Informative Advertising with Discretionary Search

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(Preliminary)

Abstract

We examine a market for a search good in which consumers are uncertain about the firm's product quality, but may search to gather information before buying. We show that credible information can be conveyed to consumers even when the firm faces a market with little or no preference heterogeneity. Rather, differences in willingness-to-pay arise from the endogenous search decisions by consumers. A fundamental assumption is that search is not required for purchase and consumers may bypass it altogether. In this case search induces a dispersion in preferences that is detrimental for the firm's ability to capture value. This provides an incentive for the firm not to overstate its quality.

When quality is common knowledge (but product fit is uncertain before search), increases in quality lead to a higher market price, but firm profits and consumer surplus are non-monotonic because of changes in the search regime. In particular if quality is high enough for search to be worthwhile the firm faces downward pressure in prices and consumers become better off. These effects reverse at higher levels of product quality.

Surprisingly, when product quality is unknown but credible information is available consumers become worse off with the probability of facing a high type firm, because this firm prefers to target high value consumers and not serve those who do not find that the product fits their needs.

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1 Introduction

The strategic communication between firms and consumers has been the focus of recent work by economists and marketing scientists. The underlying tension behind the phenomenon is the fact that consumers often appear willing to believe advertisements' claims without verifiability commitments, evidence or proof. This is especially striking because the objective of the sender of such messages is likely to be her own welfare maximization rather than the consumers'.

The paradox has found resolution in the idea that under certain circumstances the firm may have an incentive to publish truthful information in hopes of attracting some preferred segments of consumers rather than others. For example, Bagwell and Ramey (1993) proposed that low and high quality firms could differ in their preferences over segments of consumers. When firm types are significantly different the low end prefers to make small profit margins from multiple units sold to low valuation consumers, while the high end ones prefer to make high margins off high valuation consumers willing to buy only a few units. The mechanism at work is related to a more general principle of informative communication initially proposed by Farrell and Gibbons (1989). It is that the presence of different audiences can introduce a tradeoff to the sender, which ultimately grants her credibility. For example, the reason a firm can successfully attract high-type consumers in the work of Bagwell and Ramey (1993) is because its message simultaneously detracts low-type consumers from searching. Similarly, a low-type firm would prefer to attract the low-type consumers at the expense of detracting the high-type ones. This logic informs the credibility mechanism proposed by Bagwell and Ramey (1993), where advertising "matches products to buyers."

In this paper we show that audience heterogeneity is not a fundamental requirement for advertising credibility to arise, but that instead it can naturally emerge from the search decisions consumers make and the strategic pricing response by the firm. In contrast with the existing literature we decouple the search and purchase decisions and show that heterogeneity may emerge endogenously in the market, as can advertising credibility.

A fundamental assumption in the present work is that the search process by which consumers gather information about a product or service is not the property of the good itself (i.e., a 'search good' as the work by Nelson (1970) is often interpreted), but it is a decision that consumers can consider, i.e., search is not required for purchase, but discretionary. In our model information acquisition can be interpreted in at least two ways. In the classical interpretation (see Nelson (1970, 1974)) consumers incur a search cost to learn about product features and attributes before buying. The cost does not have to be intrinsically monetary, but can include other factors such as the time and effort associated with learning more about a camera online or at a physical store. A more novel interpretation is that of deliberation, proposed by Guo and Zhang (2012). In this case the consumer incurs a cost in order to learn about her own preferences and how the product characteristics map into her needs. This captures an important aspect of real life decisions. For example, while buying a camera the consumer may want to carefully consider how much portability or picture quality is most convenient for her upcoming vacation.

This feature has real-world relevance. For example, an individual may spend a significant amount of time and effort evaluating a durable camera but no time at all assessing a disposable one. The amount of effort involved may also depend on whether the product is likely to perform well and how much can information reduce uncertainty about the product's performance.

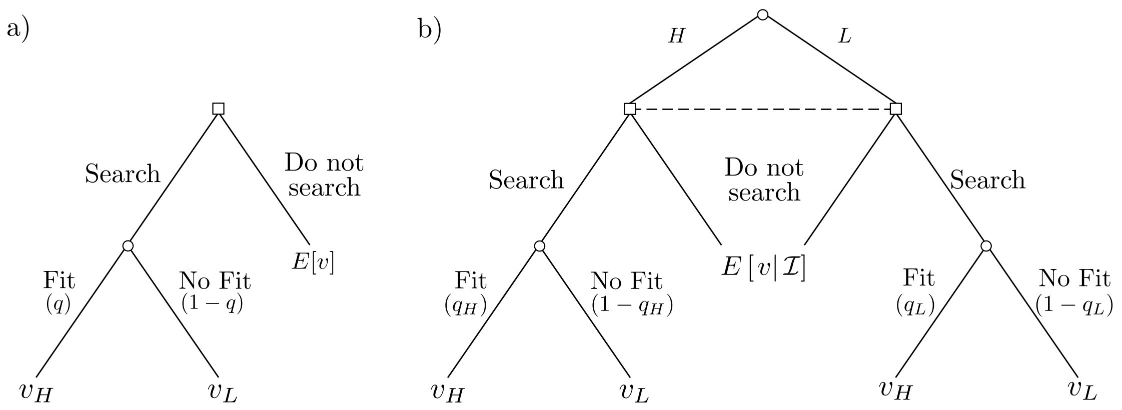
In our model the advertising message is cheap-talk in the sense that the choice of the content has no relevant cost implications for the firm (as in Crawford and Sobel, 1982). Moreover, the firm is allowed some legal freedom such that its advertising message need not correspond perfectly to reality. One instance of this phenomenon is 'puffery' in advertising, in which the firm is allowed to exaggerate its claims in a way that is reasonably expected (see Chakraborty and Harbaugh (2014) for a detailed discussion and references to the legal framework). Other work has focused on advertising disclosure, where the firm decides which information to disclose to consumers but is not allowed to misrepresent itself. Anderson and Renault (2006) and Anderson and Renault (2013) consider the disclosure decision of firms in a market for a search good with horizontal and vertical differentiation, respectively. Anand and Shachar (2009) consider the fact that advertising messages can mean different things to different consumers, and that the firm may strategically decide how precise its advertising will be. Mayzlin and Shin (2011) consider the case of bandwidth-limited consumers and show that high quality firms may prefer to refrain from advertising altogether in order to encourage search.

In the informative advertising literature Bagwell and Ramey (1993) consider the case where the firm induces search by its favorite customer segment through its advertising strategy. Relatedly, Gardete (2013) shows that firms may have an incentive to pool upwards but only to a certain extent, and that overall surplus may be maximized when they are allowed to do so.

Chakraborty and Harbaugh (2010) and Chakraborty and Harbaugh (2014) consider the case of informative multidimensional advertising. In this case the seller can attain credibility because she sells a multi-attribute product. Credibility can arise if she emphasizes some of the product characteristics, but not all.

A related literature started by Wernerfelt (1990) considers the role of cheap-talk in advertising as a means to coordinate consumers' signaling to each other. Recently, Kuksov, Shachar, and Wang (2013) investigate a related case where the firm's advertising decision is affected by the correlation between consumer preferences for functional and self-expressive attributes as well as the image these would like to project.

Figure 1: Contingent Willingness-to-pay with known and unknown Product Qualities



The innovation in this paper is that advertising credibility arises endogenously because consumers are allowed to buy irregardless of information acquisition. This assumption yields ex-post differentiated demand from an ex-ante undifferentiated one. The intuition is aided by Figure 1. Consider a product with some known quality q, which denotes the probability of a consumer finding that the product fits her needs. In particular, the consumer receives gross utility v_H if she finds a fit, but v_L if she does not, and $v_H > v_L > 0$. If fit is not known a priori the consumer's ex-ante willingness-to-pay is equal to $E\left[v\right] = q\, v_H + (1-q)\, v_L$. However, if she decides to search to learn more about the fit, her valuation will be v_H with probability q and v_L with probability $1-q$. This is depicted in panel a) of Figure 1 and summarizes a first important effect: after search takes place consumer valuations become more heterogeneous as they find whether the product fits their needs or not. This in turn creates a problem for the firm in terms of consumer surplus appropriation: If consumers do not search the firm could capture all of the consumer welfare by setting price at level $p = E[v]$ and earn profit $E[v]$. However, when some consumers search the firm faces a more heterogeneous preference distribution and will become worse off. In the limit it could receive profits $q\, v_H$ if it set price at level $p = v_H$ or it could receive profits v_L if it set price at level $p = v_L$, both of which are lower than the previous case.[1] This first case highlights the fact that as consumers search and find out whether the product fits their needs, their preferences become more heterogeneous, making it harder for the firm to appropriate value.

Consider now the case of unknown product quality, summarized in panel b) of Figure 1. In this example nature has the first move and decides whether the firm has a high or a low quality product, with $q_H > q_L$ being the respective probabilities of product fit. Consumers do not know which product quality the firm holds, but as before can determine whether the product fits their needs through costly search. When they do not search their expected valuation is given by $E[v|\mathcal{I}]$ where $\mathcal{I}$ stands for the consumers' beliefs about quality at the first information set. For example, if no advertising information is available to consumers their willingness-to-pay (if they do not search) may be equal to $E[v|\mathcal{I}] = E[v]$.

Credibility arises when the firm has an incentive to report its quality truthfully. This can happen when consumers are more likely to search when they believe the firm is of higher quality and moreover the high and low quality firm-types prefer prices close to v_H and v_L, respectively. By reporting truthfully the low-type firm reduces search and avoids the profit losses from the resulting market heterogeneity, as explained above. Moreover, if it imitated the high-type firm it would decrease its profits because the additional searching customers would apply downward pressure on the price it is able to charge. The high type firm however, is willing to induce more search because it knows searching consumers are likely to find a fit. Moreover, it can alleviate the heterogeneity concern if these customers are profitable enough, which ultimately depends on how high the quality of its product is.

[1] These profits can be thought of as an upper-bound, since at these price levels consumers would be better off not searching when facing a strictly positive search cost.

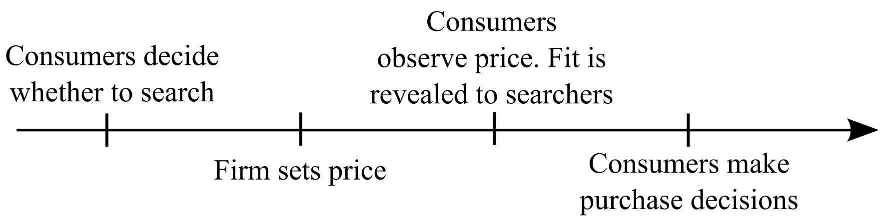
2 Known Product Quality

2.1 Preliminaries

We first consider a market where a product has quality $q \in (0,1)$. Consumers do not derive utility from product quality directly. Instead, as quality q increases so does the probability of consumers finding that the product fits their needs. In particular, with probability q the product yields gross utility v_H upon consumption and with probability $1-q$ a consumer receives gross utility v_L, where $v_H > v_L > 0$. Moreover, we normalize the utility of not buying to zero.

In this section we assume consumers know the product quality but do not know the product fit a priori. In order to learn the fit they may engage in costly search. For example, a consumer may decide to browse different websites for information on the latest trendy gadget; a manager may decide to meet with a potential supplier to assess the characteristics of a production input. In general the search process can be thought of as a costly investment that rewards the customer with product information. Consumers can decide whether they would like to engage in costly search or not. Only if they decide to search do they incur a search cost $c > 0$ and learn their own fit with the product. A fundamental feature of the model is that consumers can buy regardless of their search decision. Hence, the search decision affects their information but not their ability to buy the product. Figure 2 presents the timing of the game.

Figure 2: Timing of the Perfect Information Game



First, consumers make their search decisions. The firm makes a take-it-or-leave-it offer through price p. After this, consumers observe price and those who decided to search also observe their fit with the product. Finally, consumers make their purchase decisions.

Keeping the price disclosure to a later stage is a common assumption in the informative advertising literature in order to isolate the informational effects of the advertisement, and avoid imposing beliefs when inconsistent prices and advertising messages occur. This assumption is especially useful in Section 3 in order to focus the results on the informativeness of the advertising message.[2]

2.2 Customer Decisions

Consider the last stage of the customer decision process: the purchase decision. At this point she may have decided to search or not. If she decided not to search she will buy if

$$E\left[u_B\right] \geq 0 \Leftrightarrow E\left[v|\, q\right] - p \geq 0 \tag{1}$$

where $E\left[v|\, q\right] = q\, v_H + (1-q)\, v_L$ captures the expected gross utility from purchasing the product. If the consumer has searched, however, the decision to buy depends on whether the product fits her needs or not. In particular, if fit occurs she buys if

$$v_H - p \geq 0 \tag{2}$$

and if fit did not occur she will still decide to buy as long as

$$v_L - p \geq 0 \tag{3}$$

Consider now the decision of whether to search or not. At this stage the consumer does not yet observe the product price, but holds a belief $\widehat{p}$ about it. She decides to search if

$$E\left[U\left(Search\right)\right] \geq \max\left\{E\left[v|\, q\right] - \widehat{p}, 0\right\} \tag{4}$$

where the left hand-side captures the expected utility of searching and the right hand-side captures the best of the two possible scenarios that the consumer may face if she does not search: she receives the maximum utility between buying the product or not in the absence of product fit information. In particular, if $E\left[v|\, q\right] \geq \widehat{p}$ she believes that she will buy the product if she decides not to search, and if $E\left[v|\, q\right] < \widehat{p}$ she believes that she will not buy the product if she does not search. We now consider these possibilities in turn.

[2] See Bagwell and Ramey (1993) and Gardete (2013) for discussions.

First, let the belief over price be such that $E\left[v|\,q\right] \geq \widehat{p}$. In this case the consumer searches if

$$q\left(v_H - \widehat{p}\right) - c \geq E\left[v|\,q\right] - \widehat{p} \tag{5}$$

where the left-hand side reflects the expected utility from search, and the right-hand side captures the expected utility of buying without searching. Note that in order to describe the left-hand side one needs to consider the fact that price is bounded below by v_L. If the price were any lower the firm could increase its profits by increasing it until v_L. If the product does not provide a fit the consumer always receives zero utility either because she decided not to buy the product ($p > v_L$), or the price extracts all her product valuation ($p = v_L$). It follows from (5) that when $E\left[v|\,q\right] \geq \widehat{p}$ the consumer decides to search if and only if

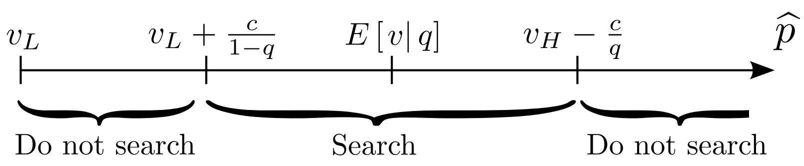
$$\widehat{p} \geq v_L + \frac{c}{1-q} \tag{6}$$

Consider now the case where the consumer believes the product is "expensive", such that $E\left[v|\,q\right] < \widehat{p}$. In this case she compares the utility from search to the utility of not searching and not buying. In particular she decides to search when $q\left(v_H - \widehat{p}\right) - c \geq 0$, which rearranged becomes:

$$\widehat{p} \leq v_H - \frac{c}{q} \tag{7}$$

Figure 3 plots how the consumer decisions depend on their beliefs over price.

Figure 3: Price Belief and Search Behavior



When the belief over price is low enough (when compared to the search cost) consumers are willing to buy without searching. As the belief increases search becomes an attractive option. However, when the belief over price is very high the option value of searching is low, and consumers prefer not to search.

Proposition 1 (Emergence of search behavior). *Search emerges only when q is intermediate ($q \in (q_0, q_1)$). Otherwise, consumers do not search independently of their belief over price. Moreover, the search cost must be low ($4c < v_H - v_L$) for search to take place.*

Proposition 1 states that when the probability of finding a fit is very high or very low, search is not valuable to consumers as they prefer to either buy without searching or not to buy at all, depending on the price the firm sets.[3] This is equivalent to the search range in Figure 3 disappearing for very low or very high values of q. The second part of the proposition states that the search cost must be small enough vis-a-vis the utility range for search to potentially arise. When this condition is not met search can never take place and the market outcome becomes trivial. We assume the search cost is low ($4c < v_H - v_L$) for the remainder of the paper. As we will see in the next section, Proposition 1 informs the nature of the market outcome.

2.3 Market Outcome

Consider the pricing behavior of the firm contingent on the search decision of customers. According to Proposition 1, when quality is very low or very high - i.e., $q \in (0, q_0) \cup (q_1, 1)$ - consumers do not search. Given consumer valuations, in this case the firm is better off charging $E[v|\, q]$, consumers buy the product without searching and no profitable deviation for either party exists. Hence, a pure strategy equilibrium with price $E[v|\, q]$ and no search by consumers exists whenever the probability of fit is either very high or very low. This is intuitive since the consumer has less to gain from search if she has a good notion of the utility she can derive from the product.

When quality is intermediate - i.e., $q \in (q_0, q_1)$ - the market outcome is more complex. Consider the demand faced by the firm:

$$D(p) = 1[p \le v_H](\alpha(q)\, q) + 1[p \le E[v|\, q]](1 - \alpha(q)) + 1[p \le v_L](\alpha(q)(1-q)) \tag{8}$$

where $\alpha(q)$ denotes the consumers' probability of searching. The demand curve suggests a set of three potentially optimal prices: v_L, $E[v|\, q]$ and v_H. The firm maximizes its profit by solving

[3] The quality limits are defined in the appendix.

the problem

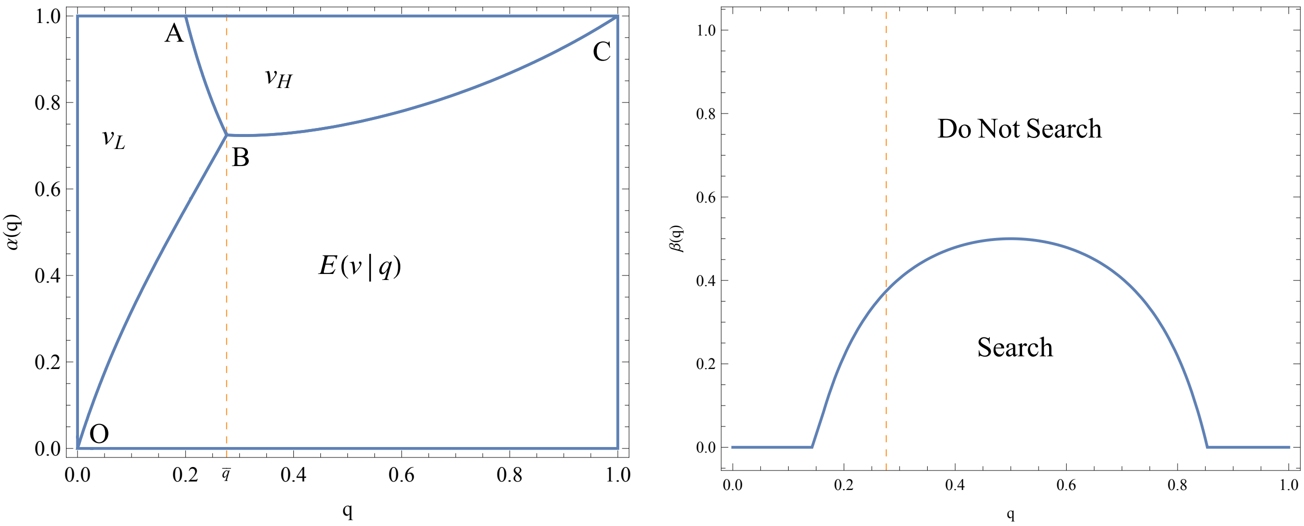
$$\max_{p} \pi\left(p\,|\alpha\left(q\right)\right) = p.D\left(p\,|\alpha\left(q\right)\right) \tag{9}$$

where it takes the search behavior of consumers $\alpha\left(q\right)$ into account. At each relevant price level the firm profit is given by

$$\pi\left(p\,|\alpha\left(q\right)\right) = \begin{cases} \alpha\left(q\right) q v_H, & p = v_H \\ \left[\alpha\left(q\right) q + 1 - \alpha\left(q\right)\right] E\left[v|\, q\right], & p = E\left[v|\, q\right] \\ v_L, & p = v_L \end{cases}$$

At price v_L, the lowest possible utility realization, the firm is able to capture the whole market. At price $E\left[v|\, q\right]$ the firm is able to capture consumers who search and find a fit, $\alpha\left(q\right) q$, as well as those who do not search, $1 - \alpha\left(q\right)$, but are willing to pay their expected valuation. Finally, at price v_H the firm serves only the proportion of consumers who search and find a fit, $\alpha\left(q\right) q$. Given a search intensity $\alpha\left(q\right)$ it is possible to determine the best response of the firm. The left panel of Figure (4) depicts the firm best-response to search behavior $\alpha\left(q\right)$.[4]

Figure 4: Best-Response Regions for Firm and Consumers



When product quality is very low the firm is generally better off charging v_L. This makes intuitive sense, since in this case the firm prefers to target consumer who do not find a fit with

[4]The parameter regions are described in the appendix.

the product, as well as all the non-searchers. As quality increases the best-response by the firm depends on the search behavior: If many consumers search the firm is better off charging price v_H and extract the surplus from consumers who search and find a fit. However, if not many consumers search the firm is better off targeting the non-searchers by pricing at $E\left[v|\, q\right]$.

In order to understand the behavior of consumers it is useful to define β_L and β_H as the probabilities of the firm setting prices equal to v_L and v_H, respectively.[5] The search decision provides consumers an option value that can be defined as

$$E\left[U\left(Search\right)\right] - E\left[U\left(\sim Search\right)\right] = \left(1 - \beta_L - \beta_H\right) q \left(v_H - E\left[v|\, q\right]\right) - c \tag{10}$$

The difference is equal to the utility of searching and receiving a fit while facing price level $E\left[v|\, q\right]$, minus the search cost. The expression above reveals that making customers indifferent between searching and not searching is equivalent to making this utility equal to zero. The reason is relatively subtle: When price is highest $(p = v_H)$ search offers no value to consumers because their maximum possible utility will be extracted by the firm independently of whether they buy or not. When price is lowest $(p = v_L)$ consumers are always willing to buy, and so the search decision does not provide additional value either. Hence, the only difference between the expected utility of searching and not searching occurs when the firm practices price $E\left[v|\, q\right]$, as denoted in expression (10). At this price consumers may be better off searching (when the search cost is low) because search offers them the option value to buy or not depending on whether they find the product satisfies their needs. At price levels v_L and v_H consumers are better off buying without searching, or not searching and not buying altogether, respectively. However, the firm has the opposite pricing incentives: When consumers search it prefers to charge v_L or v_H, and when consumers do not search it prefers to charge the expected utility $E\left[v|\, q\right]$. For this reason an equilibrium with search can only arise in mixed strategies.

In the appendix we show that no mixed strategy equilibrium occurs if the firm mixes between prices $\{v_L, v_H\}$ only, since in this case consumers would have no benefit from searching. Hence, we look for search strategies that make the firm indifferent between prices in $\{v_L, E\left[v|\, q\right]\}$ and in $\{E\left[v|\, q\right], v_H\}$. Consumers make the firm indifferent between prices $\{v_L, E\left[v|\, q\right]\}$ as they mix on the OB curve, on the left panel of Figure 4. Moreover, they make the firm indifferent between

[5] We omit the q argument of $\beta_L\left(q\right)$ and $\beta_H\left(q\right)$ for parsimony.

prices $\{E[v|\,q], v_H\}$ by searching along curve BC. We denote these search probabilities as $\alpha_L(q)$ and $\alpha_H(q)$, respectively:

$$\alpha_H^*(q) = \frac{E[v|\,q]}{qv_H + (1-q)E[v|\,q]} \tag{11}$$

$$\alpha_L^*(q) = \frac{E[v|\,q] - v_L}{(1-q)E[v|\,q]} \tag{12}$$

The firm also prices in order to make the consumer indifferent between searching and not searching. This entails keeping expression (10) equal to zero by mixing price between $\{v_L, E[v|\,q]\}$ with probabilities $\{\beta_L, 1-\beta_L\}$ and between $\{v_H, E[v|\,q]\}$ with probabilities $\{\beta_H, 1-\beta_H\}$. By comparing the firm profits of each regime one can establish the following result:

Proposition 2 (Market outcome with known quality). *When quality is low* ($q_0 < q < \overline{q}$) *consumers search according to* $\alpha_L^*(q)$ *and the firm mixes between prices* $\{v_L, E[v|\,q]\}$ *with probabilities* $\{\beta^*(q), 1-\beta^*(q)\}$*. When quality is high* ($\overline{q} < q < q_1$) *consumers search with probability* $\alpha_H^*(q)$ *and the firm mixes between prices* $\{v_H, E[v|\,q]\}$ *with the same probabilities* $\{\beta^*(q),$ $1-\beta^*(q)\}$*. When quality* q *is either very high or very low* ($q \in (0,\, q_0) \cup (q_1,\, 1)$) *consumers buy without searching and the firm sets price equal to* $E[v|\,q]$ *with probability one.*

As we had explained in the beginning of this section, when product quality is too low or too high consumers have a good idea of the utility they are likely to receive from the product, and search brings little value. In that case consumers do not search but instead buy the product immediately at price $E[v|\,q]$. When q is intermediate search provides value as long as price is equal to $E[v|\,q]$ with some probability. When quality is relatively low the firm mixes between $\{v_L, E[v|\,q]\}$ in order to target consumers who are unlikely to receive a fit, and when quality is relatively high the firm mixes between $\{v_H, E[v|\,q]\}$ to target those who do receive a fit. The quality threshold at which the switch takes place is given by $\overline{q}$, and is defined in the appendix.

The pricing policy is given by

$$\Pr(p = v_L) = \begin{cases} 0 & q > \overline{q} \\ \beta^*(q), & q_0 < q \leq \overline{q} \\ 0, & q \leq q_0 \end{cases} \tag{13}$$

$$\Pr(p = v_H) = \begin{cases} 0, & q > q_1 \\ \beta^*(q), & \overline{q} < q < q_1 \\ 0 & q \leq \overline{q} \end{cases} \tag{14}$$

$$\Pr(p = E[v|\,q]) = 1 - \beta^*(q) \tag{15}$$

where

$$\beta^*(q) \equiv 1 - \frac{c}{q(1-q)(v_H - v_L)} \tag{16}$$

and is shown on the right panel of Figure 4. Along policy $\beta^*(q)$ consumers are indifferent between searching and not searching if $q \in (q_0, q_1)$. When $q < \overline{q}$, $\beta^*(q)$ denotes the probability that the firm will set price $p = v_L$ in the mixture $\{v_L, E[v|\,q]\}$, and when $q > \overline{q}$, $\beta^*(q)$ denotes the probability that the firm will set price $p = v_H$ in the mixture $\{v_H, E[v|\,q]\}$.

The optimal search strategy for consumers is equal to

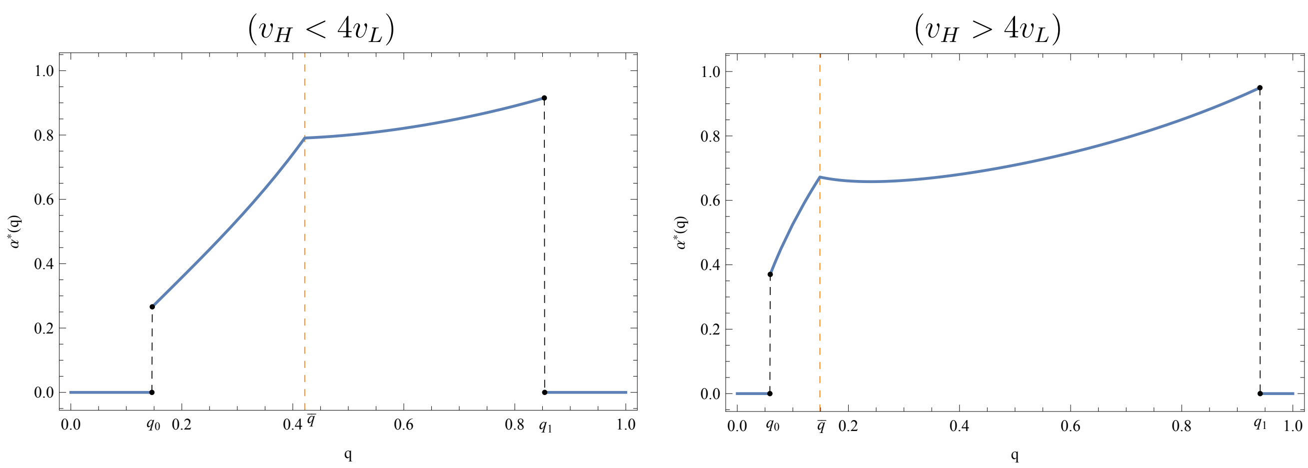
$$\alpha^*(q) = \begin{cases} 0 & q > q_1 \\ \alpha_H^*(q), & \overline{q} < q \leq q_1 \\ \alpha_L^*(q), & q_0 < q \leq \overline{q} \\ 0, & q \leq q_0 \end{cases}$$

where the search probability in the intermediate quality range (q_0, q_1) corresponds to the curve OBC on the left panel of Figure 4. We now turn to characterize the market outcome:

Proposition 3 (Characterization of market outcome with known quality). *The search probability $\alpha^*(q)$ is non-monotonic in quality. Expected price is strictly increasing in quality. Expected profits are increasing in quality except at q_0, where they decrease. Expected* *consumer welfare increases in v_H, decreases in c and v_L and is non-monotonic in q.*

Figure 5 shows the search probability at different quality levels for two different cases. In both cases the search probability is equal to zero for very low and very high levels of quality because the option value of searching is not high enough to compensate the search cost, since the outcome in either of these situations is relatively close to certain. The left panel depicts the search probability when product fit does not constitute a significant change in utility, i.e., $v_H < 4v_L$. In the intermediate region (q_0, q_1) the search probability is strictly increasing in the same range, but it is non-monotonic when $v_H > 4v_L$ (right panel of Figure 5).

Figure 5: Search Behavior and Product Quality



The search strategy can be understood in light of consumers mixing their search decision in order to make the firm indifferent across its actions. When $q < \overline{q}$ their search behavior makes the firm indifferent between prices v_L and $E[v|\, q]$. Suppose consumers attribute probability $\overline{\alpha}$ to searching. In this case the firm makes profits

$$\pi(p) = \begin{cases} v_L, & p = v_L \\ E[v|\, q](\overline{\alpha}q + 1 - \overline{\alpha}), & p = E[v|\, q] \end{cases} \tag{17}$$

since if it charges v_L all consumers buy, but if it charges price $E[v|\, q]$ only consumers who do not search, and those who search and find a fit will buy. The profit from charging v_L is independent of the search decision, so consumers can only affect the profit of the second decision when changing $\overline{\alpha}$. Consider what happens to profits when quality increases slightly, to level $q^{'} > q$. The second branch of profits increases unequivocally both through consumers who search as well as through consumers who do not search. These effects are captured by the terms $E\left[v|\,q\right]\overline{\alpha}q$ and $E\left[v|\,q\right](1-\overline{\alpha})$, respectively. Moreover, an increase in quality increases profits from the latter group faster. The reason is that while an increase in quality affects all non-searchers, it only affects some of the searchers (those who find a fit with the product). Thus, by increasing $\overline{\alpha}$ consumers can reduce $\pi\left(E\left[v|\,q\right]\right)$ back to level $\pi\left(v_L\right)$ because the loss in profits from non-searchers outweighs the gains from searching customers. This explains the positive slope of $\alpha^*\left(q\right)$ when $q < \overline{q}$.

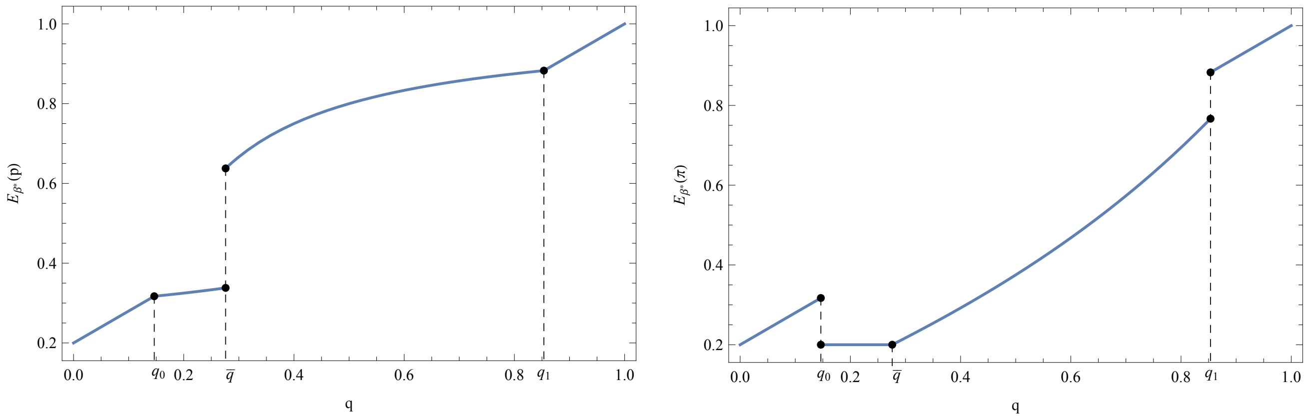
When quality is high $(q > \overline{q})$ consumers search to make the firm indifferent between prices v_H and $E\left[v|\,q\right]$. In this case the firm makes profits

$$\pi\left(p\right) = \begin{cases} \overline{\alpha}qv_H, & p = v_H \\ E\left[v|\,q\right]\left(\overline{\alpha}q + 1 - \overline{\alpha}\right), & p = E\left[v|\,q\right] \end{cases} \tag{18}$$

Consumer search affects the profits of both price levels. For example when search increases, profit $\pi\left(v_H\right)$ increases and profit $\pi\left(E\left[v|\,q\right]\right)$ decreases. However, increases in quality have mixed effects on the difference $\pi\left(v_H\right) - \pi\left(E\left[v|\,q\right]\right)$. Hence, it is not at first sight clear how consumers should search along the quality path. The answer lies on whether product fit affects utility to a large extent. When fit does not impact utility too much $(v_H < 4v_L)$ an increase in quality to level $q^{'}$ yields $\pi\left(v_H\right) < \pi\left(E\left[v|\,q^{'}\right]\right)$, and so consumers increase search to keep the firm indifferent across its price options. This happens because the firm benefits more from charging $E\left[v|\,q\right]$ and collect profits from consumers who search and find a fit as well as from non-searchers than from charging v_H and only collect profits from the first group. In other words, if product fit cannot yield that much more differential utility to consumers who find a fit, the firm benefits from targeting the non-searchers as well.

The same result holds when fit changes the customer satisfaction by a lot $(4v_L < v_H)$, but only when quality is high. When quality is close to $\overline{q}$ the firm prefers to target only the consumers who search and find a fit. As quality increases, however, the gains from charging price $E\left[v|\,q\right]$ increase faster than from charging v_H because the probability of a consumer finding no fit (and receiving utility v_L) is significantly decreased. Hence, focusing on consumers who search and find a fit through price v_H becomes less attractive. This explains the search patterns in both panels of Figure 5.

Figure 6: Expected Firm Price and Profit



The expected price and profits with respect to quality are shown in Figure 6, and they are representative of the general case. The average price increases strictly with quality: When quality is very low the firm is able to extract all surplus. When quality is low $q \in (q_0, \overline{q})$ the firm introduces price v_L in order to target consumers who search but do not find a fit. While the weight on price v_L increases with quality, it does so at a lower rate than the increase of $E[v|\, q]$, and so the average price also increases. At $\overline{q}$ the probability of fit is high enough that the firm prefers to target consumers who search and find a fit, and introduces price v_H in the mixture. Finally, after q_1 search is no longer valuable for consumers, and the firm is able to capture the whole market by pricing at $E[v|\, q]$.

Expected profit is also increasing in q except at level q_0. At this level search becomes valuable to consumers and the firm unambiguously loses profits because of the resulting preference heterogeneity. Because the firm can no longer capture the whole market at a single price, at this low level of quality the firm is better off targeting consumers who search and do not find a fit. The opposite effect happens at q_1: At this level search stops and the firm benefits from reduced preference heterogeneity. This explains the second jump in expected profits, at quality level q_1.

The result that consumer welfare (weakly) increases with v_H and decreases with the search cost c is intuitive. The non-monotonicity of welfare in q results from the fact that when $q < q_0$ or $q > q_1$ no search occurs and the firm is able to extract all the surplus. Consumers do not receive any surplus when $q \in (\overline{q}, q_1)$ either, because at this quality level the firm targets consumers who search and find a fit (as well as those who do not search). Hence, the only region where consumers have strictly positive surplus is when $q \in (q_0, \overline{q})$. In this region surplus is strictly increasing as long as the search cost c is low. When the search cost is high the average price increases at a faster rate with quality because the firm is able to extract more surplus since the search option is not as attractive. Finally, the result that consumer surplus decreases with v_L is counter-intuitive. In all regions other than $q \in (q_0, \overline{q})$ the firm is able to extract all consumer surplus and so the specific value of v_L is irrelevant. In region $q \in (q_0, \overline{q})$ the firm mixes between prices $\{v_L, E[v|\, q]\}$. The first one targets consumers who search and do not find a fit, and the second targets consumers who do not search. In this region consumers expect surplus $\beta(E[v|\, q] - v_L)$, because of the proportion of times they find price equal to v_L. However, as v_L increases so does the average price (i.e., the frequency of the low price, β, decreases), and the difference between prices $\{v_L, E[v|\, q]\}$ also naturally decreases. These two effects lead to a reduction in consumer surplus.

3 Unknown Product Quality

3.1 Preliminaries

We now investigate the case where information about product quality is asymmetric. In this case the firm's quality $q \in Q \equiv \{q_L, q_H\} > 0$ is its own private information.[6] In order to communicate its quality to consumers the firm can engage in informative advertising by sending a message $m \in Q$. The content of the message is cheap-talk in the sense that the cost of sending it as well as of deciding its content is not related to the type of firm sending the message.[7] While consumers can use advertising to inform their search decision, they still take the incentives of the firm in sending the particular message into account. In particular they understand that a firm may have an incentive for misrepresentation.

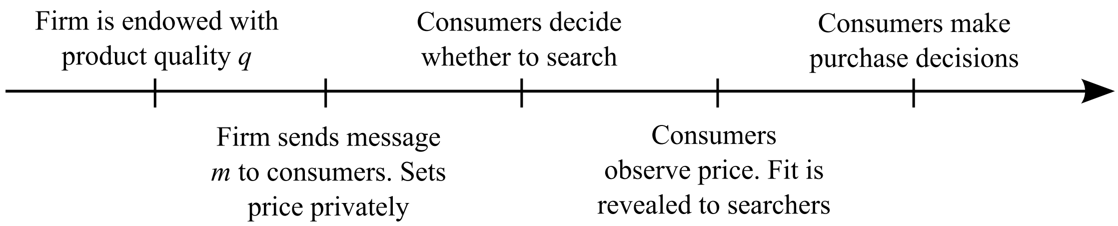
After incorporating the information asymmetry and cheap-talk advertising, the timing of the

[6] The results in this section do not depend on the probability of the firm being a high or a low type. Later in the next section we denote the probability that the firm is endowed with a high quality product by λ.

[7] This ensures that the outcome is explained purely by informational motives rather than through alternative costly signaling mechanisms. Moreover, the fact that the production cost is normalized to zero provides the same assurance.

model becomes:

Figure 7: Timing of the Imperfect Information Game



where the firm is first endowed by nature with a product of quality q. After it observes its product quality it sends a message to consumers, which can be used to inform their search decision. The rest of the game proceeds as before. After consumers receive the advertising message they form beliefs about product quality, $b(m) \in Q$. We focus on perfect Bayesian equilibria in which advertising is informational such that the firm types effectively use advertising to credibly convey their quality to consumers (i.e., separating equilibria). Hence, consumers are not naive but instead form beliefs about quality according to Bayes rule on the equilibrium path. We focus on the cases where price does not influence the consumer beliefs already formed by advertising. This ensures that all results are due to informational forces rather than by 'money burning' mechanisms a la Milgrom and Roberts (1986).

3.2 Market Outcome

When quality is not observable consumers use their beliefs to decide whether to search or not. The probability of search $\alpha(\widehat{q})$ is the same as before but now is a function of the consumers' beliefs instead of observable product quality. We are interested in the case where the firm is willing to report its type truthfully. This is captured by the following pair of incentive compatibility constraints

$$\mathrm{IC}_1 : \pi^* \left(L | \, \widehat{L} \right) \geq \max_p \pi \left(L | \, \widehat{H} \right) \tag{19}$$

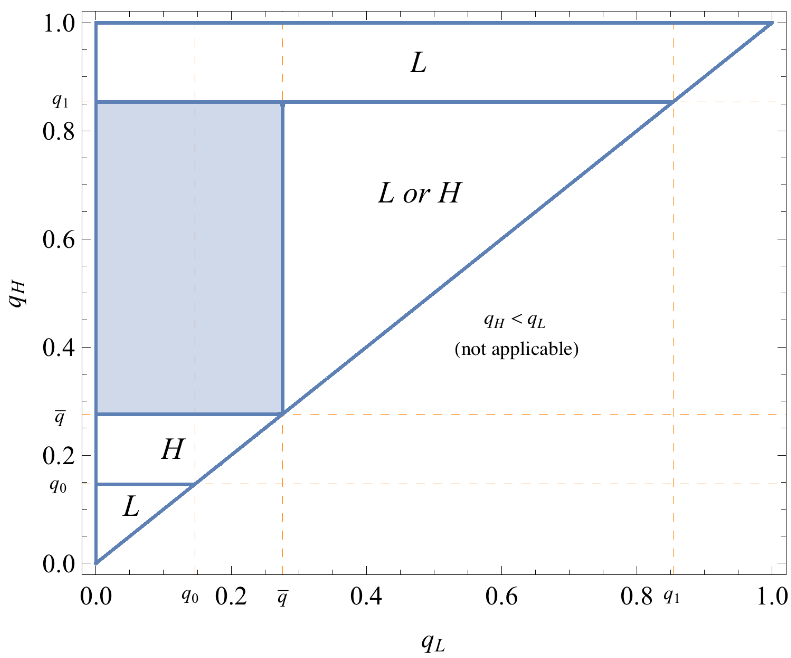
$$\mathrm{IC}_2 : \pi^* \left(H | \, \widehat{H} \right) \geq \max_p \pi \left(H | \, \widehat{L} \right) \tag{20}$$

where $\pi^*\left(L|\,\widehat{H}\right)$ denotes a low quality firm's profit as a function of price when it is believed to be of quality H, and $\pi^*\left(L|\,\widehat{H}\right)$ is the firm's profit at the equilibrium price(s). Moreover, in equilibrium consumers are willing to believe the claim of the firm, i.e., $b\,(m) = m$.

Proposition 4 (Market outcome with unknown quality). *Credible advertising can only take place in region $q_L \in (0, \overline{q})$, $q_H \in (\overline{q}, q_1)$. In sub-region $q_L < q_0$ credibility arises when $\frac{v_H - v_L}{v_L}$ is low, or if $\frac{v_H - v_L}{v_L}$ is high when q_L is also high. When $q_L \in (q_0, \overline{q})$ credibility arises when q_L is low. In both cases q_H is required to be intermediate.*

The shaded area in Figure 8 denotes the region where truthful advertising may emerge, $q_L \in (0, \overline{q})$ and $q_H \in (\overline{q}, q_1)$. The letters in the remaining regions identify the firm that benefits from misrepresentation.

Figure 8: Incentives to Provide Informative Advertising



For example, in the bottom left region the low quality firm has always an incentive to overstate its quality. Because no search takes place in this region, customers are willing to pay their ex-ante expected valuation for the product. If they believe the message of the firm the low type firm is better off overstating its quality and receiving profits $E\,[v|\,q_H]$ rather than $E\,[v|\,q_L]$.

Above this area we find that the high type firm has an incentive to understate its quality. In this region the quality of the high type firm is high but not high enough that it wants to extract profits mainly from those customers who search and find a fit with the product. Hence, if believed the high type firm is better off imitating the low type firm in order to reduce search or eliminate it altogether.

On the top region ($q_H > q_1$) the low type firm has always an incentive to imitate the high type one. By doing so it could earn profit $E\left[v|\, q_H\right]$ if believed. This is more than it can ever earn if it communicates its type truthfully.[8] The last region where credibility cannot occur is when $\overline{q} < q_L < q_H < q_1$. In this region firms mix in the price support $\left\{E\left[v|\, q_j\right], v_H\right\}$, and consumers search with some probability. Because in this region the firms target consumers who search and find a fit, the low type firm is always better off imitating the high type firm in order to extract rents from more searching customers, or from those who did not search but are now willing to pay $E\left[v|\, q_H\right]$ rather than $E\left[v|\, q_L\right]$. The high type firm may also want to imitate the low type firm if this means an increase in the search probability. As we discussed in the previous section, this can only take place if fit provides a significant increase in utility ($v_H > 4v_L$).

Credibility can emerge when $\left\{q_L \in (0, q_0), q_H \in (\overline{q}, q_1)\right\}$. In this case the low type firm earns profit $\pi\left(L|\, \widehat{L}\right) = E\left[v|\, q_L\right]$. If believed, by imitating the high type firm it can earn $\pi\left(L|\, \widehat{H}\right) = E\left[v|\, q_H\right]\left(1 - (1 - q_L)\, \alpha_H^*\left(q_H\right)\right)$ instead. In this case its margin increases from $E\left[v|\, q_L\right]$ to $E\left[v|\, q_H\right]$, but demand drops from 1 to $1 - (1 - q_L)\, \alpha_H^*\left(q_H\right)$ because by claiming to be a high type the firm induces search. Whether the tradeoff is appealing depends on the product valuations and the firms' quality levels. First, note that we can rewrite the gain in margin from misrepresentation as a function of $\frac{v_H - v_L}{v_L}$:

$$\frac{E\left[v|\, q_H\right]}{E\left[v|\, q_L\right]} = \frac{v_L + q_H\left(v_H - v_L\right)}{v_L + q_L\left(v_H - v_L\right)} = \frac{1 + q_H \frac{v_H - v_L}{v_L}}{1 + q_L \frac{v_H - v_L}{v_L}}. \tag{21}$$

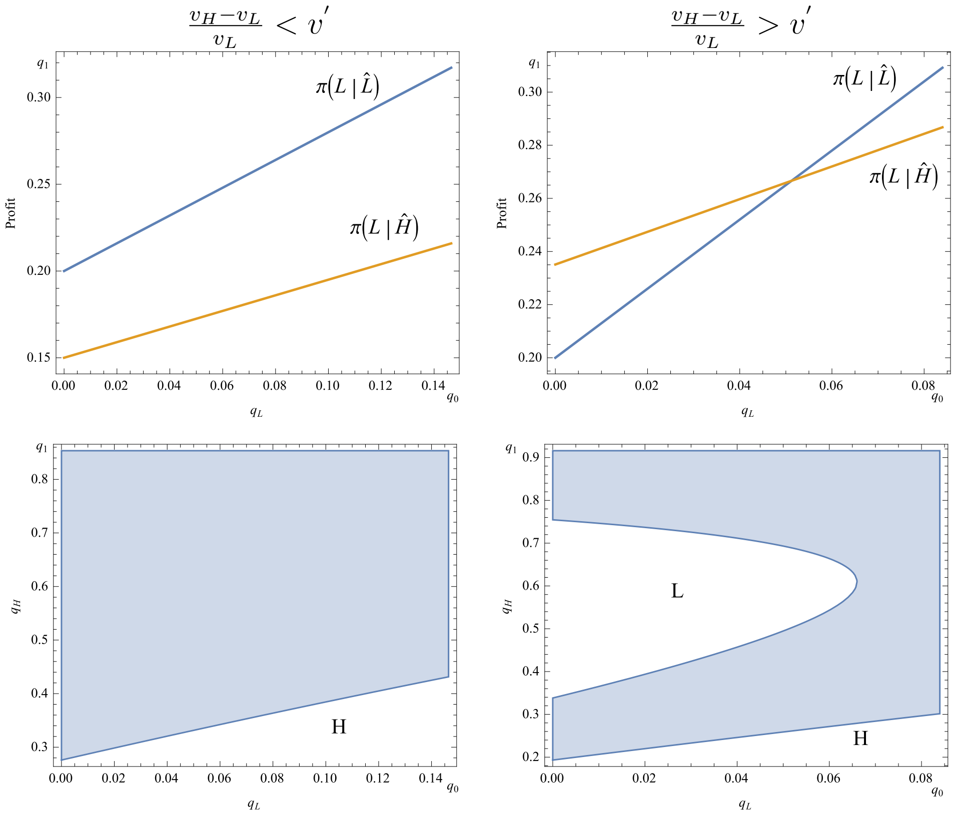
and as well as the loss in demand:

$$1 - (1 - q_L)\, \alpha_H^*\left(q_H\right) = 1 - \frac{(1 - q_L)\left(v_L + q_H\left(v_H - v_L\right)\right)}{(2 - q_H)\, q_H\left(v_H - v_L\right) + v_L} = 1 - \frac{(1 - q_L)\left(1 + q_H \frac{v_H - v_L}{v_L}\right)}{1 + (2 - q_H)\, q_H \frac{v_H - v_L}{v_L}} \tag{22}$$

It is easy to verify that both the gain in margin and the loss in demand from misrepresentation

[8] We show this in detail, in the appendix.

Figure 9: Incentives to Provide Informative Advertising, ($q_L < q_0$)



are increasing in $\frac{v_H - v_L}{v_L}$, and that the first is steeper than the second. When $\frac{v_H - v_L}{v_L}$ is low the firm does not gain enough from misrepresentation in terms of margin to compensate the loss in demand induced by search. When ratio $\frac{v_H - v_L}{v_L}$ is high the benefit from increased margins offsets the loss in search as long as q_L is low. In this case the firm is better off overstating its type in order to receive higher margins from less customers. These cases are depicted in the first row of Figure 9. In the first case (top-left cell) misrepresentation is never attractive, but in the second case (top-right cell) misrepresentation is attractive as long as q_L is low. The second row depicts the truth-telling parameter space in this sub-region, where the letters denote the firm with an incentive to misreport, as before. In the first case (bottom-left cell) the low type firm never has an incentive to misreport. In the second case however (bottom-right cell) q_L must be sufficiently high for credibility to arise. The shape of area L in the bottom-right cell derives from the fact that $\pi\left(L|\widehat{H}\right)$ is concave in q_H.

The high type firm may also have an incentive to misreport its type, because by doing so it increases demand and receives $\pi\left(H|\widehat{L}\right) = E\left[v|q_L\right]$ rather than $\pi\left(H|\widehat{H}\right) = E\left[v|q_H\right]\left(1-\left(1-q_H\right)\alpha_H^*\left(q_H\right)\right)$. Profits $\pi\left(H|\widehat{H}\right)$ are increasing in q_H such that the firm will only want to misrepresent itself when q_H is low. In the appendix we show that in this sub-region only one of the firms may have an incentive for misrepresentation.

Credibility can also emerge when $\{q_L \in (q_0, \overline{q}), q_H \in (\overline{q}, q_1)\}$. Unlike in the previous case, in this region q_L is high enough to induce some degree of search by consumers. The profit of the low-type firm under truthful advertising is $\pi\left(L|\widehat{L}\right) = v_L$ because consumers search to make the low type firm indifferent between prices v_L and $E\left[v|q_L\right]$. In this region the low-type firm's profits are independent of its quality. If the firm overstates its quality it receives profits $\pi\left(L|\widehat{H}\right) = E\left[v|q_H\right]\left(1-\left(1-q_L\right)\alpha_H^*\left(q_H\right)\right)$ which is increasing in q_L due to the increased fit of searching customers. Hence, the low-type firm is willing to advertise truthfully as long as its quality is not too high.

The high-type firm profits $\pi\left(H|\widehat{H}\right) = \alpha_H^*\left(q_H\right)q_H v_H = E\left[v|q_H\right]\left(1-\left(1-q_H\right)\alpha_H^*\left(q_H\right)\right)$ when advertising truthfully. By charging $p = v_H$ the high type firm can always profit from deviating as long as it can induce more search, since in this case it earns $\pi\left(H|\widehat{L}\right) = \alpha_L^*\left(q_L\right)q_H v_H$. When $\alpha_L^*\left(q_L\right) < \alpha_H^*\left(q_H\right)$, however, its best deviating option is to charge $p = E\left[v|q_L\right]$ and earn $\pi\left(H|\widehat{L}\right) = E\left[v|q_L\right]\left(1-\left(1-q_H\right)\alpha_L^*\left(q_L\right)\right)$. The deviation is profitable for low levels of $\frac{v_H-v_L}{v_L}$. The loss in margin and the gain in demand from misrepresentation both decrease with $\frac{v_H-v_L}{v_L}$, although demand increases at a faster rate. When $\frac{v_H-v_L}{v_L}$ is low the demand increase from imitating the low-type firm does not compensate the loss in margin, and the high-type firm is better off advertising truthfully.

Figure 10: Incentives to Provide Informative Advertising ($q_0 < q_L < \overline{q}$)

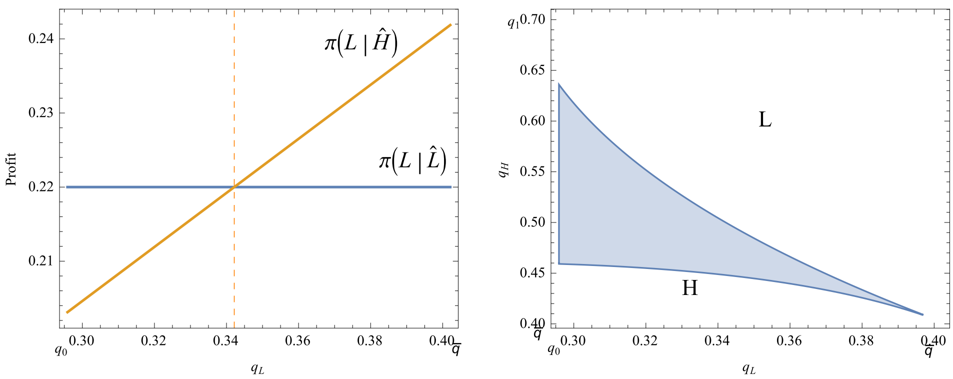


Figure 10 provides an illustration of the regions of interest at a specific set of parameter values. The left panel depicts the fact that the low-type firm is better off imitating the high-type firm as long as its quality is high enough. The right panel depicts the truth-telling region in the $\{q_L, q_H\}$ space, and the letters identify the company that is better off deviating in a particular region.[9] The low-type firm is better off advertising truthfully as long as q_L is lower than the upper bound of the shaded region on the right panel of Figure 10. The high type firm is better off advertising truthfully as long as $\frac{v_H - v_L}{v_L} > v''$. Threshold v'' is decreasing in q_H, such that the high quality firm has a higher incentive to advertise truthfully at higher values of q_H. This translates into q_H being above the lower bound of the shaded region in the same panel.

Unlike in the previous region, in this case firms may have a simultaneous incentive to misreport their types. It suffices that $\alpha_H^* (q_H) < \alpha_L^* (q_L)$. If believed the high type firm can increase profits by deviating and charging $p = v_H$. The low type firm can also increase profits in this case by charging $p = E[v| \, q_H]$ and earning $\pi \left(L | \, \widehat{H} \right) = E[v| \, q_H] \left(1 - (1 - q_L) \alpha_H^* (q_H)\right)$, which is trivially higher than what she can earn by communicating truthfully.

In summary, advertising credibility can only arise when $q_H \in (\overline{q}, q_1)$. Two cases exist in this region, $q_L < q_0$ and $q_L \in (q_0, \overline{q})$. In the first case credibility arises whenever $\frac{v_H - v_L}{v_L}$ is low or if $\frac{v_H - v_L}{v_L}$ is high, q_L is high enough. Moreover, q_H must also be high for the high-type firm not to have an incentive to understate its product quality. Otherwise, by understating its quality the high-type firm can deter search and increase profits. When $q_L \in (q_0, \overline{q})$ credibility is supported by a low level of q_L such that the low-type firm has no incentive to overstate its quality because of lost customers due to increased search. The high-type firm will advertise truthfully as long as $\frac{v_H - v_L}{v_L}$ is high, in which case a deviation would lower margins too much to be attractive. Equivalently, q_H must be high for credibility to take place. We now characterize the market outcome:

Proposition 5 (Characterization of market outcome with informative advertising). *Under truthful advertising the ex-ante price is strictly increasing in both quality levels. Ex-ante profit is strictly increasing in both quality levels, except at $q_L = q_0$, where it is decreasing. Ex-ante price and profits both increase with the probability of the firm being the high type but consumer surplus is either unaltered or decreases.*

[9] The truth-telling region can be made much smaller or larger depending on the specific parameters used.

The result follows from Proposition 3, because the informative cases are a linear combination of cases with known product quality. The difference is that for a strictly positive probability of the firm selling a high quality product - $\lambda \in (0, 1)$ - ex-ante profits are strictly increasing in both quality levels (except at $q_L = q_0$). And since the high quality firm always charges more and earns higher profits than the low quality firm, it follows that ex-ante price and profits are also increasing in λ.

The results for consumer surplus also follow from Proposition 3. A surprising result is that consumer surplus may decrease with λ, in particular when $q_L \in (q_0, \overline{q})$ because consumers can only earn positive surplus when they face the low-type firm.[10] Hence, consumers become worse off as the probability of them facing the higher quality firm increases.

4 Discussion

In this section we discuss two central features of our model, namely the emergence of mixed strategies and preference dispersion induced by search.

The mixed strategy outcome is related to the Diamond paradox, i.e., the fact that in a market with strictly positive search costs the firms' optimal strategy is to charge monopoly price, ultimately discouraging search and trade activity altogether (see Diamond (1971); see also Iyer and Kuksov (2012) for a detailed discussion). The argument is usually applicable to settings where consumers are required to incur a search cost in order to learn the price and ultimately buy. However, in our case consumers are allowed to buy without incurring the search cost: consumers can decide how much information they would like to acquire before deciding whether to buy. Together with the pricing decision by the firm this generates endogenous willingness-to-pay dispersion from otherwise homogeneous consumers.

The mixed strategy outcome does not depend the discrete/continuous nature of utility. For example, consider some consumers (indexed by i) who decide whether to search a firm with (known) product quality q. By searching consumers can learn their gross valuation $v_i = q + \varepsilon_i$, where ε_i is uniformly distributed with support $[-\gamma, \gamma]$ and is independent across consumers.

[10] When $q_L < q_0$ the firm is able to extract all of the surplus, and so λ has no effect.

Consumer i's ex-ante utility from searching is given by

$$\begin{aligned} E\left[U\left(Search\right)\right] &= Pr\left(Buy\right)E\left[U|\,Buy\right]-c \\ &= Pr\left(v_i>\widehat{p}\right)E\left[v_i-\widehat{p}|\,v_i>\widehat{p}\right]-c \\ &= \frac{\left(\gamma+q-\widehat{p}\right)^2}{4\gamma}-c \end{aligned}$$

As before, consumers may also decide not to search, from which they receive expected utility equal to $\max\left\{E\left(v_i\right)-\widehat{p},0\right\}$. The analysis in Gardete (2013) implies that an outcome with search in pure strategies exists whenever $3\gamma \geq q$, i.e., when uncertainty is high enough relative to the deterministic component of utility. However, it is easy to show that when product uncertainty is less important ($3\gamma < q$) the outcome with mixed strategies emerges. In this case when $\widehat{p} \in \left[q-\gamma+2\sqrt{c\gamma}, q+\gamma-2\sqrt{c\gamma}\right]$ consumers would rather search; however, the firm would be better off pricing at $p = q-\gamma$, at which point consumers would rather buy immediately without searching. This scenario is not an equilibrium outcome either because when consumers buy without searching the firm prefers to charge $p = q$, at which point consumers would be better off searching before buying.

The forces behind the emergence of an outcome in mixed strategies in this case are the same as in our model: When the firm has an incentive to extract all consumer surplus after search, consumers may prefer to buy without searching, at which point the firm may prefer to charge a different price, equal to the ex-ante valuation. However, at that price consumers prefer to search as long as the search cost is low, at which point the firm has an incentive to deviate again. Although Gardete (2013) does not focus on this case, it suffices that uncertainty is low and consumers be allowed to buy without searching for the mixed strategy outcome to arise.

Another fundamental feature of our model is that search induces dispersion in consumer preferences, which makes firms eager to discourage it in some conditions. To illustrate this, consider an individual who is getting interested in photography, and would like to buy a better camera. Being a novice she is uncertain about the model she should opt for: Cameras are extremely diverse and cater to different needs. Until she knows the extent of her liking for photography and what future opportunities may arise she is left to consider multiple scenarios and their likelihoods: She may discover she really loves photography to the extent of pursuing it as a career, or instead that only a few casual pictures will suffice. When no more information is available the consumer's willingness-to-pay depends on her assessment of the potential scenarios, their probabilities and her liking for each one. In this situation consumers may decide to take a photography course or procure further information to understand their preferences and evaluate the likelihood of the scenarios better. Some consumers will discover they love photography. In their case willingness-to-pay increases after information acquisition. Others' willingnesses-to-pay may decrease or stay relatively unchanged. Consumers have a relatively similar prior before search because they integrate over a number of different scenarios. After search different scenarios become important for different individuals and more diverse preferences emerge in the market.

This result can hold even when preference heterogeneity precedes information acquisition. To see this consider the case where a consumers' true utility for a product is $u = q + \varepsilon_i - p$, where $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$. Consumers do not know shock ε_i, but they can learn it through costly search. If they decide to search they learn ε_i. In this case the variance of their willingness-to-pay ("preference heterogeneity") is equal to σ_ε^2. Consumers have heterogeneous preferences before searching. For example, they may have received some information (through conversations with friends, for example) about how the product may fit their needs. Let this information be signal $s_i = \varepsilon_i + \eta_i$, $\eta_i \sim N\left(0, \sigma_\eta^2\right)$. Given this signal, their willingness-to-pay is equal to $q + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} s_i$, which depends on the specific information each consumer received, s_i. If they opt not to search the variance of their gross utility is equal to $V\left(q + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} s_i\right) = \left(\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}\right)^2 V\left(s_i\right) = \frac{\sigma_\varepsilon^4}{\sigma_\varepsilon^2 + \sigma_\eta^2}$. This variance is always lower than the variance of willingness-to-pay after search, σ_ε^2, because search 'collapses' the expectation of term ε_i into one realization, leading to a higher preference heterogeneity in willingnesses-to-pay.

5 Conclusion

We have discussed a model of strategic communication where the firm is able to convey information about its product quality to consumers, and consumers are willing to believe her. This result holds despite the firm facing consumers with homogeneous preferences. A fundamental assumption is that consumers are allowed to buy without searching. In this case they become differentiated due to the search decisions they take.

A driving force enabling credibility to arise is that search increases the heterogeneity in willingness-to-pay of consumers. This creates an incentive for firms not to overstate their quality, because doing so decreases their capability of appropriating value. In turn, they may prefer not to understate their quality because if believed this reduces consumers' willingness-to-pay. This rationale is quite general and does not depend on the discreteness of the search outcome nor on whether preferences are ex-ante homogeneous.

We have also uncovered a number of additional results. First, when quality is known the firm does not benefit from higher product quality necessarily. In particular, its profits decrease at q_0 because at this threshold consumers find search advantageous. This change introduces downward pressure on the profits of the firm.

Consumer surplus is also non-monotonic in quality because when the search regime initiates the firm prefers to target consumers who search and do not find a fit, due to its low quality. Surprisingly, consumer surplus is decreasing in v_L as this decreases the option value of searching.

We find that credible communication may arise when q_L is low and q_H is intermediate. In this situation the low quality firm has no incentive to imitate the high quality firm, as it will induce more search. Simultaneously, the high type firm has no incentive to imitate the low quality firm because it will induce preference heterogeneity, and it will be harder for her to appropriate value. While market price and profits increase with product quality, consumer surplus may decrease because the high type firm targets consumers who search and find that the product fits their needs. When these consumers are targeted they receive little surplus; only just enough to compensate their search effort. In turn, consumers who do not find a fit prefer to walk away.

The main forces at play do not depend on whether utility has discrete or continuous outcomes, or whether consumers have ex-ante homogeneous beliefs. While we have not fully characterized those situations, doing so may provide novel insights. We also abstract from reputation and the role of competition, two forces that are likely to play a role in informative advertising but that have been largely ignored by the literature so far mainly due to the complexity these factors introduce in the market interaction.

6 Appendix

6.1 Proof of Proposition 1 (Effect of quality on search behavior)

Figure 3 shows that search takes place as long as $\widehat{p} \in \left[v_L + \frac{c}{1-q}, v_H - \frac{c}{q}\right]$. The interval is non-empty as long as

$$v_H - \frac{c}{q} \geq v_L + \frac{c}{1-q} \tag{23}$$

$$\Leftrightarrow \quad q \in \left[\frac{1}{2}\left(1 - \sqrt{1 - \frac{4c}{v_H - v_L}}\right), \frac{1}{2}\left(1 + \sqrt{1 - \frac{4c}{v_H - v_L}}\right)\right] \tag{24}$$

where the condition $4c < v_H - v_L$ is necessary for search to potentially take place. Denote the lower and upper bounds by q_0 and q_1, respectively. The search cost condition can be understood by rearranging the terms in (23) to yield

$$v_H - v_L > \frac{c}{q(1-q)} \tag{25}$$

Note that the quality level q that minimizes the right hand side is $q = 1/2$. It is clear that if the condition is not met at the minimizer it cannot be met at any other quality level.

6.2 Parameter Regions for Figure 4:

The profit maximization strategy can be recovered by inspection. It is useful to denote $\alpha_i^j(q)$ as the search probability that makes the firm indifferent between practicing price i and j. For example, $\alpha_{v_L, E[v|q]}(q)$ denotes the probability of searching that makes firm of quality q indifferent between charging v_L and $E[v|q]$.[11]

Inspection of the firm's profits at each price level reveals the following pricing solution:

$$p^* = \begin{cases} v_H, & (q \leq \overline{q} \wedge \alpha(q) \geq \alpha_{v_L, v_H}) \vee \left(q > \overline{q} \wedge \alpha(q) \geq \alpha_{E[v|q], v_H}\right) \\ E[v|q], & \left(q \leq \overline{q} \wedge \alpha(q) \leq \alpha_{v_L, E[v|q]}\right) \vee \left(q > \overline{q} \wedge \alpha(q) \leq \alpha_{E[v|q], v_H}\right) \\ v_L, & q < \overline{q} \wedge \alpha_{v_L, E[v|q]} \leq \alpha(q) \leq \alpha_{v_L, v_H} \end{cases}$$

[11] Clearly, $\alpha_{i,j} = \alpha_{j,i}$. We omit argument (q) for notational simplicity.

where $\overline{q} = \frac{2v_L}{\sqrt{v_H(5v_H-4v_L)}-(v_H-2v_L)}$, $\alpha_{v_L,E[v|q]} = \frac{E[v|q]-v_L}{(1-q)E[v|q]}$, $\alpha_{E[v|q],v_H} = \frac{E[v|q]}{qv_H+(1-q)E[v|q]}$ and $\alpha_{v_L,v_H} = \frac{1}{q}\frac{v_L}{v_H}$. For example, the firm prefers to price at v_H if and only if

$$\alpha(q)\, qv_H \geq E[v|\, q]\,(1-(1-q)\,\alpha(q)) \tag{26}$$

$$\text{and} \quad \alpha(q)\, qv_H \geq v_L \tag{27}$$

$$\tag{28}$$

The firm prefers to price at v_H if search is high enough. In particular, when its quality is low ($q \leq \overline{q}$) it prefers price v_H if the search probability is higher than α_{v_L,v_H}, but when ($q > \overline{q}$) it requires a search level above $\alpha_{E[v|q],v_H}$. Clearly, quality level $\overline{q}$ makes $\alpha_{v_L,E[v|q]}$ equal $\alpha_{E[v|q],v_H}$.[12]

6.3 Proof of Proposition 2 (Market outcome with known quality)

Given the demand structures, the firm can mix among $\{v_L, E[v|\, q], v_H\}$. However, given that consumers mix over two actions a good initial guess checks the case where the firm also mixes across two actions.

6.3.1 Price mixing in $\{v_L, E[v|\, q]\}$

Let β_L be the probability the firm charges v_L for the product. The firm makes consumers indifferent between searching and not searching, i.e.,

$$\begin{aligned} & E[U(Search)] && = E[U(\sim Search)] \\ \Leftrightarrow \quad & \beta_L q(v_H - v_L) + (1-\beta_L)\, q(v_H - E[v|\, q]) - c && = \beta_L (E[v|\, q] - v_L) \\ \Leftrightarrow \quad & && \end{aligned}$$

$$\beta_L^* = 1 - \frac{c}{q(1-q)(v_H - v_L)} \tag{29}$$

[12] Solving for $\overline{q}$ yields two solutions, but one of them is always outside the unit interval and so is irrelevant.

6.3.2 Price mixing in $\{E[v|\, q], v_H\}$

Let β_H be the probability the firm charges v_H for the product. The firm makes consumers indifferent between searching and not searching, i.e.,

$$\begin{aligned} & E[U(Search)] && = E[U(\sim Search)] \\ \Leftrightarrow \quad & (1-\beta_H)\, q\, (v_H - E[v|\, q]) - c && = 0 \\ \Leftrightarrow \quad & && \end{aligned}$$

$$\beta_H^* = 1 - \frac{c}{q(1-q)(v_H - v_L)} \tag{30}$$

6.3.3 Price Mixing in $\{v_L, v_H\}$

Let $\beta_H^{'}$ be the probability the firm charges v_H for the product. The firm makes consumers indifferent between searching and not searching, i.e.,

$$\begin{aligned} & E[U(Search)] && = E[U(\sim Search)] \\ \Leftrightarrow \quad & \left(1-\beta_H^{'}\right) q\,(v_H - v_L) - c && = \left(1-\beta_H^{'}\right)(E[v|\, q] - v_L) \end{aligned}$$

which is only satisfied if $c = 0$. For a strictly positive search cost the firm cannot make the customers indifferent between searching and not searching in equilibrium.

The two possible outcomes involve mixing within $\{v_L, E[v|\, q]\}$ or within $\{E[v|\, q], v_H\}$. In order to check the conditions under which each mixing strategy dominates we need to check for profitable deviations given the consumer behavior, which we consider next.

6.3.4 Consumer Behavior

Suppose consumers search with some probability α_H^* when the firm mixes in $\{E[v|\, q], v_H\}$, and search with probability α_L^* when the firm mixes in $\{v_L, E[v|\, q]\}$. The consumers' optimal strategy is to make the firm indifferent between its prices. Hence, when the firm mixes in

$\{E[v|\, q], v_H\}$ consumers search such that

$$\begin{aligned} & \pi(p = v_H|\, \alpha_H) &= \pi(p = E[v|\, q]|\, \alpha_H) \\ \Leftrightarrow & \quad \alpha_H q v_H &= E[v|\, q](\alpha_H q + 1 - \alpha_H) \\ \Leftrightarrow & & \end{aligned}$$

$$\alpha_H^* = \frac{E[v|\, q]}{q v_H + (1-q) E[v|\, q]} \tag{31}$$

Similarly, if the firm mixes prices in $\{v_L, E[v|\, q]\}$ the consumers' best-response is to search such that

$$\begin{aligned} & \pi(p = v_L|\, \alpha_L) &= \pi(p = E[v|\, q]|\, \alpha_L) \\ \Leftrightarrow & \quad v_L &= E[v|\, q](\alpha_L^* q + 1 - \alpha_L^*) \\ \Leftrightarrow & & \end{aligned}$$

$$\alpha_L^* = \frac{E[v|\, q] - v_L}{(1-q) E[v|\, q]} \tag{32}$$

6.3.5 Market Outcome

When consumers search with probability α_L^* in response to the firm mixing in $\{v_L, E[v|\, q]\}$, no profitable deviations exist for the firm as long as

$$\max_p \pi(p|\, \alpha_L^*) \leq \pi(\{v_L, E[v|\, q]\}|\, \alpha_L^*) \tag{33}$$

where $\pi(\{v_L, E[v|\, q]\}|\, \alpha_L^*)$ is the firm's profit with mixing as above, and $\max_p \pi(p|\, \alpha_H^*)$ is the optimal deviation. The only candidate deviation is $p = v_H$ (notice that the firm is already indifferent between pricing v_L and $E[v|\, q]$). Consider the firm profits of charging each possible price:

$$\begin{aligned} \pi(p = v_L|\, \alpha_L^*) &= v_L \\ \pi(p = E[v|\, q]|\, \alpha_L^*) &= E[v|\, q](\alpha_L^* q + 1 - \alpha_L^*) \\ \pi(p = v_H|\, \alpha_L^*) &= \alpha_L^* q v_H \end{aligned}$$

Equilibrium requires that

$$\begin{aligned} & \pi\left(p=v_H \mid \alpha_L^*\right) && \leq \pi\left(p=v_L \mid \alpha_L^*\right)=\pi\left(p=E[v \mid q] \mid \alpha_L^*\right) \\ \Leftrightarrow \quad & \alpha_L^* q v_H && \leq v_L \\ \Leftrightarrow \quad & q && \leq \frac{v_L E[v \mid q]}{E[v \mid q]\left(v_H+v_L\right)-v_H v_L} \end{aligned}$$

and by expanding $E[v \mid q]$ we get

$$q \leq \bar{q} \equiv \frac{2 v_L}{\sqrt{v_H\left(5 v_H-4 v_L\right)}-\left(v_H-2 v_L\right)} \tag{34}$$

Now consider the case where the firm mixes in $\{E[v \mid q], v_H\}$ and consumers search with probability α_H^*. Consider the firm profits of charging each possible price:

$$\begin{aligned} \pi\left(p=v_L \mid \alpha_H^*\right) &= v_L \\ \pi\left(p=E[v \mid q] \mid \alpha_H^*\right) &= E[v \mid q]\left(\alpha_H^* q+1-\alpha_H^*\right) \\ \pi\left(p=v_H \mid \alpha_H^*\right) &= \alpha_H^* q v_H \end{aligned}$$

The firm is better off mixing in $\{E[v \mid q], v_H\}$ if

$$\begin{aligned} & \pi\left(p=v_L \mid \alpha_H^*\right) && \leq \pi\left(p=v_H \mid \alpha_H^*\right)=\pi\left(p=E[v \mid q] \mid \alpha_H^*\right) \\ \Leftrightarrow \quad & v_L && \leq \alpha_H^* q v_H \\ \Leftrightarrow \quad & q && \geq \frac{v_L E[v \mid q]}{E[v \mid q]\left(v_H+v_L\right)-v_H v_L} \end{aligned}$$

and by expanding $E[v \mid q]$ we get $q \geq \bar{q}$.

In the text we have also discussed that when $q \in (0, q_0) \cup (q_1, 1)$ consumers do not search, and the firm charges $E[v \mid q]$. This concludes the proof of Proposition 2.

Note that the condition $q_0 < \bar{q} < q_1$ only holds for a low enough search cost.[13]

[13] We assume this condition throughout the paper since otherwise informative advertising does not arise. The condition is $c < \frac{v_L\left(\sqrt{v_H(5v_H-4v_L)}\left(v_H^2-v_Hv_L+v_L^2\right)+v_H\left(v_H^2-5v_Hv_L+3v_L^2\right)\right)}{2(v_H-v_L)(v_H+v_L)^2}$.

6.4 Proof of Proposition 3 (Characterization of market outcome with known quality)

6.4.1 Search Probability

First, note that the search probability is strictly increasing in the range $q \in (q_0, \overline{q})$:

$$\frac{\partial \alpha_L^* (q)}{\partial q} = \frac{(v_H - v_L)(v_L + q^2 (v_H - v_L))}{(1-q)^2 (v_L + q (v_H - v_L))^2} > 0 \tag{35}$$

In the $q \in (\overline{q}, q_1)$ the search probability is affected by product quality according to

$$\frac{\partial \alpha_H^* (q)}{\partial q} = \frac{(v_H - v_L)\left(q^2 v_H - (1-q)^2 v_L\right)}{(v_L + (2-q) q (v_H - v_L))^2} \tag{36}$$

where its signal depends on the expression $q^2 v_H - (1-q)^2 v_L$, with a unique solution in the unit interval at $q^{'} \equiv \frac{\sqrt{v_L}}{\sqrt{v_H} + \sqrt{v_L}}$. It is easy to verify that $\frac{\partial \alpha_H^* (q)}{\partial q}$ is decreasing for $q < q^{'}$, and increasing otherwise. Because $q_1 > q^{'}$, the slope of search is always increasing towards q_1. However, at $q = \overline{q}$ the sign of $q^2 v_H - (1-q)^2 v_L$ depends on the relation between v_H and $4v_L$. When $4v_L > v_H$, $\left.\frac{\partial \alpha_H^* (q)}{\partial q}\right|_{q=\overline{q}} > 0$, and when $4v_L < v_H$, $\left.\frac{\partial \alpha_H^* (q)}{\partial q}\right|_{q=\overline{q}} < 0$. The underlying intuition is explained in the text.

6.4.2 Expected price

The expected price is given by

$$E[p^*(q)] = \begin{cases} E[v|\, q], & q > q_1 \\ \beta^*(q) v_H + (1 - \beta^*(q)) E[v|\, q], & \overline{q} < q \leq q_1 \\ \beta^*(q) v_L + (1 - \beta^*(q)) E[v|\, q], & q_0 < q \leq \overline{q} \\ E[v|\, q], & q \leq q_0 \end{cases}$$

It is easy to show that the average price is always monotonically increasing. When quality is very low ($q \leq q_0$) or very high ($q > q_1$) an increase in q translates to a higher willingness-to-pay by consumers who do not search, and the firm is able to charge more. In the intervals $(q_0, \overline{q})$ price is also increasing: Although $\beta^*(q)$ is increasing in q near q_0 (which means that the firm is putting more weight on price v_L rather than on price $E[v|\, q]$, the increase is less than the increase rate of $E\left[v|\,q\right]$). To see this, note that

$$\frac{\partial}{\partial q}\left(\beta^*\left(q\right)v_L+\left(1-\beta^*\left(q\right)\right)E\left[v|\,q\right]\right)=\frac{c}{\left(1-q\right)^2}>0 \tag{37}$$

On the next branch $\left(\overline{q},q_1\right)$ the partial derivative of expected price w.r.t. quality yields

$$\frac{\partial}{\partial q}\left(\beta^*\left(q\right)v_H+\left(1-\beta^*\left(q\right)\right)E\left[v|\,q\right]\right)=\frac{c}{q^2}>0 \tag{38}$$

which is also positive. Finally, inspection reveals that the expected price is continuous in q, which finishes the proof (more precisely, the average price is continuous in the $C0$ sense).

6.4.3 Expected Firm Profit

The expected firm profit is given by

$$E\left[\pi^*\left(q\right)\right]=\begin{cases}E\left[v|\,q\right], & q>q_1\\ \alpha_H^*\left(q\right)qv_H, & \overline{q}<q\leq q_1\\ v_L, & q_0<q\leq\overline{q}\\ E\left[v|\,q\right], & q\leq q_0\end{cases}$$

It is easy to verify that expected profit is always increasing within each branch. For the branch $\left(\overline{q},q_1\right)$ specifically the marginal effect of quality on expected profits is equal to

$$\frac{\partial}{\partial q}\left(\alpha_H^*\left(q\right)qv_H\right)=\frac{v_H\left(2q^2v_H^2+\left(2-3q\right)qv_Hv_L+\left(1-q\right)^2v_L^2\right)}{\left(\left(2-q\right)q\left(v_H-v_L\right)-v_L\right)^2}>0 \tag{39}$$

The sign of the expression above can be found by showing that the numerator is positive at the endpoints of quality q ($q=0$ and $q=1$) and that it is strictly increasing in q:

$$\mathrm{Num}\left(\alpha_H^*\left(q\right)qv_H\right)\big|_{q=0}=v_Hv_L^2$$

$$\mathrm{Num}\left(\alpha_H^*\left(q\right)qv_H\right)\big|_{q=1}=v_H^2\left(2v_H-v_L\right)$$

$$\frac{\partial}{\partial q}\mathrm{Num}\left(\alpha_H^*\left(q\right)qv_H\right)=2v_H\left(v_H-v_L\right)\left(2qv_H+\left(1-q\right)v_L\right)>0$$

It remains to verify evaluate the expected profit at its discontinuities. Profit always decreases near the first jump (i.e., near q_0) because of the increased heterogeneity induced by search. When consumers start searching the firm can no longer charge price $E[v|q]$ and serve the whole market. Instead, it introduces price v_L in order to target consumers who search and do not find a fit. Profit is continuous at $\overline{q}$ and at q_1 the difference in profit is always positive:

$$E[v|q_1] - \alpha_H^*(q_1)\, q_1 v_H = \left(v_H - v_L - \sqrt{(v_H - v_L)(v_H - v_L - 4c)}\right) k$$

where $k > 0$.[14] The expression is trivially positive for $c > 0$. Finally, note that the difference in payoffs is well defined as long as $4c > v_H - v_L$. This condition is equivalent to the parameter space allowing for search, i.e., $q_0 < q_1$. This completes the analysis of expected profit with respect to quality.

6.4.4 Consumer Surplus

Consumer surplus is equal to zero, except in region $q \in (q_0, \overline{q})$. In this region it is equal to

$$\begin{aligned} E(CS) &= \beta(E[v|q] - v_L) \\ &= q(v_H - v_L) - \frac{c}{1-q} \end{aligned}$$

The results in Proposition 3 follow from taking partial derivates w.r.t. the variables of interest:

$$\begin{aligned} \frac{\partial}{\partial v_H} E(CS) &= q > 0 \\ \frac{\partial}{\partial v_L} E(CS) &= -q < 0 \\ \frac{\partial}{\partial c} E(CS) &= -\frac{1}{1-q} < 0 \\ \frac{\partial}{\partial q} E(CS) &= v_H - v_L - \frac{c}{(1-q)^2} \end{aligned}$$

[14]The constant k is equal to $\frac{\left(v_H+v_L+\sqrt{(v_H-v_L)(v_H-v_L-4c)}\right)^2}{4(v_H-v_L)\left(2c+v_H+v_L+\sqrt{(v_H-v_L)(v_H-v_L-4c)}\right)}$.

The last derivative requires further inspection. First, evaluation at q_0 reveals a positive slope. The derivative has only one inflection point at $q = 1 - \frac{c}{\sqrt{c(v_H - v_L)}}$, and the second derivative is equal to $-\frac{2c}{(1-q)^3}$, which reveals a strictly concave function. Evaluation at $\overline{q}$ reveals that the sign depends on how low the search cost is: $\frac{\partial}{\partial q} E\left(CS\right)\Big|_{q=\overline{q}} = v_H - v_L - \frac{c}{(1-\overline{q})^2}$, which is positive as long as c is low enough. When c is high consumer surplus has an inverse-U shape.

6.5 Proof of Proposition 4 (Market outcome with unknown quality)

There exist 10 regions of interest. We first focus on the regions where one firm always has an incentive to deviate.

6.5.1 Region $q_L < q_H < q_0$

No search occurs in this region, and consumers are willing to pay $E\left[v|\, q_L\right]$ for a low quality product, and $E\left[v|\, q_H\right]$ for a high quality one. Given that no search occurs in this region, if believed the low type firm always has an incentive to imitate the high type and earn profits $E\left[v|\, q_H\right]$. Hence, no credible communication can take place.

6.5.2 Region $q_L < q_0 < q_H < \overline{q}$ or $q_0 < q_L < q_H < \overline{q}$

In the first sub-region the high type firm always wants to imitate the low type firm because it can increase its profits to $E\left[v|\, q_L\right]$ from v_L if it is believed. In the second sub-region the high type firm also wants to imitate the low type firm because it can receive profit $E\left[v|\, q_L\right]\left(1-\left(1-q_H\right)\alpha_L^*\left(q_L\right)\right)$ rather than v_L if believed. This is easily shown by inspection.

As for the low type firm, it never has an incentive to deviate. It earns profits $E\left[v|\, q_L\right]$ and v_L in the first and second sub-regions, respectively, by advertising truthfully. It suffices to show that $v_L > \max\left\{E\left[v|\, q_H\right]\left(1-\left(1-q_L\right)\alpha_L^*\left(q_H\right)\right), \alpha_L^*\left(q_H\right) q_L v_H\right\}$ whenever $q_H < \overline{q}$. The profit of the high type firm is useful: Because $E\left[v|\, q_H\right]\left(1-\left(1-q_H\right)\alpha_L^*\left(q_H\right)\right) = v_L$, it follows that $E\left[v|\, q_H\right]\left(1-\left(1-q_L\right)\alpha_L^*\left(q_H\right)\right) < v_L$. Also, we know that for $q_H < \overline{q}$, $\alpha_L^*\left(q_H\right) q_H v_H < v_L$, so $\alpha_L^*\left(q_H\right) q_L v_H < v_L$ as well. Hence, in these regions only the high type firm has an incentive to imitate the low type one.

6.5.3 Region $q_1 < q_H$

This region is obtained by the union of sub-regions $q_L \in (0, q_0) \cup (q_0, \overline{q}) \cup (\overline{q}, q_1) \cup (q_1, 1)$, $q_H \in (q_1, 1)$. In all sub-regions the high type firm earns profit $E[v|\, q_H]$ when its type is revealed. In sub-region $q_L \in (0, q_0) \cup (q_0, \overline{q})$ it is clear that the low type firm has an incentive to claim it is high, because $E[v|\, q_H] > \{v_L, E[v|\, q_L]\}$. When $q_L \in (\overline{q}, q_1)$ the low type firm earns profit $E[v|\, q_L](1-(1-q_L)\,\alpha_H^*(q_L))$, which is also less than $E[v|\, q_H]$. Finally, when (q_1, q_H) the low type firm earns $E[v|\, q_L]$, which again is lower than the profit it can earn if it successfully imitates the high type firm.

6.5.4 Region $\overline{q} < q_L < q_H < q_1$

In this region $\pi\left(L|\, \widehat{L}\right) = \alpha_H^*(q_L)\, q_L v_H = E[v|\, q_L](1-(1-q_L)\,\alpha_H^*(q_L))$. If it successfully imitates the high type firm, the low type firm earns $\pi\left(L|\, \widehat{H}\right) = \max\{\alpha_H^*(q_H)\, q_L v_H,$
$E[v|\, q_H](1-(1-q_L)\,\alpha_H^*(q_H))\}$. Note that if the low type firm can induce more search through imitating the high type, it prefers to deviate because it earns $\alpha_H^*(q_H)\, q_L v_H > \alpha_H^*(q_L)\, q_L v_H$. Otherwise, it is clear that $E[v|\, q_H](1-(1-q_L)\,\alpha_H^*(q_H)) > E[v|\, q_L]\ (1-(1-q_L)\,\alpha_H^*(q_L))$, and so the low type firm always prefers to deviate and imitate the high type firm if her communication is effective.

As for the high type firm, it if induces more search through imitating the low type firm then it can increase profits from $\alpha_H^*(q_H)\, q_H v_H$ to $\alpha_H^*(q_L)\, q_H v_H$. Otherwise - if $\alpha_H^*(q_L) < \alpha_H^*(q_H)$ - it always prefers to report truthfully because $E[v|\, q_H]\ (1-(1-q_H)\,\alpha_H^*(q_H)) > E[v|\, q_L]\ (1-(1-q_H)\,\alpha_H^*(q_L))$. The proof is simple but long. In particular, calculate $E[v|\, q_H]$ $(1-(1-q_H)\,\alpha_H^*(q_H)) - E[v|\, q_L](1-(1-q_H)\,\alpha_H^*(q_L))$.

The denominator becomes $(2-q_L)\, q_L\,(v_H - v_L) + v_L$, which is always positive. Hence, the sign of the expression depends on the numerator, which is linear in q_H, and so the truth-telling arises when $A + B\, q_H > 0$.[15] For truth-telling to emerge we require that $q_H > -\frac{A}{B}$ whenever $B > 0$ and that $q_H < -\frac{A}{B}$ whenever $B < 0$. It remains to verify that each of these conditions always holds. In particular, it is possible to verify that $q_L > -\frac{A}{B}$ holds whenever $B > 0$ (which in turn implies that $q_H > -\frac{A}{B}$). Moreover, when $B < 0$ it is also possible to verify that $1 < -\frac{A}{B}$,

[15] $A = -\Delta^2(1-q_L)q_L^2 + \Delta q_L v_L + v_L^2$ and $B = 2\Delta^2(1-q_L)q_L + v_L(\Delta - 2\Delta q_L) - \mathrm{vl}^2$, where $\Delta \equiv v_H - v_L$. Moreover, $B \geq 0 \Leftrightarrow q_L \leq \frac{\sqrt{v_H(v_H-2v_L)}+v_H-2v_L}{2(v_H-v_L)}$, which is possible as long as $v_H > 2v_L$.

which implies that $q_H < -\frac{A}{B}$ always holds.

Hence, the low type firm always has an incentive to deviate from truth-telling. The high type firm may also want to deviate but only if it can induce more search when doing so.

6.5.5 Region $q_L < q_0 < \overline{q} < q_H < q_1$

In this case the low type firm earns $E[v|\, q_L]$ and the high type firm earns $\alpha_H^* (q_H)\, q_H v_H$ if they advertise truthfully. To understand the conditions under which credibility may arise, we first analyze the incentives of the low type firm.

- Low type Firm:

If the low type firm deviates, it prefers to set price $p^* = E[v|\, q_H]$ rather than $p^* = v_H$. This can be shown by comparing profits

$$
\begin{aligned}
& \pi\left(L|\,\widehat{H}\right)\Big|_{p^*=E[v|q_H]} && \geq \pi\left(L|\,\widehat{H}\right)\Big|_{p^*=v_H} \\
\Leftrightarrow\quad & E[v|\,q_H]\,(1-(1-q_L)\,\alpha_H^*(q_H)) && \geq \alpha_H^*(q_H)\,q_L v_H \\
\Leftrightarrow\quad & 1-(1-q_L)\,\alpha_H^*(q_H) && \geq \frac{q_L v_H}{q_H v_H + (1-q_H)\,E[v|\,q_H]} \\
\Leftrightarrow\quad & 1 && \geq \frac{q_L v_H + (1-q_L)\,E[v|\,q_H]}{q_H v_H + (1-q_H)\,E[v|\,q_H]}
\end{aligned}
$$

which holds because $q_H > q_L$ and $v_H > E[v|\,q_H]$.

It remains to show under which conditions the low type firm is better off advertising truthfully, i.e., when

$$
\begin{aligned}
& \pi\left(L|\,\widehat{L}\right) > && \pi\left(L|\,\widehat{H}\right)\Big|_{p^*=E[v|q_H]} \\
\Leftrightarrow\quad & E[v|\,q_L] > && E[v|\,q_H]\,(1-(1-q_L)\,\alpha_H^*(q_H))
\end{aligned}
$$

The tradeoff for the low type firm is that by overstating its quality it receives a higher price but sells less units. Determining the truth-telling condition is simple but again, a relatively long process. The denominator of the difference in profits is equal to $(2-q_H)\,q_H\Delta + v_L$, which is always positive ($\Delta \equiv v_H - v_L$). The firm prefers to advertise truthfully when $A - B\,q_L > 0$.[16] It follows that $A \geq 0 \Leftrightarrow \frac{\Delta}{v_L} \leq \frac{1+\sqrt{5-4q_H}}{2(1-q_H)q_H}$ and $B \geq 0 \Leftrightarrow \frac{\Delta}{v_L} \leq \frac{-1+2q_H+\sqrt{1+4(1-q_H)q_H}}{4(1-q_H)q_H}$. When $A, B > 0$,

[16] $A \equiv -\Delta^2(1-q_H)q_H^2 + \Delta q_H v_L + v_L^2$ and $B \equiv -2\Delta^2(1-q_H)q_H - v_L\Delta(1-2q_H) + v_L^2$.

$\frac{A}{B} > 1$, and so truth-telling is always satisfied (we show this below). When $A < 0 \, (\Rightarrow B < 0)$ we require that $q_L > \frac{A}{B}$, which trivially holds since $\frac{A}{B} < 0$. Finally, the condition $A > 0 \wedge B < 0$ translates into $\frac{\Delta}{v_L} > \frac{1+\sqrt{5-4q_H}}{2(1-q_H)q_H}$ and $q_L > \frac{A}{B}$.

To verify that $\frac{A}{B} > 1$ whenever $A, B > 0$, first notice that $\frac{-1+2q_H+\sqrt{1+4(1-q_H)q_H}}{4(1-q_H)q_H} < \frac{1+\sqrt{5-4q_H}}{2(1-q_H)q_H}$, so we require $\frac{\Delta}{v_L} > \frac{1+\sqrt{5-4q_H}}{2(1-q_H)q_H}$. Moreover, $\frac{\partial}{\partial \Delta}\frac{A}{B} > 0$ and $\min_{\Delta} \frac{A}{B} = 1$ at, $\Delta = 0$. Hence, it follows that $\frac{A}{B} > 1$.

In summary, the low type firm prefers to tell the truth if $A > 0$, or if $A < 0$ and $q_L > \frac{A}{B}$. We label this condition set as C_1: $\frac{\Delta}{v_L} \leq \frac{1+\sqrt{5-4q_H}}{2(1-q_H)q_H} \vee \left(\frac{\Delta}{v_L} > \frac{1+\sqrt{5-4q_H}}{2(1-q_H)q_H} \wedge q_L > \frac{-\Delta^2(1-q_H)q_H^2+\Delta q_H v_L+v_L^2}{-2\Delta^2(1-q_H)q_H-v_L\Delta(1-2q_H)+v_L^2}\right.$.

- High type Firm:

The high type firm may also benefit from imitating the low type firm and receive $E\left[v|\, q_L\right]$. It is better off advertising truthfully when $\alpha_H^*\left(q_H\right) q_H v_H \geq E\left[v|\, q_L\right]$, which solving w.r.t. q_H yields

$$q_H \geq \frac{2\Delta^2 q_L + \Delta v_L - v_L^2 + \sqrt{4\Delta^4 q_L^2 + 4\Delta^3 q_L^2 v_L + 8\Delta^2 q_L v_L^2 + 8\Delta^3 q_L v_L + 6\Delta v_L^3 + 5\Delta^2 v_L^2 + v_L^4}}{2\Delta(\Delta + \Delta q_L + 2v_L)},$$

where $\Delta \equiv v_H - v_L$. This completes region the set of conditions C_1.

Finally, note that it cannot happen that both firms want to deviate. When the the low type firm prefers to overstate its quality, condition $E\left[v|\, q_L\right] < E\left[v|\, q_H\right]\left(1-\left(1-q_L\right)\alpha_H^*\left(q_H\right)\right)$ holds. However, this implies that $E\left[v|\, q_L\right] < E\left[v|\, q_H\right]\left(1-\left(1-q_H\right)\alpha_H^*\left(q_H\right)\right)$, which is the condition that ensures truth-telling for the high type firm.

We summarize the conditions for credible communication at the end of the next section.

6.5.6 Region $q_0 < q_L < \overline{q} < q_H < q_1$

In this region credibility may also arise, and the method of proof is similar to that of the previous section.

- Low type Firm

First, note that if the low type firm deviates, it does so using price $p^* = E\left[v|\, q_H\right]$. The proof is the same as the one in the previous section. The low type firm is better off advertising truthfully as long as its quality is low enough, i.e.,

$$\begin{aligned}
& \pi\left(L|\,\widehat{L}\right) && \geq \pi\left(L|\,\widehat{H}\right)\Big|_{p^*=E[v|q_H]} \\
\Leftrightarrow \quad & v_L \geq && E\left[v|\,q_H\right]\left(1-\left(1-q_L\right)\alpha_H^*\left(q_H\right)\right) \\
\Leftrightarrow \quad & q_L \leq && 1-\frac{1}{\alpha_H^*\left(q_H\right)}\left(1-\frac{v_L}{E\left[v|\,q_H\right]}\right).
\end{aligned}$$

The reason is that when its quality is low, the low type firm loses demand if it imitates the high type firm, in particular of consumers who search but do not find a fit. We denote this condition as C_2, applicable to the low type firm.

- High type Firm

The high type firm would prefer to understate its quality if this results in higher search. In this case it can charge $p^* = v_H$ and earn profit $\alpha_L^*\left(q_L\right) q_H v_H$ rather than $\alpha_H^*\left(q_H\right) q_H v_H$. However, when consumers search the low type firm less than the high type one ($\alpha_L^*\left(q_L\right) < \alpha_H^*\left(q_H\right)$) the firm may have an incentive to advertise truthfully. In case of deviation its best option is to charge $p^* = E\left[v|\,q_L\right]$. Solving inequality $E\left[v|\,q_H\right]\left(1-\left(1-q_H\right)\alpha_H^*\left(q_H\right)\right) > E\left[v|\,q_L\right]\left(1-\left(1-q_H\right)\alpha_L^*\left(q_L\right)\right)$ yields

$$\frac{\Delta}{v_L} > \frac{q_H(1-q_L)-q_L^2+\sqrt{q_H^2(1-q_L)(1-5q_L)+2q_H\left(2-q_L^2-q_L\right)q_L+q_L^4}}{2q_Hq_L(1-q_H+q_L)}$$

The threshold is increasing in q_H, which means that the incentive for the high-type to deviate increases with q_H. This completes the characterization of the region C_2, required for advertising credibility to emerge. As in the previous case firms never have a simultaneous incentive to deviate.

Hence, credible communicate takes place if and only if

$$C_0 \equiv \left(q_L < q_0 < \overline{q} < q_H < q_1 \wedge C_1\right) \tag{40}$$

$$\vee\left(q_0 < q_L < \overline{q} < q_H < q_1 \wedge C_2\right). \tag{41}$$

Existence can be verified by inspection: In region $q_L < q_0 < \overline{q} < q_H < q_1$ (first term of region C_0) it is easy to verify that all credibility conditions are satisfied at $v_H = 1, v_L = \frac{1}{10}, c = \frac{1}{16}, q_H = \frac{23}{128}, q_L = \frac{5}{512}$. In region $q_0 < q_L < \overline{q} < q_H < q_1$ it is easy to verify that credibility is satisfied at $v_H = 1, v_L = \frac{1}{10}, c = \frac{5}{1024}, q_H = \frac{157}{1024}, q_L = \frac{3}{256}$.

6.6 Proof of Proposition 5 (Characterization of market outcome with informative advertising)

We have already shown that expected price is strictly increasing in quality for the case of perfect information. Since the informative advertising case constitutes a linear combination of scenarios of the known quality case, it follows that ex-ante price under informative advertising is also strictly increasing in quality levels.

Expected profits depend on the particular region of interest. When $q_L < q_0$ the low type firm earns profits $E\left[v|\, q_L\right]$ and the resulting ex-ante profit becomes

$$E\left(\pi|\, q_L < q_0\right) = \left(1-\lambda\right) E\left[v|\, q_L\right] + \lambda \alpha_H^*\left(q_H\right) q_H v_H$$

When $q_L > q_0$ ex-ante profit is equal to

$$E\left(\pi|\, q_L > q_0\right) = \left(1-\lambda\right) v_L + \lambda \alpha_H^*\left(q_H\right) q_H v_H$$

In both cases profits are strictly increasing in the quality levels. However, at $q_L = q_0$ expected profits decrease due to the search regime starting for the low type firm. The statement about consumer surplus trivially follows from Proposition 3.

6.7 Parameter Values for Figures

Figures 4, 6 and 8: $v_H = 1, v_L = \frac{1}{5}, c = \frac{1}{10}$. Figure 5: $v_H = \frac{6}{10}, v_L = \frac{1}{5}, c = \frac{1}{20}$ (left panel) and $v_H = 2, v_L = \frac{1}{5}, c = \frac{1}{10}$ (right panel). Figure 9: $v_H = 1, v_L = \frac{1}{5}, c = \frac{1}{10}$ (left column) and $v_H = \frac{3}{2}, v_L = \frac{1}{5}, c = \frac{1}{10}$ (right column). Figure 10: $v_H = \frac{7}{10}, v_L = \frac{11}{50}, c = \frac{1}{10}$.

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