markets in the same country, i.e. assuming that the country features are invariant, we study whether aspects like firm size, proportion of institutional investors, liquidity or the specificity of a sector can affect the behaviour of the sentiment index. We analyse, therefore, the S&P 400 midcap index whose companies are smaller than S&P 500 firms, with a lower level of liquidity and a clear different proportion of institutional investors given the absence of analyst recommendations for these companies. We also analyse the Nasdaq index, whose core business of the companies is focused on the information technology sector.

On the other hand, we study the indexes of 6 different countries in order to observe whether a different result arises, not only due to stock market characteristics but also due to country features like culture or economic conditions. Thus, we examine the following countries and indexes: ASX 200 (Australia), TSX (Canada), Euro Stoxx 600 (Europe), Nikkei 225 (Japan), JSE (South Africa) and FTSE 100 (UK).

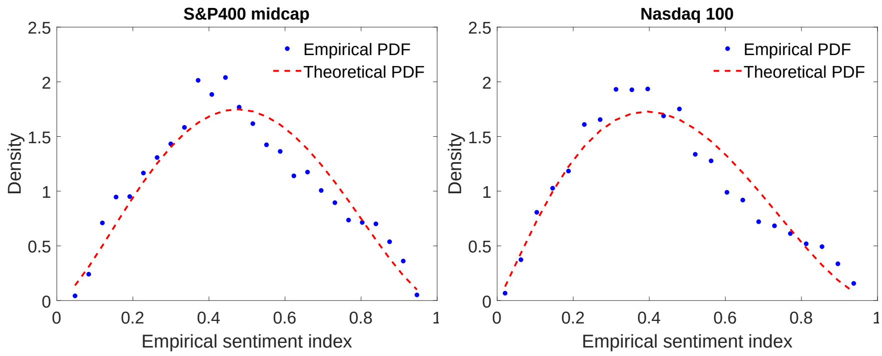
7.1 US stock markets

Investor preference is one of the main features to take into account in order to know who is investing in each type of market. In fact, Ferreira and Matos (2008) show that institutional investors prefer firms that are characterised by being large, well-governed and with high levels of stock-trading liquidity. In the same line, Aggarwal et al. (2005) find that US funds are more interested in investing in large firms with transparent accounting policies.

Table 4: Estimated parameters, ε_1, ε_2 and b for U.S. stock indices. The Langevin equation has been used to obtain the estimates excluding extreme negative events ($z_t > 0.95$).

Stock market	Date	ε_1	ε_2	b
S&P 400 midcap	1993-2018	2.55 ± 0.23	2.72 ± 0.23	0.0060 ± 0.0002
Nasdaq	1993-2018	2.18 ± 0.17	2.82 ± 0.21	0.0088 ± 0.0002

Fig. 8: Probability density function of the U.S. sentiment indices compared to the theoretical distribution. The Langevin equation has been used to obtain the estimates excluding extreme negative events ($z_t > 0.95$).



The relevance of financial reports is highlighted by Biddle et al. (2009) since those firms with higher financial reporting quality suffer less from macro-economic conditions and deviate less from predicted investment levels. Financial aspects are also crucial since domestic managers prefer companies with large dividends, low financial distress and low return variability (Covrig et al., 2006), which are features of firms with high market capitalisation. Considering these characteristics, institutional investors would prefer to invest in companies from S&P 500 rather than those from S&P 400 midcap. If we consider the median total market cap, S&P 500 is 5 times larger than S&P 400 midcap, with the following market capitalisation: 20493.91 and 4178.83 US millions for S&P 500 and S&P 400 respectively.[19]

Despite differences among these indexes it is possible to observe in Fig. (8) and Table 4 that the estimates are in line with those obtained for the S&P 500. Moreover, if we use stock markets like Nasdaq, focused on firms from the information technology sector, we keep observing very similar estimates. Therefore, even with a different type of investor, liquidity, sector, or market cap, our proxy for the sentiment of investors shows very similar statistical properties: volatility clustering in the time series of sentiment index increments and the exponential autocorrelation function of the sentiment index (the statistical analysis is shown in the supplementary material).

7.2 Worldwide stock markets

A remarkable feature of worldwide stock markets is the high level of portfolio concentration in domestic markets. This phenomena, known as "home bias", goes not only against the advantages of international diversification but also many standard asset-pricing models. This traditional feature, which was highlighted in the 90s by French and Poterba (1991), Cooper and Kaplanis (1994) and Tesar and Werner (1995), is still present nowadays

[19] Median total market capitalisation of S&P 500 and S&P 400 mid cap refer to June 2018. Factsheets of S&PDow Jones Indexes.

Table 5: Estimated parameters, ε_1, ε_2 and b for worldwide stock indices. The Langevin equation has been used to obtain the estimates excluding extreme negative events when $z_t > 0.95$.

Index	Country	Date	ε_1	ε_2	b
ASX 200	Australia	1993-2018	2.68 ± 0.23	3.09 ± 0.26	0.0060 ± 0.0001
TSX	Canada	1993-2018	3.18 ± 0.33	3.62 ± 0.36	0.0036 ± 0.0001
Nikkei 225	Japan	1993-2018	0.96 ± 0.10	1.29 ± 0.13	0.0086 ± 0.0002
Euro stoxx 600	Europe	1993-2018	1.56 ± 0.18	2.37 ± 0.25	0.0050 ± 0.0001
JSE	South Africa	2000-2018	1.70 ± 0.21	3.00 ± 0.40	0.0081 ± 0.0001
FTSE 100	UK	1993-2018	1.74 ± 0.15	2.63 ± 0.20	0.0081 ± 0.0002

despite the many improvements in terms of information channels.[20] Anderson et al. (2011) demonstrate that home bias exists in institutionally managed portfolios from more than 60 countries. In the same vein, Grinblatt and Keloharju (2001) with a database focused on Finland, contend that investors prefer those firms that are nearby with a common language and culture.

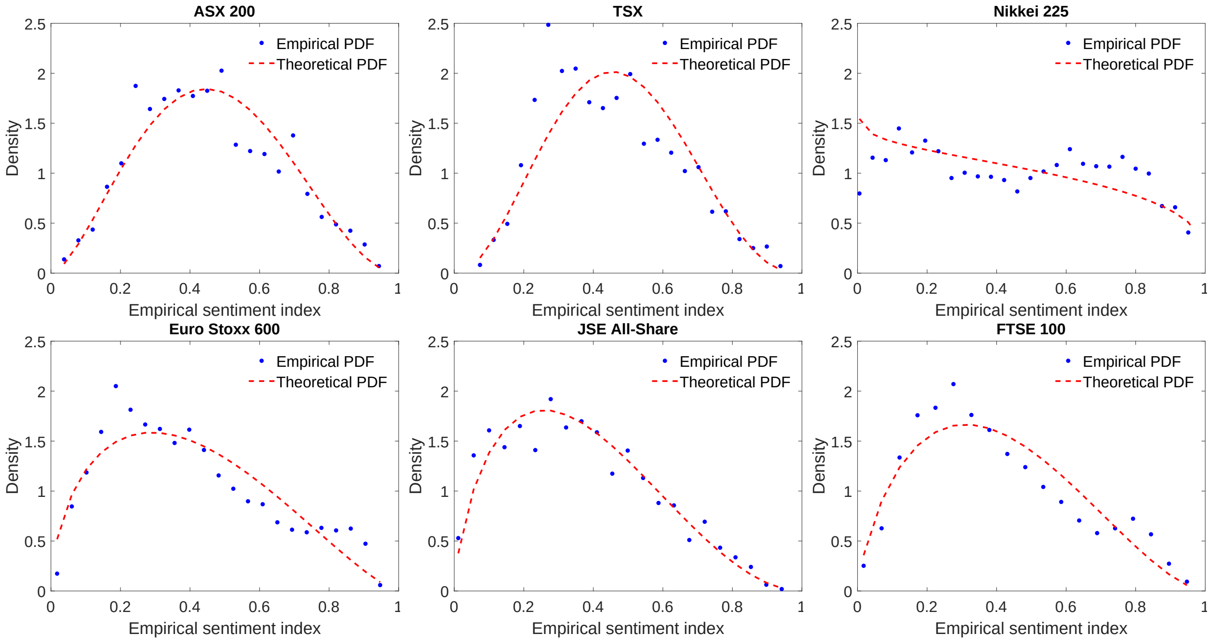
In this line, home bias underlines the fact that local agents are investing in those companies and stocks that are present in the corresponding local or domestic financial market. Therefore, we can assume that when analysing a worldwide stock market, the main proportion of capital is given by local investors. This aspect is of paramount importance for this study since the fact of observing a proper fit of the unconditional distribution of the sentiment index, regardless of the country, implies that we can use our model to describe optimism and pessimism of agents regardless of the country specific characteristics. Table 5 and Fig. (9) show how the Langevin process provides us with a fairly homogeneous description of financial markets from different countries. The estimated parameters are in fact fluctuacting in a narrow range of variability. Excluding the Nikkei 225 index, the sentiment index of all the other financial markets seem to be well characterised by an asymmetric uni-modal Beta distribution, with non-monotonic probability density. Also in the case of international markets, we observe heteroskedastic fluctuations of the index increments and an exponential decay of its autocorrelation (the results in the supplementary material).

7.3 Global financial village

The fact that all the financial markets have become a coupled complex system is not surprising given the correlation between stock market indices (Mantegna and Stanley, 1996; Forbes and Rigobon, 2002), as it is underlined by Kenett et al. (2012) when identifying that U.S., U.K., Germany and Japan indices are highly interconnected. Consequently, the "global financial village is highly prone to systemic collapses which can sweep the entire village" (Kenett et al., 2012). In the same line, by means of our early warning indicator we observe how all the stock indices are behaving in a similar manner regardless of the country specific characteristics of each index. The effect of the Russian financial crisis in 1998, the dot com crash in 2001-2003, the burst of the housing bubble in 2008-2009 and the down-trend in prices during 2015-2016, due to the weakness of China economy, are the best example of the consequences of the "global financial village", since, to a greater or lesser

[20] Although Gehrig (1993), Ivković and Weisbenner (2005) and Massa and Simonov (2006) suggest that investors can take advantage of exploiting local information, Seasholes and Zhu (2010) state that investors have not value-relevant information on local companies.

Fig. 9: Probability density function of the worldwide sentiment indices compared to the theoretical distribution. The Langevin equation has been used to obtain the estimates excluding extreme negative events ($z_t > 0.95$).

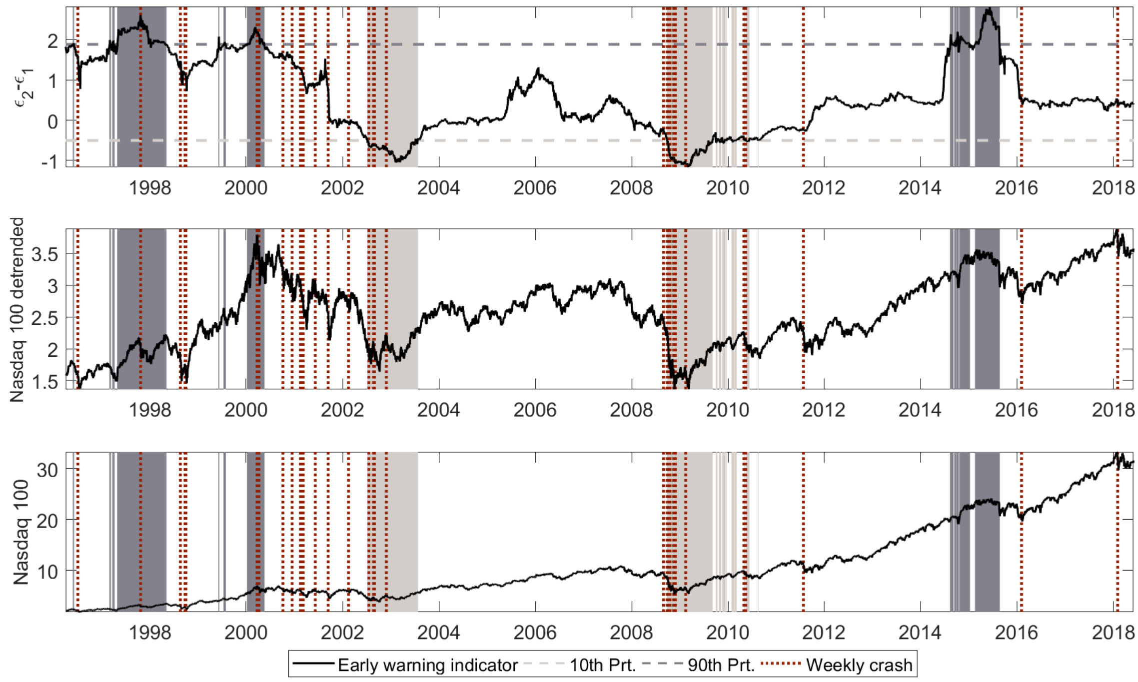


extent, it is possible to observe those events regardless of the market (see in this section: Nasdaq 100 (U.S.), Fig. (10); Eurostoxx 600 (Europe), Fig. (11) and Nikkei 225 (Japan), Fig. (12)).[21] As a result, in favour of the two hypotheses of our early warning indicator, we can identify negative weekly returns after an optimistic market phase has been detected, while a pessimistic market phase is characterised by low prices compared to the future price of the indices. In fact, we identify a high concentration of negative weekly returns in all the stock indices after the burst of the housing bubble (2008-2009), along with a lower concentration during the burst of the dot com bubble (2001-2003), and the down-trend caused by China economy (2015-2016).

However, despite the fact that all the stocks indices are generally affected by similar negative events, it is interesting to observe that a bubble originated in one specific market is generating financial downturns in other markets that were not involved in such optimistic scenario. To explain this point, we underline two main periods of the recent financial history (the dot com bubble and housing bubble) using the main stock indices (S&P 500 (U.S.), Fig. (7); Nasdaq 100 (U.S.), Fig. (10); Eurostoxx 600 (Europe), Fig. (11) and Nikkei 225 (Japan), Fig. (12)). As can be easily observed, despite the Russian financial crisis in 1998, investors trading in Nasdaq stocks maintained their positive mood giving rise to a bull market from February to May 2000. The burst of the bubble was in March 2000, as we can note due to the continuous negative weeks in Fig. (10).

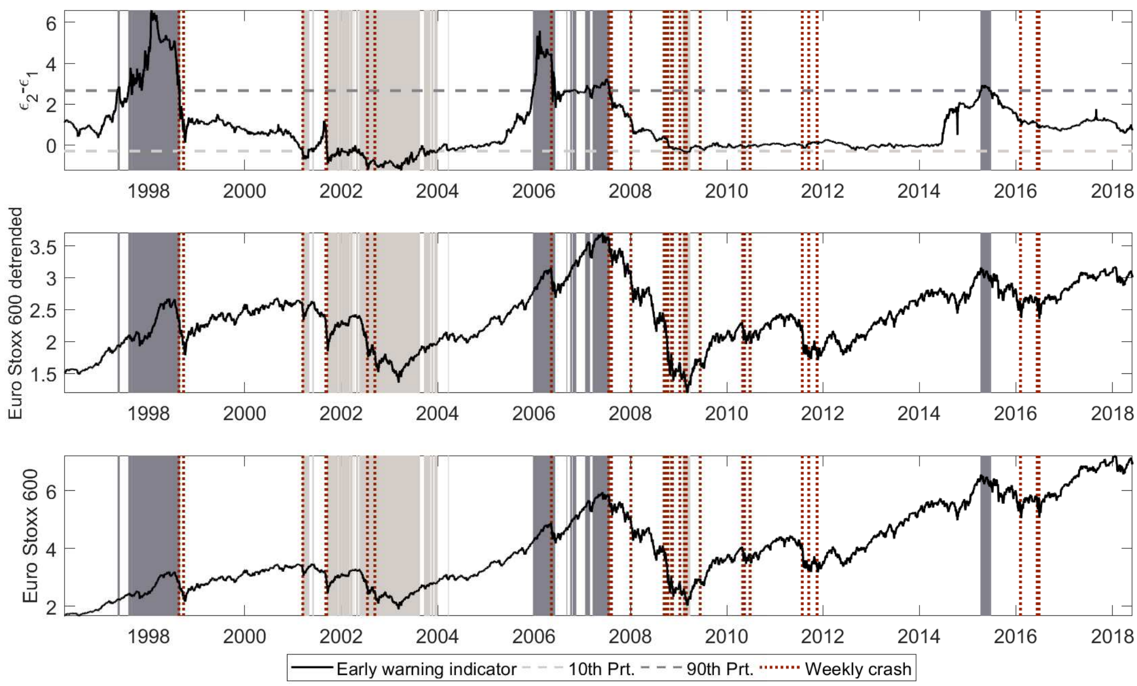
[21] In the Appendix: S&P 400 midcap (U.S.), Fig. (14); ASX 200 (Australia), Fig. (15); TSX (Canada), Fig. (16); JSE All-Share (South Africa), Fig. (17) and FTSE 100 (UK), Fig. (18)

Fig. 10: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. Nasdaq 100 index.



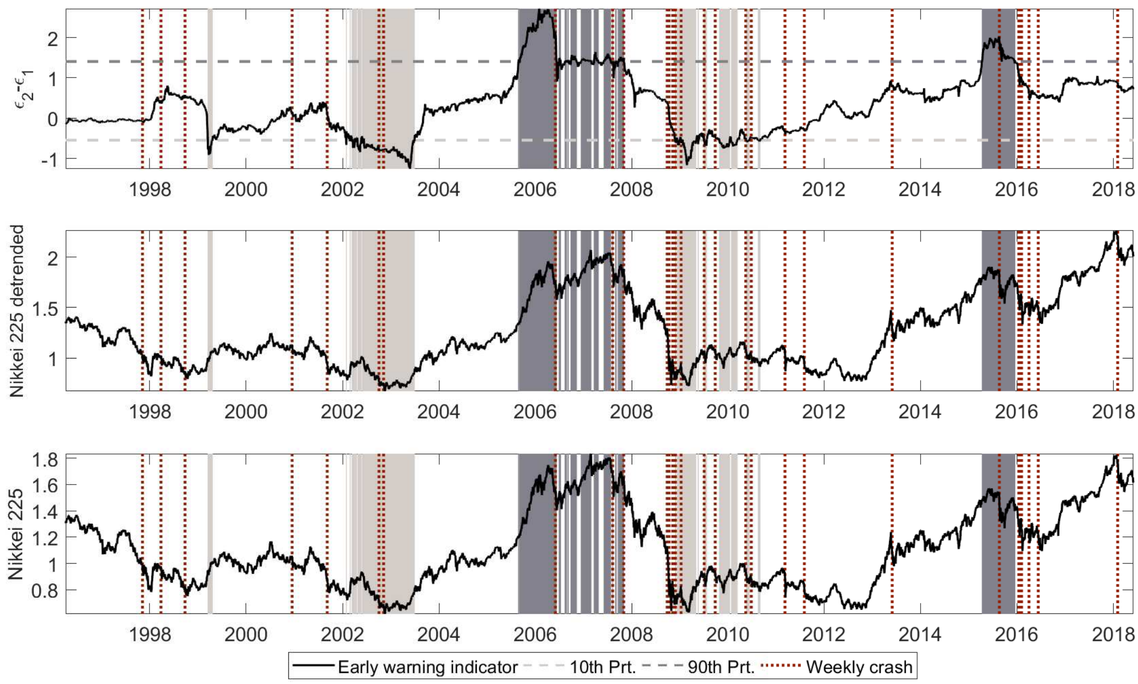
Note. Bull market phases and the subsequent negative events: 1997-1998 (Russian financial crisis), 2000 (burst of the dot com bubble) and 2015-2016 (weakness of China economy). Bear market phases: 2002-2003 (burst of the dot com bubble) and 2008-2010 (burst of the housing bubble).

Fig. 11: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. Euro stoxx 600 index.



Note. Bull market phases and the subsequent negative events: 1997-1998 (Russian financial crisis), 2006 (tightening monetary policy), 2007 (subprime mortgage crisis) and 2015 (weakness of China economy). Bear market phases: 2001-2003 (burst of the dot com bubble) and 2009 (burst of the housing bubble).

Fig. 12: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. Nikkei 225 index.



Note. Bull market phases and the subsequent negative events: 2006 (tightening monetary policy), 2007 (subprime mortgage crisis) and 2015-2016 (weakness of China economy). Bear market phases: 2002-2003 (burst of the dot com bubble) and 2008-2010 (burst of the housing bubble).

Surprisingly, we do not observe the previous optimism to the burst in the rest of indices, like S&P 500 or Euro Stoxx 600. In fact, Nikkei 225 does not even have an optimistic period before the Russian financial crisis. However all the markets suffered from the burst of the dot com bubble with a wave of pessimism during the following years, mainly, from 2001. In other words, the bubble originated in a particular market (Nasdaq) due to the optimism of their traders, and the resulting herding phenomena, gave rise to a pessimistic scenario in very different markets (S&P 500, Eurostoxx 600, Nikkei 225, and the rest of the markets in the Appendix) whose traders were not so optimistic. Interestingly, we can observe during the housing bubble the opposite scenario. A wave of optimism dominates most of the stock markets like the Eurostoxx 600 and Nikkei 225 indices, which are affected by the same negative events in May 2006 and October 2008 as in the S&P 500 index. Nevertheless, Nasdaq 100 does not have an optimistic period nor it is affected by the same negative events during this period. At any rate, and in the same line as the dot com bubble, Nasdaq and all the markets suffered from the housing crash, even though there is not an optimistic phase in the Nasdaq index, i.e. despite the fact that there is not a bubble in a specific market, this market will be affected by the mood of other markets due to financial contagion. Therefore, the early warning indicator also allows us to describe the different behaviour of each stock index in a more detail level, even though all of them are connected, as can be observed due to the effect of the financial downturns.

8 Conclusion

Inspired by the Bank of America Merrill Lynch Global Breath Rule, we have introduced an index of financial investor sentiment based on the collective movements of the stocks in a given financial market. The underlying hypothesis is that such index reflects and captures the collective behaviour of the investors influencing each other in waves of optimism and pessimism transmitted by the social interactions. The index z_t aggregates the state of each single stock, which depends on the relative position of its price with respect to the 100-day EMA. The time evolution of the index can be successfully described by the herding model introduced by Kirman (1991, 1993). In particular, the unconditional distribution of the sentiment index, the heteroskedastic fluctuations of the time series of its increments and the autocorrelation function match the analytical properties of the herding model. Based on the herding model and the sentiment index, we introduce an early warning indicator, using the distributional asymmetry of the sentiment index computed in a rolling window. Our early warning indicator can clearly identify the optimistic and pessimistic phases of the market. Thus, investors can devise strategies to effectively exploit the early warning indicator. Our results are robust when applying the early warning indicator to other indices of the US financial markets or financial market indices of other countries like Japan, Australia or Canada among others.

Acknowledgements

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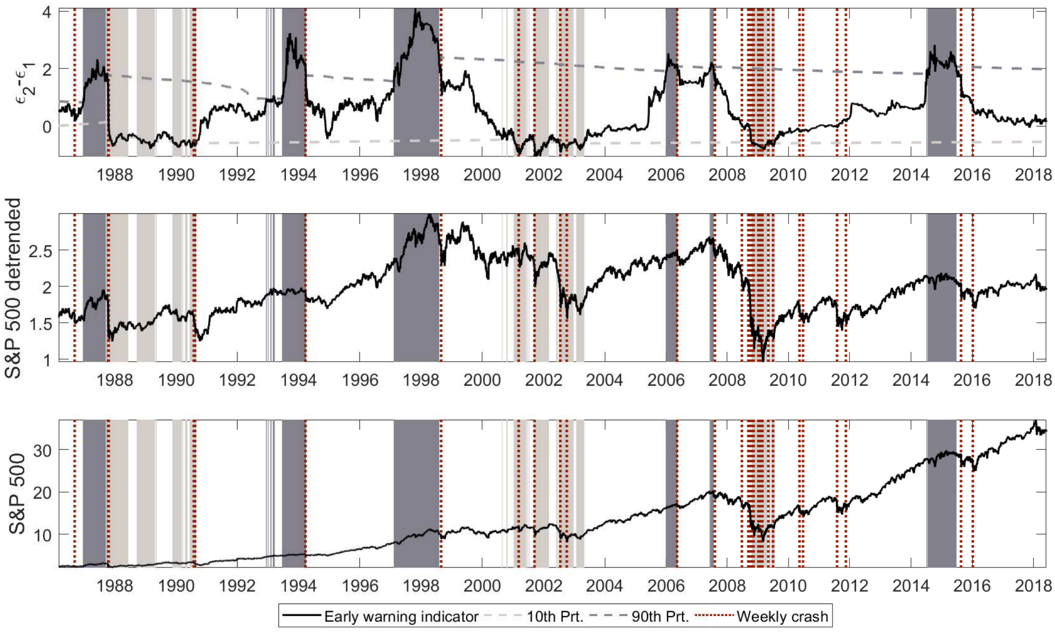
9 Appendix

9.1 A1: Robustness analysis of the determination of the thresholds

In order to study the robustness of the results of Fig. (7), we compute the two thresholds at 10^{th} and 90^{th} percentile using past data instead of the entire sample. So, we compute the percentiles considering (i) 500 data points, from day 1 to day 500; (ii) adding the other values of Λ_t, from 501 to the end of the time series and computing the new values of the thresholds. As we can see from Fig. (13), the results are essentially unchanged with respect to Fig. (7). This exercise shows that we can use the current value of the thresholds to make inference of the future behaviour of the market.

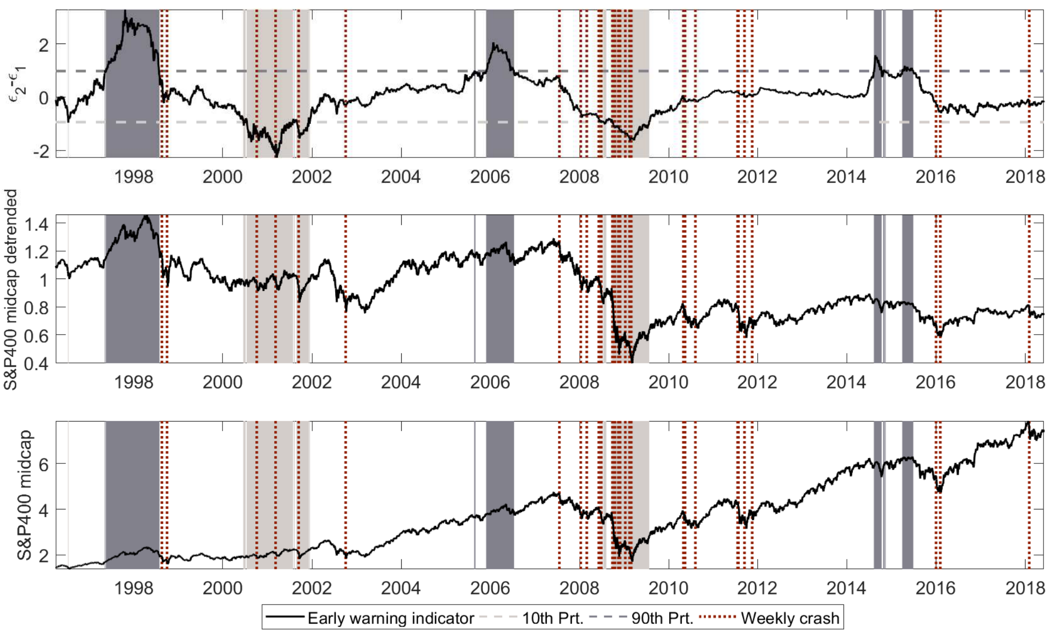
Fig. 13: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. S&P 500 index.



Note. Bull market phases and the subsequent negative events: 1987 (black monday), 1994 (tightening monetary policy), 1997-1998 (Russian financial crisis), 2006 (tightening monetary policy), 2007 (subprime mortgage crisis) and 2015-2016 (weakness of China economy). Bear market phases: 1987-1990 (black monday and Gulf war), 2001-2003 (burst of the dot com bubble) and 2008-2009 (burst of the housing bubble).

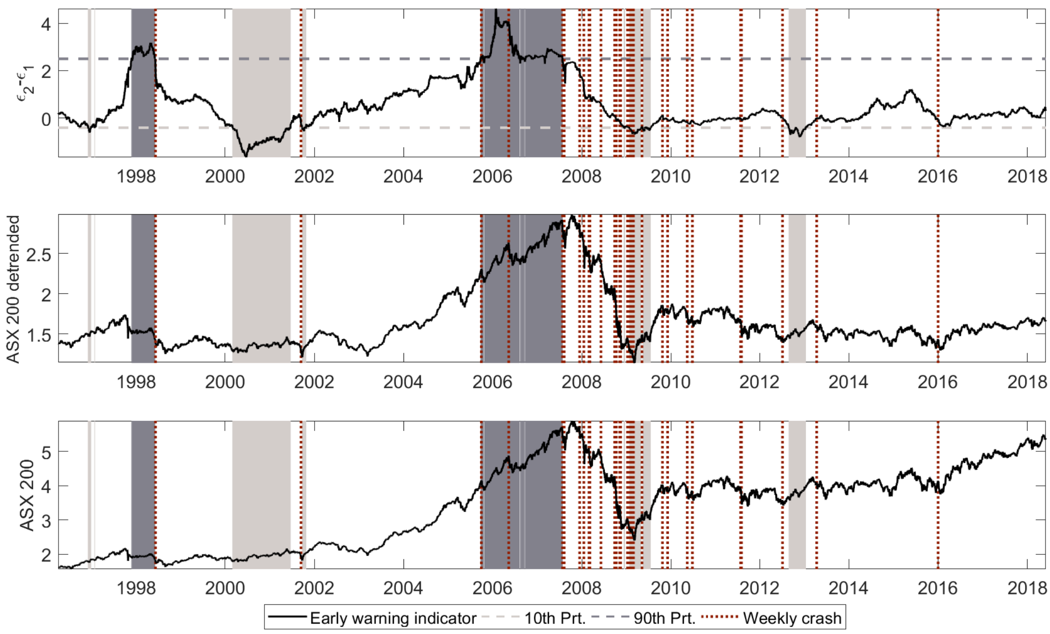
9.2 A2: Early warning indicator

Fig. 14: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. S&P 400 midcap index.



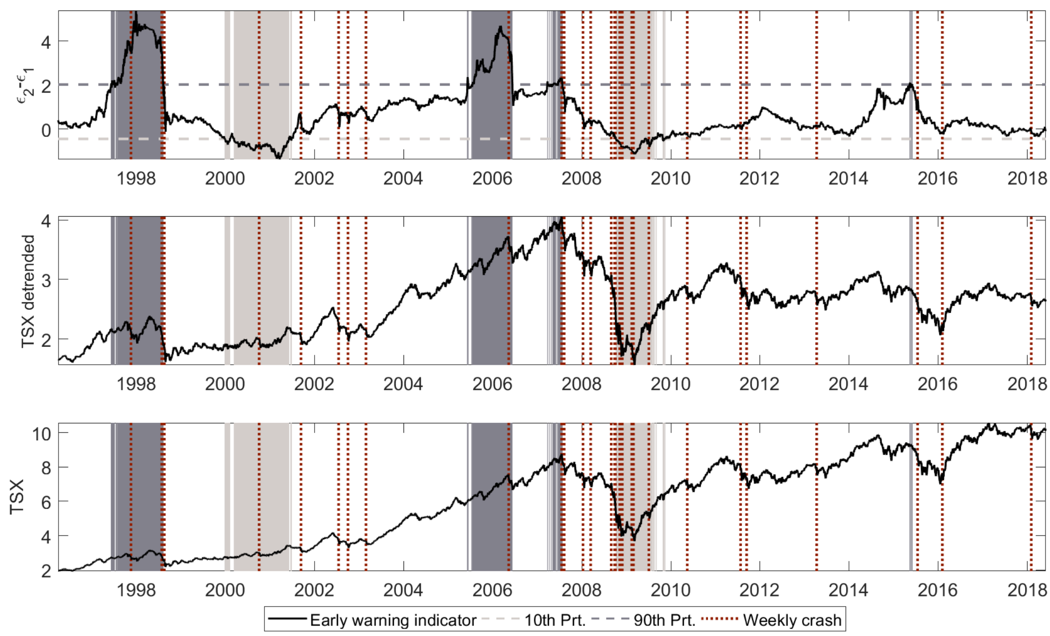
Note. Bull market phases and the subsequent negative events: 1997-1998 (Russian financial crisis), 2006 (subprime mortgage crisis) and 2014-2015 (weakness of China economy). Bear market phases: 2000-2001 (burst of the dot com bubble) and 2008-2009 (burst of the housing bubble).

Fig. 15: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. ASX 200 index.



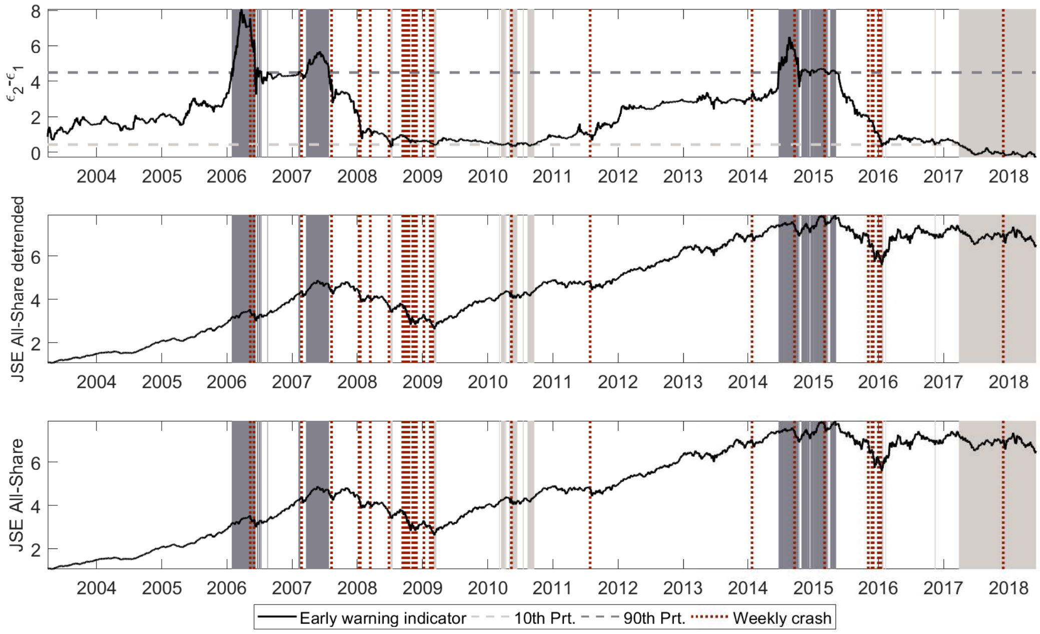
Note. Bull market phases and the subsequent negative events: 1997-1998 (Russian financial crisis), 2006 (tightening monetary policy) and 2007 (subprime mortgage crisis). Bear market phases: 2000-2001 (burst of the dot com bubble) and 2009 (burst of the housing bubble).

Fig. 16: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. TSX index.



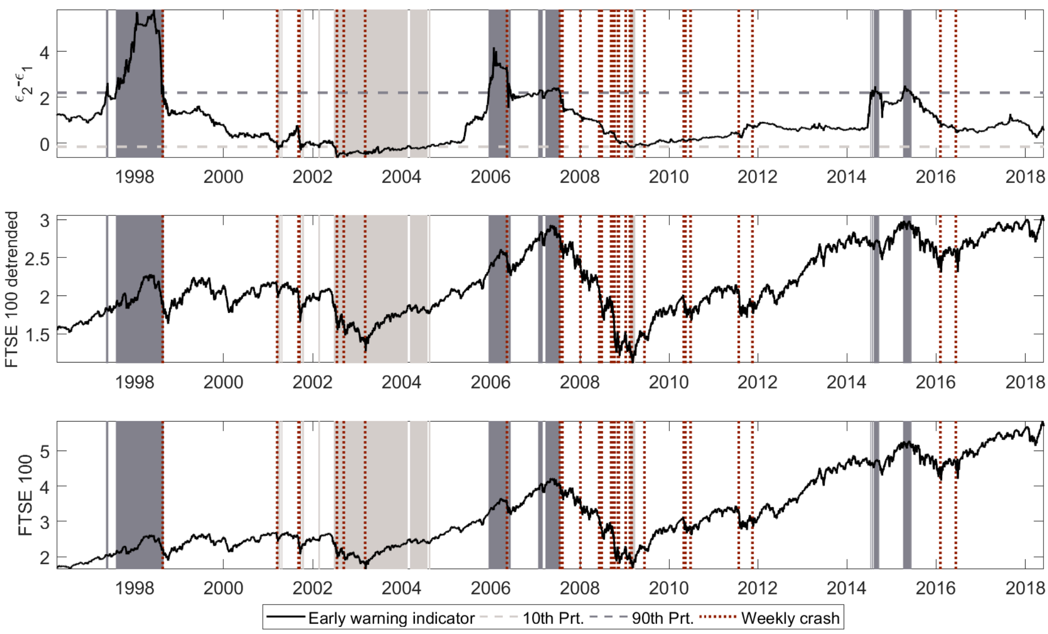
Note. Bull market phases and the subsequent negative events: 1997-1998 (Russian financial crisis), 2006 (tightening monetary policy), 2007 (subprime mortgage crisis) and 2015 (weakness of China economy). Bear market phases: 2000-2001 (burst of the dot com bubble) and 2008-2009 (burst of the housing bubble).

Fig. 17: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. JSE All-Share index.



Note. Bull market phases and the subsequent negative events: 2006 (tightening monetary policy), 2007 (subprime mortgage crisis) and 2014-2015 (weakness of China economy). Bear market phases: 2008-2009 (burst of the housing bubble) and 2016 (China financial crash).

Fig. 18: Early warning indicator using 100-day MA and a time interval of 750 days for the estimation of parameters. Light and dark grey areas represent the 10^{th} (bear market) and 90^{th} (bull market) percentiles. Dotted red line denotes the 30 most negative weekly returns. FTSE 100 index.



Note. Bull market phases and the subsequent negative events: 1997-1998 (Russian financial crisis), 2006 (tightening monetary policy), 2007 (subprime mortgage crisis) and 2014-2015 (weakness of China economy). Bear market phases: 2002-2004 (burst of the dot com bubble) and 2009 (burst of the housing bubble).

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1 Supplementary material

In the following, the time series of the index and its changes together with the fit of the autocorrelation function based on Eq. (19) are shown. The figures illustrate the statistical analysis for all the markets analysed in the paper.

1.1 Sentiment index

Fig. 1: Sentiment index and index change of S&P 400 midcap.

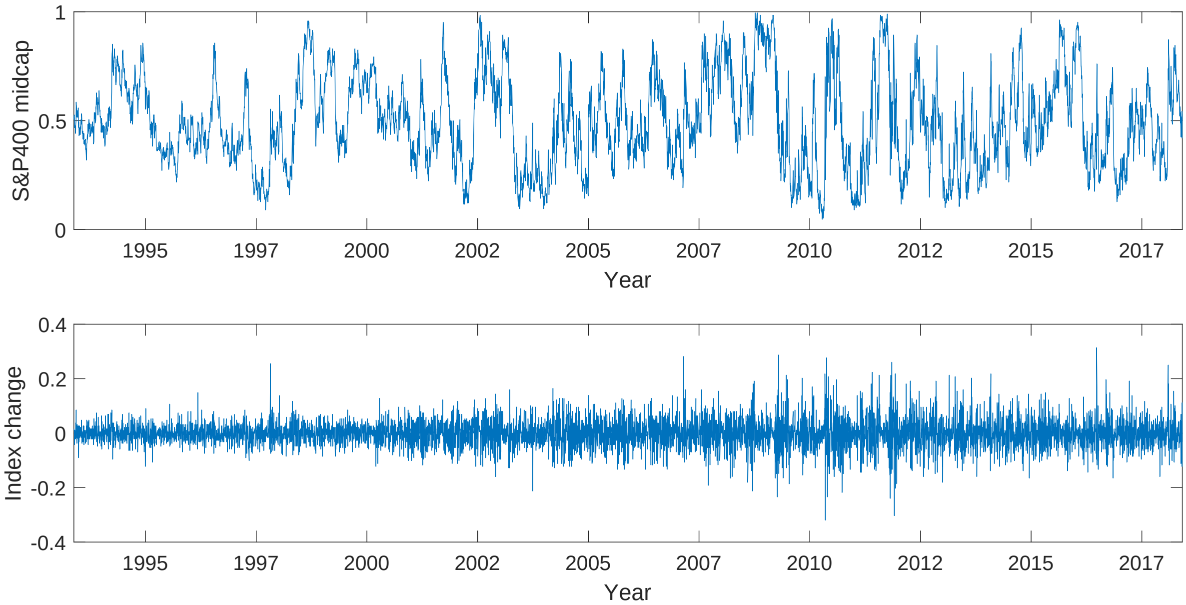


Fig. 2: Sentiment index and index change of Nasdaq 100.

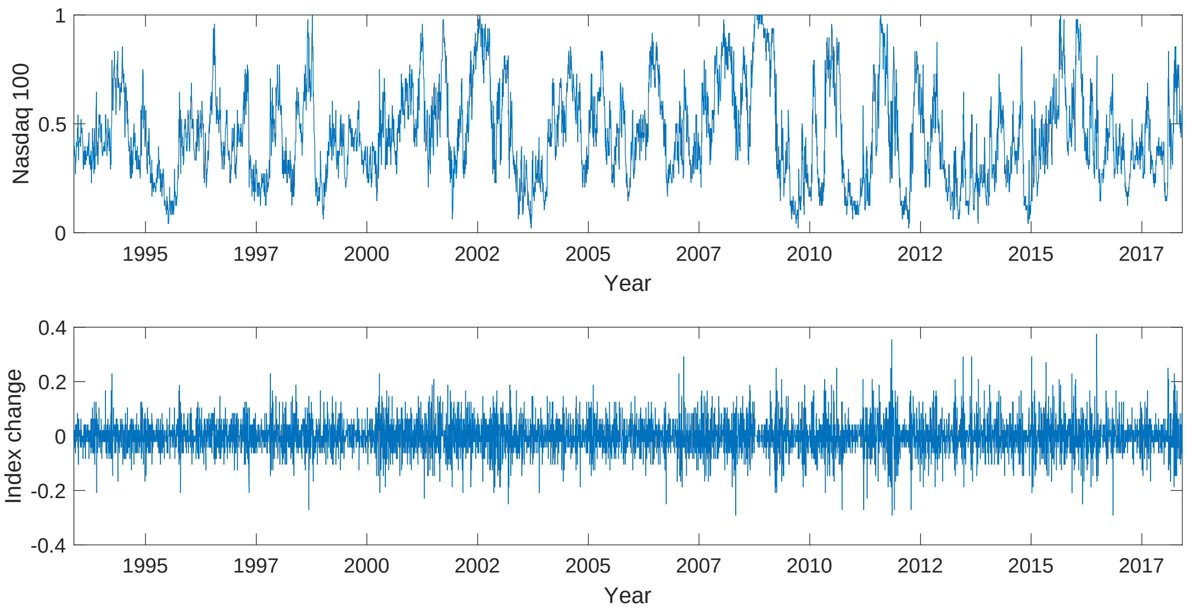


Fig. 3: Sentiment index and index change of ASX 200.

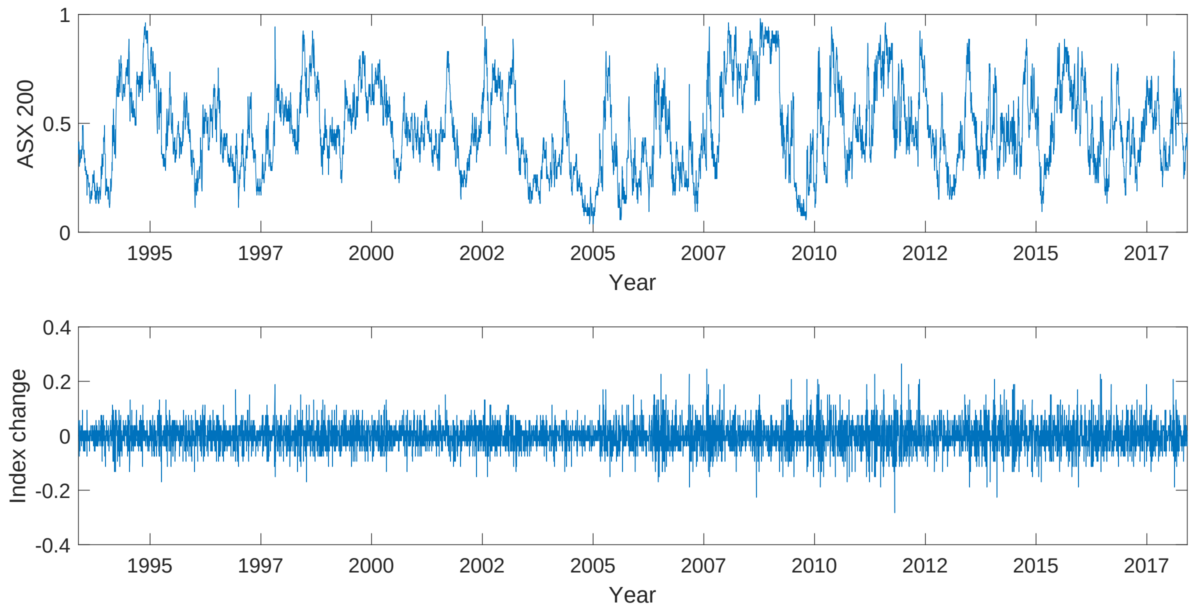


Fig. 4: Sentiment index and index change of TSX.

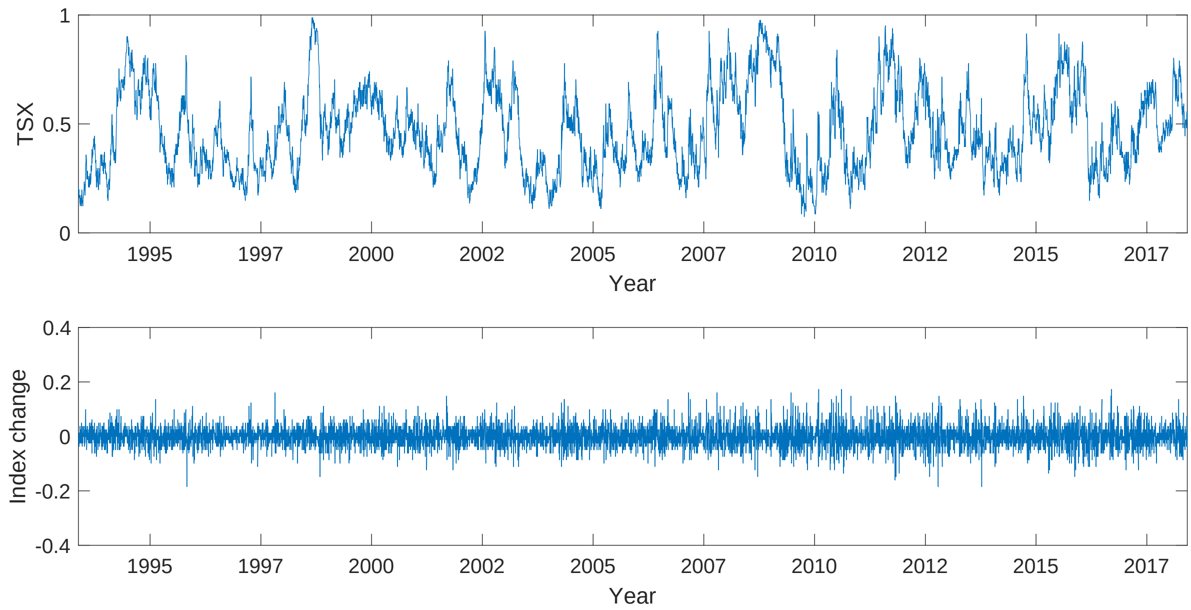


Fig. 5: Sentiment index and index change of Euro Stoxx 600.

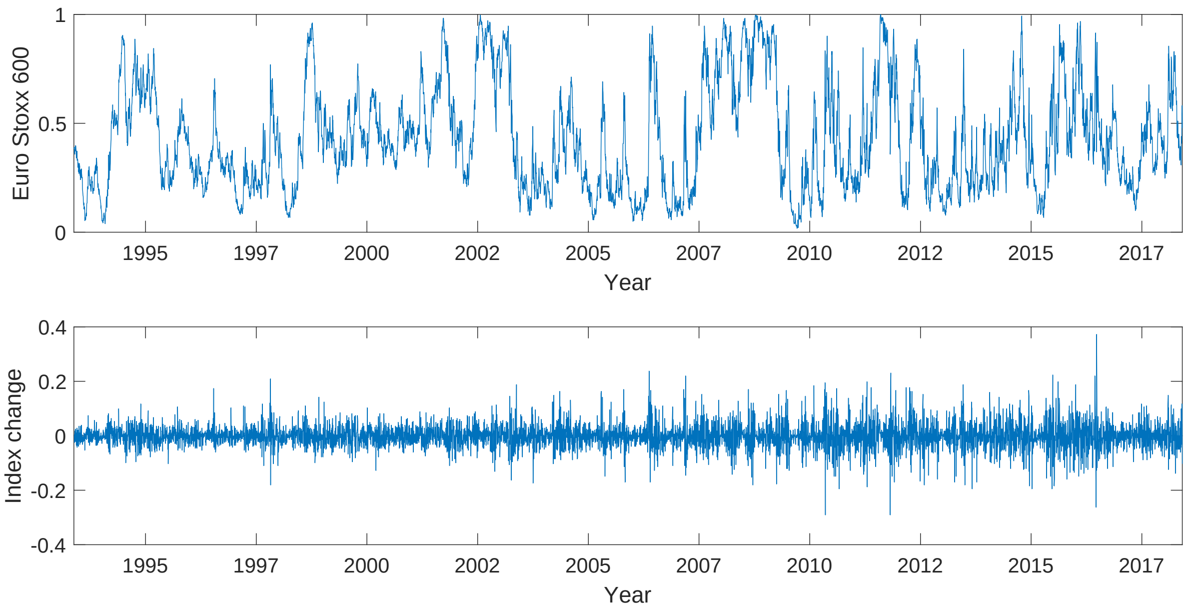


Fig. 6: Sentiment index and index change of Nikkei 225.

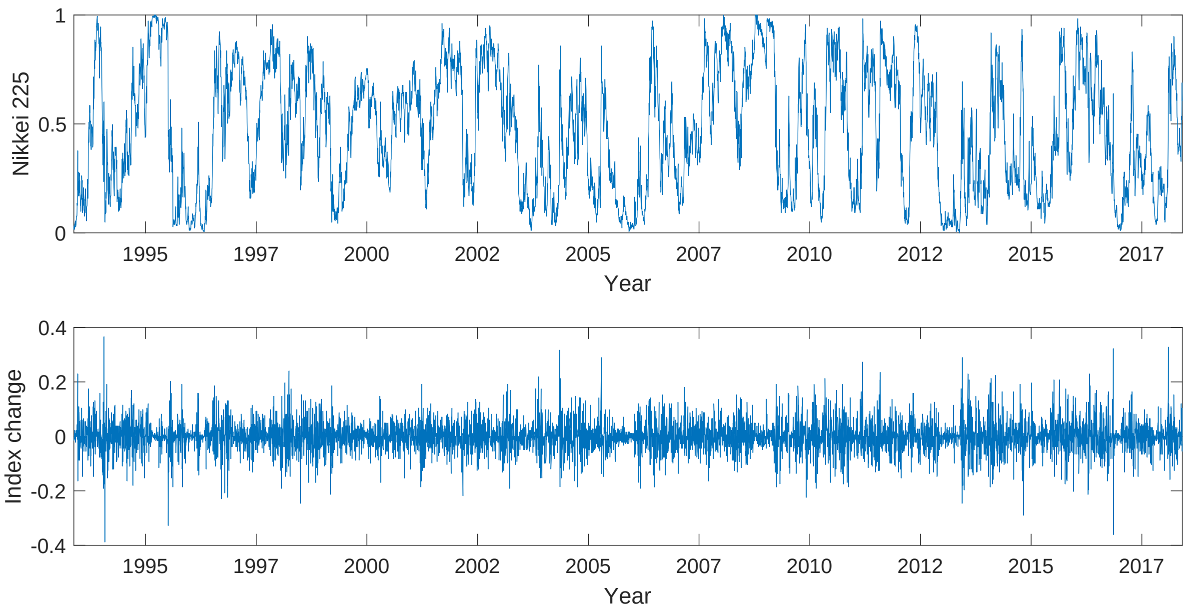


Fig. 7: Sentiment index and index change of JSE All-Share.

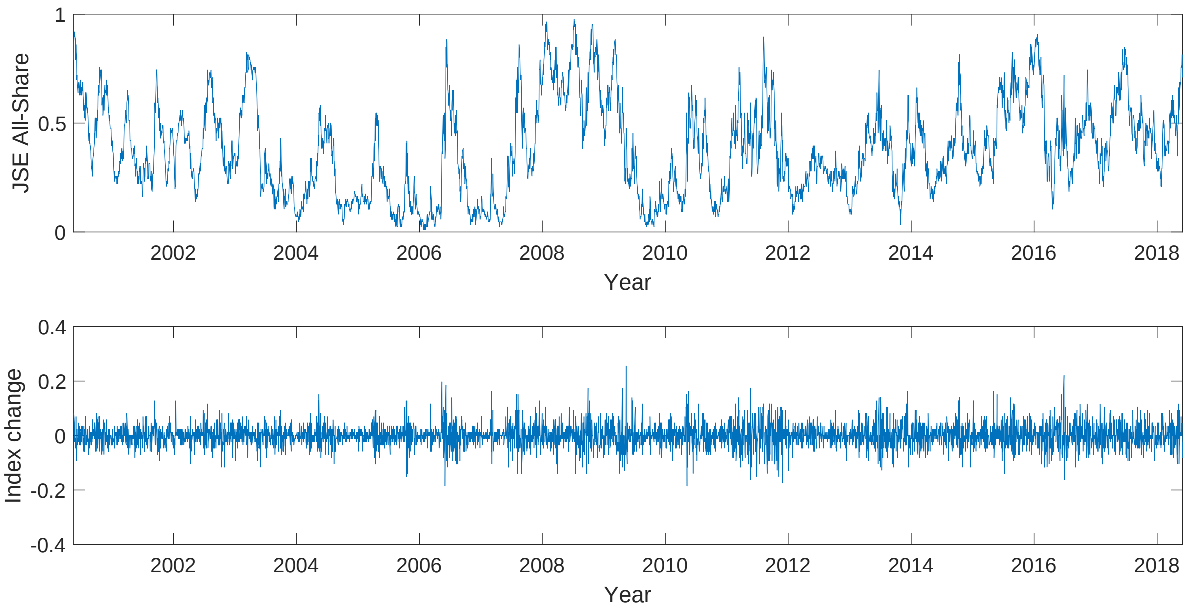
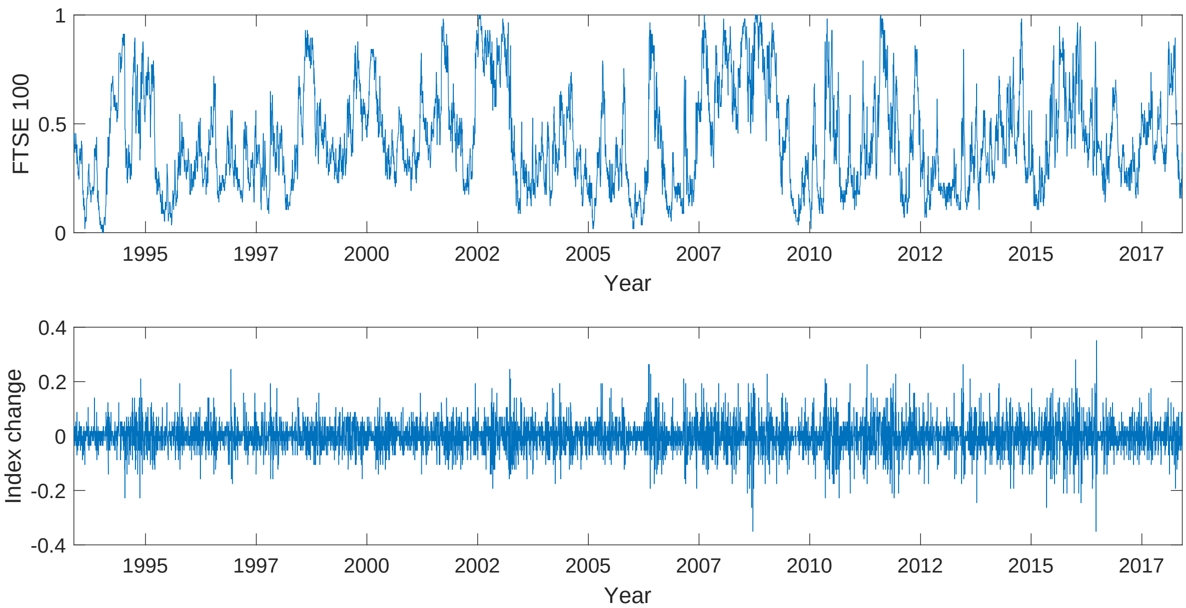


Fig. 8: Sentiment index and index change of FTSE 100.



1.2 Autocorrelation function

Fig. 9: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function. S&P 400 midcap index.

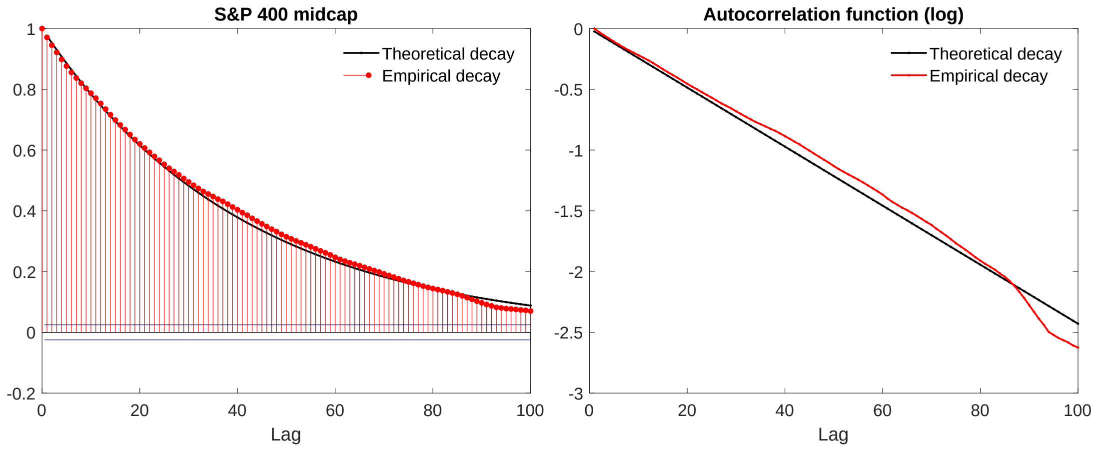


Fig. 10: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function. Nasdaq 100 index.

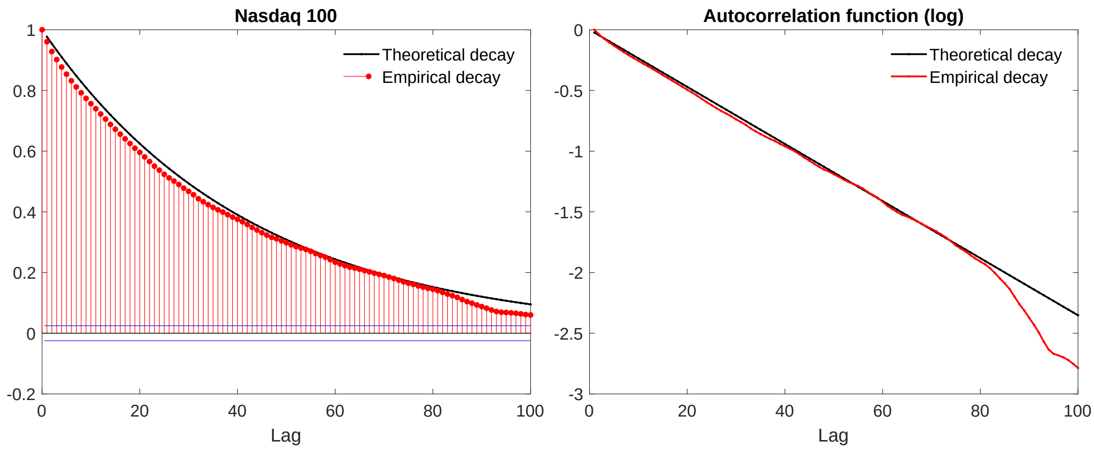


Fig. 11: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function. ASX 200 index.

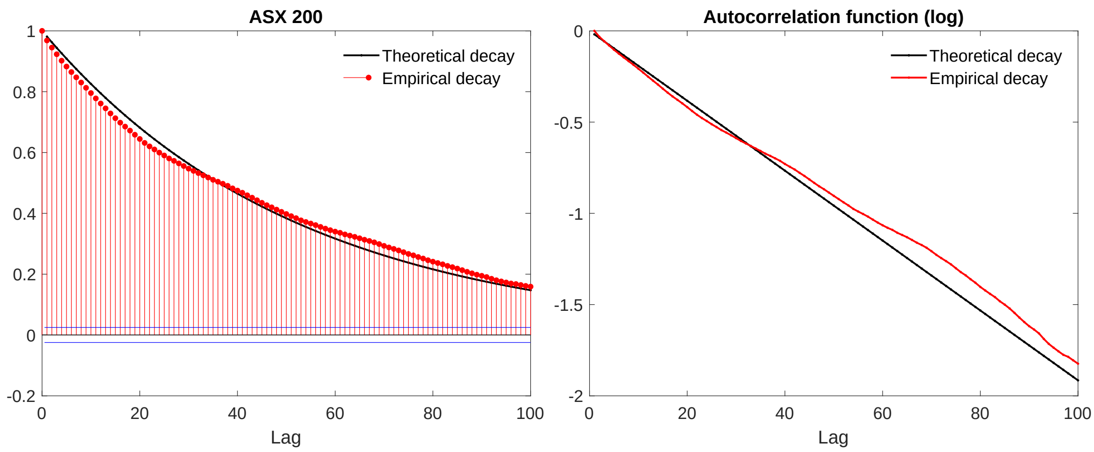


Fig. 12: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function. TSX index.

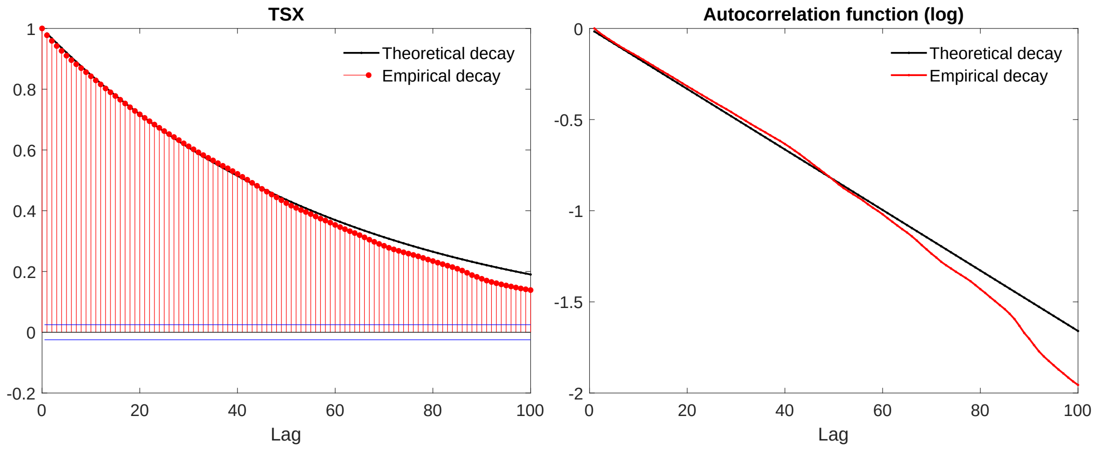


Fig. 13: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function. Nikkei 225 index.

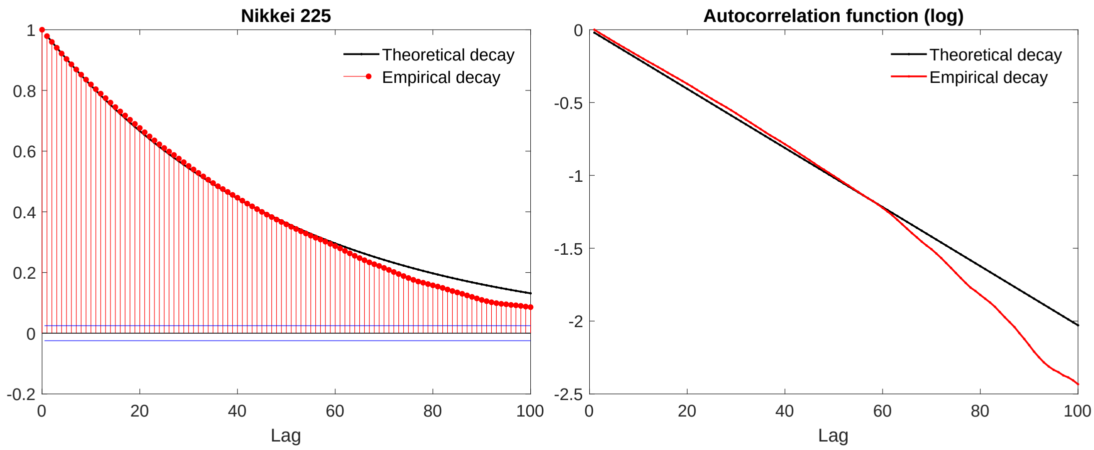


Fig. 14: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function. Euro Stoxx 600 index.

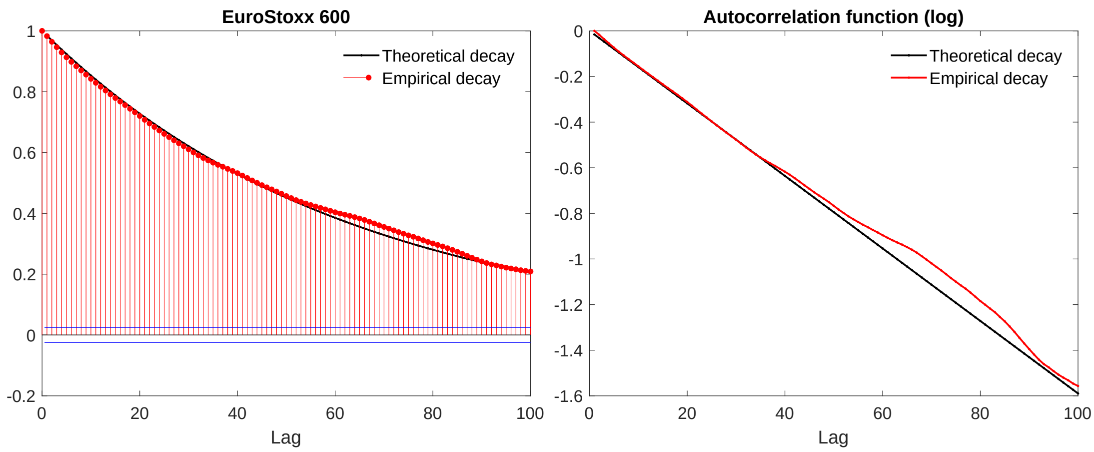


Fig. 15: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function. JSE All-Share index.

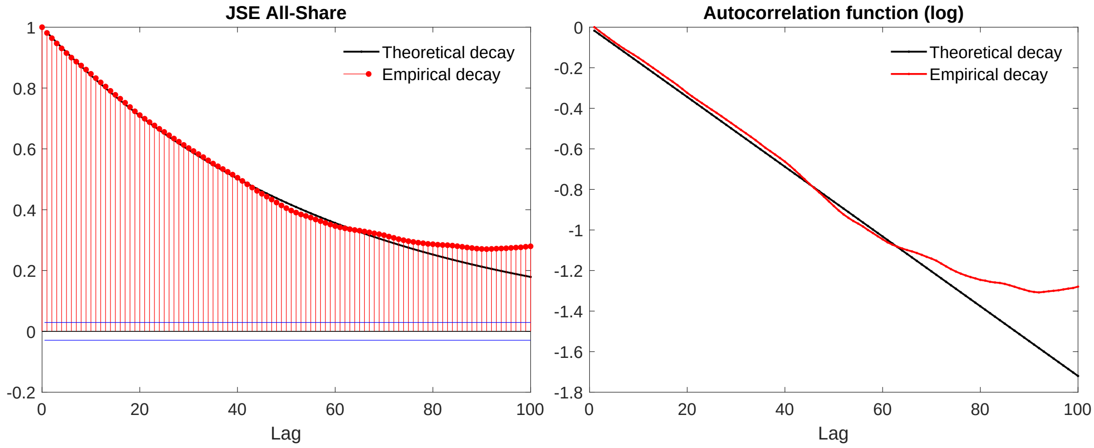


Fig. 16: Autocorrelation function of the sentiment index compared to the theoretical autocorrelation function. FTSE 100 index.