An experimental investigation of equivalent circuit for large scale WPT system with multiple receivers

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Abstract This paper demonstrates an equivalent circuit model for large scale WPT system with multiple receivers. Many articles have shown the method for determining optimum load impedance at receiver and power distribution in multiple-receiver environment. These analysis indicate requirement of an equivalent circuit when a transmitter side and receiver sides are coupled. This paper newly assumes that transmitter side is large scale so that equivalent circuit have to be combination of concentrated/distributed constant. Proposed model is experimentally investigated under the assumption that single parallel line feeder coupled with multiple receiver with different location.

Keyword Wireless power transmission, Multiple receiver, Equivalent circuit, Distributed constant circuit, Parallel line feeder

1. Introduction

Recently, wireless power transmission (WPT) technology is expected to be used in various kind of devices such as mobile terminal and electric vehicle (EV) [1]–[4]. In near future, it is anticipated that there are many situations where single source feeds electric power to multiple receivers or portable devices. The WPT system with single transmitting coupler with multiple receiving coupler with cross coupling among receiving couplers is typically modeled as single-input multiple-output (SIMO) model. References describing on SIMO-WPT system discussed about determining optimum load impedance which maximize power transmitting efficiency [5], [6] and distributing power to multi-receiver [7], [8]. These papers commonly employ equivalent circuit in order to evaluate system where transmitting and receiving couplers are magnetically coupled [9]. However, when the size of transmitter is enlarged in order to widen feeding area, equivalent circuit is required to be a combination of concentrated and distributed constant circuits because of relation between wavelength of alternative source and the size of system [10].

This paper proposes an equivalent circuit for large-scale transmitter coupled with multiple receivers. Proposal is experimentally demonstrated under the assumption that single parallel line feeder coupled with multiple receivers. Since the input impedance is required to evaluate for achieving maximum transmission efficiency based on impedance matching [11]–[13], measured input impedance is compared with theoretical one when single/multiple receivers are coupled with parallel line feeder in this paper. Experiment results show that value of input impedance change according to the positions of receivers, and it indicates that built circuit model is capable of being applied in actual environment.

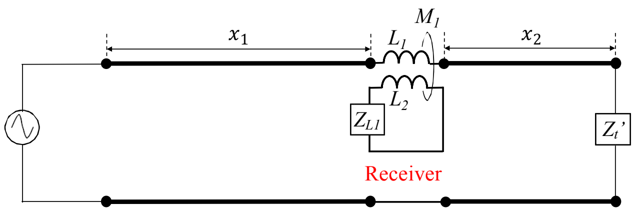


Fig. 1. System model of WPT system

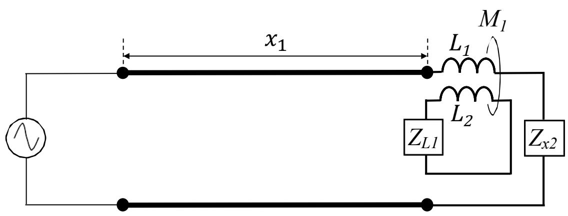


Fig. 2. Equivalent circuit converted from Fig. 1

2. Proposed Method

A large transmitter coupler with terminal impedance of Z_t' magnetically couples with a receiver locating at x_2 from end. In this model, Z_{L1} is load impedance of a receiver, L_1 and L_2, are self inductance of coils, and M_1 is mutual inductance between parallel line and receiver, respectively. In the proposed method, the areas, on a transmitter, where a receiver lies is analyzed as concentrated constant circuit, and the others are as distributed constant circuit.

Fig. 2 shows the equivalent circuit transformed from Fig. 1. Terminal impedance, Z_t', and length between terminal and receiver location can be converted to impedance by

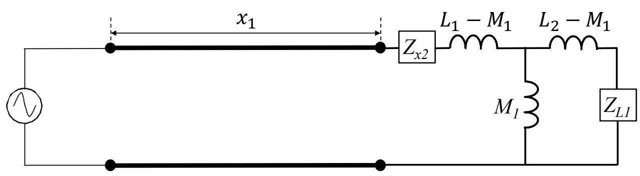


Fig. 3. Equivalent circuit converted from Fig. 2

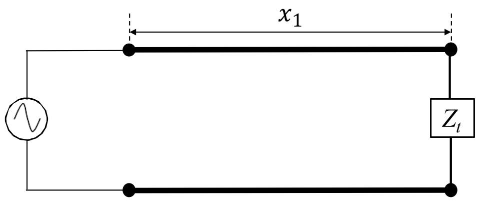


Fig. 4. Equivalent circuit converted from Fig. 3

means of transmission-line-equation as follows,

$$Z_{x_2} = Z_0 \cdot \frac{Z'_t + Z_0 \tanh(\gamma x_2)}{Z_0 + Z'_t \tanh(\gamma x_2)}. \quad (1)$$

where Z_0 and γ are characteristic impedance of parallel line and propagation constant, respectively. Fig. 3 shows equivalent circuit transformed from Fig. 2 and Fig. 4 does from Fig. 3. Conversion from Fig. 2 to Fig. 3 imply to replace coupling between coils with T-type equivalent circuit, and we merge circuit elements into one impedance in conversion between Fig. 3 and Fig. 4. Z_t is represented as

$$Z_t = Z_{x_2} + j\omega(L_1 - M_1) + \frac{j\omega M_1(Z_{L1} + j\omega(L_2 - M_1))}{Z_{L1} + j\omega L_2}. \quad (2)$$

Since transmission-line-equation can be applied to the circuit in Fig. 4, eventually, Z_{in}, can be represented as in Eqn. (3).

Consider the case of being coupled with two receivers. Fig. 5 shows the model of the system. Double receivers are coupled with parallel line feeder. The cross coupling between receivers is assumed negligible small compared to mutual inductance between parallel line and receiver. The order of circuit conversion is the same as in the single receiver case, and input impedance can be derived by transforming from terminal side. Z''_t, converted from length of x_3 in parallel line and terminal impedance, is expressed as follows,

$$Z_{x_3} = Z_0 \cdot \frac{Z''_t + Z_0 \tanh(\gamma x_3)}{Z_0 + Z''_t \tanh(\gamma x_3)}. \quad (4)$$

Coupling part of 2nd receiver can be converted into T-type circuit, and put into one impedance Z'_t represented as

$$Z'_t = Z_{x_3} + j\omega(L_3 - M_2) + \frac{j\omega M_2(Z_{L_2} + j\omega(L_4 - M_2))}{Z_{L_2} + j\omega L_4}. \quad (5)$$

Referring to Fig. 6, structure of circuit is equal to one in Fig. 1, so Eqn. (1)–(3) can be applied to this conversion too. Therefore, Z_{in} can be calculated by using Eqn. (4), (5) and (1)–(3) in order.

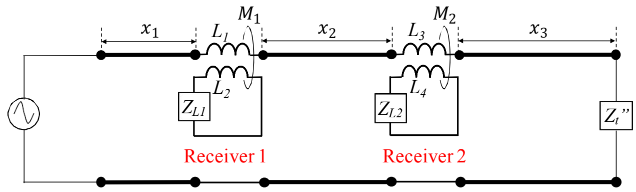


Fig. 5. System model of WPT with double receivers

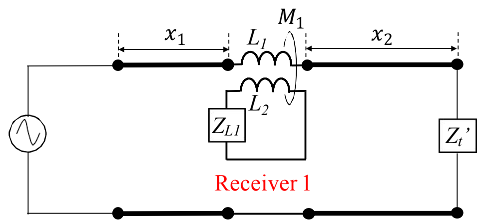


Fig. 6. Equivalent circuit converted from Fig. 5

3. Experiment Results

A. Setup and parameters

The experiment equipment,vector network analyzer, receiver, and transmitter line is shown in Fig. 7. The transmitter and receivers are made of copper tape and attached SMA connector. The frequency is set to 20 MHz. Self inductance of primary coil, L_1 and L_3, are measured value of the coil made the same size as the area lied a receiver. Self inductance of secondary coil, L_2 and L_4 and receiver impedance, Z_{L1} and Z_{L2}, are the value measured by a vector network analyzer. The coupling coefficient k_1 and mutual inductance M_1 are derived from the following equation,

$$k_1 = \sqrt{1 - \frac{L_{2\text{short}}}{L_2}} \quad (6)$$

$$M_1 = k\sqrt{L_1 L_2} \quad (7)$$

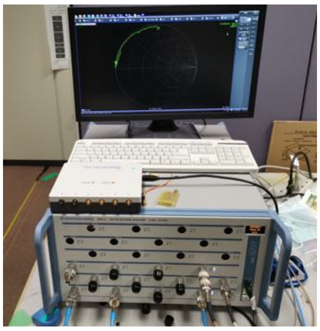
where $L_{2\text{short}}$ is the inductance of when the secondary coil L_2 is coupled with the primary coil L_1. The same is true of M_2. Incidentally, the mutual inductance is measured under the assumption that its value is regardless of the position on the transmitter. In this research, terminal of the transmission line is shorted, so Z'_t in Fig. 1 and Z''_t in Fig. 5 are $0\,\Omega$. Other parameter of transmitter and receiver is shown in TABLE I.

The characteristic impedance and propagation constant of distributed constant circuit are required to determine the input impedance. An open-short method is typically applied for calculating them, and they are calculated from the input impedance value of when the terminal is open/short, as follows,

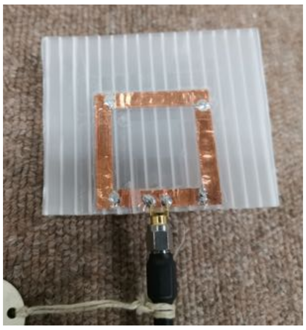
$$Z_0 = \sqrt{Z_{\text{short}} \cdot Z_{\text{open}}}, \quad (8)$$

$$Z_{in} = Z_0 \cdot \frac{Z_t + Z_0 \tanh(\gamma x_1)}{Z_0 + Z_t \tanh(\gamma x_1)}$$

$$= Z_0 \cdot \frac{Z_0 \cdot \dfrac{Z_t' + Z_0 \tanh(\gamma x_2)}{Z_0 + Z_t' \tanh(\gamma x_2)} + j\omega(L_1 - M_1) + \dfrac{j\omega M_1 (Z_{L1} + j\omega(L_2 - M_1))}{Z_{L1} + j\omega L_2} + Z_0 \tanh(\gamma x_1)}{Z_0 + \left(Z_0 \cdot \dfrac{Z_t' + Z_0 \tanh(\gamma x_2)}{Z_0 + Z_t' \tanh(\gamma x_2)} + j\omega(L_1 - M_1) + \dfrac{j\omega M_1 (Z_{L1} + j\omega(L_2 - M_1))}{Z_{L1} + j\omega L_2} \right) \tanh(\gamma x_1)}. \quad (3)$$



(a) Vector network analyzer



(b) Receiver

(c) Transmitter line

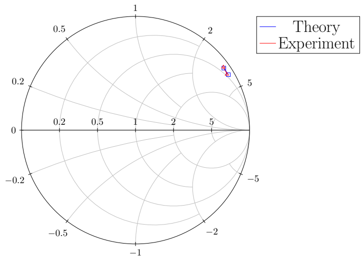
Fig. 7. Experimental equipment

TABLE I
Parameter in experiment

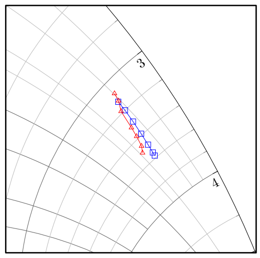
Frequency	20 MHz
Self inductance of primary coil L_1, L_3	85.8 nH
Self inductance of secondary coil L_2, L_4	65.7 nH
Inductance of secondary coil coupled with primary one $L_{2\text{short}}$, $L_{4\text{short}}$	58.0 nH
Coupling coefficient k_1, k_2	0.3414
Mutual inductance M_1, M_2	25.6 nH
Receiver impedance Z_{L1}, Z_{L2}	$(0.05 + 4.10\text{i})\,\Omega$
Terminal impedance Z_t', Z_t''	$0\,\Omega$
Length of a transmitter	3.05 m
Length of receivers	0.05 m
Width of transmitter and receivers line	6.0 mm
Distance between parallel line	30.0 mm

$$\tanh(\gamma x) = \sqrt{\frac{Z_{\text{short}}}{Z_{\text{open}}}}$$
$$\Leftrightarrow \quad \gamma = \frac{\tanh^{-1}\sqrt{\dfrac{Z_{\text{short}}}{Z_{\text{open}}}}}{x} \quad (9)$$

where Z_{short} and Z_{open} are the input impedance of when terminal of transmission line is short and open, respectively.

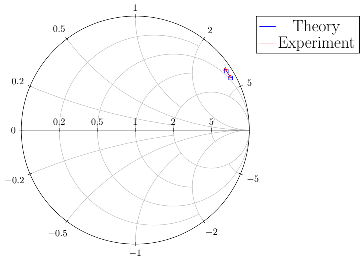


(a) Overall chart



(b) Detail chart

Fig. 8. Input impedance in case of single receiver



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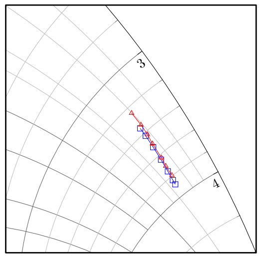

(b) Detail chart

Fig. 9. Input impedance in case of double receivers

TABLE II shows the measured value of Z_{short}, Z_{open}, and the calculated value of Z_0 and γ.

TABLE II
Secondary constant

Input impedance of short Z_{short}	$(117.5 + 806.6\text{i})\,\Omega$
Input impedance of open Z_{open}	$(4.6 - 72.8\text{i})\,\Omega$
Characteristic impedance Z_0	$(243.6 + 9.9\text{i})\,\Omega$
Propagation constant γ	$(0.0095 + 0.4271\text{i})\,\Omega$

B. Measurement results

The experimental result value of one receiver and two receivers are shown in TABLE III and TABLE IV, and plotted in Fig. 8 and Fig. 9, respectively. There are theoretical value written by blue line and experimental one by red line in each smith chart.

In Fig. 8, each plot stands for input impedance, normalized by Z_0, of when x_2 is changed from $0\,\text{m}$ to $\frac{1}{5}\lambda$ m, where λ is wavelength of fed sinusoidal wave and its length is $15.0\,\text{m}$. According to the result, the input impedance changes depending on the position of the receiver, and theoretical value matches with experiment and its error is enough small. The error between theoretical and experimental values is thought to be occurred by the surrounding environment and the assumption that M is location-independent.

TABLE III
Z_{in} in case of single receiver

$x_2[m]$	Z_{in} (Theory) [Ω]	Z_{in} (Experiment) [Ω]
0	$125.3 + 869.1i$	$138.0 + 848.7i$
$\frac{1}{30}\lambda$	$123.8 + 862.2i$	$130.2 + 838.7i$
$\frac{1}{15}\lambda$	$119.9 + 845.5i$	$123.8 + 820.3i$
$\frac{1}{10}\lambda$	$114.9 + 822.9i$	$119.1 + 803.6i$
$\frac{2}{15}\lambda$	$109.9 + 798.5i$	$111.2 + 773.1i$
$\frac{1}{6}\lambda$	$105.8 + 776.6i$	$102.0 + 759.4i$
$\frac{1}{5}\lambda$	$103.4 + 760.1i$	$98.1 + 747.6i$

In Fig. 9, each plot expresses input impedance, normalized by Z_0, of when x_3 is fixed and x_2 is varied from $0\,\mathrm{m}$ to $\frac{1}{5}\lambda$ m. Theoretical value matches with experiment similarly in single receiver experiment. Proposal can be extended to the case of more than two receivers.

TABLE IV
Z_{in} in case of two receivers

$x_3[m]$	$x_2[m]$	Z_{in} (Theory) [Ω]	Z_{in} (Experiment) [Ω]
0	0	$134.2 + 940.6i$	$126.5 + 921.0i$
	$\frac{1}{30}\lambda$	$132.0 + 930.1i$	$123.2 + 898.1i$
	$\frac{1}{15}\lambda$	$127.3 + 909.1i$	$118.3 + 879.8i$
	$\frac{1}{10}\lambda$	$121.5 + 882.2i$	$112.5 + 848.9i$
	$\frac{2}{15}\lambda$	$115.9 + 854.5i$	$107.6 + 830.7i$
	$\frac{1}{6}\lambda$	$111.6 + 830.4i$	$104.2 + 811.3i$
	$\frac{1}{5}\lambda$	$109.2 + 804.6i$	$101.4 + 786.8i$

4. Conclusion

In this paper, we proposed an equivalent circuit model when single large scale transmitter and multiple receivers are coupled, and experimentally investigated its characteristics. Experimental results indicated that equivalent circuit which combining concentrated constant and distributed constant is usable in experiment condition. It was also shown that this method can be applied even if the number of receivers is increased. This may be expected to be used in various analyses of wireless power transmission with multiple receivers in large-scale WPT system in the future.

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