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Pre-empting Plateau: the nature of topological transitions in foam

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Abstract. – When the area of a face in a dry foam approaches zero in some quasistatic processes, Plateau's rules dictate that there must be an instability. This is more subtle than generally supposed. We argue that it is generally pre-empted, that is, the instability arises before an unstable multiple vertex is formed. Experiments and calculations which simulate Plateau's wire frame experiments support this view.

Introduction. – The idealised model of a dry foam consists of faces (films) meeting in edges or lines (Plateau borders). One of Plateau's celebrated rules [1] specifies that the symmetric tetrahedral junction of four such lines is the only possible vertex in an equilibrium structure. Hence if a vertex with more than four lines is formed in any quasistatic process (by variation of boundary constraints, or by coarsening, for example), there is an immediate topological transition to a Plateau-allowed configuration [2].

The two most elementary ways in which this scenario can arise are by the vanishing of a triangular face or an edge, provoking respectively topological changes which are the inverse of each other. Figure 1 illustrates two configurations which may be transformed into each other by such changes. More complicated changes may arise in practice but they may be regarded as compounds of these two processes [2].

This much is conventional wisdom in the physics of foams. However we argue here that a significant aspect of the vanishing-face transition has been overlooked, originally by Plateau himself. It appears that in general the area of a face cannot proceed to zero in such a quasistatic process, and the transition is necessarily pre-empted. By this we mean that the system becomes unstable at some earlier point, with non-zero face area. This shrinks to zero spontaneously and the topological transition ensues. That this has not been realised may be because of undue reliance on the analogy with the simpler two-dimensional foam, where no such pre-emptive effect exists in general.

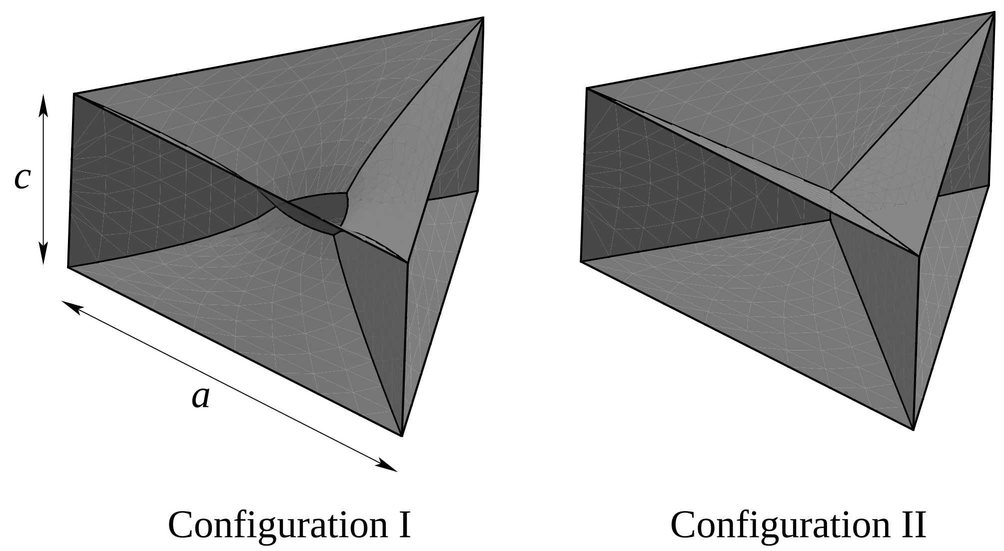

Fig. 1 – Surface Evolver simulation of the equilibrium arrangement of soap films in a wire frame in the shape of a triangular prism. Configuration I with the triangular face in the centre is unstable for an axial ratio $c/a > 0.487$.

Soap films spanning the frame of a triangular prism. – To begin with we consider a specific case, corresponding to one of the simplest demonstrations with Plateau's wire frames, the triangular prism. For a small axial ratio c/a, this displays a central triangular face, as shown in configuration I of fig. 1. As c/a is gradually increased, the face area decreases and the regime of interest may be explored. We have done this in preliminary experiments with plastic frames of variable axial ratio, as supplied by Cochranes of Oxford Ltd and dilute solution of the commercial dish-washing detergent Fairy Liquid. When such a frame with axial ratio $c/a = 0.42$ was withdrawn from the solution we obtained configuration I of fig. 1. As we then increased the ratio c/a (without re-introducing the frame into the solution) at a certain point slightly below $c/a = 0.5$ the triangular face was observed to spontaneously collapse and there was a transition to the alternative configuration II. Fig. 2 illustrates schematically the contrast between this behaviour and the topological change in 2d foam.

Fig. 1 was generated using the Surface Evolver [3]. This software enables us to accurately locate the critical value of axial ratio c/a for the ideal dry foam, which we find to be 0.487.

As shown in Fig. 3, for values of c/a between 0.413 and 0.487 there are two stable minima of total surface area, with configuration I lower in energy than configuration II up to c/a =0.472. As c/a increases to 0.487, the lowest eigenvalue of the Hessian matrix of the energy goes to zero and configuration I is no longer stable. The inset of Fig. 3 shows that at the point of instability the area of the triangular face of configuration I is indeed non-zero. The value of $c/a = 0.413$ corresponds to configuration II in which the vertical edge has shrunk to zero length.

Related cases. – Similar behaviour has been studied by Bohn [4] in configurations of bubble chains, a somewhat different context but one which can be largely treated analytically. Up to the point of instability there are *two* solutions for configurations that obey Plateau's laws of equilibrium, one being unstable. This is also the case for a soap film between two parallel rings: two types of catenoids, of which only one is stable, are possible, provided the

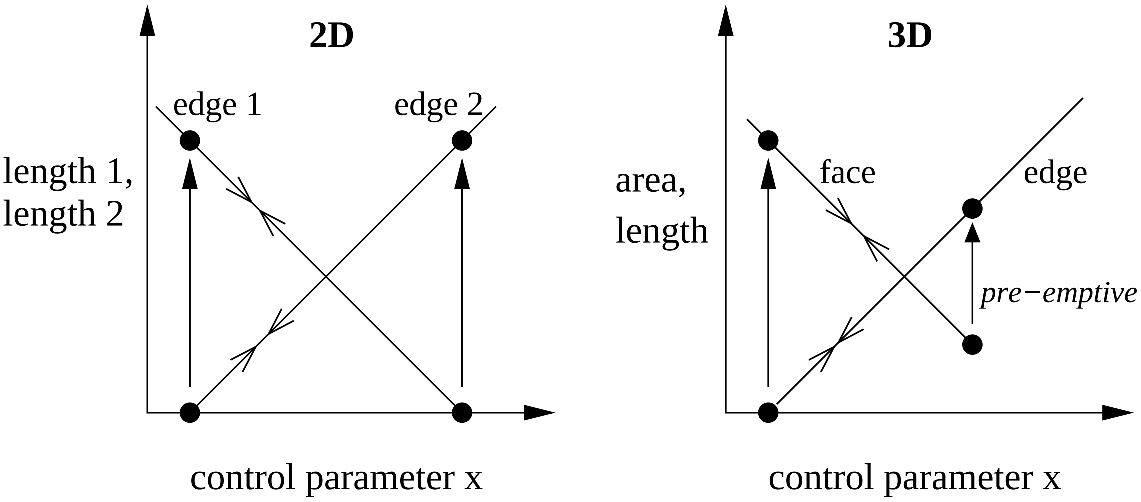

Fig. 2 – Sketch of the topological instabilities encountered in two and three dimensions. In equilibrium, edges may shrink to zero continuously (in both 2d and 3d) when some external parameter (such as the axial ratio of the frame) is varied. In 3d a non-zero face area may lead to an unstable configuration which will be resolved by a pre-emptive topological change.

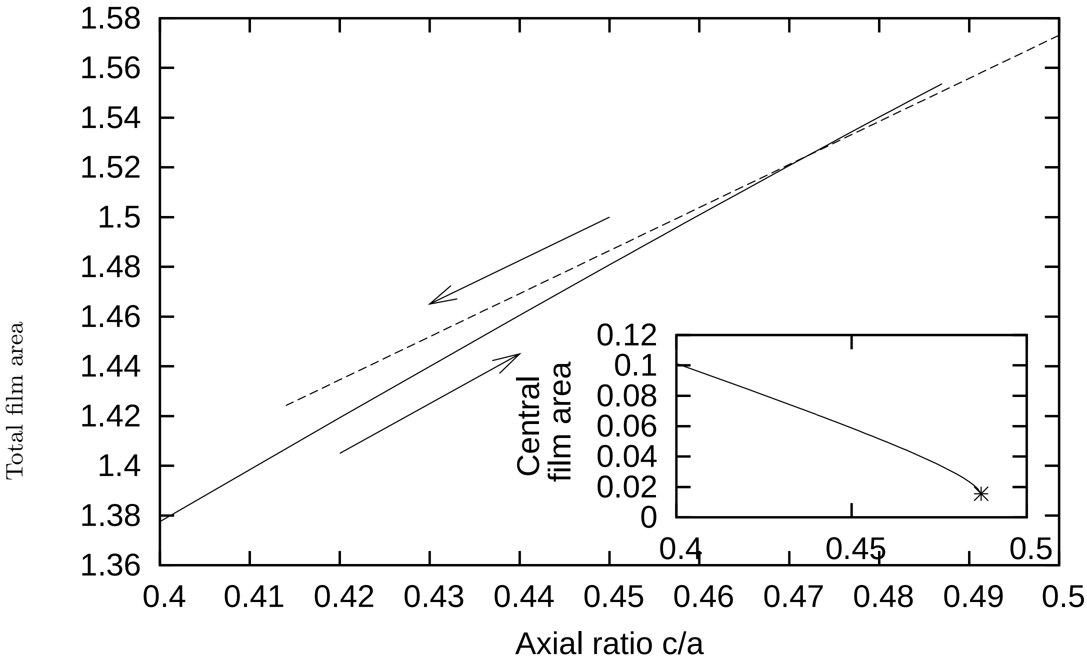

Fig. 3 – Surface Evolver calculations show that there are two stable configurations for $0.413 < c/a < 0.487$ (solid line: configuration I, dashed line: configuration II). The inset shows that the area of the triangular face of configuration I is non-zero at the point of instability, $c/a = 0.487$. There should also be an unstable equilibrium solution for $c/a < 0.487$, but this is not determined by the present calculations. (In the simulations a was kept constant, $a = 1$.)

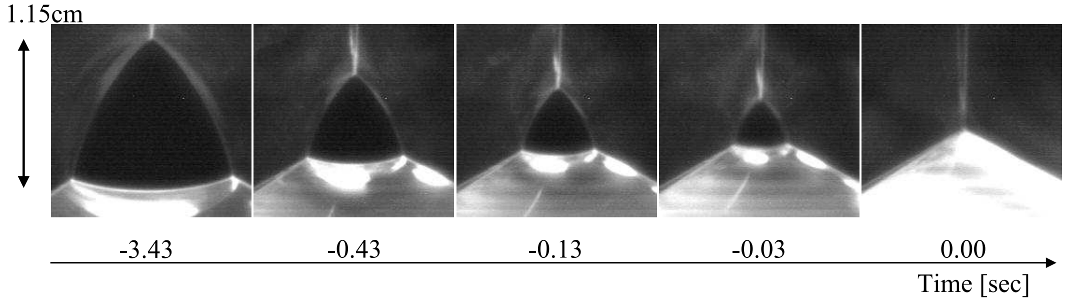

Fig. 4 – Snapshots of the vanishing triangular face of configuration I. Note the acceleration of the rate of decrease in area.(The photographs correspond to run 1 of figure 5.)

distance between the rings is not too large [5]. A yet more distant example of a pre-emptive instability was encountered in simulations of a cylindrical foam [6], and something similar is found in the uniaxial extension of a Kelvin foam [7].

In the present case, a precise explanation for why the small film is unstable eludes us. Further simulations show similar behaviour for other frames, with different numbers of sides n, although it is not as pronounced. Note also that, upon vanishing of an n fold face for $n > 3$, a more complex topological change ensues, since the stability requires that n films cannot meet in a line. We find for the tetragonal prism ($n = 4$) a critical ratio of $c/a = 1.034$ (in good agreement with our experimental value $c/a \simeq 1.03$) and for the pentagonal prism ($n = 5$) $c/a = 2.184$, although in the latter case the face area at instability is very small. It seems clear that the special conditions of these examples (exact symmetries, zero pressure differences across films) are not responsible for the pre-emptive effect, and this is confirmed by simulations of asymmetric frames. It should therefore occur rather generally in real dry foams. An analysis of the hexagonal prism ($n = 6$) and higher values of n will be included in a further paper [8], but these are probably irrelevant to real dry foams.

The dynamics of the vanishing face. – While in Surface Evolver calculations we are restricted to consideration of the static case, we can access the dynamics of the above described topological transitions by appropriate experiments. It is possible to create a transient configuration I of fig. 1 also for a ratio c/a above the critical ratio, as follows.

Withdrawing a frame with ratio $c/a = 0.5$ from surfactant solution we obtain configuration II, which is transformed into configuration I by blowing gently against one of the vertices connecting the central Plateau border. The triangular face is not stable and shrinks to zero over a time span of up to ten seconds. Fig. 4 shows the variation of its area with time, as obtained using a video camera (30 frames per second). Since the soap films are generally stable for a few minutes we can recreate the triangular face by repeated blowing, resulting in runs 2-4 in fig. 5, for which the data was obtained using the image analysis software ImageJ. We can see that the lifetime of the triangle increases with the age of the soap films (from run 1 to 4). This may be attributed to the change of liquid content due to drainage (note that in our experiment the triangular face is vertically aligned, to facilitate the imaging), which will affect the mechanisms of dissipation, but drainage itself may be a factor, too. A detailed analysis of our data close to the point where the triangle vanishes shows that its area decreases with the square root of time (see inset of fig. 5).

The amount of liquid contained in both Plateau borders and films was determined by weighing the frame with and without the spanning soap films. Together with the volume of

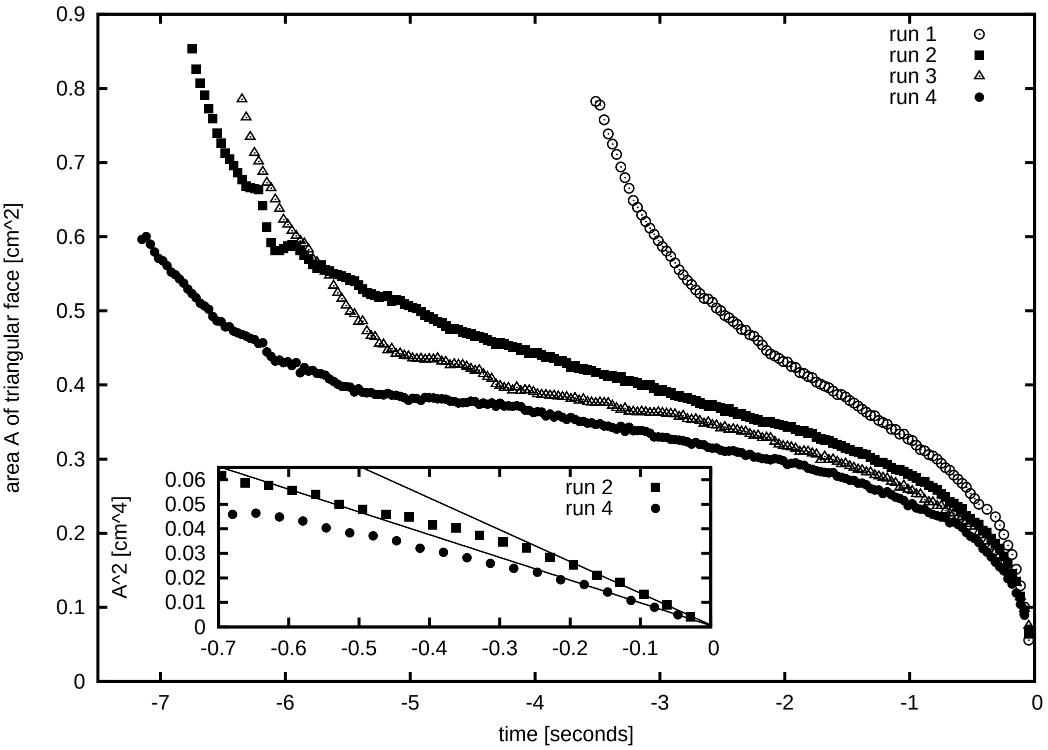

Fig. 5 – Experimental data for the decrease in area A of the central triangular face of the unstable configuration I (fig. 1) with time. The face was created by blowing against one of the central Plateau borders of configuration II for a frame with axial ratio $c/a \simeq 0.5$, where a $= 6.2$ cm. (A rescaling of the minimum area of the triangular face of figure 3 by a^2 gives A $= 0.594$.) Run 1 to 4 correspond to soap films that have been allowed to drain for an increasing amount of time (up to three minutes), before the experiment was performed. The time origin of the data has been shifted so that the vanishing of the triangular face occurs at time equals zero for all runs. The inset shows that the *square* of the area of the triangle shrinks *linearly* close to the disappearance of the face.

the triangular prism this determines a liquid fraction, defined by the total volume of liquid divided by the volume of the prism. We found values that indicate that the experiment corresponds to a (very) dry foam: the first run had a liquid fraction of roughly 0.004 ± 0.001, decreasing down to 0.002 ± 0.001 for the fourth run.

As the study of the physics of foams moves steadily from statics to dynamics [2], instabilities are becoming a focus of interest [9]. Data such as that of Fig.5 provide challenges to emerging theories of dynamic effects.

Conclusion. – Our finding of a general pre-emptive instability should constitute a useful footnote to the standard account of Plateau's laws and their practical consequences. Here we might recall that symmetric 8-fold vertices, not stable in the dry limit, were only quite recently found to be stable in foams [10, 11], if one allows for very small liquid content in the Plateau borders (liquid fraction of 0.000278 [12]). Plateau had noticed this stability in experiments with density matched emulsions. A careful study of his writings and repetition of some of his experiments might still offer a few surprises.

The pre-emptive instability also raises intriguing questions for further research. Since small isolated triangular faces are generally unstable, according to our findings, they should not be observable in an equilibrium foam, below a certain cut-off size. Since it is now possible to analyse large samples of foam both in experiment and simulation, it will be interesting to see if this cut-off can be confirmed. This conclusion may also help to explain an old observation of Schwarz [13], that diminishing quadrilateral faces generally do not form small triangular ones by topological change, but rather proceed by two elementary changes of the kind discussed here, forming a new quadrilateral face in another direction.

Finally, the pre-emptive instability also calls for an investigation into the dissipative mechanisms that might account for the square root variation of the area of the collapsing face. We have at this stage no explanation for such a power law.

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