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THE LEADING–ORDER HYDRODYNAMICS OF THE LANDAU-VLASOV KINECTIC EQUATION WITH THE NONLOCAL COLLISION INTEGRAL

V. Gorev[1], A. Sokolovsky[2], I. Shedlovsky[1]
([1]National Mining University, Dnipro, Ukraine
[2]Oles Honchar Dnipro National University, Dnipro, Ukraine)

Introduction. This paper is devoted to the hydrodynamics of a one-component gas with small potential interaction. The basis of investigation is the kinetic equation in case of small potential interaction which contains general nonlocal collision integral [1] and describes arbitrary non-uniform states. In the local approximation this equation coincides with the well-known Landau–Vlasov kinetic equation. In hydrodynamics the system is supposed to be weakly non-uniform.

Usually the system hydrodynamics is built on the basis of the local collision integral, but the use of the nonlocal collision integral is of significant importance.

Firstly, some terms of the second order in small interaction may be lost if we deal with the local collision integral. Secondly, the Burnett approximation meets difficulties in the hydrodynamics with the local collision integral. Maybe, in some cases these difficulties can be overcome on the basis of the nonlocal collision integral. The conservation laws for the considered problem were investigated in [2]. The system hydrodynamics can be based on the investigated conservation laws.

The aim of this paper is to investigate the system hydrodynamics based on the nonlocal collision integral in the leading order in small gradients.

Basic equations of the theory. The investigation is based on the kinetic equation of the second order in small potential interaction with the general nonlocal collision integral [1]:

$$\frac{\partial f(\chi_1,t)}{\partial t}=-\frac{p_{1n}}{m}\frac{\partial f(\chi_1,t)}{\partial x_{1n}}+\frac{\partial f(\chi_1,t)}{\partial p_{1n}}\frac{\partial}{\partial x_{1n}}\int d\chi_2 V_{12} f(\chi_2)+I(\chi_1,f),\quad \chi=\{\mathbf{x},\mathbf{p}\},$$
$$V_{12}\equiv V(|\,\mathbf{x}_1-\mathbf{x}_2\,|),\qquad F_{12n}=-\frac{\partial V_{12}}{\partial x_{1n}},\qquad F_{12n\tau}=-\frac{\partial}{\partial x_{1n}}V\left(\left|\mathbf{x}_1-\mathbf{x}_2+\frac{\tau}{m}(\mathbf{p}_1-\mathbf{p}_2)\right|\right), \qquad (1)$$
$$I(\chi_1,f)=\frac{\partial}{\partial p_{1n}}\int d\chi_2 F_{12n}\int_{-\infty}^{0} d\tau F_{12l\tau}\left(\frac{\partial}{\partial p_{1l}}-\frac{\tau}{m}\frac{\partial}{\partial x_{1l}}-\frac{\partial}{\partial p_{2l}}+\frac{\tau}{m}\frac{\partial}{\partial x_{2l}}\right)f(\chi_1)f(\chi_2)$$

where $V(|\,\mathbf{x}\,|)$ is the pair system potential, $f(\chi,t)$ is the one-particle distribution function, and $I(\chi_1,f)$ is the general nonlocal second-order collision integral. This kinetic equation can be obtained [1] by the Bogolyubov reduced description method and in the local approximation for the collision integral it gives the known Landau–Vlasov kinetic equation.

It should be noticed that the densities of the conserved quantities such as particle number, momentum and kinetic energy are usually used as the parameters which describe the hydrodynamic states of the system (the hydrodynamic reduced description parameters). These densities are introduced by standard definitions:

$$n(\mathbf{x},t)=\int d\mathbf{p} f(\chi,t),\qquad \pi_n(\mathbf{x},t)=\int d\mathbf{p} p_n f(\chi,t),\qquad \varepsilon_{\text{kin}}(\mathbf{x},t)=\int d\mathbf{p}(\mathbf{p}^2/2m) f(\chi,t). \qquad (2)$$

In [2] it is shown that although the system kinetic energy is conserved on the basis of the local Landau collision integral, it is not conserved on the basis of the nonlocal collision integral (1). The potential energy and total energy densities are introduced by definitions

$$\varepsilon_{\text{pot}}(\mathbf{x}_1,t)=\int d\mathbf{p}_1 d\chi_2 f_2(\chi_1,\chi_2,f(t))V_{12},\qquad \varepsilon(\mathbf{x},t)=\varepsilon_{\text{kin}}(\mathbf{x},t)+\varepsilon_{\text{pot}}(\mathbf{x},t) \qquad (3)$$

where the expression for the two-particle distribution function f_2 in terms of the one-particle distribution function f up to the first order in small potential interaction is [1]

$$f_2(\chi_1,\chi_2,f)= \qquad (4)$$

$$= f(\chi_1)f(\chi_2) - \int_{-\infty}^{0} d\tau F_{12l\tau}\left(\frac{\partial}{\partial p_{1l}} - \frac{\tau}{m}\frac{\partial}{\partial x_{1l}} - \frac{\partial}{\partial p_{2l}} + \frac{\tau}{m}\frac{\partial}{\partial x_{2l}}\right) f(\chi_1)f(\chi_2).$$

On the basis of (1) – (4) it is shown [2] that the total energy of the system is conserved up to the second order both in small potential interaction and in small gradients. So, the following set of the hydrodynamic RPDs should be used: particle density (2), momentum density (2) and total energy density (4). The use of the kinetic energy density as a reduced description parameter is unreasonable even for a model problem without the Vlasov term.

Investigation of the system hydrodynamics in the leading order in gradients. In the previous section the refined set of hydrodynamic reduced description parameters is proposed and in what follows the system hydrodynamics is built on the basis of the Chapman–Enskog method [1]. The parameters velocity υ_n and temperature T are often used instead of π_n, ε. The definitions of υ_n, T in terms of f should be the same as in equilibrium case. As is known [1,3], in equilibrium we have

$$\pi_l^{\text{eq}} = mn\upsilon_l, \qquad \varepsilon_{\text{kin}}^{\text{eq}} = \frac{3}{2}nT + \frac{1}{2}mn\upsilon^2, \qquad \varepsilon_{\text{pot}}^{\text{eq}} = \frac{n^2}{2}\int d\mathbf{r}V(r)e^{-V(r)/T} \tag{5}$$

where the temperature is given in energy units. The basic equation of the theory (1) is obtained up to the second order in small potential interaction, so the expression for $\varepsilon_{\text{pot}}^{\text{eq}}$ should be truncated up to the second order in $V(r)/T$ and on the basis of (2), (3) and (5) we obtain the definitions of the reduced description parameters n, υ_n and T in terms of the one-particle distribution function:

$$n(\mathbf{x},t) = \int d\mathbf{p}f(\chi,t), \qquad mn(\mathbf{x},t)\upsilon_n(\mathbf{x},t) = \int d\mathbf{p}p_n f(\chi,t),$$
$$\frac{3}{2}n(\mathbf{x}_1,t)T(\mathbf{x}_1,t) + \frac{1}{2}mn(\mathbf{x}_1,t)\upsilon^2(\mathbf{x}_1,t) + \frac{n^2(\mathbf{x}_1,t)}{2}\int d\mathbf{r}V(r) - \frac{n^2(\mathbf{x}_1,t)}{2T(\mathbf{x}_1,t)}\int d\mathbf{r}V^2(r) = \tag{6}$$
$$= \int d\mathbf{p}_1(p_1^2/2m)f(\chi_1,t) + \int d\mathbf{p}_1 d\chi_2 f_2(\chi_1,\chi_2,f(t))V_{12}.$$

Equations (6) with account for (4) are the additional conditions to the kinetic equation (1). It is shown that in the leading order in small gradients of hydrodynamic variables the Maxwellian distribution function

$$f^{(0)} = w_p \equiv \frac{n}{(2\pi mT)^{3/2}}\exp\left(-\frac{(\mathbf{p}-m\boldsymbol{\upsilon})^2}{2mT}\right), \qquad f^{(n)} \sim g^n, \quad g = \frac{l_{\text{fp}}}{L} << 1 \tag{7}$$

satisfies both the kinetic equation (1) and the additional conditions (6). The parameter g describes the smallness of the gradients and is given by (7) where l_{fp} is the free

path length and L is the characteristic length at which the hydrodynamic parameters are significantly changed. So, the system distribution function of the leading order in gradients is the Maxwellian one (7).

Conclusions. The definitions of system velocity and temperature are obtained in terms of the one-particle distribution function. The corresponding additional conditions to the kinetic equation are also obtained. The leading-order-in-gradients hydrodynamics of the considered system is built on the basis of the general nonlocal collision integral. It is shown that the system distribution function in the leading order in gradients is the Maxwellian one.

Our future plan is to build the system dissipative hydrodynamics and to calculate the corresponding kinetic coefficients. The results obtained here are the basis of such an investigation.

The results of the present work can be used in plasma investigations, because, as known, the Landau–Vlasov kinetic equation is widely used in plasma physics. So, the obtained results and the proposed approach have many applications. Among other things, they can be applied to the description of physical processes in telecommunication and information systems.

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