Breakdown of the Narrow Width Approximation for New Physics

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The narrow width approximation is used in high energy physics to reduce the complexity of scattering calculations. It is a fortunate accident that it works so well for the Standard Model, but in general it will fail in the context of new physics. We find numerous examples of significant corrections when the calculation is performed fully off-shell including a finite width, notably from effects from the decay matrix elements, not just phase space. If not taken into account, attempts to reconstruct the Lagrangian of a new physics discovery from data would result in considerable inaccuracies and likely inconsistencies.

The narrow-width approximation (NWA) is used extensively to calculate cross sections for production of promptly-decaying particles. It's use is justified only if five critical conditions are met: (i) the total width of a resonant particle is much smaller than its mass, $\Gamma \ll M$; (ii) daughter particles are much less massive than the parent, $m \ll M$; (iii) the scattering energy is much larger than the parent mass, $\sqrt{s} \gg M$; (iv) there is no significant interference with non-resonant processes; and (v) the resonant propagator is separable from the matrix element (ME). If these are valid, the propagator can be integrated independently over all q^2 (including unphysical values, with negligible effect) to obtain a constant:

$$\int_{-\infty}^{\infty} dq^2 \left| \frac{1}{q^2 - M^2 + iM\Gamma} \right|^2 = \frac{\pi}{M\Gamma} \qquad (1)$$

In a nutshell, the NWA assumes that the massive state is always produced exactly at its pole as an asymptotic final state, so its decay is an independent process, expressed by a simple numerical constant known as a branching ratio (BR): the fractional probability to decay to a specific final state. Parametrically, the NWA introduces an estimated error of $\mathcal{O}(\Gamma/M)$. The NWA is widely used when Γ/M is small, regardless of the other conditions. While there's some awareness of (i), (iii) and (iv), assumptions (ii) and (v) have not previously been discussed in the literature or textbooks to the best of our knowledge.

The NWA became standard at a time when numerical computation tools were not advanced enough to perform off-shell calculations of full matrix elements including decays and finite widths. Complete analytical calculations for such $2 \to n > 2$ processes are generally intractable. The NWA works well for heavy particle production (W and Z bosons and the top quark, excluding hadronic and flavor physics) above threshold in the Standard Model (SM), largely owing to the fact that the decay products are much lighter than the parent. In those cases, Γ/M is indeed small, a couple of percent at most; other uncertainties dominate. We need to include finite widths in high-energy SM calculations only in a few cases involving threshold restrictions, such as scattering at the Z pole, $e^+e^- \to W^+W^- \to 4f$ production [1], top quark pair production as a background at the Large Hadron Collider (LHC) [2], and Higgs boson decay to weak bosons [3]. Our work addresses the regime above threshold.

In the context of new physics extensions to the SM, it is *generally* the case (but not always) that massive particle widths are much smaller than their masses, $\Gamma \ll M$, leading one to conclude that the NWA is still valid. Also, collision energy is typically far above production threshold, $E_{\rm CM} - M \equiv \sqrt{s} - M \gg \Gamma$, thus avoiding a cutoff of the Breit-Wigner lineshape. We note a glaring exception to this: the proposed technique to measure new particle masses via a threshold scan at a future lepton collider [4]. Only one non-NWA study exists for such a case, and for a small subset of particles in one model [5].

Rarely does scattering of a given set of initial and final states result from only one resonant process. Interference with other resonant or non-resonant processes can generally occur, rendering the NWA technically inapplicable. In the SM at high energy, this is typically insignificant compared to other uncertainties, but this is usually not true in new physics scenarios. We have found numerous instances of significant corrections from interference effects. However, these are not the focus of this letter, so we defer that discussion to a later work [6].

Separation of the resonant particle's propagator is, strictly speaking, never valid: even when all particles are scalars, as the phase space factor for the decay particles is a function of q^2 for finite daughter mass, $m \neq 0$. However, for extremely small m the dependence may be negligible. The NWA does work very well for top quark production and decay, despite a non-trivial matrix element and massive daughter particles. But in general, momentum-dependent external wavefunctions and couplings invalidate Eq. 1. This can result in significant off-shell corrections in beyond-the-Standard Model (BSM) scenarios, where the NWA is universally adopted. A critical ingredient is massive daughters, which most BSM scenarios include, often with near-degeneracies driven by underlying symmetries. These differences from the SM yield unanticipated behavior.

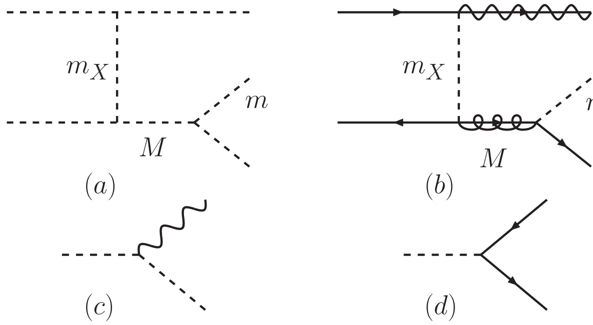

Figure 1: Feynman diagrams for various example processes discussed in the text: (a) scalar field theory toy model; (b) gluino resonance in supersymmetry; (c) VSS and (d) SFF momentum-dependent renormalizable interaction vertices.

We first consider the process of Fig. 1a, t-channel scattering in a pure scalar theory. It has no matrix elements other than the t- and s-channel propagators. This allows us to study the s-channel particle's off-shell behavior without a decay matrix element modification. We may later compare these results with those of a specific BSM model, where the decay matrix element introduces additional momentum-dependent structure. In this toy example we have only one possible contributing diagram – other topologies may be forbidden by assigning flavor or charges to the scalars – and all particles are massless except as labeled. The t-channel particle is kept massive to avoid a forward-scattering singularity, and only one final-state daughter is massive. These simplifications allow us to derive analytical expressions.

Squaring the matrix element and integrating over three-body phase space obtains the $2 \to 3$ off-shell cross section. Dividing by the on-shell $2 \to 2$ cross section times the naïve branching ratio and subtracting 1 obtains the deviation of the true leading-order rate from the NWA result, $\triangle \equiv \sigma_{OFS}/\sigma_{NWA} - 1$. We defer the rather long full result to later work [6]. Here we show the result for $\sqrt{s} \gg M$ and finite daughter mass, $m \neq 0$:

$$\triangle \sim -\tfrac{1}{2} + \tfrac{1}{\pi}\left(1 + \beta^{-2}\tfrac{\Gamma^2}{M^2}\right)\tan^{-1}\left(\beta^2\,\tfrac{M}{\Gamma}\right) - \tfrac{1}{\pi}\beta^{-2}\,\tfrac{\Gamma}{M}\left(\tfrac{M^2}{s} + \tfrac{m^2}{2\,M^2}\,\log\left(\tfrac{M^2(\beta^4\,M^2+\Gamma^2)}{m^4}\right)\right) \quad (2)$$

where $\beta = \sqrt{1 - m^2/M^2}$ is the daughter velocity. The off-shell result indeed differs from the approximate on-shell result by a quantity ostensibly of order Γ/M (with generally mild logarithmic corrections), but with an inverse velocity squared that diverges in the $m \to M$ limit, *i.e.* nearly-degenerate daughter and parent masses. This is due to the q^2-dependent t-channel propagator and decay phase space factors, and is our first principal result. Its interpretation, however, requires some care.

The partial width of a scalar particle decaying to two scalar daughters is proportional to β^2, originating from the phase space. If this is the only allowed decay, then the total width is equal to the partial width and the velocity factors cancel. This leaves only the log terms as correction factors to a coefficient which is of order $\alpha_i \times m/M$, where $\alpha_i = g_i^2/4\pi$ is the coupling strength of the decay interaction. In such a scenario, the leading-order off-shell calculation can give $\mathcal{O}(\alpha_i)$ rate corrections for decays to a nearly-degenerate daughter, a result which is interesting in its own right. The more important implication, however, is that if multiple decays are allowed, then the velocity factors for the rarest mode(s) to nearly-degenerate daughter(s) are not cancelled.

The practical impact is that the effective branching ratio, BR_{eff}, for a rarer mode can be dramatically different than the naïve BR, even by an order of magnitude. If the rare mode is taggable, as often happens in BSM scenarios, off-shell effects can alter the phenomenology significantly. Depending on the relative mass scales in the problem, the rare mode may be depleted, or enhanced to some asymptotic finite value in the limit $m \to M$. This consequence is our second principal result. For the t-channel process we considered here, the corrections can even be negative, but this is not a general rule for arbitrary topologies and particle content.

We don't observe massless scalars in nature, however, so our toy model is useful only to understand how a calculation performed fully off-shell can differ from the approximate on-shell result, simply due to other propagators which depend on q. That the decay threshold can experience sizeable corrections should not be surprising, as a production threshold can. Nevertheless, our result and its impact does not appear to be known in the literature. That finite-width effects can furthermore be enormous for rare decay modes is an important corollary, easily understood once one obtains the general analytical form of $\triangle$. We note that an s-channel all-scalar process results in a slightly different analytical result, but identical qualitative behavior.

Discoverable BSM physics cannot consist purely of new scalar vertices, although it may involve additional interactions of a vector and two scalars (VSS) or a scalar and two fermions (SFF), shown in Figs. 1c and 1d. Both types introduce additional momentum dependence into the matrix element. The decay matrix element for scalar decay to fermions (S:FF) is proportional to $q^2 - (m_1 + m_2)^2$, where q^2 is the invariant mass of the resonance and m_i are the final-state fermion masses. The VSS vertex is proportional to the difference of the scalar particles' momenta, in turn roughly proportional to q. In general, the decay matrix element alters the integration over q^2, rendering the NWA formally invalid. Our task is to see how much the off-shell result can differ from the naïve one.

We move on from a simple toy model to a realistic new-physics scenario and examine its practical phenomenology. For this purpose we choose the minimal supersymmetric Standard Model (MSSM) as our framework. Supersymmetry (SUSY) is a highly-motivated scenario, and there are several readily-available tools for performing off-shell calculations in the MSSM [7]. The MSSM spectrum contains one new particle of opposite spin statistics for every particle degree of freedom in the Standard Model, in addition to two Higgs doublets. If supersymmetry exists, however, it is a broken symmetry, as any SUSY particles that might exist must be much more massive than their SM partners. We might expect to find SUSY particles at several hundred GeV in mass, but not much above the TeV scale. Many SUSY scenarios, such as anomaly-mediated SUSY breaking (AMSB), have natural near-degeracies in their spectrum, driven by high-scale vacuum expectation values for fields, which generate the low-scale physical spectrum. Nearly-degenerate sparticles would still appear in cascade decays of heavier sparticles to the lightest sparticle, the dark matter candidate. This landscape is ripe for off-shell effects.

One MSSM process which has no possible non-resonant interference diagrams is $u\bar{d} \to \widetilde{\chi}_i^+ \widetilde{g}$ $(i = 1, 2)$, with gluino decay to strange quark plus squark, as shown in Fig. 1b. As a mixed EW–QCD process, at LHC it would be challenging to dig out from QCD SUSY processes, unless SUSY were realized in a long-lived chargino scenario such as AMSB. It is however an excellent proxy for demonstrating the physics inherent in off-shell resonance effects. That is, $\widetilde{q}\widetilde{g}$ production would exhibit a very similar matrix element effect, but the presence of QCD non-resonant interference would muddy the present lesson.

Analytically at leading order in Γ/M and $1/s$ where allowed, the off-shell to NWA cross section deviation is:

$$\triangle \sim -\frac{1}{2} + \frac{1}{\pi}\left(1 + \frac{(M^2 - 2m^2)\,\Gamma^2}{\beta^4\, M^4}\right)\tan^{-1}\left(\beta^2 \frac{M}{\Gamma}\right) + \frac{1}{2\pi}\frac{\Gamma}{M}\beta^{-4}\cdot\left(\frac{m^4}{M^4}\ln\left(\frac{\beta^4\, M^4 + M^2\Gamma^2}{m^4}\right) - \ln\left(\frac{\beta^4\, M^4 + M^2\Gamma^2}{s^4/m_X^4}\right)\right) \quad (3)$$

with parent/daughter notation as before. In the limits $m \to 0$ and $\Gamma \ll M$, the first two terms go to 0, and the remaining correction is $\mathcal{O}(\Gamma/M)$, as expected. There is also a residual $\log(s)$ dependence, which would not be noticed unless far above production threshold.

We immediately see an important difference compared to the all-scalar toy model, which has no decay matrix element: the Γ/M corrections are proportional to β^{-4} instead of β^{-2} (differences in log terms are of minor importance). This comes from the additional powers of q from the decay matrix element. This is our third principal result. We thus expect even more dramatic corrections to effective branching ratios for rare decays to nearly-degenerate daughters in spectra with dominant decays to lighter particles. These lighter particles dominate the total width, so $\Gamma_{\rm tot}$ is independent of β^4. It should be clear that the radically different behavior of off-shell BSM resonances is also due to final-state masses not significantly smaller than the resonance mass.

We define BR_{eff} as the off-shell cross section to the final state of interest, divided by the sum of all possible final state cross sections, each also calculated off-shell. For our example, this is three-body production of the final state $\widetilde{\chi}_1^+ \bar{s}\widetilde{s}_{L,R}$. We choose the MSSM parameter space benchmark point SPS1a [8], which also allows for gluino decays to lighter top quark plus stop, and bottom quark plus sbottom; these together generate a gluino partial width of 2.6 GeV, $\sim 0.4\%$ of the 600 GeV gluino mass. A typical width-to-mass ratio for MSSM gluinos lighter than about 1 TeV is $1 - 5\%$.

Numerical results for BR_{eff} are shown in Fig. 2 for a scan over daughter squark mass (we assume degenerate 1st- and 2nd-generation squarks). The shaded band delineates the region containing all 1st- and 2nd-generation squarks in all SPS benchmark scenarios with a heavier gluino. As expected, if the squarks are lighter than the stops and sbottoms, they dominate the total width, so $BR_{eff} \approx 1$. However, note that for squarks much lighter than the gluino, the *cross section* receives sizeable corrections. This tree-level effect on the overall normalization is at a level comparable to the QCD next-to-leading order uncertainty.

For $\widetilde{s}_L$ within about 10% of the gluino mass, BR_{eff} can greatly exceed the naïve NWA expectation. Where the BR becomes small, if it is a taggable rare mode (which most decays would be), then these corrections become important, lest we extract incorrect Lagrangian parameters from the relative branching ratios observed in data. At SPS1a, this decay mode would receive almost a 25% correction, an order of magnitude larger than the Γ/M estimate, and greater than the residual QCD next-to-leading order production rate uncertainty. The NWA-derived ratio of this mode's branching ratio relative to (unaffected) decays to lighter sbottoms would disagree with the mass spectra measured via other methods [9]. Results are nearly identical for decays to $\widetilde{s}_R$.

In addition to effective branching ratio corrections, we observe chirality selection in gluino decays. In the NWA, the chiral-blind Majorana-fermion gluino will decay with equal rates to left- and right-chiral squarks and anti-squarks, given equal masses (that is, up to phase space effects for the small electroweak mass splittings which typically appear). However, the initial- and final-state helicities are connected via the Dirac structure of the fermion chain. The production side of the event in our example involves a chargino, which selects left helicity. The gluino's fermion mass can flip the helicity, but does not give equal rates. This is our fourth principal result. We show it graphically in Fig. 3.

While squark masses below a couple hundred GeV are already ruled out by Tevatron searches, we see that there

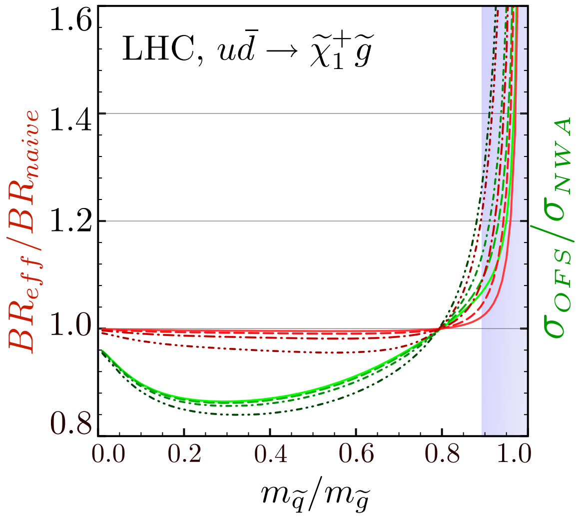

Figure 2: Ratio of effective to naïve BRs (left axis) and off-shell to NWA cross sections (right axis) for the MSSM process $u\bar{d} \to \tilde{\chi}_1^+ \bar{s} \tilde{s}_L$ at the LHC (cf. Fig. 1(b)). The resonant gluino has an additional partial width of $0.5, 1, 2, 5\%$ of its mass (solid, dashed, dashdotted and dotted curves) due to decays to stops and sbottoms. The first- and second-generation squarks lie in the shaded band for all SPS benchmark points with a heavier gluino.

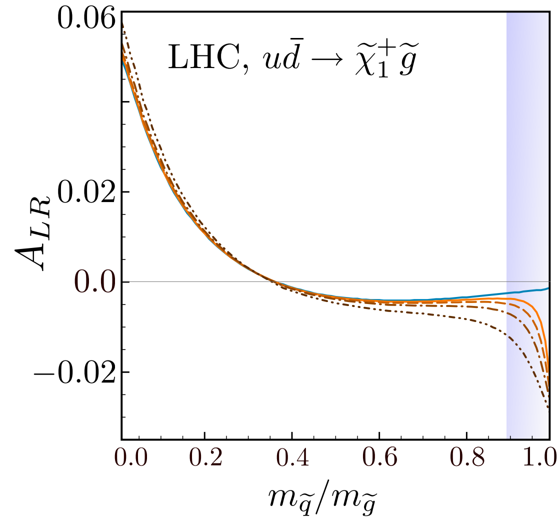

Figure 3: MSSM gluino decay asymmetry to degenerate-mass left-chiral v. right-chiral squarks as a function of the squark to gluino mass ratio, in $\tilde{\chi}_1^+ \tilde{g}$ production at the LHC, as discussed in the text. The NWA predicts exactly zero for all masses. The yellow (blue) curves are for multi-mode (single-mode) decays; line types and shaded band as in Fig. 2.

would be a few-percent asymmetry for nearly-degenerate squark masses. Left-chiral squarks prefer to decay to charginos, while $SU(2)_L$-singlet right-chiral squarks cannot. These final states differ qualitatively and can be distinguished, but a detailed study is necessary to determine what level of asymmetry would be observable.

We have examined multiple other cases involving VSS and SFF vertices. Examples include sbottoms decaying to stop plus W boson (S:SV), and squarks decaying to quark plus gluino (S:FF). Their general $\sigma_{OFS}/\sigma_{NWA}$ behavior is qualitatively similar to what we find for the F:FS case, again depending on β^{-4}, with minor variations in the log terms and coefficients. Most MSSM particle decays would exhibit the same phenomenological features of our primary example.

In summary, we investigated off-shell matrix element effects in scattering processes involving new heavy states which decay to other massive states. In general, if multiple decay modes are allowed, the accuracy of the effective branching ratios differs from the naïve $\mathcal{O}(\Gamma/M)$ expectations based on the NWA, even by orders of magnitude. Rarer modes may receive enormous corrections. If neglected, this would corrupt the extraction of model parameters from data. Additionally, massive Majorana fermions exhibit a helicity selection effect which may introduce observable asymmetries into the data. Unanticipated, such asymmetries would likely be incorrectly interpreted as signals of additional new physics, such as CP violation. Our results are based on a neglected but important aspect of tree-level cross section calculations.

Our results are far more generally applicable than just to supersymmetry. They are based on general matrix element behavior for arbitrary renormalizable interactions, phase space and integration of a heavy resonance's propagator over a range in q^2 for decay to massive daughter particles. For instance, Universal Extra Dimensions models [10] are another example of cascade decays and very close degeneracies.

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