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Nonlinear resonance of superconductor/normal metal structures to microwaves

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We study the variation in the differential conductance $G=dj/dV$ of a normal metal wire in a superconductor/normal metal heterostructure with a cross geometry under external microwave radiation applied to the superconducting parts. Our theoretical treatment is based on the quasiclassical Green's functions technique in the diffusive limit. Two limiting cases are considered: first, the limit of a weak proximity effect and low microwave frequency and second, the limit of a short dimension (short normal wire) and small irradiation amplitude.

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I. INTRODUCTION

Superconductor/normal metal (S/N) nanostructures, where the proximity effect (PE) plays an important role, have been studied very actively during last two decades. Interesting phenomena have been discovered in the course of these studies. Perhaps, the most remarkable one is an oscillatory dependence of the conductance of a normal wire attached to two superconductors which are incorporated into a superconducting loop.[1,2] This phenomenon was observed in the so-called "Andreev interferometers," i.e., in multiterminal SNS junctions (see Refs. 2–6 as well as reviews[7–10] and references therein). The reason for this oscillatory behavior of the differential conductance $G=dj/dV$ is a modification of the transport properties of the n wire due to the PE, i.e., due to the condensate induced in the n wire. The density of the induced condensate is very sensitive to an applied magnetic field H and oscillates with increasing H.

Theory[11–14] was successful in explaining the experiments and predicting new phenomena, including the re-entrance of the conductance to the normal state in mesoscopic proximity conductors[14–16] and transitions to the π state in the voltage-biased Andreev interferometers due to nonequilibrium effects.[17–19] The nonmonotonic behavior of the conductance in SN point contacts and controllable nanostructures has been observed in Refs. 20–22, and the change in the sign of the critical Josephson current in multiterminal SNS junctions has been found in Refs. 23 and 24. Many important results of the study of the SN mesoscopic structures are reviewed in Refs. 7–10. The so-called π states have also been realized in equilibrium Josephson SFS junctions with a ferromagnetic (F) layer between superconductors[25–27] or in superconductor-insulator-superconductor (SIS) Josephson junctions of high-T_c, d-wave superconductors.[28,29] A number of new phenomena have been discovered in thin one-dimensional N and S wires[30–32] (see also Ref. 33 for a recent review and references therein).

Mesoscopic SNS structures proved to be a promising alternative to superconducting quantum interference devices (SQUIDs) for certain applications, including magnetic-flux measurements and readout of quantum bits (qubits)[34] with a potential to achieve higher than state-of-the-art fidelity, sensitivity, and readout speed. To achieve such challenging aims extensive investigations of high-frequency properties of S/N nanostructures on a scale similar to that of SQUIDs are in order.

Studies undertaken to date concerned mainly the stationary properties of S/N structures. Experimental data on S/N structures under microwave radiation appeared only recently.[35,36] As to theoretical studies, one can mention two papers[11,37] where the ac impedance of a S/N structure was calculated. However, measuring the frequency dependence of the ac conductance is not an easy task. It is more convenient to measure a nonlinear dc response (dc conductance) to a microwave radiation. Recently, a numerical calculation of the dependence of the critical Josephson current I_c in SNS junction on the amplitude of an external ac radiation has been performed.[38]

In this paper, using a simple model we calculate the dc conductance of a normal (n) wire in an S/N structure (cross geometry) as a function of the frequency Ω and the amplitude of the external microwave radiation. We consider the limiting cases of a long and a short n wire and show that the response has a resonance peak at a frequency Ω close to $\varepsilon_s/\hbar$, where ε_s is the energy of a subgap in the n wire induced by the PE. Our theory predicts resonances and can help to optimize quantum devices based on hybrid SNS nanostructures.[34,39]

We employ the quasiclassical Green's-function technique in the diffusive limit. This means that we will solve the Usadel equation[40] for the retarded (advanced) Green's function $\hat{g}^{R(A)}$ and the corresponding equation for the Keldysh matrix function $\hat{g}^K$ (Sec. II). First, a weak PE will be considered when the Usadel equation can be linearized (Sec. III). We calculate the dc conductance of the n wire in this limit, assuming that the frequency of the ac radiation is low ($\Omega \ll T$). In Sec. IV, the opposite limiting case of a short n wire will be analyzed for arbitrary frequencies Ω. We present the frequency dependence of the correction to the dc conductance caused by ac radiation. In Sec. V, we discuss the obtained results.

II. MODEL AND BASIC EQUATIONS

We consider an S/N structure shown in Fig. 1. It consists of a n wire or n film which connects two N and S reservoirs

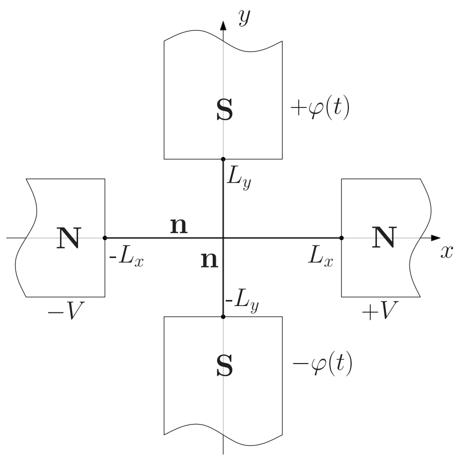

FIG. 1. Structure under consideration.

(n and N stand for a normal metal, S means a superconductor). The superconducting reservoirs may be connected by a superconducting contour. The transverse dimensions of the n wire are supposed to be smaller than characteristic dimensions of the problem but larger than the Fermi wave length and the mean-free path l (diffusive case). This implies that all quantities depend only on coordinates along the wire (the x coordinate in the horizontal direction and the y coordinate in the vertical direction). The dc voltage $2V$ is applied between the normal N reservoirs, and the phase difference 2φ exists between the superconducting reservoirs. The phase φ is assumed to be time dependent

$$\varphi(t) = \varphi_0 + \varphi_\Omega \cos(\Omega t) \tag{1}$$

and related to the magnetic flux Φ inside the superconducting contour: $\varphi(t)=\pi\Phi(t)/\Phi_0$ with $\Phi(t)=H(t)S$, where $H(t)$ is an applied magnetic field and S is the area of the superconducting contour; that is, the magnetic field contains not only a constant component but also an oscillating one.

For simplicity, we assume the structure to be symmetric both in the horizontal and vertical directions. This implies, in particular, that the interface resistances R_{nN} at $x=\pm L_x$ are equal to each other [correspondingly, $R_{nS}(L_y)=R_{nS}(-L_y)$]. Our aim is to calculate the differential dc conductance G between the N reservoirs

$$G = \frac{dj}{dV} \tag{2}$$

as a function of the amplitude of the ac signal φ_Ω and the frequency Ω.

The calculations will be carried out on the basis of the well-developed quasiclassical Green's-functions technique (see the reviews in Refs. 10 and 41–43) which successfully was applied to the study of S/N structures (see, for example, Refs. 8–11, 37, and 44–47). In this technique, all types of Green's functions (the "normal" and Gor'kov's functions as well as the retarded, advanced and Keldysh functions) are matrix elements of a 4×4 matrix

$$\check{g} = \begin{pmatrix} \hat{g}^R & \hat{g}^K \\ 0 & \hat{g}^A \end{pmatrix}, \tag{3}$$

where $\hat{g}^{R(A)}$ are matrices of the retarded (advanced) Green's functions and $\hat{g}^K$ is a matrix of the Keldysh functions. The first matrices describe thermodynamical properties of the system [the density of states (DOS), supercurrent, etc.] whereas the matrix $\hat{g}^K$ is used to describe dissipative transport and nonequilibrium properties.

The matrix $\check{g}$ satisfies the normalization condition[48]

$$(\check{g}\circ\check{g})(t,t') = \delta(t-t'), \tag{4}$$

where "$\circ$" denotes the integral product $(\check{g}\circ\check{g})(t,t') = \int dt_1 \check{g}(t,t_1)\cdot\check{g}(t_1,t')$ and "$\cdot$" is the conventional matrix product. The Fourier transform performed as $\check{g}(\varepsilon,\varepsilon') = \int dt dt' e^{i\varepsilon t - i\varepsilon' t'}\check{g}(t,t')$ yields $(\check{g}\circ\check{g})(\varepsilon,\varepsilon')=2\pi\delta(\varepsilon-\varepsilon')$ where now $(\check{g}\circ\check{g})(\varepsilon,\varepsilon')=\int\frac{d\varepsilon_1}{2\pi}\check{g}(\varepsilon,\varepsilon_1)\cdot\check{g}(\varepsilon_1,\varepsilon')$.

The matrix of Keldysh functions $\hat{g}^K$ can be expressed in terms of the matrices $\hat{g}^{R(A)}$ and a matrix of distribution functions $\hat{F}$,

$$\hat{g}^K = \hat{g}^R\circ\hat{F} - \hat{F}\circ\hat{g}^A, \tag{5}$$

where the matrix $\hat{F}$ can be assumed to be diagonal[10]

$$\hat{F} = \hat{\tau}_0 F_+ + \hat{\tau}_3 F_-. \tag{6}$$

Here $\hat{\tau}_0$ is the identity matrix and $\hat{\tau}_3$ the third Pauli matrix. The function F_- describes the charge imbalance [premultiplied with the DOS and integrated over all energies it gives the local voltage] while F_+ characterizes the energy distribution of quasiparticles.

Due to the general relation[42]

$$\hat{g}^A(\varepsilon,\varepsilon') = -\hat{\tau}_3\hat{g}^{R\dagger}(\varepsilon',\varepsilon)\hat{\tau}_3, \tag{7}$$

one can immediately calculate $\hat{g}^A$ after finding the matrix $\hat{g}^R$. That means that knowing the matrices $\hat{g}^R$ and $\hat{F}$ we can determine all entries of $\check{g}$.

The Green's functions in N and S reservoirs are assumed to have an equilibrium form corresponding to the voltages $\pm V$ and phases $\pm\varphi(t)$. For example, the retarded (advanced) Green's functions in the upper S reservoir are

$$\hat{g}_S^{R(A)}(t,t') = \hat{S}(t)\hat{g}_{S0}^{R(A)}(t-t')\cdot\hat{S}^\dagger(t'), \tag{8}$$

where $\hat{S}(t)=\exp[i\hat{\tau}_3\varphi(t)/2]$ is a unitary transformation matrix and the Fourier transform of $\hat{g}_{S0}^{R(A)}(t-t')$ equals

$$\hat{g}_{S0}^{R(A)}(\varepsilon) = \frac{1}{\xi_\varepsilon^{R(A)}}\begin{pmatrix} \varepsilon & \Delta \\ -\Delta & -\varepsilon \end{pmatrix} \tag{9}$$

with $\xi_\varepsilon^{R(A)}=\pm\sqrt{(\varepsilon\pm i0)^2-\Delta^2}$, i.e., the matrix $\hat{g}_{S0}^{R(A)}$ describes the BCS superconductor in the absence of phase. The retarded (advanced) Green's functions in the lower S reservoir are determined in the same way with the replacement $\varphi(t)\to-\varphi(t)$. The matrix $\hat{g}_N^{R(A)}$ in the right (left) N reservoirs is equal to $\hat{g}_{S0}^{R(A)}$ with $\Delta=0$, i.e., $\hat{g}_N^{R(A)}=\pm\hat{\tau}_3$.

In the reservoirs the matrix $\hat{F}(t,t')$ can be represented through the equilibrium distribution $F_{eq}(\varepsilon)=\tanh(\varepsilon/2T)$ via Eq. (8)

$$\hat{F}(t,t') = \hat{S}(t) \cdot F_{eq}(t-t')\hat{S}^{\dagger}(t'). \tag{10}$$

The phase $\varphi(t)$ in the upper S reservoir is given by Eq. (1), and for $\varphi_N(t)$ in the right N reservoir, we have $\varphi_N(t)=2eVt$. Therefore in the normal reservoir (at the right) the matrix distribution function has diagonal elements $\hat{F}_N(\varepsilon)_{11,22} = \tanh[\frac{1}{2T}(\varepsilon \pm eV)]$ and can be written as $\hat{F}_N(\varepsilon)=\hat{\tau}_0 F_{N+}(\varepsilon) + \hat{\tau}_3 F_{N-}(\varepsilon)$

$$F_{N\pm}(\varepsilon) = \frac{1}{2}\left[\tanh\frac{\varepsilon+eV}{2T} \pm \tanh\frac{\varepsilon-eV}{2T}\right]. \tag{11}$$

Thus, all Green's functions in the reservoirs are defined above.

Our task is to find the matrix $\check{g}$ in the n wire. In the considered diffusive limit it obeys the equation[48]

$$\check{\tau}_3 \cdot \frac{\partial \check{g}}{\partial t} + \frac{\partial \check{g}}{\partial t'} \cdot \check{\tau}_3 + i[eV_n(t)\check{g} - \check{g}eV_n(t')] - D\,\nabla\,(\check{g} \circ \nabla \check{g}) = 0, \tag{12}$$

where $\check{\tau}_3$ is a diagonal matrix with equal elements $(\check{\tau}_3)_{11,22} = \hat{\tau}_3$, V_n is a local electrical potential in the n wire. We dropped the inelastic collision term supposing that $E_{Th} = D/L_{max}^2 \gg \tau_{inel}^{-1}$, where D is the diffusion coefficient, $L_{max} = \max\{L_{x,y}\}$ and τ_{inel} is an inelastic-scattering time. This equation is complemented by the boundary condition[49]

$$\check{g} \circ \partial_{x,y}\check{g}|_{x,y=\pm L_{x,y}} = \pm \kappa_{N,S}[\check{g},\check{g}_{N,S}]_{\circ}, \tag{13}$$

where $\kappa_{N,S}=1/(2\sigma R_{nN,nS})$, $R_{nN,nS}$ are the nN and nS interface resistances per unit area and σ is the conductivity of the n wire. Here we introduced the commutator $[\check{g},\check{g}_{N,S}]_{\circ}=\check{g}\circ\check{g}_{N,S} - \check{g}_{N,S}\circ\check{g}$. The current in the n wire is determined by the formula

$$j = \frac{\sigma}{8e}\mathrm{Tr}\{\hat{\tau}_3 \cdot 2\pi(\check{g} \circ \partial_x \check{g})_{12}(t,t)\}. \tag{14}$$

The matrix element $(\check{g}\circ\partial_x\check{g})_{12}$ is the Keldysh component that equals $(\check{g}\circ\partial_x\check{g})_{12}=\hat{g}^R\circ\partial_x\hat{g}^K+\hat{g}^K\circ\partial_x\hat{g}^A$.

Even in a time-independent case, an analytical solution of the problem can be found only under certain assumptions.[8–11,37,44–46] In the considered case of a time-dependent phase difference, the problem becomes even more complicated. In order to solve the problem analytically, we consider two limiting cases: (a) weak proximity effect and slow phase variation in time; and (b) strong proximity effect in a short n wire and arbitrary frequency Ω of the phase oscillations.

III. WEAK PROXIMITY EFFECT; SLOW PHASE VARIATION

In this section we will assume that the proximity effect is weak and the phase difference $\varphi(t)$ is almost constant in time. The latter assumption means that the frequency of phase variation satisfies the condition $\Omega \ll T/\hbar$. The weakness of the PE means that the anomalous (Gor'kov's) part $\hat{f}^{R(A)}$ of the retarded and advanced Green's functions in the n wire $\hat{g}^{R(A)}=g^{R(A)}\hat{\tau}_3+\hat{f}^{R(A)}$ can be assumed to be small

$$|\hat{f}^{R(A)}| \ll 1. \tag{15}$$

The matrix $\hat{f}^{R(A)}$ contains only off-diagonal elements. The diagonal part obtained from the normalization is

$$g^{R(A)}\hat{\tau}_3 \approx \pm \hat{\tau}_3\left\{1 - \frac{1}{2}[\hat{f}^{R(A)}]^2\right\}. \tag{16}$$

Now we can linearize Eq. (12) for the component 11(22), that is, for the retarded (advanced) Green's functions. Then we obtain a simple linear equation

$$\nabla^2\hat{f}^{R(A)} - \kappa_{\varepsilon}^2\hat{f}^{R(A)} = 0, \tag{17}$$

where $\kappa_{\varepsilon}^{R(A)}=\sqrt{\mp 2i\varepsilon/D}$. The boundary conditions [Eq. (13)] for the matrices $\hat{f}^{R(A)}$ acquire the form

$$[\partial_x\hat{f}^{R(A)} + 2\kappa_N\hat{f}^{R(A)}]|_{x=+L_x} = 0, \tag{18}$$

$$[\partial_x\hat{f}^{R(A)} - 2\kappa_N\hat{f}^{R(A)}]|_{x=-L_x} = 0, \tag{19}$$

$$\{\partial_y\hat{f}^{R(A)} - 2\kappa_S[\hat{f}_{S,+\varphi}^{R(A)} \mp g_S^{R(A)} \cdot \hat{f}^{R(A)}]\}|_{y=+L_y} = 0, \tag{20}$$

$$\{\partial_y\hat{f}^{R(A)} + 2\kappa_S[\hat{f}_{S,-\varphi}^{R(A)} \mp g_S^{R(A)} \cdot \hat{f}^{R(A)}]\}|_{y=-L_y} = 0. \tag{21}$$

As follows from Eq. (8) the functions g_S, $\hat{f}_{S,\varphi}$ are

$$g_S^{R(A)} = \varepsilon/\xi_{\varepsilon}^{R(A)}, \tag{22}$$

$$\hat{f}_{S,\varphi}^{R(A)} = (i\hat{\tau}_2\cos\varphi + i\hat{\tau}_1\sin\varphi)\Delta/\xi_{\varepsilon}^{R(A)}. \tag{23}$$

We took into account that $\varphi(t)$ is almost constant in time.

One can show that the solution for $\hat{f}^R$ in the horizontal part of the n wire is

$$\hat{f}^R = i\hat{\tau}_2 f(x), \tag{24}$$

$$f(x) = C\cosh(\theta_x x/L_x) + \mathrm{sgn}(x)S\sinh(\theta_x x/L_x), \tag{25}$$

where $\mathrm{sgn}(x)$ is the sign function. Dropping the index R of the quantities $\kappa_{\varepsilon}, g_S(\varepsilon), \xi_{\varepsilon}$ the integration constants C and S can be written as

$$C(\varepsilon,\varphi) = \frac{(\theta_x\cosh\theta_x + r_N\sinh\theta_x)\cdot r_S\Delta\cos\varphi/\xi_{\varepsilon}}{\mathcal{D}(\varepsilon)}, \tag{26a}$$

$$S(\varepsilon,\varphi) = -\frac{(r_N\cosh\theta_x + \theta_x\sinh\theta_x)\cdot r_S\Delta\cos\varphi/\xi_{\varepsilon}}{\mathcal{D}(\varepsilon)}, \tag{26b}$$

where $\mathcal{D}(\varepsilon)=[r_S g_S(\varepsilon)\theta_x + r_N\theta_y]\cosh(\theta_x+\theta_y)+[r_S g_S(\varepsilon) r_N + \theta_x\theta_y]\sinh(\theta_x+\theta_y)$, $r_{N,S}=2\kappa_{N,S}L_{x,y}$, and $\theta_{x,y}=\kappa_{\varepsilon}L_{x,y}$.

Knowing the function $\hat{f}^R(x)$, one can find the correction to the conductance of the n wire due to the PE. In order to obtain the current, we take the (12) component (the Keldysh

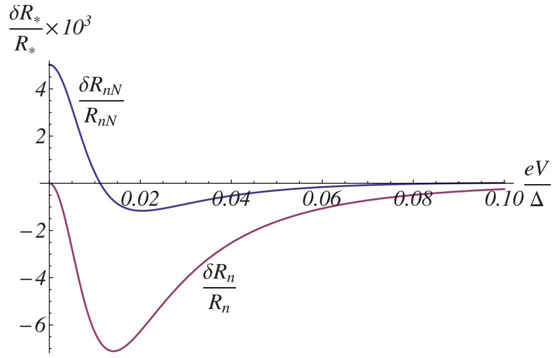

FIG. 2. (Color online) Bias voltage dependence of the normalized variations in the resistance contributions $\delta R_{nN}/R_{nN}$ and $\delta R_n/R_n$. Parameter values: $\varphi=\pi/3$, $L_y/L_x=1$, $\varepsilon_N/\Delta=2.5\times10^{-2}$, $\varepsilon_S/\Delta=5\times10^{-3}$, and $R_n/R_{nN}=1$.

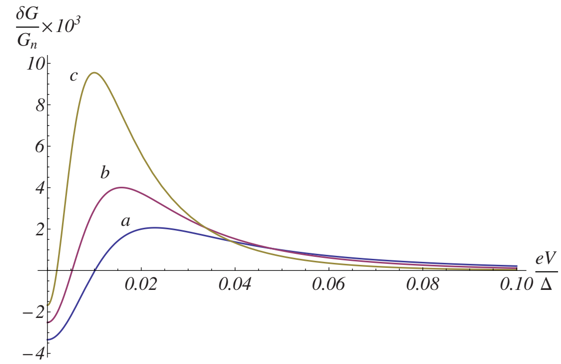

FIG. 3. (Color online) Bias voltage dependence of the normalized conductance variation $\delta G/G_n$. Parameter values: $\varphi=\pi/3$, $L_y/L_x=1$, $\varepsilon_N/\Delta=2.5\times10^{-2}$, and $\varepsilon_S/\Delta=5\times10^{-3}$. Different cases: (a) $R_n/R_{nN}=0.5$, (b) $R_n/R_{nN}=1$, and (c) $R_n/R_{nN}=2$.

component) of Eq. (12), multiply this component by $\hat{\tau}_3$ and take the trace. In the Fourier representation we get [compare with Eq. (2) of Ref. 44]

$$M_-(\varepsilon,\varphi,x)\partial_x F_-(x) = c(\varepsilon,\varphi), \tag{27}$$

where the function $M_-(\varepsilon,\varphi,x)=1+\frac{1}{4}[f(x)+f^*(x)]^2$ determines the correction to the conductivity caused by the PE and $c(\varepsilon,\varphi)$ is an integration constant that is related to the current

$$j = \frac{\sigma}{2e}\int_{-\infty}^{\infty} d\varepsilon c(\varepsilon). \tag{28}$$

It is determined from the boundary condition that can be obtained from Eq. (13)

$$M_-(\varepsilon,\varphi,x)\partial_x F_-(x) = c(\varepsilon,\varphi) = \nu[F_{N-} - F_-(L_x)], \tag{29}$$

where $\nu(\varepsilon,\varphi)=\Re\{1+\frac{1}{2}f(L_x)^2\}$ is the density of states in the n wire near the nN interface. Finding $F_-(L_x)$ and $c(\varepsilon)$ from Eq. (29), we obtain for the current density [compare with Eq. (13) of Ref. 11]

$$j(\varphi) = \frac{1}{2e}\int_{-\infty}^{\infty} d\varepsilon \frac{F_{N-}}{R_{nN}/\nu + R_n\langle M(\varepsilon,\varphi)^{-1}\rangle}. \tag{30}$$

Here F_{N-} is defined according to Eq. (11), $R_n=L_x/\sigma$ is the resistance of the n wire of the length L_x in the normal state, and $\langle M(\varepsilon,\varphi)^{-1}\rangle=(1/L_x)\int_0^{L_x}dx M_-(\varepsilon,\varphi,x)^{-1}$. The first term in the denominator is the nN interface resistance and the second term is the resistance of the $(0,L_x)$ section of the n wire modified by the PE. The expressions for the DOS $\nu(\varepsilon,\varphi)$ and the function $\langle M(\varepsilon,\varphi)^{-1}\rangle$ are given in the Appendix.

For the differential conductance $G=dj/dV$ at zero temperature we obtain

$$G[V,\varphi(t)] = [R_{nN}/\nu + R_n\langle M(eV,\varphi)^{-1}\rangle]^{-1}. \tag{31}$$

In Fig. 2 we show the dependence of the nN interface resistance variation $\delta R_{nN}=R_{nN}/\nu-R_{nN}$ and the resistance variation in the n wire $\delta R_n=R_n\langle M(eV,\varphi)^{-1}\rangle-R_n$ on the bias voltage V for a fixed phase difference. It can be seen that the δR_{nN} is either positive or negative depending on V while δR_n is always negative, i.e., the PE leads to voltage-dependent changes in the interface resistance (caused by the changes in the DOS in the n wire) and to a decrease in the resistance of the n wire.

The conductance variation $\delta G=G(V,\varphi)-G_n$, is shown in Fig. 3 for various values of R_{nN}/R_N, where $G_n=\{R_{nN}+R_N\}^{-1}$ is the conductance of the n wire in the normal state. These results have been obtained earlier.[11,14,45,46]

We are interested in the dc conductance variation averaged in time: $\delta G_{av}=(\Omega/2\pi)\int_0^{2\pi/\Omega}dt\,\delta G[V,\varphi(t)]$. First, from Eqs. (25) and (26) we can extract the dependence of the function f on the phase φ: $f(x,\varphi)=f(x,0)\cos\varphi$. Hence we obtain $M_-(\varepsilon,\varphi,x)=1+\delta M_-(\varepsilon,0,x)\cos^2\varphi$, where $\delta M_-(\varepsilon,\varphi,x)=M_-(\varepsilon,\varphi,x)-1$. At the same time, $\nu(\varepsilon,\varphi)=1+\delta\nu(\varepsilon,0)\cos^2\varphi$ with $\delta\nu(\varepsilon,\varphi)=\nu(\varepsilon,\varphi)-1$. These observations lead to the relation

$$\delta G[V,\varphi(t)] = \delta G(V,0)\cos^2\varphi(t), \tag{32}$$

which by averaging over time yields

$$\delta G_{av} = \delta G(V,0)\cdot\frac{1}{2}[1+J_0(2\varphi_\Omega)\cos(2\varphi_0)], \tag{33}$$

where J_0 is the Bessel function of the first kind and zeroth order. This oscillatory behavior of the time-averaged (dc) conductance variation δG_{av} as a function of the ac amplitude can be seen in Fig. 4.

Thus, the calculations carried out in this section under assumption of adiabatic phase variations allow us to obtain the dependence of the conductance change δG_{av} on the amplitude φ_Ω but provide no information about the frequency dependence of δG_{av}. This dependence will be found in the next section.

IV. STRONG PE; SHORT NORMAL WIRE

In this section we analyze the limiting case of a short n wire when the Thouless energy $E_{Th}=D/L_x^2$ is much larger than characteristic energies: $E_{Th}\gg D\kappa_{N,S}^2,T,eV$. In this case all the functions in Eq. (12) are almost constant in space and we can integrate this equation from $\{x,y\}=\pm0$ to $\{x,y\}=\pm L_{x,y}$ over x and y coordinates. The term $\hat{\tau}_3\cdot\partial_t\check{g}+\partial_{t'}\check{g}\cdot\hat{\tau}_3$ (in the Fourier representation $-i\varepsilon\hat{\tau}_3\cdot\check{g}+i\varepsilon'\check{g}\cdot\hat{\tau}_3$) is consid-

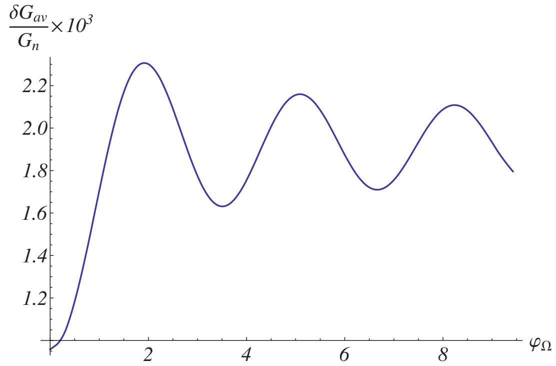

FIG. 4. (Color online) Bias voltage dependence of the normalized time-averaged conductance variation $\delta G_{av}/G_n$. Parameter values: $\varphi_0=\pi/3$, $L_y/L_x=1$, $\varepsilon_N/\Delta=2.5\times10^{-2}$, $\varepsilon_S/\Delta=5\times10^{-3}$, $eV/\Delta=5\times10^{-2}$, and $R_n/R_{nN}=1$.

ered as a constant and the term with the voltage V is omitted because we neglect the voltage drop over the n wire; the voltage drops across the nN,nS interfaces and is set to be zero in the n wire. Performing this integration and summing up the results, we obtain

$$2(L_x+L_y)\check{A}=\check{J}_x(L_x)-\check{J}_x(-L_x)-\check{J}_x(+0)+\check{J}_x(-0)+\check{J}_y(L_y)-\check{J}_y(-L_y)-\check{J}_y(+0)+\check{J}_y(-0), \tag{34}$$

where $\check{J}_x(x)=D\check{g}\circ\partial_x\check{g}|_{y=0}$, $\check{J}_y(y)=D\check{g}\circ\partial_y\check{g}|_{x=0}$, and $\check{A}=-i(\varepsilon\check{\tau}_3\cdot\check{g}-\check{g}\cdot\check{\tau}_3\varepsilon')$. Integration around the point $(x,y)=(0,0)$ yields the conservation of the "currents" (using terminology of the circuit theory[9])

$$\check{J}_x(+0)+\check{J}_y(+0)=\check{J}_x(-0)+\check{J}_y(-0). \tag{35}$$

Combining Eqs. (34) and (35) and the boundary conditions [Eq. (13)], we arrive at the equation

$$\varepsilon\check{\tau}_3\cdot\check{g}-\check{g}\cdot\check{\tau}_3\varepsilon'=i\varepsilon_N[\check{g},\check{g}_{N+}]_\circ+i\varepsilon_S[\check{g},\check{g}_{S+}]_\circ. \tag{36}$$

Here $\varepsilon_{N,S}=D/(2R_{nN,nS}\sigma L)$ is a characteristic energy related to the interface transparencies, $L=L_x+L_y$. The energy ε_N determines the damping in the spectrum of the n wire and the energy ε_S is related to a subgap induced in the n wire due to the PE. The matrices $\check{g}_{N,S+}$ are equal to $\check{g}_{N,S+}=\frac{1}{2}[\check{g}_{N,S}(L_{x,y})+\check{g}_{N,S}(-L_{x,y})]$.

In the limit of the short n wire considered in this section, we need to find only the retarded (advanced) Green's functions. Indeed, let us rewrite the expression for the current [Eq. (14)] using the boundary condition [Eq. (13)] at the right nN interface and concentrating on the dc component of the current,

$$j=\frac{1}{16eR_{nN}}\mathrm{Tr}\left\{\hat{\tau}_3\cdot\int_{-\infty}^{\infty}d\varepsilon(\hat{g}^R\cdot\hat{g}_N^K+\hat{g}^K\cdot\hat{g}_N^A-\hat{g}_N^R\cdot\hat{g}^K-\hat{g}_N^K\cdot\hat{g}^A)\right\}, \tag{37}$$

where $\hat{g}_N^{R(A)}=\pm\hat{\tau}_3$ and $\mathrm{Tr}\{\hat{\tau}_3\cdot(\hat{g}^R\cdot\hat{g}_N^K)\}=4g^RF_{N-}$. The distribution function F_{N-} in the N reservoir is defined in Eq. (11). The integral over energies from the second and third terms is zero because it is proportional to the voltage in the n wire which is set to be zero. Therefore the current can be written as

$$j=\frac{1}{2eR_{nN}}\int_{-\infty}^{\infty}d\varepsilon\,\nu(\varepsilon)F_{N-}(\varepsilon), \tag{38}$$

where $\nu(\varepsilon)=\frac{1}{2}(g^R-g^A)=\Re\{g^R(\varepsilon)\}$. This formula has an obvious physical meaning—the current through the nN interface is determined by the product of the DOS in the n wire and N reservoir ($\nu_N=1$) and the distribution function in the N reservoir (the distribution function F_- in the n wire is zero).

Using Eqs. (2), (11), and (38) we arrive at the following expression for the differential conductance:

$$G=\frac{1}{2R_{nN}}\int_{-\infty}^{\infty}\frac{d\varepsilon}{4T}\nu(\varepsilon)\left[\frac{1}{\cosh^2\frac{\varepsilon+eV}{2T}}+\frac{1}{\cosh^2\frac{\varepsilon-eV}{2T}}\right]. \tag{39}$$

In order to find the matrix $\hat{g}^R$, we can write the (11) component of Eq. (36) in the form

$$\tilde{\varepsilon}\hat{\tau}_3\cdot\hat{g}^R-\hat{g}^R\cdot\hat{\tau}_3\tilde{\varepsilon}'=i\varepsilon_S[\hat{g}^R,\hat{g}_{S+}^R]_\circ, \tag{40}$$

where $\tilde{\varepsilon}=\varepsilon+i\varepsilon_N$ and $\tilde{\varepsilon}'=\varepsilon'+i\varepsilon_N$.

According to Eqs. (1) and (8) the matrix $\hat{g}_{S+}^R$ is a function of two times, $\hat{g}_{S+}^R(t,t')$, that is, in the Fourier representation it is function of two energies: ε,ε'. Therefore, to find the matrix $\hat{g}^R(\varepsilon,\varepsilon')$ in a general case is a formidable task.

However, we can assume that the amplitude of the ac component of the phase φ_Ω is small and obtain the solution making an expansion in powers of φ_Ω,

$$\hat{g}^R=\hat{g}_0^R+\delta_1\hat{g}^R+\delta_2\hat{g}^R+\cdots. \tag{41}$$

Here and later all matrix Green's functions written without arguments are functions of two energies $(\varepsilon,\varepsilon')$. Those of them which are diagonal in energy may be also (explicitly) written with a single energy argument, e.g., $\hat{g}_{S0+}^R=\hat{g}_{S0+}^R(\varepsilon)2\pi\delta(\varepsilon-\varepsilon')$.

Similar to Eq. (41) we represent the matrix $\hat{g}_{S+}^R$ (up to the second order in φ_Ω) as $\hat{g}_{S+}^R=\hat{g}_{S0+}^R+\delta_1\hat{g}_{S+}^R+\delta_2\hat{g}_{S+}^R$ and find from Eq. (8) for the stationary part $\hat{g}_{S0+}^R$ and the corrections $\delta_1\hat{g}_{S+}^R$ (first order in φ_Ω) and $\delta_2\hat{g}_{S+}^R$ (second order in φ_Ω),

$$\hat{g}_{S0+}^R=2\pi\delta_0(\varepsilon\hat{\tau}_3+i\Delta\cos\varphi_0\hat{\tau}_2)\xi_\varepsilon^{-1}, \tag{42}$$

$$\delta_1\hat{g}_{S+}^R=-i\hat{\tau}_2\frac{\pi}{2}\varphi_\Omega\Delta\sin\varphi_0(\xi_\varepsilon^{-1}+\xi_{\varepsilon'}^{-1})(\delta_\Omega+\delta_{-\Omega}), \tag{43}$$

$$\delta_2\hat{g}_{S+}^R=-i\hat{\tau}_2\frac{\pi}{8}\varphi_\Omega^2\Delta\cos\varphi_0(P_0+P_2), \tag{44}$$

where we used the notation $\delta_\omega\equiv\delta(\varepsilon-\varepsilon'+\omega)$, $\xi_\varepsilon\equiv\xi_\varepsilon^R$ and defined the functions

$$P_0=\delta_0(2\xi_{\varepsilon'}^{-1}+\xi_{\varepsilon'+\Omega}^{-1}+\xi_{\varepsilon'-\Omega}^{-1}),$$

$$P_2=\frac{1}{2}(\delta_{2\Omega}+\delta_{-2\Omega})(\xi_{\varepsilon'}^{-1}+\xi_\varepsilon^{-1}+2\xi_{1/2(\varepsilon+\varepsilon')}^{-1}). \tag{45}$$

Using the expressions for $\delta_1\hat{g}^R_{S+}$ and $\delta_2\hat{g}^R_{S+}$ given above we can calculate the corrections to $\hat{g}^R_0$ up to the second order in φ_Ω and the corresponding modification of the DOS $\delta\nu$ in the n wire.

In the zeroth-order approximation, i.e., for $\varphi_\Omega=0$ we obtain from Eq. (40) $\hat{g}^R_0(\varepsilon,\varepsilon')=\hat{g}^R_0(\varepsilon)2\pi\delta(\varepsilon-\varepsilon')$, where the matrix $\hat{g}^R_0(\varepsilon)$ obeys the equation

$$[\hat{\tau}_3E^R_\varepsilon+i\hat{\tau}_2E^R_{sg},\hat{g}^R_0]=0, \tag{46}$$

containing $E^R_\varepsilon=\tilde{\varepsilon}+i\varepsilon_S g^R_{S0}(\varepsilon)=\varepsilon+i\varepsilon_N+i\varepsilon_S g^R_{S0}(\varepsilon)$, $E^R_{sg}=i\varepsilon_S\cos\varphi_0 f^R_{S0}(\varepsilon)$, $g^R_{S0}(\varepsilon)=\varepsilon/\xi^R_\varepsilon$, and $f^R_{S0}(\varepsilon)=\Delta/\xi^R_\varepsilon$. The solution of this equation is[11]

$$\hat{g}^R_0(\varepsilon)=\hat{\tau}_3g^R_0(\varepsilon)+i\hat{\tau}_2f^R_0(\varepsilon),$$

$$g^R_0(\varepsilon)=E^R_\varepsilon/\zeta^R_\varepsilon,\quad f^R_0(\varepsilon)=E^R_{sg,\varphi}/\zeta^R_\varepsilon, \tag{47}$$

where $\zeta^R_\varepsilon=\sqrt{(E^R_\varepsilon)^2-(E^R_{sg})^2}$. The quantity E^R_{sg} determines a subgap induced in the n wire due to the PE. Indeed, consider the most interesting case of small energies assuming that $\{\varepsilon,\varepsilon_S\}\ll\Delta$; then, $\xi^R_\varepsilon\approx i\Delta$, $f^R_{S0}(\varepsilon)\approx-i$, and $\zeta^R_\varepsilon\approx\sqrt{(\varepsilon+i\varepsilon_N)^2-(\varepsilon_S\cos\varphi_0)^2}$. This means that the spectrum of the n wire has the same form as in the BCS superconductor with a damping ε_N and a subgap $\varepsilon_S|\cos\varphi_0|$, which depends on the nS interface transparency and phase difference.

Note that the formula for the subgap induced in the N metal due to the PE in a tunnel superconductor-insulator-normal metal (SIN) junction was obtained by McMillan.[50] The obtained results for the functions $g^R_0(\varepsilon)$ and $f^R_0(\varepsilon)$ can be easily generalized for the case of asymmetric nS interfaces with different interface resistances $R_{nS1,2}$ (correspondingly, $\varepsilon_{S1,2}$). In the limit $\varepsilon_{S1,2}\ll\Delta$, we obtain for the subgap ε_{sg},

$$\varepsilon_{sg}(\varphi_0)=\frac{1}{2}\sqrt{\varepsilon^2_{S1}+\varepsilon^2_{S2}+2\varepsilon_{S1}\varepsilon_{S2}\cos 2\varphi_0}. \tag{48}$$

This formula shows that the subgap as a function of the phase difference φ varies from $\frac{1}{2}|\varepsilon_{S2}-\varepsilon_{S1}|$ for $\varphi_0=\pi/2$ to $\frac{1}{2}(\varepsilon_{S2}+\varepsilon_{S1})$ for $\varphi_0=0$.

We proceed finding the corrections of the first $(\delta_1\hat{g}^R)$ and second $(\delta_2\hat{g}^R)$ order in φ_Ω for $\hat{g}^R_0$ in a way similar to the one used in Refs. 47 and 51. The correction of the first order $\delta_1\hat{g}$ (for brevity, we drop the index R) obeys the equation

$$\zeta_\varepsilon\hat{g}_0(\varepsilon)\cdot\delta_1\hat{g}-\delta_1\hat{g}\cdot\hat{g}_0(\varepsilon')\zeta_{\varepsilon'}=i\varepsilon_S[\hat{g}_0,\delta_1\hat{g}_{S+}]_\circ, \tag{49}$$

which contains all terms of the first order in φ_Ω from Eq. (40). Note that we are making use of the relation $\hat{g}_0(\varepsilon)=\zeta^{-1}_\varepsilon[\tilde{\varepsilon}\hat{\tau}_3+i\varepsilon_S\hat{g}_{S0+}(\varepsilon)]$ evident from Eqs. (40), (46), and (47).

In order to solve Eq. (49), it is useful to employ the normalization condition [Eq. (4)] for $\hat{g}\equiv\hat{g}^R$ which for the first-order term of $\hat{g}\circ\hat{g}$ yields

$$\hat{g}_0(\varepsilon)\cdot\delta_1\hat{g}+\delta_1\hat{g}\cdot\hat{g}_0(\varepsilon')=0. \tag{50}$$

From Eqs. (49) and (50), we find

$$\delta_1\hat{g}=i\varepsilon_S\frac{\delta_1\check{g}_{S+}-\hat{g}_0(\varepsilon)\cdot\delta_1\hat{g}_{S+}\cdot\hat{g}_0(\varepsilon')}{\zeta_\varepsilon+\zeta_{\varepsilon'}}. \tag{51}$$

We determine the correction $\delta_2\hat{g}$ in the same manner. Reading off the second-order terms in Eq. (40) gives

$$[\zeta_\varepsilon\hat{g}_0,\delta_2\hat{g}]_\circ=i\varepsilon_S([\hat{g}_0,\delta_2\hat{g}_{S+}]_\circ+[\delta_1\hat{g},\delta_1\hat{g}_{S+}]_\circ). \tag{52}$$

The second-order part of the normalization condition is

$$\hat{g}_0(\varepsilon)\cdot\delta_2\hat{g}+\delta_2\hat{g}\cdot\hat{g}_0(\varepsilon')=-\delta_1\hat{g}\circ\delta_1\hat{g}. \tag{53}$$

Thus, we obtain the second-order correction

$$\delta_2\hat{g}=i\varepsilon_S\frac{\delta_2\check{g}_{S+}-\hat{g}_0(\varepsilon)\cdot\delta_2\hat{g}_{S+}\cdot\hat{g}_0(\varepsilon')}{\zeta_\varepsilon+\zeta_{\varepsilon'}}+\left\{i\varepsilon_S\frac{[\delta_1\hat{g}_{S+},\delta_1\hat{g}]_\circ}{\zeta_\varepsilon+\zeta_{\varepsilon'}}-\frac{\zeta_\varepsilon(\delta_1\hat{g}\circ\delta_1\hat{g})}{\zeta_\varepsilon+\zeta_{\varepsilon'}}\right\}\cdot\hat{g}_0(\varepsilon'). \tag{54}$$

In order to calculate the correction to the dc conductance caused by the ac radiation, δG, we need to find $\mathrm{Tr}\{\hat{\tau}_3\cdot\delta_1\hat{g}\}$ and $\mathrm{Tr}\{\hat{\tau}_3\cdot\delta_2\hat{g}\}$ and take their parts proportional to $2\pi\delta(\varepsilon-\varepsilon')$. By inspection of Eqs. (43) and (51) one recognizes that the first-order correction contains only terms proportional to $\delta(\varepsilon-\varepsilon'\pm\Omega)$ and therefore only contributes to the ac current. This is the fundamental reason why the second-order analysis is needed to determine the variation in the dc conductance.

As a result we just have to find the multiple of $2\pi\delta(\varepsilon-\varepsilon')$ contained in $\mathrm{Tr}\{\hat{\tau}_3\cdot\delta_2\hat{g}\}$ which we denote as $2\delta_{dc}g(\varepsilon)$, that is, $\delta_{dc}g(\varepsilon)2\pi\delta(\varepsilon-\varepsilon'):=\frac{1}{2}\mathrm{Tr}\{\hat{\tau}_3\cdot\delta_2\hat{g}\}_{dc}$. We represent the function $\delta_{dc}g(\varepsilon)$ as a sum

$$\delta_{dc}g(\varepsilon)=\delta^{(0)}_{dc}g(\varepsilon)+\delta^{(\Omega)}_{dc}g(\varepsilon), \tag{55}$$

where the function $\delta^{(0)}_{dc}g(\varepsilon)$ originates from the first term in Eq. (54) and the function $\delta^{(\Omega)}_{dc}g(\varepsilon)$ from the second and third terms. If we consider the case when the subgap $\varepsilon_S|\cos\varphi_0|$ is much less than Δ, i.e.,

$$\varepsilon_S|\cos\varphi_0|\ll\Delta \tag{56}$$

then, at low energies $\varepsilon\lesssim\varepsilon_S$, the function $\delta^{(0)}_{dc}g(\varepsilon)$ is almost independent of Ω whereas the function $\delta^{(\Omega)}_{dc}g(\varepsilon)$ depends strongly on Ω at $\varepsilon\approx\varepsilon_S|\cos\varphi_0|$. Assuming the validity of Eq. (56) we obtain

$$\delta^{(0)}_{dc}g(\varepsilon)=-\frac{1}{4}\varepsilon^2_S\varphi^2_\Omega\cos^2\varphi_0\frac{g_0(\varepsilon)}{\zeta^2_\varepsilon}, \tag{57}$$

$$\delta^{(\Omega)}_{dc}g(\varepsilon,\Omega)=\frac{1}{4}\varepsilon^2_S\varphi^2_\Omega\sin^2\varphi_0\sum_{\pm\Omega}\times\frac{g_0(\varepsilon)[1+f_0(\varepsilon)f_0(\varepsilon+\Omega)+g_0(\varepsilon)g_0(\varepsilon+\Omega)]}{[\zeta_\varepsilon+\zeta_{\varepsilon+\Omega}]^2}+\frac{f_0(\varepsilon)[g_0(\varepsilon)f_0(\varepsilon+\Omega)+f_0(\varepsilon)g_0(\varepsilon+\Omega)]}{\zeta_\varepsilon[\zeta_\varepsilon+\zeta_{\varepsilon+\Omega}]}, \tag{58}$$

where the functions $g_0(\varepsilon)$, $f_0(\varepsilon)$, and ζ_ε are defined in Eq. (47). The sum sign index "$\pm\Omega$" in Eq. (58) means that the given expression is added to the same one with the negative frequency $(-\Omega)$.

Using the function $\delta_{dc}g(\varepsilon,\Omega)$ we can calculate a correction to the DOS $\delta\nu(\varepsilon,\Omega)$ due to the PE and with the aid of

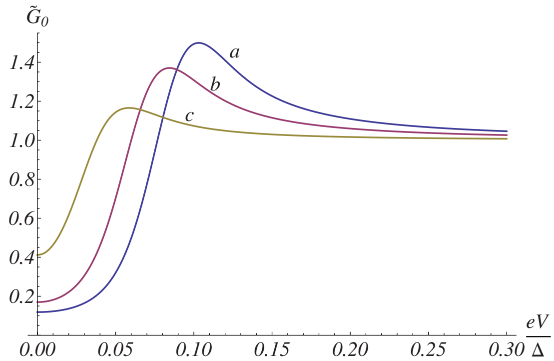

FIG. 5. (Color online) Normalized stationary differential conductance $\tilde{G}_0$ versus bias voltage V. Parameter values: $T/\Delta=10^{-2}$, $\varepsilon_S/\Delta=0.1$, and $\varepsilon_N/\Delta=10^{-2}$. Different cases: (a) $\varphi_0=\pi/8$, (b) $\varphi_0=\pi/4$, and (c) $\varphi_0=3\pi/8$.

Eq. (39) find the correction $\delta G(V,\Omega)$ to the differential dc conductance. As follows from Eq. (39), at zero temperature the normalized differential dc conductance $\tilde{G}(V,\Omega)=G(V,\Omega)R_{nN}$ is equal to

$$\tilde{G}(V,\Omega)\equiv\tilde{G}_0(V)+\delta\tilde{G}(V,\Omega)=\nu_0(eV)+\delta\nu(eV,\Omega) \tag{59}$$

with the definitions $\nu_0(eV)=\Re[g_0(eV)]$ and $\delta\nu(eV,\Omega)=\Re[\delta_{dc}g(eV,\Omega)]$.

Using the obtained formula for $g_0(\varepsilon)$ and $\delta_{dc}g(\varepsilon)$ we can calculate the conductance G_0 and its correction δG due to microwave radiation for different values of parameters (damping ε_N, phase difference $2\varphi_0$, etc.). The dependence of the conductance in the absence of radiation G_0 versus the applied voltage V is presented in Fig. 5. We see that this dependence follows the energy dependence of a SIN junction. In our case the n wire with an induced subgap plays a role of "S" with a damping ε_N in the "superconductor." Since the value of the induced subgap, $\varepsilon_{sg}=\varepsilon_S|\cos\varphi_0|$, depends on the phase difference $2\varphi_0$, the position of the peak in the dependence $G_0(V)$ is shifted downward with increasing φ_0.

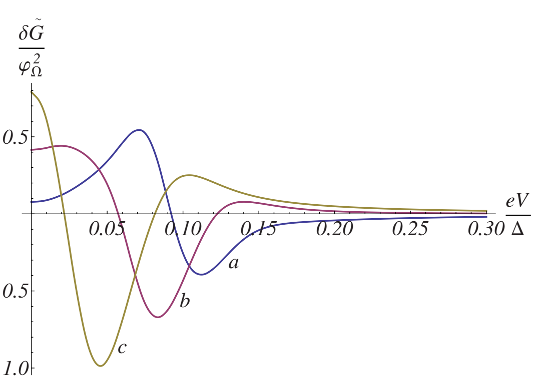

FIG. 6. (Color online) Normalized second-order correction of differential conductance $\delta\tilde{G}$ versus bias voltage V. Parameter values: $T/\Delta=10^{-2}$, $\varepsilon_S/\Delta=0.1$, $\varepsilon_N/\Delta=10^{-2}$, and $\Omega/\Delta=5\times10^{-2}$. Different cases: (a) $\varphi_0=\pi/8$, (b) $\varphi_0=\pi/4$, and (c) $\varphi_0=3\pi/8$.

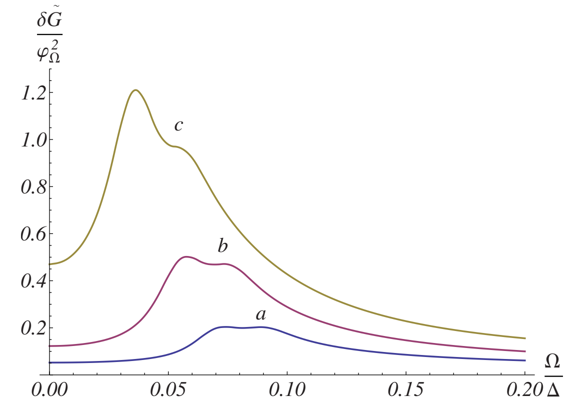

FIG. 7. (Color online) Normalized second-order correction of differential conductance $\delta\tilde{G}$ versus ac frequency Ω. Parameter values: $T/\Delta=2\times10^{-3}$, $\varepsilon_S/\Delta=0.1$, $\varepsilon_N/\Delta=10^{-2}$, and $eV/\Delta=10^{-2}$. Different cases: (a) $\varphi_0=\pi/6$, (b) $\varphi_0=\pi/4$, and (c) $\varphi_0=\pi/3$.

Note that in an asymmetrical system ($\varepsilon_{S1}\neq\varepsilon_{S2}$) the lowest value of the subgap is not zero [cf. Eq. (48)].

In Fig. 6 we show the bias voltage dependence of the conductance correction due to ac radiation $\delta_{dc}G$ (coefficient in front of φ_Ω^2) for different values of φ_0. The magnitude and the position of the arising peaks depend strongly on the values of the parameters, e.g., φ_0.

By varying the stationary phase difference φ_0 or the damping ε_N one can change the frequency dependence of the correction $\delta_{dc}G$ considerably. This is shown in Figs. 7 and 8, respectively. One can see that if $\varepsilon_N\ll\varepsilon_{sg}(\varphi_0)$, then the dependence $\delta_{dc}G(\Omega)$ has a peak located at $\approx\varepsilon_{sg}(\varphi_0)$ and split into two subpeaks. The splitting becomes more and more distinct with increasing bias voltage V. With decreasing φ_0 and increasing ε_N, the form of this dependence changes significantly. For example, the resonance curve becomes broader with increasing damping. Increasing temperature leads to a similar effect as one can see in Fig. 9.

In Fig. 10 we plot the normalized conductance correction $\delta_{dc}\tilde{G}(\varphi_0)$ as a function of φ_0 for different values of the bias voltage V. At large V this dependence is close to sinusoidal

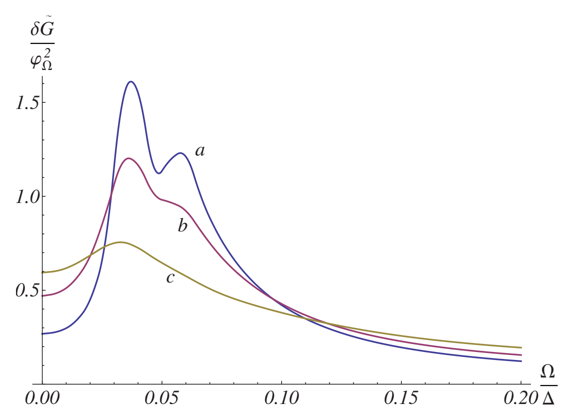

FIG. 8. (Color online) Normalized second-order correction of differential conductance $\delta\tilde{G}$ versus ac frequency Ω. Parameter values: $T/\Delta=2\times10^{-3}$, $\varepsilon_S/\Delta=0.1$, $eV/\Delta=10^{-2}$, and $\varphi_0=\pi/3$. Different cases: (a) $\varepsilon_N/\Delta=5\times10^{-3}$, (b) $\varepsilon_N/\Delta=10^{-2}$, and (c) $\varepsilon_N/\Delta=2\times10^{-2}$.

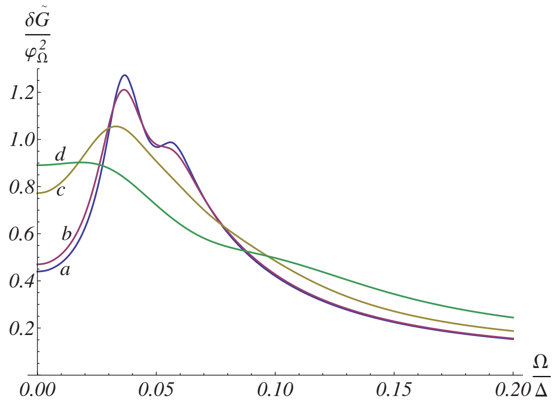

FIG. 9. (Color online) Normalized second-order correction of differential conductance $\delta\tilde{G}$ versus ac frequency Ω, Parameter values: $\varepsilon_S/\Delta=0.1$, $\varepsilon_N/\Delta=10^{-2}$, $eV/\Delta=10^{-2}$, and $\varphi_0=\pi/3$. Different cases: (a) $T=0$, (b) $T/\Delta=2\times10^{-3}$, (c) $T/\Delta=6\times10^{-3}$, and (d) $T/\Delta=10^{-2}$.

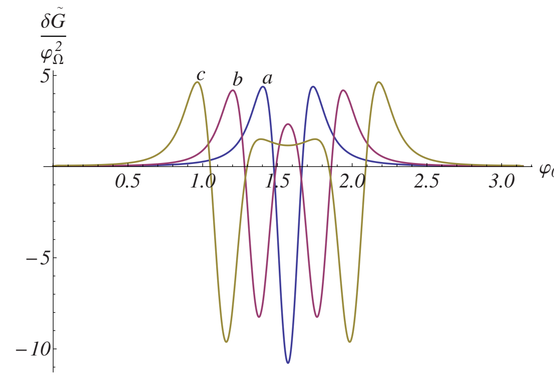

FIG. 10. (Color online) Normalized second-order correction of differential conductance $\delta\tilde{G}$ versus stationary phase difference φ_0. Parameter values: $T/\Delta=2\times10^{-3}$, $\varepsilon_S/\Delta=0.1$, $\varepsilon_N/\Delta=10^{-2}$, and $\Omega/\Delta=10^{-2}$. Different cases: (a) $V=0$, (b) $eV/\Delta=2\times10^{-2}$, and (c) $eV/\Delta=4\times10^{-2}$.

but at smaller voltages the form of the periodic function $\delta_{dc}G(\varphi_0)$ becomes more complicated.

V. CONCLUSION

We have calculated the change in the conductance in an S/N structure of the cross geometry under the influence of microwave radiation. The calculations have been carried out on the basis of quasiclassical Green's functions in the diffusive limit. Two different limiting cases have been considered: (a) a weak proximity effect and low frequency Ω of radiation; and (b) a strong proximity effect and small amplitude of radiation.

In the case (a), the conductance change δG consists of two parts. One is related to a change in the nN interface resistance due to a modification of the DOS of the n wire. At small applied voltages V_N, it is negative. Another part is caused by a modification of the conductance of the n wire due to the PE. This part is positive and consists of two competing contributions. One contribution, which is negative, stems from the a modification of the DOS of the n wire. Another contribution is positive and caused by a term, which is similar to the Maki-Thompson term.[45,46] The conductance change δG oscillates and decays with increasing amplitude of radiation.

In the case (b) a short n wire was considered so that the resistance of the n wire is negligible in comparison with the resistance of the nN interface. The correction δG has been found under assumption of a small amplitude of the radiation. We found that at small applied voltages V, the dependence $\delta G(\Omega)$ has a resonance form. It has a maximum when the frequency Ω is on the order of $\varepsilon_{sg}=\varepsilon_S|\cos\varphi_0|$ where ε_{sg} is a subgap in the spectrum of the n wire induced by the PE. With increasing V, the peak in the dependence $\delta G(\Omega)$ splits into two peaks and overall form of this dependence becomes complicated.

We assumed that the nS interface resistance is larger than the resistance of the n wire, that is, $\rho L \ll R_{nS}$. This inequality can be written in the form $\varepsilon_S \ll \varepsilon_{Th} \equiv D/L^2$, that is, the subgap energy in the n wire is much smaller than the Thouless energy ε_{Th}. In the opposite limit, $\varepsilon_S \gg \varepsilon_{Th}$, a gap on the order of ε_{Th} is induced in the n wire. This limit can be studied numerically. However, one can expect that in this limit the resonance should take place at $\omega_{res}\approx\varepsilon_{Th}/\hbar$. Experiment performed in Ref. 36 corresponds to this limit. The frequency corresponding to the Thouless energy in experiment is equal to $\varepsilon_{Th}/h\approx\frac{1}{2\pi}(400/10^{-8})s^{-1}\approx 6$ GHz whereas the resonance frequency is $\nu_{res}\approx 10$ GHz.

Note that we considered a simplified model. For example, we did not account for the change in the distribution function in the n film (heating effects). One can give estimations when the "heating" can be neglected. The change in an "effective" electron temperature δT in the n wire is approximately given by $\delta T\approx\tau_{e\text{-}ph}\sigma E^2/c_e$, where $\tau_{e\text{-}ph}$ is the electron-phonon inelastic-scattering time, $E\lesssim\delta V_S R_L/R_b L=\hbar\Omega(R_L/eLR_b)\varphi_\Omega$ is the ac electric field in the n wire, and $c_e\approx T\cdot n/\varepsilon_F$ is the heat capacity of electron gas with concentration n. Taking into account that $(R_b\sigma)^{-1}\approx Z^2/l$, we find that $\delta T/T\lesssim(\varepsilon_0/T)^2(\tau_{e\text{-}ph}/\tau)Z^4\varphi_\Omega^2$, where Z^2 is the dimensionless coefficient of electron penetration through the SN interface, which is assumed to be small, $l=v\tau$ is the mean-free path in the n wire. Therefore, the heating would be very small if the condition $\varphi_\Omega\ll(\varepsilon_0/T)\sqrt{(\tau/\tau_{e\text{-}ph})}Z^{-2}$ is fulfilled. The obtained results are useful for understanding the response of the considered and analogous SN systems to microwave radiation which can be used, for example, in Q bits.

ACKNOWLEDGMENTS

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APPENDIX

The DOS in Eqs. (29) and (30) is given by the formula $\nu(\varepsilon,\varphi)=\Re[1+\frac{1}{2}f(L_x)^2]$ with $f(L_x)$ defined in Eq. (25). Making use of the weak proximity-effect approximation we rewrite the function $\langle M(\varepsilon,\varphi)^{-1}\rangle$ in Eq. (30) as

$$\langle M(\varepsilon,\varphi)^{-1}\rangle = 1 - \frac{1}{2}\langle \Re\{f^2 + ff^*\}\rangle. \quad \text{(A1)}$$

Using Eq. (25) one can easily calculate

$$\langle f^2\rangle = \frac{C^2+S^2}{2}\frac{\sinh 2\theta_x}{2\theta_x} + \frac{C^2-S^2}{2} + CS\frac{\sinh^2\theta_x}{\theta_x}, \quad \text{(A2)}$$

$$\langle ff^*\rangle = \frac{|C|^2+|S|^2}{2}\frac{\sinh 2\theta_x'}{2\theta_x'} + \frac{|C|^2-|S|^2}{2}\frac{\sin 2\theta_x''}{2\theta_x''} + \Re\left\{C^*S\left(\frac{\sinh^2\theta_x'}{\theta_x'} + i\frac{\sin^2\theta_x''}{\theta_x''}\right)\right\}, \quad \text{(A3)}$$

where θ_x' and θ_x'' are the real and imaginary parts of θ_x, respectively, i.e., $\theta_x = \theta_x' + i\theta_x''$. We use these expressions for calculating the function $\langle M(\varepsilon,\varphi)^{-1}\rangle$ and conductance G.

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